1.1

HW #2

Necole Goodman

11.

a) $y_0 = 0.9$ billion

$y_1 = 1.03(0.9) = 0.927$

$y_2 = (1.03)^2(0.9) = 0.956$

$y_3 = (1.03)^3(0.9) = 0.983$

$y_4 = (1.03)^4(0.9) = 1.023$

b) $t = 192$ in 1900

$y_{192} = (1.03)^{192}(0.9) = 262$ billion

c) The continuous model gives a larger value than the discrete model.

6. $y' = 2y + 1 - 2t^2$, $y(0) = 2$; $y = t + t^2 + 2e^{2t}$ $y' = 1 + 2t + 4e^{2t}$

$y' = 2(t + t^2 + 2e^{2t}) + 1 - 2t^2$

$= 2t + \cancel{2t^2} + 4e^{2t} + 1 \cancel{-2t^2}$

$\boxed{y' = 2t + 4e^{2t} + 1 \checkmark}$

$y = t + t^2 + 2e^{2t}$

$y(0) = 0 + 0 + 2e^{2(0)}$

$\boxed{y = 2 \checkmark}$

7. $y = ce^{t^2}$ $y' = 2ty$; $y(0) = 2$

$y' = 2tce^{t^2}$ $2cte^{t^2} = 2t(ce^{t^2})$ ✓

$y = ce^{0^2}$

$y = c$ $\therefore \boxed{c = 2}$

12. $y' = t - y$

$y = t - 1$ → Equation based on the diagram given.

$y' = 1$

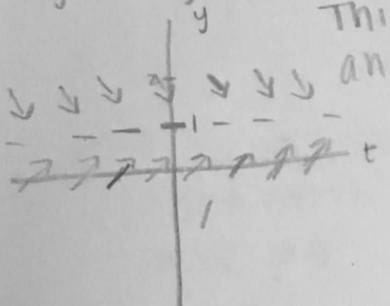
$1 + (y = t - 1 + 1$

$t = 1 + y$

$1 = t - y$ ✓

$y' = t - y$

13. $y' = 1 - y$



This graph is stable and reaches its equilibrium at $y = 1$

$(0,1) \Rightarrow y' = 0$

$(1,2) \Rightarrow y' = -1$

$(-1,1) \Rightarrow y' = 0$

$(2,0) \Rightarrow y' = 1$

$(-2,2) \Rightarrow y' = -1$

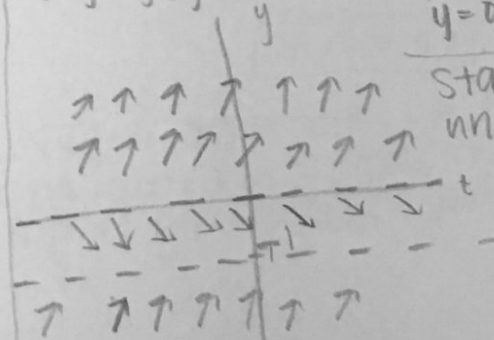
$(-2,0) \Rightarrow y' = 1$

14. $y' = y(y+1)$

Equilibrium at $y = 0$ and $y = -1$

Stable @ $y = -1$

unstable @ $y = 0$



$(0,1) \Rightarrow y' = 2$ $(1,-0.5) \Rightarrow y' = -0.75$

$(0,0) \Rightarrow y' = 0$

$(1,2) \Rightarrow y' = 6$

$(1,0) \Rightarrow y' = 0$

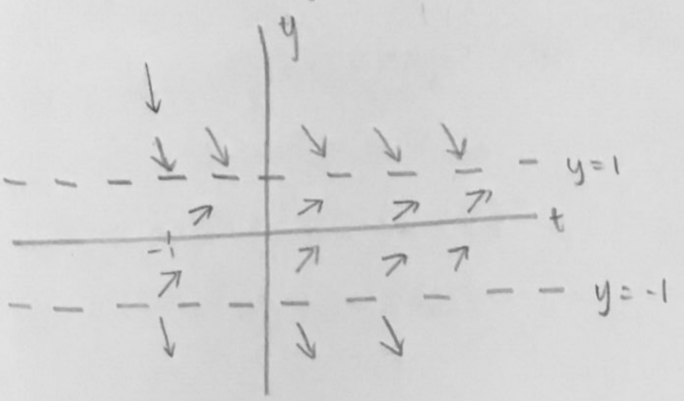
$(-1,1) \Rightarrow y' = 2$

$(-1,-4) \Rightarrow y' = +12$

$(-2,-1) \Rightarrow y' = 0$

HW #2 Necole

15. $y' = t^2(1-y^2)$



$(1, 1) \Rightarrow y' = 0$
$(1, 0.5) \Rightarrow y' = 0.5$
$(1, 2) \Rightarrow y' = -3$
$(-1, +3) \Rightarrow y' = -8$
$(-1, -2) \Rightarrow y' = -3$
$(-1, -0.5) \Rightarrow y' = +0.75$
$(1, -0.5) \rightarrow y' = 0.75$

equilibrium @ $y = 1$ and $y = -1$

stable @ $y = 1$
unstable @ $y = -1$

16. $y' = 1$, $\boxed{\text{option C}}$

I chose this option because y' is the slope and for every value of y and t there will be a positive slope in direction 1.

17. $y' = y$ $\boxed{\text{option D}}$

Option D because for any positive value of y it will lie above the t-axis, and any negative y-value will lie below the axis,

18. $y' = \frac{y}{t}$ $\boxed{\text{option F}}$

option F, because slope is going to be positive in the first and third quadrant and the second and fourth quadrants will have a negative slope.

20. $y' = t^2 + y^2$ $\boxed{\text{option E}}$

when y and $t = 0$ the slope is zero. All the other values of t and y their slopes are increasing going away from the origin. T and y approaching origin, slope is approaching zero.

1.3	Homework #2	Newele Goodman

4. $y' = \ln(ty)$

$\frac{dy}{dt} = \ln(ty)$

$dy \neq \ln(ty)\,dt$

$\boxed{\text{not seperable}}$

12. $ty' = \sqrt{1-y^2}$

$y' = \frac{\sqrt{1-y^2}}{t}$

$\frac{dy}{dt} = \frac{\sqrt{1-y^2}}{t}$

$t\,dy = \sqrt{1-y^2}\,dt$

$\frac{1}{\sqrt{1-y^2}}\,dy = \frac{1}{t}\,dt$

$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int \frac{1}{t}\,dt$

$= \ln t + C$

16. $4t\,dy = (y^2 + ty^2)\,dt$

$4t\,dy = y^2(1+t)\,dt$

$\frac{1}{y^2}\,dy = \frac{(1+t)}{4t}\,dt$

$\int \frac{1}{y^2}\,dy = \int \frac{1+t}{4t}\,dt$

$\int \frac{1}{y^2}\,dy = \int \frac{1}{4t} + \int \frac{1}{4}\,dt$

$\int \frac{1}{y^2}\,dy = \frac{1}{4}\int \frac{1}{t}\,dt + \frac{1}{4}t + C$

$-\frac{1}{y} = \frac{1}{4}\ln|t| + \frac{1}{4}t + C$

$-\frac{1}{1} = \frac{1}{4}\cancel{\ln|1|}^{\,0} + \frac{1}{4}(1) + C$

$1 = -4 + C$

$\boxed{5 = C}$

20. $y' = -\frac{1+y^2}{1+t^2}$

$\frac{dy}{dt} = -\frac{1+y^2}{1+t^2}$

$(1+t^2)\,dy = -1+y^2\,dt$

$-\frac{1}{1+y^2}\,dy = \frac{1}{1+t^2}\,dt$

$\int -\frac{1}{1+y^2}\,dy = \int \frac{1}{1+t^2}\,dt$

$\arctan(y) = \arctan(t) + C$

$\arctan(y) - \arctan(t) = C$

$\left[\arctan\left(\frac{y+t}{1-yt}\right) = C\right] \tan$

$\frac{y+t}{1-yt} = \tan(C) \qquad (-1, 0)$

$\frac{-1+0}{1-(-1)(0)} = C$

$\frac{-1}{1} = C$

$\boxed{C = -1}$

$\therefore \boxed{y = \frac{1+t}{t-1}}$

Part 2

1) $y(t) = \frac{1}{t}\,dt$

2) $y(t) = \cos t$

$y' = -\sin t$

$y'' = -\cos t$

$\boxed{y'' = -y}$

3) $y = t^2$

$y' = 2t$

$\boxed{\left(\frac{y'}{2}\right)^2 = y}$ ← make a function that is a true statement

4) $y = e^t$

$y' = e^t$

$y'' = e^t$

$\boxed{y'y'' = y^2}$ or $y\,y'\,y'' = e^{3t}$