

**Bihar Engineering University, Patna**  
**End Semester Examination – 2023**  
**Semester-IV**

Course: B.Tech.

Code: 100404

Subject: Discrete Mathematics

Time: 03 Hours

Full Marks: 70

**Instructions:-**

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **NINE** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question No. 1 is compulsory.

**Q.1 Write the answer of the following (Any seven question only):**

[2 x 7 = 14]

- a) Let A be the set odd positive integers less than 10. Then cardinality of A, |A| is
  - (i) 5
  - (ii) 9
  - (iii) 6
  - (iv) 4
- b) If m is the number of objects (pigeons) and n is the number of boxes (pigeonholes), then the function is both one – to – one and onto if
  - (i)  $m < n$  (ii)  $m = n$  (iii)  $m > n$  (iv) none of these
- c) If  $A \times B = B \times A$ , (Where A and B are general matrices) then
  - (i)  $A = d$  (ii)  $A = B'$  (iii)  $B = A$ , (iv)  $A' = B$
- d) A partial ordered relation is transitive, reflexive and
  - (i) Anti symmetric (ii) bi symmetric (iii) anti reflexive (iv) asymmetric
- e) If B is a Boolean Algebra, then which of the following is true
  - i. B is a finite but complemented lattice
  - ii. B is a finite, complemented and distributive lattice
  - iii. B is a finite, Distributive but not complemented lattice
  - iv. B is not distributive lattice
- f)  $P \rightarrow q$  is logically equivalent to
  - a)  $\sim q \rightarrow p$  b)  $\sim P \rightarrow q$
  - c)  $\sim P \wedge q$  d)  $\sim p \vee q$
- g) if  $f(x) = \cos x$  and  $g(x) = x^3$  then  $(f \circ g)(x)$  is
  - (i)  $(\cos x)^3$  (ii)  $\cos 3x$  (iii)  $x^{(\cos x)^3}$  (iv)  $\cos x^3$
- h) The number of distinguishable permutations of the letters in the word BANANA are
  - (i) 60 (ii) 36, (iii) 20, (iv) 10
- i) Which of the following pair is not congruent modulo 7?
  - (i) 10, 24
  - (ii) 25, 56
  - (iii) -31, -15
  - (iv) -64, -15
- j) Let  $N = \{1, 2, 3, \dots\}$  be ordered by divisibility, which of the following subset is totally ordered
  - (i) (2, 6, 24) (ii) (3, 5, 15) (iii) (2, 9, 16), (iv) (4, 15, 30)
- Q2. a.) Let  $A = B = \{x \mid 1 \leq x \leq 1\}$  for each of the following functions state where it is injective, surjective or bijective [7]
  - i)  $g(x) = \sin \pi x$
  - ii)  $b(x) = \frac{2x}{3}$
- (b) Let  $f(x) = x+2$ ,  $g(x) = x-2$ ,  $h(x) = 3x$  find (i) fog (ii) fogoh [7]
- Q3. a.) find the power set of each of these sets [7]
  - i)  $\{a, b\}$  ii)  $\{\phi, \{\phi\}\}$
- b.) Use Cantor's diagonal argument to prove that set F of all functions  $f: (0,1) \rightarrow \mathbb{R}$  has larger Cardinality than  $|\mathbb{R}|$  [7]
- Q4. Determine if the sets are countable or uncountable [14]
  - a.) the set A of all function  $g: \mathbb{Z}_+ \rightarrow \mathbb{Z}_+$
  - b.) The set B of all functions  $f: \mathbb{Z}_+ \rightarrow \{0,1\}$

Q5. Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ : [14]

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

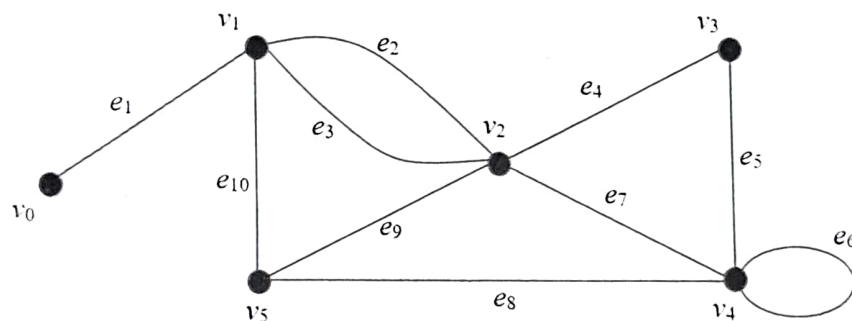
Q6. State and prove Division algorithm theorem well-ordering principle. [14]

Q7. (a) Check the validity of the following argument all integers are rational numbers. Some integers are powers of 5. Therefore, some rational numbers are powers of 5 [7]

(b) A grocery store employee is stocking apples. Each apple is a different color. There are 10 apples left in the box and the employee pulls out 2 of them at random. What is the probability that the employee pulls out one pink apple and yellow apple? [7]

Q8. Let  $\Psi : G \rightarrow H$  be a homomorphism of groups. Show that if  $a \in G$  has order  $n$ , then  $\Psi(a) \in H$  has order dividing  $n$ . [14]

Q9. Consider the following graph



(a) Does a Hamiltonian path exist? If so describe it. If not say why not. [7]

(b) Does an Eulerian path exist? If so describe it. If not say why not. [7]