

When applying the simplex method, we will use the online website to complete the elementary row operations : <https://www.zweigmedia.com/RealWorld/tutorialsf1/scriptpivot2.html>.

## Work-out Problems

*Study tip : Show all your work !*

**Exercise 1.** Determine if the specified linear programming problem is a standard maximization problem. If it is, set up the initial simplex tableau.

- Objective : Maximize  $P = 120x + 40y + 60z$   
 subject to  $x + y + z \leq 100$   
 $10x + 4y + 7z \leq 500$   
 $x + y + z \geq 60$   
 $x \geq 0, y \geq 0, z \geq 0$
- Objective : Minimize  $P = 40x + 10y$   
 subject to  $x + 3y \geq 40$   
 $14x + 4y \leq 15$   
 $x \geq 0, y \geq 0$
- Objective : Maximize  $P = a + 2b + c + 7d$   
 subject to  $24 \geq a + 2b + 3c$   
 $-3a - 6b - c \geq -42$   
 $a \geq 0, b \geq 0, c \geq 0, d \geq 0$

**Exercise 2.** Determine whether the given simplex tableau is in final form. If so, find the optimal solution to the associated linear programming problem. If not, find the pivot element to be used in the next iteration of the simplex method.

$w$	$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	$R$
1	1/2	0	1/4	0	0	0	100
0	1	-2	-1/2	1	0	0	52
0	3/2	3	-1/4	0	1	0	100
0	1/2	2	-1/4	0	0	1	800
0	-1/2	-1	1/4	0	0	1	100

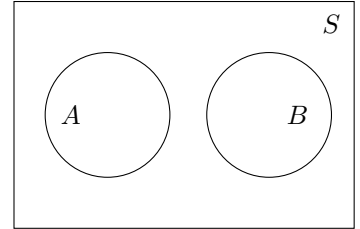
**Exercise 3.** A furniture manufacturer produces chairs, sofas, and love seats. The chairs require 5 square feet of wood, 1 pound of foam rubber, and 10 square yards of material. The sofas require 35 square feet of wood, 2 pounds of foam rubber, and 20 square yards of material. The love seats require 9 square feet of wood, 0.2 pounds of foam rubber, and 10 square yards of material. The manufacturer has in stock 405 square feet of wood, 25 pounds of foam rubber, and 410 square yards material. If the chairs yield a profit of \$300, the sofas \$200, and the love seats \$220 each, how many of each should be produced to maximize the profit? Find the maximum profit.

- Solve the linear programming problem.

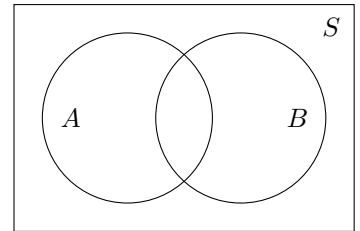
- Give an economic interpretation of the slack variables associated with the optimal solution found in Part 1, and determine which resources are in excess (if any).

**Exercise 4.** Shade each of the following in the given diagram.

1.  $A^C \cup B^C$



2.  $(A \cup B^C)^C$



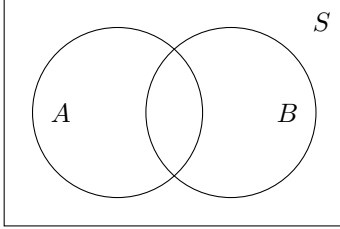
**Exercise 5.** Consider the experiment of randomly choosing one of the 26 lower-case letters from the English alphabet. Let  $S$  denote the sample space, let  $E = \{a, e, i, o, u\}$  be the event that a vowel is chosen, let  $F$  be the event that one of the remaining 21 letters is chosen, and let  $G$  be the event that one of the first 5 letters of the alphabet is chosen. How many simple events are there in this experiment? Find the events  $E \cup F \cup G$ ,  $E^C \cup F^C \cup G^C$ ,  $E \cap F \cap G$ , and  $E \cup F^C \cup G$ .

**Exercise 6.** Consider the following experiment : First, a card is drawn from an standard 52-card deck and the suit is recorded. Next, a fair 3-sided die is rolled and the number showing uppermost is recorded.

- Write the sample space for this experiment. (Use a tree diagram to help.)
- State the total number of possible events of the sample space.
- Write the outcomes in the event "A number greater than 3 is rolled."
- Write the outcomes in the event "A red card is drawn."
- Write the outcomes in the event "A 2 is rolled **and** a clubs is drawn."
- Write the outcomes in the event "A 2 is rolled **or** a clubs card is drawn."
- Which of the events from Parts 3 – 6 are simple events, if any?

**Exercise 7.** A survey of supermarket shoppers is taken to analyze which fruits they buy regularly, in order to determine which ones should be put on sale. The supermarket only sells two types of fruits : apples and bananas. In the survey, some shoppers regularly purchase bananas, and some purchase apples. Some people

purchase neither, and some people regularly purchase both apples and bananas. Let  $A$  denote the event that a shopper regularly purchases apples and let  $B$  denote the event that a shopper regularly purchases bananas. Shade the region that corresponds to the event that “A shopper regularly purchases apples or no fruit at all” in the Venn diagram below.



### Multiple Choice Problems

*Study tip : Write out all your work when you complete the multiple-choice problems.*

**Multiple Choice 1.** Determine whether the given simplex tableau is in final form. If so, find the optimal solution to the associated linear programming problem. If not, find the pivot element to be used in the next iteration of the simplex method.

$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	$P$	
$1/4$	2	1	0	0	7	0	16
1	$1/3$	0	1	0	-1	0	6
-2	-2	0	0	1	$1/3$	0	8
-2	-5	0	0	0	-1	1	80

- Yes, the simplex tableau is in final form. The system has a maximum value of 80 at  $(x, y, z) = (16, 6, 8)$ .
- Yes, the simplex tableau is in final form. The system has a maximum value of 80 at  $(x, y, z) = (0, 0, 16)$ .
- No, the simplex tableau is not in final form. The next pivot element is the 2 in the first row, second column.
- No, the simplex tableau is not in final form. The next pivot element is the -2 in the third row, second column.
- No, the simplex tableau is not in final form. The pivot element is the 7 in the first row, sixth column.

**Multiple Choice 2.** A company makes small and large desks that require wood and finish to make. The number of units of wood and finish required for each small and large desk along with the amount of wood and finish available are given in the table below :

	small	large	available
wood	2	4	100
finish	3	5	300

If the profit from each small desk is \$4.50 and the profit from each large desk is \$6, how many desks of each size should they make to maximize their profit?

In solving the above problem, the initial simplex tableau is :

$x$	$y$	$s_1$	$s_2$	$P$	
2	4	1	0	0	100
3	5	0	1	0	300
-4.5	-6	0	0	1	0

and the final simplex tableau is :

$x$	$y$	$s_1$	$s_2$	$P$	
1	2	0.5	0	0	50
0	-1	-1.5	1	0	150
0	3	2.25	0	1	225

Which of the following statements is true?

- At the optimal solution, there is 50 units of leftover wood and 150 units of leftover finish.
- At the optimal solution, there is 150 units of leftover wood and no leftover finish.
- At the optimal solution, there is no leftover wood and no leftover finish.
- At the optimal solution, there is no leftover wood and 150 units of leftover finish.
- None of these

**Multiple Choice 3.** If  $S = \{1, 2, 3, a, b, c\}$  is a uniform sample space with events  $E = \{1, a, c\}$  and  $F = \{3, a, b\}$ , which of the following is FALSE? (Note : There is only one false statement.)

- $(E \cup F) = \{1, 3, a, b, c\}$
- There are exactly 2 outcomes in  $(E \cap F^c)$ .
- $E \cap F$  is a simple event.
- The two events  $F$  and  $(E \cap F^c)$  are mutually exclusive.
- $E \cup F^c = \{1, c\}$

**Multiple Choice 4.** Find the initial simplex tableau used to find the maximum of  $P = 2x + 3y$  subject to the constraints

$$\begin{cases} 2x + 3y \leq 90 \\ 3x + y \leq 30 \\ 4x + 2y \leq 40 \\ x \geq 0, y \geq 0 \end{cases}$$

	$x$	$y$	$s_1$	$s_2$	$P$	
(a)	2	3	1	0	0	90
	3	1	0	1	0	30
	4	2	0	0	1	40
	-2	-3	0	0	0	0

(b)

$x$	$y$	$s_1$	$s_2$	$s_3$	$P$	
2	3	1	0	0	0	90
3	1	0	1	0	0	30
4	2	0	0	1	0	40
-2	-3	0	0	0	1	0

(c)

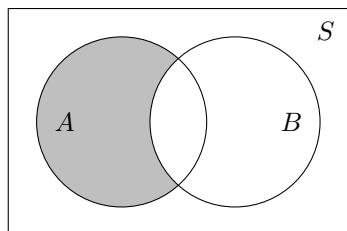
$x$	$y$	$s_1$	$s_2$	$P$	
2	3	1	0	0	90
3	1	0	1	0	30
4	2	0	0	0	40
-2	-3	0	0	1	0

(d)

$x$	$y$	$s_1$	$s_2$	$s_3$	$P$	
2	3	1	0	0	0	90
3	1	0	1	0	0	30
4	2	0	0	1	0	40
2	3	0	0	0	1	0

(e) None of these

**Multiple Choice 5.** Which of the following represents the shaded region?



- (a)  $A \cap B^C$   
 (b)  $(A \cup B)^C$   
 (c)  $(A \cap B)^C$   
 (d)  $A^C \cap B^C$   
 (e) None of these