DIP Course
MiniProject 1
Sampling Theory

Part 1: Study the Fourier Transform of Images

1. Create image series (image1, image2, …, image11) by using the below code:

```
fm = 0.12;
fn = 0.06;
image1 = zeros(127);
image2 = zeros(127);
image2((127+1)/2,(127+1)/2) = 255;
image3= zeros(127);
image3(:,(127+1)/2) = 255;
image4 = zeros(127);
image4((127+1)/2,:) = 255;
image5 = zeros(127);
for n = 1:127
 for m = 1:127
 image5(m,n) = 108+97*cos(2*pi*fm*n);
 end
end
image6 = zeros(127);
for n = 1:127
 for m = 1:127
 image6(m,n) = 108+97*cos(2*pi*fn*m);
 end
end
image7 = zeros(127);
for n = 1:127
 for m = 1:127
 image7(m,n) = 108+97*cos(2*pi*fm*n)*cos(2*pi*fn*m);
 end
end
image8 = zeros(127);
image8(:,((127+1)/2)-15:((127+1)/2)+15) = 255;
image9 = zeros(127);
image9(((127+1)/2)-15:((127+1)/2)+15,:) = 255;
image10= 255*ones(60,25);
image10 = padarray(image10, [80 100],0);
image11 = zeros(15);
image11 = padarray(image11, [1 1],255);
image11 = padarray(image11, [(127-17)/2 (127-17)/2],0);
image12= imrotate(image10,45);
```

2. Find the DFT of the image series.
3. Show the original images and their absolute value of DFT.
4. Is the DFT of images looking like you expected? Describe the result.

Part2: Working with Phase Angles

The Fourier transform is complex, so it can be expressed in polar form as

$$F(u,v) = |F(u,v)| e^{j\phi(u,v)}$$

Here,

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

is the spectrum and

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

is the phase angle. The purposes of this projects are (1) to show that the phase angle carries information about the location of image elements, (2) that the spectrum carries information regarding contrast and intensity transitions, and (3) that phase information is dominant over the spectrum regarding visual appearance of an image. Download woman . tif and test_pattern .tif (call them `f` and `g`).
It is recommended that you look up functions complex and angle before starting the project.

1. Compute the spectrum and the phase angle of image f and show the phase image. Then compute the inverse transform using only the phase term (i.e., ifft2 of $e^{j\phi(u,v)}$] and show the result. Note how the image is void of contrast, but clearly shows the woman's face and other features.

2. Compute the inverse using only the magnitude term (i.e., ifft2 of $|F(u,v)|$). Display the result and note how little structural information it contains.

3. Compute the spectrum of g. Then computer the inverse FFT using the spectrum of g for the real part and the phase of f for the imaginary part. Display the result and note how the details from the phase component dominate the image.

Part 3: Study the effect of interpolators for reconstruction

1. Create 2 synthetic images of 256x256 with f = 0.03 and f = 0.45 using following equation:

$$F(n,m) = 128+127\cos(2\pi fm)\cos(2\pi fn)$$

2. What is the Nyquist frequency for this function?
3. Show and compare images and explain what you see.
4. Find and show the DFT (Discrete Fourier Transform) of two images and explain what you see.
5. Simulate the interpolators of zero-order and bicubic of these two images and show them.
6. Use the DFT of images in 5 to explain the results.

Note: To simulate the signals in continuous domain, one approach consists of digital interpolation of discrete signals by increasing the number of points by a factor n (for this example n=4 is sufficient). The command *imresize* of MatLab allows you to do this interpolation in two modes: zero-order ('nearest') and bicubic ('bicubic').

Part 4: Evaluation of a solution for reducing the aliasing effect

1. Create a synthetic image of 256x256 consists of two regions of black and white separated by a line:

```
for i = 1:256
y(257-i) = 1.5*i-1.5;
end
for n = 1:256
    for m = 1:256
        if (n < y(m))
            fint(n,m) = 0;
        else
            fint(n,m) = 255;
        end
    end
end
```

2. Show the image and note that there are the visual distortions in image. What is the reason?
3. Increase the sampling frequency of the image by a factor 4 (resulting an image of 1024x1024). Note: use only the definition of function in 1 and not the interpolation!
4. Utilize an interpolator (command *imresize* of MatLab in bicubic mode) to reduce the size of image into 256x256.
5. Show and compare this image with the original image of 256x256 in 1.
6. Why the image in 4 has a better visual quality? Justify your answer using sampling theory.
7. Would the same result be obtained if we used zero-order interpolator in 4? Why?

Note: some Matlab commands for this project: help images, imshow, imread, imresize, fft, fft2, fftshift, mat2gray.