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1) I chose the 4th column because it contains two 0's which makes it easier to compute.

$$A = \begin{bmatrix} 2 & -2 & 3 & 1 \\ 4 & 3 & -6 & 0 \\ 0 & 3 & 2 & -1 \\ 3 & 2 & -1 & 0 \end{bmatrix} \Rightarrow a_{14}.C_{14} + a_{24}.C_{24} + a_{34}.C_{34} + a_{44}.C_{44}$$

$$a_{14}.C_{14} = 1.(-1)^5 . \begin{vmatrix} 4 & 3 & -6 \\ 0 & 3 & 2 \\ 3 & 2 & -1 \end{vmatrix} \Rightarrow -1.(4.\underbrace{C_{11}}_{-28} + 0.\underbrace{C_{21}}_{0} + 3.\underbrace{C_{31}}_{72}) = -44$$

$$a_{24}.C_{24} = 0.(-1)^6 . |\;| = 0$$

$$a_{34}.C_{34} = -1.(-1)^7 . \begin{vmatrix} 2 & -2 & 3 \\ 4 & 3 & -6 \\ 3 & 2 & -1 \end{vmatrix} \rightarrow 1.(2.\underbrace{C_{11}}_{18} + 4.\underbrace{C_{21}}_{+16} + 3.\underbrace{C_{31}}_{9}) = 43$$

$$a_{44}.C_{44} = 0.(-1)^8 . |\;| = 0$$

$\det(A) = -44 + 43 = -1$

2) 1.)

$$A = \begin{bmatrix} 2 & 2 & 4 & 6 \\ 1 & 3 & -2 & 1 \\ 2 & 8 & -4 & 2 \\ 1 & 3 & 6 & 7 \end{bmatrix} \xrightarrow{r_1.(-\frac{1}{2})} \begin{bmatrix} 2 & 2 & 4 & 6 \\ 0 & 2 & -4 & -2 \\ 2 & 8 & -4 & 2 \\ 0 & 2 & 4 & 4 \end{bmatrix} \xrightarrow{r_1.(-1)} \begin{bmatrix} 2 & 2 & 4 & 6 \\ 0 & 2 & -4 & -2 \\ 0 & 6 & -8 & -4 \\ 0 & 2 & 4 & 4 \end{bmatrix} \xrightarrow{r_2.(-3)} \begin{bmatrix} 2 & 2 & 4 & 6 \\ 0 & 2 & -4 & -2 \\ 0 & 0 & 4 & 2 \\ 0 & 2 & 4 & 4 \end{bmatrix}$$

$$\xrightarrow{r_2.(-1) \text{ to } r_4} \begin{bmatrix} 2 & 2 & 4 & 6 \\ 0 & 2 & -4 & -2 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 8 & 6 \end{bmatrix} \xrightarrow{r_3.(-2)} \begin{bmatrix} 2 & 2 & 4 & 6 \\ 0 & 2 & -4 & -2 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Determinant of a lower triangular matrix = product of the main diagonal: $2 \cdot 2 \cdot 4 \cdot 2 = 32$

2.) $2.C_{11} + 0.C_{21} + 0.C_{31} + 0.C_{41}$

$$2.(-1)^2 . \begin{vmatrix} 2 & -4 & -2 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{vmatrix} \Rightarrow 2.(2.\underbrace{C_{11}}_{8} + 0.\underbrace{C_{21}}_{0} + 0.\underbrace{C_{31}}_{0}) = 2.16 = 32$$

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3) $A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$ $A = \begin{bmatrix} 3 & 2 & 7 & 6 \\ -3 & 3 & 4 & 1 \\ 2 & 1 & -4 & -3 \\ -4 & 1 & 2 & -2 \end{bmatrix}$

$C_{11} = \begin{vmatrix} 3 & 4 & 1 \\ 1 & -4 & -3 \\ 1 & 2 & -2 \end{vmatrix} \rightarrow 3 \begin{vmatrix} -4 & -3 \\ 2 & -2 \end{vmatrix} = 42$

$-1 \begin{vmatrix} 4 & 1 \\ 2 & -2 \end{vmatrix} = 10$

$C_{11} = 44 \Leftarrow$

$+1 \begin{vmatrix} 4 & 1 \\ -4 & -3 \end{vmatrix} = -8$

$C_{12} = - \begin{vmatrix} -3 & 4 & 1 \\ 2 & -4 & -3 \\ -4 & 2 & -2 \end{vmatrix} \rightarrow +3 \begin{vmatrix} -4 & -3 \\ 2 & -2 \end{vmatrix} + 2 \begin{vmatrix} 4 & 1 \\ 2 & -2 \end{vmatrix} + 4 \begin{vmatrix} 4 & 1 \\ -4 & -3 \end{vmatrix} = -10$

$+42 \quad -20 \quad +32$

$C_{13} = + \begin{vmatrix} -3 & 3 & 1 \\ 2 & 1 & -3 \\ -4 & 1 & -2 \end{vmatrix} \rightarrow -3 \begin{vmatrix} 1 & -3 \\ 1 & -2 \end{vmatrix} - 3 \begin{vmatrix} 2 & -3 \\ -4 & -2 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ -4 & 1 \end{vmatrix} = 51$

$-3 \quad +48 \quad +6$

$C_{14} = - \begin{vmatrix} -3 & 3 & 4 \\ 2 & 1 & -4 \\ -4 & 1 & 2 \end{vmatrix} \rightarrow +3 \begin{vmatrix} 1 & -4 \\ 1 & 2 \end{vmatrix} + 2 \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} + 4 \begin{vmatrix} 3 & 4 \\ 1 & -4 \end{vmatrix} = -42$

$+18 \quad +4 \quad -64$

$C_{21} = - \begin{vmatrix} 2 & 7 & 6 \\ 1 & -4 & -3 \\ 1 & 2 & -2 \end{vmatrix} \rightarrow -2 \begin{vmatrix} -4 & -3 \\ 2 & -2 \end{vmatrix} + 1 \begin{vmatrix} 7 & 6 \\ 2 & -2 \end{vmatrix} - 1 \begin{vmatrix} 7 & 6 \\ -4 & -3 \end{vmatrix} = -57$

$-28 \quad -26 \quad -3$

$C_{22} = + \begin{vmatrix} 3 & 7 & 6 \\ 2 & -4 & -3 \\ -4 & 2 & -2 \end{vmatrix} \rightarrow 3 \begin{vmatrix} -4 & -3 \\ 2 & -2 \end{vmatrix} - 2 \begin{vmatrix} 7 & 6 \\ 2 & -2 \end{vmatrix} - 4 \begin{vmatrix} 7 & 6 \\ -4 & -3 \end{vmatrix} = 82$

$+42 \quad +52 \quad -12$

$C_{23} = - \begin{vmatrix} 3 & 2 & 6 \\ 2 & 1 & -3 \\ -4 & 1 & -2 \end{vmatrix} \rightarrow -3 \begin{vmatrix} 1 & -3 \\ 1 & -2 \end{vmatrix} + 2 \begin{vmatrix} 2 & 6 \\ 1 & -2 \end{vmatrix} + 4 \begin{vmatrix} 2 & 6 \\ 1 & -3 \end{vmatrix} = -71$

$-3 \quad -20 \quad -48$

$C_{24} = + \begin{vmatrix} 3 & 2 & 7 \\ 2 & 1 & -4 \\ -4 & 1 & 2 \end{vmatrix} \rightarrow 3 \begin{vmatrix} 1 & -4 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 7 \\ 1 & 2 \end{vmatrix} - 4 \begin{vmatrix} 2 & 7 \\ 1 & -4 \end{vmatrix} = +84$

$+18 \quad +6 \quad +60$

$C_{31} = + \begin{vmatrix} 2 & 7 & 6 \\ 3 & 4 & 1 \\ 1 & 2 & -2 \end{vmatrix} \rightarrow 2 \begin{vmatrix} 4 & 1 \\ 2 & -2 \end{vmatrix} - 3 \begin{vmatrix} 7 & 6 \\ 2 & -2 \end{vmatrix} + 1 \begin{vmatrix} 7 & 6 \\ 4 & 1 \end{vmatrix} = +41$

$-20 \quad +78 \quad -17$

$C_{32} = - \begin{vmatrix} 3 & 7 & 6 \\ -3 & 4 & 1 \\ -4 & 2 & -2 \end{vmatrix} \rightarrow -3 \begin{vmatrix} 4 & 1 \\ 2 & -2 \end{vmatrix} - 3 \begin{vmatrix} 7 & 6 \\ 2 & -2 \end{vmatrix} + 4 \begin{vmatrix} 7 & 6 \\ 4 & 1 \end{vmatrix} = +40$

$+30 \quad +78 \quad -68$

Kaan Karataş
150200081

$$C_{33} = +\begin{vmatrix} 3 & 2 & 6 \\ -3 & 3 & 1 \\ -4 & 1 & -2 \end{vmatrix} \rightarrow 3\underbrace{\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}}_{-21} + 3\underbrace{\begin{vmatrix} 2 & 6 \\ 1 & -2 \end{vmatrix}}_{-30} - 4\underbrace{\begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix}}_{+64} = +13$$

$$C_{34} = -\begin{vmatrix} 3 & 2 & 7 \\ -3 & 3 & 4 \\ -4 & 1 & 2 \end{vmatrix} \rightarrow -3\underbrace{\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}}_{-6} - 3\underbrace{\begin{vmatrix} 2 & 7 \\ 1 & 2 \end{vmatrix}}_{+9} + 4\underbrace{\begin{vmatrix} 2 & 7 \\ 3 & 4 \end{vmatrix}}_{-52} = -49$$

$$C_{41} = -\begin{vmatrix} 2 & 7 & 6 \\ 3 & 4 & 1 \\ 1 & -4 & -3 \end{vmatrix} \rightarrow -2\underbrace{\begin{vmatrix} 4 & 1 \\ -4 & -3 \end{vmatrix}}_{+16} + 3\underbrace{\begin{vmatrix} 7 & 6 \\ -4 & -3 \end{vmatrix}}_{+9} - 1\underbrace{\begin{vmatrix} 7 & 6 \\ 4 & 1 \end{vmatrix}}_{+17} = +42$$

$$\nabla \quad A^{-1} = \frac{1}{\det A} \operatorname{adj}(A)$$

$$C_{42} = +\begin{vmatrix} 3 & 7 & 6 \\ -3 & 4 & 1 \\ 2 & -4 & -3 \end{vmatrix} \rightarrow +3\underbrace{\begin{vmatrix} 4 & 1 \\ -4 & -3 \end{vmatrix}}_{-24} + 3\underbrace{\begin{vmatrix} 7 & 6 \\ -4 & -3 \end{vmatrix}}_{+9} + 2\underbrace{\begin{vmatrix} 7 & 6 \\ 4 & 1 \end{vmatrix}}_{-34} = -49$$

$$C_{43} = -\begin{vmatrix} 3 & 2 & 6 \\ -3 & 3 & 1 \\ 2 & 1 & -3 \end{vmatrix} \rightarrow -3\underbrace{\begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix}}_{+30} - 3\underbrace{\begin{vmatrix} 2 & 6 \\ 1 & -3 \end{vmatrix}}_{+36} - 2\underbrace{\begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix}}_{+32} = +98$$

$$C_{44} = +\begin{vmatrix} 3 & 2 & 7 \\ -3 & 3 & 4 \\ 2 & 1 & -4 \end{vmatrix} \rightarrow +3\underbrace{\begin{vmatrix} 3 & 4 \\ 1 & -4 \end{vmatrix}}_{-48} + 3\underbrace{\begin{vmatrix} 2 & 7 \\ 1 & -4 \end{vmatrix}}_{+45} + 2\underbrace{\begin{vmatrix} 2 & 7 \\ 3 & 4 \end{vmatrix}}_{-26} = -119$$

$$\det(A) = 44.3 + (-10).2 + 51.7 + (-42).6 + (-57).(-3) + 82.3 + (-71).4 + 84.1$$
$$+ 41.2 + 40.1 + 13.(-4) + (-49).(-3) + 42(-4) + (-49).1 + 98.2 + (-119).(-2)$$
$$= 217$$

$$* \; A^{-1} = \frac{1}{217}\begin{bmatrix} 44 & -57 & 41 & 42 \\ -10 & 82 & 40 & -49 \\ 51 & -71 & 13 & 98 \\ -42 & 84 & -49 & -119 \end{bmatrix}$$

Kaan Karataş
150200081

4) $A = \begin{bmatrix} 2 & 0 & 1 & -3 & 4 \\ 0 & 0 & 3 & 0 & 4 \\ 1 & 0 & -4 & -2 & 0 \\ 3 & 4 & 0 & 0 & 2 \\ 0 & 2 & 3 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 7 \\ 5 \\ -4 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 0 & 1 & -3 & 4 \\ 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & -9/2 & -9/2 & -2 \\ 0 & 4 & -3/2 & 9/2 & -4 \\ 0 & 2 & 3 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 1 & -3 & 4 \\ 0 & 4 & -3/2 & 9/2 & -4 \\ 0 & 0 & -9/2 & -1/2 & -2 \\ 0 & 0 & 3 & 0 & 4 \\ 0 & 2 & 3 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 1 & -3 & 4 \\ 0 & 4 & -3/2 & 9/2 & -4 \\ 0 & 0 & -9/2 & -1/2 & -2 \\ 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 15/4 & -13/4 & 3 \end{bmatrix}$

(−1/2 R_1 added to R_3, −3/2 R_1 added to R_4)

* (Swapped R_2 and R_4, det(A) changed sign)

(−1/2 R_2 added to R_4)

$\begin{bmatrix} 2 & 0 & 1 & -3 & 4 \\ 0 & 4 & -3/2 & 9/2 & -4 \\ 0 & 0 & -9/2 & -1/2 & -2 \\ 0 & 0 & 0 & -1/3 & 8/3 \\ 0 & 0 & 0 & -11/3 & 4/3 \end{bmatrix}$

($\frac{2}{3} R_3$ added to R_4, $\frac{5}{6} R_3$ added to R_5)

$\begin{bmatrix} 2 & 0 & 1 & -3 & 4 \\ 0 & 4 & -3/2 & 9/2 & -4 \\ 0 & 0 & -9/2 & -1/2 & -2 \\ 0 & 0 & 0 & -1/3 & 8/3 \\ 0 & 0 & 0 & 0 & -28 \end{bmatrix}$

(11 R_4 added to R_5)

$2 \cdot 4 \cdot \frac{-9}{2} \cdot \frac{-1}{3} \cdot -28 = -336$

$\det(A) = 336$

$X_2 = \frac{|\Delta x_2|}{|A|}$

$x_2 = \begin{bmatrix} 2 & 3 & 1 & -3 & 4 \\ 0 & 9 & 3 & 0 & 4 \\ 1 & 7 & -4 & -2 & 0 \\ 3 & 5 & 0 & 0 & 2 \\ 0 & -4 & 3 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 1 & -3 & 4 \\ 0 & 9 & 3 & 0 & 4 \\ 0 & 11/2 & -9/2 & -1/2 & -2 \\ 0 & 1/2 & -3/2 & 9/2 & -4 \\ 0 & -4 & 3 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 1 & -3 & 4 \\ 0 & 9 & 3 & 0 & 4 \\ 0 & 0 & -19/3 & -1/2 & -40/9 \\ 0 & 0 & -5/3 & 9/2 & -38/9 \\ 0 & 0 & 13/3 & -1 & 25/9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 1 & -3 & 4 \\ 0 & 9 & 3 & 0 & 4 \\ 0 & 0 & -\frac{19}{3} & -\frac{1}{2} & -\frac{40}{9} \\ 0 & 0 & 0 & \frac{88}{19} & -\frac{58}{19} \\ 0 & 0 & 0 & -\frac{51}{38} & -\frac{5}{19} \end{bmatrix}$

(−1/2 R_1 added to R_3, −3/2 R_1 added to R_4)

(−11/18 R_2 added to R_3, −1/18 R_2 added to R_4, 4/9 R_2 added to R_5)

($\frac{-5}{19} R_3$ added to R_4, $\frac{13}{19} R_3$ added to R_5)

$\begin{bmatrix} 2 & 3 & 1 & -3 & 4 \\ 0 & 9 & 3 & 0 & 4 \\ 0 & 0 & -\frac{19}{3} & -1/2 & -\frac{40}{9} \\ 0 & 0 & 0 & \frac{88}{19} & -\frac{58}{19} \\ 0 & 0 & 0 & 0 & -\frac{101}{88} \end{bmatrix}$

($\frac{51}{176} R_4$ added to R_5)

$2 \cdot 9 \cdot \frac{-19}{3} \cdot \frac{88}{19} \cdot \frac{-101}{88} = 606$

$|\Delta x_2| = 606$

$X_2 = \frac{606}{336} = \frac{101}{56}$

$X_4 = \frac{|x_4|}{|A|}$

$x_4 = \begin{bmatrix} 2 & 0 & 1 & 3 & 4 \\ 0 & 0 & 3 & 9 & 4 \\ 1 & 0 & -4 & 7 & 0 \\ 3 & 4 & 0 & 5 & 2 \\ 0 & 2 & 3 & -4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 1 & 3 & 4 \\ 0 & 0 & 3 & 9 & 4 \\ 0 & 0 & -\frac{9}{2} & \frac{11}{2} & -2 \\ 0 & 4 & -3/2 & 1/2 & -4 \\ 0 & 2 & 3 & -4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 1 & 3 & 4 \\ 0 & 4 & -3/2 & 1/2 & -4 \\ 0 & 0 & -9/2 & \frac{11}{2} & -2 \\ 0 & 0 & 3 & 9 & 4 \\ 0 & 2 & 3 & -4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 1 & 3 & 4 \\ 0 & 4 & -3/2 & 1/2 & -4 \\ 0 & 0 & -9/2 & \frac{11}{2} & -2 \\ 0 & 0 & 3 & 9 & 4 \\ 0 & 0 & \frac{15}{4} & -\frac{17}{4} & 3 \end{bmatrix}$

($\frac{-1}{2} R_1$ added to R_3, $\frac{-3}{2} R_1$ added to R_4)

* (Swapped R_2 and R_4, det. changes sign)

(−1/2 R_2 added to R_4)

$\begin{bmatrix} 2 & 0 & 1 & 3 & 4 \\ 0 & 4 & -3/2 & 1/2 & -4 \\ 0 & 0 & -\frac{9}{2} & \frac{11}{2} & -2 \\ 0 & 0 & 0 & \frac{38}{3} & 8/3 \\ 0 & 0 & 0 & \frac{1}{3} & 4/3 \end{bmatrix}$

($\frac{2}{3} R_3$ added to R_4, $\frac{5}{6} R_3$ added to R_4)

$\begin{bmatrix} 2 & 0 & 1 & 3 & 4 \\ 0 & 4 & -3/2 & 1/2 & -4 \\ 0 & 0 & -9/2 & \frac{11}{2} & -2 \\ 0 & 0 & 0 & \frac{38}{3} & 8/3 \\ 0 & 0 & 0 & 0 & \frac{24}{19} \end{bmatrix}$

$2 \cdot 4 \cdot \frac{-9}{2} \cdot \frac{38}{3} \cdot \frac{24}{19} = 576$

$|x_4| = 576$

$X_4 = \frac{576}{336} = \frac{12}{7}$

Kaan Karataş
150200081

5) $$\begin{bmatrix}2&3&7\\-2&x&-11\\0&-3&x\end{bmatrix} \longrightarrow \begin{bmatrix}2&3&7\\0&x+3&-4\\0&-3&x\end{bmatrix} \longrightarrow \begin{bmatrix}2&3&7\\0&x+3&-4\\0&0&x-\frac{12}{x+3}\end{bmatrix}$$

(R_1 added to R_2) ($\frac{3}{x+3}$ R_2 added to R_3)

∇ $2 \cdot (x+3) \cdot \left(x - \frac{12}{x+3}\right)$ must be 0 for the matrix to be non invertible.

• $x = \frac{12}{(x+3)} \rightarrow (x)(x+3) = 12 \rightarrow x^2+3x-12=0$ $\quad$ X can be $\frac{-3+\sqrt{57}}{2}$ or $\frac{-3-\sqrt{57}}{2}$

• $x \neq -3$ as it makes the equation undefined.

6) $$A = \begin{bmatrix}3&2&3&5\\0&2&4&2\\0&0&-2&-1\\0&0&0&1\end{bmatrix} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}3&2&3&5\\0&2&4&2\\0&0&1&0\\0&0&0&1\end{bmatrix} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-\frac{1}{2}&-\frac{1}{2}\\0&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}3&2&3&5\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \begin{bmatrix}1&0&0&0\\0&\frac{1}{2}&1&0\\0&0&-\frac{1}{2}&-\frac{1}{2}\\0&0&0&1\end{bmatrix}$$

(R_4 added to R_3 then $R_3 \cdot \frac{-1}{2}$)

(-2 R_4 added to R_2, -4 R_3 added to R_2, then $R_2 \cdot \frac{1}{2}$)

$$A^{-1} = \begin{bmatrix}\frac{1}{3}&-\frac{1}{3}&-\frac{1}{6}&-\frac{7}{6}\\0&\frac{1}{2}&1&0\\0&0&-\frac{1}{2}&-\frac{1}{2}\\0&0&0&1\end{bmatrix} \Leftarrow \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \begin{bmatrix}\frac{1}{3}&-\frac{1}{3}&-\frac{1}{6}&-\frac{7}{6}\\0&\frac{1}{2}&1&0\\0&0&-\frac{1}{2}&-\frac{1}{2}\\0&0&0&1\end{bmatrix}$$

(-5 R_4 added to R_1, -3 R_3 added to R_1, -2 R_2 added to R_1, then $R_1 \cdot \frac{1}{3}$)

* Since A is in lower triangular form, A^{-1} will be lower triangular too. So the lower part is already in identity shape and requires almost no computation.

7) $$A = \begin{bmatrix}1&2&7&6\\-4&-7&-17&-13\\2&2&11&7\\-3&1&-16&-21\end{bmatrix} \longrightarrow \begin{bmatrix}1&2&7&6\\0&1&11&11\\0&-2&-3&-5\\0&7&5&-3\end{bmatrix} \longrightarrow \begin{bmatrix}1&2&7&6\\0&1&11&11\\0&0&19&17\\0&0&-72&80\end{bmatrix} \longrightarrow \begin{bmatrix}1&2&7&6\\0&1&11&11\\0&0&19&17\\0&0&0&\frac{-296}{19}\end{bmatrix}$$

(4 R_1 added to R_2, -2 R_1 added to R_3, 3 R_1 added to R_4) (2 R_2 added to R_3, -7 R_2 added to R_4) ($\frac{72}{19}$ R_3 added to R_4)

$$1 \cdot 1 \cdot 19 \cdot \frac{-296}{19} = -296$$

Kaan Karataş
150200081

8)

$$A=\begin{bmatrix}5&-1&0\\-10&2&7\\0&-3&4\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&-\frac{1}{5}&0\\-10&2&7\\0&-3&4\end{bmatrix}\begin{bmatrix}1/5&0&0\\0&1&0\\0&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&-1/5&0\\0&0&7\\0&-3&4\end{bmatrix}\begin{bmatrix}1/5&0&0\\2&1&0\\0&0&1\end{bmatrix}$$

$(R_1 \cdot \frac{1}{5})$ $(10 R_1$ added to $R_2)$

$$\begin{bmatrix}1&-\frac{1}{5}&0\\0&1&-\frac{4}{3}\\0&0&7\end{bmatrix}\begin{bmatrix}\frac{1}{5}&0&0\\0&0&-\frac{1}{3}\\2&1&0\end{bmatrix}\rightarrow\begin{bmatrix}1&0&-\frac{4}{15}\\0&1&-\frac{4}{3}\\0&0&7\end{bmatrix}\begin{bmatrix}1/5&0&-\frac{1}{15}\\0&0&-1/3\\2&1&0\end{bmatrix}\rightarrow\begin{bmatrix}1&0&-\frac{4}{15}\\0&1&-\frac{4}{3}\\0&0&1\end{bmatrix}\begin{bmatrix}1/5&0&-1/15\\0&0&-1/3\\2/7&1/7&0\end{bmatrix}$$

(R_2 and R_3 swapped then new $R_2 \cdot -\frac{1}{3}$) $(\frac{1}{5} R_2$ added to $R_1)$ $(R_3 \cdot \frac{1}{7})$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\frac{29}{105}&\frac{4}{105}&-\frac{1}{15}\\\frac{8}{21}&\frac{4}{21}&-1/3\\\frac{2}{7}&\frac{1}{7}&0\end{bmatrix}\quad A^{-1}=\begin{bmatrix}29/105&4/105&-1/15\\8/21&4/21&-1/3\\2/7&1/7&0\end{bmatrix}\quad (A^{-1})^T=\begin{bmatrix}29/105&8/21&2/7\\4/105&4/21&1/7\\-1/15&-1/3&0\end{bmatrix}$$

$\left(\begin{array}{l}\frac{4}{3} R_3 \text{ added to } R_2\\ \frac{4}{15} R_3 \text{ added to } R_1\end{array}\right)$

* Determinant of $((A^{-1})^T)^3$ is equal to the third power of the determinant of $(A^{-1})^T$

$$(A^{-1})^T=\begin{bmatrix}29/105&8/21&2/7\\4/105&4/21&1/7\\-1/15&-1/3&0\end{bmatrix}\rightarrow\begin{bmatrix}29/105&8/21&2/7\\0&4/29&3/29\\0&-7/29&2/29\end{bmatrix}\rightarrow\begin{bmatrix}29/105&8/21&2/7\\0&4/29&3/29\\0&0&1/4\end{bmatrix}$$

$\left(\begin{array}{l}-\frac{4}{29} R_1 \text{ added to } R_2\\ \frac{7}{29} R_1 \text{ added to } R_3\end{array}\right)$ $(\frac{7}{4} R_2$ added to $R_3)$

* determinant $= \frac{29}{105} \cdot \frac{4}{29} \cdot \frac{1}{4} = \frac{1}{105}$

So, the determinant of $((A^{-1})^T)^3 = \left(\frac{1}{105}\right)^3 = \frac{1}{1157625}$

9)

$$A=\begin{bmatrix}1&2&7&6\\2&6&8&20\\-5&7&11&7\\0&1&-3&4\end{bmatrix}$$

If we add the forth row to the first one, R_1 becomes half of R_2. Then if we add minus two times the new R_1 to R_2, R_2 becomes a row of 0's which makes the determinant 0, and the matrix non-invertible.

* The new $R_1 = [1\ 3\ 4\ 10]$

Kaan Karataş
150200081

10) $\det(A) = -2$, $\det(B) = 1$ $\det(A^{-2}) = (-2)^{-2} = +\frac{1}{4}$ $\det(B^3) = 1^3 = 1$

$\det A^{-2}B^3 = \det(A^{-2}) \cdot \det(B^3) = \frac{1}{4}$

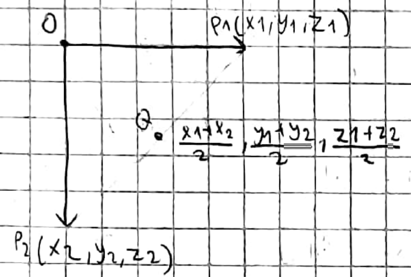
11)

$$\begin{bmatrix} x & y & z & w \\ 2x & y & 2z & 2w \\ x & y & 2z & 2w \\ -x & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} x & y & z & w \\ x & 0 & 0 & 0 \\ x & y & 2z & 2w \\ -x & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} x & y & z & w \\ -x & 0 & 0 & 0 \\ x & y & 2z & 2w \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

($-R_3$ added to R_2) (R_4 added to R_2) (optionally swapped R_4 and R_2)

* The matrix has a row of 0's. Therefore the determinant is 0, the matrix is non-invertible

12)



∇ OQ $\cdot$ ($OP_1 \times OP_2$) must be equal to 0 for the vectors to be coplanar.

$$(\vec{OP_1} \times \vec{OP_2}) = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$OQ \cdot (\vec{OP_2} \times \vec{OP_1}) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) \cdot (y_1z_2 - z_1y_2, x_1z_2 - z_1x_2, x_1y_2 - y_1x_2)$$

$$= (x_1y_1z_2 + x_2y_1z_2 - x_1y_2z_1 - x_2y_2z_1) + (x_1y_1z_2 + x_1y_2z_2 - x_2y_1z_1 - x_2y_2z_1)$$
$$+ (x_1y_2z_1 + x_1y_2z_2 - x_2y_1z_1 - x_2y_1z_2) \Big/ 2$$
$$= 0$$

13) Triangular inequality is $||A|| + ||B|| \geq ||A+B||$

$||x+y-z|| \leq ||x|| + ||y|| + ||z||$; we can write this inequality like $||x+y|| + ||-z|| \leq ||x|| + ||y|| + ||z||$. z's neutralize each other since $||-z||$ is equal to $||z||$. So we have: $||x+y|| \leq ||x|| + ||y||$ which is the triangular inequality. Thus the inequality is proven.

14) $a^T = [a_1\ a_2\ a_3]$ $\quad b^T = [b_1\ b_2\ b_3]$ $\quad c^T = [c_1\ c_2\ c_3]$

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

1)

$$b \times c = \begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = i \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - j \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + k \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$a \cdot (b \times c) = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$\det(A) = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} = a \cdot (b \times c)$$

2)

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \longrightarrow \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \longrightarrow - \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = A'$$

($R_1 - R_2$ swapped)

$$\det(A) \rightarrow -\det(A) \rightarrow \det(A) = \det(A')$$

$\forall$ $\det(A) = a \cdot (b \times c) = \det(A') = -b \cdot (a \times c)$

15) $\big|\, \|x\| + \|y\| \,\big| \leq \|x - y\|$

take $x = (3,0)$, $y = (1,0)$

$x - y = (2,0)$

$\|x - y\| = 2$ $\quad \big|\, \|x\| - \|y\| \,\big| = 2$

$2 = 2$

take $x = (0,2)$, $y = (2,0)$

$x - y = (-2,2)$

$\|x - y\| = 2\sqrt{2}$ $\quad \big|\, \|x\| - \|y\| \,\big| = 0$

$2\sqrt{2} > 0$

16) $\vec{a} = i + 2j + k$

plane $(\vec{n}) = i - 3j + 4k$

$$\text{projection } a = b \cdot \frac{a \cdot b}{\|b\| \cdot \|b\|} = (1,-3,4) \cdot \frac{(1,2,1) \cdot (1,-3,4)}{\sqrt{26} \cdot \sqrt{26}}$$

$$\downarrow$$

$$= \frac{1}{26}(1,-3,4)$$

$$\text{projection} = (1,2,1) - \left(\frac{-1}{26}, \frac{3}{26}, \frac{-4}{26}\right) = \left(\frac{27}{26}, \frac{49}{26}, \frac{30}{26}\right)$$

$$\text{unit norm vector} = \frac{a}{\|a\|} = \left(\frac{27}{\sqrt{4030}}, \frac{49}{\sqrt{4030}}, \frac{30}{\sqrt{4030}}\right)$$

17) Intersection of the planes $x+2y+2z=5$ and $x+y-3z=2$ is; $\begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 1 & 1 & -3 \end{vmatrix} = (-8, +5, -1)$

* Orthagonal to both intersection vector the vector, so $\begin{vmatrix} i & j & k \\ -8 & 5 & -1 \\ 1 & 2 & -1 \end{vmatrix} = -3i - 9j - 21k$

Thus, the unit norm vector is; $\left(\frac{\vec{U}}{\|U\|}\right)$

$\frac{-3}{\sqrt{531}} i, \frac{9}{\sqrt{531}} j, \frac{-21}{\sqrt{531}} k \Rightarrow \frac{-1}{\sqrt{59}}, \frac{3}{\sqrt{59}}, \frac{7}{\sqrt{59}}$

18) $\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ -1 & 1 & 2 \end{vmatrix} = 8i + 0j + 4k = \langle 8, 0, 4 \rangle$,

$x = 8t - 2$

$y = -1$

$z = 4t + 1$

* take $t = \frac{1}{2}$

$(2, -1, 3)$ passes

19) $P = (a, b, c)$, if $\vec{P_0 P}$ and $(2, 1, -5)$ are orthagonal, the dot product should be 0.

$(a-1, b-1, c+1) \cdot (2, 1, -5) = 0$

$2a - 2 + b - 1 - 5c - 5 = 0$

$2a + b - 5c = -8$ / $a = x+1$, $b = y+1$, $c = z-1$

$2x + y - 5z = -16$

→ Values must satisfy this equaiton to be $P \in \mathbb{R}^3$

20) Volume of the parallelpiped is $a \cdot (b \times c)$ $b \times c = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 2 & 3 & 2 \end{vmatrix} = \langle 6, 4, 0 \rangle$

$(2,0,0) \cdot (6,4,0) = 12$

Surface area $= 2(\underbrace{\|a \times b\|}_{6} + \underbrace{\|a \times c\|}_{2\sqrt{13}} + \underbrace{\|b \times c\|}_{2\sqrt{13}}) = 12 + 8\sqrt{13}$

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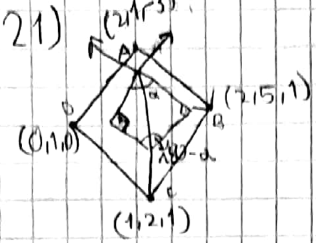
21)



$\langle 2-0, 1-1, -3-0 \rangle = \langle 2, 0, -3 \rangle$

$\langle 1-0, 2-1, 1-0 \rangle = \langle 1, 1, 1 \rangle$

$\begin{vmatrix} i & j & k \\ 2 & 0 & -3 \\ 1 & 1 & 1 \end{vmatrix} = (-3, 5, -2)$

$\langle 2-2, 5-1, 1-(-3) \rangle = \langle 0, 4, 4 \rangle$

$\langle 1-0, 2-1, 1-0 \rangle = \langle 1, 3, 0 \rangle$

$\begin{vmatrix} i & j & k \\ 0 & 4 & 4 \\ 1 & 3 & 0 \end{vmatrix} = (12, -4, 4)$

$$\cos(180-\alpha) = \frac{N_1 \cdot N_2}{\|N_1\| \cdot \|N_2\|} \rightarrow \frac{(-3,5,-2) \cdot (12,-4,4)}{\sqrt{38} \cdot \sqrt{176}} = \frac{-64}{4\sqrt{418}} = \frac{-16}{\sqrt{418}}$$

$180 - \alpha = 141{,}5 \qquad \alpha = 38{,}5$

22) Distance between a point and a plane = Projection $\overrightarrow{PQ}_{n} = \left\| \frac{|\overrightarrow{PQ} \cdot n|}{\|n\|} \right\|$

Plane $= x - 3y + z = 1$, normal $= \langle 1, -3, 1 \rangle$, the point $= (0, 1, -4)$

$$\left| \frac{(0-x)1 + (1-y) \cdot -3 + (-4-z)\, 1}{\sqrt{11}} \right| = \left| \frac{\overbrace{-x+3y-z}^{-1} - 7}{\sqrt{11}} \right| = \frac{|-8|}{\sqrt{11}} = \frac{8}{\sqrt{11}}$$

23) Scalar product of vectors is 0.

$(a+b) \cdot (a-b) = 0 \qquad a^2 - b^2 = 0 \qquad \frac{a^2}{b^2} = 1 \qquad \sqrt{\frac{a^2}{b^2}} = 1$ (Magnitudes can't be negative so the ratio can NOT be -1)

24) Three vectors form a triangle if the sum of them is equal to 0. In the question, the equation is $a+b+c=0$ so the three vectors form a triangle. Triangle is a 2 dimentional form, which satisfies all the vectors are on the same plane, in other words coplanar.

KashteColor

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25) $\|a+b\|^2 + \|a-b\|^2 = 2(\|a\|^2 + \|b\|^2)$

$$\|a\| + 2ab + \|b\|^2 + \|a\|^2 - 2ab + \|b\|^2 = 2(\|a\|^2 + \|b\|^2)$$

$$2\|a\|^2 + 2\|b\|^2 = 2(\|a\|^2 + \|b\|^2)$$

26) $\overrightarrow{P_0P_2} \cdot (\overrightarrow{P_0P_1} \times \overrightarrow{P_2P_3})$ is equal to the volume of the vectors and is equal to 0. The volume is 0, thus the vectors form a 2 dimentional and are coplanar. And since $\overrightarrow{P_0P_1} \times \overrightarrow{P_2P_3} \neq 0$ we know that $\overrightarrow{P_2P_3}$ and $\overrightarrow{P_0P_1}$ are not parallell. The vectors are coplanar and not parallell, therefore they must intersect at one point.

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