2.4

HOMEWORK #5 Necole Goodman

6.

salt = 1lb / gal
water = 300 gal
$r_i = 3$ gal / min
$r_o = 1$ gal / min

$\frac{dx}{dt}$ = RATE IN − RATE OUT

$x' = [(1\ lb/gal)(3\ gal/min)] - [\frac{x}{300(3-1)t} \frac{lb}{gal}(1\ gal/min)]$

$x' = 3\ lb/min - \frac{x}{300(3-1)t}$

$x' + \frac{x}{300(3-1)t} = 3$

$\mu = e^{\int \frac{1}{300-2t}} = e^{\frac{1}{2}\ln|300+2t|} = e^{\ln|300+2t|^{1/2}} = \sqrt{300+2t}$

$\int \frac{d}{dt}(\sqrt{300+2t}\ x) = \int 3\sqrt{300+2t}$

$\sqrt{300+2t}\ x = (300+2t)^{3/2} + C$

$y = \frac{(300+2t)^{3/2}}{(300+2t)^{1/2}} + \frac{C}{(300+2t)^{1/2}}$

$y(t) = 300 + 2t + C(300+2t)^{-1/2}$

$y(0) = 300 + 2(0) + C(300)^{-1/2} = 0$ $y(0) = 0$

$C(300)^{-1/2} = -300 \rightarrow C = -300(300)^{1/2}$

$y(t) = 300 + 2t - 300(300)^{1/2}(300+2t)^{-1/2}$

$\therefore$ 600 gal = 300gal + 2t

300gal = 2t t = 150 min

$y(150) = 300 + 2(150) - 300(300)^{1/2}(300+2(150))^{-1/2}$

$\boxed{y(150) = 387.86\ gal}$

12. V_1 -

$V_1 = 200$ gal
$V_2 = 200$ gal
$V_3 = 500$ gal
salt = 2 lb
$r_{1,0} = 5$ gal/sec
$r_3 = 10$ gal/sec

a) $\frac{dx}{dt}$ = RATE IN − RATE OUT

$= [(0\ \frac{lb}{gal})(5\ gal/sec)] - [\frac{x}{200}\ lb/gal\ (5\ gal/sec)]$

Tank 1	Tank 2
$\frac{dx}{dt} = -\frac{1}{40}x$	$\frac{dy}{dt} = -\frac{1}{40}y$
$x(0) = 20$	$y(0) = 20$
b) $x(t) = Ce^{-t/40}$	$y(t) = Ce^{-t/40}$
$20 = Ce^{-\cancel{0/40}}$	$20 = Ce^{-\cancel{0/40}}$
$C = 20$	$C = 20$
$x(t) = 20e^{-t/40}$	$y(t) = 20e^{-t/40}$

c) RATE IN = RATE OUT_1 + RATE OUT_2

$= [(1 - \frac{x}{40}\ lb/sec) - (\frac{y}{40}\ lb/sec)]$

RATE $OUT_3 = (\frac{z}{50})(10\ gal/sec)$

cont. on rear.

2.4.1

HOMEWORK 5

12. cont.

$$\frac{dz}{dt} = \frac{x}{40} + \frac{y}{40} - \frac{1}{50}z$$
$$= \frac{1}{2}e^{-t/40} + \frac{1}{2}e^{-t/40} - \frac{1}{50}z$$
$$\frac{dz}{dt} = e^{-t/40} - \frac{1}{50}z$$
$$\frac{dz}{dt} + \frac{1}{50}z = e^{-t/40}$$
$$M = e^{\int \frac{1}{50}dt}$$
$$= e^{1/50t}$$
$$\int \frac{d}{dt}\left(e^{\frac{1}{50}t} z\right) = \int e^{-1/40t}\left(e^{1/50}\right) \Rightarrow$$
$$e^{-t/50}\left[ze^{\frac{1}{50}t} = -200e^{-1/200t} + C\right]$$
$$\boxed{z = -200e^{-t/40} + Ce^{-t/50}}$$

16.

$M = 10$
$T = 70$
$t = 0.5$
$T(1) = 50$

a) $$T(t) = T_0 e^{-kt} + M(1 - e^{-kt})$$
$$= 70e^{-kt} + 10(1 - e^{-kt})$$
$$T(1) = 70e^{-k(0.5)} + 10(1 - e^{-k(0.5)})$$
$$50 = 70e^{-0.5k} + 10 - 10e^{-0.5k}$$
$$-10 \qquad\qquad -10$$
$$40 = 60e^{-0.5k}$$
$$\ln\frac{2}{3} = \ln e^{-0.5k}$$
$$\ln\left(\frac{2}{3}\right) = -0.5k$$
$$k = \frac{\ln\left(\frac{2}{3}\right)}{-0.5} = 0.8$$
$$T(t) = 70e^{-0.8t} + 10(1 - e^{-0.8t}) \Rightarrow 10 - 10e^{-0.8t}$$
$$T(1) = 60e^{-0.8} + 10$$
$$\boxed{T = 37.7}$$

b) $$T(t) = 70e^{-0.8t} + 10(1 - e^{-0.8t})$$
$$15 = 70e^{-0.8t} + 10 - 10e^{-0.8t}$$
$$-10 \qquad\qquad -10$$
$$5 = 60e^{-0.8t}$$
$$\ln\frac{5}{60} = \ln e^{-0.8t}$$
$$\ln\left(\frac{5}{60}\right) = -0.8t$$
$$t = \frac{\ln\left(\frac{5}{60}\right)}{-0.8}$$
$$t = 3.04$$

2.5

Homework #5

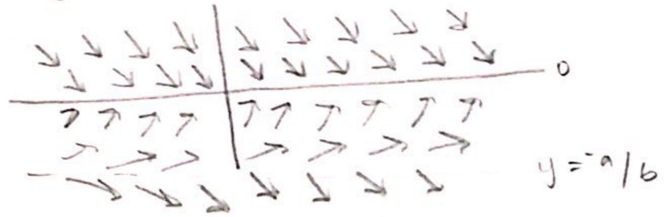
4. $y' = -ay - by^2$

$0 = -ay - by^2$

$0 = y(-a - by)$

$y = 0$

The slope is negative when $y>0$ and its positive when $-\frac{a}{b} < y = 0$. The equilibrium is stable at $y=0$. At $y = -a/b$ its not stable.



14. $y(t) = \dfrac{L}{1+\left(\frac{L}{y_0}-1\right)e^{-rt}}$

$y(t) = \dfrac{5\times10^9}{1+23e^{-rt}}$

$y(1) = \dfrac{5\times10^9}{1+23e^{-rt}}$

$0.4\times10^9 = \dfrac{5\times10^9}{1+23e^{-r}}$

$\ln e^{-r} = \dfrac{11.5}{\ln 23}$

$-r \quad \ln\left|\dfrac{11.5}{23}\right|$

$\boxed{r = 0.69}$

$y(t) - \dfrac{5\times10^9}{1+23e^{0.69t}}$

$y(4) = \dfrac{5\times10^9}{1+23e^{0.69(4)}}$

$\boxed{y(4) = 2.05\times10^9}$ ← The population after 4 hours

17.

Pop - 80,000 students

$\dfrac{dx}{dt} = kx(80000 - x)$

$\dfrac{dx}{dt} = 80000k\left(1 - \dfrac{x}{80000}\right)x$

$x(t) = \dfrac{L}{1+\left(\frac{L}{x_0}-1\right)e^{-rt}} = \dfrac{80000}{1+79e^{-rt}}$

$(1+79e^{-r})\,10000 = \dfrac{80000}{(1+79e^{-r})}(1+79e^{-r})$

$10000(1+79e^{-r} = 80000$

$1+79e^{-r} = 8$

$79e^{-r} = 7$

$r = 2.42$

$\therefore \boxed{x(t) = \dfrac{80000}{1+79e^{-2.42t}}}$

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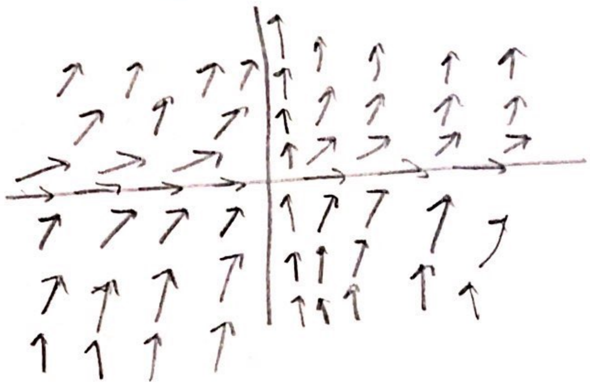
2.5

HOMEWORK #5

Necole Goodman

22.

a) $y' = y^2$



b) $y' = y^2$

$y = 0$

$y(t) = 0$ is an equilibrium solution. The values of the slopes are positive except 0. The equilibrium is semi-stable b/c its going to $y(t) = 0$ for $y < 0$ and $y > 0$

Part 2

1. $\frac{dT}{dt} = K(M-T)$ $\qquad t_0 = 0$

$$\int \frac{dT}{M-T} = \int K\,dt$$

$$e^{-\ln|M-T|} = e^{Kt + C}$$

$$M - T = Ce^{-Kt}$$

$$\boxed{T = M - Ce^{-Kt}}$$

2.

$T_0 = 35°$

$T_f = 40°$

$t = 10\text{min}$

$M = 70°$

$$T(t) = T_0 e^{-Kt} + M(1 - e^{-Kt})$$

$$T(t) = 35e^{-Kt} + 45(1 - e^{-Kt})$$

$$T(t) = 35e^{-0.16K} + 45(1 - e^{-0.16K})$$

$$70 = 35e^{-0.16K} + 45(1 - e^{-0.16K})$$

$$70 = 80e^{-0.16K}$$

$$\ln e^{-0.16K} = \ln 7/8$$

$$-0.116K = \ln 7/8$$

$$\boxed{K = 0.834}$$

Part 3 | HOMEWORK #5 Nicole Goodman

1. $y' + P(t)y = f(t)$

y_h; $\frac{dy}{dt} + P(t)y = 0$

$\frac{dy}{dt} = -P(t)y$

$dy = -P(t)y\,dt$

$\int \frac{1}{y} dy = \int -P(t)\,dt$

$e^{\cancel{\ln}|y|} = e^{-\int P(t)dt + C}$

$\boxed{y = Ce^{-\int P(t)dt}}$

2. y_p;

$v'(t) = f(t)e^{\int P(t)dt}$

$v(t) = \int f(t)e^{\int P(t)dt}$

$y_p = v(t)e^{-\int P(t)dt}$

$\boxed{y_p = \left[\int f(t)e^{\int P(t)dt}\right]e^{-\int P(t)dt}}$