

Problem 2.26

Pseudo Code

```
1 "finding height of rocket as a piecewise function
2
3 def timeLess15(time):
4     "this function produces a height if time less than 15
5     heightBefore15 = 38.1454*t + 0.13743*t*t*t
6     return heightBefore15
7
8 def timeBetween15and33(time):
9     "this function produces a height if time between 15 and 33
10    heightBefore33 = 1036 + 130.909*(time-15) + 6.18425*(time-15)*(time-15) - .428*(time-15)*(time-15)*(time-15)
11    return heightBefore33
12
13 def timeAfter33(time):
14     "this function produces a height if time is greater than 33
15    heightAfter33 = 2900 - 62.468*(time-33) - 16.9274*(time-33)*(time-33) + .41796*(time-33)*(time-33)*(time-33)
16    return heightAfter33
17
18 "passing time into correct heigh equation
19 if time < 0:
20     height = 0
21     print(height)
22 else if 0 <= time < 15
23     height = timeLess15(time)
24     print(height)
25 else if 15 <= time < 33
26     height = timeBetween15and33(time)
27     print(height)
28 else
29     height = timeAfter33(time)
30     if height < 0:
31         height = 0
32     return height
33     print(height)
```

Problem 3 Taylor series parts 1 and 2

Taylor Series |

3.A1

$$f(x) = e^x$$

$$f(x)' = e^x$$

$$f(x)'' = e^x$$

 \Downarrow

+ so on,

Taylor series is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

if $x_0 = 0$ $x = x$

then

$$\frac{f^{(0)}(0)}{0!} (x-0)^0 + \frac{f^{(1)}(0)}{1!} (x-0)^1 + \frac{f^{(2)}(0)}{2!} (x-0)^2$$

$$\text{becomes } \frac{e^0}{1} x^0 + \frac{e^0}{1!} (x-0)^1 + \frac{e^0}{2!} (x-0)^2$$

$$\text{Which is } 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots \frac{x^n}{n!} \dots$$

3.A2

$$f(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \dots$$

$$f(x) = 1$$

$$f'(x) = -x$$

$$f''(x) = 1 + \frac{x^2}{2!}$$

$$f'''(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$0 \text{ order Approx; } (1 - .5) = .5$$

$$f(1) \approx .5$$

$$\approx 1$$

1st order Approx

$$f(1) \approx 1 - (1)$$

$$\approx 0$$

2nd order Approx

$$f(1) \approx 1 - (1) + \frac{(1)^2}{2!}$$

$$\approx .5$$

3rd order Approx

$$f(1) \approx 1 - (1) + \frac{(1)^2}{2!} - \frac{(1)^3}{3!}$$

$$\approx \frac{1}{3}$$

Actual

$$f(1) = e^{-1}$$

$$= .3678$$

$$\frac{\text{True Value} - \text{Approx}}{\text{True}} \times 100 = \% \text{ err}$$

$$\frac{.3678 - 1}{.3678} \times 100 = -172\%$$

$$\frac{.3678 - 0}{.3678} \times 100 = 100\%$$

$$\frac{.3678 - .5}{.3678} \times 100 = -35.9\%$$

$$\frac{.3678 - \frac{1}{3}}{.3678} \times 100 = 9.39\%$$

Taylor Series3.81

$$f(x) = 20x^3 - 5x^2 + 7x - 80$$

$$f'(x) = 60x^2 - 10x + 7$$

$$f''(x) = 120x - 10$$

$$f'''(x) = 120$$

Taylor Series

$$f(x) \approx \dots \frac{f^{(n)}(a)}{n!} (x-a)^n + \dots$$

True Value

$$f(-1) = -112$$

Error Approx

$$\frac{\text{True Value} - \text{Approx}}{\text{True}} \times 100\%$$

0th Order Approximation ; $h = -1 - (1) = -2$

$$\begin{aligned} f(-1) &\approx 20(1)^3 - 5(1)^2 + 7(1) - 80 \\ &\approx -58 \end{aligned}$$

$$\frac{-112 - (-58)}{-112} \times 100 = 48.2\%$$

1st Order Approx ; $h = -2$

$$\begin{aligned} f(-1) &\approx -58 + [60(1)^2 - 10(1) + 7](-2) \\ &\approx -172 \end{aligned}$$

$$\frac{-112 - (-172)}{-112} \times 100 = 53\%$$

2nd Order Approx

$$\begin{aligned} f(-1) &\approx -172 + \frac{[120(1) - 10](-2)^2}{2!} \\ &\approx 48 \end{aligned}$$

$$\frac{-112 - (48)}{-112} \times 100 = 142.8\%$$

3rd Order

$$\begin{aligned} f(-1) &\approx 48 + \frac{120(-2)^3}{3!} \\ &\approx -112 \end{aligned}$$

$$\frac{-112 - (-112)}{-112} \times 100 = 0\%$$