Part 1 | HW#14 Necole Goodman

• M is $n \times n$ matrix $M\vec{v} = 0$
$\vec{v}$ is a non-zero $\det(M) = 0$
M is not invertible $|M| = 0$

2. t → be the eigenvalue
v = non-zero eigenvector
then $Av = tv$
$(A - tI)v = 0$
$A - tI$ is no invertible
$\det(A - tI) = 0$

3. 0 is an eigenvalue hence, there is a nonzero eigenvector v so that
$Av = 0v = 0$
is not invertible

4. $M_1 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ M is triangular the eigenvalues are 2, 5, 5, 1

$\begin{bmatrix} -\lambda+2 & 0 & 0 & 0 \\ 0 & -\lambda+5 & 0 & 0 \\ 0 & 0 & -\lambda+5 & 0 \\ 0 & 0 & 0 & -\lambda+1 \end{bmatrix} = 0 \quad \lambda^4 - 13\lambda^3 + 57\lambda^2 - 95\lambda + 50 = 0$

$M_2 = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 0 & 5 & 9 & 4 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\lambda+2 & 1 & 3 & 2 \\ 0 & -\lambda+5 & 9 & 4 \\ 0 & 0 & -\lambda+5 & 6 \\ 0 & 0 & 0 & -\lambda+1 \end{bmatrix} = 0 \quad \lambda^4 - 13\lambda^3 + 57\lambda^2 - 95\lambda = 0$

$M_3 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 \\ 3 & 9 & 5 & 0 \\ 5 & 8 & 6 & 1 \end{bmatrix} \begin{bmatrix} -\lambda+2 & 0 & 0 & 0 \\ 1 & -\lambda+5 & 0 & 0 \\ 3 & 9 & -\lambda+5 & 0 \\ 5 & 8 & 6 & -\lambda+1 \end{bmatrix} = 0 \quad \therefore \lambda^4 - 13\lambda^3 + 57\lambda^2 - 95\lambda = 0$

b) Eigenvalues are 1, 2, 5, 5
$\lambda_1 = 1$

$I = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 0 & 4 & 9 & 4 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad V = \begin{bmatrix} -1/8 \\ -19/8 \\ 3/2 \\ 1 \end{bmatrix}$

$\approx \begin{bmatrix} 1 & 0 & 0 & -1/8 \\ 0 & 1 & 0 & -19/8 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = 0$

b) cont.

$\lambda = 2$

$$2I - m = \begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 9 & 4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} t$$

$\lambda = 5$

$$5I - m = \begin{bmatrix} 1 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = 0 \quad \therefore v = \begin{bmatrix} \frac{1}{3} \\ 1 \\ 0 \\ 0 \end{bmatrix} t$$

$\lambda = 2$

$$2I - m_3 = \begin{bmatrix} 1 & 0 & 0 & -3/7 \\ 0 & 1 & 0 & 1/7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = 0 \quad \therefore v = \begin{bmatrix} 3/7 \\ -1/7 \\ 0 \\ 1 \end{bmatrix} t$$

$\lambda = 5$

$$5I - m_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = 0 \quad \therefore v = \begin{bmatrix} 0 \\ 0 \\ 2/3 \\ 1 \end{bmatrix} t$$

c) 5, dimensions is 1

2, dimension is 1

1, dimensions is 1

5.

a) $y'' + py' + qy = 0$

$D^2y + pdy + qy = 0$

$m^2 + pm + q = 0 \quad m = -p \pm \sqrt{p^2 - 4q}$

$m = -b + \sqrt{p^2 - 4q}, \; -b - \sqrt{b^2 - 4q}$

b) $\bar{y} = \begin{bmatrix} u \\ n \end{bmatrix} = \begin{bmatrix} y \\ y_1 \end{bmatrix} \quad \bar{y} = Ay \quad y' = \frac{dy}{dt} \quad n = \frac{dv}{dt}$

$du + bu + qy = 0 \quad v' = bv - qu$

$A = \begin{bmatrix} 0 & Q \\ -q & -b \end{bmatrix}$

c) characteristic polynomial of A

$x^2 + bx + q$

d) Eigen value of A using $x^2 + bx + q = 0$

$= \underline{a = -b \pm \sqrt{b^2 - 4q}}$

e) Both the root and the eigen value are the same.

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Homework # 14

5.3

3. $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0 \qquad \begin{vmatrix} 1-\lambda & 2 \\ 1 & 2-\lambda \end{vmatrix} = 0$

$(1-\lambda)(2-\lambda) - 2(1) = 0$

$2 - \lambda - 2\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 3\lambda = 0$

$\lambda(\lambda - 3) = 0$

$\lambda = 0, 3$

$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0\begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k_1 \\ k_1 \end{bmatrix}$

$x + 2y = 0$

$x = -2y$

$\boxed{\lambda = 0 \rightarrow = k_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix}}$

$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 3\begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k_2 \\ k_2 \end{bmatrix}$

$\boxed{\lambda = 3 \Rightarrow = k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

4. $\begin{bmatrix} 3 & 4 \\ -5 & -5 \end{bmatrix}$

$\begin{bmatrix} 3 & 4 \\ -5 & -5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 3-\lambda & 4 \\ -5 & -5-\lambda \end{bmatrix}$

$\det\left(\begin{vmatrix} 3-\lambda & 4 \\ -5 & -5-\lambda \end{vmatrix}\right)$

$(3-\lambda)(-5-\lambda) + 20 = 0$

$-15 + 2\lambda + \lambda^2 + 20 = 0$

$\lambda^2 + 2\lambda + 5 = 0$

$(\lambda+1)^2 + 4 = 0$

$\sqrt{(\lambda+1)^2} = \sqrt{-4}$

$\lambda + 1 = \pm 2i$

$\lambda = -1 \pm 2i$

$\lambda_1 = \begin{bmatrix} 3-(-1+2i) & 4 \\ -5 & -5-(-1+2i) \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 4-2i & 4 \\ -5 & -4-2i \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$v_1 + \left(\frac{4}{5} + \frac{2}{5}i\right)v_2 = 0$

$v_2 = t$

$v_1 = \left[-\frac{4}{5} - \frac{2}{5}i\right]t$

$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \left(\frac{4}{5} - \frac{2}{5}t\right) + \begin{bmatrix} -\frac{4}{5} - \frac{2}{5}i \\ 1 \end{bmatrix}$

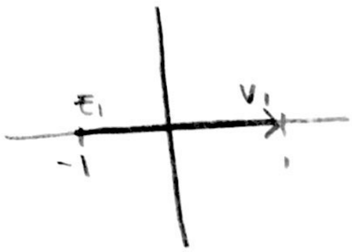
$\lambda_1 = -1 + 2i$ is $\begin{bmatrix} -\frac{4}{5} - \frac{2}{5}i \\ 1 \end{bmatrix}$

$\lambda_2 = -1 - 2i$ is $\begin{bmatrix} -\frac{4}{5} + \frac{2}{5}i \\ 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix}$

$(1-\lambda)(1-\lambda) = 0$

$\lambda = 1, 1$



$\lambda\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 1\begin{bmatrix} x \\ y \end{bmatrix}$

$x + y = x$

$y = 0$

The only value that satisfies $x+y=x$ is if $y=0$ $\therefore \bar{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

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17. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$v = \begin{bmatrix} -b \\ a-\lambda \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$

$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix}$

$(a-\lambda)(d-\lambda) - bc = 0$

$ad - a\lambda - d\lambda + \lambda^2 - bc = 0$

$\lambda^2 - (a+d)\lambda + (ad-bc) = 0$

$\begin{bmatrix} a-\lambda & b \\ 0 & d-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_1 \Rightarrow (a-\lambda)R_2 - cR_1$

$\begin{bmatrix} a-\lambda & b \\ 0 & (a-\lambda)(d-\lambda)-bc \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(a-\lambda)(d-\lambda) - bc = 0$

$\begin{bmatrix} a-\lambda & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(a-\lambda)x + by = 0$

$y = k$

$\therefore (a-\lambda)x = -bk$

$x = \frac{-bk}{a-\lambda}$

if $b = 0$, $x = 0$

$b \neq 0$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-bk}{a-\lambda} \end{bmatrix}$

$v = k\begin{bmatrix} \frac{-b}{a-\lambda} \\ 1 \end{bmatrix}$

$\boxed{\therefore \lambda \text{ is } v = \begin{bmatrix} -b \\ a-\lambda \end{bmatrix}}$

24. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 3 & 2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$

$\begin{bmatrix} 1-\lambda & 0 & 0 \\ -1 & 3-\lambda & 0 \\ 3 & 2 & 2-\lambda \end{bmatrix} = 0$

$(1-\lambda)(3-\lambda)(-2-\lambda) = 0$

$\lambda_1 = 1$

$\lambda_2 = 3$

$\lambda_3 = -2$

For λ_1 $\begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 3 & 2 & -3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 3 & 2 & -3 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 4 & 0 & -3 & 0 \\ -1 & 2 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$R_2 \Leftrightarrow R_3$ $\quad R_1 = R_1 - R_2$ $\quad R_1 \rightarrow \frac{1}{4}R_1$

$\left[\begin{array}{ccc|c} 1 & 0 & -3/4 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3/4 & 0 \\ 0 & 1 & -3/8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$R_2 = R_2 + R_1$

$R_2 = \frac{1}{2}R_2$

$x = \frac{3}{4}z$

$y = \frac{3}{8}z$

$v_1 = c\begin{bmatrix} 3/4 \\ 3/8 \\ 1 \end{bmatrix}$ c is a real #

For λ_2 $\left[\begin{array}{ccc|c} -2 & 0 & 0 & x \\ -1 & 0 & 0 & y \\ 3 & 2 & -5 & z \end{array}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x = 0$, $y = 2.5z$

$R_2 = R_2 - 3R_1$ $\quad R_2 = R_2 - 3R_1$

$\therefore v_2 = c\begin{bmatrix} 0 \\ 2.5 \\ 1 \end{bmatrix}$ c is a real #

$v_3 = c\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

$\boxed{E_1 = \text{span}\left\{\begin{bmatrix} 3/4 \\ 3/8 \\ 1 \end{bmatrix}\right\};\ E_2 = \text{span}\left\{\begin{bmatrix} 0 \\ 2.5 \\ 1 \end{bmatrix}\right\};\ E_3 = \text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}}$

- Each eigenspace has a single vector
- Eigen space is one-dimensional.

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6.1

1256 HW #14

1. $\bar{x}' = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} \bar{x}$

$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\boxed{\begin{aligned} x_1' &= x_1 + 2x^2 \\ x_2' &= 4x_1 - x_2 \end{aligned}}$

2. $\bar{x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\boxed{\begin{aligned} x_1' &= x_1 \\ x_2 &= -x_2 + 1 \end{aligned}}$

5.
$\bar{x} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \bar{v}, \quad \bar{u}(t) = \begin{bmatrix} e^{4t} \\ e^{4t} \end{bmatrix}, \quad v(t) = \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$

$u'(t) = \begin{bmatrix} 4e^{4t} \\ 4e^{4t} \end{bmatrix}, \quad v'(t) = \begin{bmatrix} -2e^{-2t} \\ 2e^{-2t} \end{bmatrix}$

$\bar{u} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \vec{u}$

$\begin{bmatrix} 4e^{4t} \\ 4e^{4t} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} e^{4t} \\ e^{4t} \end{bmatrix}$

$= \begin{bmatrix} e^{4t} + 3e^{4t} \\ 3e^{4t} + e^{4t} \end{bmatrix}$

$\begin{bmatrix} 4e^{4t} \\ 4e^{4t} \end{bmatrix} = \begin{bmatrix} 4e^{4t} \\ 4e^{4t} \end{bmatrix}$ ✓

$\bar{v} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \bar{v}$

$\begin{bmatrix} -2e^{-2t} \\ 2e^{2t} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$

$= \begin{bmatrix} e^{-2t} - 3e^{-2t} \\ 3e^{-2t} - e^{-2t} \end{bmatrix}$

$\begin{bmatrix} -2e^{-2t} \\ 2e^{-2t} \end{bmatrix} = \begin{bmatrix} -2e^{-2t} \\ 2e^{-2t} \end{bmatrix}$ ✓

$x(t) = \begin{bmatrix} e^{4t} & e^{-2t} \\ e^{4t} & -e^{-2t} \end{bmatrix}$

$\boxed{\therefore \bar{x}(t) = c_1 \begin{bmatrix} e^{4t} \\ e^{4t} \end{bmatrix} + c_2 \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}}$

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6.11

HW #11

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6. $\bar{x}' = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \bar{x}$, $\bar{u}(t) = \begin{bmatrix} e^{3t} \\ e^{3t} \end{bmatrix}$, $\bar{v}(t) = \begin{bmatrix} e^{2t} \\ 2e^{2t} \end{bmatrix}$

$\bar{u}'(t) = \begin{bmatrix} 3e^{3t} \\ 3e^{3t} \end{bmatrix}$, $\bar{v}'(t) = \begin{bmatrix} 2e^{2t} \\ 4e^{2t} \end{bmatrix}$

$u' = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \bar{u}$

$\begin{bmatrix} 3e^{3t} \\ 3e^{3t} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} e^{3t} \\ e^{3t} \end{bmatrix}$

$= \begin{bmatrix} 4e^{3t} - e^{3t} \\ 2e^{3t} + e^{3t} \end{bmatrix}$

$= \begin{bmatrix} 3e^{3t} \\ 3e^{3t} \end{bmatrix}$ ✓

$v' = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} v$

$= \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} e^{2t} \\ 2e^{2t} \end{bmatrix}$

$= \begin{bmatrix} 4e^{2t} - 2e^{2t} \\ 2e^{2t} + 2e^{2t} \end{bmatrix}$

$\begin{bmatrix} 2e^{2t} \\ 4e^{2t} \end{bmatrix} = \begin{bmatrix} 2e^{2t} \\ 4e^{2t} \end{bmatrix}$ ✓

$$\boxed{\therefore x(t) = c_1 \begin{bmatrix} e^{3t} \\ e^{3t} \end{bmatrix} + c_2 \begin{bmatrix} e^{2t} \\ 2e^{2t} \end{bmatrix}}$$