HARMONIC FUNCTIONS ON HYPERBOLIC ORBIFOLDS

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Abstract. We develop a method to understand the harmonic functions and corresponding Poisson and minimal Martin boundaries for a family of Riemannian manifolds which are nonproper as metric space (not all closed balls are compact). As an application we describe the harmonic functions and these boundaries explicitly for a class of hyperbolic orbifolds with codimension two singular branching loci.

Introduction

In the study of orbifolds, cone-manifolds, and other singular spaces, the space resulting from removing the singularity is an incomplete Riemannian manifold. Its universal cover is again an incomplete Riemannian manifold, whose completion is a metric space which is not locally compact: Finite-radius neighborhoods of the branch loci need not be compact.

Such spaces arise naturally in the study of geometric structures and their deformations; for instance, in the theory of hyperbolic Dehn surgery, an incomplete hyperbolic knot complement has a continuum of incomplete hyperbolic structures which have various types of singular loci. They also arise in the study of singular foliations on Riemann surfaces.

Harmonic theory on such spaces has not been studied. Restricting to the case of locally hyperbolic nonproper spaces which admit a cocompact group action, the aim of this paper is to characterize the geometric, Poisson, and Martin boundaries at infinity and relate them to the space of harmonic functions.

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