Math 342W/642/742W

Recitation – Day #4 (2.11.25)

I. Hyperplanes

(i) Describe/explain/define what a **hyperplane** is in the context of $\mathbb{R}^2$ or $\mathbb{R}^3$.
(*Visualizations may be useful.*)

(ii) Describe/explain/define what a **hyperplane** is in the context of $\mathbb{R}^n$.

(iii) Write the "**Hesse Normal Form**" definition of a *hyperplane*. What does it mean that it is *overparametrized*?

II. Terminology of Hyperplanes

(i) Define the set of candidate functions of interest, $\mathcal{H}$.

(ii) Define the following terms:

- $\boldsymbol{w}$
- $\boldsymbol{w}_0$
- b
- $\boldsymbol{\ell}$
- $\boldsymbol{\ell}_L$
- $\boldsymbol{\ell}_U$
- $\boldsymbol{z}$
- $\boldsymbol{z}_L$
- $\boldsymbol{z}_U$

III. Finding the Optimal Hyperplane

(i) What is the main goal in finding the optimal hyperplane among linearly separable data?

(ii) Which quantity are we *maximizing*? Which quantity are we *minimizing*?

(iii) How do we express the optimization problem for finding the optimal hyperplane among linearly separable data?

IV. Support Vector Machines (SVM)

(i) What constitutes as an **error** if we want to find the optimal hyperplane for when data are not linearly separable?

(ii) Define **Hinge Error** (HE)/**total Hinger Error**(THE).

(iii) Express/formulate the optimization problem for finding the optimal hyperplane using *Vapnik* objective/fitness/loss function to be minimized.

(iv) What category of problems are SVMs applicable for?