Necole Goodman

HOMEWORK #10

.61

. $V = \mathbb{R}^3$ $S = \{(1,0,0),(0,1,0),(2,3,1)\}$

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\\0\end{bmatrix} + c_3\begin{bmatrix}2\\3\\1\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&0&2&x\\0&1&3&y\\0&0&1&z\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&2&x\\0&1&0&-3z+y\\0&0&1&z\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&0&-2z+x\\0&1&0&-3z+y\\0&0&1&z\end{array}\right]$$

$R_2 = -3R_3 + R_2$ $R_1 = -2R_3 + R_1$

$\therefore$ S spans the vector space

5. $V = P_2$: $S = \{t+1,\ t^2+1,\ t^2-t\}$

$= c_1(t+1) + c_2(t^2+1) + c_3(t^2-t)$

$= (c_2+c_3)t^2 + (c_1-c_3)t + (c_1+c_2)$

$c_2 + c_3 = a_1$

$c_1 + 0 - c_3 = a_2$

$c_1 + c_2 + 0 = a_3$

$0 + c_2 + \cancel{c_3} = a_1$

$c_1 + 0 - \cancel{c_3} = a_2$

$c_1 + c_2 = a_1 + a_2$

$\therefore$ There is no way to solve this system in P_2

$$\left[\begin{array}{ccc|c}0&1&1&a_1\\1&0&-1&a_2\\1&1&0&a_3\end{array}\right]$$

10. $V = \mathbb{R}_3$; $S = \{(2,-1,4),(4,-2,8)\}$

$$a_1\begin{bmatrix}2\\-1\\4\end{bmatrix} + a_2\begin{bmatrix}4\\-2\\8\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{cc|c}2&4&0\\-1&-2&0\\4&8&0\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}1&2&0\\-1&-2&0\\4&8&0\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}1&2&0\\-1&-2&0\\0&0&0\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}1&2&0\\1/2&1&0\\0&0&0\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}1&0&0\\1/2&1&0\\0&0&0\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}1&0&0\\0&1&0\\0&0&0\end{array}\right]$$

$R_1 = \frac{1}{2}R_1$ $R_3 = 4R_2 + R_3$ $R_2 = -\frac{1}{2}R_2$ $R_1 = -2R_2 + R_1$ $R_2 = -\frac{1}{2}R_1 + R_2$

$\therefore$ since not all solutions equal zero. This is linearly dependent

16. $V - \mathbb{P}_2$; $S = (t+3,\ t^2-1,\ 2t^2-t-5)$

$c_1(t+3) + c_2(t^2-1) + c_3(2t^2-t-5)$

$(c_2 + 2c_3)t^2 + (c_1 - c_3)t + (3c_1 - c_2 - 5c_3)$

$$\begin{bmatrix}0&1&2\\1&0&-1\\3&-1&-5\end{bmatrix} \Rightarrow \begin{bmatrix}3&-1&-5\\1&0&-1\\0&1&2\end{bmatrix} \Rightarrow \begin{bmatrix}1&-1/3&-5/3\\1&0&-1\\0&1&2\end{bmatrix} \Rightarrow \begin{bmatrix}1&-1/3&-5/3\\0&1/3&2/3\\0&1&2\end{bmatrix} \Rightarrow \begin{bmatrix}1&-1/3&-5/3\\0&1&2\\0&1&2\end{bmatrix} \Rightarrow \begin{bmatrix}1&-1/3&-5/3\\0&1&2\\0&0&0\end{bmatrix} \Rightarrow \begin{bmatrix}1&0&-1\\0&1&2\\0&0&0\end{bmatrix}$$

$R_1 \leftrightarrow R_3$ $R_1 = \frac{1}{3}R_1$ $R_2 = -R_1 + R_2$ $R_2 = 3R_2$ $R_3 = -R_2 + R_3$ $R_1 = \frac{1}{3}R_2 + R_1$

$\therefore$ This is linearly dependent.

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HW # 10

43-46, 32

3.6

30. $\{\cos t + \sin t, \cos t - \sin t\}$

$= C_1(\cos t + \sin t) + C_2(\cos t - \sin t)$

$= (C_1 + C_2)\cos t + (C_1 - C_2)\sin t$

$= C_1 \cos t + C_2 \sin t$

$= \{\sin t, \cos t\}$ ✓ the spans are equal

43. $\{(1,1)\}$ for $\mathbb{R}_2$

Basis:
$$\left[\begin{array}{c|c} 1 & 0 \\ 1 & 0 \end{array}\right]$$

not able to put in RREF ∴ Not a basis of $\mathbb{R}_2$

44. $\{(1,2),(2,1)\}$

Basis:
$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] ✓$$

$R_2 = -2R_1 + R_2$ $\quad R_2 = -\frac{1}{3}R_2$ $\quad R_1 = -2R_2 + R_1$

∴ $\{(1,2),(2,1)\}$ is a basis for $\mathbb{R}^2$

45. $\{(-1,-1),(1,1)\}$

Basis:
$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \times$$

$R_1 = -R_1$ $\quad R_2 - R_1 = R_2$

∴ Linearly dependent, therefore not a basis for $\mathbb{R}^2$.

46. $\{(1,0),(1,1)\}$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] ✓$$

$R_1 = -R_2 + R_1$

∴ A basis for $\mathbb{R}^2$

47. $\{(1,0),(0,1),(1,1)\}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

∴ cannot be solved, not a basis for $\mathbb{R}^2$.

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3.6 HW #10

48. $\{(0,0),(1,1),(2,2),(-1,-1)\}$

$$\left[\begin{array}{cccc|c} 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] x$$

$R_2 = R_2 - R_1$

$\therefore$ linearly dependent, therefore not a basis for $\mathbb{R}^2$.

82.

a) True

b) False; set only has 2 vectors; dim(W) = 2

$$W = \left\{ r\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + s\begin{bmatrix} 3 \\ 0 \\ 0 \\ -1 \end{bmatrix} + t\begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} + y\begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \right\}$$

c) False; The basis needs two vectors

$\{(3,0,0,-1),(0,2,1,1)\}$

Part II | HOMEWORK #10 | Necole Goodman

1) The span of a set of vectors $\{\vec{v}_1, \vec{v}_2 \ldots \vec{v}_n\}$ is the set of all linear combinations of the given vectors.

2) Linear independence is when no given vector of a set can be written as a linear combination of the others.

3) The column space of a matrix A is a column that corresponds to a column in RREF of A with a leading 1.

4) The basis for a vector space is a subset of vectors in $\mathbb{V}$ that are linearly independent and span V

Part III

1. Basis:

$2x - y + z = 0$

$z = y - 2x$

$\bar{x} = (x, y, y - 2x)$

$\bar{x} = (1, 0, -2)x + (0, 1, 1)y$

$\therefore$ span $\{(1,0,-2), (0,1,1)\}$

2. $\left\{\begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$

The dimension of this vector space is 3, because their are 3 elements w/i the set.