Model Based Testing

Dr Michael Foster
Based on material from Professor Rob Hierons

Outline

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- Formal models of systems (FSMs)

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    - Unique I/O sequences (UIOs)

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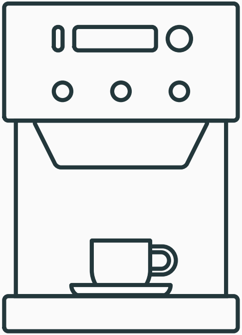
- Identifying states
    - Distinguishing sequences
    - Unique I/O sequences (UIOs)
    - The W method

Motivating Example - Simple Drinks Machine



- Select a drink

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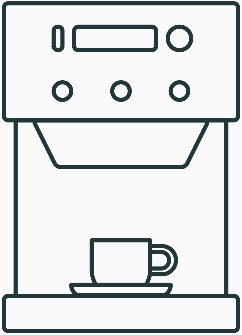

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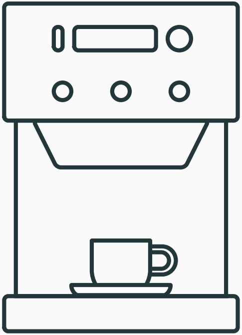

- Select a drink
- Insert coins
- Press "vend" to dispense drink

How do we test this system?

Unit testing

- Generate tests according to code components

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- Model is a specification or description of a property of interest, often an abstraction and relatively understandable

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Model based testing

- Generate tests according to a formal model
- Model is a specification or description of a property of interest, often an abstraction and relatively understandable
- Aim to achieve some level of model coverage
- Usually black-box and complements white-box testing
- Major benefits realised if the model has a formal semantics – potential for automation!

Formal Models of Systems

There are lots of different modelling notations (Z, B, state machines)

We will introduce MBT with state machines

An FSM (Mealy machine) is a sixtuple $(S, s_0, X, Y, \delta, \lambda)$ where

- S is a finite set of states

[1]DFA: Deterministic Finite Automaton

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We can extend δ and λ to take sequences giving δ^* and λ^*

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Formal Models of Systems

What types of faults can we test for with FSMs?

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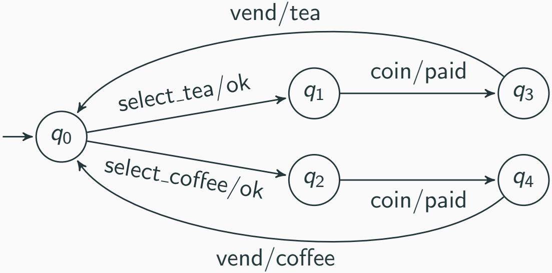
- There are extra states; can only happen if there are also state transfer faults

FSM of the Drinks Machine



- We give the system inputs and it changes state and gives us outputs

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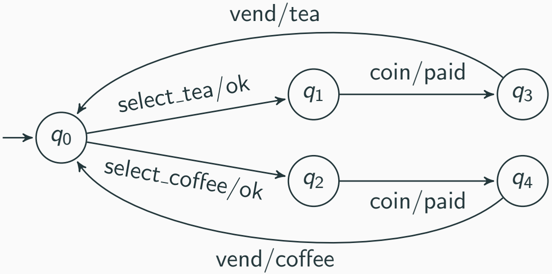

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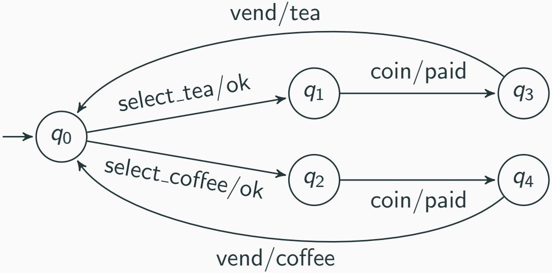

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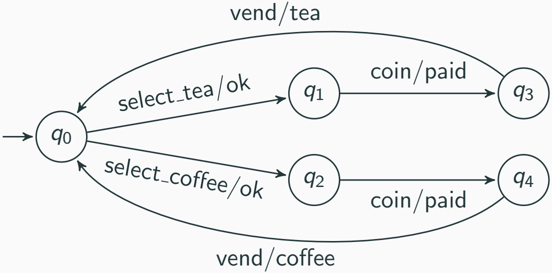

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- Transition x/y represents giving input x and getting output y, e.g.
  - inputs *select_tea, coin, vend*, gives us outputs *ok, paid, tea*
  - Machine follows path $q_0 \to q_1 \to q_3 \to q_0$

Terminology

We will focus on FSM testing with *deterministic*, *minimal*, and *completely specified* models

- An FSM is *deterministic* if there is at **most** one transition from each state for each input (implicit since δ and λ are functions)

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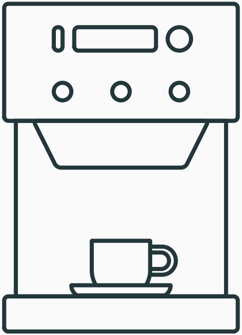
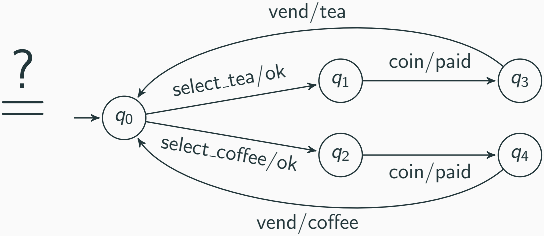
- An FSM is *minimal* if there is no trace-equivalent FSM with fewer states, i.e., if no smaller FSM in terms of number of states defines the same regular language
  - Note: we can convert any FSM into an equivalent minimal FSM

Testing from an FSM





- Assume the software behaves like an FSM model

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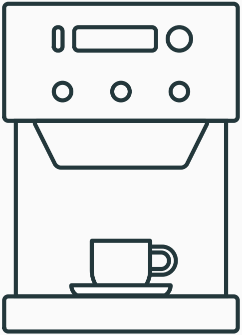

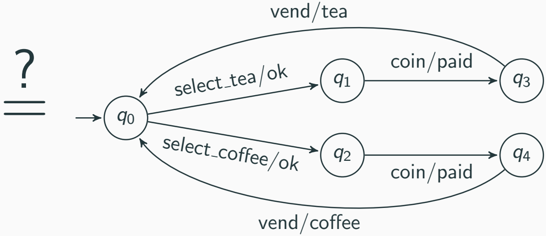

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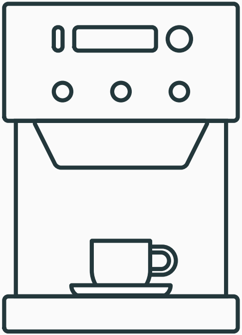

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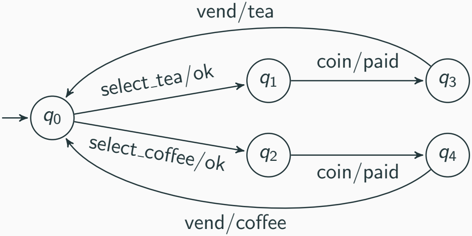
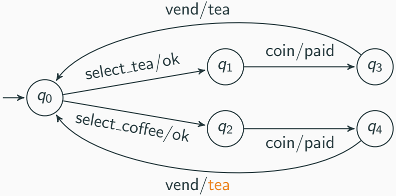


- Assume the software behaves like an FSM model
- Submit inputs to the FSM and software in parallel
- Observe and compare the outputs

Example Faults for Drinks Machine

Output faults: A transition produces the wrong output (tea)



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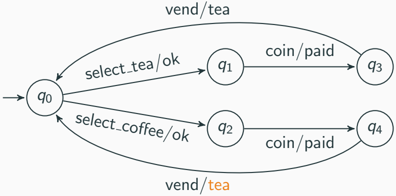
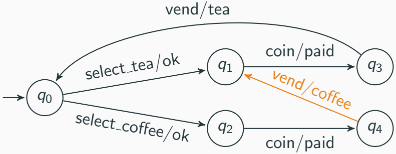


State transfer faults: A transition goes to the wrong state (q_1)



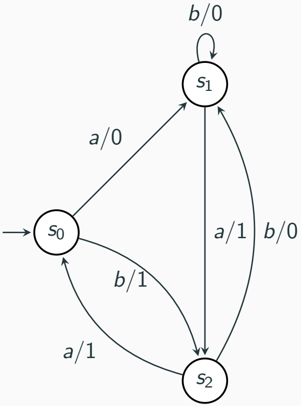
The Transition Tour Method

- Given an FSM M, the transition tour method involves:
  - Finding a path (sequence of transitions) ρ from the initial state of M that includes **all** transitions of M.
  - The test sequence is the label of ρ: the corresponding input/output sequence.

- The transition tour method is guaranteed to find **output faults** as long as there are no state transfer faults.

The Transition Tour Method: Example

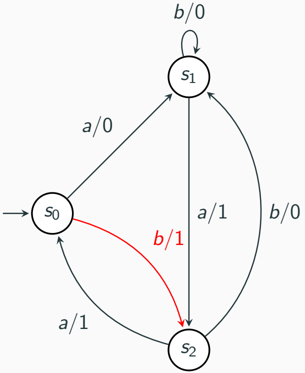
Consider a simple example



There is more than one possible transition tour

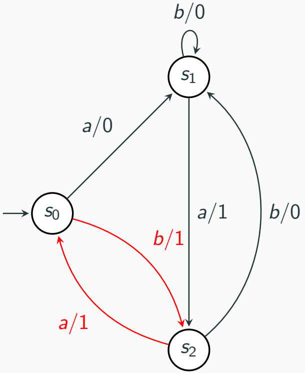
Transition Tour Method: Example

We could start with transition $(s_0, s_2, b/1)$.



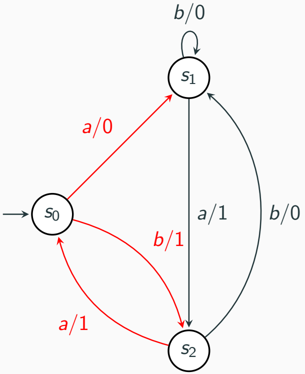
Transition Tour Method: Example

Then follow $(s_0, s_2, b/1)$ by $(s_2, s_0, a/1)$.



Transition Tour Method: Example

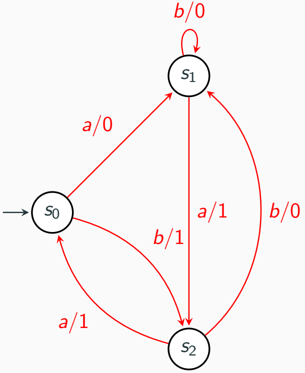
Now maybe follow $(s_0, s_1, a/0)$, giving path $(s_0, s_2, b/1)(s_2, s_0, a/1)(s_0, s_1, a/0)$.



Transition Tour Method: Example

Completing this, we could choose:

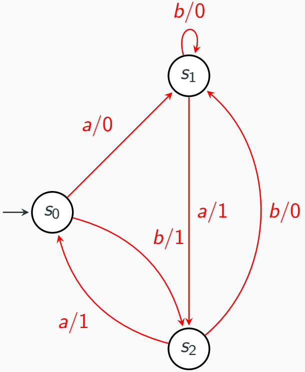
- $(s_0, s_2, b/1)(s_2, s_0, a/1)(s_0, s_1, a/0)$ followed by
- $(s_1, s_1, b/0)(s_1, s_2, a/1)(s_2, s_1, b/0)$



Transition Tour Method: Example

This gives us the following test sequence: $b/1, a/1, a/0, b/0, a/1, b/0$.

- In testing we apply input sequence *baabab* and expect to observe output sequence 110010
- The test sequence can also be represented by *baabab*/110010.



Transition Tours: Generating a Transition Tour

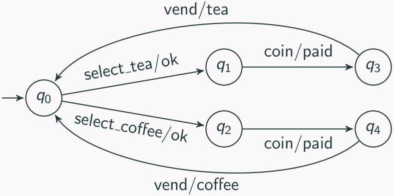
- One could apply a simple heuristic such as:
  - In the current state s, if there is a transition t from s that has not yet been included then add t the the current path and update the current state.
  - Otherwise, follow the shortest path from s that moves M to a state with at least one uncovered transition.
- Alternatively, we can use graph algorithms (we are looking for a path that includes all edges).
  - These can return an optimal (shortest) transition tour in polynomial time.

Transition Tours: Observation

- A transition tour may not find state transfer faults.
- To find state transfer faults we need to be able to *check states*.
- In black-box testing, this requires finding appropriate input sequences.
- We might follow each transition by sequences that distinguish between possible states.

Transition Tour for Drinks Machine

Assume no state transfer faults, then we can test by just executing every transition



The input sequence *select_tea*, *coin*, *vend*, *select_coffee*, *coin*, *vend* will do that for us.

We validate that the output sequence is *ok*, *paid*, *tea*, *ok*, *paid*, *coffee*.

That's Not Good Enough!

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- If there are state transfer faults, a transition tour may not find them
- Output faults may also be masked
- We want to explicitly check for state transfer faults

Identifying States

To test from an FSM, we want to check every transition $(q_i, q_j, x/y)$

- Get the FSM to state q_i

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Controllability: How do we get the FSM to q_i?

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Challenges

Controllability: How do we get the FSM to q_i?

Observability: How do we know the FSM is in q_j?

Solutions

Controllability

Find a sequence that gets the *specification* to the desired state.

Observability

Characterise states in terms of the I/O actions they can perform:

Solutions

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Observability

Characterise states in terms of the I/O actions they can perform:

- Distinguishing sequences
- Unique I/O sequences (UIOs)
- Characterising set

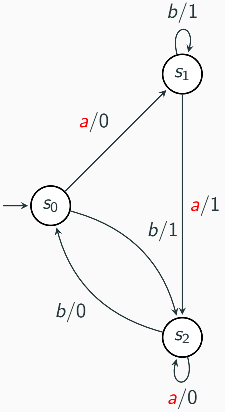
Distinguishing States

Separating two states:

- Two states s and s' of FSM M are *separated* by input sequence x if: the response of M to x is different in states s and s' ($\lambda^*(s,x) \neq \lambda^*(s',x)$)
- If there is such an input sequence x then s and s' are said to be *separable*.
- States s and s' are said to be *equivalent* if they are *not separable*.

Distinguishing States: Example 1

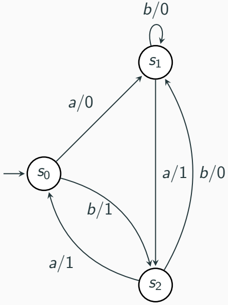
In the following FSM, input a separates states s_0 and s_1, but *does not* separate states s_0 and s_2.



Distinguishing States: Example 2

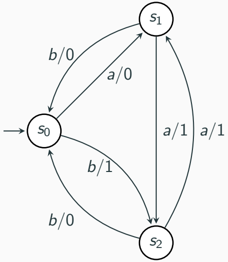
Inputs sequences of length greater than 1 are sometimes needed.

In the following FSM, no input of length 1 can separate states s_1 and s_2, but aa does ($\lambda^*(s_1, aa) = 11, \lambda^*(s_2, aa) = 10$)



Distinguishing States: Example of Equivalence

In the following FSM, no input can separate states s_1 and s_2, therefore they are *equivalent*.



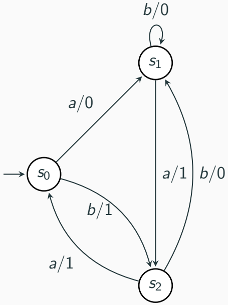
Distinguishing Sequences

A distinguishing sequence D is an input sequence that produces a different output **for each state**.

- For every pair of states s and s' of FSM M such that $s \neq s'$, we have that $\lambda^*(s, D) \neq \lambda^*(s', D))$
- This means that the output produced in response to D **identifies** the state of M.

Distinguishing Sequences: Example

- For the example FSM below, *aa* forms a Distinguishing Sequence since:
  - From state s_0 the output sequence is 01
  - From state s_1 the output sequence is 11
  - From state s_2 the output sequence is 10

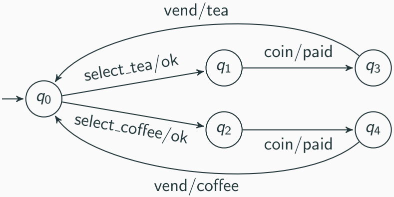


Distinguishing Sequences for Drinks Machine

Not every FSM has a distinguishing sequence! Does the drinks machine have one?

Yes! (assuming we know when an input has been refused)

$$[select\_tea, coin, vend]$$



- From q_0 we get *ok*, *paid*, *tea*
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- From q_2 we get *refuse*, *paid*, *coffee*
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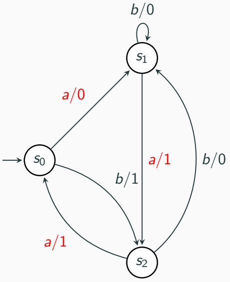
- Note: Distinguishing Sequences identify all of the states but a UIO might only identify *one* state.
- Not all FSMs have these either!

Unique I/O Sequences (UIOs): Example

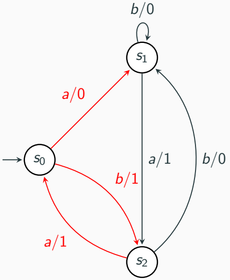
- In the example, $a/0$ forms a UIO for state s_0:
  - From state s_0 the output sequence is 0
  - From state s_1 the output sequence is 1
  - From state s_2 the output sequence is 1
- Note that a is not a Distinguishing Sequence.



Unique I/O Sequences (UIOs): Example

Using UIOs to test transitions...

- In order to test the transition $t = (s_2, s_0, a/1)$ we can build a test sequence as follows:
  - A preamble $b/1$ that reaches the initial state of t (s_2);
  - The input/output pair $a/1$ of the transition t.
  - Finally, the chosen UIO for the **end state** of the transition ($a/0$).
- This gives the test sequence $b/1, a/1, a/0$.



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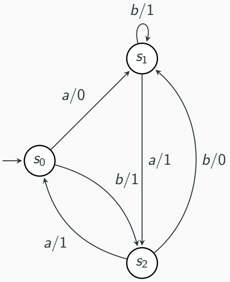
- There are polynomial time algorithms to generate W sets
- We can minimise every FSM

Characterising Sets: Usage

- If we know the output triggered by each input sequence from W, then we can identify the state.
- To check transition $t = (s_i, s_j, x/y)$ we can separately follow t by each element of W.
- Thus, using a characterising set leads to multiple tests for a transition.

Characterising Sets: Example



For the above FSM, $\{a, b\}$ is a characterising set since for every pair of states, there is an element in the set that distinguishes them:

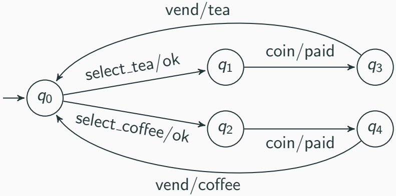
State	Response to a	Response to b
s_0	0	1
s_1	1	1
s_2	1	0

Characterising Sets: Implications

- For this example, if we are checking the final state of a transition t we separately follow it by a and b.
- So, we use two tests for a transition.
- If the response to a after t is 1 and the response to b after t is 0 the transition must have taken the implementation to a state corresponding to s_2.

Characterising Set for Drinks Machine

Again, assuming we know when an input has been refused (e.g., a loop in each state that outputs "refused"): $W = \{[select\_tea, coin, vend]\}$



State	Response to $[select\_tea, coin, vend]$
q_0	ok, paid, tea
q_1	*refused*, paid, tea
q_2	*refused*, paid, coffee
q_3	*refused*, *refused*, tea
q_4	*refused*, *refused*, coffee

The W Method

The W method produces a set of input sequences to test correctness, assuming:

- Implementation behaves like some unknown FSM with no more than n states.

[1] Note: Many real systems have a reset and sometimes this simply means switching the system off and then on again, but it may be a more complex operation.

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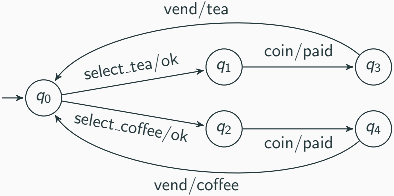

$W = \{[select\_tea,\ coin,\ vend]\}$

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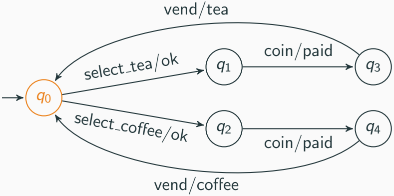

$W = \{[select\_tea,\ coin,\ vend]\}$

$V = \{\epsilon\}$

We start with the empty sequence in V, by definition.

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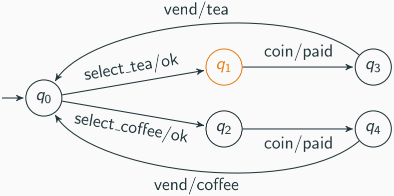

$W = \{[select\_tea,\ coin,\ vend]\}$

$V = \{\epsilon, [select\_tea]\}$

Then visit every state from the initial state; [*select_tea*] gets us to q_1.

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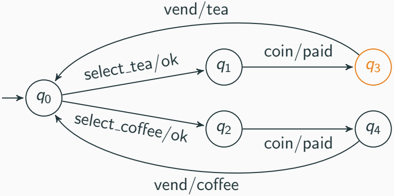

$W = \{[select\_tea,\ coin,\ vend]\}$

$V = \{\epsilon, [select\_tea], [select\_tea,\ coin]\}$

Next, $[select\_tea,\ coin]$ gets us to q_3.

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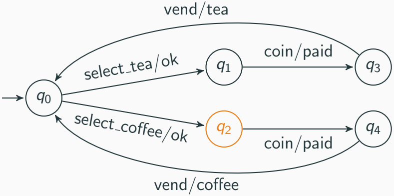
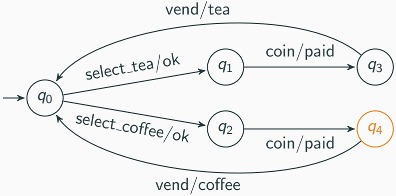


$W = \{[select\_tea,\ coin,\ vend]\}$

$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee]\}$

Next, [*select_coffee*] gets us to q_2.

The W Method for the Drinks Machine



$$W = \{[\mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}]\}$$

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Finally, $[\mathit{select\_coffee},\ \mathit{coin}]$ gets us to q_4.

For FSM with $n+1$ states, the W method produces the test set $VW \cup VXW$ (Remember X is the input alphabet)

- VW checks that each input sequence $\overline{x} \in V$ goes to the right state. The set VW contains each element of the state cover V followed by each element of the characterising set W.

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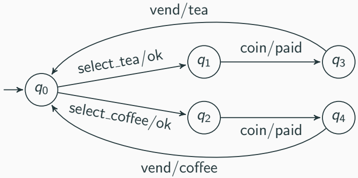
Make sure you reset before each sequence!

Applying the W Method to the Drinks Machine



$$
\begin{aligned}
V &= \{\epsilon, [\mathit{select\_tea}], [\mathit{select\_tea},\ \mathit{coin}], [\mathit{select\_coffee}], [\mathit{select\_coffee},\ \mathit{coin}]\} \\
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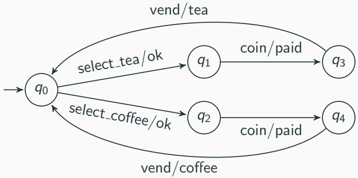

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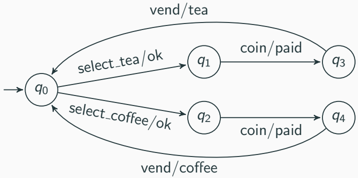

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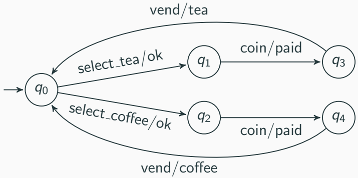
Applying the W Method to the Drinks Machine



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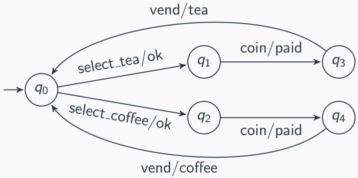
Applying the W Method to the Drinks Machine



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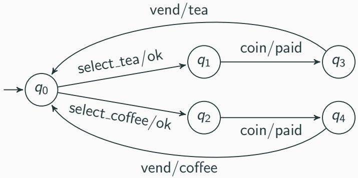
Applying the W Method to the Drinks Machine



$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$

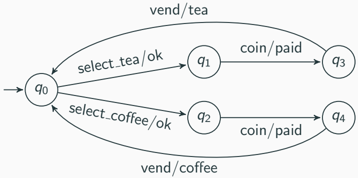
$$X = \{[select\_tea], [select\_coffee], [coin], [vend]\}$$

$$W = \{[select\_tea,\ coin,\ vend]\}$$

$$VW = \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\}$$

$$VXW = \{[select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}$$

Applying the W Method to the Drinks Machine



$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$

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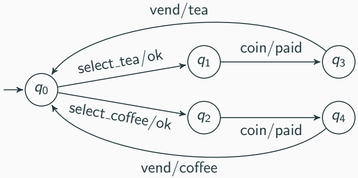
$$VW = \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\}$$

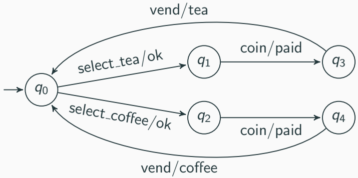
$$VXW = \{[select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}$$

Applying the W Method to the Drinks Machine



$$
\begin{aligned}
V &= \{\epsilon, [\mathit{select\_tea}], [\mathit{select\_tea},\ \mathit{coin}], [\mathit{select\_coffee}], [\mathit{select\_coffee},\ \mathit{coin}]\} \\
X &= \{[\mathit{select\_tea}], [\mathit{select\_coffee}], [\mathit{coin}], [\mathit{vend}]\} \\
W &= \{[\mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}]\} \\
VW &= \{[\mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], [\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\qquad [\mathit{select\_tea},\ \mathit{coin},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\} \\
VXW &= \{[\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\qquad [\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\qquad [\mathit{select\_tea},\ \mathit{coin},\ \mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\}
\end{aligned}
$$

Applying the W Method to the Drinks Machine



$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$

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$$W = \{[select\_tea,\ coin,\ vend]\}$$

$$\begin{aligned} VW = \{&[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], \\ &[select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \end{aligned}$$

$$\begin{aligned} VXW = \{&[select\_tea,\ select\_tea,\ coin,\ vend], \\ &[select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], \\ &[select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\} \end{aligned}$$

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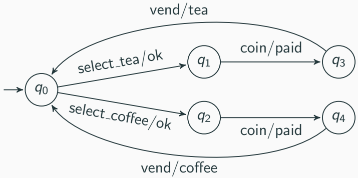

$$
\begin{aligned}
V &= \{\epsilon, [\mathit{select\_tea}], [\mathit{select\_tea},\ \mathit{coin}], [\mathit{select\_coffee}], [\mathit{select\_coffee},\ \mathit{coin}]\} \\
X &= \{[\mathit{select\_tea}], [\mathit{select\_coffee}], [\mathit{coin}], [\mathit{vend}]\} \\
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VW &= \{[\mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], [\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select\_tea},\ \mathit{coin},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\} \\
VXW &= \{[\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select\_tea},\ \mathit{coin},\ \mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\}
\end{aligned}
$$

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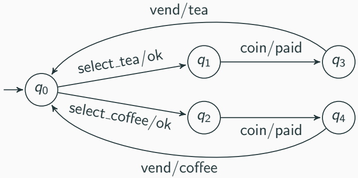

$$
\begin{aligned}
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VXW &= \{[\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select\_tea},\ \mathit{coin},\ \mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\}
\end{aligned}
$$

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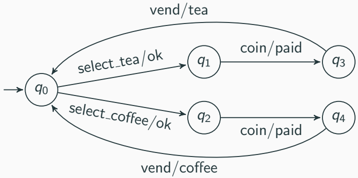
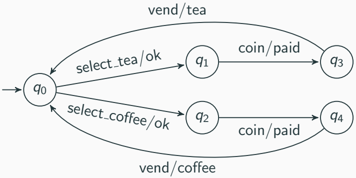


$$\begin{aligned}
V &= \{\epsilon, [\mathit{select_tea}], [\mathit{select_tea},\ \mathit{coin}], [\mathit{select_coffee}], [\mathit{select_coffee},\ \mathit{coin}]\} \\
X &= \{[\mathit{select_tea}], [\mathit{select_coffee}], [\mathit{coin}], [\mathit{vend}]\} \\
W &= \{[\mathit{select_tea},\ \mathit{coin},\ \mathit{vend}]\} \\
VW &= \{[\mathit{select_tea},\ \mathit{coin},\ \mathit{vend}], [\mathit{select_tea},\ \mathit{select_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select_tea},\ \mathit{coin},\ \mathit{select_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\} \\
VXW &= \{[\mathit{select_tea},\ \mathit{select_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select_tea},\ \mathit{select_tea},\ \mathit{select_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select_tea},\ \mathit{coin},\ \mathit{select_tea},\ \mathit{select_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\}
\end{aligned}$$

Applying the W Method to the Drinks Machine



$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$

$$X = \{[select\_tea], [select\_coffee], [coin], [vend]\}$$

$$W = \{[select\_tea,\ coin,\ vend]\}$$

$$\begin{aligned} VW = \{&[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], \\ &[select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \end{aligned}$$

$$\begin{aligned} VXW = \{&[select\_tea,\ select\_tea,\ coin,\ vend], \\ &[select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], \\ &[select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\} \end{aligned}$$

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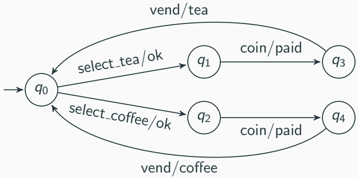

$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$

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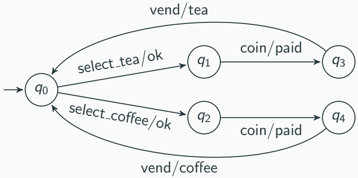

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V &= \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\} \\
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&\quad [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \\
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&\quad [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}
\end{aligned}
$$

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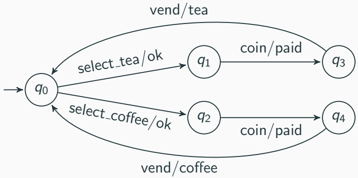

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\begin{aligned}
V &= \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\} \\
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W &= \{[select\_tea,\ coin,\ vend]\} \\
VW &= \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \\
VXW &= \{[select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}
\end{aligned}
$$

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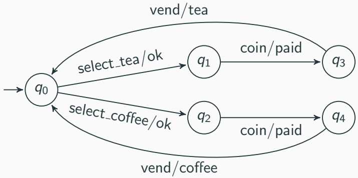

$$
\begin{aligned}
V &= \{\epsilon, [\mathit{select\_tea}], [\mathit{select\_tea},\ \mathit{coin}], [\mathit{select\_coffee}], [\mathit{select\_coffee},\ \mathit{coin}]\} \\
X &= \{[\mathit{select\_tea}], [\mathit{select\_coffee}], [\mathit{coin}], [\mathit{vend}]\} \\
W &= \{[\mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}]\} \\
VW &= \{[\mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], [\mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \\
&\quad [\mathit{select\_tea},\ \mathit{coin},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\} \\
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&\quad [\mathit{select\_tea},\ \mathit{coin},\ \mathit{select\_tea},\ \mathit{select\_tea},\ \mathit{coin},\ \mathit{vend}], \ldots\}
\end{aligned}
$$

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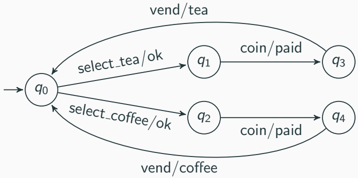

$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$

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$$VW = \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\}$$

$$VXW = \{[select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}$$

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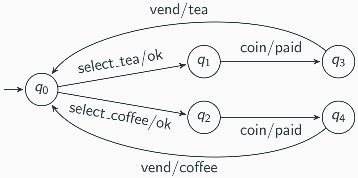

$$
\begin{aligned}
V &= \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\} \\
X &= \{[select\_tea], [select\_coffee], [coin], [vend]\} \\
W &= \{[select\_tea,\ coin,\ vend]\} \\
VW &= \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \\
VXW &= \{[select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}
\end{aligned}
$$

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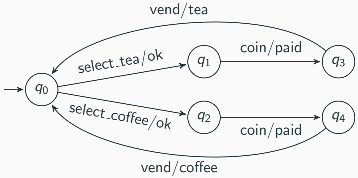

$$
\begin{aligned}
V &= \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\} \\
X &= \{[select\_tea], [select\_coffee], [coin], [vend]\} \\
W &= \{[select\_tea,\ coin,\ vend]\} \\
VW &= \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \\
VXW &= \{[select\_tea,\ select\_tea,\ coin,\ vend], \\
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\end{aligned}
$$

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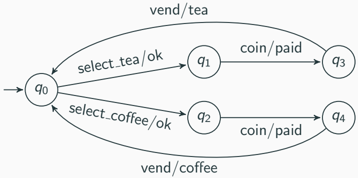

$$
\begin{aligned}
V &= \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}\\
X &= \{[select\_tea], [select\_coffee], [coin], [vend]\}\\
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VW &= \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend],\\
&\quad [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\}\\
VXW &= \{[select\_tea,\ select\_tea,\ coin,\ vend],\\
&\quad [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend],\\
&\quad [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}
\end{aligned}
$$

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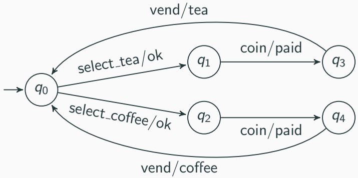

$$
\begin{aligned}
V &= \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\} \\
X &= \{[select\_tea], [select\_coffee], [coin], [vend]\} \\
W &= \{[select\_tea,\ coin,\ vend]\} \\
VW &= \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \\
VXW &= \{[select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], \\
&\quad [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}
\end{aligned}
$$

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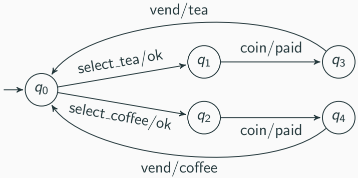

$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$
$$X = \{[select\_tea], [select\_coffee], [coin], [vend]\}$$
$$W = \{[select\_tea,\ coin,\ vend]\}$$
$$VW = \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\}$$
$$VXW = \{[select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}$$

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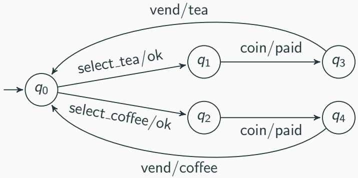

$$
\begin{aligned}
V &= \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\} \\
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&\quad [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\} \\
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&\quad [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}
\end{aligned}
$$

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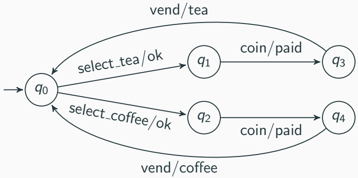

$$V = \{\epsilon, [select\_tea], [select\_tea,\ coin], [select\_coffee], [select\_coffee,\ coin]\}$$
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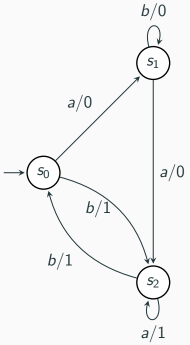
$$VW = \{[select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ coin,\ vend], \ldots\}$$
$$VXW = \{[select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ select\_tea,\ select\_tea,\ coin,\ vend], [select\_tea,\ coin,\ select\_tea,\ select\_tea,\ coin,\ vend], \ldots\}$$

A Smaller Example



$$V = \{\epsilon, a, b\}, X = \{a, b\}, \text{and } W = \{a, b\}$$

$$VW = \{\epsilon, a, b\}\{a, b\} = \{a, b, aa, ab, ba, bb\}$$

$$VXW = \{\epsilon, a, b\}\{a, b\}\{a, b\}$$

$$= \{aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb\}$$

Things to Think About

- Where do models come from? Have you ever (voluntarily) drawn a model for your software?

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    - Model inference!

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- What if we can't reliably reset?

Things to Think About

- Where do models come from? Have you ever (voluntarily) drawn a model for your software?
    - Model inference!
- What if we can't reliably reset?
    - Use transfer

Summary

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Summary

- Sometimes we do not have access to the source code of a system
- In these cases, we need to apply *black-box* testing techniques
- Model-based testing is one such technique
- We can test that the system matches an FSM specification using the W method.
    - The W method is guaranteed to find a fault if the implementation is faulty and satisfies the assumption (at most one extra state).