Work-out Problems

Study-tip: show all your work!

Exercise 1. Solve the following matrix equation for the matrix X.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{\mathrm{T}} + X = -3\begin{bmatrix} 6 & 3 \\ 7 & -1 \end{bmatrix}$$

Goal: use matrix operations to get the matrix X on one side by itself. Rewriting it as $A^T + X = -3B$, we get $X = -A^T - 3B$.

$$X = -\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{T} - 3\begin{bmatrix} 6 & 3 \\ 7 & -1 \end{bmatrix}$$

(take transpose of matrix) $$= -\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} - 3\begin{bmatrix} 6 & 3 \\ 7 & -1 \end{bmatrix}$$

(scale the matrices by the specified #s) $$= \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix} + \begin{bmatrix} -18 & -9 \\ -21 & 3 \end{bmatrix}$$

(add the matrices) $$= \begin{bmatrix} -19 & -12 \\ -23 & -1 \end{bmatrix}$$

So, $$\boxed{X = \begin{bmatrix} -19 & -12 \\ -23 & -1 \end{bmatrix}}$$

Encourage you to plug back into the OG equation and check your answer!

Exercise 2. Simplify down to a single matrix:

$$\begin{bmatrix} y-3 & x \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -4 & 7 & 1 \\ 5 & -2 & 0 \end{bmatrix} + 10 \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}^{\mathrm{T}}$$

(transpose comes before scalar multiplication)

$$= \underbrace{\begin{bmatrix} y-3 & x \\ 0 & -1 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} -4 & 7 & 1 \\ 5 & -2 & 0 \end{bmatrix}}_{2\times 3} + 10 \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$2 \times 2 \leftarrow \overset{\checkmark}{=} \rightarrow 2 \times 3$

(the multiplication of the first two matrices is defined)

(scale the third matrix by 10)

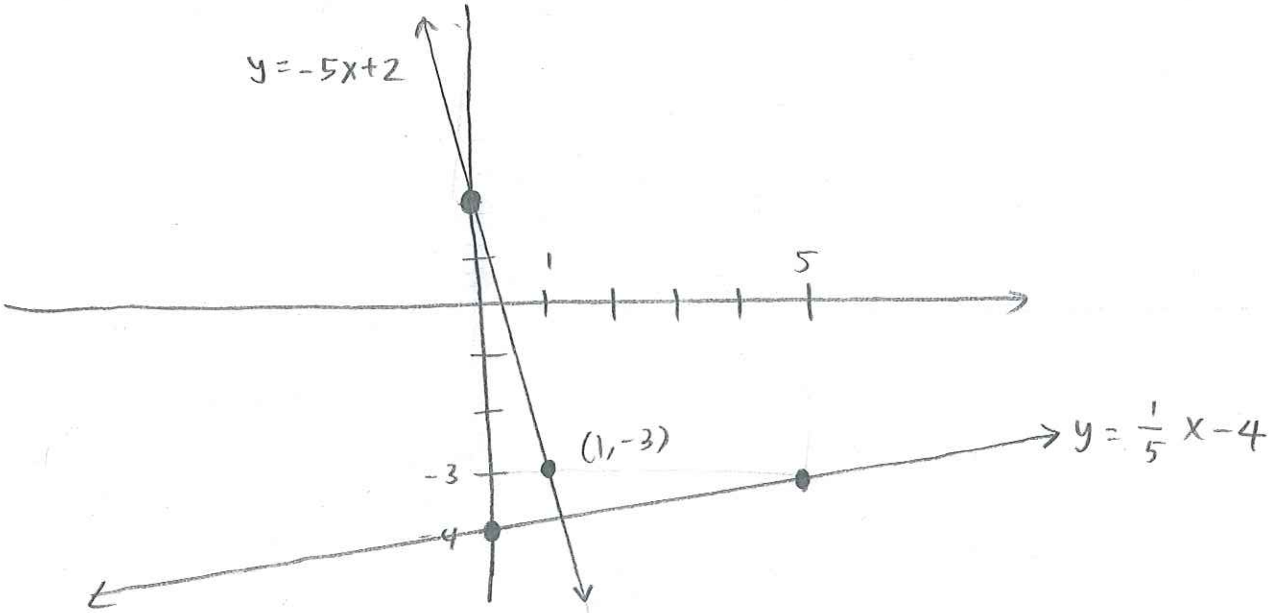
$$= \begin{bmatrix} (y-3)\cdot(-4)+x\cdot 5 & (y-3)\cdot 7 + x\cdot(-2) & (y-3)\cdot(1)+x\cdot 0 \\ 0\cdot(-4)+(-1)\cdot 5 & 0\cdot 7+(-1)\cdot(-2) & 0\cdot 1+(-1)\cdot 0 \end{bmatrix} + \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \end{bmatrix}$$

$$= \begin{bmatrix} -4y+12+5x & 7y-21-2x & y-3 \\ -5 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 5x-4y+22 & -2x+7y-1 & y+27 \\ 35 & 52 & 60 \end{bmatrix}}$$

Exercise 3. Complete each of the following. Sketch all lines, without using a calculator.

1. Plot and label the point $(1, -3)$. In which quadrant is this point?
2. Draw and label the graph of the line with equation $x - 5y = 20$.
3. Write the equation of the line that passes through the point $(1, -3)$ and has slope -5.
4. Draw and label the line found in Part 3 on the same graph.



1. The point $(1,-3)$ is plotted and labeled above, in Quadrant 4.

2. The line $x-5y=20$ can be rewritten in slope-intercept form as $y=\frac{1}{5}x-4$. It has y-intercept $(0,-4)$ and slope $\frac{1}{5}$. So, starting at $(0,-4)$, we can get to another point on the line by going up 1 unit (in y) and right 5 units (in x). It is plotted and labeled above.

3. Using point-slope form of a line through $(\overset{x_1}{1}, \overset{y_1}{-3})$ with slope $\overset{m}{-5}$, we get $y-y_1=m(x-x_1) \Longrightarrow \underline{y-(-3)=-5(x-1)}$.

4. Simplifying our answer from 3. and writing it in slope-intercept form, we have $y=-5x+2$. The line has y-intercept $(0,2)$ and another point can be found using the slope $-5=\frac{-5}{1}$: go down 5 units and right 1 unit. The line is graphed and labeled.

Exercise 4. The quantity demanded (x) of hot dogs sold at an Aggie football game is 6000 per game when the unit price (p) is \$3.25. For each decrease in unit price of \$2 below \$3.25, the quantity demanded increases by 3000 units.

1. Assuming linear demand, find the demand equation for hot dogs at the game.

We are asked to find the equation of the demand line. We are given a point (x_1, p_1) on this line, where x = # of hot dogs, p = price / hot dog (in dollars), namely $(\overset{x_1}{6000}, \overset{p_1}{3.25})$ and the slope of this line, namely

$$m = \frac{\text{change in unit price}}{\text{change in \# units demanded}} = \frac{\text{decrease in \$2}}{\text{increase by 3000}} = \frac{-2}{3000} = \frac{-1}{1500}.$$

Using point-slope form with $m = \frac{-1}{1500}$ and $(x_1, p_1) = (6000, 3.25)$, we get

$$\boxed{p - 3.25 = -\frac{1}{1500}(x - 6000)}$$

which simplifies to $\boxed{d(x) = p = \frac{-1}{1500}x + 7.25}$.

2. How many hot dogs would consumers demand if they were free?

We can restate this question as: using the demand equation, what is the value of x (# units demanded) when $p = 0$ (the unit price is \$0 = the unit is free).

We then plug $p = 0$ into the demand equation and solve for x:

$$p = \frac{-1}{1500}x + 7.25$$

$$0 = \frac{-1}{1500}x + 7.25$$

$$\frac{1}{1500}x = 7.25$$

$$x = 10875$$

The Aggie $\boxed{\text{consumers would demand 10,875 hot dogs}}$ if they were given out for free.

Exercise 5. Given the two points $(\overset{x_1}{3}, \overset{y_1}{-2})$ and $(\overset{x_2}{7}, \overset{y_2}{-2a})$.

1. Find the y-intercept of the line passing through the two points.

Plan: ① start by finding the equation of the line; ② write in slope-intercept form and read off y-intercept.

This is a line through a point $(3,-2)$ with slope $m = \frac{-2a-(-2)}{7-3} = \frac{-2a+2}{4} = \frac{2(-a+1)}{4} = \frac{-a+1}{2}$.

Using point-slope form, the line has equation

$$y-(-2) = \frac{-a+1}{2}(x-3).$$

Rewriting into slope-intercept form, we get

$$y+2 = \frac{-a+1}{2}x + \frac{-a+1}{2}\cdot(-3) \Rightarrow y+2 = \frac{-a+1}{2}x + \frac{3a-3}{2}$$

$$\Rightarrow y = \frac{-a+1}{2}x + \frac{3a-3}{2} - 2 \Rightarrow y = \frac{-a+1}{2}x + \frac{3a-3}{2} - \frac{4}{2}$$

$$\Rightarrow y = \frac{-2a+2}{4}x + \frac{3a-7}{2}.$$

The y-intercept of the line is $\boxed{\left(0, \frac{3a-7}{2}\right)}$.

2. Find the x-intercept of the line passing through the two points.

Plan: use the equation of the line found in part 1. and recall that x-intercepts happen when $y=0$.

Subbing $y=0$ into the eqn of the line $y = \frac{-a+1}{2}x + \frac{3a-7}{2}$ and solving for x, we get

$$0 = \frac{-a+1}{2}x + \frac{3a-7}{2} \Rightarrow -\frac{3a-7}{2} = \frac{-a+1}{2}x$$

$$\Rightarrow -\left(\frac{3a-7}{2}\right)\cdot\left(\frac{2}{-a+1}\right) = \left(\frac{2}{-a+1}\right)\left(\frac{-a+1}{2}\right)x$$

$$\Rightarrow -\frac{3a-7}{-a+1} = x \Rightarrow x = \frac{3a-7}{a-1}$$

The x-intercept of the line is $\boxed{\left(\frac{3a-7}{a-1}, 0\right)}$.

Exercise 6. You are the new financial advisor of Sassy Creations, the new trend-setting luxury jeweler. Unfortunately, their accounting practices are somewhat haphazard. The manager remembers that they have fixed costs of \$20,000. You noticed that a batch of 200 of their very exclusive *Queen Rev* pendants cost the company \$50,000. Their total profit from selling the 200 ~~shirts~~ pendants was \$45,000. Let x stand for the number of pendants produced and sold. Assume linear cost and revenue functions. Find the cost, revenue, and profit equations for Sassy Creations. Graph all three equations on the same graph.

want: cost function $C(x) = mx + F$, revenue function $R(x) = px$ and profit function $P(x) = R(x) - (C(x))$.

Know: $F = 20\,000$ (fixed costs), $C(200) = 50{,}000$, $P(200) = 45{,}000$.

Building $C(x)$ first, know: $C(x) = mx + 20\,000$.

Since $C(200) = 50\,000$, $C(200) = 50\,000 = m \cdot (200) + 20\,000$

(solve for m) $30\,000 = 200m$

$\Rightarrow m = \dfrac{30\,000}{200} = 150$.

cost equation

So then $\boxed{C(x) = 150x + 20\,000}$ (subbed in $m = 150$, $F = 20\,000$).

Then if $R(x) = px$, we have $P(x) = \overbrace{px}^{R(x)} - (\overbrace{150x + 20\,000}^{C(x)})$.

Since $P(200) = 45000$, $P(200) = 45\,000 = p \cdot 200 - (150 \cdot 200 + 20\,000)$

(solve for p) $45\,000 = 200p - 50\,000$

$95\,000 = 200p$

$p = \dfrac{95\,000}{200} = 475$.

revenue equation

Then $\boxed{R(x) = 475x}$ (subbed in $p = 475$), and

So $P(x) = R(x) - (C(x)) = 475x - (150x + 20\,000)$

profit equation

$\boxed{P(x) = 325x - 20\,000}$.

Exercise 7. An insurance company purchases an SUV for its employees. The original cost is \$30,500. The SUV will depreciate linearly over 5 years, after which it will have a scrap value of \$10,300.

1. What is the rate of depreciation? Answer with a complete sentence, using the correct units. Want: the slope of the line modeling the depreciation.

Know: original cost is \$30,500 $\Rightarrow$ a point $(\overset{t_1}{0}, \overset{V_1}{30500})$ on the line.

After 5 years, (scrap) value is \$10,300 $\Rightarrow$ a point $(\overset{t_2}{5}, \overset{V_2}{10300})$.

So, slope $= m = \dfrac{10\,300 - 30\,500}{5-0} = \dfrac{-20\,200}{5} = \dfrac{-4040}{1}$.

The rate of depreciation is a $\boxed{\text{loss in value of \$4040 per year}}$.

2. Find a linear model that describes the value of the SUV at the end of t years of use (denoted $V(t)$), where $0 \le t \le 5$. Want: $V(t) = mt + b$.

Know: At $t=0$, $V(0) = 30\,500 \Rightarrow b = 30\,500$. Also, $m = -4040$.

So, $\boxed{V(t) = -4040t + 30\,500}$ is a linear model for the value of the SUV, where $0 \le t \le 5$. After 5 years, the SUV is worth its scrap value of \$10,300.

3. Find and interpret the vertical intercept of $V(t)$.

The vertical intercept of $V(t)$ is $(0, 30\,500)$.

It means that at time $t=0$ years, the value of the SUV is \$30,500.

In other words, $\boxed{\text{the purchase price of the SUV is \$30,500}}$.

4. What will the SUV's value be at the end of the third year?

Want: $V(3)$. Using $V(t) = -4040t + 30\,500$ with $t=3$, we get

$V(3) = -4040 \cdot 3 + 30\,500 = 18\,380$.

$\boxed{\text{The SUV is worth \$18,380 at the end of the third year}}$.

Multiple Choice Problems

Study tip: Write out all your work when you complete the multiple-choice problems.

D

Multiple Choice 1. The demand equation for a company is $p = d(x) = 625 - 3x$, where p denotes the price per unit and x denotes the number of units demanded. Find the number of units demanded when the unit price is \$175. want: x when p = \$175.

(a) 800 units

(b) 625 units

(c) 175 units

(d) 150 units

(e) 100 units

Have: $P = d(x) = 625 - 3x$. Sub in $p = 175$, solve for x.

$175 = 625 - 3x$

$-450 = -3x$

$150 = x$

The number of units demanded is 150 units when the unit price is \$175.

C

Multiple Choice 2. Living Active, a gym accessory production company, produces foam rollers for \$10 per unit. They sell each foam roller for \$23. Their monthly fixed costs are \$136,500. Which of the following statements is false? (There is only one false statement.)

(a) Living Active earns a profit when 15,300 foam rollers are produced and sold. ✓

(b) Living Active earns a profit when 12,500 foam rollers are produced and sold. ✓

(c) Living Active undergoes a loss when 11,000 foam rollers are produced and sold. X P(11000) is positive.

(d) Living Active breaks even when 10,500 foam rollers are produced and sold. ✓

(e) Living Active undergoes a loss when 7,500 foam rollers are produced and sold. ✓

Question asks us to interpret the profit equation, so need revenue and cost equations.

foam rollers cost $\underbrace{\$10}_{m}$ to make and fixed costs of $\underbrace{\$136{,}500}_{F}$ $\Rightarrow C(x) = mx + F \Rightarrow C(x) = 10x + 136500$

Sell each foam roller for $\underbrace{\$23}_{p}$ $\Rightarrow R(x) = px \Rightarrow R(x) = 23x$.

So, $P(x) = R(x) - (C(x))$

$\Rightarrow P(x) = 23x - (10x + 136500)$

$\Rightarrow P(x) = 13x - 136500$

Plug in different values of x, as in the answer choices.

x	P(x)	profit/loss/break even
15300	+ 62400	profit
12500	+ 26 000	profit
11000	+ 6500	profit
10 500	0	breaks even
7500	-39 000	loss

D

Multiple Choice 3. The rock band *Ross's Midnight Aggies* has gained much popularity across the country. They buy a bus to travel to their destinations. The purchase price is \$185,000. The bus will depreciate linearly, and will then have a scrap value of \$75,000 after 10 years. What is the rate of depreciation of the bus?

(a) The bus loses value at a rate of \$110,000 per year.

(b) The bus loses value at a rate of \$65,065 per year.

(c) The bus loses value at a rate of \$27,500 per year.

(d) The bus loses value at a rate of \$11,000 per year.

(e) There is not enough information to determine.

want: slope of the line $V(t)$. Know: $\underset{t_1 \quad V_1}{(0, 185000)}$, $\underset{t_2 \quad V_2}{(10, 75\,000)}$ are points on the line. so

$$m = \frac{75\,000 - 185\,000}{10 - 0} = \frac{-110\,000}{10} = -11{,}000$$

The bus loses value at a rate of \$11,000 per year.

E

Multiple Choice 4. Luddington's is not too eager to supply its Wellington Boots at basement bargain rates, and accordingly controls the supply according to the formula $x = 50p - 1995$ pairs per week, where p is the price in dollars. Which of the following is true?

(a) Raising the price by \$50 results in one more pair supplied per week.

(b) Raising the price by \$50 results in 1995 more pairs supplied per week.

(c) Raising the price by \$50 results in one less pair supplied per week.

(d) Raising the price by \$1 results in 50 less pairs supplied per week.

(e) Raising the price by \$1 results in 50 more pairs supplied per week.

want: slope of the line. From $x = 50p - 1995$, we get

$$p = \frac{1}{50}x + \frac{1995}{50}.$$ So,

$$\text{slope} = \frac{\text{change in unit price } (p)}{\text{change in \# units } (x)} = \frac{+1}{+50} = \frac{\text{increase unit price by \$1}}{\text{increase \# units supplied by 50 units}}$$

there was a typo in the original answer choices...

B

Multiple Choice 5. A line has x-intercept $(3, 0)$. On the line, as y increases by 2 units, x decreases by 6 units. Find the equation of the line.

(a) $y = \frac{1}{3}x - 1$

(b) $y = -\frac{1}{3}x + 1$

(c) $y = -3x + 3$

(d) $y = -6x + 2$

(e) There is not enough information to determine.

We know a point $(3, 0)$ on the line and we can get the slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{\text{increase by 2 units}}{\text{decrease by 6 units}} = \frac{+2}{-6} = \frac{-1}{3} = m$.

Using point-slope form: $y - y_1 = m(x - x_1)$

with $m = -\frac{1}{3}$, $x_1 = 3$, $y_1 = 0$, $y - 0 = -\frac{1}{3}(x-1)$

The equation of the line is: $\boxed{y = -\frac{1}{3}x + 1}$