

Work-out Problems

Study-tip : show all your work!

Exercise 1. Three students kept track of the games they won and lost in a chess competition. They showed their results in a chart.

Ed	✓	✗	✓	✓	✗	✓	✓
Jo	✓	✓	✓	✓	✗	✓	✓
Lew	✗	✓	✗	✗	✓	✓	✗

✓ = Win ✗ = Loss

- Write a 2×3 matrix A to show the data, where each row represent the number of wins or losses and each column represents a student.

$$A = \begin{matrix} & \begin{matrix} \text{Ed} & \text{Jo} & \text{Lew} \end{matrix} \\ \begin{matrix} \text{wins} \\ \text{losses} \end{matrix} & \begin{bmatrix} \square & 6 & \square \\ 2 & \square & 4 \end{bmatrix} \end{matrix}$$

- What is the entry a_{13} ? In words, what does a_{13} represent?

Exercise 2. Given the following matrices, determine the following. If they do not exist, explain why not.

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 7 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 \\ \frac{5}{3} & 9 \end{bmatrix},$$

$$C = \begin{bmatrix} 4 & 8 & -1 \\ 5 & 0 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 7 \\ 5 \end{bmatrix} \quad 6 \quad , \quad E = \begin{bmatrix} 3 \\ -1 \\ 5 \\ 1 \end{bmatrix}.$$

- the dimensions of matrices A, B, C, D , and E
- Which of the above matrices is a square matrix?
- $A + C$
- $I_2 A - 2C$
- $CI_3 A^T$
- Let $F = -\frac{3}{2}E$. What is f_{31} ?
- $12D - 3B$
- $5DC$
- Let $G = CA^T - 6I_2$. What is g_{32} ?

Exercise 3. The Campus Bookstore's inventory of books consists of the following quantities of hardcover and paperback textbooks : Hardcover : textbooks–5280; fiction–1680; nonfiction–2320; reference–1890.

Paperback : textbooks–1940; fiction–2810; nonfiction–1490; reference–2070.

The College Bookstore's inventory of books consists of the following quantities of hardcover and paperback textbooks :

Hardcover : textbooks–6340; fiction–2220; nonfiction–1790; reference–1980.

Paperback : textbooks–2050; fiction–3100; nonfiction–1720; reference–2710.

- Represent the inventory of the Campus bookstore as a matrix.
- Represent the inventory of the College Bookstore as a matrix.
- Use matrix operations to determine a matrix that represents the inventory of a new company formed by the merger of the Campus Bookstore and the College Bookstore.

Exercise 4. The Lucrative Bank has three branches in College Station : Northgate (N), Memorial Student Center (MSC), and South College Station (SCS). matrix A shows the number of accounts of each type – checking (c), savings (s), and market (m) – at each branch office on January 1, 2019.

$$A = \begin{matrix} & \begin{matrix} c & s & m \end{matrix} \\ \begin{matrix} N \\ MSC \\ SCS \end{matrix} & \begin{bmatrix} 40039 & 10135 & 512 \\ 15231 & 8751 & 105 \\ 25612 & 12187 & 97 \end{bmatrix} \end{matrix}$$

Matrix B shows the number of accounts of each type at each branch that were opened during the first quarter of 2019, and matrix C shows the number of accounts closed during the first quarter.

$$B = \begin{matrix} & \begin{matrix} c & s & m \end{matrix} \\ \begin{matrix} N \\ MSC \\ SCS \end{matrix} & \begin{bmatrix} 5209 & 2506 & 48 \\ 1224 & 405 & 17 \\ 2055 & 771 & 21 \end{bmatrix} \end{matrix}$$

$$C = \begin{matrix} & \begin{matrix} c & s & m \end{matrix} \\ \begin{matrix} N \\ MSC \\ SCS \end{matrix} & \begin{bmatrix} 2780 & 1100 & 32 \\ 565 & 189 & 25 \\ 824 & 235 & 14 \end{bmatrix} \end{matrix}$$

- Calculate the matrix representing the number of accounts of each type at each location at the end of the first quarter.
- The sudden closing of a large textile plant has led bank analysts to estimate that all accounts will decline by 7% during the second quarter. Calculate a matrix that represents the anticipated number of each type at each branch at the end of the second quarter. Assume that no new accounts will be open or closed during the second quarter and round fractions of accounts to the nearest whole number.

Exercise 5. Find the product of the two matrices

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & y+4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ x & 2 \\ 3 & -1 \end{bmatrix}$$

Exercise 6. Find matrices A , X , and B so that the given system of equations can be written as $AX = B$.

$$\begin{cases} -3x_1 + 7x_2 + 2x_3 = 0 \\ -7x_2 + 5x_3 - 2 = 0 \\ -7x_3 + 3x_2 + 4x_1 = 4 \end{cases}$$

Exercise 7. The weighted average for a Math 101 class is calculated by weighing each of the categories by a certain percentage of the final grade : Homework, Test 1, and Test 2 each count 15% toward the final grade, Test 3 counts 25%, and the Final counts 30%. The category averages of three students, Student I, Student II, and Student III are given in the matrix below. Use matrix to calculate each student's weighted average.

	I	II	III
Homework	82	92	74
Test 1	85	88	68
Test 2	78	95	73
Test 3	75	85	82
Final	84	94	81

Exercise 8. The Metropolitan Opera is planning its last cross-country tour. It plans to perform *Carmen* and *La Traviata* in Atlanta in May. The person in charge of logistics wants to make plane reservations for the two troupes. *Carmen* has 2 stars, 25 other adults, 5 children, and 5 staff members. *La Traviata* has 3 stars, 15 other adults, and 4 staff members. There are 3 airlines to choose from. Piedmont charges round-trip fares to Atlanta of \$630 for first class, \$420 for coach, and \$250 for youth. Eastern charges \$650 for first class, \$350 for coach, and \$275 for youth. Air Atlanta charges \$700 for first class, \$370 for coach, and \$150 for youth. If stars travel first class, other adults and staff travel coach, and children travel for the youth fare, which is the most cost effective airline for each of the opera troupes?

Exercise 9. Find $a + 2b - c + d - 5x$ using the matrix equation below :

$$\begin{bmatrix} a & 5b-1 \\ c & d \end{bmatrix} - 5 \begin{bmatrix} -2 & 1 \\ 4 & x \end{bmatrix}^T = \begin{bmatrix} 6 & 4 \\ 0 & 7 \end{bmatrix}.$$

Exercise 10. Matrix L is a 5×4 matrix, matrix M is a 4×4 matrix, matrix N is a 5×5 matrix, and matrix P is a 4×5 matrix. Find the dimensions of the following matrices, or specify why they do not exist.

1. $M + N$
2. $\frac{1}{3}L + P^T$
3. $L + MI_4$
4. M^3
5. MN
6. NP^T
7. $PL + M$

Multiple Choice Problems

Study tip : Write out all your work when you complete the multiple-choice problems.

Multiple Choice 1. Given the 3×4 matrix E , what are the dimensions of matrix F for which $4E + F$ is defined?

- (a) 4×5
- (b) 4×3
- (c) 3×4
- (d) 4×4
- (e) None of these.

Multiple Choice 2. Let $A = \begin{bmatrix} -5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Find $2A - 3B$.

- (a) $\begin{bmatrix} -10 & -4 \end{bmatrix}$
- (b) $\begin{bmatrix} -2 & 2 \end{bmatrix}$
- (c) $\begin{bmatrix} -9 & -4 \end{bmatrix}$
- (d) $\begin{bmatrix} -7 & 4 \end{bmatrix}$
- (e) None of these.

Multiple Choice 3. Which of the following is the correct matrix equation used to solve the system of linear equations using inverse matrices?

$$\begin{cases} 3x - 4y + 2z = 12 \\ 2y + 4 = x + z \\ 4x + 2z = 3y + 15 \end{cases}$$

- (a) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix}$
- (b) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
- (c) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ 15 \end{bmatrix}$
- (d) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
- (e) $\begin{bmatrix} 3 & -4 & 2 \\ -1 & 2 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 15 \end{bmatrix}$

Multiple Choice 4. (*Matrix arithmetic*)

Given the matrix equation below, find the correct value of $a + b$.

$$\begin{bmatrix} 2 & 0 \\ 3b & -2 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & a \end{bmatrix}^T = 2 \begin{bmatrix} 0 & -\frac{3}{2} \\ 4 & 6 \end{bmatrix}$$

- (a) 4
- (b) 0
- (c) -6
- (d) 11
- (e) -10

Multiple Choice 5. Each day you feed your dog a mixture of three kinds of food. Matrix M shows the amount of vitamins A, B, and C (per gram) for each type of food (Kibble, Bits, Chunks). Matrix N shows the number of grams of each food consumed by the dog.

$$M = \begin{array}{cc} & \begin{array}{ccc} \text{kibble} & \text{bits} & \text{chunks} \end{array} \\ \begin{array}{c} \text{Vitamin A} \\ \text{Vitamin B} \\ \text{Vitamin C} \end{array} & \begin{bmatrix} 3 & 2 & 4 \\ 2 & 4 & 5 \\ 2 & 5 & 1 \end{bmatrix} \end{array},$$

$$N = \begin{array}{cc} & \text{grams} \\ \begin{array}{c} \text{kibble} \\ \text{bits} \\ \text{chunks} \end{array} & \begin{bmatrix} 27 \\ 55 \\ 68 \end{bmatrix} \end{array}$$

Which of the following gives the correct interpretation for the product MN ?

- (a) The number of grams of each vitamin consumed by the dog.
- (b) The number of grams of each vitamin for each food.
- (c) The number of grams of each food that your dog eats.
- (d) MN is not defined.
- (e) The product is meaningless.