

MATH 151 Lab 2

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```
In [1]: from sympy import *
        from sympy.plotting import (plot, plot_parametric, plot3d_parametric_surface, plot3d_parametric_line, plot3d)
```

```
In [2]: matplotlib notebook
```

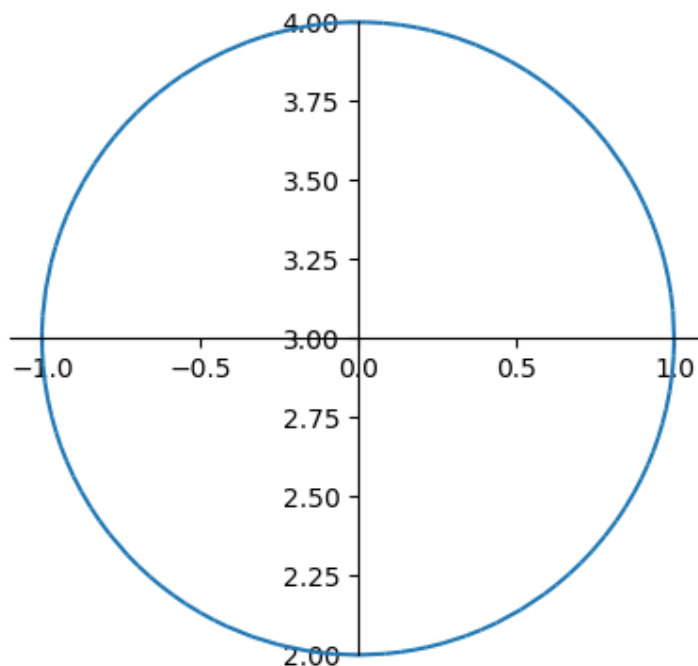
1 Let $x = \cos(t)$ and $y = 3 + \sin(t)$.

- Eliminate the parameter t to find a Cartesian equation of the curve involving x and y .
- Sketch the graph of the curve represented by the parametric equations. Use an interval of $t \in [0, 2\pi]$ and create equal axes. Use the example found at http://calclab.math.tamu.edu/Python/Equal_Axes_Example.pdf (http://calclab.math.tamu.edu/Python/Equal_Axes_Example.pdf) for help.
- Given this graph, rearrange the equation in part a) (by hand) to produce a more familiar equation of the graph in part b). Enter your answer in a print command.

```
In [3]: x,y,t=symbols('x y t')
        # 1a)
        eq1 = x-cos(t)
        eq2 = y-(3+sin(t))
        tofx = solve(eq1,t)
        #print(tofx) #we'll take the positive expression for t, so the second component
        carteq = eq2.subs(t,tofx[1]) #equation.subs(variable replaced, newvalue)
        print("A Cartesian equation of the curve is {} = 0.".format(carteq))
        # For each problem, you may insert additional cells as needed (under "Insert" above)
```

A Cartesian equation of the curve is $y - \sqrt{-x^2 + 1} - 3 = 0$.

```
In [4]: # 1b)
graph = plot_parametric(cos(t), 3+sin(t), (t, 0, 2*pi)) #plot_parametric(RHS
of x eqn, RHS of y eqn, (var, min, max))
fig = graph._backend.fig
ax = graph._backend.ax
ax.set_aspect("equal")
```



```
In [5]: # 1c)
print("Rearranging the Cartesian equation from 1a) by hand, we get the e
quation  $x^2+(y-3)^2=1$  (a circle centered at (0,3) with radius 1).")
```

Rearranging the Cartesian equation from 1a) by hand, we get the equation $x^2+(y-3)^2=1$ (a circle centered at (0,3) with radius 1).

2 Simplify the expression $\sin(2 \arcsin(x))$. In a print command, explain how this answer would be obtained by hand.

```
In [6]: # Insert code for problem #2 here.
print("The expression sin(2arcsin(x)) simplifies to {}".format(trigsimp(
    sin(2*asin(x)))))
print("To do this by hand, draw reference triangle with side x, hypotenuse 1, and other side sqrt(1-x**2), then use the trig identity sin(2x)=2sin(x)cos(x).")
```

The expression $\sin(2\arcsin(x))$ simplifies to $2x\sqrt{-x^2 + 1}$.
 To do this by hand, draw reference triangle with side x , hypotenuse 1, and other side $\sqrt{1-x^2}$, then use the trig identity $\sin(2x)=2\sin(x)\cos(x)$.

3 Given $f(x) = \frac{x^3 - 1}{\sqrt{x} - 1}$:

- Use a **for** statement to evaluate the function at $x = 0.9, 0.99, 0.999$, then at $x = 1.1, 1.01, 1.001$ to numerically estimate $\lim_{x \rightarrow 1} f(x)$.
- Plot f on the interval $[0, 2]$ to graphically verify your estimate in part a).
- Compute $\lim_{x \rightarrow 1} f(x)$ exactly.

```
In [7]: # 3a)
f = (x**3-1)/(sqrt(x)-1)
for i in [0.9, 0.99, 0.999]:
    print(f.subs(x,i))
print("As x approaches 1 from the left, the function values seem to be approaching 6.")
for j in [1.1, 1.01, 1.001]:
    print(f.subs(x,j))
print("As x approaches 1 from the right, the function values seem to be approaching 6.")
```

5.28093173771689

5.92531218695038

5.99250312468795

As x approaches 1 from the left, the function values seem to be approaching 6.

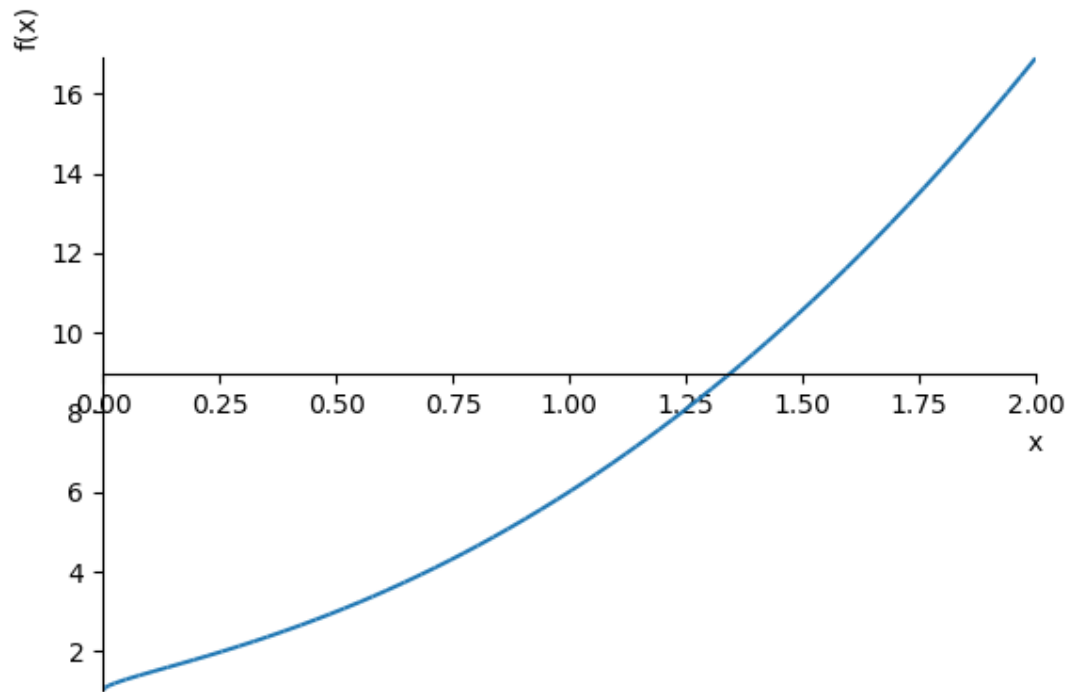
6.78155728744320

6.07531281195596

6.00750312531191

As x approaches 1 from the right, the function values seem to be approaching 6.

```
In [38]: # 3b)
plot(f,(x,0,2)) #plot_parametric(RHS of x eqn, RHS of y eqn, (var, min,
max))
```



```
Out[38]: <sympy.plotting.plot.Plot at 0x115416860>
```

```
In [14]: # 3c)
print("The limit is {}".format(limit(f,x,1))) #limit(f(x), x, x0) compu
tes the limit of f(x) as x approaches x0
```

The limit is 6.