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Linear Algebra & Applications

1)

a) $$\begin{bmatrix}1&1&-3\\0&1&-1\\-1&1&1\end{bmatrix}_{3\times3}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}_{3\times1}=\begin{bmatrix}4\\3\\2\end{bmatrix}_{3\times1}\Rightarrow\left[\begin{array}{ccc|c}1&1&-3&4\\0&1&-1&3\\-1&1&1&2\end{array}\right]\xrightarrow{R_1+R_3\to R_3}\left[\begin{array}{ccc|c}1&1&-3&4\\0&1&-1&3\\0&2&-2&6\end{array}\right]$$

$$\xrightarrow{R_3\cdot\frac{1}{2}}\left[\begin{array}{ccc|c}1&1&-3&4\\0&1&-1&3\\0&1&-1&3\end{array}\right]\xrightarrow{R_3+R_2\to R_2}\left[\begin{array}{ccc|c}1&1&-3&4\\0&1&-1&3\\0&0&0&0\end{array}\right]$$

The last one becomes a 0 row which makes the system unsolvable.

- 3 unkowns
  2 equations

b) $$\begin{bmatrix}2&1&-3\\1&0&1\\-1&2&-4\end{bmatrix}_{3\times3}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}_{3\times1}=\begin{bmatrix}1\\1\\5\end{bmatrix}_{3\times1}\Rightarrow\left[\begin{array}{ccc|c}2&1&-3&1\\1&0&1&1\\-1&2&-4&5\end{array}\right]\xrightarrow{R_3+R_1\to R_1}\left[\begin{array}{ccc|c}1&3&-7&6\\1&0&1&1\\-1&2&-4&5\end{array}\right]\xrightarrow{R_3+R_2\to R_2}\left[\begin{array}{ccc|c}1&3&-7&6\\0&2&-3&6\\-1&2&-4&5\end{array}\right]$$

$$\xrightarrow{R_1+R_3\to R_3}\left[\begin{array}{ccc|c}1&3&-7&6\\0&2&-3&6\\0&5&-11&11\end{array}\right]\xrightarrow{R_2\cdot\frac{1}{2}}\left[\begin{array}{ccc|c}1&3&-7&6\\0&1&\frac{-3}{2}&3\\0&5&-11&11\end{array}\right]\xrightarrow{-5R_2+R_3\to R_3}\left[\begin{array}{ccc|c}1&3&-7&6\\0&1&\frac{-3}{2}&3\\0&0&\frac{-7}{2}&-4\end{array}\right]\xrightarrow{R_3\cdot\frac{-2}{7}}\left[\begin{array}{ccc|c}1&3&-7&6\\0&1&\frac{-3}{2}&3\\0&0&1&\frac{8}{7}\end{array}\right]$$

$$\xrightarrow{\frac{3}{2}R_3+R_2\to R_2}\left[\begin{array}{ccc|c}1&3&-7&6\\0&1&0&\frac{33}{7}\\0&0&1&\frac{8}{7}\end{array}\right]\xrightarrow{-3R_2+7R_3+R_1\to R_1}\left[\begin{array}{ccc|c}1&0&0&\frac{-1}{7}\\0&1&0&\frac{33}{7}\\0&0&1&\frac{8}{7}\end{array}\right]$$

c) $$\begin{bmatrix}1&1&1&1\\2&0&1&2\end{bmatrix}_{2\times4}\cdot\begin{bmatrix}x\\y\\z\\w\end{bmatrix}_{4\times1}=\begin{bmatrix}1\\5\end{bmatrix}_{2\times1}$$

$\Rightarrow$ The number of equations is less than the number of the unknowns, hence the system has infinite solutions.

Kaan Karataş 150200081

2) $$A = \begin{bmatrix} 2 & 2 & 4 \\ -1 & 2 & 1 \\ -2 & -1 & 4 \end{bmatrix} \xrightarrow{R_1+R_3\to R_3} \begin{bmatrix} 2 & 2 & 4 \\ -1 & 2 & 1 \\ 0 & 1 & 8 \end{bmatrix} \xrightarrow{R_1\cdot\frac{1}{2}} \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 8 \end{bmatrix} \xrightarrow{R_1+R_2\to R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 3 \\ 0 & 1 & 8 \end{bmatrix} \xrightarrow{R_2\cdot\frac{1}{3}} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 8 \end{bmatrix}$$

$$\xrightarrow{-R_2+R_3\to R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 8 \end{bmatrix} \xrightarrow{R_3\cdot\frac{1}{8}} \underbrace{\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Row Echelon Form}} \xrightarrow{-R_3+R_2\to R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-2R_3+R_1\to R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-R_2+R_1\to R_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Reduced Row Echelon Form}}$$

3) A is in row echelon form since the first non-zero number on each row is 1 and in each row, the leading 1 is on farther right than the upper row.

B is in reduced row echelon form for the same reasons and the 0 row is at the bottom and each column that has a leading 1 has 0's everywhere else.

4) The products of AA^T and A^TA always exist because:
- If A is an $m\times n$ matrix, then A^T is an $n\times m$ matrix. In order to multiply two matrices the column number of the first must be equal to the row number of the second.

• $A_{m\times \underline{n}} \cdot A^T_{\underline{n}\times m} = (AA^T)_{m\times m}$ • $A^T_{n\times \underline{m}} \cdot A_{\underline{m}\times n} = (AA^T)_{n\times n}$

5) $A_{3\times4}$ $B_{2\times4}$ $C_{2\times3}$, so $((CAB^T)^{-1} \Rightarrow C_{2\times3}\cdot A_{3\times4} = X_{2\times4}$, $B^T = 4\times2$

$X_{2\times4}\cdot B^T_{4\times2} = y_{2\times2} \Rightarrow (y_{2\times2})^{-1}$ = Inverse of $y_{2\times2}$ and it has the same size with y. Thus the answer is 2×2

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150200081

6) For $n\times n$ matrices, following statements must be all true all false.

- The matrix A has an inverse
- $Ax=0$ has only trivial solution
- Reduced row echelon form of the matrix is I_n
- The matrix is expressible as a product of elementary matrices.

$Ax=b \rightarrow A^{-1}AX = 0A^{-1} \rightarrow X=0$,

X is given $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

The equation and the statement contradict each other as seen in the 2nd requirement. So all statements are false, the matrix has no inverse.

7) $A\cdot B = \underset{1\times 2}{[A_1 \mid A_2]} \cdot \underset{2\times 1}{\begin{bmatrix}B_1\\ \hline B_2\end{bmatrix}} = \underset{1\times 1}{[A_1B_1 + A_2B_2]}$

$$A_1\cdot B_1 = \overset{A_1}{\begin{bmatrix}1&0\\0&1\end{bmatrix}} \cdot \overset{B_1}{\begin{bmatrix}1&1\\0&1\end{bmatrix}} = \begin{bmatrix}1&1\\0&1\end{bmatrix}$$

$$A_2\cdot B_2 = \overset{A_2}{\begin{bmatrix}0&0&3\\0&1&1\end{bmatrix}} \cdot \overset{B_2}{\begin{bmatrix}-1&2\\0&-1\\2&2\end{bmatrix}} = \begin{bmatrix}6&6\\2&1\end{bmatrix}$$

$$A_1B_1 + A_2B_2 = \begin{bmatrix}7&7\\2&2\end{bmatrix}$$

$$\begin{bmatrix}1&1\\0&1\end{bmatrix} + \begin{bmatrix}6&6\\2&1\end{bmatrix}$$

$$B.A = \begin{bmatrix}B_1\\ \hline B_2\end{bmatrix} \cdot [A_1 \mid A_2] = \left[\begin{array}{c|c}B_1A_1 & B_1A_2\\ \hline B_2A_1 & B_2A_2\end{array}\right]$$

$$B_1.A_1 = \begin{bmatrix}1&1\\0&1\end{bmatrix} \qquad B_2A_1 = \begin{bmatrix}-1&2\\0&-1\\2&2\end{bmatrix}$$

$$B_1A_2 = \begin{bmatrix}0&1&4\\0&1&1\end{bmatrix} \qquad B_2A_2 = \begin{bmatrix}0&2&-1\\0&-1&-1\\0&2&8\end{bmatrix}$$

$$\Rightarrow B.A = \left[\begin{array}{cc|ccc}1&1&0&1&4\\0&1&0&1&1\\ \hline -1&2&0&2&-1\\0&-1&0&-1&-1\\2&2&0&2&8\end{array}\right]$$

$i-j>1$

8.1) $$\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\\a_{41}&a_{42}&a_{43}&a_{44}\end{bmatrix} \Rightarrow \begin{bmatrix}*&*&*&*\\0&*&*&*\\0&0&*&*\\0&0&0&*\end{bmatrix}$$

let the $*$'s be 1 $\longrightarrow$

When specifying A, we must be sure that it has no row full of 0's to be invertible.

$$\overset{A}{} \qquad \overset{A^{-1}}{}$$

$$\left[\begin{array}{cccc|cccc}1&1&1&1&1&0&0&0\\0&1&1&1&0&1&0&0\\0&0&1&1&0&0&1&0\\0&0&0&1&0&0&0&1\end{array}\right]$$

$\downarrow -R_2+R_1\rightarrow R_1$

$$\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&1&1&0&1&0&0\\0&0&1&1&0&0&1&0\\0&0&0&1&0&0&0&1\end{array}\right] \xrightarrow{-R_3+R_2\rightarrow R_2} \left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&0&0&0&1&-1&0\\0&0&1&1&0&0&1&0\\0&0&0&1&0&0&0&1\end{array}\right] \xrightarrow{-R_4+R_3\rightarrow R_3} \left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&0&0&0&1&-1&0\\0&0&1&0&0&0&1&-1\\0&0&0&1&0&0&0&1\end{array}\right]$$

$$\underset{I}{} \qquad \underset{A^{-1}}{}$$

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150200081

8.2) $Ax = I$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A^{-1} \cdot Ax = I \cdot A^{-1}$, $x = A^{-1}$

$[A \mid I] \Rightarrow [I \mid A^{-1}]$

$$\underbrace{\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right]}_{A \quad I} \xrightarrow{-\frac{c}{a} \cdot R_1 + R_2 \to R_2} \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & d - \frac{bc}{a} & -\frac{c}{a} & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \cdot \frac{1}{a}} \left[\begin{array}{cc|cc} 1 & b/a & 1/a & 0 \\ 0 & d - \frac{bc}{a} & -\frac{c}{a} & 1 \end{array}\right] \xrightarrow{R_2 \cdot \frac{a}{ad - bc}} \left[\begin{array}{cc|cc} 1 & b/a & \frac{1}{a} & 0 \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right] \xrightarrow{-\frac{b}{a} R_2 + R_1 \to R_1} \left[\begin{array}{cc|cc} 1 & 0 & \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$I \quad A^{-1}$

Thus; $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

9) $(A-I)^2 + A = 0$

$A^2 - 2AI + I^2 + A = 0$ (multiply both sides with A^{-1})

$A \cdot \underbrace{A \cdot A^{-1}}_{I} - 2 \cdot \underbrace{A \cdot I \cdot A^{-1}}_{I} + I^2 A^{-1} + \underbrace{A \cdot A^{-1}}_{I}$ (multiplying something with I is equal to itself)

$I \cdot A - 2 \cdot I^2 + A^{-1} + I$

$A - 2I + A^{-1} + I \Rightarrow A^{-1} = I - A$

10) $A \cdot A^{-1} = I$, Let us take $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ as an example matrix with a row of 0's

And $A^{-1} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, so $\underbrace{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{I}$

$ae + bg = 1$ $af + bh = 0$

$0 \cdot e + 0 \cdot g = 0$ $0 \cdot f + 0 \cdot h = 1$

This equation is equal to $0=1$, therefore impossible. Inverse of a matrix with a row of zero does not exist.

Kaan Karaloş
150200081

11) If $(A+B)^2 = A^2+2AB+B^2$, then A and B must be equal or inverses of each other since,

$$(A+B)^2 = AA + AB + BA + BB$$

$$2AB \;?\; \Rightarrow AB = BA \text{ so } \quad \text{I: } A = B \qquad \text{II: } A.B = I \Rightarrow A = B^{-1} / B = A^{-1} \qquad \text{III: } A = I / B = I$$

• Therefore A and B aren't necessarily need to be inverses of each other as they can be equal to each other, or one of them can be Identity matrix. But it is possible for them to be inverses of each other.

12) • $\begin{bmatrix}2&0&1\\1&1&0\\1&5&1\end{bmatrix} \xrightarrow[\text{1}]{2R_1+R_3\to R_3} \begin{bmatrix}2&0&1\\1&1&0\\5&5&3\end{bmatrix} \xrightarrow[\text{2}]{-5R_2+R_3\to R_3} \begin{bmatrix}2&0&1\\1&1&0\\0&0&3\end{bmatrix} \xrightarrow[\text{3}]{R_3\cdot\frac{1}{3}} \begin{bmatrix}2&0&1\\1&1&0\\0&0&1\end{bmatrix} \xrightarrow[\text{4}]{-R_3+R_1\to R_1} \begin{bmatrix}2&0&0\\1&1&0\\0&0&1\end{bmatrix}$

$\xrightarrow[\text{5}]{\frac{1}{2}\cdot R_1} \begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix} \xrightarrow[\text{6}]{-R_1+R_2\to R_2} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

6 rows operations

• $\begin{bmatrix}2&0&1\\1&1&0\\1&5&1\end{bmatrix} \xrightarrow[\text{1}]{-R_3+R_1\to R_1} \begin{bmatrix}1&-5&0\\1&1&0\\1&5&1\end{bmatrix} \xrightarrow[\text{2}]{R_1+R_3\to R_3} \begin{bmatrix}1&-5&0\\1&1&0\\2&0&1\end{bmatrix} \xrightarrow[\text{3}]{R_1-R_2\to R_2} \begin{bmatrix}1&-5&0\\0&-6&0\\2&0&1\end{bmatrix} \xrightarrow[\text{4}]{-\frac{5}{6}R_2+R_1\to R_1} \begin{bmatrix}1&0&0\\0&-6&0\\2&0&1\end{bmatrix}$

$\xrightarrow[\text{5}]{-2R_1+R_3\to R_3} \begin{bmatrix}1&0&0\\0&-6&0\\0&0&1\end{bmatrix} \xrightarrow[\text{6}]{R_2\cdot\frac{-1}{6}} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

• Changing the order of some operations mixes the sequence and might create different sequences. Even if we don't consider them, as seen in these two sequences there are totally different ways to solve the question. Therefore the sequence is NOT unique.

13) If the sum of two rows add up to the other, the reduced row echelon form of the system will have a row of 0's which makes the matrix non invertible.

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14)

a) $A=\left[\begin{array}{cccc|cccc} 2&0&0&2&1&0&0&0\\ 1&2&0&0&0&1&0&0\\ 0&1&1&0&0&0&1&0\\ 0&1&0&0&0&0&0&1 \end{array}\right] \xrightarrow{R_1\cdot\frac{1}{2}} \left[\begin{array}{cccc|cccc} 1&0&0&1&\frac{1}{2}&0&0&0\\ 1&2&0&0&0&1&0&0\\ 0&1&1&0&0&0&1&0\\ 0&1&0&0&0&0&0&1 \end{array}\right] \xrightarrow{R_4+R_1\to R_4} \left[\begin{array}{cccc|cccc} 1&0&0&1&\frac{1}{2}&0&0&0\\ 1&2&0&0&0&1&0&0\\ 0&1&1&0&0&0&1&0\\ 1&1&0&1&\frac{1}{2}&0&0&1 \end{array}\right]$

$\xrightarrow{-R_2+R_4\to R_4} \left[\begin{array}{cccc|cccc} 1&0&0&1&\frac{1}{2}&0&0&0\\ 1&2&0&0&0&1&0&0\\ 0&1&1&0&0&0&1&0\\ 0&-1&0&1&\frac{1}{2}&-1&0&1 \end{array}\right] \xrightarrow{-R_4+R_1\to R_1} \left[\begin{array}{cccc|cccc} 1&1&0&0&0&1&0&-1\\ 1&2&0&0&0&1&0&0\\ 0&1&1&0&0&0&1&0\\ 0&-1&0&1&\frac{1}{2}&-1&0&1 \end{array}\right] \xrightarrow{-R_1+R_2\to R_2} \left[\begin{array}{cccc|cccc} 1&1&0&0&0&1&0&-1\\ 0&1&0&0&0&0&0&1\\ 0&1&1&0&0&0&1&0\\ 0&-1&0&1&\frac{1}{2}&-1&0&1 \end{array}\right]$

$\xrightarrow{-R_2+R_3\to R_3} \left[\begin{array}{cccc|cccc} 1&1&0&0&0&1&0&-1\\ 0&1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1&-1\\ 0&-1&0&1&\frac{1}{2}&-1&0&1 \end{array}\right] \xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{cccc|cccc} 1&0&0&0&0&1&0&-2\\ 0&1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1&-1\\ 0&-1&0&1&\frac{1}{2}&-1&0&1 \end{array}\right] \xrightarrow{R_2+R_4\to R_4} \left[\begin{array}{cccc|cccc} 1&0&0&0&0&1&0&-2\\ 0&1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1&-1\\ 0&0&0&1&\frac{1}{2}&-1&0&2 \end{array}\right]$

b) $A=\left[\begin{array}{ccc|ccc} 1&0&-1&1&0&0\\ 0&2&2&0&1&0\\ 1&1&0&0&0&1 \end{array}\right] \xrightarrow{-R_1+R_3\to R_3} \left[\begin{array}{ccc|ccc} 1&0&-1&1&0&0\\ 0&2&2&0&1&0\\ 0&1&1&-1&0&1 \end{array}\right] \xrightarrow{R_2\cdot\frac{1}{2}} \left[\begin{array}{ccc|ccc} 1&0&-1&1&0&0\\ 0&1&1&0&\frac{1}{2}&0\\ 0&1&1&-1&0&1 \end{array}\right]$

$\xrightarrow{-R_2+R_3\to R_3} \left[\begin{array}{ccc|ccc} 1&0&-1&1&0&0\\ 0&1&1&0&\frac{1}{2}&0\\ 0&0&0&-1&-\frac{1}{2}&1 \end{array}\right]$

$\underset{o}{\nabla}$ Matrix has a row of 0's which means the determinant is 0. Therefore this matrix has no inverse.

c) $A=\left[\begin{array}{ccc|ccc} c&0&a&1&0&0\\ 0&b&0&0&1&0\\ c&b&a&0&0&1 \end{array}\right] \xrightarrow{-R_1+R_3\to R_3} \left[\begin{array}{ccc|ccc} c&0&a&1&0&0\\ 0&b&0&0&1&0\\ 0&b&0&-1&0&1 \end{array}\right] \xrightarrow{-R_2+R_3\to R_3} \left[\begin{array}{ccc|ccc} c&0&a&1&0&0\\ 0&b&0&0&1&0\\ 0&0&0&-1&-1&1 \end{array}\right]$

$\underset{o}{\nabla}$ This matrix also has a row of 0's and 0 determinant. Thus it has no inverse.

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15) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \xrightarrow{?} B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

∇ It is impossible to turn A into B with only elementary row operations since we need to swap columns of unknowns.

$A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} \xrightarrow{?} B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} \xrightarrow{-3R_2+R_1 \to R_1} \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \xrightarrow{R_1 \cdot \frac{1}{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \xrightarrow{R_2 \cdot 2} \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \xrightarrow{3R_1+R_2 \to R_2} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

16) $A = \begin{bmatrix} 2 & 0 & 4 & -4 & -1 \\ 0 & 3 & 0 & -2 & 2 \\ 0 & 0 & 5 & 0 & 1 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-R_5+R_3 \to R_3} \begin{bmatrix} 2 & 0 & 4 & -4 & -1 \\ 0 & 3 & 0 & -2 & +2 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 \cdot -\frac{1}{3}} \begin{bmatrix} 2 & 0 & 4 & -4 & -1 \\ 0 & 3 & 0 & -2 & 2 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \cdot \frac{1}{5}} \begin{bmatrix} 2 & 0 & 4 & -4 & -1 \\ 0 & 3 & 0 & -2 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$2R_4+R_2 \to R_2$
$-2R_5+R_2 \to R_2$

$\begin{bmatrix} 2 & 0 & 4 & -4 & -1 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \cdot \frac{1}{3}} \begin{bmatrix} 2 & 0 & 4 & -4 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 4R_3 + 4R_4 + R_5 \to R_1} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \cdot \frac{1}{2}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

∇ Reduced Row Echelon Form of A is a digonal matrix. All of the diagonal entries are non-zero which means the determinant of the matrix $\neq 0$ so the matrix is invertible.

17) $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & x & 3 & -1 \\ 0 & 0 & 1 & 0 \\ x+1 & 2 & 4 & 1 \end{bmatrix}$

∇ If the inverse does not exist then determinant must be 0.

$A_{33} = (-1)^6 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 1 & x & -1 \\ x+1 & 2 & 1 \end{vmatrix}$

$\downarrow$

$\begin{vmatrix} 1 & 0 & 1 \\ 1 & x & -1 \\ x+1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & x & -1 \end{vmatrix} = (x+2+0) - (x.(x+1)-2+0)$

$x+2-x^2-x+2$

$\hookrightarrow x^2 = 4 \to |x| = 2$

$|x| = 2 \to x$ can be equal to 2 or -2.

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18) A matrix is in reduced row echelon form if;

- If a row does not consist entirely of 0's, the first number is a 1 which is a leader
- Rows entirely consisting of 0's are grouped together at the bottom.
- In successive rows not entirely consisting of 0's, leader 1 of lower row is further right than the higher row
- Each column with a leader 1 has 0's everywhere else.

So all possible forms are;

i) $\begin{matrix}0&0&0&0\\0&0&0&0\end{matrix} \Big/ \begin{matrix}1&0&*&*\\0&1&*&*\end{matrix} \Big/ \begin{matrix}0&1&0&*\\0&0&1&*\end{matrix} \Big/ \begin{matrix}0&0&1&0\\0&0&0&1\end{matrix} \Big/ \begin{matrix}1&*&*&*\\0&0&0&0\end{matrix} \Big/ \begin{matrix}0&1&*&*\\0&0&0&0\end{matrix}$

$\begin{matrix}0&0&1&*\\0&0&0&0\end{matrix} \Big/ \begin{matrix}0&0&0&1\\0&0&0&0\end{matrix} \Big/ \begin{matrix}1&*&0&*\\0&0&1&*\end{matrix} \Big/ \begin{matrix}0&1&*&0\\0&0&0&1\end{matrix} \Big/ \begin{matrix}1&*&*&0\\0&0&0&1\end{matrix}$

ii) $\begin{matrix}0&0&0\\0&0&0\\0&0&0\\0&0&0\end{matrix} \Big/ \begin{matrix}1&*&*\\0&0&0\\0&0&0\\0&0&0\end{matrix} \Big/ \begin{matrix}0&1&*\\0&0&0\\0&0&0\\0&0&0\end{matrix} \Big/ \begin{matrix}0&0&1\\0&0&0\\0&0&0\\0&0&0\end{matrix} \Big/ \begin{matrix}1&0&*\\0&1&0\\0&0&0\\0&0&0\end{matrix} \Big/ \begin{matrix}1&*&0\\0&*&1\\0&0&0\\0&0&0\end{matrix} \Big/ \begin{matrix}1&0&*\\0&1&*\\0&0&0\\0&0&0\end{matrix} \Big/ \begin{matrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{matrix}$

19) $A = \begin{bmatrix}2&-1&0&3\\0&3&0&0\\0&1&2&3\\0&1&1&1\end{bmatrix}$

$2 \cdot A_{11} = \det(A)$

$-1^{(2)} \cdot \begin{vmatrix}3&0&0\\1&2&3\\1&1&1\end{vmatrix} \rightarrow \begin{vmatrix}3&0&0\\1&2&3\\1&1&1\\3&0&0\\1&2&3\end{vmatrix} = (6+0+0)-(0+9+0) = -3$

$A_{12} = 1 \cdot -3 = -3$

$\det(A) = 2 \cdot -3 = -6$

21) $A = \begin{bmatrix}1&-1&0\\0&-1&0\\0&0&1\end{bmatrix}$

$A^2 = \underbrace{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}}_{I}$ $\Rightarrow$ Multiplying a matrix with I is equal to itself.

$A^{120} = \underbrace{A.A}_{I}.\underbrace{A.A}_{I} \ldots A = (A^2)^{60} = I^{60} = I$