Exam 2 is this Friday, March 6, 2020 from 1630–1830 !

Work-out Problems

Study tip : Show all your work !

Exercise 1. Consider the following experiment : First, a card is drawn from a well-shuffled standard 52-card deck and the suit is recorded. Next, a fair 3-sided die is rolled and the number showing uppermost is recorded. Use H, D, S, C to denote the suit of the card (hearts, diamonds, spades, clubs, respectively), and use $1, 2, 3$ to record the number showing uppermost. For example, we will write $D2$ to denote the outcome that a diamonds is chosen and then a 2 is rolled.

1. Is this a uniform sample space ?
2. Find the probability of the event "A number greater than 3 is rolled."
3. Find the probability of the event "A red card is drawn."
4. Find the probability of the event "A 2 is rolled *and* a clubs is drawn."
5. Find the probability of the event "A 2 is rolled *or* a clubs card is drawn."

Exercise 2. A randomly selected sample of 646 middle school students in your town was surveyed. They were classified according to grade level and their response to the question "How do you usually get to school ?". The data collected is summarized in the table below.

	walk	bus	car
6th grade	30	120	65
7th grade	25	170	25
8th grade	40	130	41

1. Find the probability that a randomly selected middle school student usually takes the bus to school.
2. Find the probability that a randomly selected middle school student does not usually take the bus to school.
3. Find the probability that a randomly selected middle school student is in 7th grade and usually takes a car to school.
4. Find the probability that a randomly selected middle school student is at least in 7th grade or usually walks to school.
5. Find the probability that a randomly selected middle school student is not in 8th grade and does not usually walk to school.

Exercise 3. A hospital researcher is interested in the number of times the average post-op patient will ring the nurse during a 12-hour shift. For a random sample of 50 patients, the following information was obtained. Let the random variable $X =$ the number of times a patient rings the nurse during a 12-hour shift. Here, X can take on values $x = 0, 1, 2, 3, 4, 5$ (if a patient rings the nurse more than 5 times in a 12-hour shift, they are discharged from post-op care).

X	0	1	2	3	4	5
number of patients	4	8	16	14	6	2

1. Write a probability distribution table for the data, where $P(X) =$ the probability that X takes on value x.
2. Draw a histogram representing the probability distribution from Part 1.
3. What is the probability that a randomly selected post-op patient will ring the nurse 0 or 5 times during a 12-hour shift ?
4. Find the expected number of times the average post-op patient will ring the nurse during a 12-hour shift.

Exercise 4. Real estate records show that 64% of homes for sale have a garage, 21% have a swimming pool, and 17% of homes have both features.

1. Are having a pool and having a garage mutually exclusive events ? Explain.
2. Find the probability that a home for sale has either a pool or a garage.
3. Find the probability that a home for sale has neither a pool nor a garage.
4. What is the probability that a home for sale has a pool or a garage, *but not both* ?

Exercise 5. You pay \$10 to roll two fair standard five-sided dice, noting the numbers rolled on each die. If you roll a double, you win \$25. If you roll different numbers with a sum less than 5, you win \$35. Otherwise you win nothing.

1. Find the probability distribution table for your net winnings.
2. What are your expected net winnings ? Round your answer to two decimal places, and use correct units.
3. Based on your work, is this a "fair" game ?

Exercise 6. If E and F are two mutually exclusive events in a sample space S with $P(E) = 0.4$ and $P(F) = 0.3$, find each of the following probabilities.

1. $P(E \cup F)$
2. $P(F^{\mathrm{C}})$
3. $P(E \cap F)$
4. $P((E \cap F)^{\mathrm{C}})$

Exercise 7. Let $S = \{a, b, c, d, e, f\}$ with $P(\{b\}) = 0.3$, $P(\{c\}) = 0.15$, $P(\{d\}) = 0.05$, $P(\{e\}) = 0.2$, $P(\{f\}) = 0.13$. Let $E = \{a, b, c\}$ and $F = \{b, c, e, f\}$. Find each of the following probabilities.

1. $P(\{a\})$
2. $P(E)$
3. $P(F)$
4. $P(E \cap F)$

Exercise 8. A company is trying to decide how much to charge for a year-long air conditioner repair service agreement. The average cost for repairing an air conditioner is \$350 and 1 in every 100 people who purchase agreements have air conditioners that require repair in the year. What is the minimum the company should charge for this repair service agreement ?

Multiple Choice Problems

Study tip : Write out all your work when you complete the multiple-choice problems.

Multiple Choice 1. Let E and F be two events of an experiment with sample space S. Suppose that $P(E) = 0.4$, $P(F) = 0.3$, and $P(E \cup F) = 0.6$. Find $P(E^C \cap F)$.

(a) $P(E^C \cap F) = 0.1$
(b) $P(E^C \cap F) = 0.2$
(c) $P(E^C \cap F) = 0.3$
(d) $P(E^C \cap F) = 0.4$
(e) $P(E^C \cap F) = 0.5$

Multiple Choice 2. Determine whether the given simplex tableau is in final form. If so, find the optimal solution to the associated linear programming problem. If not, find the pivot element to be used in the next iteration of the simplex method.

x	y	z	s_1	s_2	s_3	P	
1/4	2	1	0	0	7	0	16
1	1/3	0	1	0	−1	0	6
−2	−2	0	0	1	1/3	0	8
−2	−5	0	0	0	−1	1	80

(a) Yes, the simplex tableau is in final form. The system has a maximum value of 80 at (16, 6, 8).
(b) Yes, the simplex tableau is in final form. The system has a maximum value of 80 at (0, 0, 16).
(c) No, the simplex tableau is not in final form. The next pivot element is the 2 in the first row, second column.
(d) No, the simplex tableau is not in final form. The next pivot element is the −2 in the third row, second column.
(e) No, the simplex tableau is not in final form. The pivot element is the 7 in the first row, sixth column.

Multiple Choice 3. The sandwich shop "That Wich!" tracks the number of veggie wraps sold each day in the last month (30 days). During the past month, they sold from zero to 8 wraps per day, with the frequencies indicated in the following table.

number of wraps sold per day	0	1	2	3	4	5	6	7	8
number of days	2	0	6	7	9	1	0	3	2

What is the probability that "That Wich!" sold at least 5 veggie wraps and less than 8 veggie wraps on a randomly selected day?

(a) 3/30
(b) 4/30
(c) 5/30
(d) 6/30
(e) 26/30

Multiple Choice 4. Solve the linear programming problem.

Minimize $P = 4x + 5y$

$$\text{subject to } \begin{cases} 5x + 3y \geq 19 \\ x + 2y \leq 8 \\ x \geq 0,\ y \geq 0\ . \end{cases}$$

(a) $P = 10$ at $(x, y) = (0, 2)$.
(b) $P = 20$ at $(x, y) = (0, 4)$.
(c) $P = 23$ at $(x, y) = (2, 3)$.
(d) There is no optimal solution.
(e) None of these

Multiple Choice 5. A fair coin is tossed three times and the face that lands uppermost is recorded. Let the random variable X represent the number of heads that is recorded. Which of the following represents the probability distribution for this experiment?

(a)

X	0	1	2	3
$P(X)$	1/4	1/4	1/4	1/4

(b)

X	0	1	2	3
$P(X)$	1/8	3/8	3/8	1/8

(c)

X	0	1
$P(X)$	1/2	1/2

(d)

X	0	1	2	3
$P(X)$	0	3/8	3/8	1/4

(e)

X	0	1	2	3	4	5	6	7	8
$P(X)$	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

Multiple Choice 6. Consider the given system of linear inequalities :

$$\begin{cases} 2x + 2y \leq 8 \\ 6x + 2y \leq 12 \\ x \geq -3, y \geq 0\ . \end{cases}$$

Which of the following statements is **false**? (There is only one false statement.)

(a) There are 3 corner points.
(b) All boundary lines of the solution set are solid.
(c) The solution set is bounded.
(d) The point $(0, 4)$ is in the solution set.
(e) The x-intercept of the boundary line $6x + 2y \leq 12$ is the point $(2, 0)$.

Multiple Choice 7. Which of the following represents an initial simplex tableau for the given linear programming problem?

Maximize $P = 3a + 2b - 4c$

$$\text{subject to } \begin{cases} 3a - 2b - c \leq 120 \\ 10b + 15c \leq 245 \\ a \geq 0,\ b \geq 0,\ c \geq 0 \end{cases}$$

(a)
a	b	c	s_1	s_2	P	
3	−2	−1	1	0	0	120
0	10	15	0	1	0	245
−3	−2	4	0	0	1	0

(b)
a	b	c	s_1	s_2	s_3	P
3	−2	−1	1	0	0	120
0	10	15	0	1	0	245
0	0	0	0	0	1	0
−3	−2	4	0	0	0	1

(c)
a	b	c	s_1	s_2	P	
3	−2	−1	1	0	0	120
10	15	0	0	1	0	245
−3	−2	4	0	0	1	0

(d)
a	b	c	s_1	s_2	P	
3	−2	−1	1	0	0	120
0	10	15	0	1	0	245
1	1	1	0	0	0	0
−3	−2	4	0	0	1	0

(e)
a	b	c	P	
3	−2	−1	0	120
0	10	15	0	245
3	2	−4	1	0

Multiple Choice 8. One constraint for a standard maximization problem is $y \geq \frac{3}{5}(x+y) - \frac{2}{5}$. What is the correct way to arrange this as an equation (including slack variable s_1) in order to enter it into an initial simplex tableau?

(a) $\frac{3}{5}x - \frac{2}{5}y + s_1 = \frac{2}{5}$
(b) $-\frac{3}{5}x - \frac{8}{5}y + s_1 = \frac{2}{5}$
(c) $-\frac{3}{5}x + \frac{2}{5}y + s_1 = -\frac{2}{5}$
(d) $\frac{3}{5}x + \frac{8}{5}y + s_1 = -\frac{2}{5}$
(e) None of these

Multiple Choice 9. A company invests money in two projects, project A and project B. If x is the amount of money a company invests in project A and y the amount invested in project B, then which of the following inequalities represents the constraint "the amount invested in B should be no more than 40% of the overall investment"?

(a) $y \geq 0.4(x+y)$
(b) $x \leq 0.4(x+y)$
(c) $x \geq 0.4(x+y)$
(d) $y \leq 0.4(x+y)$
(e) None of these

Use the following set up for both Multiple Choice 10 and Multiple Choice 11.
Oily Oil Company has decided to introduce three oil mixes made from blending two or more oils. One jar of olive-vegetable oil requires 6 oz each of olive and vegetable oils. One jar of vegetable-peanut oil requires 10 oz of vegetable oil and 6 oz of peanut oil. Finally, one jar of olive-vegetable-peanut oil requires 3 oz of olive oil, 7 oz of vegetable oil, and 2 oz of peanut oil. The company has decided to allot 15 thousand ounces of olive oil, 23 thousand ounces of vegetable oil, and 6000 oz of peanut oil for the initial production run. Its profit on one jar of olive-vegetable oil is \$1.10, its profit on one jar of vegetable-peanut oil is 70 cents and its profit on one jar of olive-vegetable-peanut oil is \$0.60. To realize a maximum profit, how many jars of each blend should Oily Oil Company produce?

Multiple Choice 10. Let
$x :=$ the number of jars of olive-vegetable oil produced
$y :=$ the number of jars of vegetable-peanut oil jars produced
$z :=$ the number of jars of olive-vegetable-peanut oil jars produced
$P :=$ Oily Oil Company's profit from sales of oil jars, in dollars.

Write a linear programming problem that can be used to answer the following question : "To realize a maximum profit, how many jars of each blend should Oily Oil Company produce?"

(a) Maximize $P = 15x + 23y + 6z$
$$\text{subject to } \begin{cases} 6x + 3z \leq (15000)\cdot(1.10) \\ 6x + 10y + 7z \leq (23000)\cdot(0.70) \\ 6y + 2z \leq (6000)\cdot(0.60) \\ x \geq 0,\ y \geq 0,\ z \geq 0 \end{cases}$$
(b) Maximize $P = 1.1x + 0.7y + 0.6z$
$$\text{subject to } \begin{cases} 6x + 3y \leq 15 \\ 6x + 10y + 7z \leq 23 \\ 6x + 2z \leq 6000 \end{cases}$$
(c) Maximize $P = 1.1x + 0.7y + 0.6z$
$$\text{subject to } \begin{cases} 6x + 3z \leq 15000 \\ 6x + 10y + 7z \leq 23000 \\ 6y + 2z \leq 6000 \\ x \geq 0,\ y \geq 0,\ z \geq 0 \end{cases}$$
(d) Maximize $P = 15x + 23y + 6000z$
$$\text{subject to } \begin{cases} 6x + 3y \leq 15000 \\ 6x + 10y + 7z \leq 23000 \\ 6x + 2z \leq 6000 \end{cases}$$
(e) None of these

Multiple Choice 11. In solving the linear programming problem "To realize a maximum profit, how many jars of each blend should Oily Oil Company produce?" from above, recall that

$x :=$ the number of jars of olive-vegetable oil produced
$y :=$ the number of jars of vegetable-peanut jars produced
$z :=$ the number of jars of olive-vegetable-peanut jars produced
$P :=$ Oily Oil Company's profit from sales of oil jars, in dollars

and suppose we chose the slack variables so that

$s_1 :=$ the excess olive oil allotted, in ounces,
$s_2 :=$ the excess vegetable oil allotted, in ounces, and
$s_3 :=$ the excess peanut oil allotted, in ounces.

You are given the following simplex tableau. Which of the following statements about the optimal solution is **true**?

$$\left[\begin{array}{cccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \\ \hline 1 & 0 & 1/2 & 1/6 & 0 & 0 & 0 & 2500 \\ 0 & 1 & 2/5 & -1/10 & 1/10 & 0 & 0 & 800 \\ 0 & 0 & -2/5 & 3/5 & -3/5 & 1 & 0 & 1200 \\ \hline 0 & 0 & 23/100 & 17/150 & 7/100 & 0 & 1 & 3310 \end{array}\right]$$

(a) Oily Oil Company should produce 1200 jars of olive-vegetable-peanut jars to maximize their profit.

(b) There was no excess peanut oil allotted.

(c) Their maximum profit cannot be determined because the optimal solution has not been reached.

(d) There is no excess olive oil and no excess vegetable oil allotted.

(e) Oily Oil Company should produce 23/100 jars of olive-vegetable-peanut jars to maximize their profit.