PDA acceptance by final state
" " " empty stack

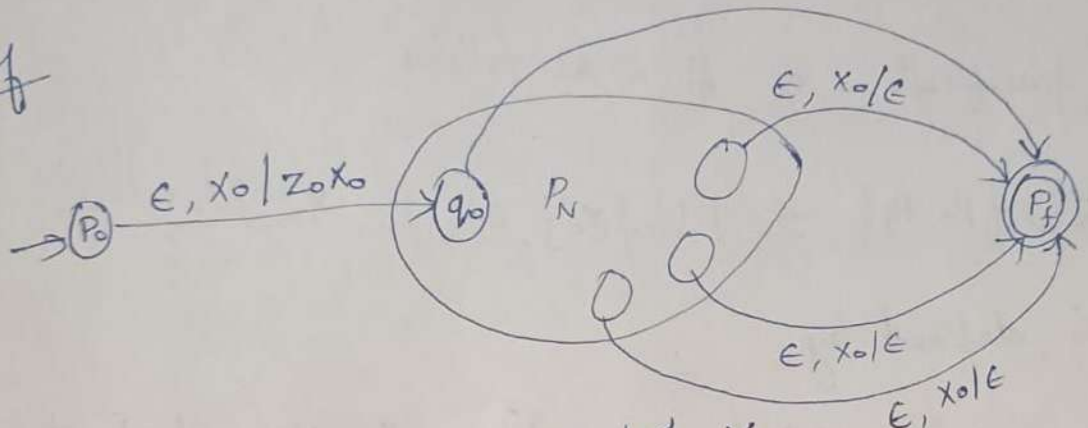
Theorem 1

If $L = N(P_N)$ for some PDA $P_N = (Q, \Sigma, \Gamma, \delta_N, q_0, Z_0)$

then $\exists$ a PDA P_F s.t $L = L(P_F)$

4
98
101
410

Proof



We use a new symbol X_0,
which must not be a symbol of Γ.
X_0 is both the start symbol of P_F and a marker
on the bottom of the stack that let us know when P_N
has reached an empty stack.
ie, if P_F sees X_0 on top of its stack
it knows that P_N would empty its stack on the same i/p.

Also we need a start state P_0
whose sole fn is to push Z_0, the start symbol of P_N
on to the top of the stack and enter state q_0, which is
the start state of P_N.

Then P_F simulates P_N, until the stack of P_N is empty,
which P_F detects because it sees X_0 on the top of the stack.

Finally we need another new state P_f, which is the accepting state of P_F.

This PDA transfers to state P_f whenever it discovers that P_N would have emptied its stack.

The specification of P_F is as follows

$$P_F = (Q \cup \{P_0, P_f\}, \Sigma, \Gamma \cup \{X_0\}, \delta_F, P_0, X_0, \{P_f\})$$

where δ_F is defined by

1. $\delta_F(P_0, \epsilon, X_0) = \{(q_0, Z_0 X_0)\}$. In its start state, P_F makes a spontaneous transition to the start state of P_N pushing its start symbol Z_0 on to the stack.

2. For all states q in Q, inputs a in Σ or $a = \epsilon$, and stack symbols Y in Γ, $\delta_F(q, a, Y)$ contains all the pairs in $\delta_N(q, a, Y)$

3. In addition to rule (2), $\delta_F(q, \epsilon, X_0)$ contains (P_f, ϵ) for every state q in Q.

We must show that w is in $L(P_F)$ iff w is in $N(P_N)$.

part

~~We are given that w is in L(P_N)~~

We are given that $(q_0, w, Z_0) \overset{*}{\underset{P_N}{\vdash}} (q, \epsilon, \epsilon)$ for some state q.

We know that

if $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is a PDA

and $(q, x, \alpha) \overset{*}{\underset{P}{\vdash}} (p, y, \beta)$ then for any strings w in Σ^*

and γ in Γ^*, it is also true that

$$(q, xw, \alpha\gamma) \overset{*}{\underset{P}{\vdash}} (p, yw, \beta\gamma).$$

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Hence we insert X_0 at the bottom of the stack

and conclude $(q_0, w, Z_0X_0) \overset{*}{\underset{P_N}{\vdash}} (q, \epsilon, X_0)$.

Since by rule (2)

P_F has all the moves of P_N.

We may also conclude that $(q_0, w, Z_0X_0) \overset{*}{\underset{P_F}{\vdash}} (q, \epsilon, X_0)$.

~~If~~ we put this sequence of moves together with the initial and final moves from rules (1) & (3) above, we get

$$(P_0, w, X_0) \underset{P_F}{\vdash} (q_0, w, Z_0X_0) \overset{*}{\underset{P_F}{\vdash}} (q, \epsilon, X_0) \underset{P_F}{\vdash} (P_f, \epsilon, \epsilon)$$

Thus P_F accepts w by final state.

Only if

The converse part requires only that the additional rule (1) & (3)

By ~~(3)~~, if the stack of P_F contains only X_0, we use rule (3). Otherwise use rule (1)

Thus any computation of P_F that accepts w ~~must look like sequence.~~

Moreover the middle of the computation all but the first and last steps must also be a computation of P_N with X_0 below the stack.

Since P_F cannot use any transition that is not also a transition of P_N, and X_0 cannot be exposed or the computation would end at the next step.

We conclude that $(q_0, w, Z_0) \overset{*}{\underset{P_N}{\vdash}} (q, \epsilon, \epsilon)$.

i.e, w is in $N(P_N)$.

Hence the Proof.