Math 342W/642/742W

Recitation – Day #3 (2.6.25)

I. Nomenclature/Terminology Review

Provide the definition/description for each of the symbols seen below.

- $z_1, \ldots, z_t$: proximal causes – the "true" drivers of the phenomenon (*unknown*)
- t: the unknown function that exactly represents the phenomenon and takes the z's as inputs
- $x_1, \ldots, x_p$: the variables/inputs that *proxy* the z's
- n: the number of data points in the given training/historical data
- p: the number of features/predictors
- f: the *target* function which produces the least amount of error approximating t
- $\mathbb{D}$: the training/historical data that the *supervised learning* will be based upon and can be expressed in two equivalent ways:
  - (i) $\{(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \ldots, (\boldsymbol{x}_n, y_n)\}$
  - (ii) $\langle X, \boldsymbol{y} \rangle$ where:
    - the $\boldsymbol{x}_i$'s make up the rows of X and have p components each
    - $\boldsymbol{y}$ is a column vector whose ith component is y_i
- $\mathcal{H}$: the set/class of candidate functions that will be considered for approximating f by the learning algorithm
- h^*: the "best" choice in $\mathcal{H}$ that approximates f
- $\mathcal{A}$: the specified *learning* algorithm that requires $\mathbb{D}$ and $\mathcal{H}$
- g: the "best" model produced from the learning algorithm based on the training data and the candidate set of functions , i.e., $g = \mathcal{A}(\mathbb{D}, \mathcal{H})$
- g_0: known as the *null* model which the model that requires no training from the training data
- X: the $n \times p$ matrix made up of rows composed of the training data $\boldsymbol{x}_i$, i.e.,

$$X = \begin{bmatrix} \leftarrow \boldsymbol{x}_1 \rightarrow \\ \leftarrow \boldsymbol{x}_2 \rightarrow \\ \vdots \\ \leftarrow \boldsymbol{x}_n \rightarrow \end{bmatrix}$$

- $\mathcal{X}$: the space of all inputs
- $x_{\cdot 1}, \ldots, x_{\cdot p}$: the column vectors of X
- $x_{1\cdot}, \ldots, x_{n\cdot}$: the rows vectors of X
- y: the response variable, i.e., labels
- $\boldsymbol{y}$: column vector whose entries are the outputs y_i
- $\mathcal{Y}$: the space of all outputs
- $\mathbb{1}_a$: $= \begin{cases} 1, & \text{if condition } a \text{ is true} \\ 0, & \text{if condition } a \text{ is false} \end{cases}$
- $\boldsymbol{w}$: a vector of p components known as *weights* which are the numerical values we are estimating given as output of the learning algorithm

II. Three Ingredients of Supervised Learning

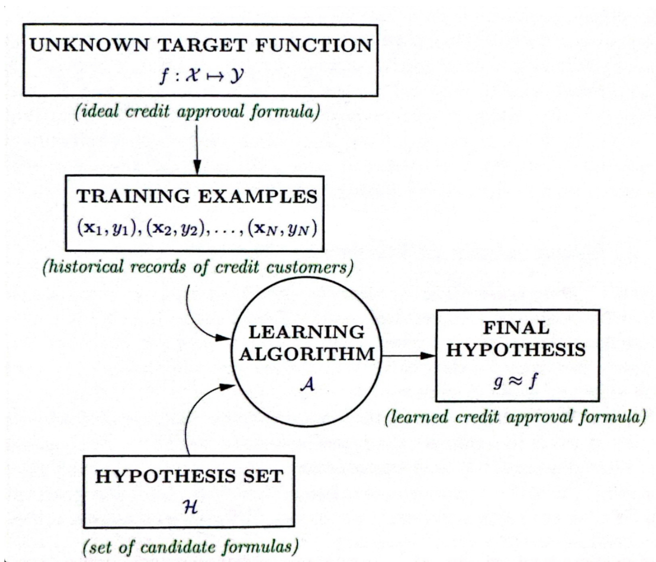
(i) List the **three** ingredients that comprises *supervised learning.*

(ii) Draw a schematic diagram/visual of this learning process.

(i) The Three Ingredients of Supervised Learning

1) Training Data, $\mathbb{D} = \langle X, \boldsymbol{y} \rangle$
2) Set of Candidate Functions, $\mathcal{H}$
3) Learning Algorithm, $\mathcal{A}$, which takes $\mathbb{D}$ and $\mathcal{H}$ as inputs

(ii) Visual of supervised learning on page 4 from *Learning from Data*:



III. Types of Errors

List and mathematically express the **errors** we take into account in *supervised learning.*

Errors in Supervised Learning:

- Ignorance Error:$= t(z_1, \ldots, z_t) - f(x_1, \ldots, x_p)$
- Misspecification Error:$= f(x_1, \ldots, x_p) - h^*(x_1, \ldots, x_p)$
- Estimation Error:$= h^*(x_1, \ldots, x_p) - g(x_1, \ldots, x_p)$
- Residuals (Model Errors): $e_i = y_i - \hat{y}_i = y_i - g(x_i) \in \{0, \pm 1\}$
- Total Error (TE): $\displaystyle\sum_{i=1}^{n} |e_i| = \sum_{i=1}^{n} \mathbb{1}_{g(x_i) \neq y_i}$
- Misclassification Error (ME): $= \dfrac{1}{n}$ TE

IV. The Perceptron Learning Algorithm (PLA)

(i) Who is credited for being the first to successfully implement the *perceptron* and when/where was it developed?

(ii) What is the underlying assumption in order to successfully implement PLA?

(iii) What are the steps/components to PLA and what is the desired result?

(iv) What are the limitations/drawbacks of PLA?

(i) In 1958, Frank Rosenblatt, a psychologist and project engineer from Cornell University, created the Mark I Perceptron which was able to recognize letters of the alphabet from a 20×20 pixel image as it iteratively *learned* the “correct” values of 400 weights.

(ii) The *Perceptron Learning Algorithm* (PLA) is guaranteed to converge under the assumption that the data is already linearly separable prior to the implementation of the algorithm.

(iii) The algorithmic steps are expressed in pseudocode below. A successful completion of PLA outputs the weights $\boldsymbol{w}$ that corresponds to a hyperplane that serves as a boundary between the two sets of linearly separable data.

(iv) The limitations/drawbacks of PLA are:

- relies on the linear separability of the two groups of data *a priori*
- it produces only one of the infinitely many possibilities of hyperplanes that *separates* the data, not an “*optimal*” one
- it is purely a computational tool that may provide insight to the correlation of the given data but it does not provide a basis for "*reasoning*”
- in 1969, Minsky & Papert, two scientists from MIT, wrote a book on the perceptron entitled *Perceptrons: An Introduction to Computational Geometry* and pointed out a specific, and rather elementary, scenario where a single perceptron is unable to correctly separate two sets of data known as the XOR (exclusive-or) problem

Algorithm 1 Perceptron Learning Algorithm

Initialize the weights $\boldsymbol{w}^{t=0} = \boldsymbol{0}^{p+1}$
while Total Error (TE) $\neq 0$ **do**
  for i = 1 to n **do**
    Compute $\hat{y}_i = \mathbb{1}_{\boldsymbol{w}\cdot\boldsymbol{x}_i \geq 0}$
    Set $w_0^{t=i} = w_0^{t=i-1} + (y_i - \hat{y}_i) \cdot 1$
    Set $w_1^{t=i} = w_1^{t=i-1} + (y_i - \hat{y}_i) \cdot x_{i,1}$
    $\vdots \quad \vdots \quad \vdots \quad \vdots$
    Set $w_p^{t=i} = w_p^{t=i-1} + (y_i - \hat{y}_i) \cdot x_{i,p}$
  end for
end while

Notable Publications on PLA: (1) "New Navy Device Learns By Doing", *New York Times* (1958), (2) "Rival", *The New Yorker* (1958), (3) "The Design of an Intelligent Automaton", *Research Trends* (1958)