Providing a Comprehensive Production Model for the WARP Shoe Company to Maximizes Profits

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Abstract

Following one of their competitors going out of business, the WARP shoe company requested a production plan for February 2006 which maximizes profit considering the rise in demand. As a result, we formulated the problem as an integer program, considering key constraints and aiming to maximize profit. After formulating the model, we estimated demand using holt-winters forecasting and solved our model utilizing gurobi. Our solution projects a profit of \$11,777,402.50, by meeting demand for expensive shoe types, coming up short of demand for some shoe types, and not producing the rest. Our solution was bound by the demand and the quantity of raw materials that were available for purchase.

1.0 Introduction

A major competitor of the WARP Shoe Company went bankrupt in the beginning of 2006. As a result, the company believes that in the month of February the demand for all shoes will double and has asked us to devise a production plan for them. In addition, all inventory for the month of January was sold, meaning that the company has no product going into the next month. We also were instructed to follow a given set of considerations [Appendix B]. The following formula was key in formulating our objective function while defining the problem.

Formula One: Profit Formula

$$Profit = Revenue - Costs$$

Since our goal is to maximize profit our first step was to attempt to identify the revenue. As we know the revenue is from the shoes being sold. This raised the question of which shoes, and how many? To answer this, we had to first view the cost of the shoes in the data tables provided, while also understanding the demand. The latter was more complex and resulted in us utilizing Holt-Winter Forecasting.

Our second step was to identify the costs which the WARP Shoe Company business was facing, and through our assumptions, which will be broken, down later we decided on four:

- 1. Worker Wages
- 2. Warehouse opening costs
- 3. Cost of Raw Materials
- 4. Machine operating Costs
- 5. Cost of not meeting demand

^{*}All code for this project can be found in our GitHub (link in Appendix A).

Once these were identified we utilized multiple programs including: Excel, R, Python and AMPL to construct our model and produce a plan to yield maximum profit.

2.0 Methods

The team chose to define this problem as a maximization integer program.

I = 557 is the total number of shoe types, J = 72 is the total number of machines, W = 8 is the total number of warehouses, and K = 165 is the total number of raw material kinds.

We defined the following decision variable:

 x_i - the number of shoes produced of the i^{th} shoe type. $\forall i = 1, ..., I$

 y_w - a binary value which takes 1 if the w^{th} warehouse is open and 0 otherwise. $\forall w = 1, ..., W$

We also defined the following parameters:

 D_i - the forecasted demand for the i^{th} shoe type. $\forall \; i = 1, ..., I$

 n_{ik} - the amount of the k^{th} raw material in one i^{th} type of shoe. $\forall \ i=1,...,I \ and \ k=1,...K$

 p_k - the unit cost of the k^{th} raw material. $\forall \; k \; = \; 1, \ldots K$

 m_i - the cost of raw materials for one i^{th} type of shoe where

$$m_i = \sum_{k=1}^K (n_{ik} \cdot p_k) \quad \forall i = 1, \dots, I$$

 t_{ij} - time to manufacture the i^{th} type of shoe on the j^{th} machine in seconds.

$$\forall i = 1, ..., I \text{ and } j = 1, ... J$$

 T_{ij} - time to manufacture the i^{th} type shoe on the j^{th} machine in hours where

$$T_{ij} = \frac{t_{ij}}{360} \ \forall \ i = 1, ..., I \ and \ j = 1, ..., J$$

 s_i - sale price of one i^{th} type shoe. $\forall i = 1, ..., I$

 C_w - capacity of the w^{th} warehouse. $\forall w = 1, ..., W$

 h_w - cost to open the w^{th} warehouse. $\forall w = 1, ..., W$

 a_i - cost per minute to operate the j^{th} machine. $\forall j = 1, ... J$

 q_k - quantity of raw material k available. $\forall k = 1, ... K$

*All parameters without a defined formula were obtained from the database

Using these parameters and variables we then defined the following model to maximize profit by computing: revenue – cost of employees – cost of not meeting demand – warehouse opening cost – cost of raw materials – cost of operating machines:

Formula Two: Objective Function

$$\max \quad Z = \left(\sum_{i=1}^{I} x_i \cdot s_i\right) - \left(25 \cdot \sum_{i=1}^{I} \sum_{j=1}^{J} T_{ij} \cdot x_i\right) - \left(\sum_{w=1}^{W} y_w \cdot h_w\right)$$

$$-\left(10 \cdot \sum_{i=1}^{I} (D_{i} - X_{i})\right) - \left(\sum_{i=1}^{I} x_{i} \cdot m_{i}\right) - \left(\sum_{i=1}^{I} \sum_{j=1}^{J} 60 \cdot a_{j} \cdot T_{ij} \cdot x_{i}\right)$$

Formula Three: Demand Constraint

$$s.t.$$
 $x_i \le D_i \ \forall \ i = 1, ..., I$ (demand)

Formula Four: Budget Constraint

s.t.
$$\sum_{i=1}^{I} x_i \cdot m_i \le 10,000,000$$
 (raw material budget)

Formula Five: Machine Hours Constraint

s.t.
$$\sum_{i=1}^{I} T_{ij} \cdot x_i \leq 336 \,\forall j = 1,...,J$$
 (available machine hours)

Formula Six: Warehouse Capacity Constraint

s.t.
$$(\sum_{i=1}^{l} x_i) - (\sum_{w=1}^{W} c_w \cdot y_w) \le 0$$
 (available warehouse capacity)

Formula Seven: Raw Materials Constraint

s.t.
$$\sum_{i=1}^{I} x_i \cdot n_{ik} \le q_k \ \forall \ k = 1, ..., K$$
 (available raw material quantities)

Formula Eight: Integer Program and Non-Negativity Constraints for Each Shoe Type

$$s.t. \quad x_i \in \mathbb{Z}^+ \ \forall \ i=1,...,I$$

Formula Nine: Binary Constraint for Warehouses

s.t.
$$y_w \in \{0,1\} \forall w = 1,...,W$$

In this model we make the following assumptions:

- 1. Workers are paid at a rate of 25\$ times the number of hours they worked (including fractional hours).
- 2. Each machine can only be operated for 12 hours a day.
- 3. Transportation costs are negligible.
- 4. Manufacturing sequence is negligible.
- 5. Machine setup times and costs are negligible.
- 6. Raw material costs are deducted from total monthly profits, not just the budget.

3.0 Results

This section details the forecasting of our demand, implementation of our model, the recommended solution, and answers to the assignment questions.

3.1 Implementation

Using the model defined in section 2.0, the team first sought to forecast the demand for the month of February 2006. To do so we first extracted the data from the table and formatted it into a CSV with our desired columns using a simple R program. Afterwards, we used a python program to visualize any trends in the data. As seen in figure 1, we found a clear seasonal trend as well as an overall trend over three years when looking at the demand for shoe type "SSH001." As a result, we decided to use python to implement Holt-Winters forecasting due to its ability to capture long-term and seasonal trends. After we completed the forecast, we doubled the values to account for the expected increase in demand.

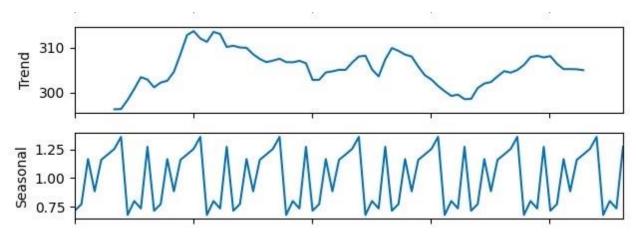


Figure 1. Trends in demand for product "SSH001."

After forecasting demand, we plugged it into our integer program and utilized AMPL to solve our model. Due to computational and time constraints, we were forced to relax our model into a linear program where $x_i \ge 0 \ \forall \ i = 1, ..., I$.

3.2 Our Solution

Due to time and computing constraints, we were forced to relax our model to a linear program and then round the received values to the nearest integer after. After solving this program using gurobi, we achieved the following objective function value:

$$Profit = $11,777,402.50$$

This suggests that by following our production plan, the WARP Shoe Company could make around this value in profit. Our model's complete recommendations for shoe production can be found in Appendix C. As seen in a snipper of our solution in figure 2, the solution involves production up to demand on some shoes, production short of demand on some shoes, and no production of the remaining shoes. Our solution was bound by shoe demand of expensive shoes and quantity of raw frequently used materials. As a result, to increase profit, it is suggested to either increase demand for expensive shoes or find a way to acquire a larger quantity of frequently used raw materials. According to our model, other changes will yield no improvement in profit.

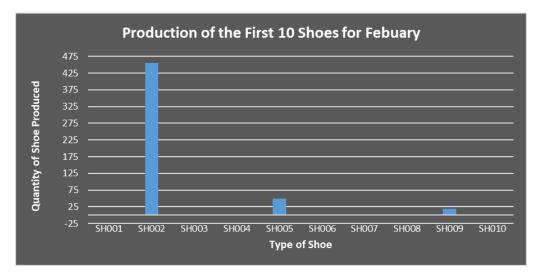


Figure 2. Recommended production for the first 10 types of shoes.

3.3 Answers to Questions

1. How should you estimate the demand for the month of February?

We started our estimation process by visualizing trends in the demand for the given three years of data where we found both seasonal and long-term trends. As a result, we utilized holt-winters forecasting due to its ability to capture seasonal and long-term dependencies. After we completed the forecast, we doubled the values to account for the expected increase in demand.

2. How many variables and constraints do you have?

We have two types of decision variables, one representing the number of each shoe type we should produce (577 total variables) and another representing a binary value for whether each warehouse is open or not (8 total variables). We had five types of constraints not including integer non-negativity and binary constraints. We had one constraint type to make sure we didn't produce more shoes than demand (577 total constraints), one to make sure the budget for raw

materials wasn't exceeded (1 total constraint), one to make sure more raw materials than we had access to weren't used (165 total constraints), one to make sure there weren't more shoes than capacity (1 total constraint), and one to make sure that available machine hours weren't exceeded (72 total constraints).

3. If you had to relax your integer program to an LP, how many constraints were violated after rounding the LP solution to the closest integer solution?

After rounding out to the nearest integer solution, we violated 68 total constraints from the demand constraint type (41 total constraints violated) and the raw materials quantity constraints (27 total constraints violated). A full list of violated constraints can be found in Appendix E.

4. Which constraints are binding, and what is the real-world interpretation of those binding constraints?

There are two binding constraints: shoe demand for some shoes and available raw material quantities for most raw materials. A list of the specific binding constraints can be found in Appendix D. The real-world interpretation for shoe demand is the fact that we produce exactly the demand for some shoes. This makes sense since we would not want to produce more than the demand. The real-world interpretation for the raw materials quantity constraint is the fact that we utilize all of the available raw materials. This makes sense since we cannot use materials that we don't have access to. Additionally, non-negativity is binding for some shoes which makes sense since we cannot make negative shoes.

5. Assume that some additional warehouse space is available at the price of \$10/box of shoes. Is it economical to buy it? What is the optimal amount of space to buy in this situation?

No, it is not economical to buy it since the warehouse capacity constraint isn't binding, has a slack of 4406. So, we have the ability to store 4406 shoes before we need to change our current solution. Incidentally, this constraint has a shadow price of 0. This means that for each additional box of capacity you buy, the objective function value will increase by \$0. This means if we spent money on buying the boxes we would net -\$10 for every additional box of capacity purchased. As a result, we should not buy any additional storage.

6. Imagine that machines were available for only 8 hours per day. How would your solution change? Which constraints are binding now? Does the new solution seem realistic to you?

After re-solving the LP with an updated machine hours constraint, our objective function and optimal solution are exactly the same as in our original formulation. This is expected since none of the machine hour constraints were originally binding. This means that in our original solution we are not utilizing all the machine hours, this could be a result of the additional cost accrued through machine operating and worker costs. The demand and raw materials quantity constraints are still the only binding constraints. This solution does not seem very realistic since it implies that we aren't using at least 112 hours of the available time on the machines in our original solution, and decreasing the available time has no effect.

7. If in addition there were a \$7,000,000 budget available to buy raw materials, what would you do? Change your formulation and solve again.

Again, the objective function does not change since we run out of available raw materials before we are able to spend our initial budget of \$10,000,000. As a result, simply adding to the budget will not increase our total profit and resolving the LP yields the same results as the initial formulation. Therefore, there is no need to attain additional raw materials budget.

4.0 Conclusions

WARP shoes are a company which gave us a very particular set of conditions for the month of February, and we have used these to output the best possible solution for them with the data provided to maximize profit. In the model constructed we introduced assumptions in the methods section, forecasted demand using Holt-Winters, and utilized multiple tools ranging from R to AMPL to find the maximum profit.

When first viewing the problem, we needed to break down the specific costs that the WARP Shoe Company's products incurred. Once we identified these costs, we were able to create an objective function which provides the profit. Our next step was to meet the set of constraints outlined by the WARP Shoe Company. These ranged from machine hours to raw materials. Following this we decided to understand the next key component in our model: the demand. In R we were able to extract the demand data from the CSV files. We then utilized a Python script to visualize trends and found a specific trend for the Shoe model with item code "SSH001". This led to us creating a Python script to complete our Holt-Winters Forecasting.

Through all of these steps we were able to conclude that the maximum profit the WARP Shoe Company can make is \$11,777,402.50.

In our model the one constraint which we found the most binding was the raw materials constraint, specifically how many raw materials we have available to us. As can be seen, even if we had a higher budget, more capacity in our warehouses, or less time on our machines: our solution doesn't change. However, due to the raw materials being limited in quantity and there not being enough demand, we shouldn't produce more shoes. Therefore, if the WARP Shoe Company wants to increase its profits further our recommendation would be to either find more suppliers or find suppliers who can provide more raw materials. Additionally, they would have to increase their marketing budget or change consumer behavior to create more demand for their products. Through both these strategies they would be able to increase profits even further.

There are also a few limitations in this model, firstly the way during our forecasting we utilized SH001 to visualize any trends meaning we may have a model which does not perfectly predict demand for all shoes. Another limitation of the model is the fact that we made a set of assumptions on worker wages specifically that they're not paid to the nearest whole hour. This might not actually be true in the real world and is a consideration which could be adjusted to reflect costs differently.

Our model is largely reflective of the real world, but if we were to deepen it, we could factor in additional costs such as transportation. Ultimately, we have largely captured the WARP Shoe Company's model for profitability and believe that the optimal profit provided is accurate to current conditions.

Appendices

Appendix A: GitHub Link

https://github.com/neckhert/WARP-Shoes-Project

Appendix B: List of Considerations

Budget for Raw Materials: \$10,000,000

Sales price on shoes remain the **same** as in the **Product Master Table**

Not meeting the demand for any type of shoe means the **loss of new potential customers** and hence has been evaluated at a **cost of \$10/pair**. Demand for each month from **1997-2003** can be found in the **Product Demand table.**

During **February 2006** the WARP shoe production plant, including all machines, will work up to **12 hours a day, 28 days a month**. The **cost** of operation of each machine is given in the **Machine_Master** table. Assume that **setup times and costs are negligible**. The **amount of time** (s) each shoe requires from each machine is given in the **Machine Assign** table.

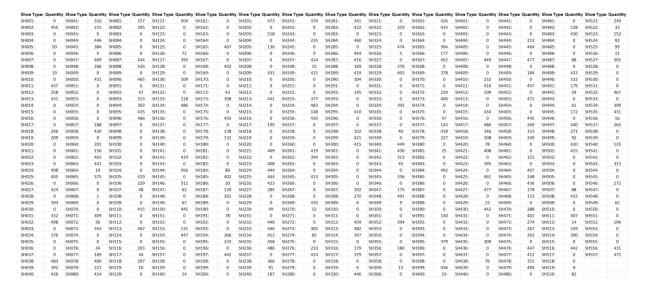
Workers are **paid** on an hourly basis at the rate of \$25/hour. Each machine has to be operated by **one worker.**

The **total** warehouse capacity can be obtained from the **Warehouse_Master table**.

Transportation costs can be ignored.

The manufacturing sequence can be ignored.

Appendix C: Recommended shoe production based on our model.



Appendix D: Complete List of Binding Constraints

Shoe Typ	oe e	Shoe Type			Shoe Type			Shoe Type			Shoe Type			Shoe Type		
SH002	demand is binding.	SH096	demand i	s binding.	SH224	demand is b	oinding.	SH312	demand is l	binding.	SH421	demand is	binding.	SH511	demand is	binding
SH011	demand is binding.	SH105	demand i	s binding.	SH226	demand is b	oinding.	SH313	demand is	binding.	SH425	demand is	binding.	SH514	demand is	binding
SH013	demand is binding.	SH110	demand i	s binding.	SH232	demand is b	oinding.	SH314	demand is l	binding.	SH427	demand is	binding.	SH516	demand is	binding.
SH025	demand is binding.	SH113	demand i	s binding.	SH233	demand is b	oinding.	SH320	demand is l	binding.	SH430	demand is	binding.	SH532	demand is	binding.
SH027	demand is binding.	SH121	demand i	s binding.	SH234	demand is b	oinding.	SH325	demand is l	binding.	SH445	demand is	binding.	SH535	demand is	binding.
SH032	demand is binding.	SH134	demand i	s binding.	SH247	demand is b	oinding.	SH329	demand is l	binding.	SH447	demand is	binding.	SH556	demand is	binding
SH034	demand is binding.	SH142	demand i	s binding.	SH249	demand is b	oinding.	SH334	demand is l	binding.	SH451	demand is	binding.	SH557	demand is	binding.
SH038	demand is binding.	SH144	demand i	s binding.	SH253	demand is b	oinding.	SH341	demand is l	binding.	SH453	demand is	binding.			
SH039	demand is binding.	SH146	demand i	s binding.	SH254	demand is b	oinding.	SH348	demand is l	binding.	SH456	demand is	binding.			
SH040	demand is binding.	SH150	demand i	s binding.	SH256	demand is b	oinding.	SH352	demand is l	binding.	SH466	demand is	binding.			
SH044	demand is binding.	SH154	demand i	s binding.	SH261	demand is b	oinding.	SH361	demand is l	binding.	SH476	demand is	binding.			
SH050	demand is binding.	SH165	demand i	s binding.	SH262	demand is b	oinding.	SH362	demand is l	binding.	SH477	demand is	binding.			
SH058	demand is binding.	SH176	demand i	s binding.	SH265	demand is b	oinding.	SH367	demand is l	binding.	SH479	demand is	binding.			
SH062	demand is binding.	SH197	demand i	s binding.	SH269	demand is b	oinding.	SH369	demand is l	binding.	SH483	demand is	binding.			
SH063	demand is binding.	SH201	demand i	s binding.	SH277	demand is b	oinding.	SH378	demand is l	binding.	SH489	demand is	binding.			
SH080	demand is binding.	SH203	demand i	s binding.	SH282	demand is b	oinding.	SH384	demand is l	binding.	SH490	demand is	binding.			
SH087	demand is binding.	SH209	demand i	s binding.	SH286	demand is b	oinding.	SH399	demand is l	binding.	SH497	demand is	binding.			
SH088	demand is binding.	SH213	demand i	s binding.	SH287	demand is b	oinding.	SH407	demand is l	binding.	SH500	demand is	binding.			
SH090	demand is binding.	SH221	demand i	s binding.	SH295	demand is b	oinding.	SH415	demand is l	binding.	SH501	demand is	binding.			
SH095	demand is binding.	SH223	demand i	s binding.	SH300	demand is b	oinding.	SH417	demand is l	binding.	SH508	demand is	binding.			

Binding Constraints:

Raw material 1 quantity is binding. Raw material 119 quantity is binding. Raw material 5 quantity is binding. Raw material 120 quantity is binding. Raw material 9 quantity is binding. Raw material 121 quantity is binding. Raw material 13 quantity is binding. Raw material 122 quantity is binding. Raw material 16 quantity is binding. Raw material 124 quantity is binding. Raw material 18 quantity is binding. Raw material 125 quantity is binding. Raw material 127 quantity is binding. Raw material 19 quantity is binding. Raw material 22 quantity is binding. Raw material 128 quantity is binding. Raw material 27 quantity is binding. Raw material 129 quantity is binding. Raw material 31 quantity is binding. Raw material 132 quantity is binding. Raw material 32 quantity is binding. Raw material 133 quantity is binding. Raw material 33 quantity is binding. Raw material 134 quantity is binding. Raw material 137 quantity is binding. Raw material 35 quantity is binding. Raw material 36 quantity is binding. Raw material 139 quantity is binding. Raw material 40 quantity is binding. Raw material 140 quantity is binding. Raw material 43 quantity is binding. Raw material 142 quantity is binding. Raw material 48 quantity is binding. Raw material 143 quantity is binding. Raw material 56 quantity is binding. Raw material 145 quantity is binding. Raw material 57 quantity is binding. Raw material 146 quantity is binding. Raw material 60 quantity is binding. Raw material 147 quantity is binding. Raw material 62 quantity is binding. Raw material 148 quantity is binding. Raw material 68 quantity is binding. Raw material 149 quantity is binding. Raw material 79 quantity is binding. Raw material 150 quantity is binding. Raw material 81 quantity is binding. Raw material 151 quantity is binding. Raw material 82 quantity is binding. Raw material 153 quantity is binding. Raw material 84 quantity is binding. Raw material 155 quantity is binding. Raw material 85 quantity is binding. Raw material 157 quantity is binding. Raw material 86 quantity is binding. Raw material 158 quantity is binding. Raw material 89 quantity is binding. Raw material 163 quantity is binding. Raw material 96 quantity is binding. Raw material 98 quantity is binding. Raw material 101 quantity is binding. Raw material 103 quantity is binding. Raw material 104 quantity is binding. Raw material 105 quantity is binding. Raw material 113 quantity is binding. Raw material 114 quantity is binding. Raw material 115 quantity is binding.

Demand Violated C	onstraints	Raw Material Violated Constraints					
List of Violated Constraints	:	Raw material 1 quantity is violated.					
SH002 demand is violated.	SH282 demand is violated.	Raw material 12 quantity is violated.					
SH013 demand is violated.	SH295 demand is violated.	Raw material 13 quantity is violated.					
SH025 demand is violated.	SH300 demand is violated.	Raw material 24 quantity is violated.					
SH032 demand is violated.	SH313 demand is violated.	Raw material 41 quantity is violated.					
SH034 demand is violated.	SH329 demand is violated.	Raw material 47 quantity is violated.					
SH038 demand is violated.	SH348 demand is violated.	Raw material 51 quantity is violated.					
SH058 demand is violated.	SH361 demand is violated.	Raw material 53 quantity is violated.					
SH062 demand is violated.	SH362 demand is violated.	Raw material 56 quantity is violated.					
		Raw material 64 quantity is violated.					
SH087 demand is violated.	SH378 demand is violated.	Raw material 69 quantity is violated.					
SH088 demand is violated.	SH407 demand is violated.	Raw material 77 quantity is violated.					
SH110 demand is violated.	SH421 demand is violated.	Raw material 80 quantity is violated.					
SH134 demand is violated.	SH425 demand is violated.	Raw material 93 quantity is violated.					
SH146 demand is violated.	SH427 demand is violated.	Raw material 96 quantity is violated.					
SH165 demand is violated.	SH451 demand is violated.	Raw material 101 quantity is violated.					
SH221 demand is violated.	SH453 demand is violated.	Raw material 107 quantity is violated.					
SH226 demand is violated.	SH456 demand is violated.	Raw material 110 quantity is violated.					
SH234 demand is violated.	SH466 demand is violated.	Raw material 113 quantity is violated.					
SH247 demand is violated.	SH501 demand is violated.	Raw material 125 quantity is violated.					
SH254 demand is violated.	SH516 demand is violated.	Raw material 126 quantity is violated.					
	SH510 demand is violated.	Raw material 132 quantity is violated.					
SH256 demand is violated.		Raw material 138 quantity is violated.					
SH277 demand is violated.		Raw material 141 quantity is violated.					
		Raw material 144 quantity is violated.					
		Raw material 148 quantity is violated.					
		Raw material 149 quantity is violated.					