MATH 151 Lab 6 solutions

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In [1]:
```
from sympy import *
from sympy.plotting import (plot, plot_parametric,plot3d_parametric_surf
ace, plot3d_parametric_line,plot3d)
```

In [2]:
```
matplotlib notebook
```

#1 Given the vector function $r(t) = \langle e^{\sin(t)}, e^{\cos(t)} \rangle$:

a) Find a vector equation for the line tangent to the curve at the point where $t = \frac{\pi}{3}$.

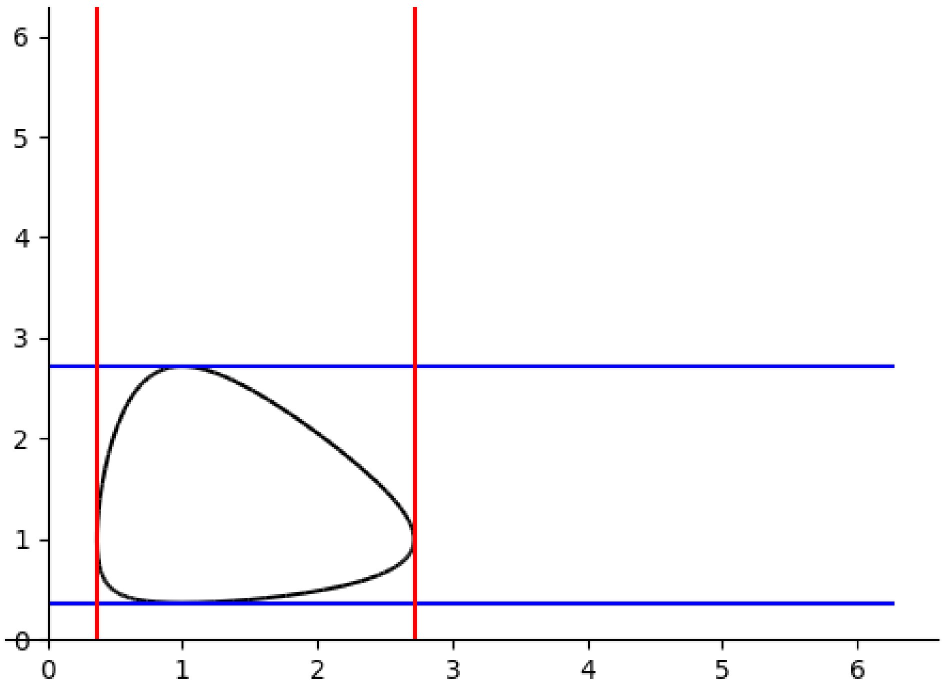
b) Find the points on the graph where the tangent line is:

```
i. Horizontal

ii. Vertical
```

c) Sketch the graph of the vector function on $t \in [0, 2\pi]$ and all tangent lines found in parts a) and b).

In [3]:
```
#1
t = symbols('t',real=True)
x = exp(sin(t))
y = exp(cos(t))

#1a
dx = diff(x,t)
slopex = dx.subs(t,pi/3)
dy = diff(y,t)
slopey = dy.subs(t,pi/3)
x0 = x.subs(t,pi/3)
y0 = y.subs(t,pi/3)
tanx = x0+slopex*t
tany = y0+slopey*t
print("1a): A vector equation for the line tangent to the given curve at
t=pi/3 is <{},{}>.\n".format(tanx,tany))

#1b
#horizontal tangents are when dy/dx=0 so when dy=0
print("1b):")
tforhtan = solve(dy,t)

xhtan = []
yhtan = []
for i in range(0,len(tforhtan)):
    xhtan.append(x.subs(t,tforhtan[i]))
    yhtan.append(y.subs(t,tforhtan[i]))
    print("There is a horizontal tangent at the point ({},{}).".format(x
.subs(t,tforhtan[i]),y.subs(t,tforhtan[i])))

    #vertical tangents are when dy/dx=0 so when dx=0
tforvtan = solve(dx,t)
xvtan = []
yvtan = []
for i in range(0,len(tforvtan)):
    xvtan.append(x.subs(t,tforvtan[i]))
    yvtan.append(y.subs(t,tforvtan[i]))
    print("There is a vertical tangent at the point ({},{}).".format(x.s
ubs(t,tforvtan[i]),y.subs(t,tforvtan[i])))

#1c
p = plot_parametric(x,y,(t,0,2*pi),line_color = 'k',show=False)
htan0 = plot_parametric(t,yhtan[0],(t,0,2*pi),line_color = 'b',show=Fals
e)
htan1 = plot_parametric(t,yhtan[1],(t,0,2*pi),line_color = 'b',show=Fals
e)
vtan0 = plot_parametric(xvtan[0],t,(t,0,2*pi),line_color = 'r',show=Fals
e)
vtan1 = plot_parametric(xvtan[1],t,(t,0,2*pi),line_color = 'r',show=Fals
e)
p.extend(htan0)
p.extend(htan1)
p.extend(vtan0)
p.extend(vtan1)
p.show()
```

```
1a): A vector equation for the line tangent to the given curve at t=pi/
3 is <t*exp(sqrt(3)/2)/2 + exp(sqrt(3)/2),-sqrt(3)*t*exp(1/2)/2 + exp
(1/2)>.

1b):
There is a horizontal tangent at the point (1,E).
There is a horizontal tangent at the point (1,exp(-1)).
There is a vertical tangent at the point (E,1).
There is a vertical tangent at the point (exp(-1),1).
```



#2 A particle moves according to a law of motion $s = t^3 - 12t^2 + 24t$, for $t \geq 0$.

a) Find the velocity at time t.

b) What is the velocity after 1 second?

c) When is the particle at rest?

d) Sketch the position function on $t \in [0, 6]$ to determine when the particle is moving in the positive direction on that interval.

e) Find the total distance traveled in the first 6 seconds (exact and approximate).

f) Find the acceleration after 1 second and at the times found in part c).

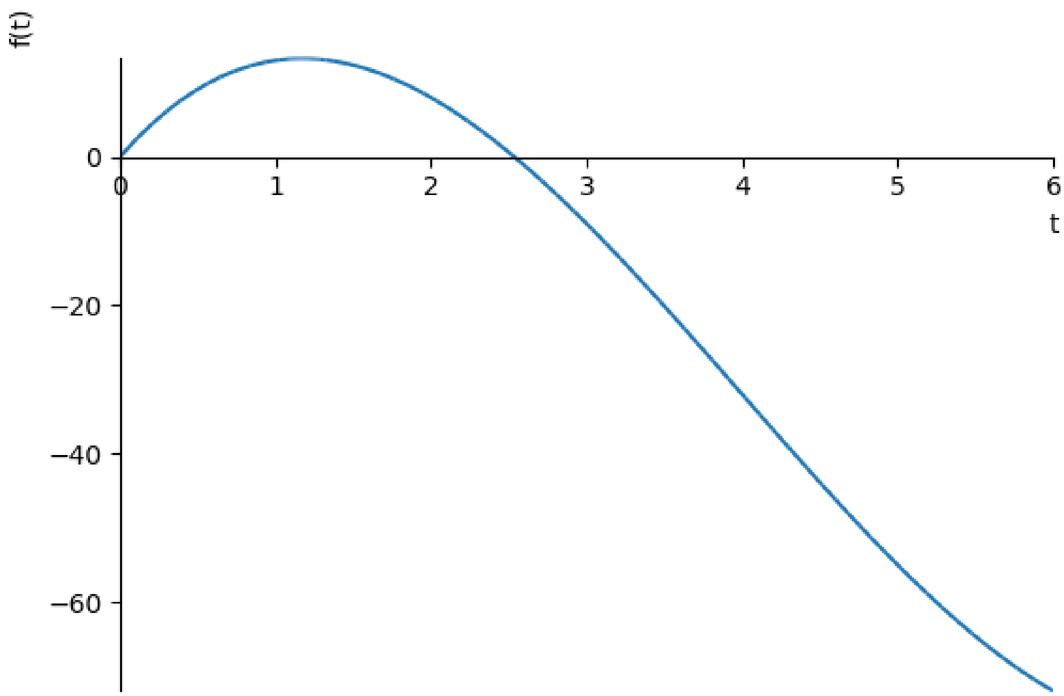
g) Graph the position, velocity, and acceleration functions on the same set of axes for $t \in [0, 8]$.

In [4]:
```
#2
t = symbols('t')
s = t**3-12*t**2+24*t
#2a
v = diff(s,t)
print("2a): The velocity of the particle is given by v(t) = {}.\n".forma
t(v))

#2b
print("2b): The velocity after 1 second is {}.\n".format(v.subs(t,1)))

#2c
print("2c):")
restt = solve(v,t)
for i in range(0,len(restt)):
    print("The particle is at rest at time t = {}.".format(restt[i]))

#2d
print("2d):")
plot(s,(t,0,6))

#2e
print("2e):")
dist = s.subs(t,restt[0])-s.subs(t,0) + s.subs(t,restt[0]) - s.subs(t,6)
print("The total distance is {}.".format(dist))
print("Approximately, the total distance traveled is {}.\n".format(dist.
evalf(5)))

#2f
print("2f):")
a = diff(v,t)
print("The acceleration after one second is {}.".format(a.subs(t,1)))
print("The acceleration when the particle is first at rest is {}. The ac
celeration when the particle is at rest next is {}.".format(a.subs(t,res
tt[0]),a.subs(t,restt[1])))

#2g
print("\n2g):")
p = plot(s,v,a,(t,0,8),show=False)
p[0].line_color = 'k'
p[1].line_color = 'b'
p[2].line_color = 'r'
p.show()
```

2a): The velocity of the particle is given by v(t) = 3*t**2 - 24*t + 2
4.

2b): The velocity after 1 second is 3.

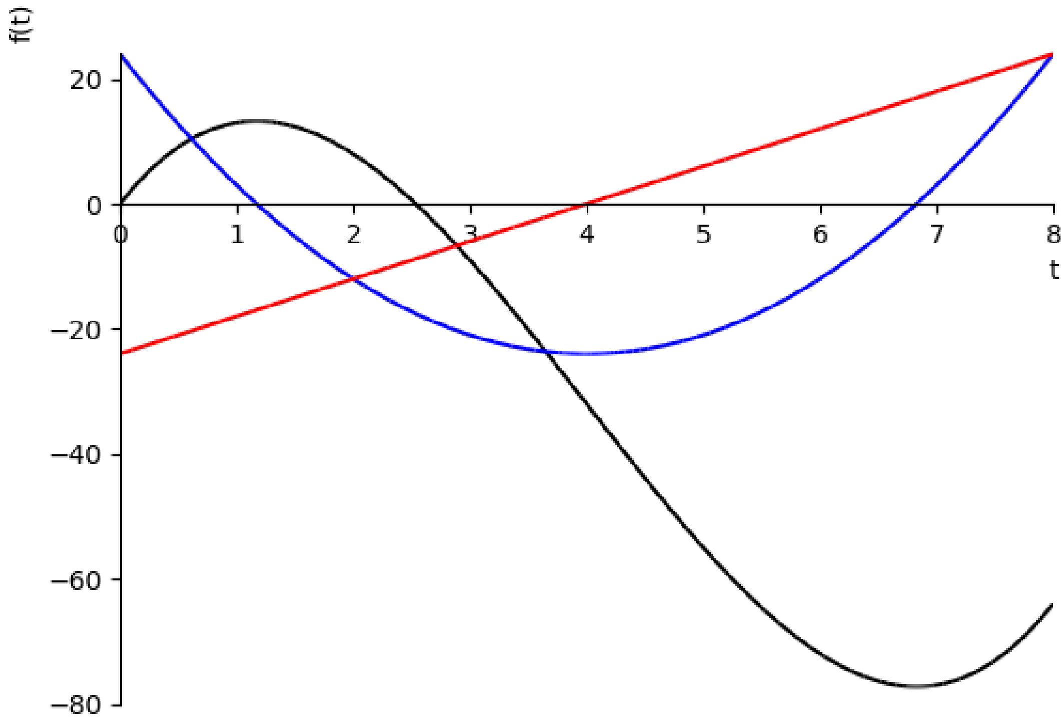
2c):
The particle is at rest at time t = -2*sqrt(2) + 4.
The particle is at rest at time t = 2*sqrt(2) + 4.
2d):



2e):
The total distance is -96*sqrt(2) - 24*(-2*sqrt(2) + 4)**2 + 2*(-2*sqrt
(2) + 4)**3 + 264.
Approximately, the total distance traveled is 98.510.

2f):
The acceleration after one second is -18.
The acceleration when the particle is first at rest is -12*sqrt(2). The
acceleration when the particle is at rest next is 12*sqrt(2).

2g):



#3 A bacteria culture grows exponentially. After 2 hours, the bacteria count was 400 and after 6 hours, the bacteria count was $20,000$.

a) Solve a system of equations to find k and y_0.

b) Use this to determine when the bacteria count reaches $2,000,000$.

c) Suppose 400 was the “initial” amount and $20,000$ the count after 4 hours. Find k and the amount of bacteria 2 hours BEFORE the “initial” time. In a print statement, explain what you notice when comparing these answers to part a).

In [5]:
```
#3
#3a
k,y0,t = symbols('k,y0,t', real=True)
pop = y0*exp(k*t)
sol = solve([400 - pop.subs(t,2),20000-pop.subs(t,6)],[k,y0])
ksol = sol[0][0]
y0sol = sol[0][1]
print("3a): To model this growth of bacteria, need k = {} and y0 = {}.\n
".format(sol[0][0],sol[0][1]))
print("\n Approximately, need k = {} and y0 = {}.\n".format(sol[0][0].ev
alf(5),sol[0][1].evalf(5)))

#3b
bacpop = pop.subs({k:ksol,y0:y0sol})
twomilt = solve(bacpop - 2000000.0, t)
print("3b): The population will reach 2,000,000 after about t = {} hour
s.\n".format(twomilt[0].evalf(5)))

#3c
solnew = solve([400 - pop.subs(t,0),20000-pop.subs(t,4)],[k,y0])
ksolnew = solnew[0][0]
y0solnew = solnew[0][1]
bacpopnew = pop.subs({k:ksolnew,y0:y0solnew})
print("3c): The new value of k is {}. Two hours before the 'initial' tim
e, there will be {} bacteria.".format(ksolnew, bacpopnew.subs(t,-2)))
print("Notice that this is exactly the growth constant k from part (a) a
nd the population at this time is the same as the initial population in
 part (a).")
```

```
3a): To model this growth of bacteria, need k = log(2)/4 + log(5)/2 and
y0 = 40*sqrt(2).


 Approximately, need k = 0.97801 and y0 = 56.569.

3b): The population will reach 2,000,000 after about t = 10.709 hours.

3c): The new value of k is log(2)/4 + log(5)/2. Two hours before the 'i
nitial' time, there will be 40*sqrt(2) bacteria.
Notice that this is exactly the growth constant k from part (a) and the
population at this time is the same as the initial population in part
(a).
```

In [ ]: