4.1 HOMEWORK #11 Nicole Goodman

3. $\ddot{x} + 9x = 0$

$x = ce^{rt}$

$\dot{x} = rce^{rt}$

$\ddot{x} = r^2ce^{rt}$

$r^2 ce^{rt} + 9ce^{rt} = 0$

$r^2 + 9 = 0$

$\sqrt{r^2} = \sqrt{-9}$

$r = \pm 3i$

$x = c_1 e^{3it} + c_2 e^{-3it}$

$1 = x(0) = c_1 e^{3it} + c_2 e^{-3it}$

$1 = c_1 + c_2$

$1 = \dot{x}(0) = 3c_1 e^{3t} - 3c_2 e^{-3t}$

$1 = 3c_1 - 3c_2$

$[1 = c_1 + c_2]3$

$1 = 3c_1 - 3c_2$

$3 = 3c_1 + 3c_2$

$1 = 3c_1 - 3c_2$

$4 = 6c_1$

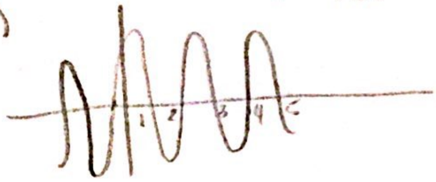
$\boxed{c_1 = \frac{2}{3}}$

$1 = \frac{2}{3} + c_2$

$\boxed{c_2 = \frac{1}{3}}$

12. $x(t) = \cos 3t + 5\sin 3t$

a)



b) amplitude ≈ 5.1

period $= \frac{2\pi}{b} = \pi$

Phase shift $[\delta = \tan^{-1}(\frac{c_2}{c_1})]$

22.

$\cos(3t - \frac{\pi}{6})$

$x(t) = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$

$c_1 = A\cos\delta$

$= \cos(\frac{\pi}{6})$

$= \frac{\sqrt{3}}{2}$

$c_2 = A\sin\delta$

$= \sin(\frac{\pi}{6})$

$= \frac{1}{2}$

$\boxed{x(t) = \frac{\sqrt{3}}{2}\cos 3t + \frac{1}{2}\sin 3t}$

25. $\ddot{x} + 9x = 0$, $x(0) = 1$, $\dot{x}(0) = 1$ (c_1 + c_2 found above)↑

$A = \sqrt{c_1^2 + c_2^2}$

$= \sqrt{(1)^2 + (\frac{1}{3})^2}$

$\boxed{A = \frac{\sqrt{10}}{3}}$

$\tan\delta = \frac{c_2}{c_1}$

$\delta = \tan^{-1}(\frac{c_2}{c_1})$

$= \tan^{-1}(\frac{1}{3})$

$\boxed{\delta \approx 0.322}$

$T = \frac{2\pi}{\omega_0}$

$\boxed{T = \frac{2\pi}{3}}$

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21. $y'' - y' = 0$, $y(0) = 2$, $y'(0) = -1$

$r^2 \cancel{ce^{rt}} - r\cancel{ce^{rt}} = 0$

$r^2 - r = 0$

$r(r-1) = 0$

$r = 0, 1$

$y = c_1 e^0 + c_2 e^t$

$2 = y(0) = c_1 + c_2$

$c_1 + c_2 = 2$

$y' = c_2 e^t$

$c_2 = -1$

$c_1 - 1 = 2$

$c_1 = 3$

$\boxed{\therefore y = 3 - e^t}$

$\Delta = b^2 - 4ac$
$= (-1)^2 - \cancel{4(1)(0)}$
$= 1$

22. $y'' - 4y' - 12y = 0$, $y(0) = 1$, $y'(0) = -1$

$r^2 \cancel{ce^{rt}} - 4r\cancel{ce^{rt}} - 12\cancel{ce^{rt}} = 0$

$r^2 - 4r - 12 = 0$

$(r+2)(r-6) = 0$

$r = -2, 6$

$y = c_1 e^{-2t} + c_2 ce^{6t}$

$1 = c_1 + c_2$

$y' = -2c_1 e^{-2t} + 6c_2 e^{6t}$

$-1 = -2c_1 + 6c_2$

$[1 = c_1 + c_2] \cdot 2$

$-1 = \cancel{-2c_1} + 6c_2$

$2 = \cancel{2c_1} + 2c_2$

$1 = 8c_2$

$c_2 = \frac{1}{8}$

$\Delta = b^2 - 4ac$
$= (-4)^2 - 4(1)(-12)$
$= 16 + 48$
$= 64$

$1 = c_1 + \frac{1}{8}$

$c_1 = \frac{7}{8}$

$\therefore \boxed{y = \frac{7}{8}e^{-2t} + \frac{1}{8}e^{6t}}$

23. $y'' - 4y' = 0$

$r^2 \cancel{ce^{rt}} - 4r\cancel{ce^{rt}} = 0$

$r^2 - 4r = 0$

$r(r-4) = 0$

$r = 0, 4$

$y = c_1 e^0 + c_2 e^{4t}$

$\boxed{\therefore \text{Basis is } \{1, e^{4t}\}}$

24. $y'' - 10y' + 25y = 0$

$r^2 \cancel{ce^{rt}} - 10r\cancel{ce^{rt}} + 25\cancel{ce^{rt}} = 0$

$r^2 - 10r + 25 = 0$

$(r-5)(r-5) = 0$

$r = 5$

$y = c_1 e^{5t} + c_2 te^{5t}$

$$\boxed{\begin{array}{l} B = \{e^{5t}, te^{5t}\} \\ \{y \mid y = c_1 e^{5t} + c_2 te^{5t} : c_1, c_2 \in \mathbb{R}\} \end{array}}$$

26. $y'' + 2\sqrt{2}\, y' + 2y = 0$

$r^2 \cancel{ce^{rt}} + 2\sqrt{2}\, r\cancel{ce^{rt}} + 2\cancel{ce^{rt}} = 0$

$r^2 + 2\sqrt{2}\, r + 2 = 0$

$x = \frac{-2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4(1)(2)}}{2(1)}$

$= \frac{-8\sqrt{2} \pm \sqrt{0}}{2}$

$x = -\sqrt{2}$

$y = c_1 e^{-\sqrt{2}t} + c_2 e^{-\sqrt{2}t}$

$$\boxed{\begin{array}{l} B = \{e^{-\sqrt{2}t}, te^{-\sqrt{2}t}\} \\ \{y \mid y = c_1 e^{\sqrt{2}t} + t c_2 e^{-\sqrt{2}t} : c_1, c_2 \in \mathbb{R}\} \end{array}}$$

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14. $ay'' + by' + cy = 0$

$\Delta = b^2 - 4ac$

$\therefore r_1, r_2 = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\boxed{r_1, r_2 = \dfrac{-b \pm \sqrt{\Delta}}{2a}}$

$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$0 = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$y' = r_1 c_1 e^{r_1 t} + r_2 c_2 e^{r_2 t} = 0$

$$\left[\begin{array}{cc|c} e^{r_1 t} & e^{r_2 t} & 0 \\ r_1 e^{r_1 t} & r_2 e^{r_2 t} & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \therefore \boxed{\begin{array}{l} c_1 = 0 \\ c_2 = 0 \end{array}}$$

RREF will prove linear independence

47. $x(t) = c_1 e^{-\left(\frac{b}{2a}\right)t} + c_2 t e^{-\left(\frac{b}{2a}\right)t}$ $\quad \frac{b}{2a} > 0$

$\lim_{t\to\infty} x(t) = c_1 \lim_{t\to\infty} e^{-(b/2a)t} + c_2 \lim_{t\to\infty} t e^{-\left(\frac{b}{2a}\right)t}$

$= c_1 \cdot 0 + c_2 \lim_{t\to\infty} t e^{-\left(\frac{b}{2a}\right)t}$

$= c_2 \lim_{t\to\infty} t e^{-\left(\frac{b}{2a}\right)t}$

$= c_2 \lim_{t\to\infty} \dfrac{t}{e^{(b/2a)t}}$

$\lim_{t\to\infty} x(t) = c_2 \lim_{t\to\infty} \dfrac{1}{(b/2a)e^{(b/2a)t}} \to 0$

$= c_2 \cdot 0$

$\boxed{\lim_{t\to\infty} x(t) = 0}$

Part II

HOMEWORK #11

1. $A = \sqrt{C_1^2 + C_2^2}$ $\tan\delta = \frac{C_2}{C_1}$

$A = \frac{C_1}{\cos\delta}$

$C_1 \tan\delta = C_2$

$C_1 = \frac{C_2}{\tan\delta}$

$A = \sqrt{\left(\frac{C_2}{\tan\delta}\right)^2 + (C_1 \tan\delta)^2}$

2. $\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$A\cos(\omega_0 t - \delta) = C_1 \cos\omega_0 t + C_2 \sin\omega_0 t$

Part III

$y'' + p(t)y + q(t)y = 0$