ExaGO™ Users Manual

Version 1.4.0

Contents

Chapter 1

Introduction

The Exascale Grid Optimization (ExaGO™) toolkit is an open source package for solving large-scale power grid optimization problems on parallel and distributed architectures, particularly targeted for exascale machines with heteregenous architectures (GPU). ExaGO™ is written in C/C++ and makes heavy use of the functionality provided by the PETSc[**?**] library. It uses RAJA [**?**]and Umpire [**?**]libraries for execution on the GPU and can make use of several optimization solvers - Ipopt [**?**], HiOp [**?, ?**]- for solving the optimization problems.

All ExaGO™ applications use a nonlinear formulation based on full AC optimal power flow. The different applications available with `ExaGO` are listed in Table **??**

Table 1.1: `ExaGO` applications

Application	Description	Notes
OPFLOW	AC optimal power flow	
SCOPFLOW	Security-constrained AC optimal power flow	Uses TCOPFLOW for multi-period contingencies
TCOPFLOW	Multi-period AC optimal power flow	
SOPFLOW	Stochastic AC optimal power flow	Uses SCOPFLOW for multi-contingency scenarios

While `ExaGO` is targeted for making use of distributed computing environments and GPUs, its full support is still under development. Table **??** lists the architecure support for ExaGO applications.

Table 1.2: `ExaGO` application execution on different hardware

Application	CPU (serial)	CPU (parallel)	GPU
OPFLOW	Y	N	Y
SCOPFLOW	Y	Y	Y
SOPFLOW	Y	Y	Y
TCOPFLOW	Y	N	N

Together, these applications cater to problems spanning the dimensions of security (contingencies), stochasticity, and time as shown in Figure **??**.
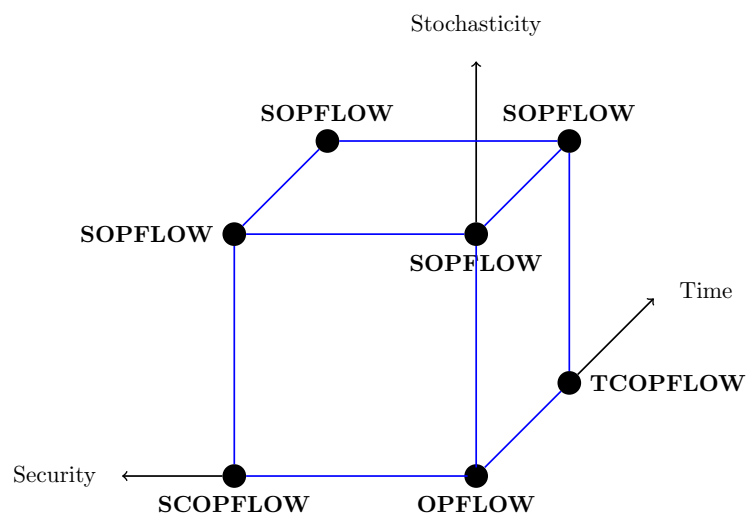
Figure 1.1: `ExaGO` provides applications along the dimensions of security (contingencies), time, and stochasticity. The label vertices denote different `ExaGO` applications available.

Chapter 2

Getting Started

2.1 System requirements

`ExaGO` is currently only built on 64b OSX and Linux machines, compiled with GCC >= 7.3. We build `ExaGO` on Intel and IBM Power9 architectures.

2.2 Prerequisites

This section assumes that you already have the `ExaGO` source code, and that the environment variable `EXAGODIR` is the directory of the `ExaGO` source code. `ExaGO` may be acquired via the PNNL git repository linked here, like so:

```
> git clone https://gitlab.pnnl.gov/exasgd/frameworks/exago.git exago
> export EXAGODIR=$PWD/exago
```

Paths to installations of third party software in examples are abbreviated with placeholder paths. For example, `/path/to/cuda` is a placeholder for a path to a valid `CUDA Toolkit` installation.

2.3 Dependencies

`ExaGO` has the dependencies listed in table **??**. The versions of dependencies listed are those we have tested with, though newer version may also be compatible.

Table 2.1: Dependency Table

Dependency	Version Constraints	Mandatory	Notes
PETSc [?]	3.16.x	✓	Core dependency
CMake	3.18	✓	Only a build dependency
MPI	3.1.3		Only tested with `openmpi` and `spectrummpi`
Ipopt [?]	3.12		
HiOp [?, ?]	0.5.1		Prefer dynamically linked
RAJA [?]	0.14.0		
Umpire [?]	6.0..0		Only when RAJA is enabled
MAGMA	2.6.1		Only when GPU acceleration is enabled
CUDA Toolkit	10.2.89 through 11.1		Only when GPU acceleration is enabled

These may all be toggled via `CMake`, which will be discussed in the section Building and installation.

2.3.1 Notes on environment modules

Many of the dependencies are available via environment modules on institutional clusters. To get additional information on your institution's clusters, please ask your institution's system administrators. Some end-to-end examples in this document will use system-specific modules and are not expected to expected to run on other clusters.

For example, the modules needed to build and run `ExaGO` on Newell, an IBM Power9 PNNL cluster, are as follows:

```
> module load gcc/7.4.0
> module load openmpi/3.1.5
> module load cuda/10.2
> module load magma/2.5.2_cuda10.2
> module load metis/5.1.0
> module load cmake/3.16.4
```

2.3.2 Additional Notes on GPU Accelerators

`CUDA` is currently the only GPU accelerator platform `ExaGO` fully supports.

2.3.3 Additional Notes on Umpire

`Umpire` is an implicit dependency of `RAJA`. If a user enables `RAJA`, they must also provide a valid installation of `Umpire`. Additionally, if a user would like to run `ExaGO` with `RAJA` and without `CUDA`, they must provide a CPU-only build of `Umpire` since an `Umpire` build with `CUDA` enabled will link against `CUDA`.

2.4 Building and installation

2.4.1 Default Build

`ExaGO` may be built with a standard `CMake` workflow:

Listing 2.1: Example `CMake` workflow

```
> cd $EXAGODIR
> export BUILDDIR=$PWD/build INSTALLDIR=$PWD/install
> mkdir $BUILDDIR $INSTALLDIR
> cd $BUILDDIR
> cmake .. -DCMAKE_INSTALL_PREFIX=$INSTALLDIR
> make install
```

The following sections will assume the user is following the basic workflow outlined above.

Note: For changes to the `CMake` configuration to take effect, the code will have to be reconfigured using `cmake` and rebuilt using `make`.

2.4.2 Additional Options

To enable additional options, `CMake` variables may be defined via `CMake` command line arguments, `ccmake`, or `cmake-gui`. `CMake` options specific to `ExaGO` have an `EXAGO_` prefix. For example, the following shell commands will build `ExaGO` with `MPI`:

```
> cmake .. -DCMAKE_INSTALL_PREFIX=$INSTALLDIR -DEXAGO_ENABLE_MPI=ON
```

`ExaGO`'s `CMake` configuration will search the usual system locations for an `MPI` installation.

For dependencies not installed to a system-wide location, users may also directly specify the location of a dependency. For example, this will build `ExaGO` with `IPOPT` enabled and installed to a user directory:

```
> cmake .. \
    -DCMAKE_INSTALL_PREFIX=$INSTALLDIR \
    -DEXAGO_ENABLE_IPOPT=ON \
    -DIPOPT_DIR=/path/to/ipopt
```

Notice that the `CMake` variable `IPOPT_DIR` does not have an `EXAGO_` prefix. This is because the variables specifying locations often belong to external `CMake` modules. `CMake` variables indicating installation directories do not have an `EXAGO_` prefix.

Some `CMake` options effect others. This is especially common when the user enables `ExaGO`'s GPU options. For example, if the user enables `EXAGO_ENABLE_GPU` and `EXAGO_ENABLE_RAJA`, the user must provide a GPU-enabled `RAJA` installation. `Umpire` is also an implicit dependency of `RAJA`, so if the user enables `EXAGO_ENABLE_GPU` they must **also** provide a GPU-enabled `Umpire` installation.

Below is a complete shell session on PNNL's cluster Newell in which a more complicated `ExaGO` configuration is built, where each dependency installation is explicitly passed to `CMake`. Environment modules specific to Newell are provided to make the example thorough, even though they are not likely to work on another machine. However, similar modules (with different version numbers) are likely to be available on other platforms.

Listing 2.2: `ExaGO` build with all options enabled

```
> module load gcc/7.4.0
> module load cmake/3.16.4
> module load openmpi/3.1.5
> module load magma/2.5.2_cuda10.2
> module load metis/5.1.0
> module load cuda/10.2
> git clone https://gitlab.pnnl.gov/exasgd/frameworks/exago.git exago
> export EXAGODIR=$PWD/exago
> cd $EXAGODIR
> export BUILDDIR=$PWD/build INSTALLDIR=$PWD/install
> mkdir $BUILDDIR $INSTALLDIR
> cd $BUILDDIR
> cmake .. \
  -DCMAKE_INSTALL_PREFIX=$INSTALLDIR \
  -DCMAKE_BUILD_TYPE=Debug \
  -DEXAGO_ENABLE_GPU=ON \
  -DEXAGO_ENABLE_HIOP=ON \
```

```
  -DEXAGO_ENABLE_IPOPT=ON \
  -DEXAGO_ENABLE_MPI=ON \
  -DEXAGO_ENABLE_PETSC=ON \
  -DEXAGO_RUN_TESTS=ON \
  -DEXAGO_ENABLE_RAJA=ON \
  -DEXAGO_ENABLE_IPOPT=ON \
  -DIPOPT_DIR=/path/to/ipopt \
  -DRAJA_DIR=/path/to/raja \
  -Dumpire_DIR=/path/to/umpire \
  -DHIOP_DIR=/path/to/hiop \
  -DMAGMA_DIR=/path/to/magma \
  -DPETSC_DIR=/path/to/petsc
> make -j 8 install
> # For the following commands, a job scheduler command may be needed.
> # Run test suite
> make test
> # Run an ExaGO application:
> $INSTALLDIR/bin/opflow
```

2.5   Usage

Each ExaGO application has the following format for execution

```
  ./app <app_options>
```

For OPFLOW for example:

```
./opflow -help
ExaGO 1.4.0 built on Jan 14 2022

Arguments for opflow:
        -help
                Print help message (type: flag)

        -version
                Print version information (type: flag)

        -config
                Print configuration options used to build ExaGO (type:
                   ↪ flag)

        -options_file /path/to/options_file
                Path to options file used to load additional ExaGO
                   ↪ configuration options (type: string)

        -opflow_model (POWER_BALANCE_POLAR|POWER_BALANCE_HIOP|
           ↪ PBPOLRAJAHIOP)
                OPFLOW model name (type: string)
```

```
-opflow_solver (IPOPT|HIOP|HIOPSPARSE)
        OPFLOW solver type (type: string)

-opflow_initialization (OPFLOWINIT_MIDPOINT|OPFLOWINIT_FROMFILE|
    ↪ OPFLOWINIT_ACPF|OPFLOWINIT_FLATSTART)
        Type of OPFLOW initialization (type: string)

-opflow_objective (MIN_GEN_COST|MIN_GENSETPOINT_DEVIATION|NO_OBJ)
        Type of OPFLOW objective (type: string)

-opflow_genbusvoltage (VARIABLE_WITHIN_BOUNDS|FIXED_WITHIN_QBOUNDS
    ↪ |FIXED_AT_SETPOINT)
        Type of OPFLOW gen bus voltage control (type: string)

-opflow_has_gensetpoint 0
        Use set-points for generator real power (type: bool)

-opflow_use_agc 0
        Use automatic generation control (AGC) (type: bool)

-opflow_tolerance 1e-06
        Optimization tolerance (type: real)

-opflow_ignore_lineflow_constraints 0
        Ignore line flow constraints? (type: bool)

-opflow_include_loadloss_variables 0
        Ignore line flow constraints? (type: bool)

-opflow_loadloss_penalty 1000
        Penalty for load loss (type: real)

-opflow_include_powerimbalance_variables 0
        Allow power imbalance? (type: bool)

-opflow_powerimbalance_penalty 10000
        Power imbalance penalty (type: real)

-hiop_compute_mode (auto|cpu|hybrid|gpu)
        Set compute mode for HiOp solver (type: string)

-hiop_verbosity_level 0
        Set verbosity level for HiOp solver, between 0 and 12 (
            ↪ type: int)

-hiop_ipopt_debug 0
```

Flag enabling debugging HIOP code with IPOPT (**type:** bool)

ExaGO applications also have an option to print many options ExaGO was configured with, which aid in reproducing build environments that a given ExaGO source tree was built with.

```
./opflow -config
ExaGO version 1.3.0 built on Jan 14 2022
Built with the following command:
$ cmake -B /Users/manc568/workspace/exago/build -S /Users/manc568/
    ↪ workspace/exago \
        -DEXAGO_ENABLE_RAJA:BOOL=OFF \
        -DEXAGO_ENABLE_PYTHON:BOOL=ON \
        -DEXAGO_ENABLE_HIP:BOOL=OFF \
        -DEXAGO_ENABLE_GPU:BOOL=OFF \
        -DEXAGO_ENABLE_HIOP_SPARSE:BOOL=ON \
        -DEXAGO_ENABLE_HIOP:BOOL=ON \
        -DEXAGO_ENABLE_IPOPT:BOOL=ON \
        -DEXAGO_ENABLE_MPI:BOOL=ON \
        -DEXAGO_BUILD_STATIC:BOOL=OFF \
        -DEXAGO_ENABLE_CUDA:BOOL=OFF \
        -DEXAGO_ENABLE_HIOP_DISTRIBUTED:BOOL=OFF \
        -DEXAGO_ENABLE_PETSC:BOOL=ON \
        -DEXAGO_BUILD_SHARED:BOOL=OFF \
        -DIPOPT_INCLUDES:STRING="" \
        -DIPOPT_ROOT_DIR:STRING="" \
        -DHiOp_DIR:STRING="/some/path" \
        -DPETSC_DIR:STRING="/some/path" \
        -DCMAKE_CXX_COMPILER:STRING="/usr/local/bin/g++-11" \
        -DIPOPT_LIBRARIES:STRING="" \
        -DCMAKE_BUILD_TYPE:STRING="Debug" \
        -DPETSC_INCLUDES:STRING="" \
        -DCMAKE_C_COMPILER:STRING="/usr/local/bin/gcc-11" \
        -DCMAKE_INSTALL_PREFIX:STRING="/Users/manc568/workspace/exago/
            ↪ install" \
        -DPETSC_LIBRARIES:STRING="" \
        -DEXAGO_OPTIONS_DIR:STRING="/Users/manc568/workspace/exago/install
            ↪ /share/exago/options"
```

Here, `app_options` are the command line options for the application. Each application has many options through which the input files and the control options can be set for the application. All application options have the form `-app_option_name` followed by the `app_option_value`. For instance,

```
./opflow -netfile case9mod.m -opflow_model POWER_BALANCE_POLAR \
-opflow_solver IPOPT
```

will execute the OPFLOW application using `case9mod.m` input file with the model POWER_BALANCE_POLAR and Ipopt [**?**]solver.

Options can also be passed to each application through an options file `-optionsfile <optionfilename` ↪ , or via command line or through a combination of the command line and options file. The option

specified via command line supersedes that given in in the options file. For example, if `-options_file` ↪ `opflowoptions` specified `-netfile case9mod.m` within its settings:

```
./opflow -options_file opflowoptions # Uses case9mod.m
./opflow -netfile case118.m -options_file opflowoptions # Uses case118.m
```

Chapter 3

Optimal power flow (OPFLOW)

OPFLOW solves the full AC optimal power flow problem and provides various flexible features that can be toggled via run-time options. It has interfaces to different optimization solvers that can be executed on CPUs or on GPUs.

3.1 Formulation

Optimal power flow is a general nonlinear programming problem with the following form

$$\text{min. } f(x) \tag{3.1}$$
$$\text{s.t.}$$
$$g(x) = 0 \tag{3.2}$$
$$h(x) \leq 0 \tag{3.3}$$
$$x^{\min} \leq x \leq x^{\max} \tag{3.4}$$

Here, x are the decision variables with lower and upper bounds $x^{\min}$ and $x^{\max}$, respectively, $f(x)$ is the objective function, $g(x)$ and $h(x)$ are the equality and inequality constraints, respectively. In the following sections we describe what constitutes these different terms as used by OPFLOW.

3.1.1 Variables and bounds

The different variables used in OPFLOW formulation are described in Table **??**.

Power imbalance variables are non-physical (slack) variables that measure the violation of power balance at buses. Having these variables (may) help in making the optimization problem easier to solve since they always ensure feasibility of the bus power balance constraints.

3.1.2 Objective Function

The objective function for OPFLOW is given in (**??**)

$$\text{min. } C_{gen}(p^{\text{g}}) + C_{dev}(\Delta p^{\text{g}}) + C_{loss}(\Delta p^{\text{l}}, \Delta q^{\text{l}}) + C_{imb}(\Delta p^{+}, \Delta p^{-}, \Delta q^{+}, \Delta q^{-}) \tag{3.5}$$

Table 3.1: Optimal power flow (OPFLOW) variables

Symbol	Variable	Bounds	Notes
p_j^{g}	Generator real power dispatch	$p_j^{\mathrm{gmin}} \le p_j^{\mathrm{g}} \le p_j^{\mathrm{gmax}}$	
q_j^{g}	Generator reactive power dispatch	$q_j^{\mathrm{gmin}} \le q_j^{\mathrm{g}} \le q_j^{\mathrm{gmax}}$	
Δp_j^{g}	Generator real power deviation	$-p_j^{\mathrm{r}} \le \Delta p_j^{\mathrm{g}} \le p_j^{\mathrm{r}}$	• Only used when `-opflow_has_gensetpoint` or `-opflow_use_agc` option is active • Δp_j^{g} is the deviation from real power generation set-point p_j^{gset}.
p_j^{gset}	Generator real power set-point	$p_j^{\mathrm{gmin}} \le p_j^{\mathrm{gset}} \le p_j^{\mathrm{gmax}}$	• Only used when `-opflow_has_gensetpoint` or `-opflow_use_agc` option is active.
ΔP	System power excess/deficit	Unbounded	Only used when `-opflow_use_agc` is active
θ_i	Bus voltage angle	$-\pi \le \theta_i \le \pi$	• Used with power balance polar model (`-opflow_model POWER_BALANCE_POLAR`) • θ_i is unbounded, except reference bus angle θ_i^{ref} which is fixed to 0
v_i	Bus voltage magnitude	$v_i^{\mathrm{min}} \le v_i \le v_i^{\mathrm{max}}$	• Used with power balance polar model (`-opflow_model POWER_BALANCE_POLAR`) • $v_i^{\mathrm{min}} = v_i^{\mathrm{max}} = v_i^{\mathrm{set}}$ if fixed generator set point voltage option is active (`-opflow_has_gensetpoint`)
$\Delta p_i^{+}, \Delta p_i^{-}$	Bus real power mismatch variables	$0 \le \Delta p_i^{+}, \Delta p_i^{-} \le \infty$	Used when power mismatch variable option is active (`-opflow_include_powerimbalance_variables1`)
$\Delta q_i^{+}, \Delta q_i^{-}$	Bus reactive power mismatch variables	$0 \le \Delta q_i^{+}, \Delta q_i^{-} \le \infty$	Used when power mismatch variable option is active (`-opflow_include_powerimbalance_variables1`)
Δp_j^{l}	Real power load loss	$0 \le \Delta p_j^{\mathrm{l}} \le p_j^{\mathrm{l}}$	Used when load loss variable option is active (`-opflow_include_loadloss_variables1`)
Δq_j^{l}	Reactive power load loss	$0 \le \Delta q_j^{\mathrm{l}} \le q_j^{\mathrm{l}}$	Used when load loss variable option is active (`-opflow_include_loadloss_variables1`)

Total generation cost $C_{gen}(p^{\mathbf{g}})$

Needs option `-opflow_objective MIN_GEN_COST`

$$C_{gen}(p^{\mathbf{g}}) = \sum_{j \in J^{\text{gen}}} C_j^{\mathbf{g}}(p_j^{\mathbf{g}}) \tag{3.6}$$

Here, $C_j^{\mathbf{g}}$ is a quadratic function of the form $C_j^{\mathbf{g}} = a_j^{\mathbf{g}} p_j^{\mathbf{g}2} + b_j^{\mathbf{g}} p_j^{\mathbf{g}} + c_j^{\mathbf{g}}$.

Total generation setpoint deviation $C(\Delta p^{\mathbf{g}})$

Needs option `-opflow_objective MIN_GENSETPOINT_DEVIATION`

$$C_{dev}(\Delta p^{\mathbf{g}}) = \sum_{j \in J^{\text{gen}}} (\Delta p_j^{\mathbf{g}2}) \tag{3.7}$$

This feature is only supported with IPOPT solver.

Load loss $C(\Delta p^{\mathbf{l}}, \Delta q^{\mathbf{l}})$

This term gets added to the objective when `-opflow_include_loadloss_variables` option is active.

$$C_{loss}(\Delta p^{\mathbf{l}}, \Delta q^{\mathbf{l}}) = \sum_{j \in J^{\text{ld}}} \sigma_j^{\mathbf{l}}(\Delta p_j^{\mathbf{l}} + \Delta q_j^{\mathbf{l}}) \tag{3.8}$$

The load loss penalty $\sigma_j^{\mathbf{l}}$ can be set via the option `-opflow_loadloss_penalty`. The default is \$1000/MW for all loads.

Power imbalance $C_{imb}(\Delta p^+, \Delta p^-, \Delta q^+, \Delta q^-)$

This term gets added to the objective when `-opflow_include_powerimbalance_variables` option is active.

$$C_{imb}(\Delta p^+, \Delta p^-, \Delta q^+, \Delta q^-) = \sum_{i \in J^{bus}} \sigma_i(\Delta p^+ + \Delta p^- + \Delta q^+ + \Delta q^-) \tag{3.9}$$

The power imbalance cost σ_i can be set via the option `-opflow_powerimbalance_penalty`. The default is \$10,000/MW^2 for all buses. Though the power imbalance variables $\Delta p^+, \Delta p^-, \Delta q^+, \Delta q^-$ are slack or non-physical, they can help in solving infeasible cases having no power flow solution, and thus provide a measure of the infeasibility.

3.1.3 Equality constraints

Nodal power balance

The nodal power balance equations for each bus i are given by

$$\sum_{\substack{j \in J^{\text{gen}} \\ A^{\text{g}}_{ij} \neq 0}} p_j^{\text{g}} = p_i^{\text{sh}} + \Delta p_i^+ - \Delta p_i^- + \sum_{\substack{j \in J^{\text{ld}} \\ A^{\text{l}}_{ij} \neq 0}} (p_j^{\text{l}} - \Delta p_j^{\text{l}}) + \sum_{\substack{j \in J^{\text{br}} \\ A^{\text{br}}_{oi} \neq 0}} p_{j_{od}}^{\text{br}} + \sum_{\substack{j \in J^{\text{br}} \\ A^{\text{br}}_{id} \neq 0}} p_{j_{do}}^{\text{br}} \tag{3.10}$$

$$\sum_{\substack{j \in J^{\text{gen}} \\ A^{\text{g}}_{ij} \neq 0}} q_j^{\text{g}} = q_i^{\text{sh}} + \Delta q_i^+ - \Delta q_i^- + \sum_{\substack{j \in J^{\text{ld}} \\ A^{\text{l}}_{ij} \neq 0}} (q_j^{\text{l}} - \Delta q_j^{\text{l}}) + \sum_{\substack{j \in J^{\text{br}} \\ A^{\text{br}}_{oi} \neq 0}} q_{j_{od}}^{\text{br}} + \sum_{\substack{j \in J^{\text{br}} \\ A^{\text{br}}_{id} \neq 0}} q_{j_{do}}^{\text{br}} \tag{3.11}$$

$$\tag{3.12}$$

where, the real and reactive power shunt consumption is given by (**??**) and (**??**)

The real and reactive power flow $p_{j_{od}}^{\text{br}}$, $q_{j_{od}}^{\text{br}}$ for line j from the origin bus o to destination bus d is given by (**??**) – (**??**) and from destination bus d to origin bus o is given by (**??**) – (**??**)

Shunt power

$$p_i^{\text{sh}} = g_i^{\text{sh}} v_i^2 \tag{3.13}$$

$$q_i^{\text{sh}} = -b_i^{\text{sh}} v_i^2 \tag{3.14}$$

Generator real power output

When using `-opflow_has_gensetpoint`, two extra variables p_j^{gset} and Δp_j^{g} are added for each generator. The generator real power output p_j^{g} is related to the power deviation Δp_j^{g} by the following relations

$$p_j^{\text{gset}} + \Delta p_j^{\text{g}} - p_j^{\text{g}} = 0 \tag{3.15}$$

$$p_j^{\text{gset}} - p_j^{\text{g}*} = 0 \tag{3.16}$$

The second equation sets the generator set-point p_j^{gset} to a fixed value $p_j^{\text{g}*}$. Here, $p_j^{\text{g}*}$ is the set-point for the generator real power output, which can be thought of as an operator set or contractual agreement set-point.

3.1.4 Inequality constraints

MVA flow on branches

MVA flow limits at origin and destination buses for each line.

$$p_{j_{od}}^{\text{br}\,2} + q_{j_{od}}^{\text{br}\,2} \leq s_j^{\text{rateA}\,2}, \quad j \in J^{\text{br}} \tag{3.17}$$

$$p_{j_{do}}^{\text{br}\,2} + q_{j_{do}}^{\text{br}\,2} \leq s_j^{\text{rateA}\,2}, \quad j \in J^{\text{br}} \tag{3.18}$$

To reduce the number of inequality constraints, only lines that are in service and having MVA A rating s_j^{rateA} less than 10000 MVA are considered.

Branch flows

In polar coordinates, the real and reactive power flow $p_{j_{od}}^{\mathrm{br}}$, $q_{j_{od}}^{\mathrm{br}}$ from bus o to bus d on line j is given by (**??**) – (**??**)

$$p_{j_{od}}^{\mathrm{br}} = g_{oo}v_o^2 + v_o v_d (g_{od} \cos(\theta_o - \theta_d) + b_{od} \sin(\theta_o - \theta_d)) \tag{3.19}$$

$$q_{j_{od}}^{\mathrm{br}} = -b_{oo}v_o^2 + v_o v_d (-b_{od} \cos(\theta_o - \theta_d) + g_{od} \sin(\theta_o - \theta_d)) \tag{3.20}$$

and from bus d to bus o is given by (**??**) – (**??**)

$$p_{j_{do}}^{\mathrm{br}} = g_{dd}v_d^2 + v_d v_o (g_{do} \cos(\theta_d - \theta_o) + b_{do} \sin(\theta_d - \theta_o)) \tag{3.21}$$

$$q_{j_{do}}^{\mathrm{br}} = -b_{dd}v_d^2 + v_d v_o (-b_{do} \cos(\theta_d - \theta_o) + g_{do} \sin(\theta_d - \theta_o)) \tag{3.22}$$

Automatic generation control (AGC)

With `-opflow_use_agc`, two additional constraints are added for each participating generator to enforce the proportional generator redispatch participation as done in automatic generation control (AGC). These two equations are

$$
\begin{aligned}
\left(\alpha_j^{\mathrm{g}} \Delta P - \Delta p_j^{\mathrm{g}} \right) \left(p_j^{\mathrm{g}} - p_j^{\mathrm{gmax}} \right) \geq 0 \\
\left(\Delta p_j^{\mathrm{g}} - \alpha_j^{\mathrm{g}} \Delta P \right) \left(p_j^{\mathrm{gmin}} - p_j^{\mathrm{g}} \right) \geq 0
\end{aligned}
\tag{3.23}
$$

Eq. **??** forces the generator set-point deviation to be equal to the generation participation when the generator has head-room available $p_j^{\mathrm{gmin}} \leq p_j^{\mathrm{g}} \leq p_j^{\mathrm{gmax}}$. Here, α_j^{g} is the generator participation factor which is the proportion of the power deficit/excess ΔP that the generator provides.

Generator bus voltage control

When the option `-opflow_genbusvoltage FIXED_WITHIN_QBOUNDS` is used, the generator bus voltage is fixed when the total reactive power generation available at the bus is within bounds. When it reaches its bounds, the voltage varies with the generator reactive power fixed at its bound. To implement this behavior, two inequality constraints are added for each generator bus

$$
\begin{aligned}
(v_i^{\mathrm{set}} - v_i)(q_i - q^{\mathrm{max}_i}) \geq 0 \\
(v_i - v_i^{\mathrm{set}})(q^{\mathrm{min}_i} - q_i) \geq 0
\end{aligned}
\tag{3.24}
$$

Here, q_i, q^{max_i}, and q^{min_i} are the generated, maximum, and minimum reactive power at the bus, respectively.

3.2   Solvers

OPFLOW can be used with a few different solvers. All the solvers solve the optimization problem via a nonlinear interior-point algorithm.

1. Ipopt [?] is a popular open-source package for solving nonlinear optimization problems. It is the most robust of the solvers implemented for solving OPFLOW. However, it can be run only on a single process and does not have GPU support.
   Option: `-opflow_solver IPOPT -opflow_model POWER_BALANCE_POLAR`

2. HiOp [?, ?] is a high-performance optimization library that implements an interior-point algorithm for solving nonlinear optimization problems. There are two solvers available from the HiOp [?, ?] library: Mixed sparse-dense formulation `-opflow_solver HIOP`, and sparse formulation `-opflow_solver HIOPSPARSE`. The library supports execution both on the CPU and the GPU. Options:
   CPU: `-opflow_solver HIOP -opflow_model POWER_BALANCE_HIOP -hiop_compute_mode CPU`
   GPU: `-opflow_solver HIOP -opflow_model PBPOLRAJAHIOP -hiop_compute_mode GPU`

3.3   Models

A 'model' in `ExaGO` describes the representation of the underlying physics. All OPFLOW models use the power balance formulation in polar coordinates for the ACOPF equations. The difference between the different models arises from their specific implementation/interface. The different models available for OPFLOW are listed in Table **??**. As discussed earlier, not every 'model' is compatible with every 'solver'. Table **??** lists the solver compatibility for the different models.

Table 3.2: OPFLOW models

Model type	OPFLOW option (**-opflow_model**)	Compatible solvers	CPU-GPU
Power balance with polar coordinates	POWER_BALANCE_POLAR	IPOPT, HIOPSPARSE	CPU
Power balance with polar coordinates used with HIOP	POWER_BALANCE_HIOP	HiOp [?, ?]	CPU/GPU
Power balance with polar coordinates used with HIOP on GPU	PBPOLRAJAHIOP	HiOp [?, ?]	GPU

Table 3.3: OPFLOW Model-solver compatibility

Model Name	Ipopt [?]	HiOp [?, ?]	HIOPSPARSE
POWER_BALANCE_POLAR	✓		✓
POWER_BALANCE_HIOP		✓	
PBPOLRAJAHIOP		✓	

3.3.1   Power balance polar

The power balance polar model (-opflow_model POWER_BALANCE_POLAR) uses the power balance formulation with polar representation for the network voltages. It runs on CPU only and is compatible with Ipopt [?] and sparse HiOp [?, ?] solvers.

3.3.2 Power balance with HiOp on CPU

This model (-opflow_model POWER_BALANCE_HIOP) implements the power balance formulation with polar coordinates used with HiOp [?, ?]solver only. The model evaluation is done only on the CPU, but the HiOp [?, ?]solver can be executed either on the CPU (-hiop_compute_mode CPU) or GPU (-hiop _compute_mode HYBRID) by setting the -hiop_compute_mode option appropriately.

3.3.3 Power balance with HiOp on GPU

The PBPOLRAJAHIOP model (-opflow_model PBPOLRAJAHIOP) computes all the model and optimization calculations on the GPU. This model uses RAJA [?]and Umpire [?] libraries to run OPFLOW calculations (objective, constraints, etc.) on the GPU.

3.4 Input and Output

The current ExaGO version only supports reading network files in MATPOWER format and can (optionally) write the output back in MATPOWER data file format.

3.5 Usage

```
  ./opflow -netfile <netfilename>  <opflowoptions>
```

3.6 Options

See table ??

3.7 Examples

Some OPFLOW example runs are provided with some sample output. Options values are the default values in table ?? unless otherwise specified. -print_output is only used in the first example to save space. Sample output is generated by running examples from the installation directory.

Example using the Ipopt [?]solver:

```
./bin/opflow -opflow_solver IPOPT -opflow_model POWER_BALANCE_POLAR -
   ↪ netfile $EXAGO_DIR/datafiles/case9/case9mod.m -print_output
[ExaGO] Creating OPFlow


*************************************************************************************
   ↪
This program contains Ipopt, a library for large-scale nonlinear
   ↪ optimization.
 Ipopt is released as open source code under the Eclipse Public License (
    ↪ EPL).
         For more information visit http://projects.coin-or.org/Ipopt
```

Table 3.4: OPFLOW options

Option	Meaning	Values (Default value)	Compatibility
-netfile	Network file name	string < 4096 characters (case9mod.m)	
-print_output	Print output to screen	0 or 1 (0)	All solvers
-save_output	Save output to file	0 or 1 (0)	All solvers
-opflow_model	Representation of network balance equations and bus voltages	See Table ?? (POWER_BALANCE_POLAR)	
-opflow_solver	Optimization solver	See section ??	
-opflow_initialization	Type of initialization	See Table ?? (MID-POINT)	All solvers
-opflow_has_gensetpoint	Uses generation set point and activates ramping variables	0 or 1 (0)	All models
-opflow_use_agc	Uses AGC formulation in OPF	0 or 1 (0)	POWER_BALANCE _POLAR only
-opflow_objective	type of objective function	See table ?? (MIN_GEN_COST)	All models
-opflow_genbusvoltage	Type of generator bus voltage control	See Table ?? (VARI-ABLE_WITHIN _BOUNDS)	POWER_BALANCE _POLAR only
-opflow_ignore_lineflow_constraints	Ignore line flow constraints	0 or 1 (0)	All models
-opflow_monitor_line_kvlevels	Monitor line flows at these KV levels	comma separated list	All models
-opflow_include_loadloss_variables	Include load loss	0 or 1 (0)	All models
-opflow_include_powerimbalance_variables	Include power imbalance	0 or 1 (0)	All models
-opflow_loadloss_penality	$ penalty for load loss	real (1000)	All models
-opflow_powerimbalance_penalty	$ penalty for power imbalance	real (10000)	All models
-opflow_tolerance	Optimization solver tolerance	real (1e-6)	All solvers

Table 3.5: OPFLOW initializations

Initialization type	Meaning
MIDPOINT	Use mid-point of bounds
FROMFILE	Use values from network file
ACPF	Run AC power flow
FLATSTART	Flat-start
DCOPF	Run DC optimal power flow

Table 3.6: OPFLOW generator bus voltage control modes

Voltage control type	Meaning	Compatibility
FIXED_WITHIN_QBOUNDS	Fixed within reactive power bounds	POWER_BALANCE_POLAR only
VARIABLE_WITHIN_BOUNDS	Variable within voltage bounds	All models

Table 3.7: OPFLOW objective function types

Objective function	Meaning	Compatibility
MIN_GEN_COST	Minimize generation cost	All models
MIN_GENSETPOINT_DEVIATION	Minimize deviation (ramp up-down) from generataor set-point	POWER_BALANCE_POLAR model only
NO_OBJ	No objective function (only feasibility)	All models

```
******************************************************************************
    ↪
This is Ipopt version 3.12.10, running with linear solver ma27.

Number of nonzeros in equality constraint Jacobian...:      114
Number of nonzeros in inequality constraint Jacobian.:       72
Number of nonzeros in Lagrangian Hessian.............:       96

Total number of variables............................:       24
                     variables with only lower bounds:        0
                variables with lower and upper bounds:       16
                     variables with only upper bounds:        0
Total number of equality constraints.................:       18
Total number of inequality constraints...............:       18
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:       18
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
   ↪ alpha_pr  ls
   0  1.0318125e+04 1.80e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e
      ↪ +00   0
   1  7.7157691e+03 1.17e+00 1.03e+02  -1.0 1.08e+00    -  6.27e-01 3.50e
      ↪ -01f  1
   2  7.6608235e+03 1.15e+00 1.01e+02  -1.0 6.28e+00    -  1.20e-02 1.37e
      ↪ -02f  1
   3  7.4466686e+03 1.09e+00 3.06e+02  -1.0 3.74e+00    -  4.15e-03 5.81e
      ↪ -02f  1
   4  5.4292675e+03 3.92e-01 4.83e+03  -1.0 7.34e-01    -  3.34e-03 6.40e
      ↪ -01f  1
   5  4.5792834e+03 2.24e-01 1.51e+03  -1.0 6.46e-01   2.0 8.77e-03 7.37e
      ↪ -01f  1
   6  4.2907579e+03 1.20e-02 3.57e+02  -1.0 3.36e-01    -  5.37e-01 1.00e
      ↪ +00f  1
   7  4.1690456e+03 4.40e-02 5.31e+01  -1.0 3.31e-01    -  9.22e-01 1.00e
      ↪ +00f  1
   8  4.1687926e+03 4.88e-04 1.93e+00  -1.0 4.79e-02   1.5 1.00e+00 1.00e
```

```
 ↪ +00h  1
  9  4.1497176e+03 1.19e-02 9.92e+00  -2.5 1.87e-01    -  8.38e-01 1.00e
 ↪ +00f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
 ↪ alpha_pr  ls
 10  4.1463942e+03 1.09e-02 5.09e-01  -2.5 1.15e-01    -  8.71e-01 1.00e
 ↪ +00h  1
 11  4.1449657e+03 1.47e-03 1.79e-02  -2.5 2.75e-02    -  1.00e+00 1.00e
 ↪ +00h  1
 12  4.1445415e+03 6.63e-04 9.12e-02  -3.8 1.48e-02    -  1.00e+00 6.30e
 ↪ -01h  1
 13  4.1444705e+03 3.43e-04 4.96e-02  -3.8 2.08e-02    -  1.00e+00 8.93e
 ↪ -01h  1
 14  4.1444809e+03 4.48e-05 1.79e-04  -3.8 6.82e-03    -  1.00e+00 1.00e
 ↪ +00f  1
 15  4.1444611e+03 1.96e-05 4.55e-03  -5.7 4.57e-03    -  1.00e+00 9.34e
 ↪ -01h  1
 16  4.1444607e+03 6.49e-06 1.17e-05  -5.7 2.60e-03    -  1.00e+00 1.00e
 ↪ +00h  1
 17  4.1444605e+03 1.19e-06 2.07e-06  -7.0 1.11e-03    -  1.00e+00 1.00e
 ↪ +00h  1
 18  4.1444605e+03 1.58e-07 3.29e-07  -7.0 4.06e-04    -  1.00e+00 1.00e
 ↪ +00h  1

Number of Iterations....: 18

                                 (scaled)                 (unscaled)
Objective...............:  9.2925122354655841e+01   4.1444604570176507e
 ↪ +03
Dual infeasibility......:  3.2927387965389691e-07   1.4685615032563802e
 ↪ -05
Constraint violation....:  2.6639677713768961e-08   2.6639677713768961e
 ↪ -08
Complementarity.........:  4.7840038622596930e-07   2.1336657225678232e
 ↪ -05
Overall NLP error.......:  4.7840038622596930e-07   2.1336657225678232e
 ↪ -05


Number of objective function evaluations             = 19
Number of objective gradient evaluations             = 19
Number of equality constraint evaluations            = 19
Number of inequality constraint evaluations          = 19
Number of equality constraint Jacobian evaluations   = 19
Number of inequality constraint Jacobian evaluations = 19
Number of Lagrangian Hessian evaluations             = 18
Total CPU secs in IPOPT (w/o function evaluations)   =      0.025
```

```
Total CPU secs in NLP function evaluations          =       0.002

EXIT: Optimal Solution Found.
================================================================
Optimal Power Flow
================================================================
Model                           POWER_BALANCE_POLAR
Solver                          IPOPT
Objective                       MIN_GEN_COST
Initialization                  MIDPOINT
Gen. bus voltage mode           VARIABLE_WITHIN_BOUNDS
Load loss allowed               NO
Power imbalance allowed         NO
Ignore line flow constraints    NO

Number of variables             24
Number of equality constraints  18
Number of inequality constraints 18

Convergence status             CONVERGED
Objective value                4144.46


----------------------------------------------------------------
Bus       Pd      Qd      Vm      Va      mult_Pmis      mult_Qmis
   ↪ Pslack         Qslack
----------------------------------------------------------------
1         0.00    0.00   1.100   0.000    2102.91         0.00
   ↪ 0.00           0.00
2         0.00    0.00   1.095   3.928    2059.18        -0.00
   ↪ 0.00           0.00
3         0.00    0.00   1.087   2.120    2065.15        -0.00
   ↪ 0.00           0.00
4         0.00    0.00   1.097  -1.993    2103.16         0.08
   ↪ 0.00           0.00
5        75.00   50.00   1.079  -3.060    2113.45         7.29
   ↪ 0.00           0.00
6        90.00   30.00   1.087  -3.927    2129.85         1.62
   ↪ 0.00           0.00
7         0.00    0.00   1.100   0.535    2059.57        -0.04
   ↪ 0.00           0.00
8       100.00   35.00   1.089  -1.720    2079.34         2.99
   ↪ 0.00           0.00
9         0.00    0.00   1.100  -0.135    2065.43        -0.09
   ↪ 0.00           0.00


----------------------------------------------------------------
   ↪
```

```
From        To       Status      Sft       Stf       Slim     mult_Sf  mult_St
--------------------------------------------------------------------------------
   ↪
1           4          1        73.18     72.98     380.00    -0.00    -0.00
2           7          1       114.18    114.68     250.00    -0.00    -0.00
3           9          1        83.57     84.60     300.00    -0.00    -0.00
4           5          1        29.68     40.50     250.00    -0.00    -0.00
4           6          1        44.86     46.03     250.00    -0.00    -0.00
5           7          1        51.29     49.04     250.00    -0.00    -0.00
6           9          1        48.94     51.43     150.00    -0.00    -0.00
7           8          1        66.61     68.11     250.00    -0.00    -0.00
8           9          1        38.86     34.15     150.00    -0.00    -0.00


--------------------------------------------------------------------------------
   ↪
Gen       Status      Fuel      Pg        Qg        Pmin      Pmax      Qmin
   ↪ Qmax
--------------------------------------------------------------------------------
   ↪
1           1       UNDEFINED   72.86      6.79      10.00    350.00   -300.00
   ↪ 300.00
2           1       UNDEFINED  114.07     -5.13      10.00    300.00   -300.00
   ↪ 300.00
3           1       UNDEFINED   80.21    -23.47      10.00    270.00   -300.00
   ↪ 300.00
[ExaGO] Finalizing opflow application.
```

Example using the HiOp [?, ?]solver on the CPU with ACPF initialization:

```
./bin/opflow -opflow_solver HIOP -opflow_model POWER_BALANCE_HIOP -netfile
   ↪   $EXAGO_DIR/datafiles/case9/case9mod.m -opflow_initialization ACPF -
   ↪ hiop_compute CPU -hiop_verbosity_level 3 -print_output
[ExaGO] Creating OPFlow

[Warning] Detected 1 fixed variables out of a total of 24.
===============
Hiop SOLVER
===============
Using 1 MPI ranks.
--------------
Problem Summary
--------------
Total number of variables: 24
     lower/upper/lower_and_upper bounds: 16 / 16 / 16
Total number of equality constraints: 18
Total number of inequality constraints: 18
     lower/upper/lower_and_upper bounds: 18 / 18 / 18
iter    objective     inf_pr     inf_du   lg(mu)  alpha_du   alpha_pr
```

```
        ↪ linesrch
   0  4.6670737e+03  3.714e-11  2.891e+03  -1.00  0.000e+00  0.000e+00
        ↪ - (-)
   1  4.6431205e+03  3.373e-04  2.791e+03  -1.00  3.013e-01  3.498e-02  1(f
        ↪ )
   2  4.4461241e+03  6.595e-02  2.105e+03  -1.00  4.910e-01  2.467e-01  1(s
        ↪ )
   3  4.1942969e+03  6.464e-02  5.808e+02  -1.00  5.986e-01  8.573e-01  1(s
        ↪ )
   4  4.1615434e+03  8.234e-03  1.159e+02  -1.00  7.556e-01  1.000e+00  1(s
        ↪ )
   5  4.1471588e+03  4.414e-03  1.291e+02  -1.00  1.000e+00  1.000e+00  1(s
        ↪ )
   6  4.1441777e+03  2.959e-02  7.889e+01  -1.00  4.818e-01  4.030e-01  1(s
        ↪ )
   7  4.1448270e+03  7.926e-03  4.949e+01  -1.00  1.000e+00  1.000e+00  1(s
        ↪ )
   8  4.1448562e+03  1.506e-03  2.576e-01  -1.00  1.000e+00  1.000e+00  1(s
        ↪ )
   9  4.1444966e+03  3.576e-04  2.859e+00  -3.82  9.631e-01  6.968e-01  1(s
        ↪ )
iter    objective      inf_pr     inf_du    lg(mu)   alpha_du    alpha_pr
    ↪ linesrch
  10  4.1444617e+03  1.671e-04  2.747e+00  -3.82  9.520e-01  7.537e-01  1(s
        ↪ )
  11  4.1444597e+03  5.733e-05  1.190e-01  -3.82  9.716e-01  1.000e+00  1(s
        ↪ )
  12  4.1444610e+03  6.929e-06  5.005e-04  -3.82  1.000e+00  1.000e+00  1(s
        ↪ )
  13  4.1444605e+03  1.681e-06  1.400e-04  -5.73  1.000e+00  1.000e+00  1(h
        ↪ )
  14  4.1444605e+03  2.574e-07  2.377e-05  -5.73  1.000e+00  1.000e+00  1(h
        ↪ )
  15  4.1444604e+03  1.411e-08  1.319e-06  -5.73  1.000e+00  1.000e+00  1(h
        ↪ )
  16  4.1444604e+03  2.396e-10  2.047e-08  -7.00  1.000e+00  1.000e+00  1(h
        ↪ )
Successfull termination.
Total time 0.933 sec
Hiop internal time:    total 0.932 sec     avg iter 0.058 sec
    internal total std dev across ranks 0.000 percent
Fcn/deriv time:    total=0.001 sec  ( obj=0.000 grad=0.000 cons=0.000 Jac
    ↪ =0.000 Hess=0.001)
    Fcn/deriv total std dev across ranks 0.000 percent
Fcn/deriv #: obj 18 grad 18 eq cons 18 ineq cons 18 eq Jac 18 ineq Jac 18
Total KKT time 0.931 sec
update init 0.848sec     update linsys 0.000 sec     fact 0.071 sec
```

```
solve rhs-manip 0.000 sec     triangular solve 0.012 sec


================================================================
Optimal Power Flow
================================================================
Model                           POWER_BALANCE_HIOP
Solver                          HIOP
Objective                       MIN_GEN_COST
Initialization                  ACPF
Gen. bus voltage mode           VARIABLE_WITHIN_BOUNDS
Load loss allowed               NO
Power imbalance allowed         NO
Ignore line flow constraints    NO

Number of variables             24
Number of equality constraints  18
Number of inequality constraints 18

Convergence status             CONVERGED
Objective value                4144.46

----------------------------------------------------------------
Bus        Pd      Qd      Vm      Va      mult_Pmis     mult_Qmis
   ↪ Pslack         Qslack
----------------------------------------------------------------
1          0.00    0.00   1.100  -0.000    2102.91        0.00
   ↪ 0.00           0.00
2          0.00    0.00   1.095   3.928    2059.18       -0.00
   ↪ 0.00           0.00
3          0.00    0.00   1.087   2.120    2065.15       -0.00
   ↪ 0.00           0.00
4          0.00    0.00   1.097  -1.993    2103.17        0.07
   ↪ 0.00           0.00
5         75.00   50.00   1.079  -3.060    2113.46        7.29
   ↪ 0.00           0.00
6         90.00   30.00   1.087  -3.927    2129.85        1.62
   ↪ 0.00           0.00
7          0.00    0.00   1.100   0.535    2059.57       -0.04
   ↪ 0.00           0.00
8        100.00   35.00   1.089  -1.720    2079.34        2.99
   ↪ 0.00           0.00
9          0.00    0.00   1.100  -0.135    2065.43       -0.09
   ↪ 0.00           0.00


----------------------------------------------------------------------------
   ↪
From       To      Status      Sft     Stf     Slim    mult_Sf  mult_St
```

```
-----------------------------------------------------------------------------------------------
    ↪
1           4           1          73.18      72.98     380.00    -0.00     -0.00
2           7           1         114.18     114.68     250.00    -0.00     -0.00
3           9           1          83.58      84.61     300.00    -0.00     -0.00
4           5           1          29.69      40.50     250.00    -0.00     -0.00
4           6           1          44.86      46.03     250.00    -0.00     -0.00
5           7           1          51.29      49.04     250.00    -0.00     -0.00
6           9           1          48.94      51.43     150.00    -0.00     -0.00
7           8           1          66.61      68.11     250.00    -0.00     -0.00
8           9           1          38.87      34.15     150.00    -0.00     -0.00


-----------------------------------------------------------------------------------------------
    ↪
Gen        Status       Fuel       Pg         Qg        Pmin      Pmax      Qmin
    ↪ Qmax
-----------------------------------------------------------------------------------------------
    ↪
1            1      UNDEFINED      72.86       6.80     10.00    350.00   -300.00
    ↪ 300.00
2            1      UNDEFINED     114.07      -5.13     10.00    300.00   -300.00
    ↪ 300.00
3            1      UNDEFINED      80.21     -23.48     10.00    270.00   -300.00
    ↪ 300.00
[ExaGO] Finalizing opflow application.
```

Example with HIOP solver on GPU with load loss activated. In this example, the load at bus 5 is increased to 750 MW leading to an infeasible power flow. Activating the load loss causes shedding of load at bus 5 and as a result makes the optimization converge.

```
./opflow -opflow_solver HIOP -opflow_model PBPOLRAJAHIOP -netfile
    ↪ $EXAGO_DIR/datafiles/case9/case9mod_loadloss.m -hiop_compute_mode
    ↪ GPU -hiop_verbosity_level 3 -print_output -
    ↪ opflow_include_loadloss_variables
[ExaGO] Creating OPFlow

[Warning] Detected 1 fixed variables out of a total of 30.
===============
Hiop SOLVER
===============
Using 1 MPI ranks.
---------------
Problem Summary
---------------
Total number of variables: 30
     lower/upper/lower_and_upper bounds: 22 / 22 / 22
Total number of equality constraints: 18
Total number of inequality constraints: 18
```

```
      lower/upper/lower_and_upper bounds: 18 / 18 / 18
iter    objective       inf_pr     inf_du    lg(mu)   alpha_du    alpha_pr
  ↪ linesrch
   0  1.4867531e+04 7.490e+00  1.000e+05  -1.00  0.000e+00  0.000e+00
  ↪ - (-)
   1  1.3403771e+04 7.490e+00  1.000e+05  -1.00  2.729e-04  1.702e-05  1(s
  ↪ )
   2  1.1742043e+04 7.489e+00  9.993e+04  -1.00  5.655e-03  1.102e-04  1(s
  ↪ )
   3  1.0453140e+04 7.484e+00  8.993e+04  -1.00  1.258e-01  6.948e-04  1(s
  ↪ )
   4  1.0933710e+04 7.247e+00  8.722e+04  -1.00  3.011e-02  3.073e-02  1(s
  ↪ )
   5  1.2262728e+04 6.691e+00  1.217e+05  -1.00  3.642e-04  7.537e-02  1(s
  ↪ )
   6  1.2718100e+04 6.509e+00  1.303e+05  -1.00  2.837e-04  2.663e-02  1(s
  ↪ )
   7  1.2721404e+04 6.508e+00  1.302e+05  -1.00  1.583e-02  1.930e-04  1(s
  ↪ )
   8  1.6122803e+04 5.696e+00  7.620e+04  -1.00  1.954e-03  1.492e-01  1(S
  ↪ )
   9  1.6366303e+04 5.692e+00  7.616e+04  -1.00  9.924e-02  8.075e-04  1(s
  ↪ )
iter    objective       inf_pr     inf_du    lg(mu)   alpha_du    alpha_pr
  ↪ linesrch
  10  1.7501119e+04 5.671e+00  7.587e+04  -1.00  3.082e-03  3.708e-03  1(s
  ↪ )
  11  1.7624878e+04 5.668e+00  8.682e+04  -1.00  9.630e-02  3.944e-04  1(s
  ↪ )
  12  2.2838548e+04 5.575e+00  8.354e+04  -1.00  1.071e-03  1.646e-02  1(s
  ↪ )
  ...
  ...
  47  7.4002786e+05 5.392e-03  5.252e+05  -1.00  1.365e-04  4.401e-03  1(s
  ↪ )
  48  7.4002761e+05 5.391e-03  5.332e+05  -1.00  2.888e-02  1.880e-03  1(s
  ↪ )
  49  7.4003537e+05 3.310e-03  4.892e+05  -1.00  3.124e-03  1.000e+00  1(s
  ↪ )
iter    objective       inf_pr     inf_du    lg(mu)   alpha_du    alpha_pr
  ↪ linesrch
  50  7.4002879e+05 3.234e-03  3.147e+05  -1.00  1.000e+00  2.273e-02  1(s
  ↪ )
  51  7.4003883e+05 1.340e-05  2.982e+03  -1.00  6.601e-01  1.000e+00  1(s
  ↪ )
  52  7.4003471e+05 3.828e-04  1.015e+01  -1.00  1.000e+00  1.000e+00  1(f
  ↪ )
```

```
  53  7.4003556e+05 2.461e-08  2.164e-01  -3.82  9.883e-01  1.000e+00  1(h
     ↪ )
  54  7.4003556e+05 2.665e-14  5.489e-07  -5.73  1.000e+00  1.000e+00  1(f
     ↪ )
Successfull termination.
Total time 1.527 sec
Hiop internal time:    total 1.465 sec    avg iter 0.027 sec
    internal total std dev across ranks 0.000 percent
Fcn/deriv time:    total=0.052 sec  ( obj=0.009 grad=0.003 cons=0.012 Jac
   ↪ =0.011 Hess=0.018)
    Fcn/deriv total std dev across ranks 0.000 percent
Fcn/deriv #: obj 97 grad 56 eq cons 97 ineq cons 97 eq Jac 56 ineq Jac 56
Total KKT time 1.249 sec
update init 0.773sec    update linsys 0.029 sec    fact 0.237 sec
solve rhs-manip 0.172 sec    triangular solve 0.038 sec


=============================================================
Optimal Power Flow
=============================================================
Model                        PBPOLRAJAHIOP
Solver                       HIOP
Objective                    MIN_GEN_COST
Initialization               MIDPOINT
Gen. bus voltage mode        VARIABLE_WITHIN_BOUNDS
Load loss allowed            YES
Load loss penalty ($)        1000.
Power imbalance allowed       NO
Ignore line flow constraints  NO


Number of variables          30
Number of equality constraints    18
Number of inequality constraints  18


Convergence status           CONVERGED
Objective value              740035.56


-------------------------------------------------------------------
Bus       Pd     Qd     Vm     Va     mult_Pmis     mult_Qmis
   ↪ Pslack        Qslack
-------------------------------------------------------------------
1        0.00   0.00  1.100   0.000      4182.05          0.00
   ↪ 0.00          0.00
2        0.00   0.00  1.100  12.398      4028.36          0.00
   ↪ 0.00          0.00
3        0.00   0.00  1.100  16.811      6041.35          0.00
   ↪ 0.00          0.00
4        0.00   0.00  1.021  -4.918      5930.37      19899.38
```

```
  ↪ 0.00            0.00
5      417.50  109.68   0.900  -15.480    100000.00     100000.00
  ↪ 0.00            0.00
6       90.00   30.00   1.018   -2.414      7827.64      16824.43
  ↪ 0.00            0.00
7        0.00    0.00   1.052    5.271     19366.68      29169.45
  ↪ 0.00            0.00
8      100.00   35.00   1.053    5.150     15594.67      21709.27
  ↪ 0.00            0.00
9        0.00    0.00   1.083    9.964      7193.17       9437.04
  ↪ 0.00            0.00


-----------------------------------------------------------------------------
  ↪
From      To      Status      Sft      Stf      Slim    mult_Sf  mult_St
-----------------------------------------------------------------------------
  ↪
1         4         1       230.52   213.89   380.00     -0.00     -0.00
2         7         1       250.00   239.12   250.00   2404.19      0.00
3         9         1       246.68   242.96   300.00      0.00      0.00
4         5         1       250.00   227.52   250.00  14113.38      0.00
4         6         1        47.20    50.98   250.00     -0.00     -0.00
5         7         1       210.17   239.03   250.00      0.00      0.00
6         9         1       137.83   144.52   150.00      0.00      0.00
7         8         1        10.10     7.53   250.00     -0.00     -0.00
8         9         1       100.92    98.82   150.00     -0.00     -0.00


-----------------------------------------------------------------------------
  ↪
Gen      Status      Fuel      Pg       Qg       Pmin      Pmax      Qmin
  ↪ Qmax
-----------------------------------------------------------------------------
  ↪
1          1     UNDEFINED   167.37   158.52    10.00    350.00   -300.00
  ↪ 300.00
2          1     UNDEFINED   229.90    98.21    10.00    300.00   -300.00
  ↪ 300.00
3          1     UNDEFINED   242.50    45.19    10.00    270.00   -300.00
  ↪ 300.00
[ExaGO] Finalizing opflow application.
```

Chapter 4

Multi-period optimal power flow (TCOPFLOW)

TCOPFLOW solves a full AC multi-period optimal power flow problem with the objective of minimizing the total cost over the given time horizon while adhering to constraints for each period and between consecutive time-periods (ramping constraints).

4.1 Formulation

The multi-period optimal power flow problem is a series of optimal power flow problems coupled via temporal constraints. The generator real power deviation $(p^{\mathrm{g}}_{jt} - p^{\mathrm{g}}_{jt-\Delta t})$ constrained within the ramp limits form the temporal constraints. An illustration of the temporal constraints is shown in Fig. **??** with four time steps. Each time-step t is coupled with its preceding time $t - \Delta t$, where Δt is the time-step where the objective is to find a least cost dispatch for the given time horizon.
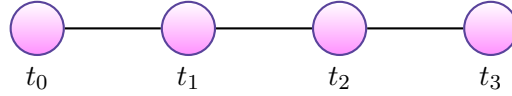
Figure 4.1: Multi-period optimal power flow example with four time-steps. The lines connecting the different time-periods denote the coupling between them.

In general form, the equations for multi-period optimal power flow are given by (**??**) – (**??**). TCOPFLOW solves to minimize the total generation cost $\sum_{t=0}^{N_t-1} f(x_t)$ over the time horizon, where N_t is the number of time-steps. At each time-step, the equality constraints $(g(x_t))$, inequality $h(x_t)$, and the lower/upper limit (x^-, x^+) constraints need to be satisfied. Equation (**??**) represents the coupling between the consecutive time-steps. It is the most common form of coupling that limits the deviation of the real power generation at time t from its preceding time-step $t - \Delta t$ to within its ramping capability Δx_t.

$$\min \sum_{t=0}^{N_t-1} f(x_t) \tag{4.1}$$

$$\text{s.t.}$$

$$g(x_t) = 0, \qquad\qquad t \in [0, N_t - 1] \tag{4.2}$$

$$h(x_t) \leq 0, \qquad\qquad t \in [0, N_t - 1] \tag{4.3}$$

$$x^- \leq x_t \leq x^+, \qquad\qquad t \in [0, N_t - 1] \tag{4.4}$$

$$-\Delta x_t \leq x_t - x_{t-\Delta t} \leq \Delta x_t, \qquad\qquad t \in [1, N_t - 1] \tag{4.5}$$

4.2   Solvers

Currently, `ExaGO` only supports solving TCOPFLOW using Ipopt [?] on on a single rank.

4.3   Input and Output

- **Network file:** The network file describing the network details. Only MATPOWER format files are currently supported.

- **Load data:** One file for load real power and one for reactive power. The files need to be in CSV format. An example of the format for the 9-bus case is here.

- **Wind generation:** The wind generation time-series described in CSV format. See an example of the format here.

If the load data and/or wind generation profiles are not set then a flat profile is assumed, i.e., the load and wind generation for all hours is constant.

The TCOPFLOW output is saved to a directory named `tcopflowout`. This directory contains N_t files, one for each time-step, in MATPOWER data file format.

4.4   Usage

```
./bin/tcopflow -netfile $EXAGO_DIR/datafiles/case9/case9mod.m -
    ↪ tcopflow_ploadprofile $EXAGO_DIR/datafiles/case9/load_P.csv -
    ↪ tcopflow_qloadprofile $EXAGO_DIR/datafiles/case9/load_Q.csv -
    ↪ print_output -tcopflow_dT 5 -tcopflow_duration 0.5
[ExaGO] TCOPFLOW: Application created
[ExaGO] TCOPFLOW: Duration = 0.500000 hours, timestep = 5.000000 minutes,
    ↪ number of time-steps = 7
[ExaGO] TCOPFLOW: Using IPOPT solver
[ExaGO] TCOPFLOW: Setup completed


**************************************************************************
    ↪
This program contains Ipopt, a library for large-scale nonlinear
    ↪ optimization.
```

This is Ipopt version 3.12.10, running with linear solver ma27.

Number of nonzeros in equality constraint Jacobian...:   798
Number of nonzeros in inequality constraint Jacobian.:   540
Number of nonzeros in Lagrangian Hessian.............:   672

Total number of variables............................:   168
                     variables with only lower bounds: 0
                variables with lower and upper bounds: 112
                     variables with only upper bounds: 0
Total number of equality constraints.................:   126
Total number of inequality constraints...............:   144
        inequality constraints with only lower bounds: 0
   inequality constraints with lower and upper bounds: 144
        inequality constraints with only upper bounds: 0

iter   objective   inf_pr   inf_du lg(mu) ||d|| lg(rg) alpha_du
    ↪ alpha_pr ls
   0 7.2226875e+04 1.80e+00 1.00e+02 -1.0 0.00e+00   - 0.00e+00 0.00e
    ↪ +00   0
   1 5.1460539e+04 1.04e+00 1.60e+02 -1.0 1.08e+00   - 6.27e-01 4.21e
    ↪ -01f 1
   2 4.8692257e+04 9.13e-01 1.60e+03 -1.0 1.42e+00   2.0 2.84e-03 1.29e
    ↪ -01f 1
   3 4.1695179e+04 5.82e-01 8.63e+02 -1.0 1.09e+00   - 3.86e-03 3.78e
    ↪ -01f 1
   4 3.8223523e+04 4.01e-01 5.86e+02 -1.7 7.91e-01   - 2.00e-01 3.10e
    ↪ -01f 1
   5 3.1081047e+04 9.06e-02 1.98e+03 -1.7 5.58e-01   - 4.80e-01 1.00e
    ↪ +00f 1
   6 3.0468392e+04 1.83e-01 1.34e+02 -1.7 5.84e-01   - 2.75e-01 1.00e
    ↪ +00f 1
   7 3.0405940e+04 9.33e-02 4.47e+00 -1.7 3.04e-01   - 4.96e-01 1.00e
    ↪ +00f 1
   8 3.0382232e+04 3.12e-02 7.07e-01 -2.5 1.76e-01   - 7.33e-01 1.00e
    ↪ +00h 1
   9 3.0369124e+04 2.45e-03 4.69e-02 -2.5 4.46e-02   - 1.00e+00 1.00e
    ↪ +00h 1
iter   objective   inf_pr   inf_du lg(mu) ||d|| lg(rg) alpha_du
    ↪ alpha_pr ls

```
  10  3.0364520e+04 9.81e-04 7.50e-02  -3.8 2.34e-02     -  9.84e-01 6.75e
      ↪ -01h  1
  11  3.0364301e+04 1.91e-04 1.14e-02  -3.8 1.67e-02     -  8.91e-01 9.19e
      ↪ -01h  1
  12  3.0364426e+04 4.32e-05 9.33e-05  -3.8 6.82e-03     -  1.00e+00 1.00e
      ↪ +00f  1
  13  3.0364326e+04 1.45e-05 4.77e-03  -5.7 3.90e-03     -  1.00e+00 9.44e
      ↪ -01h  1
  14  3.0364329e+04 1.12e-06 2.32e-06  -5.7 1.10e-03     -  1.00e+00 1.00e
      ↪ +00h  1
  15  3.0364328e+04 7.91e-09 3.33e-08  -7.0 9.28e-05     -  1.00e+00 1.00e
      ↪ +00h  1

Number of Iterations....: 15

                                    (scaled)                 (unscaled)
Objective...............:  6.8081453631121144e+02    3.0364328319480031e
    ↪ +04
Dual infeasibility......:  3.3330834427063850e-08    1.4865552154470478e
    ↪ -06
Constraint violation....:  1.5610245006347778e-09    1.5610245006347778e
    ↪ -09
Complementarity.........:  1.0802025033126202e-07    4.8177031647742860e
    ↪ -06
Overall NLP error.......:  1.0802025033126202e-07    4.8177031647742860e
    ↪ -06


Number of objective function evaluations            = 16
Number of objective gradient evaluations            = 16
Number of equality constraint evaluations           = 16
Number of inequality constraint evaluations         = 16
Number of equality constraint Jacobian evaluations  = 16
Number of inequality constraint Jacobian evaluations = 16
Number of Lagrangian Hessian evaluations            = 15
Total CPU secs in IPOPT (w/o function evaluations)  =      0.036
Total CPU secs in NLP function evaluations          =      0.014

EXIT: Optimal Solution Found.
================================================================
Multi-Period Optimal Power Flow
================================================================
OPFLOW Model                    POWER_BALANCE_POLAR
Solver                          IPOPT
Duration (minutes)              30.00
Time-step (minutes)             5.00
Number of steps                 7
```

```
Active power demand profile          /people/abhy245/projects/exaSGD/exago/
    ↪ datafiles/case9/load_P.csv
Rective power demand profile         /people/abhy245/projects/exaSGD/exago/
    ↪ datafiles/case9/load_Q.csv
Wind generation profile              NOT SET
Load loss allowed                    NO
Power imbalance allowed              NO
Ignore line flow constraints         NO

Number of variables                  168
Number of equality constraints       126
Number of inequality constraints     126
Number of coupling constraints       18

Convergence status                   CONVERGED
Objective value                      30364.33

--------------------------------------------------------------------
Bus        Pd        Qd       Vm       Va       mult_Pmis       mult_Qmis
    ↪ Pslack        Qslack
--------------------------------------------------------------------
1         0.00     0.00    1.089    0.000      2102.34          -0.00
    ↪ 0.00          0.00
2         0.00     0.00    1.092    3.904      2059.04          -0.00
    ↪ 0.00          0.00
3         0.00     0.00    1.087    2.077      2065.00          -0.00
    ↪ 0.00          0.00
4         0.00     0.00    1.095   -2.016      2102.60          -0.04
    ↪ 0.00          0.00
5        75.00    30.00    1.088   -3.150      2112.60           1.16
    ↪ 0.00          0.00
6        90.00    30.00    1.085   -3.962      2129.51           1.68
    ↪ 0.00          0.00
7         0.00     0.00    1.100    0.501      2059.43          -0.06
    ↪ 0.00          0.00
8       100.00    35.00    1.089   -1.757      2079.19           2.98
    ↪ 0.00          0.00
9         0.00     0.00    1.100   -0.176      2065.27          -0.09
    ↪ 0.00          0.00

--------------------------------------------------------------------
    ↪
From       To       Status     Sft      Stf      Slim     mult_Sf   mult_St
--------------------------------------------------------------------
    ↪
1          4         1        73.35    73.70    380.00    -0.00     -0.00
2          7         1       114.60   115.46    250.00    -0.00     -0.00
```

3	9	1	83.29	84.28	300.00	-0.00	-0.00
4	5	1	28.85	31.90	250.00	-0.00	-0.00
4	6	1	44.89	45.71	250.00	-0.00	-0.00
5	7	1	49.16	50.83	250.00	-0.00	-0.00
6	9	1	49.36	51.07	150.00	-0.00	-0.00
7	8	1	66.70	68.20	250.00	-0.00	-0.00
8	9	1	38.80	34.06	150.00	-0.00	-0.00

Gen	Status	Fuel	Pg	Qg	Pmin	Pmax	Qmin	Qmax

1	1	UNDEFINED	72.83	-8.69	10.00	350.00	-300.00	300.00
2	1	UNDEFINED	114.06	-11.12	10.00	300.00	-300.00	300.00
3	1	UNDEFINED	80.20	-22.47	10.00	270.00	-300.00	300.00

[ExaGO] Finalizing tcopflow application.

Chapter 5

Security-constrained optimal power flow (SCOPFLOW)

SCOPFLOW solves a contingency-constrained optimal power flow problem. The problem is set up as a two-stage optimization problem where the first-stage (base-case) represents the normal operation of the grid and the second-stage comprises N_c contingency scenarios. Each contingency scenario can be single or multi-period.

5.1 Formulation

5.1.1 Single-period

The contingency-constrained optimal power flow (popularly termed as security-constrained optimal power flow (SCOPF) in power system parlance) attempts to find a least cost dispatch for the base case (or no contingency) while ensuring that if any of contingencies do occur then the system will be secure. This is illustrated in Fig. **??** for a SCOPF with a base-case c_0 and three contingencies.
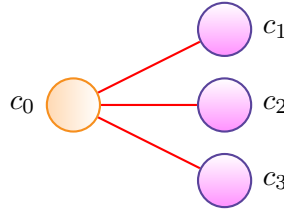
Figure 5.1: Contingency constrained optimal power flow example with three contingencies. c_0 represents the base case (or no contingency case). c_1, c_2, c_3 are the three contingency cases. Each of the contingency states is coupled with the base-case through ramping constraints (denoted by red lines)

In general form, the equations for contingency-constrained optimal power flow are given by (**??**) – (**??**). This is a two-stage stochastic optimization problem where the first stage is the base case c_0 and each of the contingency states $c_i, i \in [1, N_c]$ are second-stage subproblems. SCOPFLOW aims to minimize the objective $\sum_{c=0}^{N_c} f(x_c)$, while adhering to the equality $g_c(x_c)$, inequality $h_c(x_c)$, and the lower/upper bound (x^-, x^+) constraints. Equation (**??**) represents the coupling between the base-case c_0 and each of the contingency states c_i. Equation (**??**) is the most typical form of coupling that limits the deviation of the contingency variables x_c from the base x_0 to within Δx_c. An example of this

constraint could be the allowed real power output deviation for the generators constrained by their ramp limit, which is currently the only constraint SCOPFLOW supports.

$$\min \sum_{c=0}^{N_c} f_c(x_c) \tag{5.1}$$

$$\text{s.t.}$$

$$g_c(x_c) = 0, \qquad\qquad c \in [0, N_c] \tag{5.2}$$

$$h_c(x_c) \leq 0, \qquad\qquad c \in [0, N_c] \tag{5.3}$$

$$x^- \leq x_c \leq x^+, \qquad\qquad c \in [0, N_c] \tag{5.4}$$

$$-\Delta x_c \leq x_c - x_0 \leq \Delta x_c, \qquad\qquad c \in [1, N_c] \tag{5.5}$$

5.1.2   Multiperiod

In the multi-period version, each contingency is comprised of multiple time-periods. The multiple periods have variables and constraints as described in chapter **??**. An example of multi-contingency multi-period optimal power flow is illustrated in Fig. **??** for multi-period SCOPFLOW with three contingencies c_1, c_2, and c_3 coupled to the base case c_0. Each state is multi-period with two time-periods. Each time-step is coupled with its adjacent one through ramping constraints. We assume that the contingency is incident at the first time-step, i.e. at t_0. This results in the coupling between the contingency cases $c_i, i \in [1, N_c]$ and the base-case c_0 only at time-step t_0.
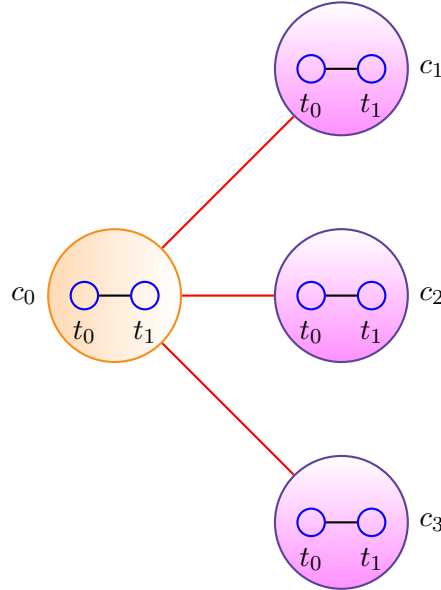
Figure 5.2: Multi-period contingency constrained optimal power flow example with two contingencies c_0 and c_1, each with two time-periods t_0, t_1. State c_0 represents the base case (no contingency) case. The contingency states c_1, c_2, c_3 are coupled with the no-contingency state c_0. The red line denotes the coupling between the contingencies.

The overall objective of this contingency-constrained multi-period optimal power flow is to find a secure dispatch for base-case c_0 while adhering to contingency and temporal constraints. The general formulation of this problem is given in Eqs. (**??**) – (**??**).

$$\min \sum_{c=0}^{N_c} \sum_{t=0}^{N_t-1} f_{ct}(x_{c,t}) \tag{5.6}$$

$$\text{s.t.}$$

$$g_{ct}(x_{c,t}) = 0, \qquad\qquad c \in [0, N_c], t \in [0, N_t - 1] \tag{5.7}$$

$$h_{ct}(x_{c,t}) \leq 0, \qquad\qquad c \in [0, N_c], t \in [0, N_t - 1] \tag{5.8}$$

$$x^- \leq x_{c,t} \leq x^+, \qquad\qquad c \in [0, N_c], t \in [0, N_t - 1] \tag{5.9}$$

$$-\Delta x_t \leq x_{c,t} - x_{c,t-\Delta t} \leq \Delta x_t, \qquad\qquad c \in [0, N_c], t \in [1, N_t - 1] \tag{5.10}$$

$$-\Delta x_c \leq x_{c,0} - x_{0,0} \leq \Delta x_c, \qquad\qquad c \in [1, N_c] \tag{5.11}$$

$$\tag{5.12}$$

In this formulation, the objective is to reduce the cost for the base-case time horizon, where $f(x_{0,t})$ is the objective cost for contingency c_0 at time t. Equation (**??**) represents the coupling between the base case c_0 and each contingency c_i at time-step t_0. We use a simple box constraint Δx_c to restrict the deviation of decision variables $x_{c,0}$ from the base-case $x_{0,0}$. The bound Δx_c could represent here, for example, the allowable reserve for each generator.

5.2   Solvers

SCOPFLOW supports solving the optimization problem via Ipopt [**?**], HiOp [**?, ?**], or *EMPAR*. Ipopt [**?**]can solve SCOPFLOW on single rank only. HiOp [**?, ?**]supports solving the problem in parallel using a primal-decomposition algorithm. With HIOP, one can solve the subproblem either on the CPU or GPU by selecting the appropriate subproblem model and solver (see options table below). The *EMPAR* solver does not solve the security-constrained ACOPF problem, but it only solves the base-case and the contingencies independently with OPFLOW. It distributes the contingencies to different processes when executed in parallel.

5.3   Input and Output

To execute SCOPFLOW, the following files are required:

- **Network file:** The network file describing the network details. Only MATPOWER format files are currently supported.

- **Contingency file:** The file describing the contingencies. Contingencies can be single or multiple outages. The contingency file needs to be described in PTI format.

If the multi-period option is chosen, then additional files describing the load and wind generation can be (optionally) set.

- **Load data:** One file for load real power and one for reactive power. The files need to be in CSV format. An example of the format for the 9-bus case is here.

- **Wind generation:** The wind generation time-series described in CSV format. See an example of the format here.

The SCOPFLOW output is saved to a directory named `scopflowout`. This directory contains N_c files to save the solution for each contingency in MATPOWER datafile format. Each file has the name `cont_xx` where `xx` is the contingency number.

If the multi-period option is chosen then N_c subdirectories are created (one for each contingency), and each subdirectory contains N_t output files, one for each time-period. The subdirectories have the naming convention `cont_xx` and the output file are named as `t_yy` where `yy` is the time-step number.

5.4   Usage

```
./scopflow -netfile <netfilename> -ctgcfile <ctgcfilename> \
<scopflowoptions> [-scopflow_enable_multiperiod 1]
```

5.5   Options

See table **??**. In addition, all OPFLOW options in Table **??** and TCOPFLOW options in Table **??** can be used.

Table 5.1: SCOPFLOW options

Option	Meaning	Values (Default value)	Compatibility
-netfile	Network file name	string < 4096 characters (case9mod_gen3_wind.m)	
-ctgcfile	Contingency file name	string < 4096 characters (case9.cont)	
-print_output	Print output to screen	0 or 1 (0)	
-save_output	Save output to directory	0 or 1 (0)	
-scopflow_solver	Set solver for scopflow	IPOPT, HIOP, or EMPAR (IPOPT)	
-scopflow_subproblem_solver	Set solver for subproblem	IPOPT or HIOP (IPOPT)	Only when using HIOP solver for SCOPFLOW
-scopflow_subproblem_model	Set model for subproblem	See OPFLOW chapter	Only when using HIOP solver for SCOPFLOW
-scopflow_Nc	Number of contingencies	int (0. Passing -1 results in all contingencies in the file used)	
-scopflow_mode	Operation mode: Preventive or corrective	0 or 1 (0)	
-scopflow_enable_multiperiod	Multi-period SCOPFLOW	0 or 1 (0)	IPOPT solver only
-scopflow_pload_profile	Real power load profile	string (load_P.csv)	
-scopflow_qload_profile	Reactive power load profile	string (load_Q.csv)	
-scopflow_windgenprofile	Wind generation profile	string (case9/scenarios_9bus.csv)	
-scopflow_dT	Length of time-step (minutes)	double (5.0)	
-scopflow_duration	Total duration (hours)	double (0.5)	

Depending on the value chosen for `-scopflow_mode`, SCOPFLOW can operate either in *preventive* (mode = 0) or *corrective* (mode = 1) mode. In the preventive mode, the PV and PQ generator real power is fixed to its corresponding base-case values. The generators at the reference bus pick up any make-up power required for the contingency. The corrective mode allows deviation of the PV and PQ generator real power from the base-case dispatch constrained by its 30-min. ramp rate capability. The optimization decides the optimal redispatch. One can have AGC control instead of having the generators proportionally share the deficit/excess power by using the option *(-opflow˙use˙agc)*.

The option `-scopflow_enable_multiperiod 1` must be used in order to enable any of the options listed in table **??** for multiperiod analysis.

5.6   Examples

Some SCOPFLOW example runs are provided with some sample output. Options are the default options given in table **??**, **??** and **??** unless otherwise specified. Sample output is generated by running examples in the installation directory.

Example using the Ipopt [**?**]solver:

```
./bin/scopflow -netfile $EXAGO_DIR/datafiles/case9/case9mod.m -ctgcfile
    ↪ $EXAGO_DIR/datafiles/case9/case9.cont -scopflow_Nc 4 -
    ↪ scopflow_solver IPOPT -print_output
[ExaGO] SCOPFLOW: Application created
[ExaGO] SCOPFLOW running with 5 subproblems (base case + 4 contingencies)
[ExaGO] SCOPFLOW: Using IPOPT solver
[ExaGO] SCOPFLOW: Setup completed


******************************************************************************
    ↪
This program contains Ipopt, a library for large-scale nonlinear
    ↪ optimization.
 Ipopt is released as open source code under the Eclipse Public License (
    ↪ EPL).
        For more information visit http://projects.coin-or.org/Ipopt
******************************************************************************
    ↪

This is Ipopt version 3.12.10, running with linear solver ma27.

Number of nonzeros in equality constraint Jacobian...:      598
Number of nonzeros in inequality constraint Jacobian.:      328
Number of nonzeros in Lagrangian Hessian.............:      660

Total number of variables............................:      144
                    variables with only lower bounds:        0
                variables with lower and upper bounds:      104
                    variables with only upper bounds:        0
Total number of equality constraints.................:      114
Total number of inequality constraints...............:       82
```

```
           inequality constraints with only lower bounds:        0
      inequality constraints with lower and upper bounds:       82
           inequality constraints with only upper bounds:        0

iter     objective     inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
   ↪ alpha_pr  ls
   0  1.0318125e+04 1.80e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e
   ↪ +00    0
   1  9.2868867e+03 1.56e+00 8.62e+01  -1.0 1.08e+00    -  6.27e-01 1.31e
   ↪ -01f  1
   2  9.0292788e+03 1.50e+00 3.50e+02  -1.0 2.60e+00    -  1.37e-03 4.28e
   ↪ -02f  1
   3  6.8151659e+03 9.02e-01 3.07e+03  -1.0 1.03e+00    -  4.57e-02 4.09e
   ↪ -01f  1
   4  5.6911245e+03 5.63e-01 2.09e+03  -1.0 7.85e-01   2.0 1.22e-01 3.87e
   ↪ -01f  1
   5  5.6219674e+03 5.39e-01 2.00e+03  -1.0 8.34e-01    -  1.99e-01 4.29e
   ↪ -02f  1
   6  4.7820782e+03 2.34e-01 7.42e+02  -1.0 7.27e-01    -  5.45e-01 5.75e
   ↪ -01f  1
   7  4.1986838e+03 3.84e-02 2.59e+02  -1.7 4.11e-01    -  3.64e-01 1.00e
   ↪ +00f  1
   8  4.1593634e+03 6.26e-02 1.52e+01  -1.7 3.77e-01    -  5.76e-01 1.00e
   ↪ +00f  1
   9  4.1598457e+03 3.63e-04 1.53e+00  -2.5 4.57e-02   1.5 8.25e-01 1.00e
   ↪ +00h  1
iter     objective     inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
   ↪ alpha_pr  ls
  10  4.1472241e+03 4.61e-02 5.44e+00  -3.8 1.15e-01    -  3.15e-01 1.00e
   ↪ +00f  1
  11  4.1437514e+03 3.32e-02 3.50e+00  -3.8 1.95e-01    -  5.75e-01 3.47e
   ↪ -01h  1
  12  4.1440398e+03 1.07e-02 1.64e+00  -3.8 7.68e-02    -  2.42e-01 6.18e
   ↪ -01h  1
  13  4.1444455e+03 1.33e-03 4.38e-02  -3.8 3.29e-02    -  9.49e-01 1.00e
   ↪ +00h  1
  14  4.1444954e+03 5.41e-06 2.57e-02  -3.8 2.31e-03   1.0 1.00e+00 1.00e
   ↪ +00h  1
  15  4.1444778e+03 1.47e-05 1.41e-02  -3.8 3.80e-03   0.6 1.00e+00 1.00e
   ↪ +00h  1
  16  4.1444804e+03 1.82e-05 4.42e-03  -3.8 3.58e-03   0.1 1.00e+00 1.00e
   ↪ +00h  1
  17  4.1444799e+03 9.05e-05 3.06e-03  -3.8 7.43e-03  -0.4 1.00e+00 1.00e
   ↪ +00h  1
  18  4.1444616e+03 1.08e-05 5.76e-03  -5.7 3.72e-03  -0.9 1.00e+00 9.26e
   ↪ -01h  1
  19  4.1444608e+03 5.66e-06 1.11e-04  -5.7 2.43e-03  -1.3 1.00e+00 1.00e
```

```
                                    ↪ +00h  1
iter    objective     inf_pr     inf_du lg(mu)   ||d||   lg(rg) alpha_du
   ↪ alpha_pr  ls
  20  4.1444608e+03 2.21e-08 3.62e-05  -5.7 2.38e-03  -1.8 1.00e+00 1.00e
       ↪ +00H  1
  21  4.1444608e+03 3.35e-07 2.93e-05  -5.7 5.76e-03  -2.3 1.00e+00 1.00e
       ↪ +00H  1
  22  4.1444608e+03 2.25e-04 1.95e-05  -5.7 1.15e-02  -2.8 1.00e+00 1.00e
       ↪ +00h  1
  23  4.1444608e+03 5.41e-04 1.11e-05  -5.7 1.96e-02  -3.2 1.00e+00 1.00e
       ↪ +00h  1
  24  4.1444608e+03 2.03e-02 1.22e-04  -5.7 1.15e-01  -3.7 1.00e+00 1.00e
       ↪ +00h  1
  25  4.1444608e+03 8.07e-04 2.02e-05  -5.7 2.32e-02  -3.3 1.00e+00 1.00e
       ↪ +00h  1
  26  4.1444608e+03 6.02e-03 3.97e-05  -5.7 6.75e-02  -3.8 1.00e+00 1.00e
       ↪ +00h  1
  27  4.1444608e+03 1.93e-02 1.75e-04  -5.7 1.17e-01  -4.3 1.00e+00 1.00e
       ↪ +00h  1
  28  4.1444608e+03 4.88e-02 7.07e-05  -5.7 1.71e-01  -4.7 1.00e+00 1.00e
       ↪ +00h  1
  29  4.1444608e+03 8.42e-04 6.07e-06  -5.7 3.53e-02  -4.3 1.00e+00 1.00e
       ↪ +00h  1
iter    objective     inf_pr     inf_du lg(mu)   ||d||   lg(rg) alpha_du
   ↪ alpha_pr  ls
  30  4.1444608e+03 1.16e-02 4.10e-06  -5.7 1.11e-01  -4.8 1.00e+00 1.00e
       ↪ +00h  1
  31  4.1444608e+03 1.54e-03 1.71e-06  -5.7 3.87e-02  -4.4 1.00e+00 1.00e
       ↪ +00h  1
  32  4.1444608e+03 2.12e-02 9.93e-06  -5.7 1.38e-01  -4.8 1.00e+00 1.00e
       ↪ +00h  1
  33  4.1444608e+03 1.52e-01 8.57e-05  -5.7 5.25e-01  -5.3 1.00e+00 1.00e
       ↪ +00h  1
  34  4.1444608e+03 5.89e-03 2.06e-05  -5.7 1.27e-01  -4.9 1.00e+00 1.00e
       ↪ +00h  1
  35  4.1444608e+03 2.60e-01 1.06e-04  -5.7 1.23e+00     -  7.38e-01 1.00e
       ↪ +00H  1
  36  4.1444608e+03 4.78e-02 3.16e-05  -5.7 3.18e-01     -  1.00e+00 1.00e
       ↪ +00h  1
  37  4.1444608e+03 1.18e-03 3.22e-07  -5.7 7.50e-02     -  1.00e+00 1.00e
       ↪ +00h  1
  38  4.1444608e+03 3.36e-05 2.96e-09  -5.7 1.62e-02     -  1.00e+00 1.00e
       ↪ +00h  1
  39  4.1444608e+03 3.14e-10 5.68e-13  -5.7 5.87e-05     -  1.00e+00 1.00e
       ↪ +00h  1
iter    objective     inf_pr     inf_du lg(mu)   ||d||   lg(rg) alpha_du
   ↪ alpha_pr  ls
```

```
  40  4.1444605e+03 7.54e-08 1.83e-07  -7.0 2.81e-04    -  1.00e+00 1.00e
     ↪ +00h  1


Number of Iterations....: 40

                                    (scaled)                 (unscaled)
Objective..............:   9.2925124417578417e+01   4.1444605490239974e
    ↪ +03
Dual infeasibility......:   1.8347495281748061e-07   8.1829828956596351e
    ↪ -06
Constraint violation....:   1.3290786859965209e-08   1.3290786859965209e
    ↪ -08
Complementarity.........:   2.6885276874539710e-07   1.1990833486044711e
    ↪ -05
Overall NLP error.......:   2.6885276874539710e-07   1.1990833486044711e
    ↪ -05


Number of objective function evaluations           = 44
Number of objective gradient evaluations           = 41
Number of equality constraint evaluations          = 44
Number of inequality constraint evaluations        = 44
Number of equality constraint Jacobian evaluations = 41
Number of inequality constraint Jacobian evaluations = 41
Number of Lagrangian Hessian evaluations           = 40
Total CPU secs in IPOPT (w/o function evaluations) =      0.078
Total CPU secs in NLP function evaluations         =      0.026

EXIT: Optimal Solution Found.
==============================================================
Security-Constrained Optimal Power Flow
==============================================================
Number of contingencies          4
Multi-period contingencies?      NO
Solver                           IPOPT
Initialization                   MIDPOINT
Load loss allowed                NO
Power imbalance allowed          NO
Ignore line flow constraints     NO


Convergence status              CONVERGED
Objective value (base)          4144.46


--------------------------------------------------------------
Bus       Pd      Qd      Vm      Va      mult_Pmis     mult_Qmis
    ↪ Pslack        Qslack
```

```
--------------------------------------------------------------------------------
1          0.00      0.00    1.100    0.000       2102.91          0.00
    ↪ 0.00          0.00
2          0.00      0.00    1.095    3.928       2059.18         -0.00
    ↪ 0.00          0.00
3          0.00      0.00    1.087    2.120       2065.15         -0.00
    ↪ 0.00          0.00
4          0.00      0.00    1.097   -1.993       2103.17          0.08
    ↪ 0.00          0.00
5         75.00     50.00    1.079   -3.060       2113.46          7.29
    ↪ 0.00          0.00
6         90.00     30.00    1.087   -3.927       2129.85          1.62
    ↪ 0.00          0.00
7          0.00      0.00    1.100    0.535       2059.57         -0.04
    ↪ 0.00          0.00
8        100.00     35.00    1.089   -1.720       2079.34          2.99
    ↪ 0.00          0.00
9          0.00      0.00    1.100   -0.135       2065.43         -0.09
    ↪ 0.00          0.00
```

From	To	Status	Sft	Stf	Slim	mult_Sf	mult_St
1	4	1	73.18	72.98	380.00	-0.00	-0.00
2	7	1	114.18	114.68	250.00	-0.00	-0.00
3	9	1	83.58	84.61	300.00	-0.00	-0.00
4	5	1	29.69	40.50	250.00	-0.00	-0.00
4	6	1	44.86	46.03	250.00	-0.00	-0.00
5	7	1	51.29	49.04	250.00	-0.00	-0.00
6	9	1	48.94	51.43	150.00	-0.00	-0.00
7	8	1	66.61	68.11	250.00	-0.00	-0.00
8	9	1	38.87	34.15	150.00	-0.00	-0.00

Gen	Status	Fuel	Pg	Qg	Pmin	Pmax	Qmin	Qmax
1	1	UNDEFINED	72.86	6.79	10.00	350.00	-300.00	300.00
2	1	UNDEFINED	114.07	-5.13	10.00	300.00	-300.00	300.00
3	1	UNDEFINED	80.21	-23.48	10.00	270.00	-300.00	300.00

```
[ExaGO] Finalizing scopflow application.
```

Example using HIOP solver with IPOPT subproblem solver.

```
bin/scopflow -netfile $EXAGO_DIR/datafiles/case9/case9mod.m -ctgcfile
    ↪ $EXAGO_DIR/datafiles/case9/case9.cont -scopflow_Nc 4 -
    ↪ scopflow_solver IPOPT -print_output -scopflow_solver HIOP -
    ↪ scopflow_subproblem_solver IPOPT -scopflow_subproblem_model
    ↪ POWER_BALANCE_POLAR
[ExaGO] SCOPFLOW: Application created
[ExaGO] SCOPFLOW running with 5 subproblems (base case + 4 contingencies)
[ExaGO] SCOPFLOW: Using HIOP solver
Failed to read option file 'hiop_pridec.options'. Hiop will use default
    ↪ options.
[ExaGO] SCOPFLOW: Setup completed
total number of recourse problems  4


****************************************************************************
    ↪
This program contains Ipopt, a library for large-scale nonlinear
    ↪ optimization.
 Ipopt is released as open source code under the Eclipse Public License (
    ↪ EPL).
         For more information visit http://projects.coin-or.org/Ipopt
****************************************************************************
    ↪


This is Ipopt version 3.12.10, running with linear solver ma27.

Number of nonzeros in equality constraint Jacobian...:      114
Number of nonzeros in inequality constraint Jacobian.:       72
Number of nonzeros in Lagrangian Hessian.............:       96

Total number of variables............................:       24
                     variables with only lower bounds:        0
                variables with lower and upper bounds:       16
                     variables with only upper bounds:        0
Total number of equality constraints.................:       18
Total number of inequality constraints...............:       18
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:       18
        inequality constraints with only upper bounds:        0

iter    objective     inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
    ↪ alpha_pr  ls
   0  1.0318125e+04 1.80e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e
      ↪ +00    0
   1  7.7157691e+03 1.17e+00 1.03e+02  -1.0 1.08e+00    -  6.27e-01 3.50e
```

```
   ↪ -01f  1
  2  7.6608235e+03 1.15e+00 1.01e+02  -1.0 6.28e+00    -  1.20e-02 1.37e
   ↪ -02f  1
  3  7.4466686e+03 1.09e+00 3.06e+02  -1.0 3.74e+00    -  4.15e-03 5.81e
   ↪ -02f  1
  4  5.4292675e+03 3.92e-01 4.83e+03  -1.0 7.34e-01    -  3.34e-03 6.40e
   ↪ -01f  1
  5  4.5792834e+03 2.24e-01 1.51e+03  -1.0 6.46e-01  2.0 8.77e-03 7.37e
   ↪ -01f  1
  6  4.2907579e+03 1.20e-02 3.57e+02  -1.0 3.36e-01    -  5.37e-01 1.00e
   ↪ +00f  1
  7  4.1690456e+03 4.40e-02 5.31e+01  -1.0 3.31e-01    -  9.22e-01 1.00e
   ↪ +00f  1
  8  4.1687926e+03 4.88e-04 1.93e+00  -1.0 4.79e-02  1.5 1.00e+00 1.00e
   ↪ +00h  1
  9  4.1497176e+03 1.19e-02 9.92e+00  -2.5 1.87e-01    -  8.38e-01 1.00e
   ↪ +00f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
  ↪ alpha_pr  ls
 10  4.1463942e+03 1.09e-02 5.09e-01  -2.5 1.15e-01    -  8.71e-01 1.00e
  ↪ +00h  1
 11  4.1449657e+03 1.47e-03 1.79e-02  -2.5 2.75e-02    -  1.00e+00 1.00e
  ↪ +00h  1
 12  4.1445415e+03 6.63e-04 9.12e-02  -3.8 1.48e-02    -  1.00e+00 6.30e
  ↪ -01h  1
 13  4.1444705e+03 3.43e-04 4.96e-02  -3.8 2.08e-02    -  1.00e+00 8.93e
  ↪ -01h  1
 14  4.1444809e+03 4.48e-05 1.79e-04  -3.8 6.82e-03    -  1.00e+00 1.00e
  ↪ +00f  1
 15  4.1444611e+03 1.96e-05 4.55e-03  -5.7 4.57e-03    -  1.00e+00 9.34e
  ↪ -01h  1
 16  4.1444607e+03 6.49e-06 1.17e-05  -5.7 2.60e-03    -  1.00e+00 1.00e
  ↪ +00h  1
 17  4.1444605e+03 1.19e-06 2.07e-06  -7.0 1.11e-03    -  1.00e+00 1.00e
  ↪ +00h  1
 18  4.1444605e+03 1.58e-07 3.29e-07  -7.0 4.06e-04    -  1.00e+00 1.00e
  ↪ +00h  1

Number of Iterations....: 18

                                (scaled)                   (unscaled)
Objective...............:  9.2925122354655841e+01   4.1444604570176507e
   ↪ +03
Dual infeasibility......:  3.2927387965389691e-07   1.4685615032563802e
   ↪ -05
Constraint violation....:  2.6639677713768961e-08   2.6639677713768961e
   ↪ -08
```

```
Complementarity.........:   4.7840038622596930e-07   2.1336657225678232e
   ↪ -05
Overall NLP error.......:   4.7840038622596930e-07   2.1336657225678232e
   ↪ -05


Number of objective function evaluations             = 19
Number of objective gradient evaluations             = 19
Number of equality constraint evaluations            = 19
Number of inequality constraint evaluations          = 19
Number of equality constraint Jacobian evaluations   = 19
Number of inequality constraint Jacobian evaluations = 19
Number of Lagrangian Hessian evaluations             = 18
Total CPU secs in IPOPT (w/o function evaluations)   =      0.027
Total CPU secs in NLP function evaluations           =      0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.12.10, running with linear solver ma27.

Number of nonzeros in equality constraint Jacobian...:      118
Number of nonzeros in inequality constraint Jacobian.:       64
Number of nonzeros in Lagrangian Hessian.............:      141

Total number of variables............................:       30
                     variables with only lower bounds:        0
                variables with lower and upper bounds:       22
                     variables with only upper bounds:        0
Total number of equality constraints.................:       24
Total number of inequality constraints...............:       16
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:       16
        inequality constraints with only upper bounds:        0

iter    objective     inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
   ↪ alpha_pr  ls
   0  0.0000000e+00 1.80e+00 0.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e
      ↪ +00    0
   1  0.0000000e+00 1.41e+00 3.55e+01  -1.0 9.96e-01    -  6.48e-01 2.16e
      ↪ -01h  1
   2  0.0000000e+00 1.36e+00 5.74e+01  -1.0 3.86e+00    -  3.79e-03 3.37e
      ↪ -02f  1
   3  0.0000000e+00 1.12e+00 3.23e+02  -1.0 1.01e+00    -  1.98e-02 1.76e
      ↪ -01f  1
   4  0.0000000e+00 4.03e-01 7.31e+02  -1.0 7.32e-01    -  2.71e-02 9.03e
      ↪ -01f  1
   5  0.0000000e+00 8.41e-02 1.58e+02  -1.0 6.95e-01    -  4.70e-01 8.52e
      ↪ -01h  1
```

```
  6  0.0000000e+00 4.93e-03 3.36e+01  -1.0 1.85e-01    -  1.00e+00 1.00e
    ↪ +00f  1
  7  0.0000000e+00 2.20e-02 2.50e+01  -1.0 6.50e-01    -  4.45e-01 2.50e
    ↪ -01h  3
  8  0.0000000e+00 1.07e-01 5.13e+00  -1.0 3.57e-01   0.0 1.00e+00 1.00e
    ↪ +00h  1
  9  0.0000000e+00 1.07e-01 4.86e+00  -1.7 9.45e-01    -  3.30e-01 1.98e
    ↪ -01h  2
iter    objective     inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
  ↪ alpha_pr  ls
 10  0.0000000e+00 9.21e-03 3.47e+00  -1.7 1.17e-01  -0.5 8.84e-01 1.00e
    ↪ +00h  1
 11  0.0000000e+00 2.70e-02 9.88e-01  -1.7 1.67e-01    -  1.00e+00 1.00e
    ↪ +00h  1
 12  0.0000000e+00 8.46e-03 1.62e-01  -1.7 1.58e-01  -1.0 1.00e+00 1.00e
    ↪ +00h  1
 13  0.0000000e+00 7.11e-04 1.09e-01  -2.5 4.58e-02  -1.4 1.00e+00 1.00e
    ↪ +00h  1
 14  0.0000000e+00 1.11e-01 9.70e-02  -2.5 1.43e+00    -  1.00e+00 1.00e
    ↪ +00H  1
 15  0.0000000e+00 2.40e-02 5.99e-02  -2.5 2.27e-01  -1.9 1.00e+00 1.00e
    ↪ +00h  1
 16  0.0000000e+00 3.45e-02 2.41e-01  -2.5 3.69e-01    -  5.82e-01 1.00e
    ↪ +00h  1
 17  0.0000000e+00 1.85e-02 1.46e-01  -2.5 3.58e-01    -  1.00e+00 1.00e
    ↪ +00h  1
 18  0.0000000e+00 8.37e-03 2.78e-02  -2.5 9.85e-02    -  1.00e+00 1.00e
    ↪ +00h  1
 19  0.0000000e+00 6.33e-06 5.45e-04  -3.8 6.81e-03    -  1.00e+00 1.00e
    ↪ +00h  1
iter    objective     inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
  ↪ alpha_pr  ls
 20  0.0000000e+00 1.42e-07 2.95e-06  -5.7 1.02e-03    -  1.00e+00 1.00e
    ↪ +00h  1
 21  0.0000000e+00 1.44e-06 1.25e-07  -7.0 3.43e-03    -  1.00e+00 1.00e
    ↪ +00h  1

Number of Iterations....: 21

                                (scaled)                 (unscaled)
Objective...............:  0.0000000000000000e+00   0.0000000000000000e
    ↪ +00
Dual infeasibility......:  1.2466133995485293e-07   1.2466133995485293e
    ↪ -07
Constraint violation....:  1.8383034489088956e-07   1.8383034489088956e
    ↪ -07
Complementarity.........:  1.0156847334169061e-07   1.0156847334169061e
```

```
      ↪ -07
Overall NLP error.......:   1.8383034489088956e-07    1.8383034489088956e
      ↪ -07


Number of objective function evaluations         = 28
Number of objective gradient evaluations         = 22
Number of equality constraint evaluations        = 28
Number of inequality constraint evaluations      = 28
Number of equality constraint Jacobian evaluations   = 22
Number of inequality constraint Jacobian evaluations = 22
Number of Lagrangian Hessian evaluations         = 21
Total CPU secs in IPOPT (w/o function evaluations)   =      0.032
Total CPU secs in NLP function evaluations       =      0.003

EXIT: Optimal Solution Found.

...
...


================================================================
Security-Constrained Optimal Power Flow
================================================================
Number of contingencies          4
Multi-period contingencies?       NO
Solver                            HIOP
Initialization                    MIDPOINT
Load loss allowed                 NO
Power imbalance allowed           NO
Ignore line flow constraints      NO



Convergence status               CONVERGED
Objective value (base)           4144.46


----------------------------------------------------------------------
Bus        Pd      Qd      Vm      Va      mult_Pmis      mult_Qmis
    ↪ Pslack        Qslack
----------------------------------------------------------------------
1        0.00    0.00   1.100   0.000     2102.91          0.00
    ↪ 0.00          0.00
2        0.00    0.00   1.095   3.928     2059.18         -0.00
    ↪ 0.00          0.00
3        0.00    0.00   1.087   2.120     2065.15         -0.00
    ↪ 0.00          0.00
4        0.00    0.00   1.097  -1.993     2103.16          0.08
    ↪ 0.00          0.00
```

```
5          75.00    50.00    1.079   -3.060       2113.45           7.29
    ↪ 0.00           0.00
6          90.00    30.00    1.087   -3.927       2129.85           1.62
    ↪ 0.00           0.00
7           0.00     0.00    1.100    0.535       2059.57          -0.04
    ↪ 0.00           0.00
8         100.00    35.00    1.089   -1.720       2079.34           2.99
    ↪ 0.00           0.00
9           0.00     0.00    1.100   -0.135       2065.43          -0.09
    ↪ 0.00           0.00


-----------------------------------------------------------------------------
    ↪
From        To         Status       Sft       Stf       Slim     mult_Sf  mult_St
-----------------------------------------------------------------------------
    ↪
1            4          1          73.18     72.98     380.00     -0.00     -0.00
2            7          1         114.18    114.68     250.00     -0.00     -0.00
3            9          1          83.57     84.60     300.00     -0.00     -0.00
4            5          1          29.68     40.50     250.00     -0.00     -0.00
4            6          1          44.86     46.03     250.00     -0.00     -0.00
5            7          1          51.29     49.04     250.00     -0.00     -0.00
6            9          1          48.94     51.43     150.00     -0.00     -0.00
7            8          1          66.61     68.11     250.00     -0.00     -0.00
8            9          1          38.86     34.15     150.00     -0.00     -0.00


-----------------------------------------------------------------------------
    ↪
Gen       Status       Fuel       Pg        Qg        Pmin      Pmax      Qmin
    ↪ Qmax
-----------------------------------------------------------------------------
    ↪
1           1      UNDEFINED     72.86      6.79      10.00    350.00   -300.00
    ↪ 300.00
2           1      UNDEFINED    114.07     -5.13      10.00    300.00   -300.00
    ↪ 300.00
3           1      UNDEFINED     80.21    -23.47      10.00    270.00   -300.00
    ↪ 300.00
[ExaGO] Finalizing scopflow application.
```

Chapter 6

Stochastic optimal power flow (SOPFLOW)

SOPFLOW solves a stochastic security-constrained multi-period optimal power flow problem. The problem is set up as a two-stage optimization problem where the first-stage (base-case) represents the normal operation of the grid (or the most likely forecast) and the second-stage comprises N_s scenarios of forecast deviation. Each scenario can have multiple contingencies and each contingency can be multi-period.

6.1 Formulation

An illustration of SOPFLOW is shown in Fig. **??** for a case with two scenarios s_0 and s_1 with three contingencies each, and each scenario/contingency with two time-periods. We assume that any contingency is incident at the first time-step, i.e., at t_0.

The full formulation for the stochastic security-constrained multi-period optimal power flow is given in (**??**) – (**??**). In this formulation, the objective is to reduce the expected cost, where $f(x_{s,c,t})$ is the cost for scenario s with contingency c at time t. π_s is the probability of scenario s.

$$\min \ \sum_{s=0}^{N_s-1} \pi_s \sum_{c=0}^{N_c-1} \sum_{t=0}^{N_t-1} f(x_{s,c,t}) \tag{6.1}$$

$$\text{s.t.}$$

$$g(x_{s,c,t}) = 0, \qquad s \in [1, N_s - 1], c \in [0, N_c - 1], t \in [0, N_t - 1] \tag{6.2}$$

$$h(x_{s,c,t}) \leq 0, \qquad s \in [1, N_s - 1], c \in [0, N_c - 1], t \in [0, N_t - 1] \tag{6.3}$$

$$x^- \leq x_{s,c,t} \leq x^+, \qquad s \in [1, N_s - 1], c \in [0, N_c - 1], t \in [0, N_t - 1] \tag{6.4}$$

$$-\Delta x_t \leq x_{s,c,t} - x_{s,c,t-\Delta t} \leq \Delta x_t, \qquad s \in [1, N_s - 1], c \in [0, N_c - 1], t \in [1, N_t - 1] \tag{6.5}$$

$$-\Delta x_c \leq x_{s,c,0} - x_{s,0,0} \leq \Delta x_c, \qquad s \in [1, N_s - 1], c \in [1, N_c - 1] \tag{6.6}$$

$$-\Delta x_s \leq x_{s,0,0} - x_{0,0,0} \leq \Delta x_s, \qquad s \in [1, N_s - 1] \tag{6.7}$$

The modeling details used for an optimial power flow problem are also used for a SOPFLOW problem, i.e., each of the circles shown in Fig. **??** has the modeling details of an optimal power flow problem (OPFLOW). Incorporating the probabilities π_s for each scenario is not implemented yet which leads to each scenario having an equal probability.
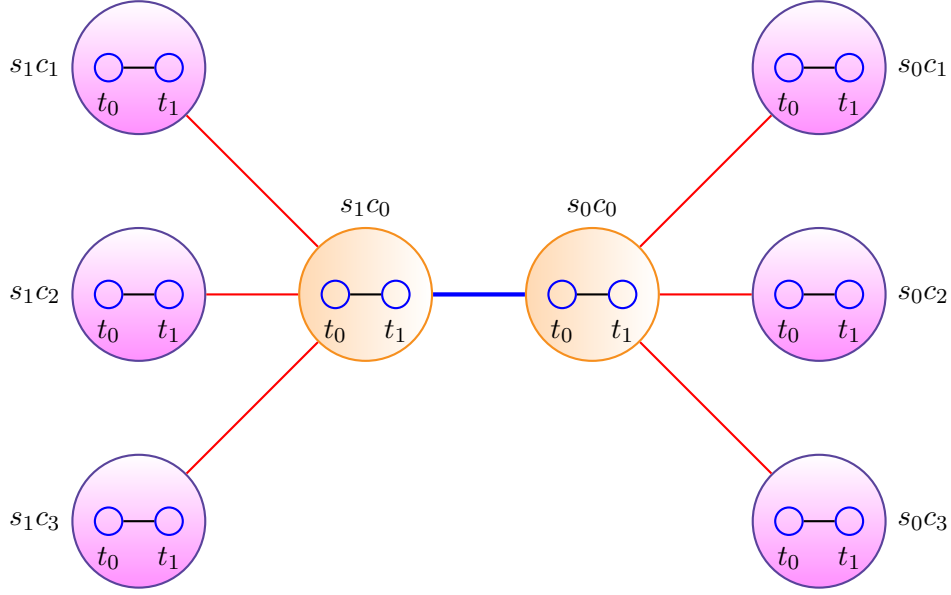
Figure 6.1: Stochastic multi-period contingency constrained structure with two scenarios s_0 and s_1. Each scenario has three contingencies c_1, c_2, and c_3. s_0c_0 and s_1c_0 denote the base-cases for the two scenarios. Each scenario and contingency has two time-periods t_0, and t_2, t_2. The red line denotes the coupling between the contingencies and their respective base-case scenarios. The blue line denotes the coupling between the scenarios

Depending on the options selected, SOPFLOW can be used to solve

- Single-period stochastic optimal power flow : No contingencies or time-periods

- Single-period contingency-constrained stochastic optimal power flow : No time-periods

- Multi-period security-constrained stochastic optimal power flow : Full formulation

Currently, SOPFLOW uses wind power generation as the stochastic variables and each scenario is a realization of the power output from wind generators. A zero fuel cost is used for wind power generation to ensure wind generation would be the dispatched to the given target level (upper limit).

For the contingecy-constrained stochastic optimal power flow, SOPFLOW flattens out the contingencies and scenarios to a two-level formulation. In this formulation, all the scenarios (and their contingencies) are coupled to a base scenario problem.

For contingencies, SOPFLOW supports generation and/or transmission outages. A contingency can have multiple outages, but, it should not cause any islanding. The coupling between the no-contingency and the contingency case for each scenario is also the difference in real power output ($p_{jsct}^{\mathrm{g}} - p_{js0t}^{\mathrm{g}}$, $j \in J^{\mathrm{gen}}$) that must be within the 30 minute generator ramp rate. Refer to ?? for details on the contingency modeling.

For multi time-period, we use ramping constraints on the generator real power output between successive time steps.

SOPFLOW can be run in two modes: preventive and corrective. In the preventive mode, generator real power output is fixed to the base-case values for generators at PV bus(es). In this mode, the generators at the reference bus provide/absorb any deficit/surplus power. The corrective mode allows deviation of the PV and PQ generator real power from the base-case dispatch constrained by its 30-min.

ramp rate capability. Note that the preventive/corrective mode is only applied at the first step t_0. In the successive time-steps, the generator dispatch is dictated by the previous step dispatch and the ramp limits.

6.2 Solvers

SOPFLOW supports solving the optimization problem via Ipopt [?], HiOp [?, ?], or *EMPAR*. Ipopt [?]can solve SOPFLOW on single rank only. HiOp [?, ?]supports solving the problem in parallel using a primal-decomposition algorithm. With HIOP, one can solve the subproblem either on the CPU or GPU by selecting the appropriate subproblem model and solver (see options table below). The *EMPAR* solver does not solve the stochastic ACOPF problem, but it only solves the base-case and the stochastic scenarios independently with OPFLOW. It distributes the scenarios and contingencies to different processes when executed in parallel. Table **??** lists the compatibility of the different solvers for different variations of SOPFLOW .

Table 6.1: SOPFLOW solver compatibility

Solver	Stochastic scenarios	Include contingencies	Include multi-period
IPOPT	Y	Y	Y
HIOP	Y	Y	N
EMPAR	Y	Y	Y

6.3 Input and Output

The following files are needed for executing SOPFLOW.

- **Network file:** The network file describing the network details. Only MATPOWER format files are currently supported.

- **Scenario file:** SOPFLOW only supports reading wind generation scenarios in a CSV format. An example of this format for the 9-bus case is here.

- **Contingency file:** Contingencies can be specified via PTI format file as described in chapter **??**. The option `-sopflow_enable_multicontingency` should be set for multi-contingency problems.

- **Load data:** One file for load real power and one for reactive power. The files need to be in CSV format. An example of the format for the 9-bus case is here.

The SOPFLOW output is saved to a directory named `sopflowout`. This directory contains N_s subdirectories to save the solution for each scenario. Each of these subdirectories contain N_c subdirectories, one for each contingency. Each contingency subdirectory has N_t MATPOWER format files to store the output for each time-period for the given contingency and scenario. The subdirectories have the directory name format `scen_x` where x is the scenario number, `cont_y` where y is the contingency number, and the output files have the file name format `t_z` where z is the time-step number.

6.4 Usage

```
    ./sopflow -netfile <netfilename>  -scenfile <scenfilename> \
    [-sopflow_enable_multicontingency 1] <sopflowoptions>
```

6.5 Options

Table 6.2: SOPFLOW options

Option	Meaning	Values (Default value)	Compatibility
-netfile	Network file name	string 4096 characters (case9mod_gen3_wind.m)	
-scenfile	Scenario file name	string 4096 characters (case9/10scenarios_9bus.csv)	
-ctgcfile	Contingency file name	string (case9.cont)	
-sopflow_mode	Operation mode: Preventive or corrective	0 or 1 (0)	
-sopflow_solver	Set solver for sopflow	IPOPT, HIOP, or EMPAR	
-sopflow_subproblem_solver	Set solver for subproblem	IPOPT or HIOP (IPOPT)	Only when using HIOP solver for SOPFLOW
-sopflow_subproblem_model	Set model for subproblem	See OPFLOW chapter	Only when using HIOP solver for SOPFLOW
-sopflow_enable_multicontingency	Multi-contingency SOPFLOW	0 or 1 (0)	
-sopflow_Ns	Number of scenarios	int (Default 0. Use -1 to select all scenarios from the scenario file)	

6.6 Examples

Some SOPFLOW example runs are provided with some sample output. Options are the default options given in Tables **??**, **??**, **??** and **??** unless otherwise specified. Sample output is generated by running examples in the installation directory.

Example using the Ipopt [**?**]solver:

```
bin/sopflow -netfile $EXAGO_DIR/datafiles/case9/case9mod.m -scenfile
    ↪ $EXAGO_DIR/datafiles/case9/10_scenarios_9bus.csv -sopflow_Ns 4 -
    ↪ sopflow_enable_multicontingency 0 -sopflow_solver IPOPT -
    ↪ print_output
[ExaGO] SOPFLOW: Application created
Rank[0],s = 0, scen_num = 0
Rank[0],s = 1, scen_num = 1
Rank[0],s = 2, scen_num = 2
Rank[0],s = 3, scen_num = 3
[ExaGO] SOPFLOW: Using IPOPT solver
[ExaGO] SOPFLOW: Setup completed
```

```
    ****************************************************************************
        ↪
    This program contains Ipopt, a library for large-scale nonlinear
        ↪ optimization.
     Ipopt is released as open source code under the Eclipse Public License (
        ↪ EPL).
             For more information visit http://projects.coin-or.org/Ipopt
    ****************************************************************************
        ↪

    This is Ipopt version 3.12.10, running with linear solver ma27.

    Number of nonzeros in equality constraint Jacobian...:      513
    Number of nonzeros in inequality constraint Jacobian.:      288
    Number of nonzeros in Lagrangian Hessian.............:      564

    Total number of variables............................:      120
                         variables with only lower bounds:        0
                    variables with lower and upper bounds:       88
                         variables with only upper bounds:        0
    Total number of equality constraints.................:       96
    Total number of inequality constraints...............:       72
            inequality constraints with only lower bounds:        0
       inequality constraints with lower and upper bounds:       72
            inequality constraints with only upper bounds:        0

    iter    objective      inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
        ↪ alpha_pr  ls
      0  3.2548812e+04 1.80e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e
        ↪ +00    0
      1  2.6023632e+04 1.18e+00 1.21e+02  -1.0 1.07e+00    -  5.64e-02 3.40e
        ↪ -01f  1
      2  2.0106844e+04 4.83e-01 2.57e+03  -1.0 6.99e-01   2.0 1.02e-02 6.07e
        ↪ -01f  1
      3  2.0011119e+04 4.70e-01 2.50e+03  -1.0 5.84e-01   2.4 1.32e-01 2.67e
        ↪ -02f  1
      4  1.9372689e+04 3.88e-01 1.99e+03  -1.0 7.94e-01   1.9 2.37e-01 1.97e
        ↪ -01f  1
     ...
     18  1.6587232e+04 4.15e-05 1.21e+03  -5.7 5.76e-03    -  1.00e+00 9.18e
        ↪ -01h  1
     19  1.6587229e+04 8.62e-06 1.55e-05  -5.7 3.00e-03    -  1.00e+00 1.00e
        ↪ +00h  1
    iter    objective      inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
        ↪ alpha_pr  ls
     20  1.6587228e+04 1.50e-06 2.56e-06  -7.0 1.25e-03    -  1.00e+00 1.00e
```

```
    ↪ +00h  1
  21  1.6587228e+04 2.18e-07 4.53e-07  -7.0 4.77e-04    -  1.00e+00 1.00e
    ↪ +00h  1


Number of Iterations....: 21

                                (scaled)                  (unscaled)
Objective..............:   3.7191094895317639e+02    1.6587228323311669e
    ↪ +04
Dual infeasibility......:   4.5286779256921672e-07    2.0197903548587065e
    ↪ -05
Constraint violation....:   3.6703967934426096e-08    3.6703967934426096e
    ↪ -08
Complementarity.........:   6.2406713469536533e-07    2.7833394207413294e
    ↪ -05
Overall NLP error.......:   6.2406713469536533e-07    2.7833394207413294e
    ↪ -05


Number of objective function evaluations             = 22
Number of objective gradient evaluations             = 22
Number of equality constraint evaluations            = 22
Number of inequality constraint evaluations          = 22
Number of equality constraint Jacobian evaluations   = 22
Number of inequality constraint Jacobian evaluations = 22
Number of Lagrangian Hessian evaluations             = 21
Total CPU secs in IPOPT (w/o function evaluations)   =      0.043
Total CPU secs in NLP function evaluations           =      0.013


EXIT: Optimal Solution Found.
==============================================================
Stochastic Optimal Power Flow
==============================================================
Number of scenarios              4
Multi-contingency scenarios?     NO
Solver                           IPOPT
Initialization                   MIDPOINT
Load loss allowed                NO
Power imbalance allowed          NO
Ignore line flow constraints     NO

Convergence status              CONVERGED
Objective value (base)          4149.15


--------------------------------------------------------------
Bus       Pd       Qd       Vm       Va       mult_Pmis      mult_Qmis
   ↪ Pslack         Qslack
```

```
-----------------------------------------------------------------------------
1          0.00    0.00   1.100    0.000       2152.67          0.00
    ↪ 0.00           0.00
2          0.00    0.00   1.095    3.935       2108.81         -0.00
    ↪ 0.00           0.00
3          0.00    0.00   1.087    1.637       2117.68         -0.00
    ↪ 0.00           0.00
4          0.00    0.00   1.097   -2.054       2152.95          0.18
    ↪ 0.00           0.00
5         75.00   50.00   1.079   -3.127       2163.77          7.53
    ↪ 0.00           0.00
6         90.00   30.00   1.087   -4.084       2181.34          1.69
    ↪ 0.00           0.00
7          0.00    0.00   1.100    0.455       2109.21         -0.04
    ↪ 0.00           0.00
8        100.00   35.00   1.089   -1.907       2130.61          3.09
    ↪ 0.00           0.00
9          0.00    0.00   1.100   -0.471       2117.94         -0.09
    ↪ 0.00           0.00
```

From	To	Status	Sft	Stf	Slim	mult_Sf	mult_St
1	4	1	75.41	75.22	380.00	-0.00	-0.00
2	7	1	117.10	117.61	250.00	-0.00	-0.00
3	9	1	78.55	79.51	300.00	-0.00	-0.00
4	5	1	29.85	40.63	250.00	-0.00	-0.00
4	6	1	46.99	47.91	250.00	-0.00	-0.00
5	7	1	51.14	48.90	250.00	-0.00	-0.00
6	9	1	47.20	49.41	150.00	-0.00	-0.00
7	8	1	69.69	70.99	250.00	-0.00	-0.00
8	9	1	36.50	31.07	150.00	-0.00	-0.00

Gen	Status	Fuel	Pg	Qg	Pmin	Pmax	Qmin	Qmax
1	1	UNDEFINED	75.12	6.61	10.00	350.00	-300.00	300.00
2	1	UNDEFINED	116.99	-5.04	10.00	300.00	-300.00	300.00
3	1	UNDEFINED	75.00	-23.36	10.00	85.00	-300.00	300.00

```
[ExaGO] Finalizing sopflow application.
```

Example using the *IPOPT* solver with multicontingency enabled:

```
bin/sopflow -netfile $EXAGO_DIR/datafiles/case9/case9mod.m -scenfile
    ↪ $EXAGO_DIR/datafiles/case9/10_scenarios_9bus.csv -sopflow_Ns 4 -
    ↪ sopflow_enable_multicontingency 1 -sopflow_solver IPOPT -
    ↪ print_output -sopflow_Nc 4 -ctgcfile $EXAGO_DIR/datafiles/case9/
    ↪ case9.cont -sopflow_Nc 4
[ExaGO] SOPFLOW: Application created
Rank[0],s = 0, scen_num = 0, cont_num = 0
Rank[0],s = 1, scen_num = 0, cont_num = 1
Rank[0],s = 2, scen_num = 0, cont_num = 2
Rank[0],s = 3, scen_num = 0, cont_num = 3
Rank[0],s = 4, scen_num = 0, cont_num = 4
Rank[0],s = 5, scen_num = 1, cont_num = 0
Rank[0],s = 6, scen_num = 1, cont_num = 1
Rank[0],s = 7, scen_num = 1, cont_num = 2
Rank[0],s = 8, scen_num = 1, cont_num = 3
Rank[0],s = 9, scen_num = 1, cont_num = 4
Rank[0],s = 10, scen_num = 2, cont_num = 0
Rank[0],s = 11, scen_num = 2, cont_num = 1
Rank[0],s = 12, scen_num = 2, cont_num = 2
Rank[0],s = 13, scen_num = 2, cont_num = 3
Rank[0],s = 14, scen_num = 2, cont_num = 4
Rank[0],s = 15, scen_num = 3, cont_num = 0
Rank[0],s = 16, scen_num = 3, cont_num = 1
Rank[0],s = 17, scen_num = 3, cont_num = 2
Rank[0],s = 18, scen_num = 3, cont_num = 3
Rank[0],s = 19, scen_num = 3, cont_num = 4
[ExaGO] SOPFLOW: Using IPOPT solver
[ExaGO] SOPFLOW: Setup completed


****************************************************************************
    ↪
This program contains Ipopt, a library for large-scale nonlinear
    ↪ optimization.
 Ipopt is released as open source code under the Eclipse Public License (
    ↪ EPL).
         For more information visit http://projects.coin-or.org/Ipopt
****************************************************************************
    ↪

This is Ipopt version 3.12.10, running with linear solver ma27.

Number of nonzeros in equality constraint Jacobian...:     2449
Number of nonzeros in inequality constraint Jacobian.:     1312
Number of nonzeros in Lagrangian Hessian.............:     2820
```

```
Total number of variables.............................:       600
                   variables with only lower bounds:         0
              variables with lower and upper bounds:       440
                   variables with only upper bounds:         0
Total number of equality constraints..................:       480
Total number of inequality constraints................:       328
        inequality constraints with only lower bounds:         0
   inequality constraints with lower and upper bounds:       328
        inequality constraints with only upper bounds:         0

iter    objective      inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
   ↪ alpha_pr  ls
  0  1.6274406e+05 1.80e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e
   ↪ +00    0
  1  1.4901965e+05 1.56e+00 8.69e+01  -1.0 1.07e+00    -  1.73e-01 1.31e
   ↪ -01f  1
  2  1.4480394e+05 1.49e+00 1.32e+02  -1.0 1.80e+00    -  1.34e-03 4.72e
   ↪ -02f  1
  3  1.3529809e+05 1.31e+00 1.15e+02  -1.0 1.22e+00    -  4.90e-02 1.18e
   ↪ -01f  1
  4  1.1656024e+05 9.25e-01 9.13e+01  -1.0 8.50e-01    -  9.55e-02 2.99e
   ↪ -01f  1
  5  1.0783333e+05 7.18e-01 9.48e+01  -1.7 7.97e-01    -  2.05e-01 2.24e
   ↪ -01f  1
  6  9.9954510e+04 5.10e-01 9.64e+01  -1.7 7.94e-01    -  3.55e-01 2.89e
   ↪ -01f  1
  7  9.3292251e+04 3.16e-01 9.31e+01  -1.7 7.35e-01    -  3.05e-01 3.78e
   ↪ -01f  1
  8  9.2973069e+04 3.05e-01 9.00e+01  -1.7 4.70e-01  2.0 3.84e-02 3.33e
   ↪ -02f  1
 ...
 30  8.3618318e+04 1.07e-05 1.74e+03  -5.7 3.88e-03    -  1.00e+00 8.83e
   ↪ -01f  1
 31  8.3618280e+04 5.27e-06 2.36e-04  -5.7 2.35e-03    -  1.00e+00 1.00e
   ↪ +00h  1
 32  8.3618280e+04 6.40e-07 1.28e-06  -5.7 8.17e-04    -  1.00e+00 1.00e
   ↪ +00h  1
 33  8.3618274e+04 1.95e-07 1.70e-06  -7.0 4.52e-04    -  1.00e+00 1.00e
   ↪ +00h  1
 34  8.3618274e+04 8.21e-09 1.71e-08  -7.0 9.25e-05    -  1.00e+00 1.00e
   ↪ +00h  1

Number of Iterations....: 34

                              (scaled)                 (unscaled)
Objective...............:    1.8748492036555979e+03    8.3618274483039670e
```

```
↪ +04
Dual infeasibility......:   1.7101740512915904e-08   7.6273762687604936e
   ↪ -07
Constraint violation....:   1.3811524007811826e-09   1.3811524007811826e
   ↪ -09
Complementarity.........:   1.1101524615042143e-07   4.9512799783087964e
   ↪ -06
Overall NLP error.......:   1.1101524615042143e-07   4.9512799783087964e
   ↪ -06


Number of objective function evaluations           = 35
Number of objective gradient evaluations           = 35
Number of equality constraint evaluations          = 35
Number of inequality constraint evaluations        = 35
Number of equality constraint Jacobian evaluations   = 35
Number of inequality constraint Jacobian evaluations = 35
Number of Lagrangian Hessian evaluations           = 34
Total CPU secs in IPOPT (w/o function evaluations)  =      0.135
Total CPU secs in NLP function evaluations          =      0.094


EXIT: Optimal Solution Found.
===============================================================
Stochastic Optimal Power Flow
===============================================================
Number of scenarios             4
Multi-contingency scenarios?    YES
Contingencies per scenario      4
Solver                          IPOPT
Initialization                  MIDPOINT
Load loss allowed               NO
Power imbalance allowed         NO
Ignore line flow constraints    NO

Convergence status              CONVERGED
Objective value (base)          4149.15


---------------------------------------------------------------------
Bus       Pd       Qd       Vm       Va      mult_Pmis      mult_Qmis
   ↪ Pslack        Qslack
---------------------------------------------------------------------
1        0.00     0.00    1.100    0.000      2152.67           0.00
   ↪ 0.00          0.00
2        0.00     0.00    1.095    3.935      2108.81          -0.00
   ↪ 0.00          0.00
3        0.00     0.00    1.087    1.637      2117.68          -0.00
   ↪ 0.00          0.00
```

```
4        0.00     0.00   1.097  -2.054     2152.95          0.18
  ↪ 0.00          0.00
5       75.00    50.00   1.079  -3.127     2163.77          7.53
  ↪ 0.00          0.00
6       90.00    30.00   1.087  -4.084     2181.34          1.69
  ↪ 0.00          0.00
7        0.00     0.00   1.100   0.455     2109.22         -0.04
  ↪ 0.00          0.00
8      100.00    35.00   1.089  -1.907     2130.61          3.09
  ↪ 0.00          0.00
9        0.00     0.00   1.100  -0.471     2117.94         -0.09
  ↪ 0.00          0.00


--------------------------------------------------------------------------------
  ↪
From      To         Status      Sft      Stf      Slim    mult_Sf  mult_St
--------------------------------------------------------------------------------
  ↪
1         4          1          75.41    75.22    380.00   -0.00    -0.00
2         7          1         117.10   117.61    250.00   -0.00    -0.00
3         9          1          78.55    79.51    300.00   -0.00    -0.00
4         5          1          29.85    40.63    250.00   -0.00    -0.00
4         6          1          46.99    47.91    250.00   -0.00    -0.00
5         7          1          51.14    48.90    250.00   -0.00    -0.00
6         9          1          47.20    49.41    150.00   -0.00    -0.00
7         8          1          69.69    70.99    250.00   -0.00    -0.00
8         9          1          36.50    31.07    150.00   -0.00    -0.00


--------------------------------------------------------------------------------
  ↪
Gen     Status      Fuel      Pg       Qg       Pmin     Pmax      Qmin
  ↪ Qmax
--------------------------------------------------------------------------------
  ↪
1          1     UNDEFINED    75.12     6.61    10.00    350.00   -300.00
  ↪ 300.00
2          1     UNDEFINED   116.99    -5.04    10.00    300.00   -300.00
  ↪ 300.00
3          1     UNDEFINED    75.00   -23.36    10.00     85.00   -300.00
  ↪ 300.00
[ExaGO] Finalizing sopflow application.
```

Appendices

Appendix A

Symbol reference

Units of measurement are given in Table (**??**), indices and index sets in Table (**??**), subsets in Table (**??**), special set elements in Table (**??**), and real-valued parameters in Table (**??**).

Table A.1:   Units of measurement

Symbol	Description
1	dimensionless. Dimensionless real number quantities are indicated by a unit of 1.
USD	US dollar. Cost, penalty, and objective values are expressed in USD.
h	hour. Time is expressed in h.
pu	per unit. Voltage magnitude is expressed in a per unit system under given base values, and the unit is denoted by pu
rad	radian. Voltage angles are expressed in rad.
MW	megawatt. Real power is expressed in MW.
MVar	megavolt-ampere-reactive. Reactive power is expressed in MVar.
MVA	megavolt-ampere. Apparent power is expressed in MVA.
MW at 1 pu	megawatt at unit voltage. Conductance is expressed in MW at 1 pu, meaning the conductance is such as to yield a real power flow equal to the indicated amount when the voltage is equal to 1 pu
MVar at 1 pu	megavolt-ampere-reactive at unit voltage. Susceptance is expressed in MVar at 1 pu, meaning the susceptance is such as to yield a reactive power flow equal to the indicated amount when the voltage is equal to 1 pu

Table A.2:   Index sets

Symbol	Description
$a \in A$	areas
$i \in I$	buses

Symbol	Description
$j \in J$	bus-connected grid components, i.e. loads, shunts, generators, stochastic resources, branches
$k \in K$	security contingencies, i.e. NERC $(n-1)$- or $(n-k)$-style contingencies, different from the severe event we are modeling
$s \in S$	stochastic scenarios
$t \in T$	time periods

Table A.3:   Subsets

Symbol	Description
$J^{\mathrm{gen}} \subset J$	generators
$J^{\mathrm{ld}} \subset J$	loads
$J^{\mathrm{br}} \subset J$	branches, i.e. lines, transformers
$J^{\mathrm{sh}} \subset J$	shunts
$J_i^{\mathrm{o}} \subset J$	branches with origin bus at bus i
$J_i^{\mathrm{d}} \subset J$	branches with destination bus at bus i

Table A.4:   Special set elements

Symbol	Description
$a_i \in A$	area of bus i
$i_j^{\mathrm{d}} \in I$	destination bus of branch $j \in J$
$i_j^{\mathrm{o}} \in I$	origin bus of branch $j \in J^{\mathrm{br}}$

Table A.5:   Real-valued parameters

Symbol	Description
b_j^{max}	maximum susceptance for shunt $j \in J^{\mathrm{sh}}$ (MVar at 1 pu)

Symbol	Description
b_j^{min}	minimum susceptance for shunt $j \in J^{\text{sh}}$ (MVar at 1 pu)
b_j^{ch}	charging susceptance for branch $j \in J^{\text{br}}$ (MVar at 1 pu)
b_j^{s}	series susceptance for branch $j \in J^{\text{br}}$ (MVar at 1 pu)
p_j^{gset}	real power set point of generator $j \in J^{\text{gen}}$ (MW)
p_j^{gmax}	maximum real power output for generator $j \in J^{\text{gen}}$ (MW)
p_j^{gmin}	minimum real power output for generator $j \in J^{\text{gen}}$ (MW)
p_j^{r}	maximum ramp rate for generator $j \in J^{\text{gen}}$ (MW/h)
q_j^{gmax}	maximum reactive power output for generator $j \in J^{\text{gen}}$ (MVar)
q_j^{gmin}	minimum reactive power output for generator $j \in J^{\text{gen}}$ (MVar)
v_i^{max}	maximum voltage magnitude for bus $i \in I$ (pu)
v_i^{min}	minimum voltage magnitude for bus $i \in I$ (pu)
π_s	probability of scenario s (1)