Work-out Problems

Study tip: Show all your work!

Exercise 1. Find each of the following for the given rational function f:

$$f(x) = \frac{2x^2 - 3x - 9}{x^2 - x - 6}.$$

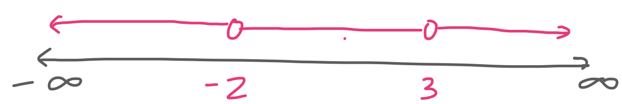
1. The domain (is for a rational function $f(x) = \frac{p(x)}{q(x)}$, domain is set of all x for which $q(x) \neq 0$)

$f(x) = \frac{2x^2 - 3x - 9}{x^2 - x - 6}$, so set $x^2 - x - 6 \neq 0$ and solve for x.

$\Rightarrow (x-3)(x+2) \neq 0$

$\Rightarrow x - 3 \neq 0$ or $x + 2 \neq 0$

$\Rightarrow x \neq 3$ or $x \neq -2$



$\boxed{\text{Domain: } (-\infty, -2) \cup (-2, 3) \cup (3, \infty)}$

2. The coordinates of all hole(s) (x-coordinates of any holes are given by any factors that cancel away from denom.)

Factor f and identify which any factor that $p(x)$ and $q(x)$ have in common.

$$f(x) = \frac{2x^2 - 3x - 9}{x^2 - x - 6} = \frac{(2x+3)(x-3)}{(x-3)(x-2)}$$

Both numerator and denominator have a common factor of $(x-3)$, so there is a hole at $x - 3 = 0$. To get the y-coord, plug $x = 3$ into simplified form of $f(x)$.

$\Rightarrow f(x) = \frac{2x+3}{x+2} \leftarrow$ simplified form of $f(x)$

when $x = 3 \Rightarrow f(3) = \frac{2(3)+3}{(3)+2} = \frac{9}{5}$. There is a hole at: $\boxed{\left(3, \frac{9}{5}\right)}$.

3. The vertical asymptote(s) (vertical asymptotes are given by any factors $(x-a)$ that remain in denominator in simplified form)

The simplified form of f is $f(x) = \frac{2x+3}{x+2}$ so there is a vertical asymptote when $x + 2 = 0$, so at the vertical line $\boxed{x = -2}$. (solve for x)

☆Recall: All intercepts are found after reducing the rational function to lowest terms.

$$f(x) = \frac{2x^2 - 3x - 9}{x^2 - x - 6} = \frac{2x+3}{x+2}$$

4. The x-intercept(s) x-intercepts happen when $y=0$

Domain: $(-\infty, -2) \cup (-2, 3) \cup (2, \infty)$

Plug $y = f(x) = 0$ into numerator of reduced form $f(x) = \dfrac{2x+3}{x+2}$

then solve for x: $0 = 2x+3$

$\Rightarrow x = -\frac{3}{2}$ (check: $x = -\frac{3}{2}$ is in the domain ✓)

So, there is an x-intercept $\left(-\frac{3}{2}, 0\right)$.

5. The y-intercept y-intercepts happen when $x=0$

Domain: $(-\infty, -2) \cup (-2, 3) \cup (2, \infty)$

Plug $x=0$ if it is in the domain into the reduced form

$y = f(x) = \dfrac{2x+3}{x+2}$ and solve for y.

$x=0$ is in the domain of f, so can plug in.

$$y = f(0) = \frac{2(0)+3}{(0)+2} = \frac{3}{2}.$$

So, there is a y-intercept $\left(0, \frac{3}{2}\right)$.

Exercise 2. Simplify the rational expressions:

1. $\frac{-3}{x+1} - 4\left(\frac{x}{x-6}\right) = \frac{-3}{x+1} + \frac{-4x}{x-6}$

common denom: $(x+1)(x-6)$

$$= \frac{-3\cdot(x-6)}{(x+1)\cdot(x-6)} + \frac{-4x\cdot(x+1)}{(x-6)\cdot(x+1)}$$

(multiply each terms num/denom by the factors its denom is missing)

$$= \frac{-3x+18}{(x+1)(x-6)} + \frac{-4x^2-4x}{(x-6)(x+1)}$$

(multiply out numerators, leave denoms in factored form)

$$= \frac{-3x+18+(-4x^2-4x)}{(x+1)(x-6)}$$

(add fractions with same denom by adding num. terms)

$$= \boxed{\frac{-4x^2-7x+18}{(x+1)(x-6)}}$$

(combine like terms in num.)

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2. $(-7)\cdot\dfrac{\frac{2}{a}+\frac{3}{b}}{\frac{5}{b}-\frac{6}{a^2}} = (-7)\dfrac{\frac{2\cdot b}{a\cdot b}+\frac{3\cdot a}{b\cdot a}}{\frac{5\cdot a^2}{b\cdot a^2}+\frac{-6\cdot b}{a^2\cdot b}}$

$$= (-7)\frac{\frac{2b+3a}{ab}}{\frac{5a^2-6b}{a^2b}} = (-7)\cdot\frac{2b+3a}{ab}\cdot\frac{a^2b}{5a^2-6b}$$

dividing by a fraction is equivalent to multiplying by its reciprocal

$$= \frac{(-7)(2b+3a)(a^2b)}{(ab)(5a^2-6b)} = \frac{(-7)(2b+3a)(a)}{(5a^2-6b)}$$

$$= \boxed{\frac{-7a(2b+3a)}{5a^2-6b}}$$

3. $\dfrac{\dfrac{4-2(x+h)}{3(x+h)-1}-\dfrac{4-2x}{3x-1}}{h} = \left(\dfrac{(4-2x-2h)}{(3x+3h-1)} + \dfrac{-(4-2x)}{(3x-1)}\right)\cdot\dfrac{1}{h}$

$$= \left(\frac{(4-2x-2h)\cdot(3x-1)}{(3x+3h-1)\cdot(3x-1)} + \frac{(-4+2x)\cdot(3x+3h-1)}{(3x-1)\cdot(3x+3h-1)}\right)\cdot\frac{1}{h}$$

$$= \left(\frac{12x-4-6x^2+2x-6xh+2h}{(3x+3h-1)(3x-1)} + \frac{-12x-12h+4+6x^2+6xh-2x}{(3x-1)(3x+3h-1)}\right)\cdot\frac{1}{h}$$

$$= \left(\frac{+2h-12h}{(3x+3h-1)(3x-1)}\right)\cdot\frac{1}{h}$$

$$= \frac{-10\cancel{h}}{(3x+3h-1)(3x-1)}\cdot\frac{1}{\cancel{h}}$$

$$= \boxed{\frac{-10}{(3x+3h-1)(3x-1)}}$$

Exercise 3. Find and completely simplify the difference quotient for $f(x) = \dfrac{4x}{x-5}$.

Want: $\dfrac{f(x+h) - f(h)}{h}$ (the difference quotient)

$$= \left(\frac{4(x+h)}{(x+h)-5} - \frac{4x}{x-5} \right) \cdot \frac{1}{h}$$

$$= \left(\frac{4x+4h}{x+h-5} + \frac{-4x}{x-5} \right) \cdot \frac{1}{h}$$

$$= \left(\frac{(4x+4h)(x-5)}{(x+h-5)(x-5)} + \frac{(-4x)(x+h-5)}{(x-5)(x+h-5)} \right) \cdot \frac{1}{h}$$

$$= \left(\frac{4x^2 - 20x + 4xh - 20h}{(x+h-5)(x-5)} + \frac{-4x^2 - 4xh + 20x}{(x-5)(x+h-5)} \right) \cdot \frac{1}{h}$$

$$= \frac{-20\cancel{h}}{(x+h-5)(x-5)} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{\frac{-20}{(x+h-5)(x-5)}}$$

rewrite in radical form first $g(x) = \sqrt{9-5x}$

Exercise 4. Find and completely simplify the difference quotient for $g(x) = (9 - 5x)^{\frac{1}{2}}$.

Want: $\dfrac{g(x+h) - g(x)}{h}$ (the difference quotient)

$$= \frac{\sqrt{9-5(x+h)} - \sqrt{9-5x}}{h} = \frac{\sqrt{9-5x-5h} - \sqrt{9-5x}}{h}$$

$$= \frac{\left(\sqrt{9-5x-5h} - \sqrt{9-5x}\right) \cdot \left(\sqrt{9-5x-5h} + \sqrt{9-5x}\right)}{(h) \cdot \left(\sqrt{9-5x-5h} + \sqrt{9-5x}\right)}$$

$$= \frac{(9-5x-5h) - (9-5x)}{(h) \cdot \left(\sqrt{9-5x-5h} + \sqrt{9-5x}\right)}$$

$$= \frac{9-5x-5h-9+5x}{(h) \cdot \left(\sqrt{9-5x-5h} + \sqrt{9-5x}\right)}$$

$$= \frac{-5\cancel{h}}{(\cancel{h}) \cdot \left(\sqrt{9-5x-5h} + \sqrt{9-5x}\right)}$$

$$= \boxed{\frac{-5}{\left(\sqrt{9-5x-5h} + \sqrt{9-5x}\right)}}$$

Exercise 5. Rationalize the denominator: $\dfrac{\sqrt{x}}{3\sqrt{x}+\sqrt{x-2}}$

$$\frac{(\sqrt{x})}{(3\sqrt{x}+\sqrt{x-2})}\,\frac{(3\sqrt{x}-\sqrt{x-2})}{(3\sqrt{x}-\sqrt{x-2})}$$

$$= \frac{\sqrt{x}\cdot 3\sqrt{x} - \sqrt{x}\cdot\sqrt{x-2}}{3\sqrt{x}\cdot 3\sqrt{x} - 3\sqrt{x}\cdot\sqrt{x-2} + \sqrt{x-2}\cdot 3\sqrt{x} - \sqrt{x-2}\cdot\sqrt{x-2}}$$

$$= \frac{3x - \sqrt{x(x-2)}}{9x - (x-2)}$$

$$= \frac{3x - \sqrt{x^2-2x}}{9x - x + 2}$$

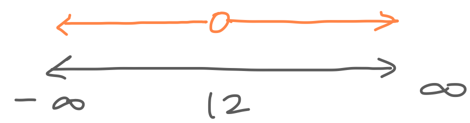
$$= \boxed{\frac{3x - \sqrt{x^2-2x}}{8x+2}}$$

Exercise 6. Write the domain of each algebraic function in interval notation.

1. $f(x) = \dfrac{12x+4}{-6x+72}$

rational function, so just need denominator $\neq 0$.

$-6x+72 \neq 0 \Rightarrow -6x \neq -72 \Rightarrow x \neq 12.$

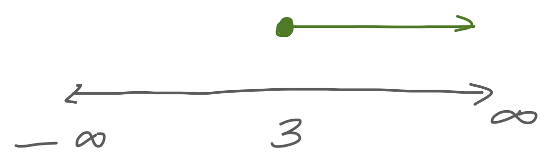


Domain of f: $\boxed{(-\infty, 12) \cup (12, \infty)}$

2. $g(s) = \sqrt{15-5s}$

radical function with even index (index = 2) so need argument ≥ 0.

$15-5s \geq 0 \Rightarrow -5s \geq -15 \Rightarrow s \leq 3$



Domain of g: $\boxed{[3, \infty)}$

3. $h(x) = (87x+16)^{\frac{1}{3}} = \sqrt[3]{(87x+16)^{1}} = \sqrt[3]{87x+16}$

radical function with odd index (index = 3) so domain is determined by domain of argument.

Since $87x+16$ is a polynomial, it exists for any real number $\Rightarrow$ the odd radical is defined for any real number.

Domain of h: $\boxed{(-\infty, \infty)}$

4. $j(t) = \sqrt{5}t^{14} - \frac{2}{3}$

polynomial in t, so is defined for any real #.

Domain of j: $\boxed{(-\infty, \infty)}$

algebraic function. Here,
- Numerator: is a polynomial
  AND
- Denominator: has an odd radical.

5. $k(x) = \dfrac{x+5}{\sqrt[7]{4x-16}}$

domain of numerator: AND domain of denominator:

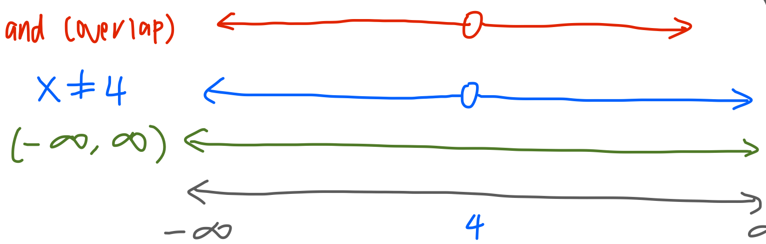
$(-\infty, \infty)$

$\sqrt[7]{4x-16} \neq 0$ and domain of $4x-16$

$4x-16 \neq 0$ and $(-\infty,\infty)$

$x \neq 4$ and $(-\infty,\infty)$ (need overlap)

$x \neq 4$



$\Rightarrow$ Domain of k: $\boxed{(-\infty, 4) \cup (4, \infty)}$

6. $m(x) = \dfrac{\sqrt[8]{x+3} - \sqrt[3]{4x+15} - 4x + 20}{x(x^3 - 5x^2 - 4x + 20)}$ another algebraic function.

domain of $\sqrt[8]{x+3}$ AND domain of $\sqrt[3]{4x+15}$ AND domain of denominator

even radical (index = 8)

$\Rightarrow x+3 \geq 0$

$\Rightarrow x \geq -3$

$[-3, \infty)$

AND

odd radical (index = 3)
argument is polynomial

$(-\infty, \infty)$

AND

$x(x^3 - 5x^2 - 4x + 20) \neq 0$

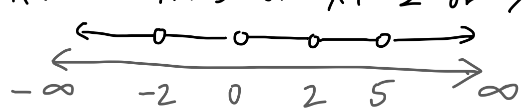
$x\left[x^2(x-5) - 4(x-5)\right] \neq 0$

$x \quad (x-5)(x^2-4) \neq 0$

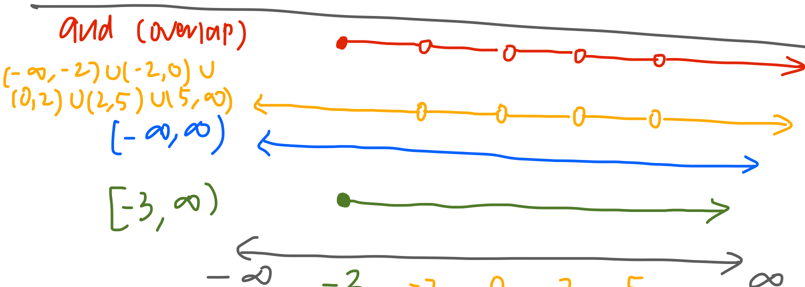
$x(x-5)(x+2)(x-2) \neq 0$

$x \neq 0$ or $x-5 \neq 0$ or $x+2 \neq 0$ or $x-2 \neq 0$

$x \neq 0$ or $x \neq 5$ or $x \neq -2$ or $x \neq 2$



$(-\infty,-2) \cup (-2,0) \cup (0,2) \cup (2,5) \cup (5,\infty)$



So, Domain of m: $\boxed{[-3,-2) \cup (-2,0) \cup (0,2) \cup (2,5) \cup (5,\infty)}$

7. $n(x) = |3x-4|$

can be written as a piecewise-defined function:

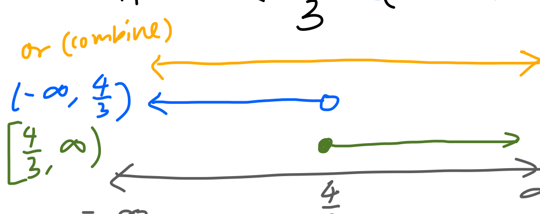
$$n(x) = \begin{cases} 3x-4 & \text{if } 3x-4 \geq 0 \\ -(3x-4) & \text{if } 3x-4 < 0 \end{cases} = \begin{cases} 3x-4 & \text{if } x \geq \frac{4}{3} \quad \text{(piece A)} \\ -3x+4 & \text{if } x < \frac{4}{3} \quad \text{(piece B)} \end{cases}$$

domain of piece A $\cup$ domain of piece B

domain of poly. and $x \geq \frac{4}{3}$ — domain of poly. and $x < \frac{4}{3}$

$(-\infty, \infty)$ and $[\frac{4}{3}, \infty)$ — $(-\infty, \infty)$ and $(-\infty, \frac{4}{3})$

$[\frac{4}{3}, \infty) \cup (-\infty, \frac{4}{3})$



Domain of n: $\boxed{(-\infty, \infty)}$

write:

Exercise 7. Use the given piecewise-defined function to find the following.

Plan: find the domain of each piece of the function on its specified interval and then take the union of each of these domains to get the final answer.

$$f(x) = \begin{cases} \dfrac{5x-7}{9x^2-9} & \text{if } x < 0 \quad (A) \\ \sqrt[8]{5x+9} & \text{if } 0 \le x \le 2 \quad (B) \\ 2x^5 + \sqrt[3]{3x-20} & \text{if } x > 4 \quad (C) \end{cases}$$

$f_A(x) = \dfrac{5x-7}{9x^2-9}$

$f_B(x) = \sqrt[8]{5x+9}$

$f_C(x) = 2x^5 + \sqrt[3]{3x-20}$

1. The domain of f

domain of f_A on A: domain of $\dfrac{5x-7}{9x^2-9}$ AND $x<0$.

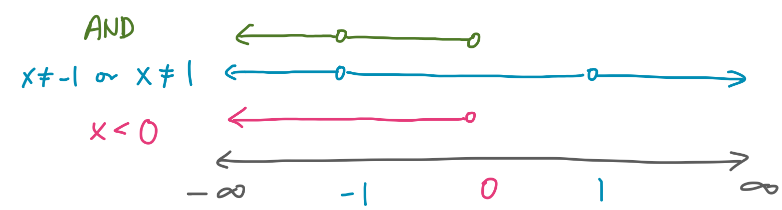
$9x^2 - 9 \neq 0$ and $x < 0$

$9(x^2-1) \neq 0$

$9(x+1)(x-1) \neq 0$

$(9 \neq 0$ or $x+1 \neq 0$ or $x-1 \neq 0)$ ...

$(x \neq -1$ or $x \neq 1)$ and $x < 0$



$\Rightarrow$ domain of f_A on A:

$\boxed{(-\infty, -1) \cup (-1, 0)}$

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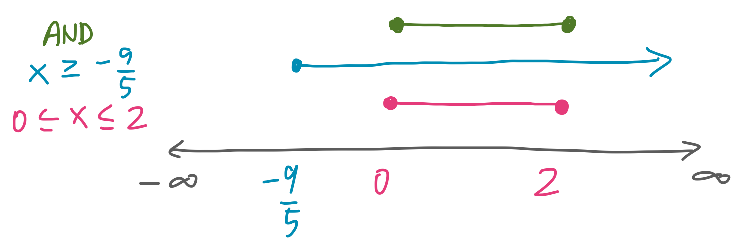
domain of f_B on B: domain of $\sqrt[8]{5x+9}$ AND $0 \le x \le 2$.

$5x+9 \ge 0 \Rightarrow 5x \ge -9 \Rightarrow x \ge -\frac{9}{5}$ ...

$5x+9 \ge 0$ and $0 \le x \le 2$

$x \ge -\frac{9}{5}$ and $0 \le x \le 2$



$\Rightarrow$ domain of f_B on B:

$\boxed{[0, 2]}$

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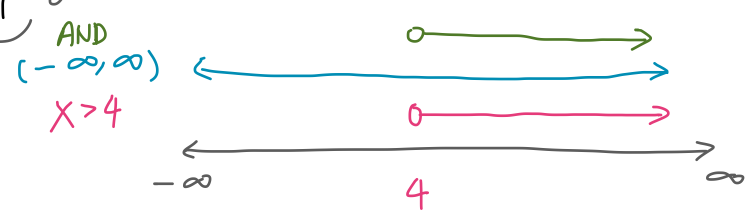
domain of f_C on C: domain of $2x^5 + \sqrt[3]{3x-20}$ AND $x > 4$

$2x^5$ is a poly

$\sqrt[3]{2x-5}$ is an odd index radical ...

$(-\infty, \infty)$ and $x > 4$



$\Rightarrow$ domain of f_C on C:

$\boxed{(4, \infty)}$

Domain of f: $\boxed{(-\infty, -1) \cup (-1, 0) \cup [0, 2] \cup (4, \infty)}$

$$f(x) = \begin{cases} \dfrac{5x-7}{9x^2-9} & \text{if } x < 0 \\ \sqrt[8]{5x+9} & \text{if } 0 \le x \le 2 \\ 2x^5 + \sqrt[3]{3x-20} & \text{if } x > 4 \end{cases}$$

2. The y-intercept

domain of f: $(-\infty,-1) \cup (-1,0) \cup \left[-\frac{9}{5}, 2\right] \cup (4,\infty)$

Since y-intercept happens when $x=0$ and $x=0$ is in the domain, need

$f(0) = f_B(0) = \sqrt[8]{5(0)+9} = \sqrt[8]{9}$. The y-intercept is: $\boxed{(0, \sqrt[8]{9})}$.

$x=0$ is in interval B

3. $f(-1)$

domain of f: $(-\infty,-1) \cup (-1,0) \cup \left[-\frac{9}{5}, 2\right] \cup (4,\infty)$

$x=-1$ is not in the domain of f $\Rightarrow$

$\boxed{f(-1) \text{ is not defined}}$.

4. $f(2)$

domain of f: $(-\infty,-1) \cup (-1,0) \cup \left[-\frac{9}{5}, 2\right] \cup (4,\infty)$

$x=2$ is in the domain of f and is in interval B, so use f_B:

$f(2) = f_B(2) = \sqrt[8]{5(2)+9} = \boxed{\sqrt[8]{19}}$

5. $f(3)$

domain of f: $(-\infty,-1) \cup (-1,0) \cup \left[-\frac{9}{5}, 2\right] \cup (4,\infty)$

$x=3$ is not in the domain of f $\Rightarrow$

$\boxed{f(3) \text{ is not defined}}$.

6. $f(5)$

domain of f: $(-\infty,-1) \cup (-1,0) \cup \left[-\frac{9}{5}, 2\right] \cup (4,\infty)$

$x=5$ is in the domain and it's in interval C, so use f_C:

$f(5) = f_C(5) = 2(5)^5 + \sqrt[3]{3(5)-20} = 2 \cdot 32 + \sqrt[3]{15-20} = \boxed{64 + \sqrt[3]{-5}}$

Exercise 8. A T-shirt printer sells custom-printed shirts for $12.50 each for the first 20 shirts, and drops the price to $11.00 for each additional shirt, up to a maximum order of 50 shirts. Let the function $p(s)$ represent the price (in dollars) of ordering s shirts.

1. Write a piecewise-defined function to model $p(s)$.

$$p(s) = \begin{cases} 12.50\, s & \text{if } 0 \le s \le 20 \quad (A) \\ (12.50)(20) + 11.00\, s & \text{if } 20 < s \le 50 \quad (B) \end{cases}$$

price per shirt changes after 20 shirts are purchased, but still have to buy first 20 shirts at the original price then each additional shirt costs less

2. Find and interpret $p(14)$.

since $0 \le 14 \le 20$

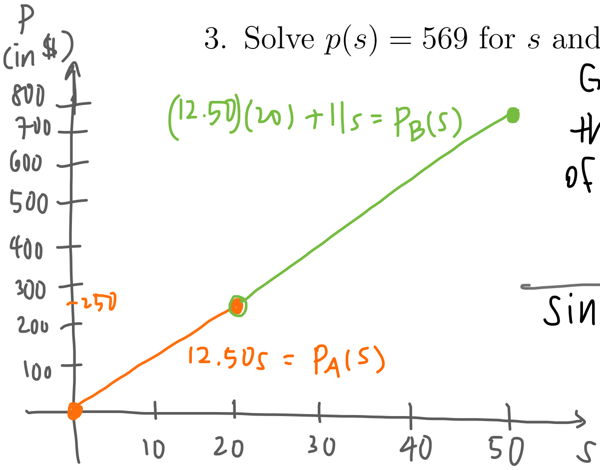
$s = 14$ is in interval A so use the first piece to evaluate

$$p(14) = 12.50 \cdot (14) = 175.$$

$p(s)$ calculates price of purchasing s shirts, in dollars

$\boxed{\text{It costs \$175 to order 14 shirts.}}$

3. Solve $p(s) = 569$ for s and interpret your answer.



Graphing the piecewise-defined function, we see that the line $P = 569$ intersects the second piece of the function, so solve $P_B(s) = 569$ for s:

$$(12.50)(20) + 11.00s = 569$$

$$250 + 11s = 569 \Rightarrow 11s = 319$$

$$\Rightarrow \underline{s = 29}.$$

Since s denotes # shirts ordered, $P(s) = 569$ means:

$\boxed{\text{"It costs \$569 to order 29 T-shirts."}}$

Exercise 9. Simplify completely

$$\sqrt[5]{\frac{32y^7}{x^{20}}} \div \frac{\left(\sqrt[2]{121x^{\frac{9}{7}}}\right)^3}{y^{-\frac{2}{3}}}$$

$$= \left(\frac{32y^7}{x^{20}}\right)^{\frac{1}{5}} \cdot \frac{y^{-\frac{2}{3}}}{\left(\sqrt[2]{121x^{\frac{9}{7}}}\right)^3}$$

$$= \frac{(32)^{\frac{1}{5}}\; y^{7\cdot\frac{1}{5}}}{(x^{20})^{\frac{1}{5}}} \cdot \frac{1}{y^{\frac{2}{3}} \cdot \left[\left(121x^{\frac{9}{7}}\right)^{\frac{1}{2}}\right]^3}$$

$$= \frac{(2^5)^{\frac{1}{5}}}{x^{20\cdot\frac{1}{5}}} \cdot \frac{y^{\frac{7}{5}}}{y^{\frac{2}{3}}} \cdot \frac{1}{\left[(121)^{\frac{1}{2}} \cdot x^{\frac{9}{7}\cdot\frac{1}{2}}\right]^3}$$

$$= \frac{2^{5\cdot\frac{1}{5}}}{x^4} \cdot \frac{y^{\frac{7}{5}-\frac{2}{3}}}{1} \cdot \frac{1}{\left[\sqrt{121} \cdot x^{\frac{9}{14}}\right]^3}$$

$$= \frac{2^1}{x^4} \cdot \frac{y^{\frac{21}{15}-\frac{10}{15}}}{1} \cdot \frac{1}{\left[11\, x^{\frac{9}{14}}\right]^3}$$

$$= \frac{2}{x^4} \cdot \frac{y^{\frac{11}{15}}}{1} \cdot \frac{1}{11^3 \cdot x^{\frac{27}{14}}}$$

$$= \frac{2y^{\frac{11}{15}}}{1331\, x^{4+\frac{27}{14}}} = \frac{2y^{\frac{11}{15}}}{1331\, x^{\frac{56}{14}+\frac{27}{14}}} = \boxed{\frac{2y^{\frac{11}{15}}}{1331\, x^{\frac{83}{14}}}}$$

Multiple Choice Problems

Study tip: Write out all your work when you complete the multiple-choice problems.

C

Multiple Choice 1. Simplify the following and express the answer using no negative exponents:

$$\frac{a^{-4}b^{7}}{(2a^{4}b^{-1}c^{-2})^{5}} = \frac{a^{-4}\,b^{7}}{2^{5}\,a^{4\cdot 5}\,b^{-1\cdot 5}\,c^{-2\cdot 5}}$$

$$= \frac{a^{-4}\,b^{7}}{32\,a^{20}\,b^{-5}\,c^{-10}}$$

$$= \frac{b^{7}\cdot b^{5}\cdot c^{10}}{a^{4}\cdot 32\cdot a^{20}}$$

$$= \boxed{\frac{b^{12}\,c^{10}}{32\,a^{24}}}$$

(a) $\dfrac{b^{12}c^{10}}{2a^{24}}$

(b) $\dfrac{b^{3}}{32a^{13}c^{3}}$

(c) $\boxed{\dfrac{b^{12}c^{10}}{32a^{24}}}$

(d) $\dfrac{a^{-4}b^{7}}{10a^{20}b^{-5}c^{-10}}$

(e) None of the given answer choices are correct.

E

Multiple Choice 2. Find the domain of

$$f(x) = \frac{\sqrt{x+5}}{(x+8)(x-9)}.$$

numerator restriction:

even index radical $\Rightarrow$ need

$x + 5 \geq 0 \Rightarrow x \geq -5$

AND

denominator restriction:

need $(x+8)(x-9) \neq 0$

$\Rightarrow x+8 \neq 0 \quad x-9 \neq 0$

$\Rightarrow x \neq -8$ and $x \neq 9$

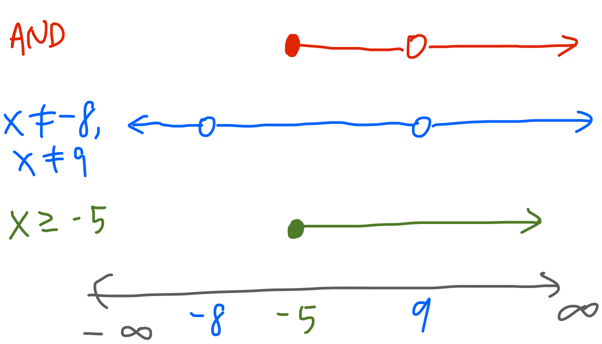
(a) $(-\infty, -8) \cup (-8, -5) \cup (-5, 9) \cup (9, \infty)$

(b) $(0, \infty)$

(c) $(-\infty, -8) \cup (-8, -5] \cup [-5, 9) \cup (9, \infty)$

(d) $(-5, 9) \cup (9, \infty)$

(e) $\boxed{[-5, 9) \cup (9, \infty)}$

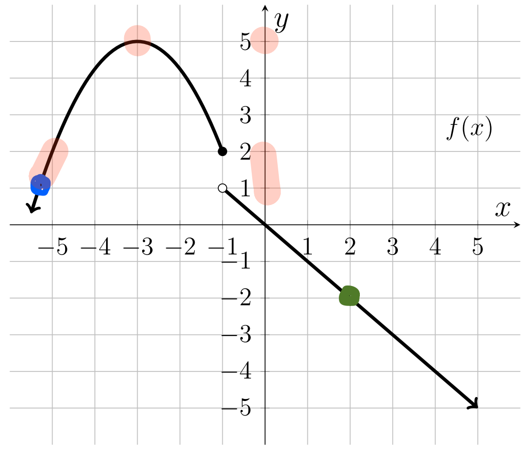


Domain of f:

$$\boxed{[-5, 9) \cup (9, \infty)}$$

E

Multiple Choice 3. Use the graph of $f(x)$ below to determine which of the following statements is FALSE. (There is only one false statement.)



(a) The domain of $f(x)$ is $(-\infty, \infty)$.

True: the graph continues forever on the left and right ends, and each x-value represented in the figure produces a point on the graph.

(b) There is only one value of x for which $f(x) = 1$.

(c) $f(2) = -2$.

(d) $f(x)$ does not have a minimum value.

$\boxed{\text{(e) The range of } f(x) \text{ is } (-\infty, 1) \cup (2, 5).}$

(a) is true: every x-value represented in the figure produces a point on the graph and the graph continues forever on the left and right ends.

(b) is true: the unique point that has y-coordinate 1 is indicated in blue.

(c) is true: when $x = 2$, the y-value is -2 as indicated in green

(d) is true: the graph has end behavior $\swarrow \cdots \searrow$ so $f(x) \to -\infty$ as $x \to \pm\infty$

(e) is false: the range of $f(x)$ is $(-\infty, 5]$, since there are points with $1 \le f(x) \le 2$ and $f(x) = 5$.

B

Multiple Choice 4. Given $H(x)$ below, find $H(8)$.

$$H(x) = \begin{cases} \sqrt{x+8} & \text{if } -7 < x < 0 \\ x^2 - 2x & \text{if } 0 \le x < 8 \\ |x - 10| & \text{if } 8 \le x \end{cases}$$

$x = 8$ is in the interval $8 \le x$, so use the third piece to evaluate:

(a) $H(8) = -2$

$\boxed{\text{(b) } H(8) = 2}$

(c) $H(8) = 4$

(d) $H(8) = 48$

(e) $H(8)$ is undefined.

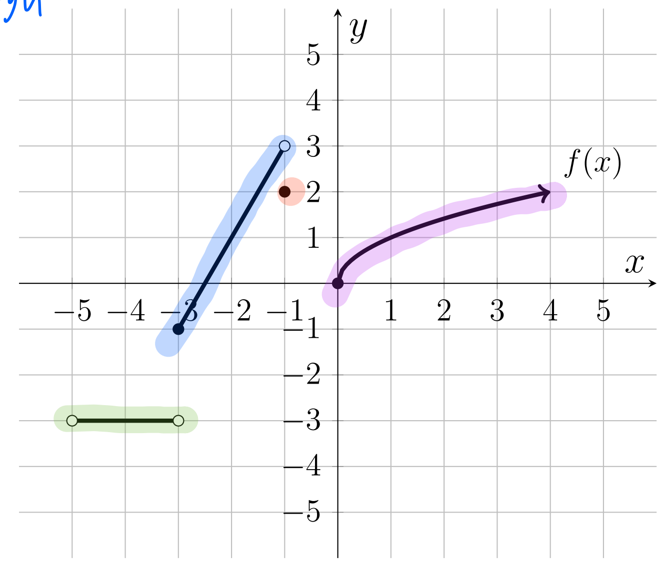
$$H(8) = |8 - 10|$$
$$= |-2|$$
$$\Rightarrow \boxed{H(8) = 2}$$

D

Multiple Choice 5. Which of the following could be an equation for the piecewise-defined function graphed below?

line with slope 2 through $(-3,-1)$:

$$y-(-1) = 2(x-(-3))$$
$$y+1 = 2(x+3)$$
$$y+1 = 2x+6$$
$$y = 2x+5$$



shape of radical function.
check:

x	$f(x)$
0	0
1	1
4	2

$\Rightarrow y = \sqrt{x}$

(a) $\begin{cases} -3 & \text{if } -5 < x < -3 \\ 2x+3 & \text{if } -3 \le x < -1 \\ 2 & \text{if } x = -1 \\ \sqrt{x} & \text{if } 0 \le x \end{cases}$

$$f(x) = \begin{cases} -3 & \text{if } -5 < x < -3 \\ 2x+5 & \text{if } -3 \le x < -1 \\ 2 & \text{if } x = -1 \\ \sqrt{x} & \text{if } 0 \le x \end{cases}$$

(b) $\begin{cases} -3 & \text{if } -5 < x < -3 \\ 2x+3 & \text{if } -3 \le x < -1 \\ 2 & \text{if } x = -1 \\ \sqrt{x} & \text{if } 0 \le x < 4 \end{cases}$

(c) $\begin{cases} -3 & \text{if } -5 < x \le -3 \\ 2x+5 & \text{if } -3 \le x < -1 \\ -1 & \text{if } x = 2 \\ \sqrt{x} & \text{if } 0 \le x \end{cases}$

$\boxed{\text{(d) } \begin{cases} -3 & \text{if } -5 < x < -3 \\ 2x+5 & \text{if } -3 \le x < -1 \\ 2 & \text{if } x = -1 \\ \sqrt{x} & \text{if } 0 \le x \end{cases}}$

(e) None of these; this is not the graph of a function.