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Lab1-Logit

Individual Choice Modeling and Market Analysis (2015)

Oskar Västberg: oskar.blom@abe.kth.se

0760-287364

Recall from the lecture

$$P_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}} = \frac{e^{\beta' x_{ni}}}{\sum_j e^{\beta' x_{nj}}}$$

P_{ni}, probability that decision − maker n chooses alternative i.
V_{ni}, deterministic utility of alternative i.

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Recall from the lecture

Log-likelihood function

$$LL(\beta) = \sum_{n=1}^{N} \sum_{i \in C_n} y_{ni} \ln P_{ni}$$

Maximum likelihood estimation

$$\max_{\beta} LL(\beta) = \max_{\beta} \sum_{n=1}^{N} \sum_{i \in C_n} y_{ni} \ln P_{ni}$$

Model selection

- Hypothesis testing
  - Standard t-test

$$t - statistic = \frac{\hat{\beta}_k - \beta_k^*}{S_k}$$

where $\hat{\beta}_k$ is the estimate for the k^{th} parameter,

β_k^* is the hypothesized value for the k^{th} parameter and

S_k is the standard error of the estimate.

  - Typically want to check if parameter is significantly different from zero
  - Probability to by chance get |t|>2 when there is no effect is ~5%.

- Likelihood ratio test:

$$R = L(\hat{\beta}^H)/\mathrm{L}(\hat{\beta}) \ , \ \ -2logR \sim \chi^2_{d.f.}$$

$$-2logR = -2(\mathrm{LL}(\hat{\beta}^H) - LL(\hat{\beta}\ \ ))$$

- **Example**
- $U_1 = \beta_{cost} \cdot cost + \beta_{time} \cdot time$
- $U_0 = \beta_{cost} \cdot cost$
- $2\left(\mathrm{LL}(\hat{\beta}^H) - LL(\hat{\beta}\ \ )\right) = 4$
- Is this difference significant?

Q: How many degrees of freedom?

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Degrees of freedom (df)	χ^2 value[18]										
1	0.004	0.02	0.06	0.15	0.46	1.07	1.64	2.71	3.84	6.64	10.83
2	0.10	0.21	0.45	0.71	1.39	2.41	3.22	4.60	5.99	9.21	13.82
3	0.35	0.58	1.01	1.42	2.37	3.66	4.64	6.25	7.82	11.34	16.27
4	0.71	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	13.28	18.47
5	1.14	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.07	15.09	20.52
6	1.63	2.20	3.07	3.83	5.35	7.23	8.56	10.64	12.59	16.81	22.46
7	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.02	14.07	18.48	24.32
8	2.73	3.49	4.59	5.53	7.34	9.52	11.03	13.36	15.51	20.09	26.12
9	3.32	4.17	5.38	6.39	8.34	10.66	12.24	14.68	16.92	21.67	27.88
10	3.94	4.87	6.18	7.27	9.34	11.78	13.44	15.99	18.31	23.21	29.59
P value (Probability)	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.01	0.001

Value of Time

- Value of time: money/time, e.g., kr/min

$$\text{VofT} = \frac{\partial V_i}{\partial Time_i} \Big/ \frac{\partial V_i}{\partial Cost_i}$$

$$V_i = \ldots + \beta_{TVT} TVT_i + \beta_{Cost} Cost_i + \ldots$$

the value of time is given by

$$\text{VofT} = \beta_{TVT} / \beta_{Cost}$$

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Current data base

park_poss_wp	cheap_parking_wp	company_car	pt_subs_wp	dist_OK_m	car_time_hi	car_time_low	pt_aux_time_hi	pt_fstwait_hi
0	0	0	0					
0	0	0	0					
0	0	0	0	6550	22.38	24.79	6.93	2.733
0	0	0	0	6550	22.38	24.79	6.93	2.733
0	0	0	0	900	7.323	7.205	10.8	
0	0	0	0	3950	15.27	14.43	6.36	1.435
1	1	0	0	10630	14.97	15.25	13.8	6
0	0	0	0	6950	25.71	20.57	8.22	0.811
1	0	0	0	4820	16.63	12.49	11.28	2.143
1	1	0	0	4820	16.63	12.49	11.28	2.143
0	0	0	0	22390	31.18	31.86	12.6	0.556
0	0	0	0	13970	21.28	18.37	14.4	7.5
0	0	0	0	9000	19.3	17.82	7.56	1.765
0	0	0	0	16030	22.91	22.03	19.56	15
0	0	0	0	17070	35.58	29.29	10.94	1.765

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NOTE: examples are taken from
A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models (Koppelman & Bhat,2006)