MATH 151 Lab 7

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In [1]:
```
from sympy import *
from sympy.plotting import (plot, plot_parametric,plot3d_parametric_surf
ace, plot3d_parametric_line,plot3d)
```

In [2]:
```
matplotlib notebook
```

#1. Given $f(x) = x^{\frac{4}{5}}(x-4)^2$.

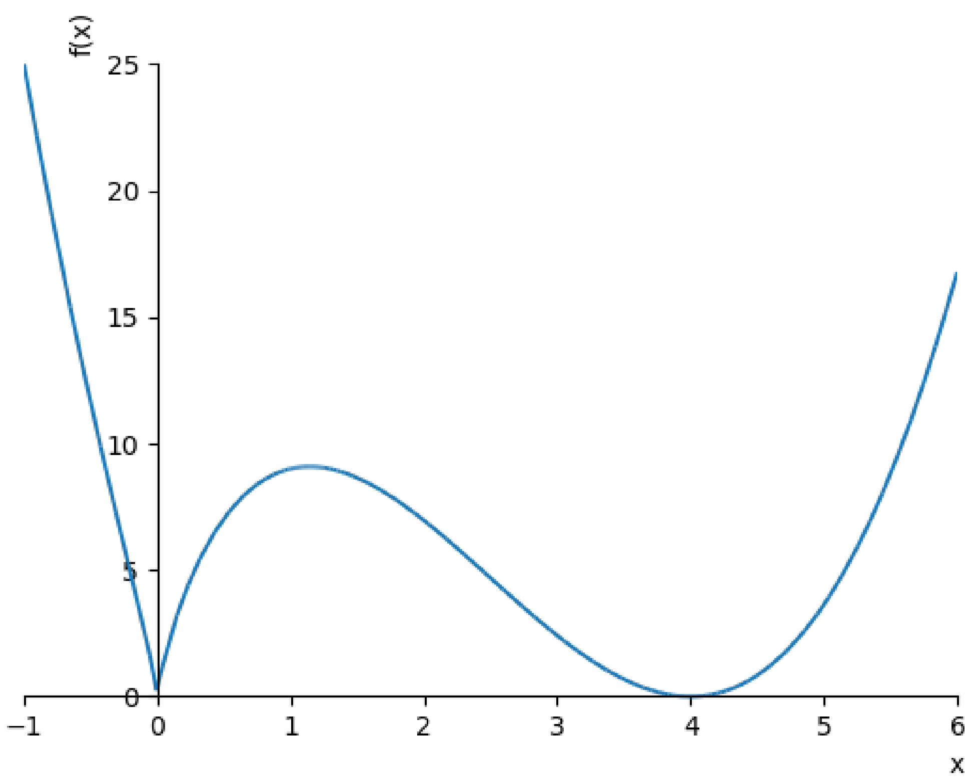
a) State f' in factored form and list the critical values of f

b) Plot f on the interval $x \in [-1, 6]$. (NOTE: In order to see the graph for $x < 0$, plot $g(x) = |x|^{\frac{4}{5}}(x-4)^2$).

c) In a print command, state which critical values are local maxima and which are local minima.

In [3]:
```
#1
x = symbols('x', real = True)
f = x**Rational(4,5)*(x-4)**2 #notice this trick for writing 4/5 as a fr
action
#1a
df = diff(f,x)
print("1a): The derivative of f is {}.".format(df.factor()))
crits = solve(df,x)
#print(len(crits)) #there are two solutions
#Oh, but x=0 is also a critical value! Let's append it.
crits.append(0)
print("1a): The critical values of f are x={}, x={}, and x={}.".format(c
rits[0],crits[1],crits[2]))
#1b
print("1b):")
plot(abs(x)**Rational(4,5)*(x-4)**2,(x,-1,6))
#1c
print("1c): The critical value x=0 produces a local minimum, x=8/7 produ
ces a local maximum, and x=4 produces a local minimum.")
```

```
1a): The derivative of f is 2*(x - 4)*(7*x - 8)/(5*x**(1/5)).
1a): The critical values of f are x=8/7, x=4, and x=0.
1b):
```




```
1c): The critical value x=0 produces a local minimum, x=8/7 produces a
local maximum, and x=4 produces a local minimum.
```

#2. Given $f(x) = e^x + e^{-2x}$:

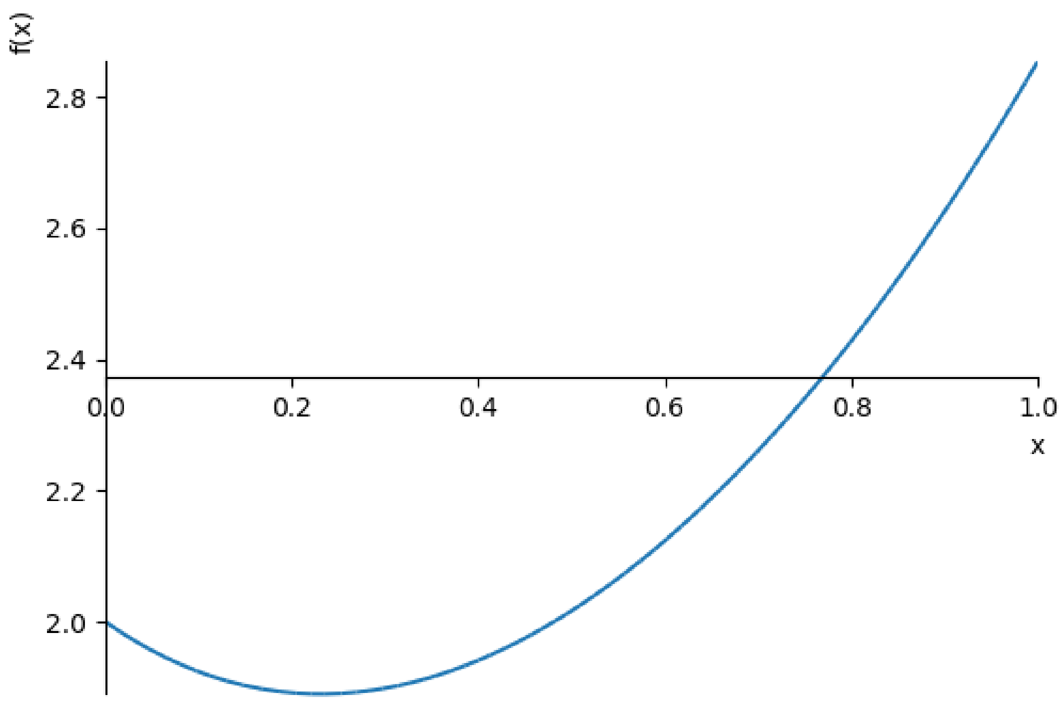
a) Find the critical values of f (exact and approximate).

b) Find the (approximate) absolute maximum and absolute minimum of f on $[0, 1]$.

c) Plot the graph of the function on the interval above.

In [4]:
```
#2
x = symbols('x', real = True)
f = exp(x)+exp(-2*x)
df = diff(f,x)
crits = solve(df,x)
#print(len(crits)) #there is only 1 critical value
#also not that this time there is no critical value coming from the deno
minator being equal to 0
print("2a): The critical value of f is exactly x={}.".format(crits[0]))
print("2a): The critical value of f is approximately x={}.".format(crits
[0].evalf(5)))
crits.append(0)
crits.append(1)
posext = [(f.subs({x:xnew})).evalf(5) for xnew in crits]
max(posext)
min(posext)
print("2b): The absolute maximum is approximately {} and it occurs at x=
{}.".format(max(posext),crits[posext.index(max(posext))]
))
print("2b): The absolute minimum is approximately {} and it occurs at ap
proximately x={}.".format(min(posext),crits[posext.index(min(posext))].e
valf(5)
))
#2c
print("2c):")
plot(f,(x,0,1))
```

```
2a): The critical value of f is exactly x=log(2**(1/3)).
2a): The critical value of f is approximately x=0.23105.
2b): The absolute maximum is approximately 2.8536 and it occurs at x=1.
2b): The absolute minimum is approximately 1.8899 and it occurs at appr
oximately x=0.23105.
2c):
```



Out[4]: `<sympy.plotting.plot.Plot at 0x1148580b8>`

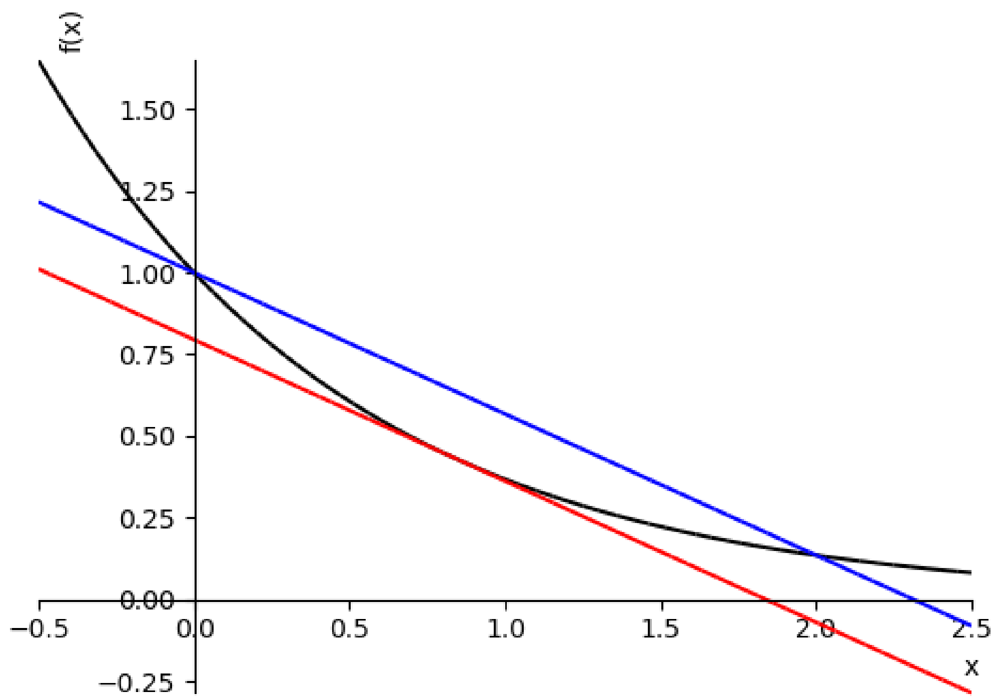
#3. Given $f(x) = e^{-x}$:

a) Find the (exact and approximate) value of c for which the Mean Value Theorem is satisfied on the interval $[0, 2]$.

b) Plot f, the the tangent line at $x = c$, and the secant line from $x = 0$ to $x = 2$ on the same set of axes. Use the domain $x \in [-0.5, 2.5]$.

In [5]:
```
#3
x, c = symbols('x c', real = True)
f = exp(-x)
#3a
a = 0
b = 2
fa = f.subs(x,a) #f(a)
fb = f.subs(x,b) #f(b)
df = diff(f,x)
fprimec = df.subs(x,c) #f'(c)
csol = solve(fprimec - (fb-fa) / (b-a), c) #f'(c) = (f(b)-f(a)) / (b-a)
len(csol)
cmean = csol[0]
print("3a): On the interval [0,2], the Mean Value Theorem is satisfied a
t c={}, which is approximately c={}.".format(cmean,cmean.evalf(5)))
#3b
tline = df.subs(x,cmean)*(x-cmean)+f.subs(x,cmean)
sline = (fb-fa)/(b-a)*(x-0)+f.subs(x,0)
#I'm going to plot each separately to emphasize with color the differenc
e between the curves plotted.
p = plot(f,(x,-0.5,2.5), line_color = 'k', show = False)
pt = plot(tline,(x,-0.5,2.5), line_color = 'r', show = False)
ps = plot(sline,(x,-0.5,2.5), line_color = 'b', show = False)
p.extend(pt)
p.extend(ps)
print("3b):")
p.show()
```

```
3a): On the interval [0,2], the Mean Value Theorem is satisfied at c=lo
g(2*exp(2)/((-1 + E)*(1 + E))), which is approximately c=0.83856.
3b):
```



#4. Given $f(x) = \arcsin \frac{x-1}{x+1}$ and $g(x) = 2 \arctan \sqrt{x}$:

a) Find, then simplify the derivative of $f(x) - g(x)$. (NOTE that $x > 0$. This can be assumed in the symbols statement).

b) In a print command, explain what this tells you about $f(x) - g(x)$.

c) Find specifically what $f(x) - g(x)$ is equal to.

In [7]:
```
#4
x = symbols('x', real = True, positive = True)
f = asin((x-1)/(x+1))
g = 2*atan(sqrt(x))
#4a
h = f - g
dh = diff(h,x)
print("4a): The derivative of f(x)-g(x) is {}.".format(simplify(dh)))
print("4b): We have that f'(x) - g'(x) = 0, which means that f'(x) =
 g'(x), that is, f(x) = g(x) + constant.")
print("4c): Specifically, f(x) - g(x) = {}.".format(h.subs(x,1)))
```

```
4a): The derivative of f(x)-g(x) is 0.
4b): We have that f'(x) - g'(x) = 0, which means that f'(x) = g'(x), th
at is, f(x) = g(x) + constant.
4c): Specifically, f(x) - g(x) = -pi/2.
```

All done! :)