

$$24. \begin{aligned} x + y &= 4 \\ x - y &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$R_2 = R_2 - R_1$$

$$R_2 = -\frac{1}{2} R_2$$

$$R_1 = R_1 - R_2$$

\therefore unique solutions @ $x = 2$ and $y = 2$

$$27. \begin{aligned} 2x + 4y - 2z &= 0 \\ 5x + 3y + z &= 0 \end{aligned}$$

$$\begin{bmatrix} 2 & 4 & -2 & | & 0 \\ 5 & 3 & 0 & | & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & | & 0 \\ 5 & 3 & 0 & | & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & | & 0 \\ 0 & -7 & 5 & | & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & | & 0 \\ 0 & 1 & -5/7 & | & 0 \end{bmatrix} \Rightarrow$$

$$R_1 = \frac{1}{2} R_1$$

$$R_2 = R_2 + (-5)R_1$$

$$R_2 = -\frac{1}{7} R_2$$

$$R_1 = R_1 + (-2)R_2$$

$$\begin{bmatrix} 1 & 0 & 3/7 & | & 0 \\ 0 & 1 & -5/7 & | & 0 \end{bmatrix} \text{ can no longer simplify}$$

no unique solutions

$$x + \frac{3}{7}z = 0 \Rightarrow x = -\frac{3}{7}z$$

$$y - \frac{5}{7}z = 0 \Rightarrow y = \frac{5}{7}z$$

28.

$$x - y - 2z = 1$$

$$2x + 3y + z = 2$$

$$5x + 4y + 2z = 4$$

$$\begin{bmatrix} 1 & -1 & -2 & | & 1 \\ 2 & 3 & 1 & | & 2 \\ 5 & 4 & 2 & | & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & -2 & | & 1 \\ 0 & 5 & 5 & | & 0 \\ 5 & 4 & 2 & | & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & -2 & | & 1 \\ 0 & 5 & 5 & | & 0 \\ 0 & 9 & 12 & | & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & | & 1 \\ 0 & 1 & 1 & | & 0 \\ 0 & 9 & 12 & | & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & | & 1 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 3 & | & -1 \end{bmatrix}$$

$$R_2 = R_2 + (-2)R_1$$

$$R_3 = R_3 + (-5)R_1$$

$$R_2 = \frac{1}{5} R_2$$

$$R_3 = R_3 + (-9)R_2$$

$$R_3 = \frac{1}{3} R_3$$

$$\begin{bmatrix} 1 & 0 & -1 & | & 1 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & | & -1/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 2/3 \\ 0 & 1 & 0 & | & 1/3 \\ 0 & 0 & 1 & | & -1/3 \end{bmatrix} \checkmark$$

$$R_1 = R_1 + R_3$$

\therefore unique solutions @ $x = 2/3, y = 1/3, z = -1/3$

3.21

HOMEWORK #17

Nicole Goodman

$$\begin{aligned}
 34. \quad & x + 2y + z = 2 \\
 & 2x - 4y - 3z = 0 \\
 & -x + 6y - 4z = 2 \\
 & x - y + 0 = 4
 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -4 & -3 & 0 \\ -1 & 6 & -4 & 2 \\ 1 & -1 & 0 & 4 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -4 & -3 & 0 \\ -1 & 6 & 4 & 2 \\ 0 & -3 & -1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -4 & -3 & 0 \\ 0 & 8 & 5 & 4 \\ 0 & -3 & -1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -8 & -5 & -4 \\ 0 & 8 & 5 & 4 \\ 0 & -3 & -1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 5/8 & 1/2 \\ 0 & 8 & 5 & 4 \\ 0 & -3 & -1 & 2 \end{array} \right]$$

$$R_4 = R_4 - R_1 \quad R_3 = R_3 + R_1 \quad R_2 = -2R_1 + R_2 \quad R_2 = -\frac{1}{8}R_2 \quad R_3 = R_3 - 8R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 5/8 & 1/2 \\ 0 & 0 & -8 & 0 \\ 0 & -3 & -1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 5/8 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & -3 & -1 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 5/8 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 15/8 & 7/2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 5/8 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7/2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7/2 \end{array} \right]$$

$$R_3 = -\frac{1}{8}R_3 \quad R_4 = R_4 + 3R_2 \quad R_4 = R_4 + (-\frac{15}{8})R_3 \quad R_2 = R_2 + (-\frac{5}{8})R_3 \quad R_1 = -2R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7/2 \end{array} \right] \therefore \text{NO SOLUTION}$$

66.

$$\begin{aligned}
 a) \quad & \left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 3 \\ 1 & -1 & 1 & 1 & 3 \\ 2 & -3 & 4 & 4 & 9 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 2 & 1 & 0 & 0 & 3 \\ 2 & -3 & 4 & 4 & 9 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 0 & 3 & -2 & -2 & -3 \\ 0 & -1 & 2 & 2 & 3 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & -1 & 2 & 2 & 3 \end{array} \right] \\
 & R_1 \leftrightarrow R_2 \quad R_2 = R_2 - 2R_1 \quad R_2 = -\frac{1}{3}R_2 \quad R_3 = R_3 + R_2
 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & 0 & 4/3 & 4/3 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3/2 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & 2 & 0 & 0 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/2 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & 2 & 0 & 0 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/2 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{array} \right]$$

$$R_3 = R_3 + 2R_2 \quad R_1 = R_1 + \frac{1}{2}R_2 \quad R_1 = R_1 + \frac{1}{4}R_3 \quad R_2 = \frac{1}{2}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/2 \\ 0 & 0 & 1 & 1 & 3/2 \\ 0 & 1 & 0 & 0 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 3/2 \end{array} \right]$$

infinitely many solutions

$$R_2 = (-\frac{3}{2})R_2 \quad R_2 \leftrightarrow R_3$$

3.2

HOMEWORK #1

NEWELL Goodman

66.

$$b) \left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 3 \\ 1 & -1 & 1 & 1 & 3 \\ 1 & 2 & -1 & -1 & -6 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3/2 \\ 1 & -1 & 1 & 1 & 3 \\ 1 & 2 & -1 & -1 & -6 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3/2 \\ 1 & -1 & 1 & 1 & 3 \\ 0 & 3/2 & -1 & -1 & -6 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3/2 \\ 0 & -3/2 & 1 & 1 & 3/2 \\ 0 & 3/2 & -1 & -1 & -15/2 \end{array} \right]$$

$R_1 = \frac{1}{2}R_1$ $R_3 = R_3 - R_1$ $R_2 = R_2 - R_1$ $R_2 = (-\frac{2}{3})R_2$

$$\left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3/2 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & 3/2 & -1 & -1 & -15/2 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3/2 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & 0 & 0 & 0 & -6 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3/2 \\ 0 & 1 & -2/3 & -2/3 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \therefore \text{No solution and inconsistent}$$

$R_3 = R_3 - \frac{3}{2}R_2$ $R_3 = -\frac{1}{6}R_3$ can do no more

67.

RREF $\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & c \end{array} \right]$ if $c \neq 0$ there are no solutions

$$\begin{aligned} 2x_2 + x_3 &= 0 \\ 3x_1 + 4x_2 + x_3 &= 12 \\ x_1 + 4x_2 + 2x_3 &= 15 \end{aligned} \quad \left[\begin{array}{ccc|c} 0 & 2 & 1 & 0 \\ 3 & 1 & 1 & 12 \\ 1 & 4 & 2 & 15 \end{array} \right]$$

if $c = 0$ there is one solution / if RREF $\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 0 \end{array} \right]$ there are infinite solutions

$$0 + 2x_2 + x_3 = 8$$

$$3x_1 + x_2 + x_3 = 12$$

$$x_1 + 4x_2 + 2x_3 = 15$$

$$1. \begin{bmatrix} 0 & 2 & 1 & | & 8 \\ 3 & 1 & 1 & | & 12 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 1 & 1 & | & 12 \\ 0 & 2 & 1 & | & 8 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1/3 & 1/3 & | & 4 \\ 0 & 2 & 1 & | & 8 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1/3 & 1/3 & | & 4 \\ 0 & 2 & 1 & | & 8 \\ 0 & 11/3 & 5/3 & | & 11 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$ $R_1 = \frac{1}{3} R_1$ $R_3 = -R_1 + R_3$ $R_2 = \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 1/3 & 1/3 & | & 4 \\ 0 & 1 & 1/2 & | & 2 \\ 0 & 11/3 & 5/3 & | & 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1/3 & 1/3 & | & 4 \\ 0 & 1 & 1/2 & | & 2 \\ 0 & 0 & -1/6 & | & -17/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1/3 & | & 10/3 \\ 0 & 1 & 1/2 & | & 2 \\ 0 & 0 & -1/6 & | & -17/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1/3 & | & 10/3 \\ 0 & 1 & 1/2 & | & 2 \\ 0 & 0 & 1 & | & 34 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1/3 & | & 10/3 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 34 \end{bmatrix}$$

$R_3 = -11/3 R_2 + R_3$ $R_1 = -\frac{1}{3} R_2 + R_1$ $R_3 = -6 R_3$ $R_2 = -\frac{1}{2} R_3 + R_2$ $R_1 = \frac{1}{3} R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & -7 \\ 0 & 0 & 1 & | & 22 \end{bmatrix}$$

$$2. \begin{bmatrix} 0 & 2 & 1 & | & 8 \\ 3 & 1 & 1 & | & 12 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 3 & | & 23 \\ 3 & 1 & 1 & | & 12 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 3 & | & 23 \\ 0 & -17 & -8 & | & 81 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 3 & | & 23 \\ 0 & -17 & -8 & | & 81 \\ 0 & -2 & -1 & | & -8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 3 & | & 23 \\ 0 & -17 & -8 & | & 81 \\ 0 & 0 & 0 & | & -5 \end{bmatrix}$$

$R_1 = R_1 + R_3$ $R_2 = -3R_1 + R_2$ $R_3 = -R_1 + R_3$

$$\begin{bmatrix} 1 & 6 & 3 & | & 23 \\ 0 & -17 & -8 & | & 81 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 3 & | & 23 \\ 0 & -17 & -8 & | & 81 \\ 0 & -2 & -1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 3 & | & 23 \\ 0 & 1 & 3/17 & | & 51/17 \\ 0 & -2 & -1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 3/17 & | & 49/17 \\ 0 & 1 & 3/17 & | & 51/17 \\ 0 & -2 & -1 & | & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3/17 & | & 49/17 \\ 0 & 1 & 3/17 & | & 51/17 \\ 0 & 0 & -1/17 & | & -22/17 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 0 & | & -1 \\ 0 & 1 & 8/17 & | & 59/17 \\ 0 & 0 & -1/17 & | & -22/17 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 8/17 & | & 59/17 \\ 0 & 0 & 7/17 & | & -22/17 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & -7 \\ 0 & 0 & 0 & | & 22 \end{bmatrix}$$

Part II

HOMEWORK #7

$$3. \begin{bmatrix} 0 & 2 & 1 & | & 8 \\ 3 & 1 & 1 & | & 12 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 2 & | & 15 \\ 3 & 1 & 1 & | & 12 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 2 & | & 15 \\ 0 & -11 & -5 & | & -33 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 2 & | & 15 \\ 0 & 1 & -5/11 & | & -3 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2/11 & | & 3 \\ 0 & 1 & -5/11 & | & -3 \\ 0 & 2 & 1 & | & 8 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$ $R_2 = -3R_1 + R_2$ $R_3 = -\frac{1}{11}R_2$ $R_2 = \dots$

$$\begin{bmatrix} 1 & 0 & 2/11 & | & 3 \\ 0 & 1 & -5/11 & | & -3 \\ 0 & 0 & 1/11 & | & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 3/11 & | & 3 \\ 0 & 0 & 1/11 & | & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & -7 \\ 0 & 0 & 1/11 & | & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & -7 \\ 0 & 0 & 1 & | & 22 \end{bmatrix}$$

$$4. \begin{bmatrix} 0 & 2 & 1 & | & 8 \\ 3 & 1 & 1 & | & 12 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 2 & | & 15 \\ 3 & 1 & 1 & | & 12 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 2 & | & 15 \\ 0 & -11 & -5 & | & -33 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 2 & | & 15 \\ 0 & 1 & 1 & | & 9 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & | & 51 \\ 0 & 1 & 1 & | & 9 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 1 & | & 9 \\ 0 & 0 & 1 & | & 26 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & -7 \\ 0 & 0 & 1 & | & 22 \end{bmatrix}$$

$$5. \begin{bmatrix} 0 & 2 & 1 & | & 8 \\ 3 & 1 & 1 & | & 12 \\ 1 & 4 & 2 & | & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 2 & | & 15 \\ 3 & 1 & 1 & | & 12 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 3 & 1 & 1 & | & 12 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 1 & | & 15 \\ 0 & 2 & 1 & | & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 1 & | & 15 \\ 0 & 0 & -1 & | & -22 \end{bmatrix}$$