MATH 151 Lab 5 Solutions

TA: Nida Obatake

In [22]:
```
from sympy import *
from sympy.plotting import (plot, plot_parametric,plot3d_parametric_surf
ace, plot3d_parametric_line,plot3d)
```

In [23]:
```
matplotlib notebook
```

#1 Find the values of r for which $y = e^{rx}$ is a solution to the following differential equations:

a) $y'' - 4y' + 3y = 0$

b) $y'' - 4y' + 13y = 0$

c) Note the solutions in (b) are complex. Compute $y'' - 4y' + 13y$ when $y = e^{2x}(\cos(3x) + \sin(3x))$. What can you conclude based on your answers to b) and c)?

In [24]:
```
#1
r,x = symbols('r x')
y = exp(r*x)
dy = diff(y,x)
ddy = diff(y,x,2)
#1a
eqa = ddy-4*dy+3*y
sola = solve(eqa,r)
print("The function y = e^(rx) is a solution to the differential equatio
n y''-4y'+3y=0 when r={} or r={}.\n".format(sola[0],sola[1]))
#1b
eqb = ddy-4*dy+13*y
solb = solve(eqb,r)
print("The function y = e^(rx) is a solution to the differential equatio
n y''-4y'+13y=0 when r={} or r={}.\n".format(solb[0],solb[1]))
#1c
y = exp(2*x)*(cos(3*x)+sin(3*x))
dy = diff(y,x)
ddy = diff(y,x,2)
eqc = ddy-4*dy+13*y
print("The function y = e^(2x)(cos(3x)+sin(3x) is a solution to the diff
erential equation y''-4y'+13y=0 since the output is {}.\n".format(eqc.si
mplify()))
print("We can conclude that e^(rx) involves sines and cosines when r is
 a complex number.")
```

```
The function y = e^(rx) is a solution to the differential equation y''-
4y'+3y=0 when r=1 or r=3.

The function y = e^(rx) is a solution to the differential equation y''-
4y'+13y=0 when r=2 - 3*I or r=2 + 3*I.

The function y = e^(2x)(cos(3x)+sin(3x) is a solution to the differenti
al equation y''-4y'+13y=0 since the output is 0.

We can conclude that e^(rx) involves sines and cosines when r is a comp
lex number.
```

#2 Given the equation $y(y^2-1)(y-2) = x(x-1)(x-2)$:

a) Find $\dfrac{dy}{dx}$.

b) Find the points (x and y coordinates) where the graph of the equation has a horizontal tangent line.

c) Find the equations of the tangent lines at the point $(0, 1)$ and $(0, 2)$.

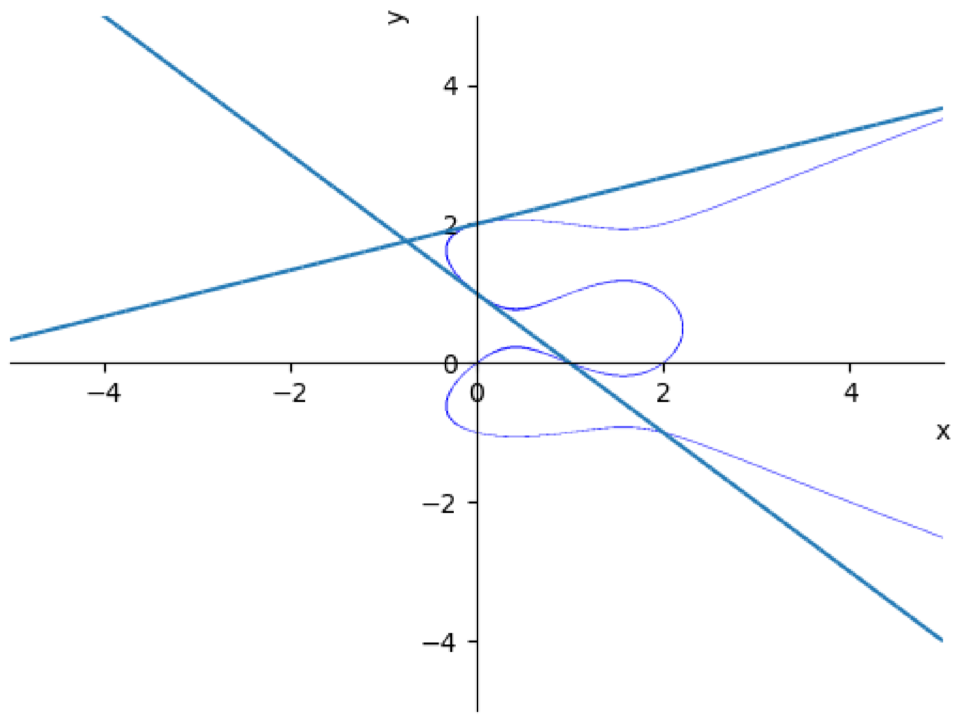
d) Plot the graph of the equation (using **plot_implicit**) and both tangent lines from part c). Use the domain $x \in [-5, 5]$.

In [26]:
```
#2
x,y = symbols('x y')
#2a
impeq = y*(y**2-1)*(y-2)-x*(x-1)*(x-2)
dy = idiff(impeq,y,x)
print("The derivative dy/dx is {}.\n".format(simplify(dy)))
#2b
htanpts = solve([dy,impeq],x,y,real=True)
for i in range(0,len(htanpts)):
    print("Horizontal tangent {} is at about ({},{}).".format(i+1,htanpt
s[i][0].evalf(4),htanpts[i][1].evalf(4)))
#2c
tan1 = dy.subs({x:0,y:1})*(x-0)+1
tan2 = dy.subs({x:0,y:2})*(x-0)+2
print("\n")
print("The tangent line at (0,1) is y = {} and the tangent line at (0,2)
 is y = {}.".format(tan1,tan2))
#2d
plot1 = plot_implicit(impeq,(x,-5,5),show=False)
plot2 = plot(tan1,tan2,(x,-5,5),show=False)
plot1.extend(plot2)
plot1.show()
```

```
The derivative dy/dx is (3*x**2 - 6*x + 2)/(2*(2*y**3 - 3*y**2 - y +
1)).

Horizontal tangent 1 is at about (0.4226,0.2295).
Horizontal tangent 2 is at about (0.4226,2.058).
Horizontal tangent 3 is at about (0.4226,0.7705).
Horizontal tangent 4 is at about (0.4226,-1.058).
Horizontal tangent 5 is at about (1.577,-0.1824).
Horizontal tangent 6 is at about (1.577,1.182).
Horizontal tangent 7 is at about (1.577,1.926).
Horizontal tangent 8 is at about (1.577,-0.9263).


The tangent line at (0,1) is y = -x + 1 and the tangent line at (0,2) i
s y = x/3 + 2.
```



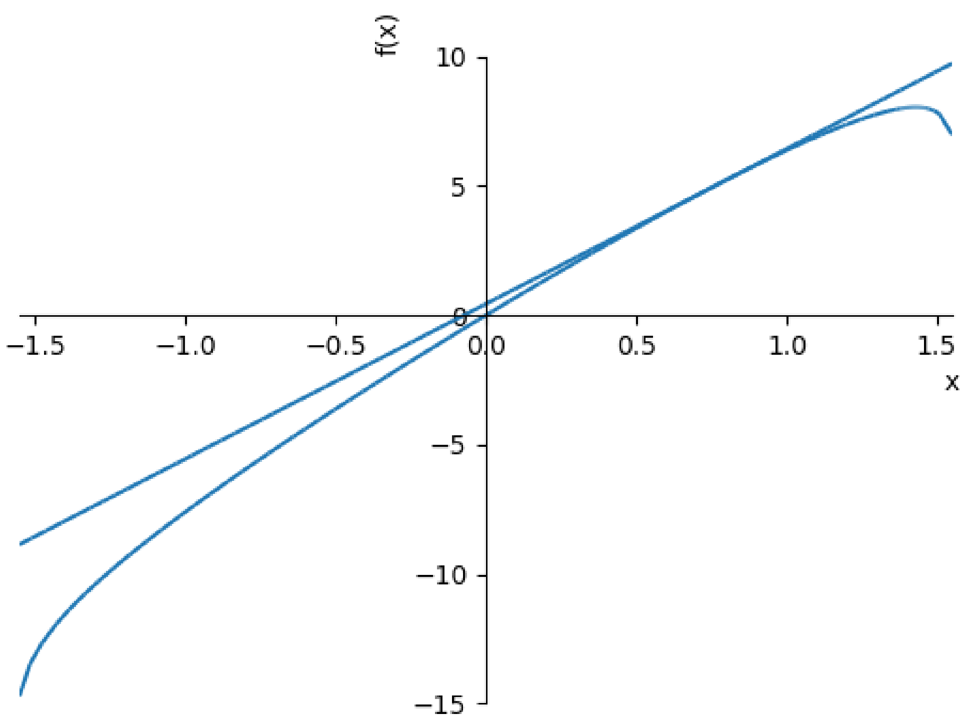
#3 . Given $f(x) = cx + \ln(\cos(x))$:

a) Find the value of c for which $f'(\frac{\pi}{4}) = 6$.

b) Plot f (using the value of c from part a)) and the tangent line at $x = \frac{\pi}{4}$. Use the domain $x \in [-1.55, 1.55]$.

In [32]:
```
#3
x,c = symbols('x c')
f = c*x + ln(cos(x))
#3a
df = diff(f,x)
dfevald = df.subs(x,pi/4)
sol = solve(dfevald-6,c,real=True)
for i in range(0,len(sol)):
    print("A value c for which f'(pi/4)=6 is {}.".format(sol[i]))
#3b
tan = 6*(x-pi/4)+f.subs({x:pi/4,c:sol[i]})
plot(f.subs(c,sol[0]),tan,(x,-1.55,1.55))
```

```
A value c for which f'(pi/4)=6 is 7.
```



Out[32]: `<sympy.plotting.plot.Plot at 0x11f051b00>`

#4 Given the equation $x^y = y^x$, use logarithmic differentiation to find $\frac{dy}{dx}$. (NOTE: The logarithm step can be done by hand).

In [50]:
```
#4
x,y=symbols('x y')
eq = x**y - y**x
dy = idiff(eq,y,x)
print("The derivative dy/dx is {}.".format(factor(dy)))
```

```
The derivative dy/dx is -y*(x*y**x*log(y) - x**y*y)/(x*(x*y**x - x**y*y
*log(x))).
```