Work-out Problems

Study-tip: show all your work!

Exercise 1. Three students kept track of the games they won and lost in a chess competition. They showed their results in a chart.

Ed	✓	✗	✓	✓	✗	✓	✓
Jo	✓	✓	✓	✓	✗	✓	✓
Lew	✗	✓	✗	✗	✓	✓	✗

← Ed has 5 wins, 2 losses
← Jo has 6 wins, 1 losses
← Lew has 3 wins, 4 losses

✓= Win ✗= Loss

1. Write a 2×3 matrix A to show the data, where each row represent the number of wins or losses and each column represents a student.

(Use the table to determine # wins and # losses for each player and fill in appropriate entry in A.)

$$A = \begin{array}{c} \\ \text{wins} \\ \text{losses} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Ed} & \text{Jo} & \text{Lew} \end{array} \\ \begin{bmatrix} \square & 6 & \square \\ 2 & \square & 4 \end{bmatrix} \end{array}$$

$$A = \begin{array}{c} \\ \text{wins} \\ \text{losses} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Ed} & \text{Jo} & \text{Lew} \end{array} \\ \begin{bmatrix} \boxed{5} & 6 & \boxed{3} \\ 2 & \boxed{1} & 4 \end{bmatrix} \end{array}$$

2. What is the entry a_{13}? In words, what does a_{13} represent?

a_{13} represents the entry of matrix A that is in row 1 and column 3. In this case, $\boxed{a_{13} = 3}$, and a_{13} represents the number of wins (row 1 represents # wins) that Lew had (column 3 represents Lew).

other terms for dimensions are shape or size

Exercise 2. Given the following matrices, determine the following. If they do not exist, explain why not.

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 7 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 \\ \frac{5}{3} & 9 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 8 & -1 \\ 5 & 0 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} \frac{7}{5} & 6 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \\ -1 \\ 5 \\ 1 \end{bmatrix}.$$

1. the dimensions of matrices A, B, C, D, and E

(the dimensions of a matrix is #rows x #cols)

A is 2×3. B is 2×2. C is 2×3. D is 1×2. E is 4×1.

2. Which of the above matrices is a square matrix?

$\boxed{B}$ is the only square matrix above.

(A square matrix has #rows = #cols)

3. $A+C$

first notice that this operation is defined because the two matrices have the same shape (both 2×3). The result is another 2×3 matrix

$$= \overbrace{\begin{bmatrix} -1 & 0 & 2 \\ 3 & 7 & -2 \end{bmatrix}}^{A} + \overbrace{\begin{bmatrix} 4 & 8 & -1 \\ 5 & 0 & -2 \end{bmatrix}}^{C}$$

(add matrices by adding corresponding entries)

$$= \begin{bmatrix} -1+4 & 0+8 & 2+(-1) \\ 3+5 & 7+0 & -2+(-2) \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 8 & 1 \\ 8 & 7 & -4 \end{bmatrix} = A+C}$$

4. ~~$I_2A - 2C$~~ $A - 2C$

Again, this operation is defined: multiplying a matrix by a # doesn't change its size, so again we are working with 2×3 matrices (same shape), so can subtract, and will get a new 2×3 matrix.

$$= \overbrace{\begin{bmatrix} -1 & 0 & 2 \\ 3 & 7 & -2 \end{bmatrix}}^{A} - 2\overbrace{\begin{bmatrix} 4 & 8 & -1 \\ 5 & 0 & -2 \end{bmatrix}}^{C}$$

$$= \begin{bmatrix} -1 & 0 & 2 \\ 3 & 7 & -2 \end{bmatrix} + \begin{bmatrix} -8 & -16 & 2 \\ -10 & 0 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} -9 & -16 & 4 \\ -7 & 7 & 2 \end{bmatrix}}.$$

(trick: multiply C by −2 and then just have to add!)

5. ~~CI_3A^T~~ CA^T

First, check if can multiply.

C is $\boxed{2}$×3 • A^T is 3×$\boxed{2}$ (need to transpose before mult.)

since the inner dimensions match, can multiply and will get a $\boxed{2}$×$\boxed{2}$ matrix as a result.

$$= \begin{bmatrix} 4 & 8 & -1 \\ 5 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ 0 & 7 \\ 2 & -2 \end{bmatrix}$$

(Need to transpose A first, then multiply: think of transpose like exponentiation; it comes before multiplication/addition)

$$= \begin{bmatrix} 4\cdot(-1) + 8\cdot 0 + (-1)\cdot 2 & 4\cdot 3 + 8\cdot 7 + (-1)\cdot(-2) \\ 5\cdot(-1) + 0\cdot 0 + (-2)\cdot 2 & 5\cdot 3 + 0\cdot 7 + (-2)\cdot(-2) \end{bmatrix}$$

$$CA^T = \boxed{\begin{bmatrix} -6 & 70 \\ -9 & 19 \end{bmatrix}}$$

just multiply every entry of E by $-\frac{3}{2}$

f_{31} is the entry of F in row 3, column 1

6. Let $F = -\frac{3}{2}E$. What is f_{31}?

$$F = -\frac{3}{2} \cdot \begin{bmatrix} 3 \\ -1 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} \\ \frac{3}{2} \\ -\frac{15}{2} \\ -\frac{3}{2} \end{bmatrix}.$$

so $\boxed{f_{31} = -\frac{15}{2}}$.

7. $12D - 3B$

D is 1×2, so 12 D is also 1×2

B is 2×2, so $-3B$ is also 2×2

This operation is $\boxed{\text{not defined}}$ since the two matrices do not have the same shape, so they cannot be added / subtracted.

8. $5DC$

$\boxed{1}\times (2)$ $(2)\times \triangle{3}$

(inner dimensions match, so can multiply. Result is a $\boxed{1}\times 3$ matrix.)

(can multiply D by 5 first or multiply D and C and then the result by 5)

$$= 5 \cdot \begin{bmatrix} \frac{7}{5} & 6 \end{bmatrix} \cdot \begin{bmatrix} 4 & 8 & -1 \\ 5 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 30 \end{bmatrix} \begin{bmatrix} 4 & 8 & -1 \\ 5 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 7\cdot 4 + 30\cdot 5 & 7\cdot 8 + 30\cdot 0 & 7\cdot(-1) + (30)\cdot(-2) \end{bmatrix}$$

$$\boxed{5DC = \begin{bmatrix} 178 & 56 & -39 \end{bmatrix}}$$

9. Let $G = CA^T$ ~~$- 6I_2$~~. What is g_{32}?

g_{32} is the entry of G in row 3, column 2.

We calculated $G = CA^T$ in part 5 and got that it was a 2×2 matrix. So, $\boxed{g_{32} \text{ is not defined}}$, since G doesn't even have a row 3!

Exercise 3. The Campus Bookstore's inventory of books consists of the following quantities of hardcover and paperback textbooks:
Hardcover: textbooks–5280; fiction–1680; nonfiction–2320; reference–1890.
Paperback: textbooks–1940; fiction–2810; nonfiction–1490; reference–2070.

The College Bookstore's inventory of books consists of the following quantities of hardcover and paperback textbooks:
Hardcover: textbooks–6340; fiction–2220; nonfiction–1790; reference–1980.
Paperback: textbooks–2050; fiction–3100; nonfiction–1720; reference–2710.

1. Represent the inventory of the Campus bookstore as a matrix. Call it A.

Campus:
$$A = \begin{array}{c} \\ \text{hardcover} \\ \text{paperback} \end{array} \begin{array}{c} \begin{array}{cccc} tb & f & nf & ref \end{array} \\ \begin{bmatrix} 5280 & 1680 & 2320 & 1890 \\ 1940 & 2810 & 1490 & 2070 \end{bmatrix} \end{array}$$

2. Represent the inventory of the College Bookstore as a matrix. Call it B.

College:
$$B = \begin{array}{c} \\ \text{hardcover} \\ \text{paperback} \end{array} \begin{array}{c} \begin{array}{cccc} tb & f & nf & ref \end{array} \\ \begin{bmatrix} 6340 & 2220 & 1790 & 1980 \\ 2050 & 3100 & 1720 & 2710 \end{bmatrix} \end{array}$$

3. Use matrix operations to determine a matrix that represents the inventory of a new company formed by the merger of the Campus Bookstore and the College Bookstore.

Merger:
$$A + B = \begin{array}{c} \\ \text{hard cover} \\ \text{paperback} \end{array} \begin{array}{c} \begin{array}{cccc} tb & f & nf & ref \end{array} \\ \begin{bmatrix} 11620 & 3900 & 4110 & 3870 \\ 3990 & 5910 & 3210 & 4780 \end{bmatrix} \end{array}$$

can add since same shape and same row/column labels

can use calculator to add quickly!

Exercise 4. The Lucrative Bank has three branches in College Station: Northgate (N), Memorial Student Center (MSC), and South College Station (SCS). matrix A shows the number of accounts of each type – checking (c), savings (s), and market (m) – at each branch office on January 1, 2019.

$$A = \begin{array}{c} \\ N \\ MSC \\ SCS \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 40039 & 10135 & 512 \\ 15231 & 8751 & 105 \\ 25612 & 12187 & 97 \end{bmatrix} \end{array}$$

Matrix B shows the number of accounts of each type at each branch that were opened during the first quarter of 2019, and matrix C shows the number of accounts closed during the first quarter.

$$B = \begin{array}{c} \\ N \\ MSC \\ SCS \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 5209 & 2506 & 48 \\ 1224 & 405 & 17 \\ 2055 & 771 & 21 \end{bmatrix} \end{array} \qquad C = \begin{array}{c} \\ N \\ MSC \\ SCS \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 2780 & 1100 & 32 \\ 565 & 189 & 25 \\ 824 & 235 & 14 \end{bmatrix} \end{array}$$

1. Calculate the matrix representing the number of accounts of each type at each location at the end of the first quarter. Call it D

$$D = A + B - C = \boxed{\begin{array}{c} \\ N \\ MSC \\ SCS \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 42468 & 11541 & 528 \\ 15890 & 8967 & 97 \\ 26843 & 12723 & 104 \end{bmatrix} \end{array}}$$

accounts @ beginning of first quarter

new accounts opened in first quarter

loss of

accounts closed in first quarter

(use calculator)

Note: this operation is defined since all matrices have same shape and same row/column labels.

2. The sudden closing of a large textile plant has led bank analysts to estimate that all accounts will decline by 7% during the second quarter. Calculate a matrix that represents the anticipated number of each type at each branch at the end of the second quarter. Assume that no new accounts will be open or closed during the second quarter and round fractions of accounts to the nearest whole number. Call the matrix E.

$$E = D - 0.07D = 0.93D = \begin{bmatrix} 39495.24 & 10733.13 & 491.04 \\ 14777.7 & 8339.31 & 90.21 \\ 24963.99 & 11832.39 & 96.72 \end{bmatrix}$$

@ end of first quarter have this many accounts

by end of second quarter will lose 7% of accounts

(use calculator)

round to nearest whole #

$$= \boxed{\begin{array}{c} \\ N \\ MSC \\ SCS \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 39495 & 10733 & 491 \\ 14778 & 8339 & 90 \\ 24964 & 11832 & 97 \end{bmatrix} \end{array}}$$

Operation is defined, since matrices have same size and same row/column labels (a $\boxed{3}\times 3$ times 3×2)

(operation is defined and result is 3×2.)

Exercise 5. Find the product of the two matrices

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & y+4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ x & 2 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-2)\cdot 1 + 1\cdot x + 2\cdot 3 & (-2)\cdot 3 + 1\cdot 2 + 2\cdot(-1) \\ 3\cdot 1 + 2\cdot x + 4\cdot 3 & 3\cdot 3 + 2\cdot 2 + 4\cdot(-1) \\ 0\cdot 1 + (-2)\cdot x + (y+4)\cdot(3) & 0\cdot 3 + (-2)\cdot 2 + (y+4)\cdot(-1) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} x+4 & -6 \\ 2x+15 & 9 \\ -2x+3y+12 & -y-8 \end{bmatrix}}$$

Exercise 6. Find matrices A, X, and B so that the given system of equations can be written as $AX = B$.

$$\begin{cases} -3x_1 + 7x_2 + 2x_3 = 0 \\ -7x_2 + 5x_3 - 2 = 0 \\ -7x_3 + 3x_2 + 4x_1 = 4 \end{cases}$$

pull all variables to left and write "in order"; push all constants to right hand side.

$$\begin{cases} -3x_1 + 7x_2 + 2x_3 = 0 \\ 0x_1 - 7x_2 + 5x_3 = 2 \\ 4x_1 + 3x_2 - 7x_3 = 4 \end{cases}$$

this looks like the result of a matrix multiplication!

$$\underbrace{\begin{bmatrix} -3 & 7 & 2 \\ 0 & -7 & 5 \\ 4 & 3 & -7 \end{bmatrix}}_{\substack{A \\ (3\times 3)}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\substack{X \\ (3\times 1)}} = \underbrace{\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}}_{\substack{B \\ (3\times 1)}}$$

Check for yourself: when you multiply out $A\cdot X$, you do in fact get the matrix B!

Exercise 7. The weighted average for a Math 101 class is calculated by weighing each of the categories by a certain percentage of the final grade: Homework, Test 1, and Test 2 each count 15% toward the final grade, Test 3 counts 25%, and the Final counts 30%. The category averages of three students, Student I, Student II, and Student III are given in the matrix below. Use matrix to calculate each student's weighted average.

Want a 3×1 matrix:

$$\text{students}\ \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} \overset{\text{grade (weighted average)}}{\begin{bmatrix} \ \\ \ \\ \ \end{bmatrix}}$$

$$\begin{array}{l} \\ \text{Homework} \\ \text{Test 1} \\ \text{Test 2} \\ \text{Test 3} \\ \text{Final} \end{array} \begin{array}{c} \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\ \begin{bmatrix} 82 & 92 & 74 \\ 85 & 88 & 68 \\ 78 & 95 & 73 \\ 75 & 85 & 82 \\ 84 & 94 & 81 \end{bmatrix} \end{array} \overset{\text{call this}}{=} M$$

We are given info about two matrices, M from above and another matrix

$$N = \begin{array}{l} \text{Homework} \\ \text{Test 1} \\ \text{Test 2} \\ \text{Test 3} \\ \text{Final} \end{array} \overset{\%\text{ of grade}}{\begin{bmatrix} 0.15 \\ 0.15 \\ 0.15 \\ 0.25 \\ 0.30 \end{bmatrix}}.$$

The matrix we want

$$\text{students}\ \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} \overset{\text{grade}}{\begin{bmatrix} \ \\ \ \\ \ \end{bmatrix}} = \underbrace{\begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} \overset{\begin{array}{ccccc} \text{HW} & \text{Test1} & \text{Test2} & \text{Test3} & \text{Final} \end{array}}{\begin{bmatrix} 82 & 85 & 78 & 75 & 84 \\ 92 & 88 & 95 & 85 & 94 \\ 74 & 68 & 73 & 82 & 81 \end{bmatrix}}}_{\substack{M^T \\ 3\times 5 \\ (\text{students})\times(\text{categories})}} \cdot \underbrace{\begin{array}{c} \text{HW} \\ \text{T1} \\ \text{T2} \\ \text{T3} \\ \text{Final} \end{array} \overset{\text{grade}}{\begin{bmatrix} 0.15 \\ 0.15 \\ 0.15 \\ 0.25 \\ 0.30 \end{bmatrix}}}_{\substack{N \\ 5\times 1 \\ (\text{categories})\times(\text{grade})}}$$

= ✓ inner dimensions and labels match

$$= M^T \cdot N$$

$$= \text{students}\ \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} \overset{\text{grade}}{\begin{bmatrix} 80.7 \\ 90.7 \\ 77.05 \end{bmatrix}}.$$

So,
Student I averaged 80.7 %
Student II averaged 90.7 %
and Student III averaged 77.05 %.

Exercise 8. The Metropolitan Opera is planning its last cross-country tour. It plans to perform *Carmen* and *La Traviata* in Atlanta in May. The person in charge of logistics wants to make plane reservations for the two troupes. *Carmen* has 2 stars, 25 other adults, 5 children, and 5 staff members. *La Traviata* has 3 stars, 15 other adults, and 4 staff members. There are 3 airlines to choose from. Piedmont charges round-trip fares to Atlanta of \$630 for first class, \$420 for coach, and \$250 for youth. Eastern charges \$650 for first class, \$350 for coach, and \$275 for youth. Air Atlanta charges \$700 for first class, \$370 for coach, and \$150 for youth. If stars travel first class, other adults and staff travel coach, and children travel for the youth fare, which is the most cost effective airline for each of the opera troupes?

Want:

$$\begin{array}{cc} & \text{opera troupes} \\ \text{Airline cost} & \begin{bmatrix} \quad & \quad \\ & \\ & \end{bmatrix} \end{array} \quad \text{a } 3\times 2 \text{ matrix.}$$

$$\begin{array}{cc} & \text{Opera Troupe} \\ \text{Airline cost} & \begin{bmatrix} \quad & \quad \\ & \\ & \end{bmatrix} \end{array} = \text{airline} \begin{array}{cc} & \begin{array}{ccc} \text{first class} & \text{coach} & \text{youth} \end{array} \\ \begin{array}{c} \text{Piedmont} \\ \text{Atlanta} \\ \text{Eastern} \end{array} & \begin{bmatrix} 630 & 420 & 250 \\ 650 & 350 & 275 \\ 700 & 370 & 150 \end{bmatrix} \end{array} \begin{array}{cc} & \text{opera troupe} \\ & \begin{array}{cc} \text{Carmen} & \text{La Traviata} \end{array} \\ \begin{array}{c} \text{first class} \\ \text{coach} \\ \text{youth} \end{array} & \begin{bmatrix} 2 & 3 \\ 25+5 & 15+4 \\ 5 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{ccc} 3 \times 3 & & 3 \times 2 \\ \text{airline} \times \text{fare type} & & \text{fare type} \times \text{opera troupe} \end{array}$$

$= \checkmark$

$$= \begin{bmatrix} 630 & 420 & 250 \\ 650 & 350 & 275 \\ 700 & 370 & 150 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 30 & 19 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{array}{cc} & \begin{array}{cc} \text{Carmen} & \text{La Traviata} \end{array} \\ \begin{array}{c} \text{Piedmont} \\ \text{Eastern} \\ \text{Air Atlanta} \end{array} & \begin{bmatrix} 15110 & 9870 \\ 13175 & 8600 \\ 13250 & 9130 \end{bmatrix} \end{array}$$

$\boxed{\text{Eastern}}$ is the most cost-effective airline for both Carmen (\$ 13175) and La Traviata (\$ 8600).

Exercise 9. Find $a + 2b - c + d - 5x$ using the matrix equation below:

Plan: can only compare matrices when there is exactly one matrix on each side of the eqn, so do the operations until have "matrix 1 = matrix 2"

$$\begin{bmatrix} a & 5b-1 \\ c & d \end{bmatrix} - 5\begin{bmatrix} -2 & 1 \\ 4 & x \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 6 & 4 \\ 0 & 7 \end{bmatrix}.$$

transpose first!

$$\begin{bmatrix} a & 5b-1 \\ c & d \end{bmatrix} - 5\begin{bmatrix} -2 & 4 \\ 1 & x \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 0 & 7 \end{bmatrix}$$

multiply by -5

$$\begin{bmatrix} a & 5b-1 \\ c & d \end{bmatrix} + \begin{bmatrix} 10 & -20 \\ -5 & -5x \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 0 & 7 \end{bmatrix}$$

add the matrices on left

$$\begin{bmatrix} a+10 & 5b-1+(-20) \\ c+(-5) & d+(-5x) \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 0 & 7 \end{bmatrix}$$

simplify

$$\begin{bmatrix} a+10 & 5b-21 \\ c-5 & d-5x \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 0 & 7 \end{bmatrix}$$

Now, two matrices are equal if the corresponding entries in matching spots are equal. So here if

$$\begin{cases} a+10 = 6 \xrightarrow{\text{solve for } a} a = -4 \\ 5b-21 = 4 \xrightarrow{\text{solve for } b} b = 5 \\ c-5 = 0 \xrightarrow{\text{solve for } c} c = 5 \\ d-5x = 7 \xrightarrow{\text{solve for } d-5x} d-5x = 7 \end{cases}$$

use the above equations to get a, b, c, $d-5x$

Goal: find $a + 2b - c + d - 5x$

$$a+2b-c+(d-5x) = (-4)+2(5)-(5)+(7)$$

plug all the known values in

$$= -4 + 10 - 5 + 7$$

$$= \boxed{8}$$

Exercise 10. Matrix L is a 5×4 matrix, matrix M is a 4×4 matrix, matrix N is a 5×5 matrix, and matrix P is a 4×5 matrix. Find the dimensions of the following matrices, or specify why they do not exist.

1. $M + N$

4x4 5x5

NO, because they don't have the same size.

2. $\frac{1}{3}L + P^T$

5x4 5x4 (because P is 4x5)

yes, because they are the same size and the result is a 5x4 matrix.

3. ~~$L + MI_4$~~ L+M

5x4 4x4

NO, because they are not the same size.

4. M^3 = (M · M) · M

$\boxed{4}$x4 4x$\boxed{4}$

defined and is a 4x4 matrix

4x4

Yes, because inner dimensions match. The result is a 4x4 matrix.

5. MN

4x4 5x5

not the same

NO, because inner dimensions don't match.

6. NP^T

5x5 5x4 (since P is 4x5)

Yes, because the inner dimensions, and the result is 5x4.

7. $PL + M$

4x5 5x4 4x4

inner dimensions match and result is 4x4

Yes, can add since PL and M have the same dimensions (4x4), and the result is a 4x4 matrix.

Multiple Choice Problems

Study tip: Write out all your work when you complete the multiple-choice problems.

C

Multiple Choice 1. Given the 3×4 matrix E, what are the dimensions of matrix F for which $4E + F$ is defined?

(a) 4×5

(b) 4×3

(c) 3×4

(d) 4×4

(e) None of these.

$4E + F$

3×4

need F to be the same size as E in order to add them, so F has to be $\underline{3 \times 4}$.

E

Multiple Choice 2. Let $A = \begin{bmatrix} -5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Find $2A - 3B$.

(a) $\begin{bmatrix} -10 & -4 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} -9 & -4 \end{bmatrix}$

(d) $\begin{bmatrix} -7 & 4 \end{bmatrix}$

(e) None of these.

$$2A - 3B$$

$$= 2\begin{bmatrix} -5 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -13 & 4 \end{bmatrix}$$

E

Multiple Choice 3. Which of the following is the correct matrix equation used to solve the system of linear equations using inverse matrices?

$$\begin{cases} 3x - 4y + 2z = 12 \\ 2y + 4 = x + z \\ 4x + 2z = 3y + 15 \end{cases} \xrightarrow{\text{clean-up equations first!}} \begin{cases} 3x - 4y + 2z = 12 \\ -x + 2y - z = -4 \\ 4x - 3y + 2z = 15 \end{cases}$$

(a) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ 15 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(e) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & -3 & 2 \end{bmatrix}}_{\text{coefficient matrix}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{variable matrix}} = \underbrace{\begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix}}_{\text{constant matrix}}$$

E

Multiple Choice 4. ***(Matrix arithmetic)***
Given the matrix equation below, find the correct value of $a + b$.

$$\begin{bmatrix} 2 & 0 \\ 3b & -2 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & a \end{bmatrix}^T = 2\begin{bmatrix} 0 & -\frac{3}{2} \\ 4 & 6 \end{bmatrix}$$

(a) 4

(b) 0

(c) −6

(d) 11

(e) −10

- do all operations to reduce left-hand side down to a single matrix.
- do all operation on right-hand side to reduce to a single matrix
- compare the two matrices entry-by-entry.

$$\begin{bmatrix} 2 & 0 \\ 3b & -2 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & a \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 2-2 & 0-3 \\ 3b-4 & -2-a \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 \\ 3b-4 & -2-a \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 12 \end{bmatrix} \Rightarrow$$

$3b - 4 = 8$
$\hookrightarrow b = 4$
$-2 - a = 12$
$\hookrightarrow a = -14$

Since $a = -14$, $b = 4$, we get that $a + b = -14 + 4 = \boxed{-10}$

A

Multiple Choice 5. Each day you feed your dog a mixture of three kinds of food. Matrix M shows the amount of vitamins A, B, and C (per gram) for each type of food (Kibble, Bits, Chunks). Matrix N shows the number of grams of each food consumed by the dog.

$$M = \begin{array}{r} \\ \text{Vitamin A} \\ \text{Vitamin B} \\ \text{Vitamin C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{kibble} & \text{bits} & \text{chunks} \end{array} \\ \left[\begin{array}{ccc} 3 & 2 & 4 \\ 2 & 4 & 5 \\ 2 & 5 & 1 \end{array}\right] \end{array}, \quad N = \begin{array}{r} \\ \text{kibble} \\ \text{bits} \\ \text{chunks} \end{array} \begin{array}{c} \text{grams} \\ \left[\begin{array}{c} 27 \\ 55 \\ 68 \end{array}\right] \end{array}$$

Which of the following gives the correct interpretation for the product MN?

(a) The number of grams of each vitamin consumed by the dog.

(b) The number of grams of each vitamin for each food. ← that's matrix M

(c) The number of grams of each food that your dog eats. ← that's matrix N

(d) MN is not defined.

(e) The product is meaningless.

M $\qquad\qquad\qquad\qquad$ N

$3 \times 3 \qquad\qquad 3 \times 1$

vitamins × foods $\qquad$ foods × # grams consumed

=

⇒ MN <u>is</u> defined since inner dimensions <u>and</u> inner labels are equal.

The result is a 3×1 matrix

vitamins × # grams consumed

So, the result is a matrix representing the # of grams of each vitamin consumed by the dog. (A)