Finite State Machine Testing – Sample Solutions

Example FSMs

In the exercises, we will use two FSMs. Both have input alphabet $X = \{a, b\}$ and output alphabet $Y = \{0, 1\}$. The first FSM will be called M_1 and is represented by the following directed graph.

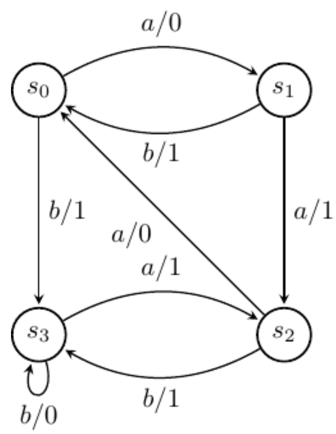

Figure 1: FSM Example 1

The second, called M_2, is represented by the following directed graph.

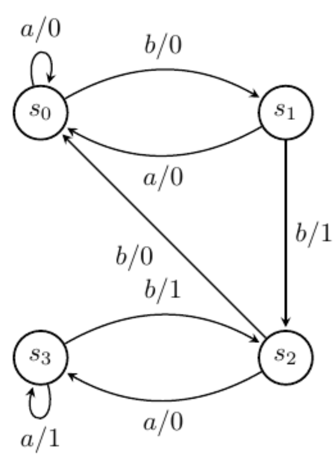

Figure 2: FSM Example 2

In both cases, s_0 is the initial state.

Exercises

For each FSM M_i given above:

1. Determine the resulting state and the output produced after taking *abbab* as input.
2. For every state s, find a set of input sequences V (state cover) that takes the FSM from its initial state s_0 to state s.
3. Derive a transition tour.
4. Determine whether there is a Unique Input Output Sequence (UIO) for each state.
5. Find a characterising set W.
6. Apply the W-method (no extra states).

Sample Solutions

Answers for M_1

1. The resulting state is given by $\delta^*(s_0, abbab)$ and the output is given by $\lambda^*(s_0, abbab)$:

$$\begin{aligned}
\delta^*(s_0, abbab) &= \delta^*(\delta(s_0, a), bbab) \\
&= \delta^*(s_1, bbab) \\
&= \delta^*(\delta(s_1, b), bab) \\
&= \delta^*(s_0, bab) \\
&= \delta^*(\delta(s_0, b), ab) \\
&= \delta^*(s_3, ab) \\
&= \delta^*(\delta(s_3, a), b) \\
&= \delta^*(s_2, b) \\
&= \delta^*(\delta(s_2, b), \epsilon) \\
&= \delta^*(s_3, \epsilon) \\
&= s_3
\end{aligned}$$

$$
\begin{aligned}
\lambda^*(s_0, abbab) &= \lambda(s_0, a)\lambda^*(\delta(s_0, a), bbab) \\
&= 0\lambda^*(s_1, bbab) \\
&= 0\lambda(s_1, b)\lambda^*(\delta(s_1, b), bab) \\
&= 01\lambda^*(s_0, bab) \\
&= 01\lambda(s_0, b)\lambda^*(\delta(s_0, b), ab) \\
&= 011\lambda^*(s_3, ab) \\
&= 011\lambda(s_3, a)\lambda^*(\delta(s_3, a), b) \\
&= 0111\lambda^*(s_2, b) \\
&= 0111\lambda(s_2, b)\lambda^*(\delta(s_2, b), \epsilon) \\
&= 01111\lambda^*(s_3, \epsilon) \\
&= 01111
\end{aligned}
$$

2. There are a number of choices for the state cover, including $V = \{\epsilon, a, aa, b\}$ This is because ϵ reaches s_0, a reaches s_1, b reaches s_3 and aa reaches s_2.

   In this case, the sequences are minimal (they could result from the use of a breadth-first search).

3. Transition tours include: *aaaabbbabaa*. To see that this covers each transition we can simply trace the behaviour through M_1 getting:

$$
s_0 \xrightarrow{a} s_1 \xrightarrow{a} s_2 \xrightarrow{a} s_0 \xrightarrow{a} s_1 \xrightarrow{b} s_0 \xrightarrow{b} s_3 \xrightarrow{b} s_3 \xrightarrow{a} s_2 \xrightarrow{b} s_3 \xrightarrow{a} s_2 \xrightarrow{a} s_0
$$

   To see that this includes each transition we simply need to check that for each state s_i, each input has been received when M_1 is in state s_i.

4. There are UIOs for each state, including the following.

State	UIO
s_0	$a/0\ a/1$
s_1	$b/1\ a/0$
s_2	$a/0\ a/0$
s_3	$b/0$

   To see that this is the case, for each UIO we can consider the output produced, in response to the input sequence from the UIO, for every other state. Consider, for example, the UIO for state s_1. We get the following:

State	Response to *ba*
s_0	11

State	Response to ba
s_1	10
s_2	11
s_3	01

From this, we can see that the output produced from s_1 is different from that produced from any other state. Thus, to check that the FSM is in s_1 it is sufficient to input ba: if we get output 10 then the FSM (before we applied ba) was in state s_1 and otherwise it was in some other state.

Note: from the above table we can see that ba also defines a UIO for state s_3, however the shorter sequence b also gives a UIO for s_3.

5. There are a number of possible characterising sets, including one defined by the UIOs. A smaller example is $W = \{aa, b\}$.

6. Applying the W-Method, we use: $V = \{\epsilon, a, aa, b\}$, $W = \{aa, b\}$. We therefore obtain the following:

$$\{\epsilon, a, aa, b\}\{a, b\}\{aa, b\}$$

We can expand this out to get:

$$\{aaa, ab, baa, bb, aaaa, aab, abaa, abb, aaaaa, aaab, aabaa, aabb, baaa, bab, bbaa, bbb\}$$

Answers for M_2

1. The resulting state is given by $\delta^*(s_0, abbab)$ and the output is given by $\lambda^*(s_0, abbab)$:

$$\begin{aligned}
\delta^*(s_0, abbab) &= \delta^*(\delta(s_0, a), bbab) \\
&= \delta^*(s_0, bbab) \\
&= \delta^*(\delta(s_0, b), bab) \\
&= \delta^*(s_1, bab) \\
&= \delta^*(\delta(s_1, b), ab) \\
&= \delta^*(s_2, ab) \\
&= \delta^*(\delta(s_2, a), b) \\
&= \delta^*(s_3, b) \\
&= \delta^*(\delta(s_3, b), \epsilon) \\
&= \delta^*(s_2, \epsilon) \\
&= s_2
\end{aligned}$$

$$
\begin{aligned}
\lambda^*(s_0, abbab) &= \lambda(s_0, a)\lambda^*(\delta(s_0, a), bbab) \\
&= 0\lambda^*(s_0, bbab) \\
&= 0\lambda(s_0, b)\lambda^*(\delta(s_0, b), bab) \\
&= 00\lambda^*(s_1, bab) \\
&= 00\lambda(s_1, b)\lambda^*(\delta(s_1, b), ab) \\
&= 001\lambda^*(s_2, ab) \\
&= 001\lambda(s_2, a)\lambda^*(\delta(s_2, a), b) \\
&= 0010\lambda^*(s_3, b) \\
&= 0010\lambda(s_3, b)\lambda^*(\delta(s_3, b), \epsilon) \\
&= 00101\lambda^*(s_3, \epsilon) \\
&= 00101
\end{aligned}
$$

2. State cover: $V = \{\epsilon, b, bb, bba\}$. Again, this set is minimal (same reason).

3. Transition tours include: *ababbaabb*

4. There is no UIO for state s_1. To see this, let us suppose that there was a UIO for s_1. We find the following.

   - This UIO cannot start with input a since transitions with $a/0$ leave s_0 and s_1 and end at the same state (i.e. such a UIO could not distinguish s_0 from s_1 and thus could not be a UIO for s_1).

   - This UIO cannot start with input b since transitions with $b/1$ leave s_3 and s_1 and end at the same state (i.e. such a UIO could not distinguish s_3 from s_1 and thus could not be a UIO for s_1).

   Both cases lead to a contradiction and so the assumption, that there is a UIO for s_1, must be false.

5. Characterising set: $W = \{b, aa\}$

   To see that this is a characterising set, it is sufficient to consider the output produced, from each state, in response to both b and aa and see that this pair of values identifies each state.

State	Response to b	Response to aa
s_0	0	00
s_1	1	00
s_2	0	01
s_3	1	11

6. Applying the W-Method, we use $\{\epsilon, b, bb, bba\}$ and $W = \{b, aa\}$.

   We therefore obtain: $\{\epsilon, b, bb, bba\}\{a, b\}\{b, aa\}$

We can expand this out to:

$$\{ab, aaa, bb, baa, bab, baaa, bbb, bbaa, bbab, bbaaa, bbbb, bbbaa, bbaab, bbaaaa, bbabb, bbabaa\}$$