Matematical Analysis

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Contents

1 Calculate Area under Curve

Find the are under the cruve bounded by $y = 8 + 2x - x^2$ & $2x - y + 4 = 0$.

1.1 Finding Points of Intersection

To find the area between the curves, we first need to determine where they intersect. We set the two equations equal to each other:

$$8 + 2x - x^2 = 2x + 4$$

Simplifying:

$$\begin{aligned} 8 + 2x - x^2 &= 2x + 4 \\ 8 - x^2 &= 4 \\ -x^2 &= -4 \\ x^2 &= 4 \end{aligned}$$

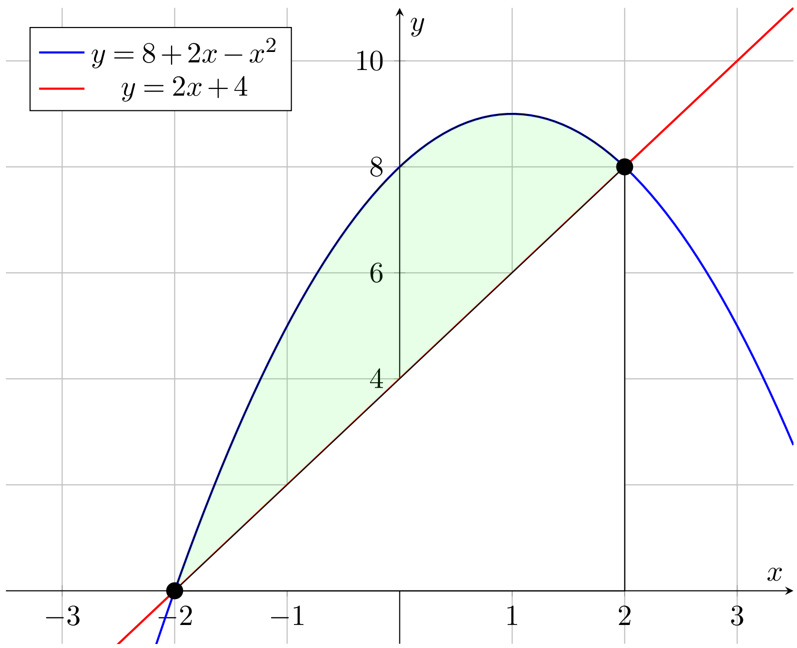
Therefore:

$$x = \pm 2$$

The curves intersect at $x = -2$ and $x = 2$.

1.2 Graphical Representation

The shaded region represents the area to be calculated between the parabola and the line.



1.3 Computing the Area

The area between two curves is computed by subtracting the area under the lower curve from the area under the upper curve:

$$S_1 - S_2$$

$$S_1 = \int_{-2}^{2} 8 + 2x - x^2 dx = \left[8x + x^2 - \frac{x^3}{3}\right]_{-2}^{2} = \frac{80}{3}$$

$$S_2 = \int_{-2}^{2} 2x + 4 dx = \left[x^2 + 4x\right]_{-2}^{2} = 16$$

$$S_1 - S_2 = \boxed{\frac{32}{3}}$$

2 Show that function satisfies the given formula

Given the function:

$$z = \ln x^2 + xy + y^2$$

Show that this function satisfies the formula below:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2$$

To verify this, we'll compute both partial derivatives and plug them into the formula.

2.1 Differentiate function for x

We apply the chain rule to find the partial derivative with respect to x:

$$\frac{\partial}{\partial x}\ln x^2 + xy + y^2 = \frac{\partial(x^2 + xy + y^2)}{\partial x} \cdot \frac{1}{x^2 + xy + y^2} = \frac{2x + y}{x^2 + xy + y^2}$$

2.2 Differentiate function for y

Similarily:

$$\frac{\partial}{\partial y}\ln x^2 + xy + y^2 = \frac{x + 2y}{x^2 + xy + y^2}$$

2.3 Plugging back to the formula

Now we substitute the partial derivatives into the original formula and simplify:

$$x\frac{2x + y}{x^2 + xy + y^2} + y\frac{x + 2y}{x^2 + xy + y^2} = \frac{x(2x + y) + y(x + 2y)}{x^2 + xy + y^2}$$

$$= \frac{2x^2 + xy + yx + 2y^2}{x^2 + xy + y^2} = \frac{2x^2 + 2xy + 2y^2}{x^2 + xy + y^2} = \frac{2(x^2 + xy + y^2)}{x^2 + xy + y^2} = \boxed{2}$$

3 Find Convergence/Divergence of the given Series

Given the Series:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$$

Find wether or not this series converges or diverges.

3.1 Find a similar series to compare to

We use the limit comparison test. Because as n grows larger, $\frac{1}{\sqrt{n(n+1)}}$ behaves similarly to $\frac{1}{n}$, we choose:

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

First, we verify that this comparison series diverges using the integral test:

$$\int_{1}^{\infty} \frac{1}{x} dx = [\ln |x|]_{1}^{\infty} = \infty$$

Since the integral diverges, the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ also diverges.

3.2 Limit comparison test

We compute the limit of the ratio of the two series:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n(n+1)}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n(n+1)}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2(1+\frac{1}{n})}} = \lim_{n\to\infty} \frac{n}{n\sqrt{1+\frac{1}{n}}} = \frac{1}{\sqrt{1+0}} = 1$$

Since the limit equals a positive constant, both series share the same convergence behavior. Because $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, we conclude:

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}} \text{ diverges}}$$