

Math 342W/642/742W

Recitation – Day #7 (2.25.25)

I. Linear Algebra Basics for Vectors

Define the following for a set of vectors $V = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\} \in \mathbb{R}^n$:

(i) $\text{span}(V)$:

(ii) linearly independent set V :

II. Linear Algebra Basics for Matrices

Given an $n \times p$ matrix $A \in \mathbb{R}^{n \times p}$, define the following:

(i) $\text{col}(A)$:

(iii) $\text{rank}(A)$:

(ii) $\text{row}(A)$:

(iv) $\text{null}(A)$:

Define the matrix multiplication of two matrices $A \in \mathbb{R}^{n \times p}$, and $B \in \mathbb{R}^{p \times m}$:

Define the **matrix-column representation** for the matrix product AB :

Define the **matrix-row representation** for the matrix product AB :

III. More on Rank

Provide justification for the following statements on rank:

1. Given $A \in \mathbb{R}^{n \times p}$, $\text{rank}(A) = \text{rank}(A^T)$.
2. Given $A \in \mathbb{R}^{n \times p}$, $\text{rank}(A) = \text{rank}(UA) = \text{rank}(AV)$ whenever U, V are invertible matrices.
3. Given $A \in \mathbb{R}^{n \times p}$, and $B \in \mathbb{R}^{p \times m}$, (i) $\text{rank}(AB) \leq \text{rank}(A)$, and (ii) $\text{rank}(AB) \leq \text{rank}(B)$

IV. Equivalent Statements

Complete the following list of equivalent statements for $A \in \mathbb{R}^{n \times p}$ (assume $n > p$):

- | | |
|---------------------------|----|
| 1. $\text{rank}(A) = p$. | 4. |
| 2. | 5. |
| 3. | 6. |