Math 342W/642/742W

Recitation – Day #22 (5.6.25)

I. Asymmetric Cost Binary Classification

(i) What is the typical output space of binary classification models?

$$\mathcal{Y} = \{0, 1\}$$

(ii) What are the two erroneous decisions/classifications that can be made?

- False Positive (FP): When $\hat{y} = 1$ but $y = 0$. (a.k.a. "*false alarm, overestimation*")
- False Negative (FN): When $\hat{y} = 0$ but $y = 1$. (a.k.a "*miss, underestimation*")

(iii) What are the two correct decisions/classifications that can be made?

- True Positive (TP): When $\hat{y} = 1$ and $y = 1$. (a.k.a. "*hit*")
- True Negative (TN): When $\hat{y} = 0$ and $y = 0$. (a.k.a. "*correct rejection*")

(iv) Write out the meanings of each of the following abbreviations:

- N = total # of negatives
- P = total # of positives
- PN = total # of predicted negatives
- PP = total # of predicted positives
- FP = # of false positives
- FN = # of false negatives
- TP = # of true positives
- TN = # of true negatives
- C_{FP} = cost of a false positive
- C_{FN} = cost of a false negative

(v) What is the hyperparameter we are considering in the analysis of a binary classifier?

$$\hat{y} = \mathbb{1}_{\hat{p} \geq \theta} \quad \text{where } \theta = \text{ the probability estimate threshold}$$

(v) Create the 2×2 Confusion Table/Matrix below:

	$\hat{y} = 0$	$\hat{y} = 1$	Total
$y = 0$	TN	FP	N
$y = 1$	FN	TP	P
Total	PN	PP	n

II. Binary Classification Performance Measures

Define the following binary classification performance measures:

(i) Error: $\text{ME} = \dfrac{\text{FN} + \text{FP}}{n}$

(ii) Precision: $= \dfrac{\text{TP}}{\text{PP}}$

(iii) Accuracy: $\text{ACC} = \dfrac{\text{TN} + \text{TP}}{n} = 1 - \text{ME}$

(iv) Recall: $= \dfrac{\text{TP}}{\text{P}}$ (a.k.a *sensitivity*)

(v) F1 Score: $= \dfrac{2}{\frac{1}{\text{recall}} + \frac{1}{\text{precision}}}$

(vi) False Discovery Rate: $\text{FDR} = \dfrac{\text{FP}}{\text{PP}}$

(vii) False Omission Rate: $\text{FOR} = \dfrac{\text{FN}}{\text{PN}}$

(viii) Total Cost: $C = C_{\text{FP}} \cdot \text{FP} + C_{\text{FN}} \cdot \text{FN}$

(ix) Specificity: $= \dfrac{\text{TN}}{\text{N}}$

(x) False Positive Rate:

$$\text{FPR} = \frac{\text{FP}}{\text{N}} = 1 - \text{Specificity}$$

III. The ROC Curve

(i) What is the *receiver operating characteristics* (ROC) curve?

The ROC curve is a curve plotted on graph whose horizontal axis representing FPR and the vertical axis representing Recall/Sensitivity. The points on the curve are ordered pairs of (FPR, Recall) based upon a specified threshold θ for the probability of a positive class $\hat{p}$.

(ii) How is the *area under the curve* (AUC) related to the ROC curve?

The AUC is the area underneath the ROC curve of a given classifier. The AUC is helpful when comparing multiple classifiers by comparing their AUCs with each others.

(iii) What does a diagonal line of a ROC curve represent?

A diagonal line of a ROC curve is a default/baseline of performance for a given classifier. It is the result of having a classifier be the same as flipping a fair coin for predicting either a positive or negative class.

(iv) What is the ideal standard for a classifier using the ROC curve? How do we compare different classifiers knowing their ROC curves?

The ideal classifier would have Recall $= 1$, i.e. TP=P $\implies$ FP=0 $\implies$ FPR=0. Therefore, the closer that the ROC curves are to the coordinate $(0, 1)$ are considered better indicators of performances. Equivalently, the closer the ROC curves have an AUC closer to 1, the better the overall performance that the classifier posesses. Thus, comparing classifiers with either the ROC or AUC metrics are equivalent.