Exam 3 is this Friday, April 17, 2020!

Disclaimer: while this week-in-review includes some Exam 3 review, **not everything is covered on this week-in-review**. Use the set of review questions in eCampus for a comprehensive review for all of Chapter 5. Also use previous Week-in-Review worksheets for more practice.

Work-out Problems

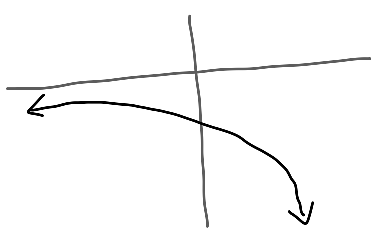
Study tip: Show all your work!

$f(x) = -7(e^4)^x$

Exercise 1. Consider the exponential function $f(x) = -7e^{4x}$. Find the domain, range, end behavior, horizontal asymptote, zeros, and y-intercept.

Have an exponential function $f(x) = ab^x$ with $a = -7$, $b = e^4$.

Since $a < 0$, $b > 1$, the graph of the function $f(x) = -7e^{4x}$ looks like



. From this, we see that:

domain: $(-\infty, \infty)$

range: $(-\infty, 0)$

end behavior: $f(x) \to -\infty$ as $x \to \infty$
$f(x) \to 0$ as $x \to -\infty$

horizontal asymptote: $y = 0$ (since $f(x) \to 0$ as $x \to -\infty$)

zeros: none (The range does not include $y=0$. Equivalently, there is no x-value such that $0 = -7e^{4x}$)

y-intercept: since $x=0$ is in the domain $(-\infty, \infty)$, plug in $x=0$ to see $y = f(0) = -7e^{4\cdot 0} = -7\cdot(e^0) = -7\cdot(1) = -7$

$\Rightarrow$ There is a y-intercept at $(0, -7)$.

Exercise 2. Solve the following equations for the exact value(s) of x.

1. $e^{2x-5} = e^3 \cdot e^4$

$$e^{2x-5} = e^{3+4}$$
$$e^{2x-5} = e^{7}$$
$$2x-5 = 7$$
$$2x = 12$$
$$\boxed{x=6}$$

check: started with $e^{2x-5} = e^3 \cdot e^4$

since the domain of the exponential function is $(-\infty, \infty)$, $x=6$ is in the domain so don't have to worry about extraneous solutions.

2. $\dfrac{8^{-3}}{8^{-2x}} = \left(\dfrac{1}{4^2}\right)^x \cdot 16^2$

Notice. 8, 4 and 16 can all be written as powers of 2.

$$8^{-3+2x} = (4^{-2})^x \cdot 16^2$$
$$8^{-3+2x} = 4^{-2x} \cdot 16^2$$
$$(2^3)^{-3+2x} = (2^2)^{-2x} \cdot (2^4)^2$$
$$2^{3\cdot(-3+2x)} = 2^{2\cdot(-2x)} \cdot 2^{4\cdot 2}$$
$$2^{-9+6x} = 2^{-4x+8}$$
$$-9+6x = -4x+8$$
$$10x = 17$$
$$\boxed{x = \frac{17}{10}}$$

check: started with $\dfrac{8^{-3}}{8^{-2x}} = \left(\dfrac{1}{4^2}\right)^x \cdot 16^2$

since the domain of the exponential functions are $(-\infty,\infty)$, $x = 17/10$ is in the domains so don't have to worry about extraneous solutions.

3. $2^x \cdot x^2 - 5 \cdot 2^x \cdot x = 14 \cdot 2^x$

Notice: all terms have a 2^x in common. Pull to one side and factor it out.

$$2^x \cdot x^2 - 5 \cdot 2^x \cdot x - 14 \cdot 2^x = 0$$
$$2^x (x^2 - 5x - 14) = 0$$

$\Rightarrow$ $2^x = 0$ or $x^2-5x-14=0$

NO solution (since $2^x > 0$ for all x)

$(x-7)(x+2) = 0$

$\Rightarrow x-7=0$ or $x+2=0$

$\Rightarrow x = 7$ or $x=-2$

So $\emptyset \cup \{x=7\} \cup \{x=-2\}$

$\Rightarrow$ $\boxed{x=7 \text{ or } x=-2}$

check: started with $2^x \cdot x^2 - 5 \cdot 2^x \cdot x = 14 \cdot 2^x$

These x are in the domain of the original functions, so don't have to worry about extraneous solutions.

Need to extract x; it's stuck in the exponents.

4. $5^x = 3^{1-2x}$

check: started with $5^{x} = 3^{1-2x}$

$$\ln(5^{x}) = \ln(3^{1-2x})$$

$$x \cdot \ln(5) = (1-2x) \cdot \ln(3)$$

$$x \cdot \ln(5) = \ln(3) - 2x \cdot \ln(3)$$

$$x\ln(5) + 2x\ln(3) = \ln(3)$$

$$x(\ln(5) + 2\ln(3)) = \ln(3)$$

This x is in the domain of the original functions, so don't have to worry about extraneous solutions.

$$\Rightarrow \boxed{x = \frac{\ln(3)}{\ln(5) + 2\ln(3)}}$$

5. $\log_3(81) - 11^{\log_{11}(5)} + \log_4(4^x) = 4^{3/2}$

clean up using log/exponential properties.

$a^{\log_a(b)} = b$ and $\log_a a^b = b$

$$\log_3(3^4) - 5 + x = \sqrt[2]{4^3}$$

$$4 - 5 + x = \sqrt[2]{64}$$

$$-1 + x = 8$$

$$\boxed{x = 9}$$

check: started with $\log_3(81) - 11^{\log_{11}(5)} + \log_4(4^{x}) = 4^{\frac{3}{2}}$

This x is in the domain of the original function, so don't have to worry about extraneous solutions.

6. $\log_c(3x - 1) = 3$, where $c > 0$

Need to extract x; it's stuck in the log. Convert to an exponential equation.

$$c^3 = 3x - 1$$

$$c^3 + 1 = 3x$$

$$\boxed{\frac{c^3+1}{3} = x}$$

Don't box your answer yet! Since the domain of $\log_c(3x-1)$ is restricted to $(\frac{1}{3}, \infty)$ need to confirm that this x is in the domain. Since $c > 0$, $c^3 > 0 \Rightarrow \frac{c^3}{3} > 0$, then $\frac{c^3}{3} + \frac{1}{3} = \frac{c^3+1}{3} = x > \frac{1}{3}$, so this solution is in the domain, and is not extraneous.

7. $\ln(x + 1) = -\ln(x) + \ln(6)$

$$\ln(x+1) + \ln(x) = \ln(6)$$

$$\ln((x+1) \cdot x) = \ln(6)$$

$$\ln(x^2 + x) = \ln(6)$$

$$x^2 + x = 6$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$\cancel{x = -3} \text{ or } \boxed{x = 2}$$

Don't box your answer yet! Since the domain of $\ln(x+1)$ is restricted to $(-1, \infty)$ and the domain of $\ln(x)$ is $(0, \infty)$, need to confirm to these x-values are in the domains of the OG functions.

Since $x = -3$ is not in $(-1, \infty)$ it is an extraneous solution.

Since $x = 2$ is in $(-1, \infty)$ AND $(0, \infty)$ it is safe.

Exercise 3. Let $f(x) = \frac{1}{4}e^{3x-7}$ and $g(x) = \sqrt{\log_2(x) - 1}$, and $h(x)$ have values in the table given below

x	-3	-2	-1	0	1	2	3	4	5	6
$h(x)$	0	5	0	-15	6	3	5	1	39	4

$h(-3) = 0$

Find each of the following.

1. $(g+h)(-3)$ $= g(-3) + h(-3) = \sqrt{\log_2(-3) - 1} + 0$

   $\boxed{\text{is not defined}}$.

   not defined; domain of $\log_2(x)$ is $(0, \infty)$

2. $(fg)(\sqrt{3})$ $= f(\sqrt{3}) \cdot g(\sqrt{3}) = \boxed{\left(\frac{1}{4}e^{3(\sqrt{3})-7}\right) \cdot \left(\sqrt{\log_2(\sqrt{3}) - 1}\right)}$

3. $(f-g)(4)$ $= f(4) - g(4) = \left(\frac{1}{4}e^{3(4)-7}\right) - \left(\sqrt{\log_2(4) - 1}\right)$

   $= \left(\frac{1}{4}e^{12-7}\right) - \sqrt{\log_2(2^2) - 1} = \frac{1}{2}e^5 - \left(\sqrt{2-1}\right) = \boxed{\frac{1}{2}e^5 - 1}$

4. $\left(\frac{f}{g}\right)(0)$ $= \frac{f(0)}{g(0)}$ $\boxed{\text{is not defined}}$.

   $g(0)$ is not defined since the domain of g is $(0, \infty)$ (which excludes $x = 0$).

5. $(f \circ g)(x)$ $= f(g(x)) = \boxed{\frac{1}{4}e^{3\left(\sqrt{\log_2(x) - 1}\right) - 7}}$

6. $(h \circ h)(6)$ $= h(h(6)) = h(4) = \boxed{1}$

   $h(6) = 4$

   $h(4) = 1$

7. $(f \circ h)(3)$ $= f(h(3)) = f(5) = \frac{1}{4}e^{3(5)-7}$

   $= \boxed{\frac{1}{4}e^8}$

Exercise 4. Express the domain of the given function in interval notation.

1. $f(x) = e^{\sqrt[4]{3x-27}}$ Domain of $e^{\sim}$ is domain of $\sim$.

need: domain of $\sqrt[4]{3x-27}$. Even index radical $\Rightarrow$ need its argument to be nonnegative:

$3x-27 \geq 0 \Rightarrow 3x \geq 27 \Rightarrow x \geq 9 \Rightarrow [9, \infty)$

So, the domain of f is $\boxed{[9, \infty)}$.

2. $g(t) = \dfrac{8 \cdot 4^t}{3t(2-t)^{\frac{1}{8}}} = \dfrac{8 \cdot 4^t}{3t \cdot \sqrt[8]{(2-t)^1}} = \dfrac{8 \cdot 4^t}{3t \cdot \sqrt[8]{2-t}}$

no restrictions from numerator: 4^t is an exponential function (with domain $(-\infty, \infty)$).

AND

denominator restrictions: need denominator $\neq 0$

$3t \sqrt[8]{2-t} \neq 0$

$\Rightarrow 3t \neq 0$ or $\sqrt[8]{2-t} \neq 0$

$\Rightarrow t \neq 0$ or $2-t \neq 0$

$\Rightarrow t \neq 0$ or $t \neq 2$

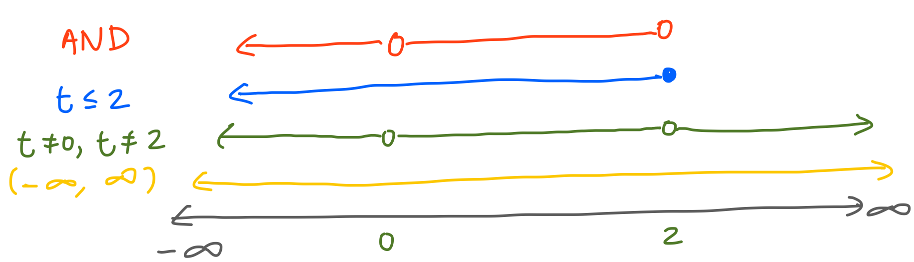
$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

AND

even index radical: need argument ≥ 0.

$2-t \geq 0 \Rightarrow -t \geq -2 \Rightarrow t \leq 2 \Rightarrow (-\infty, 2]$

Taking overlap, get



domain of g is: $\boxed{(-\infty, 0) \cup (0, 2)}$

3. $h(s) = \dfrac{55s+78}{e^{12s-22}}$

no restrictions from numerator: $55s+78$ is a polynomial (so domain is $(-\infty,\infty)$)
AND
no restrictions from exponential: e^{12s-22} has argument a polynomial, $12s-22$, so its domain is $(-\infty,\infty)$.
AND
denominator restrictions: need denominator $\neq 0$.
$e^{12s-22} \neq 0$, but this is true for all s in $(-\infty,\infty)$ since the range of an exponential growth function is $(0,\infty)$.

So, domain of h: $\boxed{(-\infty,\infty)}$.

4. $j(t) = \log_8(12-4t)$

log restrictions: need argument > 0.
$12-4t>0 \Rightarrow -4t > -12 \Rightarrow t < 3 \Rightarrow (-\infty, 3)$

So, domain of j: $\boxed{(-\infty,3)}$

5. $h(x) = \dfrac{\ln(x+15)}{\sqrt[7]{x+3}}$

log restrictions: need argument >0: $x+15>0 \Rightarrow x>-15 \Rightarrow (-15,\infty)$
AND
odd index radical: domain of odd index radical is domain of its argument. Here, it's argument is a polynomial, $x+3$, so its domain is $(-\infty,\infty)$
AND
denominator restrictions: need denominator $\neq 0$:
$\sqrt[7]{x+3} \neq 0 \Rightarrow x+3 \neq 0 \Rightarrow x \neq -3 \Rightarrow (-\infty,-3)\cup(-3,\infty)$.

Taking overlap of the restrictions, get:
domain of h: $\boxed{(-\infty,-3)\cup(-3,\infty)}$.

Exercise 5. *Aggie Venture Associates* is encouraging you to deposit money into their account with an interest rate of 3.5% that compounds quarterly. How long would it take to increase your principal to 23 times your initial investment? Round your final answer to two decimal places, if necessary.

Reliance Reveille offers you an interest rate of 4% compounded continuously. How much would you have to invest in the account today to have a balance of \$230,000 after 40 years? Round your final answer to two decimal places.

Aggie Venture Associates: Want t for which $A = 23P$.

compound interest formula: $A = P\left(1+\frac{r}{m}\right)^{mt}$.

Have: $A = 23P$, $r = 0.035$, $m = 4$. want t. Plug in and solve:

$$23P = P\left(1+\frac{r}{m}\right)^{mt}$$

$$23 = \left(\left(1+\frac{r}{m}\right)^{m}\right)^{t}$$

$$\ln(23) = \ln\left(\left(\left(1+\frac{r}{m}\right)^{m}\right)^{t}\right)$$

$$\ln(23) = t \cdot \ln\left(\left(1+\frac{r}{m}\right)^{m}\right)$$

$$t = \frac{\ln(23)}{\ln\left(\left(1+\frac{r}{m}\right)^{m}\right)}$$

$$t = \frac{\ln(23)}{\ln\left(\left(1+\frac{0.035}{4}\right)^{4}\right)}$$

$$t = 89.976916\ldots$$

$$\Rightarrow t \approx 89.98$$

$\Rightarrow$ It will take approximately $\boxed{89.98 \text{ years}}$ to increase your principal to 23 times your initial investment in the Aggie Venture Associates acct.

Reliance Reveille: Want: P for which $A = 230000$.

continuous compounding formula: $A = Pe^{rt}$.

Have: $A = 230000$, $r = 0.04$, $t = 40$. want P. Plug in & solve.

$$A = Pe^{rt} \Rightarrow \frac{A}{e^{rt}} = P \Rightarrow P = \frac{230000}{e^{(0.04)(40)}} = 46436.199138\ldots$$

$$\Rightarrow P \approx 46,436.20$$

$\Rightarrow$ You need to deposit a principal of $\boxed{\$46,436.20}$ today to increase to a balance of \$230,000 after 40 years in the Reliance Reveille account.

Exercise 6. Let $f(x) = \sqrt[3]{-x+\frac{3}{2}}$ and $g(x) = x^2 - 4$. Write down the formula of $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x))$$

$$= f(x^2 - 4)$$

$$= \boxed{\sqrt[3]{-(x^2-4)+\frac{3}{2}}}$$

Exercise 7. Find two functions f and g such that $(f \circ g)(x) = \dfrac{1}{\sqrt{7-2x^2}}$ and neither $f(x) = x$ nor $g(x) = x$. No explanation necessary.

Many answers here. Plug your g (inside func.) into your f (outside func) to confirm that your answer is correct. Four possible options:

$(f \circ g)(x) = \dfrac{1}{\sqrt{7-2x^2}}$

$g(x) = \sqrt{7-2x^2}$

$f(x) = \dfrac{1}{x}$

$(f \circ g)(x) = \dfrac{1}{\sqrt{7-2x^2}}$

$g(x) = 2x^2$

$f(x) = \dfrac{1}{\sqrt{7-x}}$

$(f \circ g)(x) = \dfrac{1}{\sqrt{7-2x^2}}$

$g(x) = 7-2x^2$

$f(x) = \dfrac{1}{\sqrt{x}}$

$(f \circ g)(x) = \dfrac{1}{\sqrt{7-2x^2}}$

$g(x) = x^2$

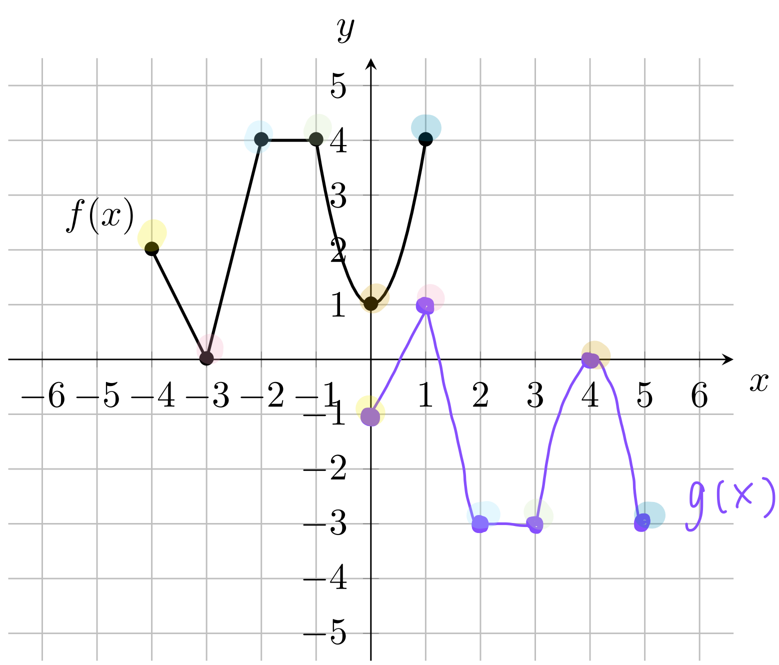
$f(x) = \dfrac{1}{\sqrt{7-2x}}$

Exercise 8. 1. Describe in words the transformations necessary to go from the graph of a parent function $f(x)$ to the graph of $g(x)$, where $g(x) = -f(x-4)+1$.

Start with f.

- shift f(x) right 4 units : $y_2 = f(x-4)$
- reflect $y_2(x)$ over the x-axis : $y_3 = -f(x-4)$
  (multiply all y-coordinates by −1)
- shift $y_3(x)$ up 1 unit: $y_4(x) = -f(x-4)+1$
  $= g(x)$

2. The graph of $f(x)$ is given below. <u>On the same plane below</u>, sketch the graph of the following transformation: $g(x) = -f(x-4)+1$. Indicate clearly at least 6 points on the graph of $g(x)$.



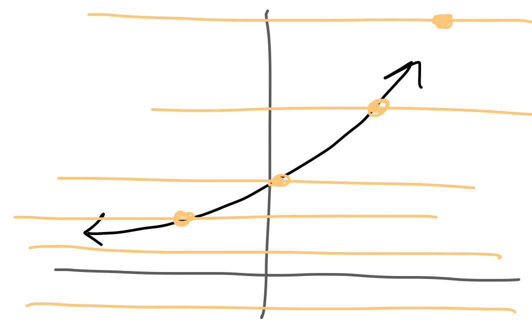
Starting points on f(x)	Points on $y_2(x)$ [shift f(x) right 4 units]	Points on $y_3(x)$ [multiply y-coords of $y_2(x)$ by −1]	Points on $y_4(x) = g(x)$ [shift $y_3(x)$ up 1 unit]
(−4, 2)	(0, 2)	(0, −2)	(0, −1)
(−3, 0)	(1, 0)	(1, 0)	(1, 1)
(−2, 4)	(2, 4)	(2, −4)	(2, −3)
(−1, 4)	(3, 4)	(3, −4)	(3, −3)
(0, 1)	(4, 1)	(4, −1)	(4, 0)
(1, 4)	(5, 4)	(5, −4)	(5, −3)

The final graph is plotted above.

Exercise 9. Use the graphs of the following parent functions to determine which are one-to-one functions: (1) exponential growth, (2) logarithmic decay, (3) quadratic, (4) absolute value.

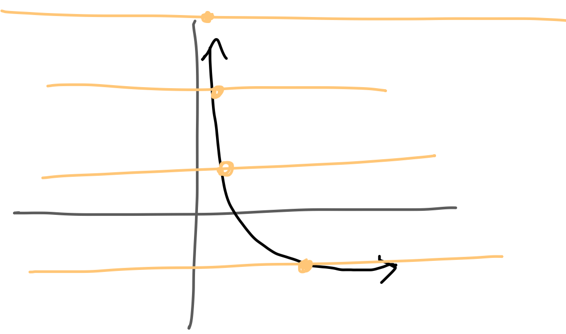
(1) exponential growth function:

It passes the horizontal line test (every horizontal line intersects it at most once) so it is a one-to-one function.



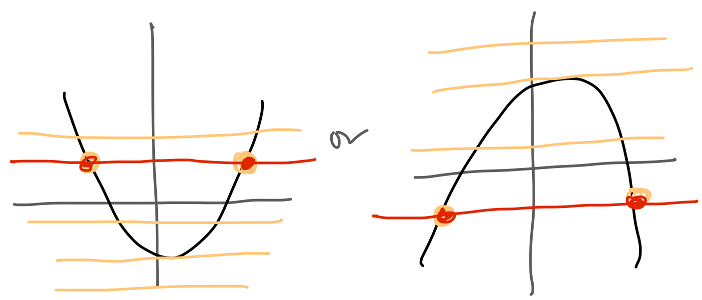
(2) logarithmic decay function.

It passes the horizontal line test (every horizontal line intersects it at most once) so it is a one-to-one function.



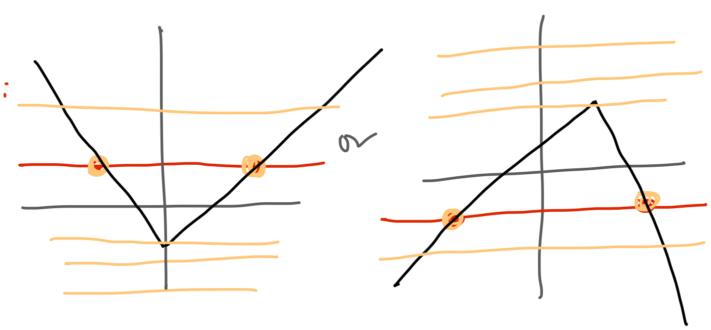
(3) quadratic function:

It <u>fails</u> the horizontal line test: can find at least one line that intersects the function more than once, so it is <u>not</u> a one-to-one function.



(4) absolute value function:

It <u>fails</u> the horizontal line test: can find at least one line that intersects the function more than once, so it is <u>not</u> a one-to-one function.



Recall: • $\log_b(M \cdot N) = \log_b(M) + \log_b(N)$ • $\log_b(b) = 1$

• $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$ • $\log_b(M^n) = n\log_b(M)$

Exercise 10. Expand or condense the following using properties of logarithms and simplify. Assume when necessary that all quantities represent positive numbers.

1. $\log\left(\dfrac{x^3\sqrt{x+1}}{10^5(x-2)^2}\right)$

$= \log\left(x^3 \cdot \sqrt{x+1}\right) - \log\left(10^5 \cdot (x-2)^2\right)$

$= \log(x^3) + \log(\sqrt{x+1}) - \left[\log(10^5) + \log(x-2)^2\right]$

$= 3\log(x) + \log\left((x+1)^{\frac{1}{2}}\right) - \left[5\log(10) + 2\log(x-2)\right]$

$= 3\log(x) + \frac{1}{2}\log(x+1) - 5 \cdot 1 + 2\log(x-2)$

$= \boxed{3\log(x) + \frac{1}{2}\log(x+1) - 5 + 2\log(x-2)}$

2. $\frac{1}{3}\log_b(z) - 4\log_b(z^2) + \log_b(x) + \dfrac{\log_b(y+9)}{2}$

$= \log_b\left(z^{\frac{1}{3}}\right) - \log_b\left((z^2)^4\right) + \log_b(x) + \frac{1}{2} \cdot \log_b(y+9)$

$= \left(\log_b\left(z^{\frac{1}{3}}\right) - \log_b(z^8)\right) + \left(\log_b(x) + \log_b(y+9)^{\frac{1}{2}}\right)$

$= \log_b\left(\dfrac{z^{\frac{1}{3}}}{z^8}\right) + \log_b\left((x) \cdot (y+9)^{\frac{1}{2}}\right)$

$= \log_b\left(\dfrac{1}{z^{8-\frac{1}{3}}}\right) + \log_b\left(x \cdot (y+9)^{\frac{1}{2}}\right)$

$= \log_b\left(\dfrac{1}{z^{23/3}}\right) + \log_b\left(x \cdot (y+9)^{\frac{1}{2}}\right) = \boxed{\log_b\left(\dfrac{x \cdot (y+9)^{\frac{1}{2}}}{z^{23/3}}\right)}$

Multiple Choice Problems

Study tip: Write out all your work when you complete the multiple-choice problems.

E

Multiple Choice 1. Suppose $b > 0$ and $c < 0$. Which of the following is NOT a transformation that can be used to graph the function $g(x) = -8\sqrt[5]{x-b} + c$ from corresponding parent function?

(a) Vertical shift down c units

(b) Vertical stretch by a factor of 8

(c) Reflection over the x-axis

(d) Horizontal shift right b units

$\boxed{\text{(e) All of these transformations are needed.}}$

To transform $f(x) = \sqrt[5]{x}$ to $g(x) = -8\sqrt[5]{x-b} + c$

where $b > 0$ and $c > 0$

need to

- shift right b units (right since $b>0$)
- reflect over the x-axis, by multiplying y-coords by -1.
- vertical stretch by a factor of 8
- shift down c units (down since $c < 0$)

A

Multiple Choice 2. The expression $\frac{2e^{x+4}}{4^{x+2}e^{2x-7}}$ is equivalent to which of the following?

(a) $\boxed{2^{-2x-3}e^{-x+11}}$

(b) $4^{-x}e^{-x+11}$

(c) $2^{-2x-4}e^{-x+11}$

(d) $2^{-2x+5}e^{-x-3}$

(e) $2^{5-2x}e^{-x+11}$

$$\frac{2e^{x+4}}{4^{x+2}\cdot e^{2x-7}} = \frac{2\,e^{(x+4)-(2x-7)}}{(2^2)^{x+2}}$$

$$= \frac{2^1\,e^{x+4-2x+7}}{2^{2x+4}}$$

$$= 2^{1-(2x+4)}\,e^{-x+11}$$

$$= 2^{1-2x-4}\,e^{-x+11}$$

$$= \boxed{2^{-2x-3}\,e^{-x+11}}$$

D **Multiple Choice 3.** Given $g(x) = -x^2 + 3x - 4$, find and simplify $\dfrac{g(x+h) - g(x)}{h}$ completely.

(a) $\dfrac{2x^3 + 2xh + h^2 + 3h}{h}$

(b) 1

(c) $3 - h + 2x$

$\boxed{\text{(d) } -2x - h + 3}$

(e) None of these

$$\frac{g(x+h) - g(x)}{h} = \frac{\left(-(x+h)^2 + 3(x+h) - 4\right) - \left(-x^2 + 3x - 4\right)}{h}$$

$$= \frac{\left(-(x^2 + 2xh + h^2) + 3x + 3h - 4\right) + x^2 - 3x + 4}{h}$$

$$= \frac{\cancel{-x^2} - 2xh - h^2 \cancel{+3x} + 3h \cancel{-4} \cancel{+x^2} \cancel{-3x} \cancel{+4}}{h}$$

$$= \frac{-2xh - h^2 + 3h}{h}$$

$$= \frac{\cancel{h}(-2x - h + 3)}{\cancel{h}} = \boxed{-2x - h + 3}$$

D

Multiple Choice 4. What amount will an account have after 10 years if \$3,000 is invested at an annual rate of 3.15% compounded continuously? (Round to the nearest penny.)

(a) \$7,110.78

(b) \$4,109.08

(c) \$70,008.19

$\boxed{\text{(d) \$4,110.78}}$

(e) None of the above

Compounded continuously, so:

Use $A = Pe^{rt}$, want A when $t=10$, $r=0.0315$, $P=3000$.

$$A = 3000 \cdot e^{(0.0315)(10)}$$

$$= 4110.777\ldots$$

$$\boxed{A \approx \$4110.78}$$

E

Multiple Choice 5. If $f(x) = \sqrt[3]{2x+1}$ and $h(x) = x + 3$, find $(h \circ f)(13)$.

(a) 48

(b) 3

(c) $\sqrt[3]{33}$

(d) 16

$\boxed{\text{(e) } 6}$

$$(h \circ f)(13) = h\left(f(13)\right)$$

$$= h\left(\sqrt[3]{2(13)+1}\right)$$

$$= h\left(\sqrt[3]{26+1}\right)$$

$$= h\left(\sqrt[3]{27}\right)$$

$$= h(3)$$

$$= (3) + 3$$

$$= \boxed{6}$$

D

Multiple Choice 6. Which of the following are properties of the graph of $g(x) = \ln x$?

A: The graph has a vertical asymptote at $x = 1$. False

B: The graph has a vertical asymptote at $x = 0$.

C: The graph goes through $(e, 0)$. False

D: The graph goes through $(1, 0)$.

E: The graph has a horizontal asymptote. False

F: $g(x) \to \infty$ as $x \to \infty$.

(a) A, C, and E

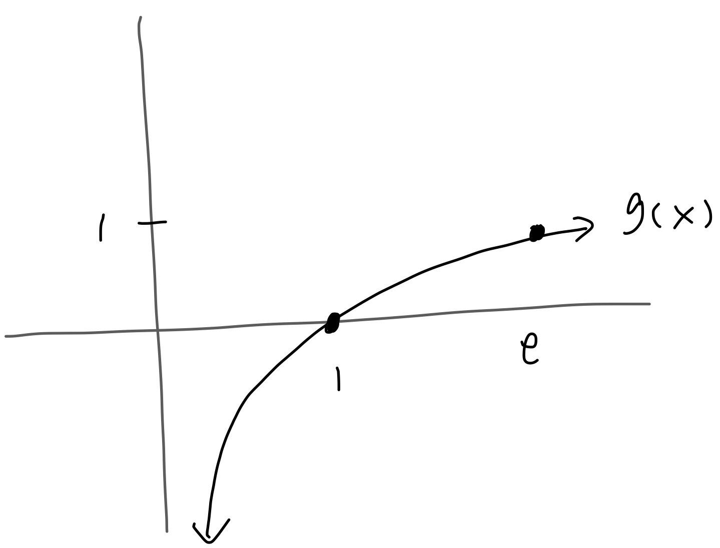
(b) B, D, and E

(c) A, D, and E

$\boxed{\text{(d) B, D, and F}}$

(e) A, C, and F

$g(x) = \ln(x) = \log_e(x)$ has base $b = e > 1$.



domain: $(0, \infty)$

range: $(-\infty, \infty)$

end-behavior:

$g(x) \to \infty$ as $x \to \infty$

$g(x) \to -\infty$ as $x \to 0$

vertical asymptote: $x = 0$

$g(e) = \ln(e) = 1$

$\Rightarrow$ graph passes through $(e, 1)$

$g(1) = \ln(1) = 0$

$\Rightarrow$ graph passes through $(1, 0)$

B

Multiple Choice 7. If $\ln 2 = a$ and $\ln 3 = b$, then $\ln \sqrt[5]{6} =$

(a) $\frac{1}{5}ab$

(b) $\frac{1}{5}(a+b)$

(c) $5ab$

(d) $5(a+b)$

(e) None of these

$$\ln\left(\sqrt[5]{6}\right) = \ln\left(6^{\frac{1}{5}}\right) = \frac{1}{5}\ln(6)$$

$$= \frac{1}{5}\ln(2\cdot 3)$$

$$= \frac{1}{5}\left(\ln(2) + \ln(3)\right)$$

$$= \boxed{\frac{1}{5}(a+b)}$$

B

Multiple Choice 8. The equation $\log_\pi x = \frac{1}{2}$ can be written in exponential form as

(a) $x = \left(\frac{1}{2}\right)^\pi$

$\boxed{\text{(b) } x = \pi^{1/2}}$

(c) $x^\pi = 1/2$

(d) $\pi = x^{1/2}$

(e) $\pi = \left(\frac{1}{2}\right)^x$

$$\log_{\pi}(x) = \frac{1}{2} \iff \boxed{\pi^{\frac{1}{2}} = x}$$

D

Multiple Choice 9. Determine the domain of the following function. $f(x) = \dfrac{\sqrt[3]{x-3}}{x^2-7x+12}$.

(a) $(3,4)\cup(4,\infty)$

(b) $[3,4)\cup(4,\infty)$

(c) $(-\infty,4)\cup(4,\infty)$

(d) $\boxed{(-\infty,3)\cup(3,4)\cup(4,\infty)}$

(e) None of these

- Odd index radical: has domain equal to the domain of its argument. It's argument is the polynomial $x-3$ $\Rightarrow$ the domain is $(-\infty,\infty)$.

AND

- denominator $\neq 0$:

$x^2-7x+12 \neq 0 \Rightarrow (x-4)(x-3)\neq 0$

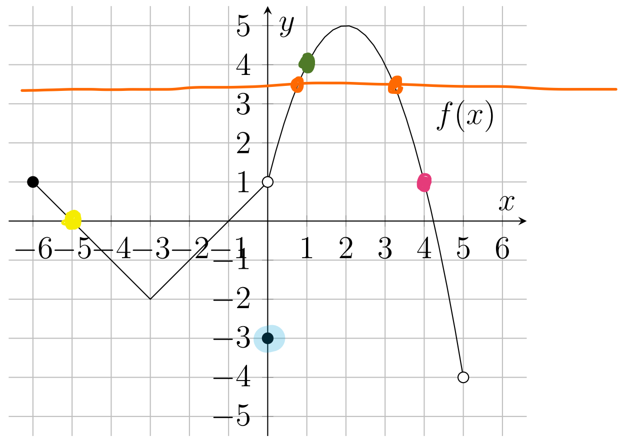
$\Rightarrow x\neq 4$ or $x\neq 3$

$\Rightarrow (-\infty,3)\cup(3,4)\cup(4,\infty)$

Taking the overlap, get

domain of f: $\boxed{(-\infty,3)\cup(3,4)\cup(4,\infty)}$

For Multiple Choice 10, 11, and 12, use the graph of $f(x)$ below.



D

Multiple Choice 10. Which of the following statements is **FALSE**?

(a) f is not one-to-one. True, f is not one-to-one; it fails the HLT.

(b) f is a function. True, f is a function; it passes the VLT.

(c) $f(1) = 4$. True, when x=1, the y-coord is 4.

(d) $f(0)$ does not exist. FALSE: $f(0) = -3$.

(e) $f(-5) = 0$. True, x = -5 is a zero of f.

A

Multiple Choice 11. What is the domain of f?

(a) $[-6, 5)$

(b) $[-6, 1) \cup (1, 5)$

(c) $(-\infty, \infty)$

(d) $(-4, 5]$

(e) None of these answer choices are correct.

f has domain:

$[-6, 0) \cup \{0\} \cup (0, 5)$

$= \boxed{[-6, 5)}$

every x-value in this interval yields a point on the graph of f.

B

Multiple Choice 12. Suppose $g(x) = |x - 7|$. What is $f(g(3))$?

(a) None of these answer choices are correct.

(b) $f(g(3)) = 1$.

(c) $f(g(3)) = 0$.

(d) $f(g(3)) = 4$.

(e) $f(g(3)) = 3$.

$$f(g(3)) = f(|(3) - 7|)$$
$$= f(|-4|)$$
$$= f(4)$$
$$= 1$$

when x=4 on graph of f, y = f(4) = 1

B

Multiple Choice 13. The cost, in dollars, of manufacturing x units of a product is given by $C(x) = 4x + 15000$. The demand equation for the same product is given by $p = -\frac{1}{500}x + 22$ where x is the quantity demanded at a unit price of \$$p$. If the selling price of the item is determined by the demand function, what is the maximum profit that this manufacturer can obtain?

(a) \$15,000

$\boxed{\text{(b) \$25,500}}$

(c) \$25,000

(d) \$30,000

(e) None of these

want: maximum value of $P(X) = R(X) - C(X)$.

since $R(x) = (p(x)) \cdot x = \left(\frac{-1}{500}X + 22\right) \cdot X$

$\Rightarrow R(X) = \frac{-1}{500}X^2 + 22X$

$\Rightarrow P(X) = \left(\frac{-1}{500}X^2 + 22X\right) - \left(4X + 15000\right)$

$P(X) = \frac{-1}{500}X^2 + 18X - 15000$

$P(x)$ is a quadratic $\Rightarrow$ max of $P(x)$ happens at vertex.

$X = \frac{-b}{2a} = \frac{-18}{2\left(\frac{-1}{500}\right)} = 4500$

$\Rightarrow P\left(\frac{-b}{2a}\right) = P(4500) = \frac{-1}{500}(4500)^2 + 18(4500) - 15000$

$= \boxed{\$25,500}.$

E

Multiple Choice 14. Let h be the function $h(x) = -(39630\sqrt{3} + 118560)x^2 + (30\sqrt{3} - 3975)x^4 + (3975\sqrt{3} + 39630)x^3 - 30x^5 + (118560\sqrt{3} + 102240)x - 102240\sqrt{3}$ which factors into:

$$h(x) = -15(x - \sqrt{3})(x + 142)(2x - 3)(x - 4)^2.$$

Which of the following statements is **FALSE**?

(a) h has a y-intercept of $(0, -102240\sqrt{3})$.

(b) The leading coefficient of h is -30.

(c) $h(x) \to \infty$ as $x \to -\infty$.

(d) h has a zero at $(-142, 0)$.

(e) $\boxed{\text{The domain of } h \text{ is } (-\infty, -142) \cup (-142, \frac{3}{2}) \cup (\frac{3}{2}, \sqrt{3}) \cup (\sqrt{3}, 4) \cup (4, \infty).}$ FALSE!

- h is a polynomial.
- It's constant term is $-102240\sqrt{3} \Rightarrow$ it's y-intercept is $(0, -102240\sqrt{3})$.

- The leading term of h is $-30x^5 \Rightarrow$

  degree: $n = 5$ (odd)

  leading coefficient: $a_n = -30$ (<0)

  end behavior: $h(x) \to -\infty$ as $x \to \infty$

  $h(x) \to \infty$ as $x \to -\infty$

- Using the factored form of h, get that it has zeros at $x = \sqrt{3}$, $x = -142$, $x = \frac{3}{2}$, $x = 4$.

- h has domain $(-\infty, \infty)$ since it is a polynomial.