MATH 151 Lab 1 - Solutions

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In [1]:
```
from sympy import *
#from sympy.plotting import (plot, plot_parametric,plot3d_parametric_surface, plot3d_parametric_line,plot3d)
```

#1 A constant force with vector representation $F = 10i + 18j$ moves an object along a straight line from the point $P(2, 3)$ to the point $Q(4, 9)$.

(a) Compute the displacement vector $D = \overrightarrow{PQ}$ from P to Q. (b) Find the work done if the distance is measure in meters (m) and the magnitude of the force is measured in Newtons (N). Compute the dot produc two ways: i. Using the component formula given in class (p56 of the Supplement) ii. Using Python's **dot** command. Give each answer in a **print** command with proper units.

In [2]:
```
# 1(a).
P=Matrix([2,3])
Q=Matrix([4,9])
F=Matrix([10,18])
D=Q-P #D is the displacement vector from P to Q
#print('The displacement vector from P to Q is',(D[0],D[1]),'.')
print("The displacement vector from P to Q is <{},{}>.".format(D[0],D[1]))
```

```
The displacement vector from P to Q is <2,6>.
```

In [3]:
```
#1(b) i.
worki = F[0]*D[0]+F[1]*D[1]
#print('The work done by the force is', worki, 'Joules.')
print("The work done by the force is {} Joules.".format(worki))
```

```
The work done by the force is 128 Joules.
```

In [4]:
```
#1(b) ii.
workii = F.dot(D)
print("The work done by the force is {} Joules.".format(workii))
```

```
The work done by the force is 128 Joules.
```

#2 A pulley system is suspended by ropes. The system weighs 350 N. The ropes, fastened at different heights make angles of 50° and 38° with the horizontal. Use matrices to find the magnitude of the tension force in each rope, and the (vector) tension forces.

In [5]:
```
#2
T1x = cos(50*pi/180).evalf(6) #horizontal component of tension in T1
T1y = sin(50*pi/180).evalf(6) #vertical component of tension in T1
T2x = cos(38*pi/180).evalf(6) #horizontal component of tension in T2
T2y = sin(38*pi/180).evalf(6) #vertical component of tension in T2

ForceMatrix = Matrix([[-T1x,T2x],[T1y,T2y]]) #formula from Ex. 5, Sup
p 1.1
ForceSum = Matrix([0,350]) #horizontal components yield 0 since no ho
rizontal movement, vertical components sum to 350 since there is a 35
0N weight pulling directly downward.
Tensions = ForceMatrix**(-1)*ForceSum

print("The magnitude of tension in the rope T1 is {} Newtons \n and t
he magnitude of tension in rope T2 is {} Newtons.".format(Tensions[0
],Tensions[1]))
print("The tension force in T1 is given by the vector <{},{}>.".forma
t(Tensions[0]*-T1x,Tensions[0]*T1y))
print("The tension force in T2 is given by the vector <{},{}>.".forma
t(Tensions[1]*T2x,Tensions[1]*T2y))
```

```
The magnitude of tension in the rope T1 is 275.972 Newtons
 and the magnitude of tension in rope T2 is 225.113 Newtons.
The tension force in T1 is given by the vector <-177.391,211.407>.
The tension force in T2 is given by the vector <177.391,138.593>.
```

#3 Given the vectors $\mathbf{a} = \langle 4, 3 \rangle$ and $\mathbf{b} = \langle 2, -1 \rangle$:

(a) Find a vector of unit length in the direction of $2\mathbf{a} + \mathbf{b}$.

(b) Find the angle between $\mathbf{a}$ and $\mathbf{b}$.

(c) Find the scalar and vector projection of $\mathbf{b}$ onto $\mathbf{a}$.

In [6]:
```
#3(a)
a=Matrix([4,3])
b=Matrix([2,-1])
sum = a+2*b
vect = sum/sum.norm()
print("A vector of unit length in the direction of a+2b is the vector
({},{}).".format(vect[0],vect[1]))
cosineofangle = a.dot(b)/(a.norm()*b.norm())
angle = acos(a.dot(b)/(a.norm()*b.norm()))
print("The cosine of the angle is {} and the angle is approximately
{} radians, or {} degrees.".format(cosineofangle,angle.evalf(5),(angl
e*180/pi).evalf(5)))
scalarprojbona = a.dot(b)/a.norm()
vectorprojbona = a.dot(b)/a.norm()**2*a
print("The scalar projection of vector b onto vector a is {} and the
 vector projection is ({},{}).".format(scalarprojbona,vectorprojbona[
0],vectorprojbona[1]))
```

```
A vector of unit length in the direction of a+2b is the vector (8*sqr
t(65)/65,sqrt(65)/65).
The cosine of the angle is sqrt(5)/5 and the angle is approximately
1.1071 radians, or 63.435 degrees.
The scalar projection of vector b onto vector a is 1 and the vector p
rojection is (4/5,3/5).
```