Math 342W/642/742W

Recitation – Day #9 (3.4.25)

I. Geometric Proof for R^2

(i) Define the following terms:

- SST =
- SSE =
- SSR =
- R^2 =

(ii) Define what ***mean-centering*** is:

(iii) Project the mean-centered vector onto the column space of X:

(iv) Draw an illustration of this projection:

(v) Using geometric principles, show that $R^2 \in [0, 1]$.

IV. QR-Decomposition

(i) What is the goal behind the QR-decomposition/factorization of a matrix? What is accomplished?

(ii) What is the process that creates the QR-decomposition of a matrix called?

(iii) How will we now define the orthogonal projection matrix H?

III. QR-Decomposition in Action

Let W be a subspace of $\mathbb{R}^4$ defined as $W = \text{span}\left(\left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}\right\}\right)$. Let V be a 4×3 matrix whose columns are the vectors that span W.

(i) Construct an orthogonal basis for W.

(ii) Find the QR-decomposition(factorization) of V.