4.3 | Homework # 12 Necole Goodman

1. $y'' + 9y = 0$

$r^2 + 9 = 0$

$= A\cos() + B\sin()$

$$\boxed{\begin{array}{l} y(t) = c_1\cos(3t) + c_2\sin(3t) \\ B = \{c_1\cos(3t),\ c_2\sin(3t)\} \end{array}}$$

4. $y'' + 2y' + 8y = 0$

$r^2 + 2r + 8 = 0$

$\Delta = b^2 - 4ac$

$0 = 2^2 - 4(1)(8)$

$0 \neq -28$

$x = \dfrac{-2 \pm \sqrt{-28}}{2}$

$= \dfrac{-2 \pm \sqrt{7}\sqrt{4}}{2}$

$= -1 \pm \sqrt{7}$

$$\boxed{y(t) = c_1 e^{-t}\cos\sqrt{7}t + c_2 e^{-t}\sin\sqrt{7}t}$$

14. $y'' - y' + y = 0 \quad y(0) = 0,\ y'(0) = 1$

$r^2 - r + 1 = 0$

$r = \dfrac{-(-1) \pm \sqrt{-1^2 - 4(1)(1)}}{2}$

$= 1 \pm \sqrt{-3} = \boxed{\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}}$

$y(t) = e^{t/2}(c_1\cos(\frac{\sqrt{3}}{2})t + c_2\sin(\frac{\sqrt{3}}{2})t)$

$y'(t) = e^{t/2}\left(\frac{\sqrt{3}}{2}c_2\cos\frac{\sqrt{3}}{2} + -\frac{\sqrt{3}}{2}c_1\sin\frac{\sqrt{3}}{2}t\right) + \frac{1}{2}e^{t/2}(c_1\cos\frac{\sqrt{3}}{2}t + c_2\sin\frac{\sqrt{3}}{2}t)$

$y(0) = c_1 = 0$

$y'(0) = \frac{\sqrt{3}}{2}c_2 + \frac{1}{2}c_1 = \frac{\sqrt{3}}{2}c_2 = 1$

$$\boxed{\therefore y(t) = e^{t/2}\left(\frac{2\sqrt{3}}{2}\sin\frac{\sqrt{3}}{2}t\right)}$$

22. $y'' + y' = 0$

Graph B

$\Delta = b^2 - 4ac$

$\Delta = (1)^2 - 4(1)(0) = 1$

$r_1 = \dfrac{-1 - \sqrt{1}}{2} = -1$

$r_2 = \dfrac{-1 + \sqrt{1}}{2} = 0$

$y(t) = c_1 e^{-t} + c_2 \quad \therefore$ Graph B

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Homework #12

23. $y'' + 3y' + 2y = 0$

$\Delta = 3^2 - 4(1)(2) = 1$

$r_1 = \frac{-3-\sqrt{1}}{2} = -2$

$r_2 = \frac{-3+\sqrt{1}}{2} = -1$

$\boxed{y(t) = c_1 e^{-2t} + c_2 e^{-t}}$

$\therefore$ Graph A

24. $y'' - 5y' + 6y = 0$

$\Delta = (-5)^2 - 4(1)(6) = 1$

$r_1 = \frac{-(-5)-\sqrt{1}}{2} = 2$

$r_2 = \frac{-(-5)+\sqrt{1}}{2} = 3$

$\boxed{y(t) = c_1 e^{2t} + c_2 e^{3t}}$ graph C

25. $y'' + y' + y = 0$

$\Delta = (1)^2 - 4(1)(1)$
$= -3$

$\alpha = \frac{-1}{2}$

$\beta = \frac{\sqrt{-(-3)}}{2} = \frac{\sqrt{3}}{2}$

$\boxed{y(t) = e^{-t/2}\left[c_1 \cos(\tfrac{\sqrt{3}}{2})t + c_2 \sin(\tfrac{\sqrt{3}}{2})t\right]}$

31. $r_1 < 0,\ r_2 = 0$

$y(t) = c_1 e^{rt} + c_2$

First term goes to zero as $t \to \infty$ $\therefore$ $c_2 t \to \infty$

32. $r = \alpha \pm \beta i$

$y(t) = e^{\alpha t}(c_1 \cos\beta t + c_2 \sin\beta t)$

If $a > 0$, solution goes to ∞ as $t \to \infty$

If $a < 0$, the solution goes to 0 as $t \to \infty$

If $a = 0$, the solution remains periodic for all t.

4.4 | HOMEWORK #12 Necole Goodman

$\cdot\ y'' - y = t$

$y_p(t) = -t$

Let $y_p = At^2 + Bt + C$

$y'_p = +2At + B$

$y''_p = 2A$

$C - 2A = 0$

$A = 0$

$B = -1$

$C = 0$

$\boxed{y_p(t) = -t}$

$2A - At^2 - Bt - C = t$

$-At^2 - Bt - (C - 2A) = 0$

$-A = 0$

$-B = 1$

4. $ty'' + y' = 4t$

$t^2 y'' + ty = 4t^2$

$g(t) = 4t^2$

$y_p = At^2 + Bt + C$

$y_p' = 2At + B \quad y_p'' = 2A$

$ty''_p + y'_p = 4t \Rightarrow t(2A) + 2At + B = 4t$

$4At + B = 4t$

$A = 1,\ B = 0$

$\boxed{\therefore y_p = t^2 + C}$

10. $f(t) = te^t$

$y'' + 2y' + 5y = f(t)$

$y'' + 2y' + 5y = 0$

$r^2 + 2r + 5 = 0, \quad a = 1,\ b = 2,\ c = 5$

$\Delta = 2^2 - 4(1)(5) = -16 < 0$

$y(t) = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$

$\alpha = \frac{-b}{2a} \qquad \alpha = \frac{-2}{2(1)} = -1$

$\beta = \frac{\sqrt{-\Delta}}{2a} \qquad \beta = \frac{\sqrt{-(-16)}}{2(1)} = 2$

$\therefore y_h(t) = e^{-t}(C_1 \cos 2t + C_2 \sin 2t)$

$\boxed{y_h(t) = C_1 e^{-t} \cos 2t + C_2 e^{-t} \sin 2t}$

14. $f(t) = te^{3t}$

$y'' - 6y' + 9y = te^{3t}$

$y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$

$r_1 = 3 \quad r_2 = 3$

$y_h(t) = C_1 e^{3t} + tC_2 e^{3t}$

The solution set is $\{e^{3t}, te^{3t}\}$

$\therefore y'' - 6y' + 9y = te^{3t}$ is $y_p(t) = t^2 e^{3t}$

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4.4 | Homework # 12

23. $y''+4y'=t$

$r^2+4r=0$

$r(r+4)=0$

$r=0,4 \quad y_h(t)=C_1+C_2e^{-4t}$

$y'_p=2At+B$

$2A+4(2At+B)=t$

$2A+8At+4B=t$

$8At+(2A+4B)=t$

$8A=1 \qquad A=1/8, \; B=-1/16$

$2A+4B=0$

$y_p(t)=\frac{1}{8}t^2-\frac{1}{16}t \qquad \boxed{y(t)=C_1+C_2e^{-4t}+\frac{1}{8}t^2-\frac{1}{16}t}$

33. $y''-4y'+4y=te^{2t}$

$r^2-4r+4=0$

$(r-2)(r-2)=0$

$r_1=2, r_2=2 \quad y_h(t)=C_1e^{2t}+tC_2e^{2t}$

$6Ate^{2t}+12At^2e^{2t}+4At^3e^{2t}+2Be^{2t}+8Bte^{2t}+4Bt^2e^{2t}$

$=-4(3At^2e^{2t}+2At^3e^{2t}+2Bte^{2t}+2Bt^2e^{2t})+(At^3e^{2t}+Bt^2e^{2t})=te^{2t}$

$6Ate^{2t}+2Be^{2t}=te^{2t}$

$6A=1 \qquad A=1/6 \qquad y_p=\frac{1}{6}t^3e^{2t}$

$2B=0 \qquad B=0$

$\boxed{y(t)=C_1e^{2t}+C_2te^{2t}+\frac{1}{6}t^3e^{2t}}$

41. $y''+y'-2y=3-6t, \; y(0)=-1, \; y'(0)=0$

$r^2+r-2=0$

$(r+2)(r-1)$

$r=-2, r=1$

$y_h(t)=C_1e^{-2t}+C_2e^t$

$0+A-2(At+B)=3-6t$

$-2At+(A+B)=-6t+3$

$-2A=-6$

$A=3$

$B=0$

$y_p(t)=3t$

$y(t)=C_1e^{-2t}+C_2e^t+3t$

$C_1e^{-2(0)}+C_2e^{(0)}+3(0)=1$

$C_1+C_2=1$

$y'(0)=0$

$-2C_1e^{-2(0)}+C_2e^{(0)}+3=0$

$-2C_1+C_2+3=0$

$C_1=2/3, C_2=-5/3$

$\boxed{\therefore y(t)=\frac{2}{3}e^{-2t}-\frac{5}{3}e^t+3t}$

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PG 4.4 | Homework #12 Necole Goodman

a) 43. $y'' + 4y = t$; $y(0)=1$; $y'(0)=1$

$r^2 + 4 = 0$

$r(r+4)$

$r = \pm 2i$ $y_h(t) = c_1 \cos 2t + c_2 \sin 2t$

b) $0 + 4At = t$

$4A = 1$ $y_P(t) = 1/4 t$

$A = 1/4$ $y(t) = c_1 \cos 2t + c_2 \sin 2t + \frac{1}{4}t$

$y'(t) = -2c_1 \sin 2t + 2c_2 \cos 2t + \frac{1}{4}$

$c_1 \cos 2(0) + c_2 \cancel{\sin 2(0)}^{\,0} + \cancel{\frac{1}{4}(0)}^{\,0} = 1$

c) $c_1 = 1$

$y'(0) = -1$

$= -2c_1 \sin 2(0) + 2c_2 \cos 2(0) + \frac{1}{4} = -1$

=

$\boxed{y(t) = \cos 2t - \frac{5}{8}\sin 2t + \frac{1}{4}t}$

=

d

a) $y(t) = v(t)e^{-\frac{pt}{2}}$

$y'(t) = -\frac{p}{2} v(t)e^{-\frac{pt}{2}} + v'(t)e^{-\frac{pt}{2}}$

$\boxed{y'(t) = \left(v'(t) - \frac{p}{2} v(t)\right) e^{-\frac{pt}{2}}}$

b) $y''(t) = -\frac{p}{2}\left(v'(t) - \frac{p}{2} v(t)\right)e^{-\frac{pt}{2}} + e^{-\frac{pt}{2}}\left[v''(t) - \frac{p}{2} v'(t)\right]$

$= \left[v''(t) - pv'(t) + \frac{p^2}{4} v(t)\right]e^{-\frac{pt}{2}}$

c) $y'' + py' + qy = 0$

$= \left[v''(t) - pv'(t) + \frac{p^2}{4} v(t)\right]e^{-\frac{pt}{2}} + p\left[v'(t) - \frac{p}{2} v(t)\right]e^{-\frac{pt}{2}} + q\left[v(t)e^{-\frac{pt}{2}}\right]$

$= \left[v''(t) - \cancel{pv'(t)} + \frac{p^2}{4} v(t) + \cancel{pv'(t)} - \frac{p^2}{2} v(t) + qv(t)\right]e^{-\frac{pt}{2}}$

$\boxed{= \left[v''(t) - \frac{p^2}{4} v(t) + qv(t)\right]e^{-\frac{pt}{2}} = 0}$

d) $p^2 = 4q$; $e^{-\frac{pt}{2}} \neq 0$

$\left[v''(t) - \frac{p^2}{4} v(t) + qv(t)\right]e^{-\frac{pt}{2}} = 0$

$= \left[v''(t) - \cancel{\frac{4q}{4} v(t)} + \cancel{qv(t)}\right]e^{-\frac{pt}{2}} = 0$

$\boxed{v''(t) = 0}$

e) $v(t) = ct + d$

$v''(t) = 0$

$v'(t) = t + c$

$\boxed{v(t) = ct + d}$

Part III | Homework #12 | Necole Goodman

1) $y_1(t) = e^{\alpha t}\cos\beta t$

$y_1'(t) = \alpha e^{\alpha t}\cos\beta t - \beta e^{\alpha t}\sin\beta t$

$\boxed{y_1'(t) = [\alpha\cos\beta t - \beta\sin\beta t]e^{\alpha t}}$

$y_1''(t) = \alpha[\alpha\cos\beta t - \beta\sin\beta t]e^{\alpha t} + e^{\alpha t}[-\alpha\beta\sin\beta t - \beta^2\cos\beta t]$

$= [\alpha^2\cos\beta t - \alpha\beta\sin\beta t - \alpha\beta\sin\beta t - \beta^2\cos\beta t]e^{\alpha t}$

$= [\alpha^2\cos\beta t - 2\alpha\beta\sin\beta t - \beta^2\cos\beta t]e^{\alpha t}$

$y_2(t) = e^{\alpha t}\sin\beta t$

$y_2'(t) = \alpha e^{\alpha t}\sin\beta t + \beta e^{\alpha t}\cos\beta t$

$= [\alpha\sin\beta t + \beta\cos\beta t]e^{\alpha t}$

$y_2''(t) = \alpha[\alpha\sin\beta t + \beta\cos\beta t]e^{\alpha t} + e^{\alpha t}[\alpha\beta\cos\beta t - \beta^2\sin\beta t]$

$= [\alpha^2\sin\beta t + \alpha\beta\cos\beta t + \alpha\beta\cos\beta t - \beta^2\sin\beta t]e^{\alpha t}$

$= [\alpha^2\sin\beta t + 2\alpha\beta\cos\beta t - \beta^2\sin\beta t]e^{\alpha t}$

$\alpha = -\frac{\rho}{2} \quad \beta = \frac{\sqrt{49-\rho^2}}{2}$