1.4 | .3

Homework #3

Necole Goodman

4. $y' = t^2 + e^{-y}$, $y(0) = 0$; $[0, 2]$ $y_{n+1} = y_n + hf(t_n, y_n)$

$h = 0.01$

	t	y
0	0	0.0
1	0.1	0.1
2	0.2	0.192
3	0.3	0.278
4	0.4	0.363

- According to xcel @ t=2 the value is 3.348
- Not seperable

5.

	t	y
0	1	1
1	1.1	1.1414
2	1.2	1.2911
3	1.3	1.449
4	1.4	1.6143
5	1.5	1.788

According to xcel the value @ t=5 is 12.252

The equation is not seperable

$h = 0.01$

$y' = \sqrt{t+y}$ $y(1) = 1$: $[1, 5]$

12. $y' = y^2$, $y(0) = 1$ $y(t) = \frac{1}{1-t}$

$h = 0.25$

	t	y
0	0	1
1	0.25	1.25
2	0.5	1.641
3	0.75	2.314
4	1	3.652

$y' = y^2$

$\int \frac{1}{y^2} dy = \int dt$

$-\frac{1}{y} = t + C$

$-\frac{1}{1} = 0 + C$

$C = -1$

$\therefore -\frac{1}{y} = t - 1$

$y = \frac{1}{1-t}$

$y(1) \to \infty$ ✓

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1.4

HW #3

Nicole Goodman

15. $y' = y \quad y(0) = A \quad y(t) = Ae^t$

$y' = y$

$\int \frac{1}{y}\,dy = \int dt$

$e^{\ln|y|} = e^{(t+c)}$

$y = e^t + e^c$

$y = \pm e^c e^t \quad \rightarrow e^c = K$

$y = Ke^t$

$y(0) = Ke^t$

$A = Ke^0$

$\therefore K = A$

$y = Ae^t$

1.5

HW # 3

Newle Goodman

2. $ty' + y = 2$; $y(0) = 0$

Solve for y' : $ty' = 2 - y$

$$y' = \frac{2-y}{t} = \frac{2-0}{0} = \text{und} \quad \boxed{\therefore \text{ not continuous}}$$

3. $y' = y^{4/3}$; $y(0) = 0$

$y = 0^{4/3} = 0$ $\therefore$ continuous

$\frac{\partial f}{\partial y} = \frac{3}{7} y^{7/3}$

15. All of B, C and D are unique solutions to the given graph on the interval A

18. All unique solutions on the interval of all values of t

22.

a) $\frac{dy}{dt} = |y|$

$f(t, y) = 3t^2 (y+1)$

$f_y = 3t^2 y$

$\therefore$ unique at $y = -1$ → According to graph I plotted

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2.1

HW #3

Newle Goodman

4. $t\frac{d^2y}{dt^2}+\frac{dy}{dt}+ty=1$

$L(cy)=tcy''+cy'+tcy\overset{?}{=}1$

$=c(\underbrace{ty''+cy'+ty}_{=1})\overset{?}{=}1$

$=c(1)\overset{?}{=}1$

$L(y_1+y_2)=t(y_1+y_2)''+(y_1+y_2)'+t(y_1+y_2)\overset{?}{=}1$

$=ty_1''+ty_2''+y_1'+y_2'+ty_1+ty_2\overset{?}{=}1$

$=\underbrace{[ty_1''+y_1'+ty_1]}_{=1}\overset{?}{=}\underbrace{[ty_2''+y_2'+ty_2]}_{=1}\overset{?}{=}1$

∴ The equation is linear, nonhomogenous, and the coefficient is a variable. order is (2)

5. $\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=0$

$y'''+y''+y'+y\overset{?}{=}0$

$L(cy)=cy'''+cy''+cy'+cy\overset{?}{=}0$

$=c(\underbrace{y'''+y''+y'+y}_{=0})\overset{?}{=}0$

$=c(0)=0$

$L(y_1+y_2)=(y_1+y_2)'''+(y_1+y_2)''+(y_1+y_2)'+(y_1+y_2)\overset{?}{=}0$

$\begin{cases}=y_1'''+y_1''+y_1'+y\overset{?}{=}0\\=y_2'''+y_2''+y_2'+y_2\overset{?}{=}0\end{cases}$

→ Both $=0$

∴ The equation is linear, homogenous, and the coefficient is constant. and the order is 3

8. $\frac{d^2w}{dt^2}+w^2\frac{dw}{dt}+w=0$

$w''+w^2w'+w\overset{?}{=}0$

$L(cw)=cw''+w^2(cw')+cw\overset{?}{=}0$

$=c(\underbrace{w''+w^2w'+w}_{=0})\overset{?}{=}0$

$=c(0)=0$

$L(w_1+w_2)=(w_1+w_2)''+(w_1+w_2)^2(w_1+w_2)'+(w_1+w_2)$

$=w_1''+(w_1^2+2w_1w_2+w_2^2)(w_1+w_2)'+w_1+w_2\overset{?}{=}0$

12. $L(y)=y'+2y$

$L(cy)=cy'+2(cy)$

$=c(y'+2y)$

$L(y_1+y_2)=(y_1+y_2)'+2(y_1+y_2)$

$\begin{cases}=y_1'+2y_1\\=y_2'+2y_2\end{cases}$

∴ Equation is linear

$\boxed{2.1}$ HW #3 NewLe Goodman

13. $L(y) = y' + y^2$

$L(cy) = cy' + (cy)^2$

$= cy' + c^2y^2$

$\times = c(y_1 + cy^2)$

$L(y_1+y_2) = (y_1+y_2)' + (y_1+y_2)^2$

$= \cancel{y_1'} + \cancel{y_2'} + \cancel{y_1^2} + 2y_1y_2 + \cancel{y_2^2}$

×

Extra term

∴ Equation is nonlinear.

14. $L(y) = y' + 2ty$

$L(cy) = cy' + 2t(cy)$

$= c(y' + 2ty)$ ✓

$L(y_1+y_2) = (y_1+y_2)^3 + 2t(y_1+y_2)$

$\begin{cases} = y_1' + 2ty_1 ✓ \\ = y_2' + 2ty_2 ✓ \end{cases}$

∴ Equation is linear

15. $L(y) = y' - e^t y$

$L(cy) = cy' - e^t(cy)$

$= c(y' - e^t y)$ ✓

$L(y_1+y_2) = (y_1+y_2)' - e^t(y_1+y_2)$

$\begin{cases} = y_1' - e t y_1 \\ = y_2' - e t y_2 \end{cases}$ ✓

∴ Equation is linear.

16. $L(y) = y'' + (\sin t)y$

$L(cy) = cy'' + (\sin t)(cy)$

$= c(y'' + (\sin t)y)$ ✓

∴ Equation is linear

$L(y_1+y_2) = (y_1+y_2)'' + (\sin t)(y_1+y_2)$

$\begin{cases} = y_1'' + y_1 \sin t ✓ \\ = y_2'' + y_2 \sin t ✓ \end{cases}$

17. $L(y) = y'' + (1-y^2)y' + y$

$L(cy) = cy'' + (1-(cy)^2)(cy)' + cy$

$= cy'' + (1-c^2y^2)cy' + cy$

$= c(y'' + y'(1-c^2y^2) + y)$

×

∴ Equation not linear

24. $y' + P(t)y = 0$

$L(y_1+y_2) = (y_1+y_2)' + P(t)(y_1+y_2)$

Are equal $\begin{cases} = y_1' + P(t)y_1 \\ = y_2' + P(t)y_2 \end{cases}$

∴ $y_1 + y_2$ is a solution

$L(cy) = cy' + P(t)cy$

$= c(y' + P(t)y)$ ✓

∴ $cy_1(t)$ is a solution for any constant of c.

Part 2 | HW #3

1. a) $y' = y$ $y(t) = ce^t$

$ce^t = ce^t$ $y' = ce^t$

where $c \geq 0$

b) $y(t) = ce^t$

$\frac{\partial f}{\partial t} = ce^t$

∴ continuous

. $y' = -y$ $y(t) = ce^{-t}$

$-ce^{-t} = -ce^{-t}$ $y' = -ce^{-t}$

where $c < 0$

) $y(t) = ce^{-t}$

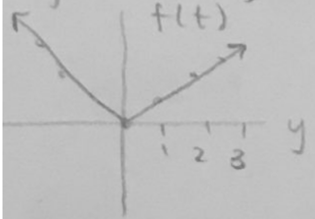
$\frac{\partial f}{\partial t} = -ce^{-t}$

∴ continuous

art 3

$y' = f(t,y) = |y| = \begin{cases} y & \text{if } y > 0 \\ -y & \text{if } y \leq 0 \end{cases}$; $y(0) = 0$

. yes, $f(t,y)$ is continuous for all values of t and y



← According to graph this represents all values of y and t.

$\frac{\partial f}{\partial t} |y| = 0$, meaning that for any value of y there will always be a solution.

$y' = |y|$

$(t) = ce^t$

$y'(t) = ce^t$

$ce^t = |ce^t|$

4. $f_y(t,y)$

$\frac{\partial f}{\partial y} ce^t = ce^t$

$\frac{\partial f}{\partial y} ce^{-t} = -ce^{-t}$

The region is always continuous.

5. $y(0) = 0$

$y(0) = ce^0$

$y(0) = c$ ← not always zero.

∴ a unique solution

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Part IV

HW #3

Nicole Goodman

1. $\vec{y} = [y'', y', y]$ $\quad L(\vec{y}) = 5y'' + (\cos t)y' + e^t y$.

$L(cy) = 5cy'' + (\cos t)(cy') + e^t(cy)$

✓ $= c(5y'' + (\cos t)y' + e^t y)$

$L(y_1 + y_2) = 5(y_1 + y_2)'' + \cos(t)(y_1 + y_2)' + e^t(y_1 + y_2)$

$= \underline{5y_1''} + 5y_2'' + \underline{y_1'\cos t} + y_2'\cos t + \underline{y_1 e^t} + y_2 e^t$

$= \begin{cases} 5y_1'' + y_1'\cos t + y_1 e^t \\ 5y_2'' + y_2'\cos t + y_2 e^t \end{cases}$ ✓

$\therefore$ Equation is linear