## Work-out Problems

*Study tip: Show all your work!*

**Exercise 1.** Let $x$ denote the input variable and $y$ denote the output variable. Determine which of the following gives $y$ as a function of $x$.

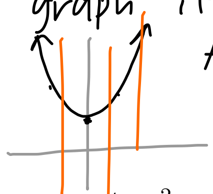
1. $\{(-1,0),(0,-3),(2,-3),(3,0),(4,5)\}$

Each input value $(x)$ produces exactly one output value $(y)$, so yes this defines a function of $x$.

2. $\{(6,10),(-7,3),(0,4),(6,-4)\}$

Here, one input value $x=6$ produces two different output values ($y=10$ and $y=-4$), so this does not define a function.

3. $y=x^2+1$ We identify this function as a quadratic. We can graph it and use the vertical line test to determine the answer.



Any vertical line intersects the function (at most) once so by the vertical line test, yes this equation defines $y$ as a function of $x$.

4. $y^2=x+1$

Notice that for an input value $x=0$, $y^2=x+1 \Rightarrow y^2=1$. This means we have two different possible output values $y=1$, $y=-1$ for a single input value, so this equation does not define a function.

5. $x^2+y^2=4$

Again, let's just test this out. Notice that for the input $x=0$, $x^2+y^2=4 \Rightarrow y^2=4 \Rightarrow y=2$ or $y=-2$.

So the one input value $x=0$ produces two different output values $(y=2,\ y=-2)$, so this does not define a function.

Means "In the evaluation of f, replace every x by (−10)."

**Exercise 2.** Given $f(x) = -6x^2 + ax - 7$ and $g(x) = -x^2 - \frac{4}{5}x$ evaluate each of the following.

1. $f(-10)$ $= -6(-10)^2 + a(-10) - 7$

   $= -6(100) + (-10a) - 7 = -600 - 10a - 7 = \boxed{-10a - 607}$

2. $f(0)$ $= -6(0)^2 + a(0) - 7$

   $= -6(0) + 0 - 7 = \boxed{-7}$

3. $f(t)$ $= -6(t)^2 + a(t) - 7$

   $= \boxed{-6t^2 + at - 7}$

4. $f(t+1)$ $= -6(t+1)^2 + a(t+1) - 7$

   $= -6(t+1)(t+1) + at + a - 7$

   $= -6(t^2 + 2t + 1) + at + a - 7 = \boxed{-6t^2 - 12t + at + a - 13}$

5. $f(x+1)$ $= -6(x+1)^2 + a(x+1) - 7$

   $= -6(x+1)(x+1) + ax + a - 7$

   $= -6(x^2 + 2x + 1) + ax + a - 7 = \boxed{-6x^2 - 12x + ax + a - 13}$

6. $f(x) + 1$ $= (-6x^2 + ax - 7) + 1$

   $= \boxed{-6x^2 + ax - 6}$

7. $g(x+h)$ $= -(x+h)^2 - \frac{4}{5}(x+h)$

   $= -(x+h)(x+h) - \frac{4}{5}x - \frac{4}{5}h = -(x^2 + 2xh + h^2) - \frac{4}{5}x - \frac{4}{5}h$

   $= \boxed{-x^2 - \frac{4}{5}x - 2xh - \frac{4}{5}h + h^2}$

8. $g(x^2 - 5)$

   $= -(x^2-5)^2 - \frac{4}{5}(x^2-5) = -((x^2-5)(x^2-5)) - \frac{4}{5}x^2 - \frac{4}{5}(-5)$

   $= -(x^4 - 10x^2 + 25) - \frac{4}{5}x^2 + 4 = -x^4 + 10x^2 - \frac{4}{5}x^2 - 21$

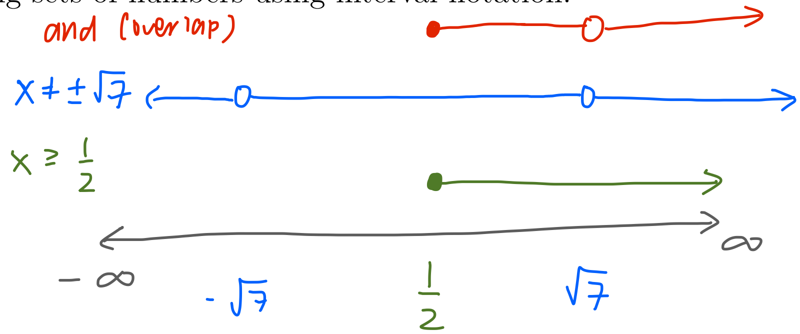
   $= \boxed{-x^4 + \frac{46}{5}x^2 - 21}$

$[\frac{1}{2}, \sqrt{7}) \cup (\sqrt{7}, \infty)$

**Exercise 3.** Express the following sets of numbers using interval notation.

1. $\{x \mid x \geq \frac{1}{2} \text{ and } x \neq \pm\sqrt{7}\}$

and, so take over lap



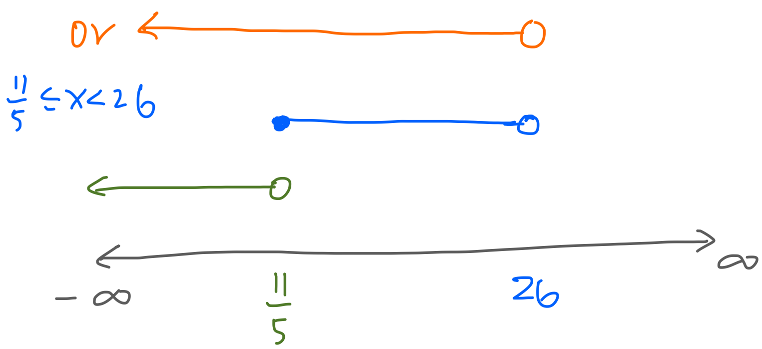

Answer: $[\frac{1}{2}, \sqrt{7}) \cup (\sqrt{7}, \infty)$

$(-\infty, 26)$

2. $\{x \mid \frac{11}{5} > x \text{ or } \frac{11}{5} \leq x < 26\}$

or, so combine

(same as $\frac{11}{5} > x$ $x < \frac{11}{5}$)




Answer: $(-\infty, 26)$

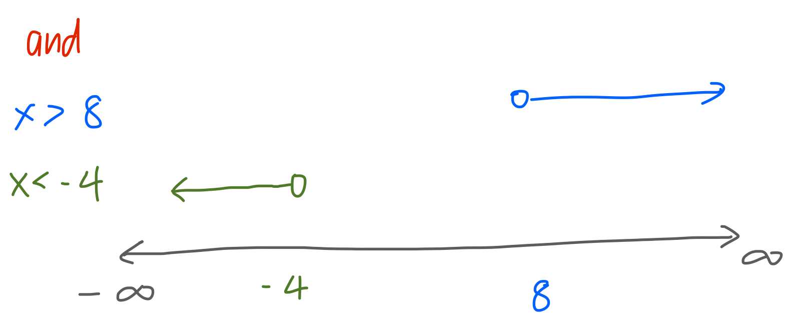
$\emptyset$ [there is no overlap!]

3. $\{x \mid x < -4 \text{ and } 2x + 5 > 21\}$

$2x + 5 > 21$

$\Rightarrow 2x > 16$

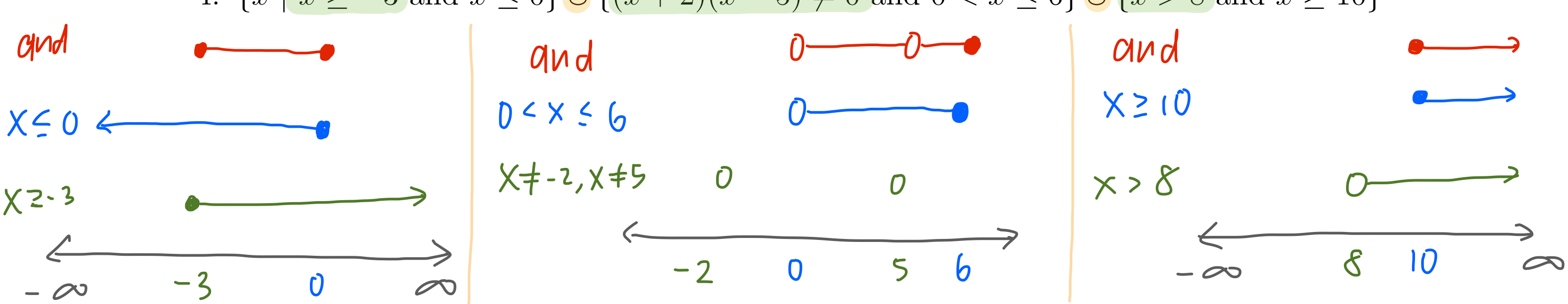
$\Rightarrow x > 8$




Answer: $\emptyset$

unions, so find each of the three pieces separately, then combine using Union symbol

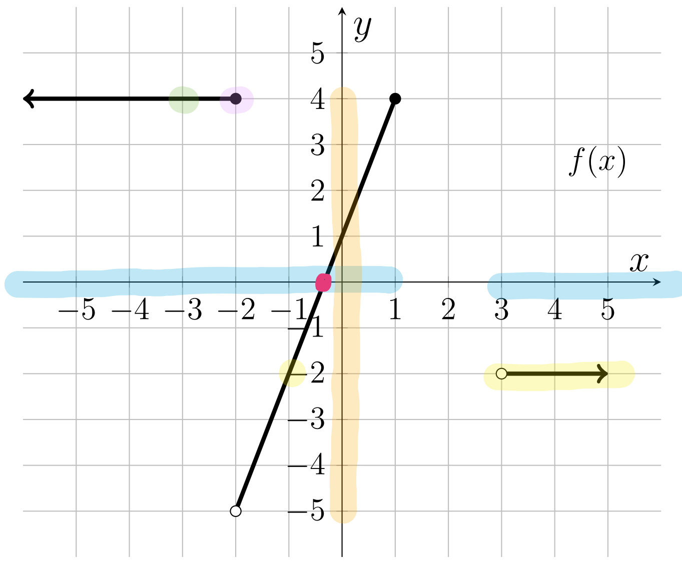
4. $\{x \mid x \geq -3 \text{ and } x \leq 0\} \cup \{(x+2)(x-5) \neq 0 \text{ and } 0 < x \leq 6\} \cup \{x > 8 \text{ and } x \geq 10\}$




Answer: $\boxed{[-3, 0] \cup (0,5) \cup (5, 6] \cup [10, \infty)}$

**Exercise 4.** The graph given below is the graph of a function $f(x)$ (why?).

By the vertical line test since any vertical line intersects the graph at most once (so 0 or 1 times).




Determine the following.

1. The domain of $f$ (in interval notation)

   $(-\infty, 1] \cup (3, \infty)$

2. The range of $f$ (in interval notation)

   $[-5, 4]$

3. $f(-3) = 4$ (The y-value on the graph of f when $x=-3$ is $y=4$.)

4. $f(-2) = 4$ (The y-value on the graph of f when $x=-2$ is $y=4$.)

5. $f(2)$ does not exist. There is no point on the graph with x-coordinate $x=2$. In other words, $x=2$ is not in the domain of the function.

6. All zeros of $f$

Want: the x-coordinates of all points where the graph of f touches the x-axis. There is one. To find it, find the equation of this line segment, $y=3x+1$, and set $y=0$, solve for x to get $\boxed{x=-\frac{1}{3}}$ as the zero.

7. Where $f(x) = -2$

Want: All x for which $y=-2$. From the graph we see that this happens for all x in: $\{x \mid x=-1 \text{ or } 3<x\}$.

**Exercise 5.** Anakin Skywalker, the consultant for the company "It's Lit!" determines that if the company sells $x$ lightsabers, their demand equation (where $x$ is the number of lightsabers demanded and $p$ is the price in dollars) is the function $p(x) = -\frac{1}{50}x + 1000$.

1. Determine the company's revenue equation. Recall: $R(x) = p \cdot x$

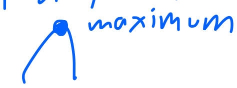
$$R(x) = p \cdot x = \left(-\frac{1}{50}x + 1000\right) \cdot x$$

$$\Rightarrow \boxed{R(x) = -\frac{1}{50}x^2 + 1000x}$$

$a = -\frac{1}{50}$

$b = 1000$

2. Find the number of lightsabers that should be produced to yield a maximum revenue. Round to the nearest whole number, if necessary. *Answer with a complete sentence, using the correct units.* Want: x-value of vertex of $R(x)$, since $R(x)$ is a quadratic, with graph a parabola that opens downward:




$$x = \frac{-b}{2a} = \frac{-(1000)}{2\left(-\frac{1}{50}\right)} = 25\,000$$

It's Lit should produce $\boxed{25{,}000 \text{ lightsabers}}$ to maximize revenue.

3. Calculate the maximum revenue of the company in dollars. Round your answer to 2 decimal places, if necessary. *Answer with a complete sentence, using the correct units.*

Want the revenue when $x = 25\,000$ (i.e., the y-coordinate of the vertex of $R(x)$)

$$R\left(\frac{-b}{2a}\right) = R(25\,000) = -\frac{1}{50}(25\,000)^2 + 1000(25\,000) = 12\,500\,000.$$

Their maximum revenue is $\boxed{\$\,12{,}500{,}000}$.

4. Find the price at which each lightsaber should be sold to maximize their revenue. Round your answer to 2 decimal places, if necessary. *Answer with a complete sentence, using the correct units.* want: $p$ when $x = 25000$.

Since $p(x) = -\frac{1}{50}x + 1000$, when $x = 25000$ we get

$$p = -\frac{1}{50}(25000) + 1000 = 500.$$

To maximize their revenue, the unit price of a lightsaber should be $\boxed{\$500}$.

5. The company's cost equation is given by $C(x) = \frac{1568}{5}x + 1{,}139{,}950$. How many lightsabers should Anakin sell to break even? want: x-value for which $R(x) = C(x)$.

$$R(x) = C(x) \Rightarrow -\frac{1}{50}x^2 + 1000x = \frac{1568}{5}x + 1\,139\,950$$

solve for x:

$$-\frac{1}{50}x^2 + \frac{3432}{5}x - 1139950 = 0$$

use quadratic formula with $a = -\frac{1}{50}$, $b = \frac{3432}{5}$, $c = -1139950$:

$$x = \frac{-\frac{3432}{5} \pm \sqrt{\left(\frac{3432}{5}\right)^2 - 4\left(-\frac{1}{50}\right)(-1139950)}}{2\left(-\frac{1}{50}\right)} \Rightarrow$$

$x = 1750$

or

$x = 32570$

To break-even, Anakin should sell $\boxed{1750 \text{ lightsabers or } 32{,}750 \text{ lightsabers}}$.

**Exercise 6.** A degree 3 polynomial $f(x) = ax^3 + bx^2 + cx + d$ has $y$-intercept $(0, -4)$ and exactly three real zeros at $x = 8$, $x = -3$, and $x = \frac{1}{2}$.

1. Find the equation of $f$.
2. Describe the end behavior of the graph of $f(x)$ (both symbolically and with a quick sketch).
3. What is the domain of $f$?

1. Need to find values of $a, b, c, d$. $y$-intercept of $(0,-4) \Rightarrow f(0) = -4$, so $f(0) = \boxed{d = -4}$. $f$ is a polynomial with exactly three real zeros at $x = 8$, $x = -3$, $x = \frac{1}{2}$, so can write $f(x) = a(x-8)(x+3)(x-\frac{1}{2})$.

$$f(x) = a(x-8)(x-(-3))(x-\tfrac{1}{2}) = a(x-8)(x+3)(x-\tfrac{1}{2})$$
$$= a(x^2 - 5x - 24)(x - \tfrac{1}{2}) = a\left(x^3 - \tfrac{1}{2}x^2 - 5x^2 + \tfrac{5}{2}x - 24x + 12\right)$$
$$= a\left(x^3 - \tfrac{11}{2}x^2 - \tfrac{43}{2}x + 12\right) = ax^3 - \tfrac{11}{2}ax^2 - \tfrac{43}{2}ax + 12a$$

But we also know $f(x) = ax^3 + bx^2 + cx + d$ since $d = -4$

$$= ax^3 + bx^2 + cx + (-4)$$

Equating the two expressions for $f(x)$ we have

$$ax^3 - \tfrac{11}{2}ax^2 - \tfrac{43}{2}ax + 12a = ax^3 + bx^2 + cx + (-4)$$

(Now match up the coefficients)

$a = a$, $\quad -\frac{11}{2}a = b$, $\quad -\frac{43}{2}a = c$, $\quad 12a = -4$

$\Downarrow$

$b = -\frac{11}{2}\left(-\frac{1}{3}\right) \Leftarrow c = -\frac{43}{2}\left(-\frac{1}{3}\right) \Leftarrow a = -\frac{1}{3}$

$b = \frac{11}{6}$ $\qquad c = \frac{43}{6}$

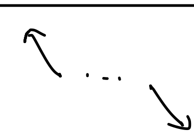
So, $f(x) = ax^3 + bx + cx + d = \boxed{-\frac{1}{3}x^3 + \frac{11}{6}x^2 + \frac{43}{6}x - 4 = f(x)}$

2. degree: $n = 3$ odd

leading coefficient: $a_n = -\frac{1}{3} < 0$

$f(x) \to -\infty$ as $x \to \infty$

$f(x) \to \infty$ as $x \to -\infty$



3. $f$ is a polynomial, so its domain is $(-\infty, \infty)$.

**Exercise 7.** Determine if each of the following is a polynomial. If it is a polynomial, specify its degree, leading coefficient, end behavior, and domain. If it is not a polynomial, state a reason for your answer.

1. $f(x) = 3x^{1/2} + x\sqrt{2}$

No, this is not a polynomial since polynomials do not have fractional exponents on its variable (like the $\frac{1}{2}$ here).

2. $g(x) = x + 2x^{-1}$

No, this is not a polynomial since polynomials do not have negative exponents on variable (like the $-1$ here).

3. $h(x) = x^3 + 2x + \frac{\sqrt{6}}{7}$

Yes, this is a polynomial.

degree: $n = 3$ (odd)

leading coefficient: $a_n = 1$ (positive)

end behavior:

$h(x) \to \infty$ as $x \to \infty$

$h(x) \to -\infty$ as $x \to -\infty$

domain: $(-\infty, \infty)$ A polynomial has domain all real numbers.

4. $j(x) = 130x - \frac{5^{2/3}}{4}\sqrt{3}x^{23} + 17x^{12}$

Yes, this is a polynomial.

degree: $n = 23$ (odd)

leading coeff: $a_n = -\frac{5^{\frac{2}{3}}}{4}\sqrt{3}$ (negative)

$j(x) \to -\infty$ as $x \to \infty$

$j(x) \to \infty$ as $x \to -\infty$

domain: $(-\infty, \infty)$ A polynomial has domain all real numbers.

5. $k(x) = ax^6 + bx^{58} - cx^5 + dx^{11} + 18x^2$ where $a < 0$, $b < 0$, $c < 0$, and $d > 0$.

Yes, this is a polynomial.

end behavior:

degree: $n = 58$ (even)

leading coeff: $b$ (neg)

$k(x) \to -\infty$ as $x \to \infty$

$k(x) \to -\infty$ as $x \to -\infty$

domain: $(-\infty, \infty)$ A polynomial has domain all real numbers.

this is a polynomial of degree 2, i.e., a quadratic, so its graph is a parabola

**Exercise 8.** Given $y = -5x^2 + 10x - 4$, without graphing determine the vertex, axis of symmetry, maximum value, minimum value, domain, range, $y$-intercept, and $x$-intercept(s) of the function. write $y = f(x) = -5x^2 + 10x - 4$.

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$. Here: $a = -5$, $b = +10$, $c = -4$

$\Rightarrow x = \frac{-b}{2a} = \frac{-(10)}{2(-5)} = \frac{-10}{-10} = 1$. Then $f\left(\frac{-b}{2a}\right) = f(-1) = -5(-1)^2 + 10(-1) - 4$

$= -5(1) - 10 - 4 = -19$

The vertex is $(1, -19)$.

axis of symmetry: since the x-coordinate of the vertex is 1, the axis of symmetry is the vertical line $x = 1$.

maximum value: since the leading coefficient $a_n = a = -5$ is negative, the parabola opens downward so its maximum value is the y-coordinate of the vertex, $-19$.

minimum value: since the parabola opens downwards it has no minimum value.

domain: $(-\infty, \infty)$ A polynomial has domain all real numbers.

range: since the parabola opens downwards and has a max value of $-19$, the range is $(-\infty, -19]$.

y-intercept: sub $x=0$ and solve for $y$: $y = -5x^2 + 10x - 4$

So the y-intercept is $(0, -4)$. $y = -5(0) + 10(0) - 4 = -4$.

x-intercept(s): sub $y=0$ and solve for $x$: $y = -5x^2 + 10x - 4$

$0 = -5x^2 + 10x - 4 \Rightarrow x = \frac{-10 \pm \sqrt{(10)^2 - 4(-5)(-4)}}{2(-5)} = \frac{-10 \pm \sqrt{100 - 80}}{-10}$

(use the quadratic formula with $a=-5$, $b=10$, $c=-4$)

$= \frac{-10 \pm \sqrt{20}}{-10}$

So, the x-intercepts are $\left(\frac{-10 + \sqrt{20}}{-10}, 0\right)$ and $\left(\frac{-10 - \sqrt{20}}{-10}, 0\right)$.

**Exercise 9.** Let $f(x) = -x^2 + 3x$. Find and simplify the following completely.

$$\frac{f(x+h) - f(x)}{h}$$

*This expression is called a* _difference quotient_. *This important concept is introduced in Section 5.2 and we will see it throughout Chapter 5.*

$$\frac{f(x+h) - f(x)}{h} = \frac{\left(-(x+h)^2 + 3(x+h)\right) - \left(-x^2 + 3x\right)}{h}$$

$$= \frac{\left(-(x+h)(x+h) + 3x + 3h\right) - \left(-x^2 + 3x\right)}{h}$$

$$= \frac{\left(-(x^2 + 2xh + h^2) + 3x + 3h\right) + x^2 - 3x}{h}$$

$$= \frac{-x^2 - 2xh - h^2 + 3x + 3h + x^2 - 3x}{h}$$

combine like terms

$$= \frac{-2xh - h^2 + 3h}{h}$$

factor out common h

$$= \frac{h(-2x - h + 3)}{h}$$

cancel common factor of h from numerator and denominator

$$= \boxed{-2x - h + 3}$$

## Multiple Choice Problems

*Study tip: Write out all your work when you complete the multiple-choice problems.*

B

**Multiple Choice 1.** It has been determined that the revenue function for a stapler is given by $R(x) = -0.025x^2 + 8.25x$ and the cost function is given by $C(x) = 1.25x + 500$ where $R(x)$ and $C(x)$ are in dollars and $x$ represents the number of staplers produced and sold. What is the selling price of the stapler when the profit is maximized? (Answers are given to the nearest penny.)

want: $p(x)$ for the $x$ that maximizes the revenue.

(a) \$140.00

(b) \$4.75

(c) \$4.13

(d) \$10.00

(e) None of these

First, find the x-coordinate of the vertex of profit eq.

From $P(X) = R(X) - C(X)$

$= (-0.025X^2 + 8.25X) - (1.25X + 500)$

$P(X) = -0.025X^2 + 7X - 500$

x-coord of vertex: $X = \frac{-b}{2a} = \frac{-7}{2(-0.025)} = 140.$

The profit is maximized when $X = 140$.

The selling price equation $p(x)$ comes from $R(X) = p \cdot X$, so since $R(X) = -0.025X^2 + 8.25X = (-0.025x + 8.25) \cdot X \Rightarrow p(x) = -0.025x + 8.25$

Then $p(140) = -0.025(140) + 8.25 = \$4.75$.

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D

**Multiple Choice 2.** What is the range of the function

$$h(x) = -(x-3)^2 + 18\ ?$$

(a) $(-\infty, \infty)$

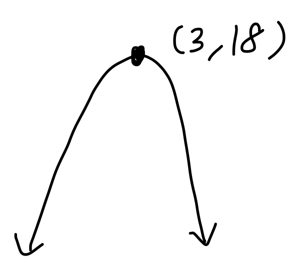
(b) $[18, \infty)$

(c) $(-\infty, 3]$

(d) $(-\infty, 18]$

(e) $(-\infty, 18)$

This is a quadratic written in vertex form.
It's vertex is $(3, 18)$, and it opens downward since the leading coefficient is negative.
So, the graph looks like




So, the range is $(-\infty, 18]$

A

**Multiple Choice 3.** Find all the exact zeros of the quadratic function:

$$f(x) = 10x^2 - 3x - 4$$

(a) $x = \frac{4}{5}, -\frac{1}{2}$

(b) $x = -\frac{4}{5}, \frac{1}{2}$

(c) $x = \frac{3}{20}$

(d) $x = -4$

(e) None of the given answer choices are correct.

To find zeroes, set $f(x) = 0$ and solve for $x$.

$$10x^2 - 3x - 4 = 0$$

(use quad. formula or factor)

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(10)(-4)}}{2(10)}$$

$$= \frac{3 \pm \sqrt{9+160}}{20} = \frac{3 \pm \sqrt{169}}{20}$$

$$= \frac{3 \pm 13}{20}$$

$$\to x = \frac{3+13}{20} = \frac{16}{20} = \frac{4}{5}$$

$$\to x = \frac{3-13}{20} = \frac{-10}{20} = \frac{-1}{2}$$

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D

**Multiple Choice 4.** Let $h$ be the function $h(x) = -5x^8 + 85x^7 + 45x^6 - 6345x^5 + 15120x^4 + 121500x^3 - 349920x^2$ which factors into:

$$h(x) = -5(x-0)^2(x+6)^2(x-3)(x-9)^2(x-6).$$

Which of the following statements is **FALSE**?

(a) The domain of the function is $(-\infty, \infty)$. True, since $h$ is a polynomial.

(b) $h(9) = 0$ True: $h(9) = -5(9-0)^2(9+6)^2(9-3)(9-9)^2(9-6) = 0$

(c) The function $h(x)$ is a polynomial. True.

(d) $h(x) \to \infty$ as $x \to -\infty$. False: degree: $n = 8$ (even) leading coeff: $a_n = -5$ (neg.)

end behavior: $h(x) \to -\infty$ as $x \to \infty$

(e) The function $h(x)$ has zeros at $x = -6, 0, 3, 6, 9$. True, since $h(x) = 0 \Rightarrow$

$$-5(x-0)^2(x+6)^2(x-3)(x-9)^2(x-6) = 0$$

(by the zero factor property)

$$x - 0 = 0, \quad x + 6 = 0, \quad x - 3 = 0, \quad x - 9 = 0, \quad x - 6 = 0$$

$$\Rightarrow \quad x = 0, \quad x = -6, \quad x = 3, \quad x = 9, \quad x = 6$$

C

**Multiple Choice 5.** Elena's Biking Company manufactures and sells bikes. Each bike costs \$40 to make and the company's fixed costs are \$5000. Elena knows that the company's cost is given by a linear function, and that the unit price (in dollars) of each bike is a linear function of the number of bikes sold. Based on her sales data, when the unit price of a bike is \$280, she knows that 10 bikes will be sold, and if the unit price drops by \$60, then 40 bikes will be sold. Which of the following statements is **FALSE**? (All answers are rounded to the *nearest* whole number).

(a) Elena's cost equation is given by $C(x) = 40x + 5000$, $x$ denotes the number of bikes sold.

(b) The graph of the profit equation is a parabola that opens downward.

(c) $(x, p) = (40, 60)$ is a point on the graph of the linear demand function where the unit price $p$ is a function of number of bikes sold, $x$

(d) Elena will maximize her profit if 65 bikes are sold

(e) To break even, Elena should sell approximately 23 bikes or 107 bikes.

Cost eq: fixed costs are $b = 5000$, variable cost is $m = 40$ and linear cost equation is given by $C(x) = mx + b = 40x + 5000$.

price eq: know: $(x_1, P_1) = (10, 280)$ and $(x_2, P_2) = (40, 280-60) = (40, 220)$ are 2 points on the unit price graph. since its known to be linear:

$$m = \frac{P_2 - P_1}{x_2 - x_1} = \frac{220 - 280}{40 - 10} = \frac{-60}{30} = -2 \Rightarrow p - p_1 = m(x - x_1)$$

$$p - 280 = -2(x - 10)$$

$$p(x) = -2x + 300$$

Revenue eq: $R(x) = p \cdot x = (-2x + 300) \cdot x = -2x^2 + 300x = R(x)$

Profit eq: $P(x) = R(x) - (C(x)) = (-2x^2 + 300x) - (40x + 5000)$

$$P(x) = -2x^2 + 260x - 5000.$$

Now,

(a) is True since $C(x) = 40x + 5000$.

(b) is True since $P(x) = -2x^2 + 260x - 5000$ is a quadratic with negative leading coefficient $(-2)$.

(c) is False since when $x = 40$, $p(40) = -2(40) + 300 = -80 + 300 = 220$, so actually $(40, 220)$ will be a point on the unit price graph.

(d) is true since for $P(x)$, $x = \frac{-b}{2a} = \frac{-(260)}{2(-2)} = 65$ is the x-coord. of the vertex.

(e) is true since $P(x) = 0 \Rightarrow -2x^2 + 260x - 5000 = 0$

$$\Rightarrow x = \frac{-260 \pm \sqrt{(260)^2 - 4(-2)(-5000)}}{2(-2)} \Rightarrow \begin{array}{l} x \approx 23.466\ldots \\ x \approx 106.533\ldots \end{array}$$