Math 342W/642/742W

Recitation – Day #10 (3.13.25)

I. The Projection Matrix H with QR-Decomposition

(i) What is the matrix Q that comes from decomposing our model/design matrix X into its QR Factorization?

(ii) What do we know about the colsp[Q] and the colsp[X]?

(iii) What is an equivalent way to define our projection matrix H onto the column space of X?

(iv) How do we express our vector of predicted responses $\hat{\boldsymbol{y}}$ with H? now with Q? with projections?

(v) How does the pythagorean theorem help us with understanding more about $\|\hat{\boldsymbol{y}}\|^2$?

II. Insights of OLS through QR-Decomposition

(i) What makes $\left\|\text{proj}_{\boldsymbol{q_0}}(\boldsymbol{y})\right\|^2$ special?

(ii) How is SSR related to the projection of $\boldsymbol{y}$ along the orthogonal columns of Q?

(iii) What insight does this all give us with including a new feature(s) $\boldsymbol{x}_{\cdot\star}$ to the model/design matrix X under OLS?

(iv) What is the relationship between R^2, SSR, SSE and RMSE?