WIR Problem Set #11 April 28, 2020 Sections 6.1–6.3, Exam 3 Review

Please note that while this week-in-review contains some review for Exam 3, it is **not** comprehensive. Use review worksheets on eCampus and past week-in-reviews for more practice.

Work-out Problems

Study tip: Show all your work!

Exercise 1. Congratulations! You just graduated high school! Your rich aunt has given you a high school graduation gift of \$700,000. The gift, however, is in the form of a 37-year bond with an annual interest rate of 2.5% compounded annually. The bond will be worth \$700,000 in 37 years. What is this gift worth at the present time? (Round to the nearest cent.)

Want: PV.

N = (1)(37) Know: m = 1, t = 37

I% = 2.5

PV = ? → −280 746.933

PMT = 0

FV = 700 000

P/Y = C/Y = 1 Know: m = 1

The gift is worth $\boxed{\$280,746.93}$ at the present time.

Exercise 2. You are now officially an Aggie! You're going to work hard for the next 4 years, and already plan to celebrate your graduation from Texas A&M University by going on a vacation backpacking in Europe. You put $200 in your account today, and decide to set aside $150 at the end of each month for the trip. If you started depositing the money when you were a freshman at the end of each month into a savings account paying interest at the rate of 8% per year, compounded monthly, how much money will be in your travel fund at the end of the 4th year when you graduate? (Round to the nearest cent.)

want: FV when $t = 4$

$N = (12)(4)$ know: $m = 12$, $t = 4$

$I\% = 8$

$PV = -200$ ← note that these are both negative #s; at this time I am "losing" these amounts

$PMT = -150$

$FV = ?$ → 8727.6205

$P/Y = C/Y = 12$ know: $m = 12$

After 4 years, I will have $8727.62 in my travel fund.

Exercise 3. Whoop! You just graduated, went on your backpacking trip to Europe, and are now hired at a firm working as a business consultant! With all the financial skills you learned in Math 140 you know that you already should start saving up for retirement. You wish to accumulate a retirement fund of \$550,000. How much should you deposit each month into your retirement account, if the account pays interest at a rate of 5.5%/year compounded monthly, and if you want to retire in 40 years? (Round to the nearest cent.)

Want: PMT

$N = (12)(40)$ Know: $m = 12$, $t = 40$

$I\% = 5.5$

$PV = 0$

$PMT = ?$ → -315.90321

$FV = 550\,000$

$P/Y = C/Y = 12$ Know: $m = 12$

I should deposit **\$315.90** in my retirement account at the end of each month to attain my desired amount after 40 years.

Exercise 4. You plan to purchase your first home! You need to take out a mortgage loan for $150,000. Your finance company has offered you two options:

- Option A: A fixed-rate mortgage at an interest rate of 6.5% per year compounded monthly, payable over a 30-year period in 360 equal monthly installments.
- Option B: A fixed-rate mortgage at an interest rate of 6.25% per year compounded monthly, payable over a 12-year period in 144 equal monthly installments.

How much interest would you save if you choose the 12-year mortgage instead of the 30-year mortgage?

want: (interest paid in option A) − (interest paid in option B)

know: interest paid on a loan = (PMT) * N − PV.

So, calculate these for each of the options:

need: PMT for option A

N = (12)(30) (m = 12, t = 30)
I% = 6.5
PV = 150 000
PMT = ? → −948.102...
FV = 0
P/Y = C/Y = 12 (m = 12)

PMT for option A: 948.10

need: PMT for option B

N = (12)(12) (m = 12, t = 12)
I% = 6.25
PV = 150 000
PMT = ? → −1483.2552...
FV = 0
P/Y = C/Y = 12 (m = 12)

PMT for option B: 1483.26

interest paid in option A = (PMT) * N − PV = (948.10)(12·30) − 150 000
= 191 316

interest paid in option B = (PMT) * N − PV = (1483.26)(12·30) − 150 000
= 63 589.44

If we choose option B, we save ourselves from paying
$191 316 − $63589.44 = $ 127,726.56 extra in interest.

Exercise 5. Suppose I have $5000 dollars of credit card debt accruing interest at a rate of 27% per year compounded monthly. What will my outstanding debt be after 3 years if at the end of each month if I pay $150 to the credit card company?

Want: FV when $t = 3$

$N = (12)(3)$ Know: $m = 12$, $t = 3$

$I\% = 27$

$PV = 5000$

$PMT = -150$

$FV = ?$ $\rightsquigarrow -2953.6393\ldots$

$P/Y = C/Y = 12$ Know: $m = 12$

After 3 years of payments on my credit card debt, I will still owe $\boxed{\$2953.64}$.

Exercise 6. Skye wishes to purchase a $360,000 house. She will make a down payment of 14% of the purchase price, and take out a mortgage loan on the remaining balance. The mortgage is to be amortized through monthly payments for a term of 30 years, with an interest rate of 3%/year compounded monthly on the unpaid balance.

1. What monthly payment will Skye be required to make? (Round to the nearest cent.)
   want: PMT
2. Skye plans to sell her house in 10 years. What will Skye's outstanding balance be at that time? (Round to the nearest dollar.) want: FV when $t = 10$

① know: $m = 12$, $t = 30$

$N = (12)(30)$

$I\% = 3$

$PV = 360\,000 - 0.14(360\,000) = 309\,600$ (purchase price minus down payment = loan amount)

$PMT = ? \rightsquigarrow -1305.2861\ldots$

$FV = 0$

$P/Y = C/Y = 12$ know: $m = 12$

Her required monthly mortgage payment is $\boxed{\$1305.29}$.

② leave everything from above in calc, just change t

know: $m = 12$, $t = 10$

$N = (12)(10)$

$I\% = 3$

$PV = 360\,000 - 0.14(360\,000) = 309\,600$ (purchase price minus down payment = loan amount)

$PMT = -1305.2861\ldots$ (left in calc from last step)

$FV = ? \rightsquigarrow -235\,357.33\ldots$

$P/Y = C/Y = 12$ know: $m = 12$

Skye's outstanding balance after 10 years is $\boxed{\$235,357}$.

After 10 years, skye's equity in her home is $\boxed{\$124,463}$:

$\text{Equity} = (\text{value of item}) - (\text{outstanding balance}) = (360\,000) - (235,537)$

$= 124,463$

Exercise 7. Alexa purchased a new home for $230,000 and financed the purchase price at 5.25% annual interest compounded monthly for 30 years. The bank figured that Alexa's monthly payment is $1,270.07. How much of Alexa's first payment will go toward interest and how much will go toward the current principal?

Need: interest paid on first payment $= I_1$

amount paid toward principal on first payment $= PMT - I_1$.

Know: $r = 0.0525$, $m = 12$, $PV = 230000$,

$PMT = 1270.07$

$$I_1 = \left(\frac{r}{m}\right) \cdot PV_0 = \left(\frac{0.0525}{12}\right) \cdot (230000)$$

$$= 1006.25$$

$$PMT - I_1 = 1270.07 - 1006.25$$

$$= 263.82$$

Of Alexa's first payment of $1270.07,

$\boxed{\$1006.25}$ went towards paying interest,

and $\boxed{\$263.82}$ went towards the principal.

Exercise 8. If you deposit \$10,000 into an account that compounds interest quarterly for 40 years, you will have a balance of \$211,307.65. ① What is the interest rate for this investment? If you have a chance to put the same deposit in a continuously compounded account at the same interest rate, ② how much faster will you get to a balance of \$211,307.65? Round your **final** answers to one decimal place.

① want: I%

N = (4)(40) know: m = 4, t = 40

I% = ? ⇝ 7.7

PV = −10 000

PMT = 0

FV = 211 307.65

P/Y = C/Y = 4 know: m = 4

The interest rate for this account is **7.7%**.

② want: t in $A = Pe^{rt}$ since this account compounds continuously.

know: P = 10 000, r = 0.077, A = 211 307.65

↑ since the account has the same interest rate as the other account

$$A = Pe^{rt} \Rightarrow \frac{A}{P} = e^{rt} \Rightarrow \ln\left(\frac{A}{P}\right) = rt$$

$$\Rightarrow t = \frac{\ln\left(\frac{A}{P}\right)}{r} = \frac{\ln\left(\frac{211\,307.65}{10\,000}\right)}{0.077} = 39.6198\ldots$$

The second account takes 39.6 years to accumulate to \$211,307.65 while the first account takes 40 years to do the same, so the second account will achieve our goal **0.4 years = 4.8 months faster** than the first account.

Multiple Choice Problems

Study tip: Write out all your work when you complete the multiple-choice problems.

E

Multiple Choice 1. A bank advertises a nominal rate of 8.2% compounded daily. What is the annual percentage yield? (Round your answer to 3 decimal places.)

(a) There is not enough information to determine the annual percentage yield.

(b) 0.085%

(c) 8.203%

(d) 8.515%

(e) 8.545%

$m = 12$, $I\% = 8.2$

In calculator:

Eff (8.2, 12) = 8.545

D

Multiple Choice 2. What is the domain of $g(x) = \dfrac{\ln(x-2)}{e^{x-3}}$?

(a) $(-\infty, \infty)$

(b) $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

(c) $[2, \infty)$

(d) $(2, \infty)$

(e) $(2, 3) \cup (3, \infty)$

① $\ln(x-2) \Rightarrow x-2 > 0 \Rightarrow x > 2 \Rightarrow (2, \infty)$

AND

② e^{x-3} has domain equal to domain of $x-3$, which is a polynomial, so $(-\infty, \infty)$

AND

③ denominator $\neq 0$: so $e^{x-3} \neq 0$, but this has no solution, since $e^{x-3} > 0$ for any x. So the domain of the denominator is $(-\infty, \infty)$.

Domain of g: $(2, \infty) \cap (-\infty, \infty) \cap (-\infty, \infty)$

$= \boxed{(2, \infty)}$

C

Multiple Choice 3. Lauren found an ordinary annuity that pays 4.5% annual interest compounded monthly. If she deposits \$100.00 each month into this annuity for the next twenty-five years, how much interest will she have earned?

(a) \$2615.80

(b) \$25,607.18

(c) \$25,299.80

(d) \$55,299.80

(e) \$30,000

want: interest earned = FV − (total amount invested)

= FV − (PMT * N)

Need: FV

N = (12)(25) Know: m = 12, t = 25

I% = 4.5

PV = 0

PMT = −100

FV = ? → 55299.80

P/Y = C/Y = 12 Know: m = 12

Then:

interest earned = FV − (total amount invested)

= FV − (PMT * N)

= 55299.81 − (100 · (12)(25))

= 25299.81

Lauren earns \$25299.81 in interest.

D

Multiple Choice 4. The United States paid about 4 cents an acre for the Louisiana Purchase in 1803. Suppose the value of this property grew at an annual rate of 5.5% compounded annually. What was an acre of land worth 200 years later, in 2003? (Round to the nearest cent.)

(a) None of these

(b) \$178875.94

(c) \$2.56

(d) \$1788.76

(e) \$1.50

Can use TVM solver: want FV when $t = 200$

or

can use continuous compounding formula: want A in $A = P\left(1 + \frac{r}{m}\right)^{mt}$

Using TVM solver:

want: FV

$N = (1)(200)$ Know: $m = 1$, $t = 200$

$I\% = 5.5$

$PV = -0.04$

$PMT = 0$

$FV = ?$ → 1788.7594...

$P/Y = C/Y = 1$ Know: $m = 1$

An acre of the land is worth \$1788.76 200 years later.

A

Multiple Choice 5. Find the accumulated amount at the end of 11 months on a $1200 bank deposit paying simple interest at a rate of 6%/year. (Round your answer to the nearest cent.)

(a) $1266.00

(b) $1206.55

(c) $80400.00

(d) $7800.00

(e) None of these

want: A in the simple interest

formula $A = P + I$.

$= P + P \cdot r \cdot t$

know: $P = 1200$, $r = 0.06$, $t = \frac{11}{12}$

r: annual interest rate; t: convert to years

$A = P + I = P + P \cdot r \cdot t$

$= 1200 + (1200)(0.06)\left(\frac{11}{12}\right)$

$= 1266$

The account accumulates to $\boxed{\$\,1266.00}$.

D

Multiple Choice 6. Suppose I have an account with the amount \$1325 at the beginning of January 2018. If at the end of December 2019, it has the amount \$1523, what rate of interest was earned, to two decimal places (assuming simple interest and no other deposits or withdrawals in this time period)?

(a) 1.14%

(b) 1.75%

(c) 7.21%

(d) 7.47%

(e) 14.94%

want: r in the simple interest

formula $A = P + I$.

$A = P + P \cdot r \cdot t$

Know: $P = 1325$, $A = 1523$, $t = 2$

beginning Jan 2018 to end of Dec 2019 is 2 years.

$$A = P + P \cdot r \cdot t$$

$$\Rightarrow A - P = P \cdot r \cdot t$$

$$\Rightarrow r = \frac{A - P}{P \cdot t}$$

$$= \frac{1523 - 1325}{(1325)(2)}$$

$$\Rightarrow r = 0.074716\ldots$$

$$\Rightarrow \approx 7.4716\ldots\%$$

$$\approx 7.47\%$$

The account pays simple interest at an annual rate of $\boxed{7.47\%}$.

B

Multiple Choice 7. Which of the following is equivalent to $\frac{f(x+h)-f(x)}{h}$ if $f(x) = \sqrt{x-5}$?

(a) $\frac{5}{\sqrt{x+h-5}-\sqrt{x-5}}$

(b) $\frac{1}{\sqrt{x+h-5}+\sqrt{x-5}}$

(c) $\frac{1}{\sqrt{x+h-5}-\sqrt{x-5}}$

(d) $\frac{h-10}{h(\sqrt{x+h-5}-\sqrt{x-5})}$

(e) None of these

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{(x+h)-5}-\sqrt{x-5}}{h}$$

$$= \frac{\left(\sqrt{x+h-5}-\sqrt{x-5}\right)}{(h)} \cdot \frac{\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}{\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$= \frac{(x+h-5)-(x-5)}{(h)\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$= \frac{x+h-5-x+5}{(h)\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$= \frac{h}{h\left(\sqrt{x+h-5}+\sqrt{x-5}\right)} = \boxed{\frac{1}{\sqrt{x+h-5}+\sqrt{x-5}}}$$

E

Multiple Choice 8. Find the product of the solutions of

$$2e^{4-x^2} = 3$$

(a) $\sqrt{\frac{5}{2}}$

(b) $\ln\left[\left(\frac{2}{3}\right)^4\right]$

(c) $4 - \ln\frac{3}{2}$

(d) $4\ln 3 - 4\ln 2$

(e) $-4 + \ln 3 - \ln 2$

$$2e^{4-x^2} = 3$$

$$e^{4-x^2} = \frac{3}{2}$$

$$\ln\left(e^{4-x^2}\right) = \ln\left(\frac{3}{2}\right)$$

$$4 - x^2 = \ln\left(\frac{3}{2}\right)$$

$$-x^2 = -4 + \ln\left(\frac{3}{2}\right)$$

$$x^2 = 4 - \ln\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad x = \pm\sqrt{4 - \ln\left(\frac{3}{2}\right)}$$

To get the product, just multiply the two solutions:

$$(\text{first solution}) \cdot (\text{second solution})$$

$$= \left(\sqrt{4 - \ln\left(\frac{3}{2}\right)}\right) \cdot \left(-\sqrt{4 - \ln\left(\frac{3}{2}\right)}\right)$$

$$= -\left(4 - \ln\left(\frac{3}{2}\right)\right) = -4 + \ln\left(\frac{3}{2}\right)$$

$$= \boxed{-4 + \ln(3) - \ln(2)}$$

B

Multiple Choice 9. Suppose the demand equation for a particular product is given by $x = 12000 - 1000p$, where x is the number of units sold and p is the unit price in dollars. Find the company's **maximum revenue**.

(a) \$48000

(b) \$36000

(c) \$24000

(d) \$12000

(e) \$6000

want : y-coordinate of vertex of $R(x)$.

need : equation of $R(x)$.

know : $R(x) = p \cdot x$ ↑ price eqn

And $x = 12000 - 1000p$

$\Rightarrow \quad 1000p = 12000 - x$

$\Rightarrow \quad p = \dfrac{12000 - x}{1000} = -\dfrac{1}{1000}x + 12 = p$

So, $R(x) = p \cdot x = \left(-\dfrac{1}{1000}x + 12\right) \cdot x = -\dfrac{1}{1000}x^2 + 12x$

(the revenue equation is a quadratic, so its graph is a parabola open downwards, so the max happens at the vertex)

$a = -\frac{1}{1000}$, $b = 12$

$\Rightarrow$ x-coord of vertex of $R(x)$: $x = \dfrac{-b}{2a} = \dfrac{-(12)}{2\left(-\frac{1}{1000}\right)} = 6000$

$\Rightarrow$ y-coord of vertex of $R(x)$: $R\left(\dfrac{-b}{2a}\right) = R(6000)$

$= -\dfrac{1}{1000}(6000)^2 + 12(6000)$

$= 36000.$

The company's maximum revenue is **\$36 000**.

D

Multiple Choice 10. Find the domain of the following function

$$f(x) = \frac{\sqrt{x+1}}{(x^2-9)\sqrt[5]{60+12x}}.$$

(a) $(-\infty, -5) \cup (-3, 3) \cup (3, \infty)$

(b) $(-5, -3) \cup (3, \infty)$

(c) $(9, \infty)$

(d) $[-1, 3) \cup (3, \infty)$

(e) $(-1, 3) \cup (3, \infty)$

- $\sqrt{x+1}$: even radical so domain: $x+1 \geq 0 \Rightarrow x \geq -1$

AND

- x^2-9 : polynomial so domain $(-\infty, \infty)$

AND

- $\sqrt[5]{60+12x}$: odd radical, so domain equal to domain of argument, $60+12x$, which is a polynomial, so domain $(-\infty, \infty)$.

AND

- denominator : $\neq 0$, so $(x^2-9)\left(\sqrt[5]{60+12x}\right) \neq 0$

$\Rightarrow x^2-9 \neq 0$ or $\sqrt[5]{60+12x} \neq 0$

$\Rightarrow x^2 \neq 9$ $\quad 60+12x \neq 0$

$\Rightarrow x \neq \pm 3$ or $x \neq -5$

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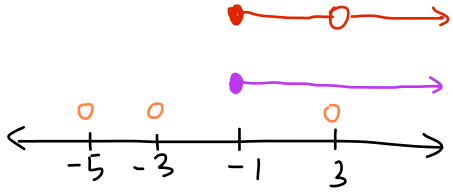
domain of f: need $x \geq -1$ and $x \neq -5, x \neq -3, x \neq 3$

AND = overlap



$\Rightarrow$ $[-1, 3) \cup (3, \infty)$