MATH 151 Lab 4 Solutions

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Use Python to answer all the questions!!

In [1]:
```
from sympy import *
from sympy.plotting import *
```

In [2]:
```
matplotlib notebook
```

In [3]:
```
x = symbols('x')
```

#1 Let $f(x) = x^4 + 2e^x$:

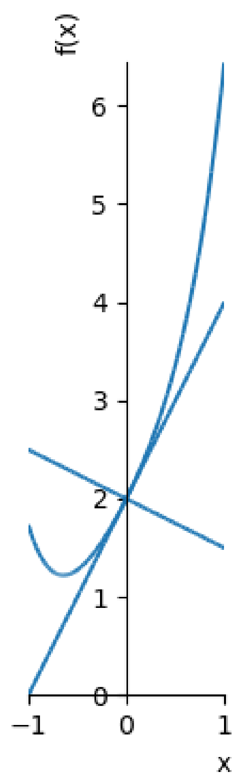
a) Find the equation of the tangent line at the point $(0, 2)$

b) Find the equation of the normal line (the line perpendicular to the curve) at the point $(0, 2)$.

c) Graph the function, tangent line, and normal line on the same set of axes in the interval $[-1, 1]$. Use the technique outlined in Lab 2 to show an equal axis (showing the lines are perpendicular).

In [4]:
```
#1
f = x**4 + 2*exp(x)
df = diff(f,x)
m = df.subs(x,0)
yat1 = f.subs(x,0)
#1a
tan = m*(x-0) + yat1
print("The tangent line is y = {}.".format(tan))
#1b
norm = -1/m*(x-0) + yat1
print("The normal line is y = {}.".format(norm))
#1c
plot1 = plot(f,tan,norm,(x,-1,1))
fig = plot1._backend.fig
ax=plot1._backend.ax
ax.set_aspect("equal")
```

```
The tangent line is y = 2*x + 2.
The normal line is y = -x/2 + 2.
```



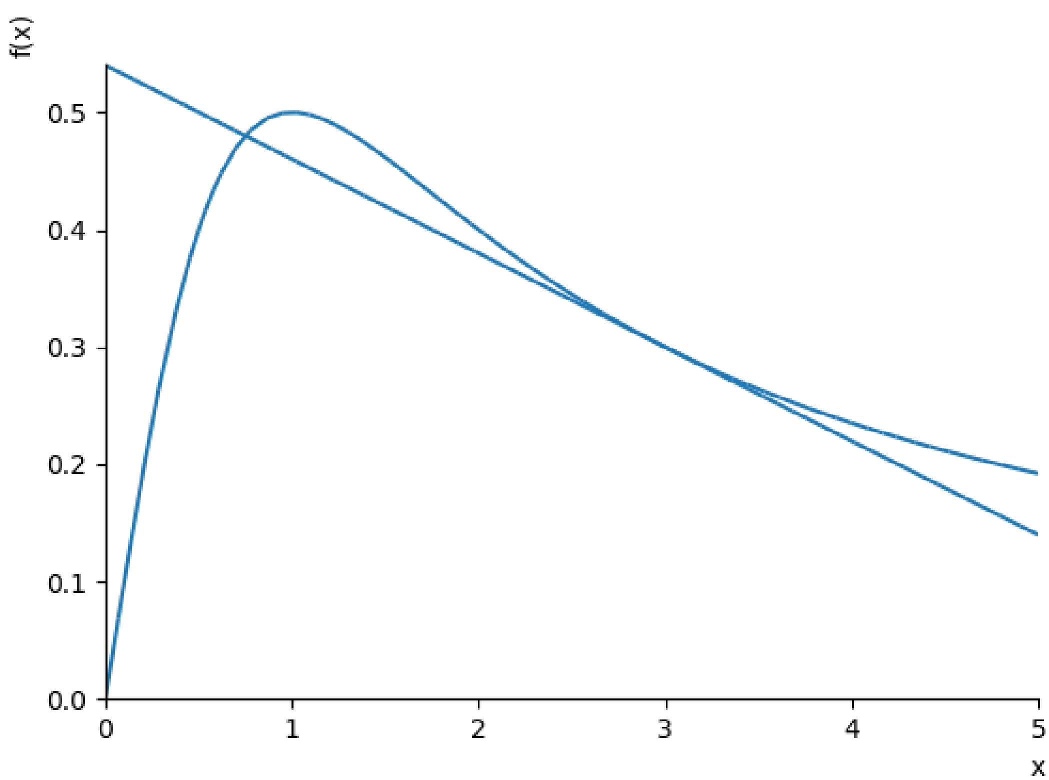
#2 Given $f(x) = \dfrac{x}{1+x^2}$:

a) Find the equation of the line tangent to f at the point where $x = 3$.

b) Graph the function and the tangent line on the same set of axes in the interval $[0, 5]$.

In [5]:
```
#2
f = x/(1+x**2)
df = diff(f,x)
m = df.subs(x,3)
yat3 = f.subs(x,3)
#2a
tan2 = yat3 + m*(x-3)
print("The tangent line is y = {}.".format(tan2))
#2b
plot(f,tan2,(x,0,5))
```

```
The tangent line is y = -2*x/25 + 27/50.
```



Out[5]: <sympy.plotting.plot.Plot at 0x1211f1f98>

#3 Given $g(x) = \dfrac{x}{e^x}$:

a) Find and (if necessary) simplify the first five derivatives of g.

b) In a print command, state the formula for the nth derivative of g.

In [6]:
```
#3
g = x / exp(x)
#3a
for i in [1,2,3,4,5]:
    print("The {}th derivative of g is {}.".format(i,simplify(diff(g,x,i
))))
#3b
print("The nth derivative of g is (-1)^n*(x-n)*exp(-x).")
```

```
The 1th derivative of g is (-x + 1)*exp(-x).
The 2th derivative of g is (x - 2)*exp(-x).
The 3th derivative of g is (-x + 3)*exp(-x).
The 4th derivative of g is (x - 4)*exp(-x).
The 5th derivative of g is (-x + 5)*exp(-x).
The nth derivative of g is (-1)^n*(x-n)*exp(-x).
```

#4 Given $f(x) = \dfrac{x}{\sqrt{1 - \cos(2x)}}$:

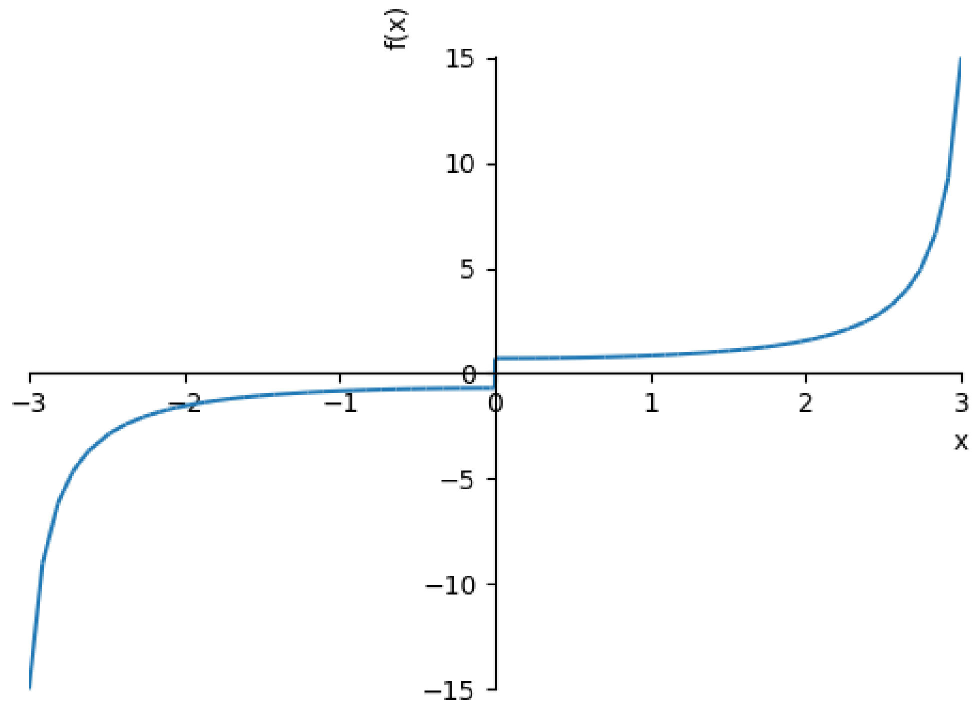
a) Graph $f(x)$ on the interval $[-3, 3]$. What appears to happen at $x = 0$?

b) Use the for command (list comprehension) to evaluate the function at $x = -0.1, -0.01, -0.001$, then at $x = 0.1, 0.01, 0.001$ to numerically estimate $\lim\limits_{x \to 0^-} f(x)$ and $\lim\limits_{x \to 0^+} f(x)$.

c) Compute the left and right-hand limits of f as $x \to 0$. (NOTE: to obtain accurate results, you MUST simplify the expression when taking the limit!)

d) Let $g(x) = (f(x))^2$. Compute $\lim\limits_{x \to 0} g(x)$. In a print command, explain why this limit exists even though the limit of f does not exist.

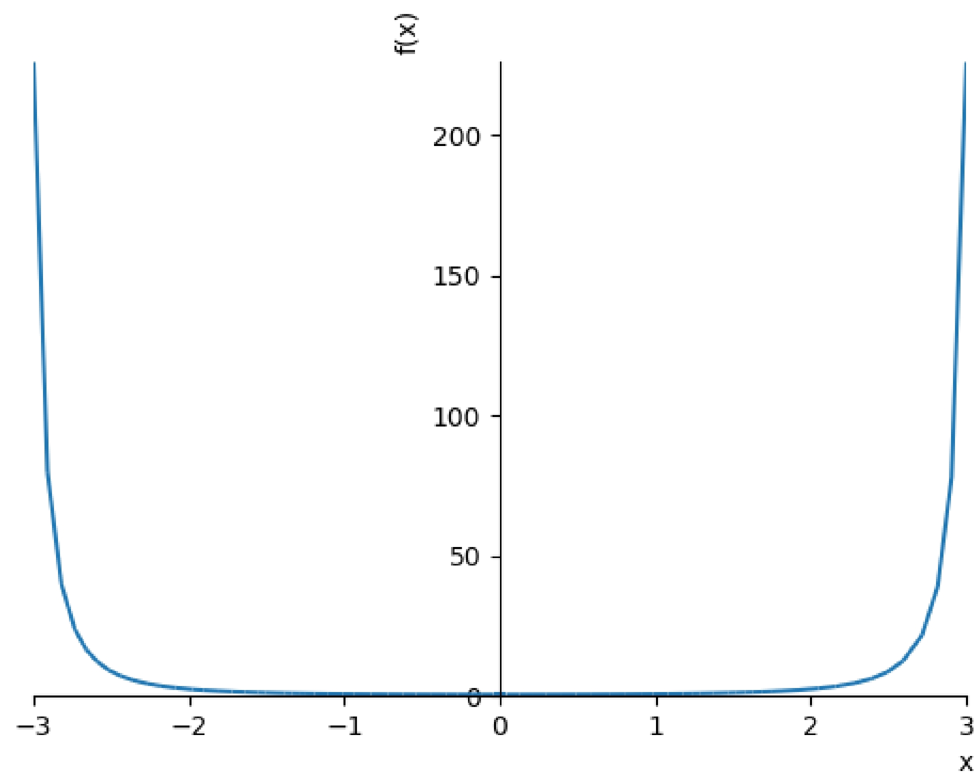
In [7]:
```
#4
f = x / sqrt(1-cos(2*x))
#4a
plot(f,(x,-3,3))
print("In the graph is looks like f is a vertical line at x=0... weird!"
)
#4b
leftvals = [f.subs({x:xnew}) for xnew in [-0.1,-0.01,-0.001]]
rightvals = [f.subs({x:xnew}) for xnew in [0.1,0.01,0.001]]
print("As x approaches 0 from the left, the y-values look like:")
print(leftvals)
print("As x approaches 0 from the right, the y-values look like:")
print(rightvals)
#4c
print("The left-hand limit of f as x approaches 0 is {}".format(limit(si
mplify(f),x,0,'-')))
print("The right-hand limit of f as x approaches 0 is {}".format(limit(s
implify(f),x,0,'+')))
#4d
g = f**2
print("The limit of g as x approaches 0 is {}.".format(limit(simplify(g
),x,0)))
```




```
In the graph is looks like f is a vertical line at x=0... weird!
As x approaches 0 from the left, the y-values look like:
[-0.708286668869620, -0.707118566437023, -0.707106899031475]
As x approaches 0 from the right, the y-values look like:
[0.708286668869620, 0.707118566437023, 0.707106899031475]
The left-hand limit of f as x approaches 0 is -sqrt(2)/2
The right-hand limit of f as x approaches 0 is sqrt(2)/2
The limit of g as x approaches 0 is 1/2.
```

(d) Why does squaring f and obtaining g fix the problem - that is why does the limit exist for g at 0, when it didn't for f? The reason the limit of f does not exist is because the limit is a negative number when approaching from the left, but a positive number when approaching from the right, and certainly those numbers are the same. In other words, f has a jump discontinuity at $x = 0$. However, when you square f to get g, the values get squared and become positive, so the left and right hand limits are the same, hence the limit exists for g. You can see this clearly when you look at the graph of g below.

In [8]:
```
plot(g,(x,-3,3))
```



Out[8]: <sympy.plotting.plot.Plot at 0x1206d01d0>