

DIP Course

Homework 1

Lecture 1

Discrete convolution in two dimensions

1- Determine the convolution of $x(m, n)$ with each of the following arrays, where the boxed element denotes the (0, 0) location.

$$x(m, n) = \begin{bmatrix} 1 & 4 & 1 \\ \boxed{2} & 5 & 3 \end{bmatrix}$$

a) $\begin{bmatrix} 0 & -1 & 1 \\ -1 & \boxed{4} & -1 \\ 0 & -1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 \\ \boxed{-1} & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & \boxed{1} \\ -1 & 1 \end{bmatrix}$

d) compare the result of b and c.

Fourier Transform

2- An image determined by the following function and is sampled such that $\Delta_x = \Delta_y = 0.125$

$$f(x, y) = 4 \sin(8x + 10y)$$

a) compute the Fourier transform $F(u, v)$ and plot $F(u, v)$.

b) What is the Nyquist frequency for this function? Has the Nyquist rate been met?

c) compute the sampled image spectrum $F_s(u, v)$ and plot $F_s(u, v)$ accurately (Up to 5 harmony).

d) apply a low-pass filter to the sampled image spectrum and determine the reconstructed image $\hat{f}(x, y)$.

(Low-pass filter: have a rectangular region of support with cutoff frequency at half the sampling frequencies).