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1) $\|\text{proj}_{u\times v}(u)\| = ?$ $\underset{0}{\nabla}$ Since u is orthogonal to the vector $u\times v$, the result must be equal to 0.

2) $\|u-v\| = \sqrt{3} \rightarrow \|u-v\|^2 = 3 \rightarrow \sqrt{u^2 - 2uv\cos\theta + v^2}^{\,2} = 3$

$u^2 + v^2 - 2u.v.\cos\theta = 3$

$1 + 1 - 2\cos\theta = 3$ $\qquad \|u\times v\| = u\,v\,\sin\theta = \dfrac{\sqrt{3}}{2}$

$\cos\theta = -\dfrac{1}{2}$ $\qquad$ (1, 1, $\frac{\sqrt{3}}{2}$)

$\sin\theta = \dfrac{\sqrt{3}}{2}$

3) Cauchy Schwarz $\Rightarrow |u.v| \leq \|u\|.\|v\|$

Assume that $u = (1, \frac{1}{2}, \frac{1}{3} \dots \frac{1}{n})$ and $v = (\frac{1}{e}, \frac{1}{e^2}, \frac{1}{e^3} \dots \frac{1}{e^n})$

$\|u\| = \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots \frac{1}{n^2}}$ $\qquad \|v\| = \sqrt{\frac{1}{e^2} + \frac{1}{e^4} + \frac{1}{e^6} \dots \frac{1}{e^{2n}}}$

$= \sqrt{\sum_{n=1}^{\infty} \frac{1}{n^2}} = \sum_{n=1}^{\infty} \frac{1}{n} = \sqrt{\frac{\pi^2}{6}}$ $\qquad = \sqrt{\sum_{n=1}^{\infty} (e^2)^{-n}} = \sqrt{\frac{e^2}{1-e}} = \sum_{n=1}^{\infty} e^{-n}$

$\sum_{n=1}^{\infty} \frac{e^{-n}}{n} = \sqrt{\frac{\pi^2}{6}} \cdot \sqrt{\frac{e^2}{1-e^2}} = \frac{\pi.e}{\sqrt{6}.\sqrt{1-e^2}}$

4) $c_1(1,1,2,1) + c_2(1,-2,-2,0) + c_3(0,1,1,-2) = (3,-1,1,4)$

$c_1 + c_2 = 3$

$c_1 - 2c_2 + c_3 = -1$

$2c_1 - 2c_2 + c_3 = 1$

$c_1 - 2c_3 = 4$

$\Big\rangle\ c_1 = 2,\ c_2 = 1,\ c_3 = -1$

* It can be represented with these coefficients

5) $\left[\begin{array}{ccc|c} -1 & -1 & 1 & w_1 \\ 2 & -1 & -3 & w_2 \\ 0 & -3 & -1 & w_3 \end{array}\right] \xrightarrow[(R_2+2R_1)]{} \left[\begin{array}{ccc|c} -1 & -1 & 1 & w_1 \\ 0 & -3 & -1 & w_2+2w_1 \\ 0 & -3 & -1 & w_3 \end{array}\right] \xrightarrow[(R_3-R_2)]{} \left[\begin{array}{ccc|c} -1 & -1 & 1 & w_1 \\ 0 & -3 & -1 & w_2+2w_1 \\ 0 & 0 & 0 & w_3-w_2-2w_1 \end{array}\right]$

* The domain is R^3

The codomain is R^3

* The standard matrix is; $\begin{bmatrix} -1 & -1 & 1 \\ 2 & -1 & -3 \\ 0 & -3 & -1 \end{bmatrix}$

* The range is any set of (w_1, w_2, w_3) satisfying $2w_1 + w_2 = w_3$

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6)

a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3+2\sqrt{3} \\ -2+3\sqrt{3} \end{bmatrix}$

e) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 \end{bmatrix}$

7) $T_1 \circ T_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos & -\sin \\ 0 & \sin & \cos \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sin & \cos \end{bmatrix}$

$T_2 \circ T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos & -\sin \\ 0 & \sin & \cos \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sin \\ 0 & 0 & \cos \end{bmatrix}$

$\Big\} \; T_1 \circ T_2 \neq T_2 \circ T_1$

8)

a) $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & -1 \\ 5 & 6 & -4 \end{bmatrix} \xrightarrow[(R_3-5R_1)]{(R_2-2R_1)} \begin{bmatrix} 1 & 1 & -1 \\ 0 & -5 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow[(\text{then } R_2 \cdot \frac{1}{6})]{(R_2+5R_3)} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow[(\text{then } R_2-R_3)]{(R_2, R_3 \text{ swap})} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[(R_1+R_3)]{(R_1-R_2)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ independent one-to-one

b) $\begin{bmatrix} 1 & -3 & -1 \\ 1 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \xrightarrow[(R_3-R_1)]{(R_2-R_1)} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 4 & 0 \\ 0 & 3 & 0 \end{bmatrix} \xrightarrow{(R_3-\frac{3}{4}R_2)} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$ No pivots every column, row of zeros, determinant 0, ∞ solutions, Many-to-one, Has no inverse.

inverse of a

$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 2 & -3 & -1 & 0 & 1 & 0 \\ 5 & 6 & -4 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -5 & -1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -5 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -5 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -5 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -5 & 0 & 1 \\ 0 & -5 & 1 & -2 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 6 & 0 & -1 \\ 0 & 1 & 1 & -5 & 0 & 1 \\ 0 & 0 & 6 & -27 & 1 & 5 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 6 & 0 & -1 \\ 0 & 1 & 1 & -5 & 0 & 1 \\ 0 & 0 & 1 & -\frac{9}{2} & \frac{1}{6} & \frac{5}{6} \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & -1/2 & -1/6 & 1/6 \\ 0 & 0 & 1 & -\frac{9}{2} & \frac{1}{6} & \frac{5}{6} \end{array}\right]$ inverse transformation of a is $\begin{bmatrix} -3 & \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{1}{6} \\ -\frac{9}{2} & \frac{1}{6} & \frac{5}{6} \end{bmatrix}$

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The range is a subset of the codomain which is R^3.

9) $T(x_1,x_2) + T(a,b) \overset{?}{=} T(x_1+a, x_2+b)$

$(2x_1+2a+x_2+b, 2-x_1-a) \neq (2x_1+2a+x_2+b, 1-x_1-a)$

*The transformation is not linear.

10) $\underline{L}x + \underline{t} = y \qquad A \cdot \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 1 \end{bmatrix}$

$$\begin{bmatrix} \underline{L} & \underline{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} \underline{L}x + \underline{t} \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 1 \end{bmatrix}$$

11) $\begin{bmatrix} 1/2 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{-\sqrt{3}}{2} & 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 0 & 0 \\ \frac{-\sqrt{3}}{2} & 0 & 1/2 \end{bmatrix} \rightsquigarrow$ The standard matrix

12) T_1 = many to one, dependent, has row of 0's, non invertible

T_2 = one-to-one, independent, invertible.

Assume $T_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad T_1 \circ T_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad T_2 \circ T_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

determinant = 0 determinant = 0

not invertible

13) Vector space $\rightarrow$ $a, b, a+b, ka \in V$

a)

$a = (x_1, y_1, z_1) \qquad b = (x_2, y_2, z_2) \qquad a+b = (x_1+x_2, y_1+y_2, z_1+z_2)$

$x+z = 2y \Rightarrow x_1 + z_1 = 2y_1 , \; x_2 + z_2 = 2y_2 , \; x_1+x_2+z_1+z_2 = 2y_1+2y_2$

$\downarrow$

$kx_1 + kz_1 = 2ky_1$

$kx_2 + kz_2 = 2ky_2$ $\longrightarrow$ Vector Space

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b) $x^2+y^2+z^2=0 \Rightarrow x_1^2+y_1^2+z_1^2=0$, $x_2^2+y_2^2+z_2^2=0$

$(x_1+x_2)^2+(y_1+y_2)^2+(z_1+z_2)^2 = x_1^2+2x_1x_2+x_2^2+y_1^2+2y_1y_2+y_2^2+z_1^2+2z_1z_2+z_2^2$

$\downarrow$

* As in real space all the values can only $\leftarrow 2x_1x_2+2z_1z_2+2y_1y_2$ be 0, it is a vector space.

c) $x-y=1 \Rightarrow x_1-y_1=1$, $x_2-y_2=1$

$\rightarrow x_1+x_2-y_1-y_2=2$

$\downarrow$

Not Vector Space

d) $x+y<1 \Rightarrow x_1+y_1<1$, $x_2+y_2<1$

$\rightarrow x_1+y_1+x_2+y_2<2$

$\downarrow$

Not Vector Space

14)

a- $\begin{bmatrix} a & b \\ a+b & 0 \end{bmatrix} + \begin{bmatrix} d & e \\ d+e & 0 \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ a+b+d+e & 0 \end{bmatrix}$, $k \cdot \begin{bmatrix} a & b \\ a+b & 0 \end{bmatrix} = \begin{bmatrix} ka & kb \\ k(a+b) & 0 \end{bmatrix} \Rightarrow$ Subspace

b- $\begin{bmatrix} a & b \\ ab & 0 \end{bmatrix} + \begin{bmatrix} d & e \\ de & 0 \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ ab+de & 0 \end{bmatrix} \rightarrow$ Not Subspace

c- $\begin{bmatrix} a & b \\ a+b & 1 \end{bmatrix} + \begin{bmatrix} d & e \\ d+e & 1 \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ a+b+d+e & 2 \end{bmatrix} \rightarrow$ Not Subspace

15) Subspace axioms require functions to ensure $k \cdot w \in W$. If $k=0$, then the result becomes equal to zero which is not a subspace.

16) Because of the requirements of subspace axioms, $k \cdot w \in W$ must be true. So if $k=0$, the result is equal to 0 matrix and not invertible. Hence, they can not form a subspace.

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Let's assume $V = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Not invertible $W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Not invertible $V+W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ invertible

$v \in W$, $w \in W$ but $v+w \notin W$ so they don't form a subspace.

17)

a) $\begin{bmatrix} 1 & 0 & -2 & 2 \\ 2 & 1 & 4 & -4 \\ 1 & -1 & 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 2 & 1 & 4 & -4 & 0 \\ 1 & -1 & 3 & 0 & 0 \end{array}\right] \xrightarrow[(R_3 - R_1)]{(R_2 - 2R_1)} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 8 & -8 & 0 \\ 0 & -1 & 5 & -2 & 0 \end{array}\right] \xrightarrow{(R_3 + R_2)} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 8 & -8 & 0 \\ 0 & 0 & 13 & -10 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & \frac{-24}{13} & 0 \\ 0 & 0 & 1 & \frac{-10}{13} & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{6}{13} & 0 \\ 0 & 1 & 0 & \frac{-24}{13} & 0 \\ 0 & 0 & 1 & \frac{-10}{13} & 0 \end{array}\right] \Rightarrow$$

$(R_3 \cdot \frac{1}{13})$ (then $R_2 - 8R_3$) $\quad (R_1 + 2R_3)$

$x_1 = -\frac{6}{13} x_4$

$x_2 = \frac{24}{13} x_3$

$x_3 = 10/13\, x_4$

b) $\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 2 & 1 & 4 & -4 & 0 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 8 & -8 & 0 \end{array}\right]$ $\quad x_1 = 2x_3 - 2x_4$, $x_2 = 8x_4 - 8x_3$

c) $$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 2 & 1 & 4 & -4 & 0 \\ 0 & 1 & 6 & -6 & 0 \end{array}\right] \xrightarrow{(R_2 - 2R_1)} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 8 & -8 & 0 \\ 0 & 1 & 6 & -6 & 0 \end{array}\right] \xrightarrow{(R_3 - R_2)} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 8 & -8 & 0 \\ 0 & 0 & -2 & 2 & 0 \end{array}\right] \xrightarrow[(R_2 - 8R_3)]{(R_3 \cdot \frac{1}{-2})} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right] \xrightarrow{(R_1 + 2R_3)} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$x_1 = 0$
$x_2 = 0$
$x_4 = x_3$

d) $$\begin{bmatrix} 1 & 0 & -2 \\ 2 & 0 & -4 \\ 0 & 1 & 6 \\ -1 & 0 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$(R_2 - 2R_1)$ $(R_4 + R_1)$ $(R_4 \cdot \frac{1}{6})$ $\quad$ (R_2 swapped R_3) (R_3 swapped R_4) $(R_2 - 6R_4)$

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

18)

a) $$\left[\begin{array}{ccc|c} 4 & -2 & 0 & x \\ 3 & 0 & 3 & y \\ 2 & -1 & 0 & z \\ 1 & 0 & 1 & w \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 4 & -2 & 0 & x \\ 3 & 0 & 3 & y \\ 0 & 0 & 0 & z - \frac{x}{2} \\ 0 & 0 & 0 & w - y/3 \end{array}\right]$$

$(R_3 - \frac{1}{2}R_1)$ $(R_4 - \frac{1}{3}R_2)$

$x, y, z, w \in R^4$

$w = \frac{y}{3}$

$z = \frac{x}{2}$

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150200081

b) $c_1(4,3,2,1) + c_2(-2,0,-1,0) + c_3(0,3,0,1) = 0$

$4c_1 = 2c_2 \quad c_2 = 2c_1$

$c_1 = -c_3 \quad c_3 = -c_1$

* The equation has more than only trivial solution. Also, the Matrix form has row of 0's. Thus, dependent.

c) $c_1(0,3,0,1) + c_2(-2,0,-1,0) = (4,3,2,1)$

$a_2 = -2 \quad a_1 = 1$

* Basis of R^4 must have 4 vectors so two more vectors must be added which are not in the span.

$$\begin{bmatrix}-2\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\3\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

1.9) $W = \begin{vmatrix} F(x) & F_1(x) \\ F'(x) & F_1'(x) \end{vmatrix}$ $\quad$ $W \neq$ dependent $\quad$ $W =$ independent $\quad$ $\begin{vmatrix} \cos(2x) & \cos^2 x - \sin^2 x \\ -2\sin(2x) & -2\sin x\cos x - 2\sin x\cos x \end{vmatrix}$

$-\cos(2x) \cdot 4\sin x \cdot \cos x + (\cos 2x) 4\sin x \cdot \cos x = 0 \rightarrow$ dependent

20) $\begin{bmatrix} 1 & 1 & 1 & 0 & k_1 \\ -2 & 2 & 0 & 0 & k_2 \\ 0 & 4 & -1 & 7 & k_3 \end{bmatrix} \cdot \begin{bmatrix} a\\b\\c\\d \end{bmatrix} = \begin{bmatrix} k_1\\k_2\\k_3 \end{bmatrix}$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & k_1 \\ -2 & 2 & 0 & 0 & k_2 \\ 0 & 4 & -1 & 7 & k_3 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & k_1 \\ 0 & 2 & 1 & 0 & k_1 + k_2/2 \\ 0 & 4 & -1 & 7 & k_3 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & k_1 \\ 0 & 1 & \frac{1}{2} & 0 & k_1/2 + k_2/4 \\ 0 & 0 & -3 & 7 & k_3 - k_2 - 2k_1 \end{array}\right]$$

$(R_2 \cdot \frac{1}{2})$
(then $R_2 + R_1$)

$(R_2 \cdot \frac{1}{2})$
$(R_3 - 4R_2)$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & k_1 \\ 0 & 1 & \frac{1}{2} & 0 & k_1/2 + k_2/4 \\ 0 & 0 & 1 & \frac{-7}{3} & \frac{k_3 - k_2 - 2k_1}{-3} \end{array}\right]$$

Basis is

$$\begin{bmatrix}1\\-2\\0\end{bmatrix}, \begin{bmatrix}1\\2\\4\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}$$ with dimension R^3

$$a_1\begin{pmatrix}1\\-2\\0\end{pmatrix} + a_2\begin{pmatrix}1\\2\\4\end{pmatrix} + a_3\begin{pmatrix}1\\0\\-1\end{pmatrix} = \begin{pmatrix}0\\0\\7\end{pmatrix}$$

$a_1 + a_2 + a_3 = 0$

$a_1 = +a_2$

$4a_2 - a_3 = 7$

$a_1 = a_2$

$a_1 = a_2 = \frac{7}{6}$

$a_3 = \frac{-7}{3}$

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21) Dependent Vectors → $c_1v_1+c_2v_2+c_3v_3+c_4v_4=0$ has more than only trivial solution. And if one of the remaining vectors is the combination of other two vectors, then there will be a possibility of the set being independent.

- Basis of R^4 must consist of 4 vectors, therefore 3 vectors are not enough to be basis hence no basis.

22) R^5 can be written with 5 independent vectors at most. Thus at most only 1 vector can be added.

23) U, V and W are not on the same plane if they are independent. $\text{Proj}_{u\times v} w$ can be equal to w which makes $\text{proj}_{u\times v} w$, v and u independent.

If $u\times v$ and w are orthagonal, then $\text{proj}_{u\times v} w$ becomes 0. Thus $\text{proj}_{u\times v} w$ and u, v becomes dependent because the sets contains containing zero vector is linearly dependent.

24) Basis of 2x2 matrices consists of 4 vectors. If the set has 5, at least 1 vector is linearly dependent. Hence it can be witten as a linear combination of the other vectors.

- Basis of 3x3 matrices consists of 9 vectors. If the set has 5 vectors, then every vector can be independent. Therefore, they can not be written as a combination of the other vectors.

25)

a) 4 vectors are not enough to be basis for R^6 since basis of R^6 has 6 vectors.

b) $\begin{vmatrix} -x & y \\ x & -y \end{vmatrix} \rightarrow \begin{vmatrix} 0 & 0 \\ x & -y \end{vmatrix}$ determinant is 0, dependent → can't form basis for R^2

c) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 2 & 0 & 0 \\ -1 & 2 & 6 & 0 \end{array}\right] \xrightarrow[(R_3+R_1)]{(R_2-R_1)} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & -3 & 9 & 0 \end{array}\right] \xrightarrow[(R_3 \cdot -1/3)]{(R_2 \cdot \frac{1}{4})} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \xrightarrow[(R_1+R_2-R_3)]{(R_3+3R_2)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$a_1=0$
$a_2=0$ → Only the trivial solution so independent can be basis for R^3.
$a_3=0$

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d) Any set containing 0 vector is dependent. Therefore can't be basis for R^n.

26)
$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & 1 & 5 \\ 0 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ -2 & 1 & 5 & 1 \\ 0 & -1 & 4 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ 0 & 1 & 6 & 2 \\ 0 & -1 & 4 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 10 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & 3/10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7/10 \\ 0 & 1 & 0 & 2/10 \\ 0 & 0 & 1 & 3/10 \end{array}\right] \Rightarrow k_1 = \frac{7}{20} \quad k_2 = \frac{1}{5} \quad k_3 = \frac{3}{10}$$

$$\frac{7}{20}a + \frac{1}{5}b + \frac{3}{10}c = w$$

$(7/20, 1/5, 3/20) \rightarrow$ coordinates

27)

a) All the given vectors are in 3 dimension which can have an linearly independent set of maximum 3 vectors. 4 vectors can not form a linearly independent set.

b)
$$\begin{bmatrix} 2 & 0 & 1 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 3 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \left[\begin{array}{cccc|c} 2 & 0 & 1 & 1 & x \\ -1 & 1 & -1 & 0 & y \\ -1 & 3 & -2 & 1 & z \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1/2 & 1/2 & x/2 \\ -1 & 1 & -1 & 0 & y \\ 0 & 2 & -1 & 1 & z-y \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1/2 & 1/2 & x/2 \\ 1 & -1 & 1 & 0 & -y \\ 0 & 1 & -1/2 & 1/2 & \frac{(z-y)}{2} \end{array}\right]$$

$(R_1 \cdot \frac{1}{2}), (R_3 - R_2)$ $\quad$ $(R_2 \cdot -1)$ $(R_3 \cdot \frac{1}{2})$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1/2 & 1/2 & x/2 \\ 0 & -1 & 1/2 & -1/2 & -y-x/2 \\ 0 & 1 & -1/2 & 1/2 & (z-y)/2 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1/2 & 1/2 & x/2 \\ 0 & 1 & -1/2 & 1/2 & y+x/2 \\ 0 & 0 & 0 & 0 & (z-3y)/2 - x/2 \end{array}\right] \quad z = 3y + x$$

$(R_2 - R_1)$ $\quad$ $(R_3 + R_2)$ $(R_2 \cdot -1)$ $\quad$ $x \rightarrow a \quad y \rightarrow b$

$$\begin{pmatrix} a \\ b \\ a+3b \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

c) The space has 2 basis vectors thus the dimension is 2.

d) $\text{span} = \begin{pmatrix} a \\ b \\ a+3b \end{pmatrix} \rightsquigarrow a\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

e) $a(1,0,1) + b(0,1,3) \stackrel{?}{=} (2,2,0)$ $\quad$ $a=2,\ b=2,\ a+3b \neq 0$

$\downarrow$

* The system is inconsistent so w is not in the span of $\{v_1, v_2, v_3, v_4\}$

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28) $x - y + z = 2$, $x \Rightarrow a_1$ $y \Rightarrow a_2$ $z \Rightarrow 2 - a_1 + a_2$

$a_1\begin{pmatrix}1\\0\\-1\end{pmatrix} + a_2\begin{pmatrix}0\\1\\1\end{pmatrix} + \begin{pmatrix}0\\0\\2\end{pmatrix}$ → Since the system has a constant, it can't form a basis

29)

$$\left[\begin{array}{cccc|c}1&1&-1&1&0\\2&-1&2&-1&0\end{array}\right] \to \underset{(R_2-2R_1)}{\left[\begin{array}{cccc|c}1&1&-1&1&0\\0&-3&4&-3&0\end{array}\right]} \to \underset{(R_2\cdot -1/3)}{\left[\begin{array}{cccc|c}1&1&-1&1&0\\0&1&-\frac{4}{3}&1&0\end{array}\right]} \to \underset{(R_1-R_2)}{\left[\begin{array}{cccc|c}1&0&1/3&0&0\\0&1&-\frac{4}{3}&1&0\end{array}\right]}$$

$x \to a_1$ $y \to a_2$
$-3x \to a_3$
$-4x-y \to a_4$

$a_1 + \frac{a_3}{3} = 0$

$a_2 - \frac{4a_3}{3} + a_4 = 0$

$$\begin{pmatrix}a_1\\a_2\\a_3\\a_4\end{pmatrix} = \begin{pmatrix}x\\y\\-3x\\-4x-y\end{pmatrix} = x\begin{pmatrix}1\\0\\-3\\-4\end{pmatrix} + y\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

Dimension is 2.

30) $t(3,1,-3) + (-1, -\frac{1}{3}, 1)$

$$\begin{pmatrix}3t\\t\\-3t\end{pmatrix} = t\begin{pmatrix}3\\1\\-3\end{pmatrix}$$

→ Basis is $\begin{pmatrix}3\\1\\-3\end{pmatrix}$

31)

$$\begin{bmatrix}1&1&0&3\\0&-2&1&2\\1&3&-1&1\end{bmatrix} \cdot \begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix} \Rightarrow \left[\begin{array}{cccc|c}1&1&0&3&x\\0&-2&1&2&y\\1&3&1&1&z\end{array}\right] \to \left[\begin{array}{cccc|c}1&1&0&3&x\\0&-2&&&y\\0&2&-1&-2&z-x\end{array}\right] \to \left[\begin{array}{cccc|c}1&1&0&3&x\\0&-2&1&2&y\\0&0&0&0&z+y-x\end{array}\right]$$

$-x+y+z=0$ $x \to a$ $y \to b$ $z \to a-b$

$$\begin{pmatrix}a\\b\\a-b\end{pmatrix} = a\begin{pmatrix}1\\0\\1\end{pmatrix} + b\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

Span of subspace P^2

* Since the vector number is 2 and not 3, it does not span second order polynomials.

$\begin{pmatrix}0\\0\\1\end{pmatrix}$ is not in the span of the vectors, so if it is added to the basis, it will span the second order polynomials.