Please note that while this week-in-review is a review for Exam 1 and Exam 2, it is **not** comprehensive. Use review worksheets on eCampus and past week-in-reviews for more practice.

Work-out Problems

Study tip: Show all your work!

Exercise 1. A company collects a revenue of \$45,864 when 84 units are sold. The cost for producing each unit is \$385 and in total (including fixed costs) it costs \$37,675 to produce 53 units. Assuming a linear revenue and linear cost, how many units should the company sell to make a profit of \$73,373? Let x = # units produced and sold.

Want: x such that $P(x) = 73373$. Need: $P(x)$ (profit eqn).

Know: $R(84) = 45864$, variable cost: 385; $C(53) = 37675$.

Need: An equation for $P(x)$. Know: $P(x) = R(x) - (C(x))$, so first find $R(x)$ and $C(x)$.

Revenue equation: Linear revenue equation: $R(x) = p \cdot x$ where p = unit price (in \$). Then since $R(84) = 45864$,

$R(84) = p \cdot 84 = 45864 \Rightarrow p = 546$. So $\boxed{R(x) = 546x}$.

Cost equation: Linear cost equation: $C(x) = mx + b$ ← fixed cost. (variable cost: m)

Since $m = 385$, $C(x) = 385x + b$ and since $C(53) = 37675$:

$C(53) = 385(53) + b = 37675 \Rightarrow b = 17270$. So $\boxed{C(x) = 385x + 17270}$

Profit equation:
$$P(x) = R(x) - (C(x))$$
$$= 546x - (385x + 17270)$$
$$P(x) = 161x - 17270.$$

Set $P(x) = 73373$ and solve for x:

$P(x) = 161x - 17270 = 73373$

$\Rightarrow 161x = 90643 \Rightarrow \boxed{x = 563}$.

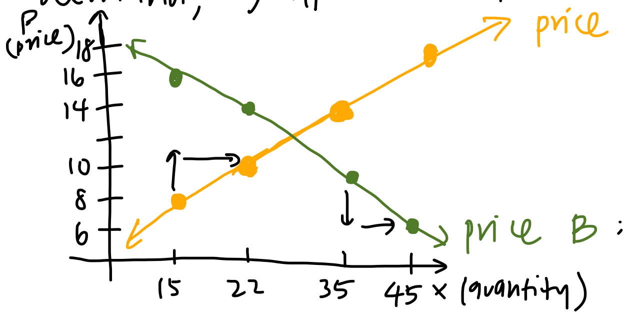
The company must sell $\boxed{\text{563 units}}$ to obtain a profit of \$73,373.

Exercise 2. One of the rows of the following table represents a supply ~~curve~~ line and the other represents a demand ~~curve~~ line:

quantity	x	15	22	35	45
price A	p	8	10	14	18
price B	p	16	14	10	6

(1) Which price represents the demand and which represents the supply? (2) Find and interpret the equilibrium point.

(1) To determine which price represents supply and which represents demand, graph the points. Use slope to determine.



price A: for price A, as price increases, quantity increases ⇒ price A represents the supply equation.

price B: for price B, as price decreases, quantity increases ⇒ price B represents the demand equation.

(2) To find the equilibrium point, find the intersection of the supply line and the demand line. First need the eqns of these lines.

supply eqn: $(x_1, p_1) = (15, 8)$ and $(x_2, p_2) = (22, 10)$ are two points on the supply line. It has slope $m = \frac{p_2 - p_1}{x_2 - x_1} = \frac{10-8}{22-15} = \frac{2}{7}$.

Then $p - p_1 = m(x - x_1) \Rightarrow p - 8 = \frac{2}{7}(x - 15) \Rightarrow$ $\boxed{-\frac{2}{7}x + p = \frac{26}{7}}$ supply equation

demand eqn: $(x_1, p_1) = (15, 16)$ and $(x_2, p_2) = (22, 14)$ are two points on the demand line. It as slope $m = \frac{p_2 - p_1}{x_2 - x_1} = \frac{14-16}{22-15} = \frac{-2}{7}$.

Then $p - p_1 = m(x - x_1) \Rightarrow p - 16 = -\frac{2}{7}(x - 15) \Rightarrow$ $\boxed{\frac{2}{7}x + p = \frac{142}{7}}$ demand equation

Now find equilibrium point by solving the system:

$$\begin{cases} -\frac{2}{7}x + p = \frac{26}{7} \\ \frac{2}{7}x + p = \frac{142}{7} \end{cases} \xrightarrow{\text{aug. matrix}} \begin{array}{ccc} x & p & \# \end{array} \left[\begin{array}{cc|c} -2/7 & 1 & 26/7 \\ 2/7 & 1 & 142/7 \end{array}\right] \xrightarrow{\star\ \text{rref( )}} \begin{array}{ccc} x & p & \# \end{array} \left[\begin{array}{cc|c} 1 & 0 & 29 \\ 0 & 1 & 12 \end{array}\right]$$

$\Rightarrow \begin{cases} x = 29 \\ p = 12 \end{cases}$

The equilibrium point is $(x, p) = (29, 12)$. This means that when 29 units are sold at a unit price of \$12, there is no shortage nor surplus; there is market equilibrium.

★ Guide to matrices in calculator: https://www.math.tamu.edu/~kahlig/calc/141-topic-list.html
click "Reduced Row Echelon Form" on left.

Exercise 3. Andrea sold photographs at an art fair. She priced the photos according to size: small photos cost \$10, medium photos cost \$15, and large photos cost \$40. She sold as many small photos as medium and large photos combined. She also sold twice as many medium photos as large. A booth at the art fair cost \$300.

Set up and solve a system of equations to answer the following: How many of each size photo did she sell in order to pay for the booth?

Let x = # small photos sold, y = # medium photos sold, z = # large photos sold.

Set up a system of equations by translating the sentences:

$$\begin{cases} 10x + 15y + 40z = 300 \\ x = y + z \\ y = 2z \end{cases} \xrightarrow{\text{clean up}} \begin{cases} 10x + 15y + 40z = 300 \\ x - y - z = 0 \\ y - 2z = 0 \end{cases}$$

(line up variables on left, constants on right)

Solve the system using matrices / rref():

$$\begin{array}{cccc} x & y & z & \# \end{array}$$
$$\left[\begin{array}{ccc|c} 10 & 15 & 40 & 300 \\ 1 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \xrightarrow{\text{rref()}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow[\text{to equations}]{\text{translate back}} \begin{cases} x = 9 \\ y = 6 \\ z = 3 \end{cases}$$

Interpret answer using defined variables:

$$\boxed{\text{Andrea sold 9 small photos, 6 medium photos, and 3 large photos in order to pay for the art booth.}}$$

Exercise 4. Find the solution set for the given system of linear equations.

$$\begin{cases} 2x + 2y + 6z = 14 \\ 2x - y + 3z = 5 \end{cases}$$

use matrices / rref() to solve

set up augmented matrix

$$\begin{array}{cccc} x & y & z & \# \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 2 & 2 & 6 & 14 \\ 2 & -1 & 3 & 5 \end{array}\right] \xrightarrow{\text{rref()}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

translate back into equations

$$\begin{cases} x + 2z = 4 \\ y + z = 3 \end{cases}$$

solve for x, solve for y

$$\begin{cases} x = -2z + 4 \\ y = -z + 3 \end{cases}$$

Once z is determined, know precise value of x and y, but z is unrestricted / free to be any real number.

Set $z = t$ where t is any real number.

Then
$$\begin{cases} x = -2z + 4 \\ y = -z + 3 \end{cases} \xrightarrow[z=t]{\text{sub in}} \begin{cases} x = -2t + 4 \\ y = -t + 3 \end{cases}$$

Solution:

$$(x, y, z) = (-2t + 4, \; -t + 3, \; t)$$

where t is any real number.

Exercise 5. Solve the following matrix equation for x, y, and z:

Do the specified matrix operations to simplify to one matrix on each side of equal sign.

$$\begin{bmatrix} 9 & 2 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} x & 9 & 2 \\ 10 & 5 & 9 \end{bmatrix} - 3 \begin{bmatrix} 7 & 5 & 6y \\ 9-z & 4 & 2 \end{bmatrix} = \begin{bmatrix} 35 & 71 \\ 76 & 68 \\ 126 & 67 \end{bmatrix}^{T}$$

$2 \times 2 \quad 2 \times 3$

$=$

so can multiply, will result in 2×3 matrix

$$\begin{array}{cccc} & \text{col 1} & \text{col 2} & \text{col 3} \\ \begin{matrix}\text{row 1}\\ \text{row 2}\end{matrix} & \multicolumn{3}{c}{\begin{bmatrix} 9\cdot x + 2\cdot 10 & 9\cdot 9 + 2\cdot 5 & 9\cdot 2 + 2\cdot 9 \\ 5\cdot x + 7\cdot 10 & 5\cdot 9 + 7\cdot 5 & 5\cdot 2 + 7\cdot 9 \end{bmatrix}} \end{array} + \begin{bmatrix} -3\cdot 7 & -3\cdot 5 & -3\cdot 6y \\ -3(9-z) & -3\cdot 4 & -3\cdot 2 \end{bmatrix} = \begin{bmatrix} 35 & 71 \\ 76 & 68 \\ 126 & 67 \end{bmatrix}^{T}$$

$$\begin{bmatrix} 9x+20 & 91 & 36 \\ 5x+70 & 80 & 73 \end{bmatrix} + \begin{bmatrix} -21 & -15 & -18y \\ -27+3z & -12 & -6 \end{bmatrix} = \begin{bmatrix} 35 & 76 & 126 \\ 71 & 68 & 67 \end{bmatrix}$$

$$\begin{bmatrix} 9x+20-21 & 91-15 & 36-18y \\ 5x+70-27+3z & 80-12 & 73-6 \end{bmatrix} = \begin{bmatrix} 35 & 76 & 126 \\ 71 & 68 & 67 \end{bmatrix}$$

$$\begin{bmatrix} 9x-1 & 76 & 36-18y \\ 5x+3z+43 & 68 & 67 \end{bmatrix} = \begin{bmatrix} 35 & 76 & 126 \\ 71 & 68 & 67 \end{bmatrix}$$

Compare the two matrices entry-by-entry now:

$9x-1 = 35$

$9x = 36$

$x = 4$

$36-18y = 126$

$-18y = 90$

$y = -5$

$5x+3z+43 = 71$

$5x+3z = 28$

$3z = 28-5x$

$3z = 28-5(4)$

$3z = 8$

$z = \frac{8}{3}$

Sub in $x=4$

So $\boxed{x=4,\ y=-5,\ z=\frac{8}{3}}$ make the matrix equation true.

Exercise 6. Solve the following system of equations.

$$\begin{cases} 4 + 2z + y = x \\ 4x - 4y - z = 2 \\ -x = -y - 2z + 3 \end{cases} \xrightarrow{\text{clean up}} \begin{cases} -x + y + 2z = -4 \\ 4x - 4y - z = 2 \\ -x + y + 2z = 3 \end{cases}$$

(line up variables on left, constants on right)

Solve the system using matrices / rref():

$$\begin{array}{cccc} x & y & z & \# \\ \end{array}$$
$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & -4 \\ 4 & -4 & 1 & 2 \\ -1 & 1 & 2 & 3 \end{array}\right] \xrightarrow{\text{rref()}} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\text{to equations}]{\text{translate back}} \begin{cases} x - y = 0 \\ z = 0 \\ 0 = 1 \end{cases}$$

The last equation says that $0 = 1$ which is false no matter what the values of x, y, z are, hence the system has $\boxed{\text{NO solution}}$.

Exercise 7. Which of the following matrices are in reduced row echelon form? (Here, * denotes *any* real number.)

$$A = \left[\begin{array}{cc|c} \boxed{1} & 3 & 5 \\ 2 & 3 & 0 \\ 1 & 0 & 0 \end{array}\right], \quad B = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & * & 0 \\ 0 & 0 & 0 & 1 \end{array}\right],$$

$$C = \left[\begin{array}{ccc|c} 1 & 0 & * & * \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right], \quad D = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

A is <u>not</u> in RREF: the leading entry $a_{11} = \boxed{1}$ does not have only zeros in its column.

B <u>is</u> in RREF, for any value of *.

C is <u>not</u> in RREF: the leading entry $a_{22} = \boxed{1}$ does not have only zeros in its column.

D is <u>not</u> in RREF: the row of all zeros should be the last row of the matrix.

Exercise 8. What is the next row operation that should be used to pivot on the matrix element a_{22}?

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & \frac{1}{2} & -\frac{3}{5} \\ 0 & \frac{4}{3} & 2 & 0 \end{array}\right]$$

$a_{22} = 1$. We are not done pivoting on a_{22} because we need the rest of the entries in its column to be 0. To accomplish this use the row operation $\left(-\frac{4}{3}\right) \cdot R_2 + R_3 \longrightarrow R_3$.

will get:

$$\begin{array}{ll} \left(-\frac{4}{3}\right) \cdot R_2 & \left[\begin{array}{ccc|c} 0 & -\frac{4}{3} & -\frac{2}{3} & \frac{4}{5} \end{array}\right] \\ + \; R_3 & \left[\begin{array}{cccc} 0 & \frac{4}{3} & 2 & 0 \end{array}\right] \\ \hline \text{new } R_3 & \left[\begin{array}{ccc|c} 0 & 0 & \frac{4}{3} & \frac{4}{5} \end{array}\right] \end{array}$$

So A will then become:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & \frac{1}{2} & -\frac{3}{5} \\ 0 & 0 & \frac{4}{3} & \frac{4}{5} \end{bmatrix}.$$

Exercise 9. A school sells shirts, hats, and decals at two booths during a football game. Use matrix multiplication to determine the total amount of money collected from sales at each of the booths during the game.

$$M = \begin{array}{c} \\ \text{shirts} \\ \text{hats} \\ \text{decals} \end{array} \begin{array}{c} \text{price} \\ \text{(in \$)} \\ \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \end{array}, \quad N = \begin{array}{c} \\ \text{booth 1} \\ \text{booth 2} \end{array} \begin{array}{c} \begin{array}{ccc} \text{shirts} & \text{hats} & \text{decals} \end{array} \\ \begin{bmatrix} 70 & 90 & 80 \\ 100 & 50 & 90 \end{bmatrix} \end{array}$$

Want: a matrix

$$\begin{array}{c} \\ \text{booths} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} \quad \end{bmatrix} \end{array}$$

Notice:

$$N = \begin{array}{c} \\ \text{booths} \end{array} \begin{array}{c} \text{goods} \\ \begin{bmatrix} \quad \end{bmatrix} \end{array} \qquad M = \begin{array}{c} \\ \text{goods} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} \quad \end{bmatrix} \end{array}$$

$$2 \times 3 \qquad\qquad 3 \times 1$$

inner dimensions/ labels match

So want the matrix:

$$N \cdot M = \begin{array}{c} \\ \text{booths} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 70 & 90 & 80 \\ 100 & 50 & 90 \end{bmatrix} \end{array} \cdot \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{array}{c} \\ \text{booths} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 70 \cdot 10 + 90 \cdot 8 + 80 \cdot 7 \\ 100 \cdot 10 + 50 \cdot 8 + 90 \cdot 7 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ \text{booths} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 1980 \\ 2030 \end{bmatrix} \end{array}$$

$\boxed{\text{Booth 1 makes \$1980 in sales and Booth 2 makes \$2030 in sales.}}$

Exercise 10. Let S be defined by

① graph S
② identify corner points of S
③ test corner points
④ determine optimal value of objective function.

$$\begin{cases} x + 2y \geq 40 \\ x + y \geq 30 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

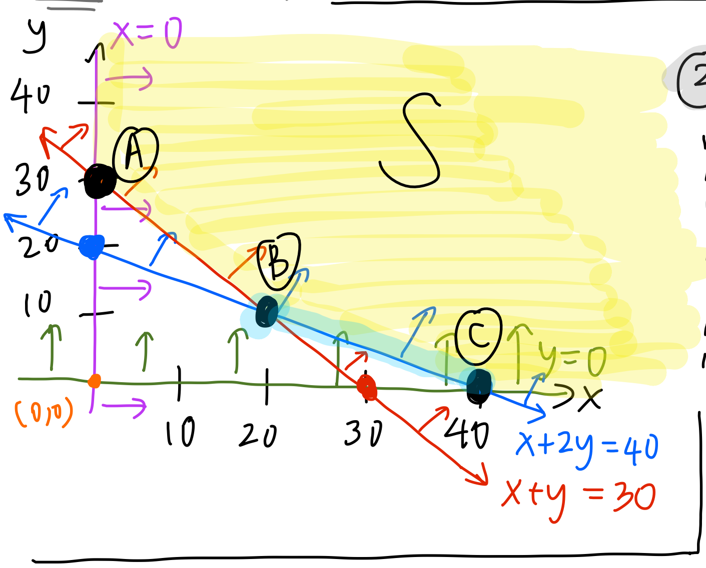
in all cases we consider, minimum always exists.

1. Find the *minimum* of the objective function $C = 3x + 6y$ in the feasible region S. At how many points of S is the minimum achieved?

①

inequality:	$X + 2y \boxed{\geq} 40$ (solid)	$X + y \boxed{\geq} 30$ (solid)	$X \boxed{\geq} 0$ (solid)	$y \boxed{\geq} 0$ (solid)
boundary line:	$X + 2y = 40$	$X + y = 30$	$X = 0$	$y = 0$
x-intercept:	$(\underline{40}, 0)$	$(\underline{30}, 0)$	vertical line (y-axis)	horizontal line (x-axis)
y-intercept:	$(0, \underline{20})$	$(0, \underline{30})$		
test point:	$(0,0)$ ↳ $0 + 2(0) > 40$; $0 > 40$? FALSE	$(0,0)$ $0 + 0 > 30$? $0 > 30$? FALSE	points to right to $X=0$	points above $y=0$

(true shading)



② • S is unbounded
• S has 3 corner points Ⓐ, Ⓑ, Ⓒ

Ⓐ: $(0, 30)$, Ⓒ: $(40, 0)$

Ⓑ: $\begin{cases} X + 2y = 40 \\ X + y = 30 \end{cases} \rightarrow \begin{array}{ccc} x & y & \# \\ \end{array} \left[\begin{array}{cc|c} 1 & 2 & 40 \\ 1 & 1 & 30 \end{array}\right] \xrightarrow{\text{rref}} \begin{array}{ccc} x & y & \# \\ \end{array} \left[\begin{array}{cc|c} 1 & 0 & 20 \\ 0 & 1 & 10 \end{array}\right] \Rightarrow \begin{cases} X = 20 \\ y = 10 \end{cases}$

$\Rightarrow$ Ⓑ: $(20, 10)$

③

corner pt	value of obj. func $C = 3X + 6y$	goal: minimize
Ⓐ $(0, 30)$	$3(0) + 6(30) = 180$	
Ⓑ $(20, 10)$	$3(20) + 6(10) = 120$	
Ⓒ $(40, 0)$	$3(40) + 6(0) = 120$	

④ The minimum value of the objective function C is 120. It occurs at the infinitely many points on the line segment connecting corner points Ⓑ & Ⓒ.

Let S be defined by

$$\begin{cases} x + 2y \geq 40 \\ x + y \geq 30 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

2. Find the *maximum* of the objective function $C = 3x + 6y$ in the feasible region S. At how many points of S is the maximum achieved?

we graphed S above and saw that S was unbounded. So, any objective function with a feasible region S will have no maximum value.

That is, C has no maximum in S (the max occurs at zero points of S).

Exercise 11. Heather and Tony decide to start a cookie business. Heather is going to contribute chocolate chip cookies and Tony is going to make Tony's Special Secret cookies. One batch of 100 of Heather's cookies takes 3/4 of an hour of preparation time, and one hour in the oven. One batch of 100 of Tony's cookies takes one hour of prep time and a full two hours in the oven. Combined, Heather and Tony are willing to put in 30 hours of prep time, and 50 hours of oven time. They will collect a profit of $60.00 on each 100 of Heather's cookies and a profit of $90.00 on each 100 of Tony's cookies. How many cookies of each type do they make in order to maximize profits?

Set up and solve the linear programming problem using the simplex method (if possible). Are there any leftover resources at the optimal production level? Explain. Then solve using the method of corners (if possible).

Variables: $x :=$ # of batches of 100 of Heather's cookies
$y :=$ # of batches of 100 of Tony's cookies
$P :=$ profit from cookie sales (in $)

Objective: Maximize $P = 60x + 90y$

subject to
$$\begin{cases} \frac{3}{4}\cdot x + 1\cdot y \le 30 & \text{(total prep time allotted)} \\ 1\cdot x + 1\cdot y \le 50 & \text{(total oven time allotted)} \\ x \ge 0, \quad y \ge 0 & \text{(nonnegativity constraints)} \end{cases}$$

This is a standard maximization problem, so can use the simplex method to solve.

modified constraints/objective:

$$\frac{3}{4}x + y + s_1 = 30$$
$$x + y + s_2 = 50$$
$$-60x - 90y + P = 0$$

$\Longrightarrow$ initial simplex tableau:

$$\begin{array}{cccccc|c} x & y & s_1 & s_2 & P & & \# \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 3/4 & 1 & 1 & 0 & 0 & 30 \\ 1 & 1 & 0 & 1 & 0 & 50 \\ \hline -60 & -90 & 0 & 0 & 1 & 0 \end{array}\right]$$

First pivot element:

$$\begin{array}{ccccc|c} x & y & s_1 & s_2 & P & \# \end{array}$$
$$\left[\begin{array}{ccccc|c} 3/4 & 1 & 1 & 0 & 0 & 30 \\ 1 & 1 & 0 & 1 & 0 & 50 \\ \hline -60 & -90 & 0 & 0 & 1 & 0 \end{array}\right]$$

pivot col

nonneg ratios
$30/1 = 30 \leftarrow$ smallest $\Rightarrow$ pivot row
$50/1 = 50$

$\Rightarrow$ Pivot on the 1 in row 1, col 2.

★ Pivot using online tool: https://www.zweigmedia.com/RealWorld/tutorialsf1/scriptpivot2.html

Next tableau:

Additional page for work.

$$\begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccccc|c} x & y & s_1 & s_2 & P & \# \end{array}$$

$$\left[\begin{array}{ccccc|c} 3/4 & 1 & 1 & 0 & 0 & 30 \\ 1/4 & 0 & -1 & 1 & 0 & 20 \\ \hline 15/2 & 0 & 90 & 0 & 1 & 2700 \end{array}\right]$$

Done pivoting since there are no negative #s left in the bottom row. To read off answer, identify basic and non basic variables.

non-basic variables:

$x = 0$

$s_1 = 0$

Set all nonbasic variables equal to 0.

basic variables:

$y = 30$

$s_2 = 20$

$P = 2700$

Use final simplex tableau to read off values of basic variables.

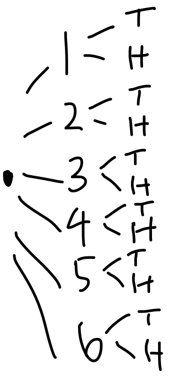
Interpret variables to answer question:

To maximize their profits and obtain a profit of \$2700, they should sell 0 batches of Heather's cookies and 30 batches of Tom's cookies, ~~for~~ a total of 300 of Tom's cookies sold. They will use up all of their prep time allotted (since $s_1=0$), but they'll have an excess of 20 hours of oven time (since $s_2=20$)

★ You will get the same answer if you solve using the method of corners; try it!

Exercise 12. A game consists of rolling a fair six-sided die, noting the number that lands uppermost, and flipping a fair coin, noting the side facing up. Let E denote the event "*An even number is rolled*" and let F denote the event "*The coin lands on heads.*"

1. Find the sample space associated with this experiment.



$$\boxed{S = \{1T,\ 1H,\ 2T,\ 2H,\ 3T,\ 3H,\ 4T,\ 4H,\ 5T,\ 5H,\ 6T,\ 6H\}}$$

2. How many possible events are there?

$n = 12$ simple events in S $\Rightarrow$ $2^n = \boxed{2^{12} \text{ possible events}}$ in S

3. List out the outcomes in the event $E \cap F$ and give a written description of the event.

$E \cap F = \{2H,\ 4H,\ 6H\}$

is the event that an even number is rolled <u>and</u> the coin lands on heads.

4. Express the event "*An even number is rolled or the coin lands on heads*" using the correct symbols, and then find the probability of this event.

"An even number is rolled or the coin lands on heads"

or, so combine

is $E \cup F$.

$P(E \cup F) = P(\{2H, 2T, 4H, 4T, 6H, 6T, 1H, 3H, 5H\})$

$$= \frac{9}{12} = \boxed{\frac{3}{4}}$$

Exercise 13. Let S be a sample space and let E and B be events in this sample space such that $P(A) = \frac{6}{11}$, $P(A \cap B^C) = \frac{2}{9}$, and $P(B) = \frac{1}{3}$.

1. Find $P((A \cap B^C)^C)$

$$P((A\cap B^c)^c) = 1 - P(A \cap B^c) = 1 - \frac{2}{9} = \boxed{\frac{7}{9}}$$

(by complement rule)

2. Find $P(A \cup B^C)$.

$$P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$$

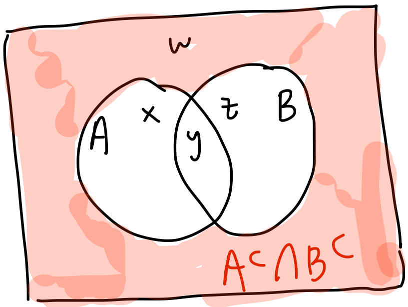
by union rule

(by complement rule

$$= \frac{6}{11} + (1 - P(B)) - \frac{2}{9}$$

$$= \frac{6}{11} + 1 - \frac{1}{3} - \frac{2}{9} = \boxed{\frac{98}{99}}$$

3. Shade the region $A^C \cap B^C$ in a Venn Diagram.

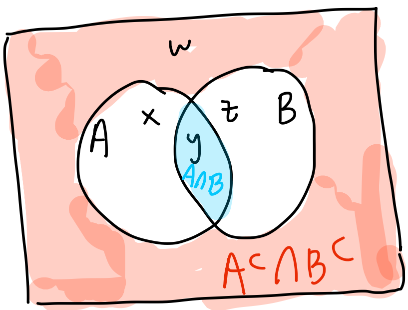


$A^c = \{w, z\}$

$\cap \; B^c = \{w, x\}$

$A^c \cap B^c = \{w\}$

4. Are the events $A \cap B$ and $A^C \cap B^C$ mutually exclusive? Explain.



$A \cap B$ and $A^c \cap B^c$ are mutualy exclusive because they have no common overlap.

Exercise 14. You pay $\$p$ to roll two fair standard four-sided dice, noting the numbers rolled on each die. If you roll a double, you win \$16. If you roll different numbers with a sum less than 4 or a sum greater than 6, you win \$12. Otherwise you win nothing.

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

1. Find the probability distribution table for your net winnings.

Let X = net winnings (in \$). Say $p>0$.

outcome	X value of R.V.	$P(X)$ probability of X
roll doubles	$-p+16$	$\frac{4}{16}$
roll diff. #s with sum <4 or sum >6	$-p+12$	$\frac{4}{16}$
lose	$-p$	$\frac{8}{16}$

2. What should the price to play, $\$p$, be set to in order for this to be a fair game?

Fair game if expected value is 0.

$$E(X) = x_1 \cdot p_1 + x_2 \cdot p_2 + x_3 \cdot p_3$$

$$= (-p+16)\cdot\left(\frac{4}{16}\right) + (-p+12)\cdot\frac{4}{16} + (-p)\cdot\frac{8}{16}$$

$$= -\frac{4}{16}p + \frac{64}{16} - \frac{4p}{16} + \frac{48}{16} - \frac{8p}{16}$$

$$= -p + \frac{112}{16} \overset{\text{set}}{=} 0 \Rightarrow p = \frac{112}{16} = 7.$$

$$\boxed{\text{The price to play should be set to \$7 to make this a fair game.}}$$