Please note that while this week-in-review is a review for Exam 1 and Exam 2, it is **not** comprehensive. Use review worksheets on eCampus and past week-in-reviews for more practice.

Work-out Problems

Study tip: Show <u>*all*</u> *your work!*

Exercise 1. A company collects a revenue of \$45,864 when 84 units are sold. The cost for producing each unit is \$385 and in total (including fixed costs) it costs \$37,675 to produce 53 units. Assuming a linear revenue and linear cost, how many units should the company sell to make a profit of \$73,373?

Exercise 2. One of the rows of the following table represents a supply line and the other represents a demand line:

quantity	x	15	22	35	45
price A	p	8	10	14	18
price B	p	16	14	10	6

Which price represents the demand and which represents the supply? Find and interpret the equilibrium point.

Exercise 3. Andrea sold photographs at an art fair. She priced the photos according to size: small photos cost \$10, medium photos cost \$15, and large photos cost \$40. She sold as many small photos as medium and large photos combined. She also sold twice as many medium photos as large. A booth at the art fair cost \$300.

Set up and solve a system of equations to answer the following: How many of each size photo did she sell in order to pay for the booth?

Exercise 4. Find the solution set for the given system of linear equations.

$$\begin{cases} 2x + 2y + 6z = 14 \\ 2x - y + 3z = 5 \end{cases}$$

Exercise 5. Solve the following matrix equation for x, y, and z:

$$\begin{bmatrix} 9 & 2 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} x & 9 & 2 \\ 10 & 5 & 9 \end{bmatrix} - 3 \begin{bmatrix} 7 & 5 & 6y \\ 9-z & 4 & 2 \end{bmatrix} = \begin{bmatrix} 35 & 71 \\ 76 & 68 \\ 126 & 67 \end{bmatrix}^{\mathrm{T}}$$

Exercise 6. Solve the following system of equations.

$$\begin{cases} 4 + 2z + y = x \\ 4x - 4y - z = 2 \\ -x = -y - 2z + 3 \end{cases}$$

Exercise 7. Which of the following matrices are in reduced row echelon form? (Here, * denotes *any* real number.)

$$A = \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & 3 & 0 \\ 1 & 0 & 0 \end{array}\right] \quad , \quad B = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & * & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad ,$$

$$C = \left[\begin{array}{ccc|c} 1 & 0 & * & * \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right] \quad , \quad D = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Exercise 8. What is the next row operation that should be used to pivot on the matrix element a_{22}?

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{5} \\ 0 & \frac{4}{3} & 2 & 0 \end{array}\right]$$

Exercise 9. A school sells shirts, hats, and decals at two booths during a football game. Use matrix multiplication to determine the total amount of money collected from sales at each of the booths during the game.

$$M = \begin{array}{c} \\ \text{shirts} \\ \text{hats} \\ \text{decals} \end{array} \begin{array}{c} \text{price} \\ \text{(in \$)} \\ \left[\begin{array}{c} 10 \\ 8 \\ 7 \end{array}\right] \end{array}, \quad N = \begin{array}{c} \\ \text{booth 1} \\ \text{booth 2} \end{array} \begin{array}{c} \begin{array}{ccc} \text{shirts} & \text{hats} & \text{decals} \end{array} \\ \left[\begin{array}{ccc} 70 & 90 & 80 \\ 100 & 50 & 90 \end{array}\right] \end{array}$$

Exercise 10. Let S be defined by

$$\begin{cases} x + 2y \geq 40 \\ x + y \geq 30 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

1. Find the *minimum* of the objective function $C = 3x + 6y$ in the feasible region S. At how many points of S is the minimum achieved?

Let S be defined by

$$\begin{cases} x + 2y \geq 40 \\ x + y \geq 30 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

2. Find the *maximum* of the objective function $C = 3x + 6y$ in the feasible region S. At how many points of S is the maximum achieved?

Exercise 11. Heather and Tony decide to start a cookie business. Heather is going to contribute chocolate chip cookies and Tony is going to make Tony's Special Secret cookies. One batch of 100 of Heather's cookies takes 3/4 of an hour of preparation time, and one hour in the oven. One batch of 100 of Tony's cookies takes one hour of prep time and a full two hours in the oven. Combined, Heather and Tony are willing to put in 30 hours of prep time, and 50 hours of oven time. They will collect a profit of $60.00 on each 100 of Heather's cookies and a profit of $90.00 on each 100 of Tony's cookies. How many cookies of each type do they make in order to maximize profits?

Set up and solve the linear programming problem using the simplex method (if possible). Are there any leftover resources at the optimal production level? Explain. Then solve using the method of corners (if possible).

Additional page for work.

Exercise 12. A game consists of rolling a fair six-sided die, noting the number that lands uppermost, and flipping a fair coin, noting the side facing up. Let E denote the event "*An even number is rolled*" and let F denote the event "*The coin lands on heads.*"

1. Find the sample space associated with this experiment.

2. How many possible events are there?

3. List out the outcomes in the event $E \cap F$ and give a written description of the event.

4. Express the event "*An even number is rolled or the coin lands on heads*" using the correct symbols, and then find the probability of this event.

Exercise 13. Let S be a sample space and let E and B be events in this sample space such that $P(A) = \frac{6}{11}$, $P(A \cap B^{\mathrm{C}}) = \frac{2}{9}$, and $P(B) = \frac{1}{3}$.

1. Find $P((A \cap B^{\mathrm{C}})^{\mathrm{C}})$

2. Find $P(A \cup B^{\mathrm{C}})$.

3. Shade the region $A^{\mathrm{C}} \cap B^{\mathrm{C}}$ in a Venn Diagram.

4. Are the events $A \cap B$ and $A^{\mathrm{C}} \cap B^{\mathrm{C}}$ mutually exclusive? Explain.

Exercise 14. You pay $\$p$ to roll two fair standard four-sided dice, noting the numbers rolled on each die. If you roll a double, you win \$16. If you roll different numbers with a sum less than 4 or a sum greater than 6, you win \$12. Otherwise you win nothing.

1. Find the probability distribution table for your net winnings.

2. What should the price to play, $\$p$, be set to in order for this to be a fair game?