Homework #1 Math 024 Necole Goodman

1. $\int x\cos x\,dx$

$u = x$ $dv = \cos x$
$du = dx$ $v = \sin x$

LIATE
Logarithmic
Inverse
Algebraic
Trig
Exponent

$= x\sin x - \int \sin x\,dx$

$\boxed{= x\sin x + \cos x + C}$

2. $\int t^2 e^{-t}\,dt$

$u = t^2$ $dv = e^{-t}$
$du = 2t\,dt$ $v = -e^{-t}$

$= (t^2)(-e^{-t}) + 2\int te^{-t}\,dt$

$u = t$ $dv = e^{-t}$
$du = dt$ $v = -e^{-t}$

$= (t^2)(-e^{-t}) - 2\{te^{-t} + \int e^{-t}\}$

$\boxed{= -t^2e^{-t} - 2te^{-t} - 2e^{-t} + C}$

3. $\int \ln t\,dt$

$u = \ln t$ $dv = 1$
$du = \frac{1}{t}\,dt$ $v = t$

$= t\ln t - \int t\left(\frac{1}{t}\right)dt$

$\boxed{= t\ln t - t + C}$

4. $\int \tan^{-1} x\,dx$

$u = \tan^{-1} x$ $dv = 1\,dt$
$du = \frac{1}{x^2+1}$ $v = x$

$= x\arctan x - \int \frac{x}{x^2+1}\,dx$

$u = x^2+1$ $du = 2x$
$\frac{1}{2}du = x\,dx$

$= x\arctan x - \frac{1}{2}\int \frac{1}{u}\,du$

$= x\arctan x - \frac{1}{2}\ln(u)$

$\boxed{= x\arctan x - \frac{1}{2}\ln(x^2+1) + C}$

5. $\int x^3 e^{x^2}\,dx$

$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}u\,du = dx$

$n = u$ $dv = e^u$
$dn = du$ $v = e^u$

$= \int ue^u\,du$

$= ue^u - \int e^u\,du$

$= ue^u - e^u + C$

$\boxed{= \frac{1}{2}(x^2e^{x^2} - e^{x^2}) + C}$

6. $\int \frac{x}{x^2+1}\,dx$ $\quad u = x^2+1 \quad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,dx$

$= \frac{1}{2}\ln|u| + C$

$\boxed{= \frac{1}{2}\ln|x^2+1| + C}$

7. $\int \frac{z-1}{z+2}\,dz$ $\quad u = z+2 \quad du = 1dz$

$z = u-2$

$= \int \frac{z-1}{u}\,du$

$= \int \frac{u-2-1}{u}$

$= \int \frac{u-3}{u}\,du$

$= \int 1 - \frac{3}{u}$

$= u - 3\ln|u|$

$\boxed{= (z+2) - 3\ln(z+2) + C}$

8. $\int \frac{x+2}{(x-1)(x+3)}\,dx$

$x+2 = \frac{A}{x-1} + \frac{B}{x+3}\left(\frac{(x-1)(x+3)}{1}\right)$

$x+2 = A(x+3) + B(x-1)$

$x+2 = Ax+3A+Bx-B$

$x+2 = (A+B)x + 3A - B$

$A+B=1 \quad 3A-B=2$

$A+3A-2=1 \quad B=3A-2$

$4a=3 \quad B = 3\left(\frac{3}{4}\right)-2$

$\boxed{A=\frac{3}{4}} \quad B = \frac{9}{4} - \frac{8}{4}$

$\boxed{B = \frac{1}{4}}$

$\int \frac{x+2}{(x-1)(x+3)}\,dx$ $\quad u = x-1 \quad du = 1dx \quad u = x+3 \quad du = 1dx$

$= \frac{1}{4}\int\left(\frac{3}{x-1} + \frac{5}{x+3}\right)$

$\boxed{= \frac{1}{4}\left(3\ln|x-1| + \ln|x+3|\right) + C}$

HW#1 cont.

9. $\int \frac{2v+1}{v(v^2+4)}\,dv$

$\frac{2v+1}{v(v^2+4)} = \left(\frac{A}{v} + \frac{Bx+C}{v^2+4}\right)\left(\frac{v(v^2+4)}{1}\right)$

$0v^2+2v+1 = A(v^2+4) + (Bv+C)(v)$

$0v^2+2v+1 = Av^2+4A+Bv^2+Cv$

$= (A+B)v^2 + 4A + Cv$

$A+B=0$ $\boxed{C=2}$ $4A=1$

$\frac{1}{4}+B=0$ $\boxed{A=\frac{1}{4}}$

$\boxed{B=\frac{-1}{4}}$

$\int \frac{2v+1}{v(v^2+4)}$

$= \int \frac{\frac{1}{4}}{v} + \frac{-\frac{1}{4}v+2}{v^2+4}\,dv$

$u=v^2+4$

$du=2v\,dv$

$\frac{1}{2}du = v\,dv$

$= \frac{1}{4}\ln|v| - \frac{\frac{1}{4}}{4}du + \frac{2}{u}$

$= \boxed{\frac{1}{4}\ln|v| - \frac{1}{4}\ln|v^2+4| + \arctan\left(\frac{2}{v}\right) + C}$

10. $\int \frac{x^2+5}{(x-1)(x+4)}\,dx$

$x^2+5 = \frac{A}{x-1} + \frac{B}{x+4}\left(\frac{(x-1)(x+4)}{1}\right)$

$x^2+0x+5 = A(x+4) + B(x-1)$

$-4^2+5 = B(-5)$ $A(5)=6$

$-5B = 16+5$ $A=\frac{6}{5}$

$B=\frac{-21}{5}$

$\int \frac{x^2+5}{(x-1)(x+4)}\,dx$

$u=x-1$ $u=x+4$

$du=dx$ $du=dx$

$= \boxed{\frac{6}{5}\ln|x-1| - \frac{21}{5}\ln|x+4| + C}$

11. $\int \frac{x+3}{2x-1}\,dx$

$u=x+3$ $du=1\,dx$

$x=u-3$

$= \int \frac{u}{2(u-3)-1}$

$= \int \frac{u}{2u-7}$

$u=2x-1$

$\frac{du}{2} = \frac{2}{2}dx$

$\frac{du}{2} = dx$

$\frac{1}{2}\int \frac{u+3}{u}\,du$

12. $\int \frac{2y+3}{y^2+3y+4}\,dy$ $\quad u = y^2+3y+4$

$du = 2y+3$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$\boxed{= \ln|y^2+3y+4| + C}$

13. $\int \frac{3x+2}{x(x+1)^2}\,dx$

$3x+2 = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\left(\frac{x(x+1)}{1}\right)$

$0x^2+3x+2 = A(x+1) + Bx + C(x+1)x$

$= Cx^2 + (A+B+C)x + A$

$C = 0 \qquad A = 2 \qquad A+B+C = 3$

$2 + B + 0 = 3$

$B = 1$

14. $\int \frac{1}{\sqrt{4-y^2}}\,dy$

$y = a\sin\theta$

$y = 2\sin\theta$

$dy = 2\cos\theta\,d\theta$

$\theta = \arcsin\left(\frac{y}{2}\right)$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1-\sin^2\theta$

$= \int \frac{1}{\sqrt{4-4\sin^2\theta}}(2\cos\theta)$

$= \int \frac{2\cos\theta}{\sqrt{4(1-\sin^2\theta)}}$

$= \int \frac{\cancel{2}\cos\theta}{\cancel{2}\sqrt{\cos^2\theta}}\,d\theta$

$\int \frac{\cos\theta}{\cos\theta}\,d\theta$

$= \int d\theta$

$= \theta + C$

$\boxed{= \arcsin\left(\frac{y}{2}\right) + C}$

HW #1 cont

15. $\int \frac{1}{u^2+2u+2}\, du$

$\int \frac{1}{(u^2+2u+1)+2-1}$

$= \int \frac{1}{(u+1)^2+1}$ $\quad u = u+1$
$\quad du = 1du$

$\int \frac{1}{u^2+1}$

$= \arctan(u) + C$

$\boxed{= \arctan(u^2+1) + C}$

16. $\int \frac{1}{x \ln x}\, dx$ $\quad u = \ln x$
$\quad du = \frac{1}{x}\, dx$

$\int \frac{1}{u}\, du$

$= \ln|u| + C$

$\boxed{= \ln|\ln x| + C}$

17. $\int \tan t\, dt$

$= \int \frac{\sin t}{\cos t}\, dt$ $\quad u = \cos t \quad du = -\sin t\, dt$

$= \int -\frac{1}{u}\, du$

$= -\ln|u| + C$

$\boxed{= -\ln|\cos x| + C}$

18. $\int \frac{x+1}{x^2-x-6}\, dx$

$= \int \frac{x+1}{(x-3)(x+2)}\, dx$

$x+1 = \frac{A}{x-3} + \frac{B}{x+2} \left(\frac{(x-3)(x+2)}{1} \right)$

$x+1 = A(x+2) + B(x-3)$

$x+1 = (A+B)x + 2A - 3B$

$A+B = 1 \qquad 2A-3B = 1$

$B = 1-A \qquad 2A-3(1-A) = 1$

$B = 1-\frac{4}{5} \qquad 2A-3+3A = 1$

$5A = 4$

$\boxed{B = \frac{1}{5}} \qquad \boxed{A = \frac{4}{5}}$

$\int \frac{x+1}{x^2-x-6}\, dx$

$= \int \frac{\frac{4}{5}}{x-3} + \frac{\frac{1}{5}}{x+2}$ $\quad u = x-3 \quad du = dx \qquad u = x+2 \quad du = dx$

$\boxed{= \frac{4}{5}\ln|x-3| + \frac{1}{5}\ln|x+2| + C}$

HW #1 cont.

19. $\int \cos^2 x \, dx$

$\cos^2 x = \frac{1+\cos(2x)}{2}$

$= \int \frac{1+\cos(2x)}{2}$

$= \frac{1}{2}\int 1+\cos(2x)$

$= \frac{1}{2}\int 1\,dx + \int \cos(2x)\,dx$

$u = 2x \quad du = 2\,dx$

$\frac{1}{2}du = dx$

$= \frac{1}{2}[x + \frac{1}{2}\int \cos(u)\,du$

$= \frac{1}{2}[x + \frac{1}{2}\sin u]$

$\boxed{= \frac{1}{2}[x + \frac{1}{2}\sin(2x)] + C}$

20. $\int \sqrt{1-x^2}\,dx$

$x = 1\sin\theta$

$dx = \cos\theta\,d\theta$

$\theta = \arcsin x$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$= \int \sqrt{1-\sin^2\theta}\,(\cos\theta\,d\theta)$

$= \int \sqrt{\cos^2\theta}\,(\cos\theta\,d\theta)$

$= \int \cos\theta\,(\cos\theta)\,d\theta$

$= \int \cos^2\theta\,d\theta$

$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$= \frac{1}{2}\int 1+\cos(2\theta)$

$= \frac{1}{2}\int 1\,d\theta + \int \cos(2\theta)\,d\theta$

$u = 2\theta \quad du = 2\,d\theta$

$\frac{1}{2}du = d\theta$

$= \frac{1}{2}\int 1\,d\theta + \frac{1}{2}\int \cos u\,du$

$= \frac{1}{2}[\theta + \frac{1}{2}\sin u] + C$

$= \frac{1}{2}[\theta + \frac{1}{2}\sin(2\theta)] + C$

$\boxed{\frac{1}{2}[\arcsin x + \frac{1}{2}\sin(2\arcsin x)] + C}$

HW #1 cont

21. $\int e^{3x}\sin 2x\,dx$

$u = e^{3x}$ $dv = \sin(2x)\,dx$

$du = 3e^{3x}$ $v = -\frac{1}{2}\cos(2x)$

LIATE

$u = 2x$ $du = 2dx$

$\frac{1}{2}du = dx$

$= -\frac{1}{2}e^{3x}\cos(2x) - \int -\frac{3}{2}e^{3x}\cos(2x)\,dx$

$u = e^{3x}$ $dv = \cos(2x)$

$du = 3e^{3x}$ $v = \frac{1}{2}\sin(2x)$

$= -\frac{1}{2}e^{3x}\cos(2x) + \frac{3}{2}\left[\frac{1}{2}e^{3x}\sin(2x) - \frac{3}{2}\int e^{3x}\sin(2x)\right]$

$\int e^{3x}\sin 2x = -\frac{1}{2}e^{3x}\cos(2x) + \frac{3}{2}\left[\frac{1}{2}e^{3x}\sin(2x)\right] - \frac{3}{2}\int e^{3x}\sin(2x)]$

$= \frac{2\int e^{3x}\sin(2x)}{2} = \frac{-\frac{1}{2}e^{3x}\cos(2x) + \frac{3}{2}\left[\frac{1}{2}e^{3x}\sin(2x)\right]}{2}$

$\int e^{3x}\sin(2x) = \boxed{-\frac{1}{13}e^{3x}(2\cos(2x) - 3\sin(2x)) + C}$

22. $\int e^{y}\sin^2 y\,dy$

$u = e^{y}$

$du = \ln y\,dy$

$y = \ln(u)$

$= \int \sin^2(\ln u)\,du$

$= \int \frac{1-\cos(2\ln u)}{2}\,du$

$-\frac{e^{x}(2\sin(2x) + \cos(2x)) - 5}{10} + C$

$= \frac{1}{2}\int 1 - \cos(2\ln u)\,du$

$= \frac{1}{2}\int 1\,du - \int \cos(2\ln u)\,du$

$u = \ln(u)$

$du = e^{u}du$

$= \frac{1}{2}[u] - [e^{u}\frac{1}{2}\sin(2u) - \int e^{u}(\frac{1}{2}\sin 2u)]$

$u = e^{u}$ $dv = \cos(2u)$

$= \frac{1}{2}[u] - e^{u}\frac{1}{2}\sin(2u) - \frac{1}{2}[e^{u}(-\frac{1}{2}\cos 2u)$

$du = e^{u}$ $v = \frac{1}{2}\sin(2u)$

$- \int e^{u}(-\frac{1}{2}\cos(2u))\,du)$

$u = e^{u}$ $dv = \sin 2u$

$du = e^{u}$ $v = -\frac{1}{2}\cos(2u)$

$= \frac{1}{2}u - e^{u}(\frac{1}{2}\sin(2u)) - \frac{1}{2}(-\frac{1}{2}e^{u}\cos(2u) - \int -\frac{1}{2}e^{u}\cos(2u)\,du)$

$= \frac{1}{5}e^{u}(2\sin(2u) + \cos(2u))$

$= \frac{1}{5}e^{\ln u}(2\sin(2\ln(u)) + \cos(2\ln(u)))$

$= \frac{1}{5}u(2\sin(2\ln u) + \cos(2\ln u))$

$= \frac{1}{2}(u - \frac{1}{5}u(2\sin(2\ln u)) + \cos(2\ln(u)))$

$= \frac{1}{2}(e^{y} - \frac{1}{5}e^{y}(2\sin(2y) + \cos(2y)))$

$\boxed{= \frac{1}{2}(e^{y} - \frac{1}{5}e^{y}(2\sin(2y) + \cos(2y))) + C}$

HW #1 Part 2

1. $2y' + 2y = 4$

   $y' + 5y = 16$

2. $y'' + \sin y = e^t$

   $y'' = 2t - y^2 y'$

3. $3ty' + 2\cos y = 1$

   $y' = y^2$

4. $y'' = -2t(y')^2$

   $y'' = \frac{4}{3} y^3 y'$

5. $y''' + t^5 y'' - \cos t y = 0$

   $2y''' + \cos y' = e^y$

6. $y'''' \sin t + e^t y'' = 3$

   $y'''' = 16e^y + ty'''$