Exam 1 is on Friday, February 7 from 4:30pm – 6:30pm!

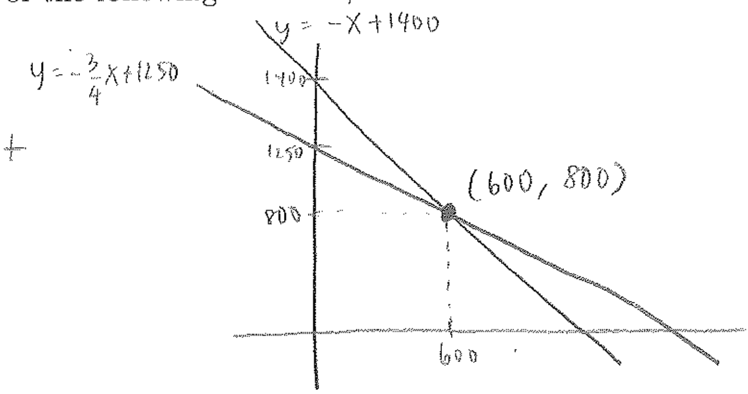
Work-out Problems

Study tip: Show all your work!

Exercise 1. Suppose you are given the following system of equations:

$$\begin{cases} 0.03x + 0.04y = 50 \\ x + y = 1400 \end{cases}.$$

Solve the system of equations by using each of the following methods, and then determine the type of the system.



Plan:

1. the method of graphing

Write both equations in slope-intercept form and then put in calculator. Use 2nd, calc, intersect to find intersection point.

$$\begin{cases} 0.03x + 0.04y = 50 \\ x + y = 1400 \end{cases} \Rightarrow \begin{cases} y = -\frac{3}{4}x + 1250 \\ y = -x + 1400 \end{cases}$$

The solution to the system is $(x, y) = (600, 800)$.

2. the method of substitution,

Plan: Solve equation ② for x and then sub into ①. Solve for y, then back substitute to get x.

$x + y = 1400 \Rightarrow x = 1400 - y$. Sub this into $0.03x + 0.04y = 50$, get $0.03(1400 - y) + 0.04y = 50 \Rightarrow 42 - 0.03y + 0.04y = 50$

$\Rightarrow 0.01y = 8 \Rightarrow y = 800$. Subbing $y = 800$ back into $x = 1400 - y$, we get $x = 1400 - 800 = 600$.

The solution is $x = 600$, $y = 800$.

3. the method of addition, and

Goal: eliminate one variable by multiplying equation(s) by (a) scalar(s) and adding:

$$\begin{cases} 0.03x + 0.04y = 50 \\ x + y = 1400 \end{cases} \xrightarrow[\text{to eliminate } x]{\text{multiply equation ② by } -0.03} \begin{cases} 0.03x + 0.04y = 50 \\ -0.03x - 0.03y = -42 \end{cases}$$

$\xrightarrow{\text{add the equations}}$ $0.01y = 8 \Rightarrow \underline{y = 800}$. Substitute $y = 800$ back into one of the original equations and solve for x. Since $x + y = 1400$, then with $y = 800$, get $x + 800 = 1400 \Rightarrow \underline{x = 600}$.

The solution is $x = 600$, $y = 800$.

4. rref() in a calculator.

Plan: write the system as an augmented matrix, apply rref() and then read off the solution.

$$\begin{cases} 0.03x + 0.04y = 50 \\ x + y = 1400 \end{cases} \xrightarrow[\text{matrix}]{\text{write as augmented}} \begin{array}{ccc} x & y & \# \end{array} \left[\begin{array}{cc|c} 0.03 & 0.04 & 50 \\ 1 & 1 & 1400 \end{array}\right]$$

$$\xrightarrow{\text{RREF()}} \begin{array}{ccc} x & y & \# \end{array} \left[\begin{array}{cc|c} 1 & 0 & 600 \\ 0 & 1 & 800 \end{array}\right] \xrightarrow[\text{equations}]{\text{translate back to}} \begin{cases} x = 600 \\ y = 800 \end{cases}$$

The solution is $x = 600$, $y = 800$.

5. Is this system: *independent*, *inconsistent*, or *dependent*?

Since the system has a unique solution, the system is called $\underline{\text{independent}}$.

Exercise 2. Suppose you are given the following system of equations:

$$\begin{cases} x - 3y = 5 \\ -2x + 6y = -10 \ . \end{cases}$$

1. Solve the system of equations. If there are infinitely many solutions, write a parametric solution using t and/or s. If there is no solution, write "No Solution".

We could use any of the 4. methods outlined in exercise 1, but here I'll use rref( ).

$$\begin{cases} x - 3y = 5 \\ -2x + 6y = -10 \end{cases} \xrightarrow[\text{matrix}]{\text{write as augmented}} \begin{array}{ccc|c} x & y & & \# \end{array} \left[\begin{array}{cc|c} 1 & -3 & 5 \\ -2 & 6 & -10 \end{array}\right] \xrightarrow{\text{RREF}} \begin{array}{ccc|c} x & y & & \# \end{array} \left[\begin{array}{cc|c} 1 & -3 & 5 \\ 0 & 0 & 0 \end{array}\right]$$

Translating back to equations, we get

$$\begin{cases} x - 3y = 5 \\ 0 = 0 \end{cases}$$

$\leftarrow$ this is true for any value of x, y

so we just need x, y-values that satisfy this first equation

once y is fixed, can use equation ① to get x. So, let $y = t$

Then $x - 3y = 5 \rightarrow x = 3y + 5 \rightarrow x = 3t + 5$ since $y = t$.

So, there are infinitely many solutions $(x, y) = (3t + 5, t)$ where t is any real #.

2. Is this system: *independent*, *inconsistent*, or *dependent*?

There are infinitely many solutions, so the lines defining the system intersect in infinitely many points. In other words, they are the same line, hence the system is called dependent.

Is there another way you could show the system is dependent algebraically?

Exercise 3. Let x be the number of Popeyes chicken sandwiches made and sold, and let the cost and revenue (in dollars) be given by the equations $C(x) = 0.99x + 252$ and $R(x) = 3.99x$.

1. Sketch the cost, revenue, and profit equations on the same graph.

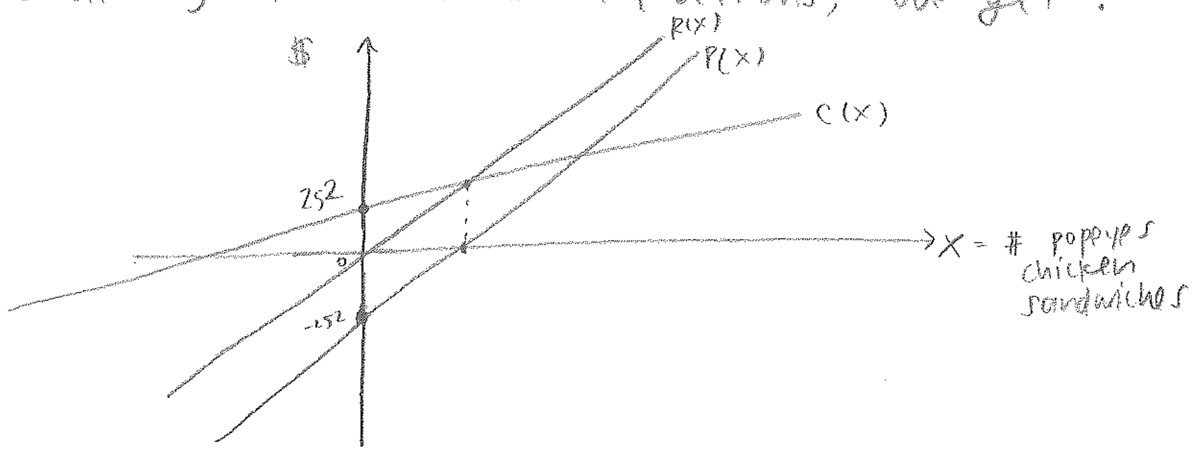
cost equation: $C(x) = 0.99x + 252$

Revenue equation: $R(x) = 3.99x$

Profit equation: $P(x) = R(x) - \left(C(x)\right) = 3.99x - \left(0.99x + 252\right)$

$\Rightarrow \quad P(x) = 3x - 252$

Graphing the three equations, we get:



2. Find and interpret the break-even quantity.

Plan: Break-even happens when $R(x) = C(x)$ (Revenue = cost), or equivalently $P(x) = 0$. Can use the graph to solve for this: find the x-value of the point where the revenue and cost lines intersect (since we're asked for the break-even quantity). Using 2nd + calc + intersect, get that $R(x) = C(x)$ when $x = 84$.

This means that the company's revenue will be exactly equal to its costs when 84 chicken sandwiches are sold.

(Can you solve this using a different method?)

Exercise 4. Pivot the given augmented matrix about the boxed element. *Do not* completely reduce the matrix to reduced row-echelon form. Specify clearly, *using the correct notation*, what row operation you are doing in each step.

$$\left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 0 & \boxed{3} & -9 & 12 \\ 0 & -2 & 3 & -7 \end{array}\right]$$

Goal: when pivoting on an element, want to make that element a 1 and all other elements in that column into 0, using row operations.

make this a 1

$$\left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 0 & \boxed{3} & -9 & 12 \\ 0 & -2 & 3 & -7 \end{array}\right] \xrightarrow{\frac{1}{3}\cdot R_2 \to R_2} \left[\begin{array}{cccc} 1 & 4 & -2 & 1 \\ 0 & \boxed{1} & -3 & 4 \\ 0 & -2 & 3 & -7 \end{array}\right]$$

next, make this a 0

now, make this a 0

$$\xrightarrow{2\cdot R_2 + R_3 \longrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 0 & \boxed{1} & -3 & 4 \\ 0 & 0 & -3 & 1 \end{array}\right]$$

$2R_2$:	0	2	-6	8
$+R_3$:	0	-2	3	-7
new R_3:	0	0	-3	1

$$\xrightarrow{-4\cdot R_2 + R_1 \longrightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & 10 & -15 \\ 0 & \boxed{1} & -3 & 4 \\ 0 & 0 & -3 & 1 \end{array}\right]$$

$-4\cdot R_2$:	0	-4	12	-16
$+R_1$:	1	4	-2	1
new R_1:	1	0	10	-15

We have completed pivoting on the boxed element!

Exercise 5. Aggie Success scholarship fund receives a gift of \$145,000. The money is invested in stocks, bonds, and CDs. CDs pay 3.3% interest, bonds pay 4.1% interest, and stocks pay 7.7% interest. Aggie Success invests \$30,000 more in bonds than in CDs. If the annual income from the investments is \$8072.50, how much was invested in each account? Round to the nearest cent.

Goal: Start with the question: want to find how much was invested in each account (stocks, bonds, and CDs). Define variables:

Let s = amount invested in stocks (measured in dollars)

b = amount invested in bonds (measured in dollars)

c = amount invested in CDs (measured in dollars)

Now, we translate the sentences into equations.

Total of \$145,000 invested in the three accounts $\Rightarrow$ $s + b + c = 145\,000$

CDs pay 3.3% interest, bonds pay 4.1% interest, stocks pay 7.7% interest and total income from investments is \$8072.50 $\Rightarrow$ $.077s + .041b + .033c = 8072.50$

They invest \$30,000 more in bonds than CDs $\Rightarrow$ bonds > CDs $\Rightarrow$ $b = c + 30\,000$

Organizing this into a system of equations with variables aligned, and then applying rref(), we'll get our solution:

$$\begin{cases} s + b + c = 145000 \\ .077s + .041b + .033c = 8072.50 \\ b - c = 30000 \end{cases} \longrightarrow \begin{array}{cccc} s & b & c & \# \\ \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 145000 \\ .077 & .041 & .033 & 8072.50 \\ 0 & 1 & -1 & 30000 \end{array}\right]$$

$$\xrightarrow{\text{RREF}} \begin{array}{cccc} s & b & c & \# \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 64687.5 \\ 0 & 1 & 0 & 55156.25 \\ 0 & 0 & 1 & 25156.25 \end{array}\right] \Rightarrow s = 64687.5,\ b = 55156.25,\ c = 25156.25$$

$\boxed{\text{Aggie Success invested \$64,687.50 in stocks, \$55,156.25 in bonds, and \$25,156.25 in CDs.}}$

Plan: first translate back into a system of equations and use it to determine the solution.

Exercise 6. Find the solution(s) to the systems corresponding to the following augmented matrices. If there are infinitely many solutions, write a parametric solution using t and/or s. If there is no solution, write "No Solution".

Using the variables x, y, and z, translating back,

1. $$\begin{array}{cccc} x & y & z & \# \\ \end{array}$$ $$\left[\begin{array}{ccc|c} 1 & 0 & 5 & -2 \\ 0 & 1 & -8 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \begin{cases} x + 5z = -2 \\ y - 8z = 9 \\ 0 = 0 \end{cases}$$

Need all three equations to be satisfied in order to have a solution. $0=0$ is true for any value of x, y, z.

Notice that we can solve $x+5z=-2$ for x: $x = -5z-2$ and similarly $y-8z=9$ for y: $y=8z+9$. What we notice is that once we know what z is, then x and y are determined by the above expressions. So, let $z=t$ (where t is any real number) and then

$x = -5z-2 \Rightarrow x = -5t-2.$

$y = 8z+9 \Rightarrow y = 8t+9.$

So, the solution is $(x,y,z) = \boxed{(-5t-2,\ 8t+9,\ t)}$ where t is any real #.

Notice: there are infinitely many solutions, one for any choice of t.

2. $$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{array}\right]$$

Same idea as above: translate back to equations first. Let's use x and y as our variables (notice, there are only two!)

$$\begin{array}{c} \begin{array}{ccc} x & y & \# \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{array}\right] \end{array} \longrightarrow \begin{cases} x = 5 \\ y = -1 \\ 0 = 1 \end{cases}$$

To get a solution, need all three equations to be satisfied.

The first equation is satisfied when $x=5$, the second when $y=-1$, but the last says $0=1$ which is False no matter what the values of x or y are. So, there is no way to satisfy all three equations simultaneously, hence the system has no solution.

Exercise 7. At a county fair, adult tickets sold for \$5.50, senior tickets for \$4.00, and child tickets for \$1.50. On the opening day, the number of child and senior tickets sold was 30 more than half the number of adult tickets sold. The number of senior tickets sold was 5 more than four times the number of child tickets. How many of each type of ticket were sold if the total receipts from the ticket sales were \$14,970?

Goal: (Start with question): Want to know how many of each type of ticket (child, adult, senior) were sold. Define variables.

Let a = # adult tickets sold

c = # child tickets sold

s = # senior tickets sold.

Translate sentences into equations:

Adult tickets sell for \$5.50, senior tickets for \$4, child tickets for \$1.50 and total sales were \$14970 :

$$5.50\,a + 4\,s + 1.50\,c = 14970$$

# of child and senior tickets sold was 30 more than half the # of adult tickets :

$$c + s = \frac{1}{2} \cdot a + 30$$

senior tickets sold was 5 more than four times the # of child tickets :

$$s = 4 \cdot c + 5$$

Clean-up the equations by lining up the variables on the left hand side, write as a matrix, apply rref().

$$\begin{cases} 5.50\,a + 4\,s + 1.50\,c = 14970 \\ -\frac{1}{2}a + s + c = 30 \\ s - 4c = 5 \end{cases} \longrightarrow \begin{array}{c} \begin{array}{cccc} a & c & s & \# \end{array} \\ \left[\begin{array}{ccc|c} 5.50 & 4 & 1.50 & 14970 \\ -\frac{1}{2} & 1 & 1 & 30 \\ 0 & 1 & -4 & 5 \end{array}\right] \end{array}$$

$$\xrightarrow{\text{RREF}} \begin{array}{c} \begin{array}{cccc} a & c & s & \# \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2050 \\ 0 & 1 & 0 & 845 \\ 0 & 0 & 1 & 210 \end{array}\right] \end{array} \Rightarrow$$

$a = 2050$, $s = 845$, $c = 210$

The county fair sold 2050 adult tickets, 845 senior tickets, and 210 child tickets.

Exercise 8. The supply and demand equations for the world famous cupcake shop *For Heaven's Cakes!* are known to be linear. When cupcakes are priced at \$2 each, the supplier produces 200 cupcakes. However, when cupcakes prices are increased to \$8 each, the supplier produces 400 cupcakes. Above \$8, consumers are not willing to buy a cupcake, and they would be willing to snatch up 800 cupcakes if the shop gave them out for free.

1. When the price is \$3.50 per cupcake, is there a shortage of cupcakes or surplus of cupcakes? The question is asking us if there are more cupcakes supplied or demanded when $p = 3.50$. We'll need the two equations.

supply: know: $(\overset{x_1}{200}, \overset{p_1}{2})$ and $(\overset{x_2}{400}, \overset{p_2}{8})$ are two points on the supply line, so using point-slope form,

$$p - p_1 = m(x - x_1) \Rightarrow p - 2 = \frac{8-2}{400-200}(x - 200) \Rightarrow \underbrace{p = 0.03x - 4}_{\text{supply equation}}.$$

demand: know $(\overset{x_1}{0}, \overset{p_1}{8})$ is the y-intercept and $(\overset{x_2}{800}, \overset{p_2}{0})$ is the x-intercept of the demand line. Using slope-intercept form,

$$p = mx + b \Rightarrow p = \frac{0-8}{800-0}x + 8 \Rightarrow \underbrace{p = -0.01x + 8}_{\text{demand equation}}$$

When $p = 3.5$, $\underset{\text{supply eqn}}{3.5 = 0.03x - 4} \Rightarrow 7.5 = 0.03x \Rightarrow x = 250$ cupcakes supplied

When $p = 3.5$, $3.5 = -0.01x + 8 \Rightarrow -4.5 = -0.01x \Rightarrow x = 450$ cupcakes demanded.

When cupcakes are \$3.50 each, more cupcakes are demanded that supplied, so there is a shortage of cupcakes.

2. How many cupcakes must be sold and at what price should they be sold in order to achieve market equilibrium? Plan: Market equilibrium happens when supply = demand

So, solve the system

$$\begin{cases} p = 0.03x - 4 \\ p = -0.01x + 8 \end{cases}.$$

Solving by addition, we get

$$\begin{cases} -p = -0.03x + 4 & \left(\begin{matrix}\text{multiply} \\ \text{first eqn} \\ \text{by } -1\end{matrix}\right) \\ p = -0.01x + 8 \end{cases}$$

$(+)$

$$0 = -0.04x + 12$$
$$-12 = -0.04x$$
$$300 = x$$

Sub $x = 300$ back into $p = 0.03x - 4$, get

$$p = 0.03 \cdot (300) - 4$$
$$p = 5$$

So, 300 cupcakes must be sold at \$5 each in order for the number of cupcakes supplied to be exactly the number of cupcakes demanded.

Exercise 9. Reduce matrix A below to reduced row-echelon form, without using a calculator. Then, check your answer using `rref()` in a calculator.

$$A = \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & -3 & 6 \end{array}\right]$$

Goal: use row operations to get the matrix in the form $\left[\begin{array}{cc|c} 1 & 0 & \# \\ 0 & 1 & \# \end{array}\right]$.

Start by pivoting on the top left element. It's done, because the element is 1 and the rest of the entries in its column are 0. Now move down one row and over one column to a_{22} and pivot on it.

now make it a 0

$$A = \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & \boxed{-3} & 6 \end{array}\right] \xrightarrow{-\frac{1}{3}\cdot R_2 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & \boxed{1} & -2 \end{array}\right]$$

make it a 1

$$\xrightarrow{2\cdot R_2 + R_1 \longrightarrow R_1} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -2 \end{array}\right]$$

$2\cdot R_2$	0	2	-4
$+ R_1$	1	-2	4
new R_1	1	0	0

Done!

Check using rref() on your calculator!

Multiple Choice Problems

Study tip: Write out all your work when you complete the multiple-choice problems.

A

Multiple Choice 1. A farmer will supply organic carrots to a restaurant. The restaurant's demand equation for organic carrot bunches is given by $p(x) = -0.1x + 6$, and its supply equation is given by $p(x) = 0.125x + 1.5$ where p is measured in dollars, and x is the number of bunches of organic carrots. What is the unit price at which a bunch of organic carrots should be sold to achieve market equilibrium? (Round to the nearest cent, if necessary.)

(a) \$4

(b) \$5.20

(c) \$20

(d) \$5.60

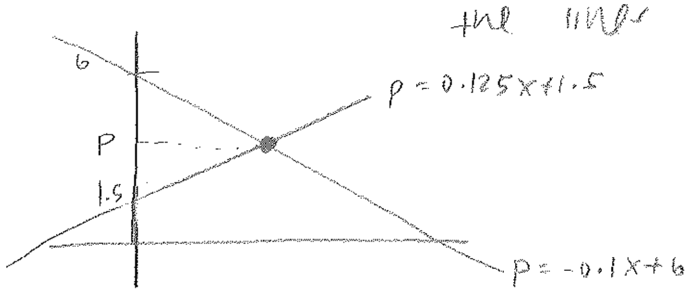
(e) None of these.

↳ supply = demand, so solve the system, but question only wants the value of p.

$$\begin{cases} p = -0.1x + 6 \\ p = 0.125x + 1.5 \end{cases}$$

Graph the lines in calculator, get that the lines intersect at $(\overset{x}{20}, \overset{p}{4})$.

So, @ market equilibrium, the unit price is \$4.



B

Multiple Choice 2. Leia is arranging for a concert to be held in the student center. The use of the hall will be free but they have several costs they will incur: They will have security costs (\$300), the cost of the main band (\$2,500), and the cost of the supporting band (\$420). They will also incur a cost of \$1 per person, since, on arrival, every ticket holder will be given a bottle of water. Leia has decided to sell tickets for \$15 per person. What is the break-even number of tickets for this event? Let x = # tickets sold

(a) 245 tickets

(b) 230 tickets

(c) 215 tickets

(d) 4 tickets

(e) None of these.

Want x for which $R(x) = C(x)$.

$C(x) = 1 \cdot x + 3220$

$R(x) = 15x$

$R(x) = C(x)$

$\Rightarrow 15x = x + 3220$

$\Rightarrow 14x = 3220$

$\Rightarrow x = 230$

Leia should sell 230 tickets to break even.

A

Multiple Choice 3. Which of the following matrices are in reduced row-echelon form (RREF)?

leading 1s, staircase, 0s in rest of column with leading 1s

yes

$$\text{I.} \quad \left[\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

NO

$$\text{II.} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

leading 1s don't form a "staircase"

NO

$$\text{III.} \quad \left[\begin{array}{ccc|c} 1 & 0 & \cancel{2} & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

need all 0s in columns with a leading 1.

(a) I only.

(b) II only.

(c) III only.

(d) I and II only.

(e) All three are in RREF.

E

Multiple Choice 4. Using x, y, and z as the variables, find the solution to the system that has the following augmented matrix in row reduced echelon form

translate to equations

$$\begin{array}{cccc} x & y & z & \# \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{cases} x = 1 \\ y = -3 \\ z = 0 \end{cases}$$

want: (x, y, z) that satisfy all equations, get $(x, y, z) = (1, -3, 0)$.

(a) There is no solution.

(b) $(x, y, z) = (1 + 4t, \ -3 - 2t, \ t)$, where t is any real number

(c) $(x, y, z) = (1 - 4t, \ -3 + 2t, \ t)$, where t is any real number

(d) $(x, y, z) = (-3, \ -1, \ 1)$

(e) $(x, y, z) = (1, \ -3, \ 0)$

A

Multiple Choice 5. Given a matrix below, which row operation must be performed to complete the process of pivoting about the entry in row one, column one?

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 3 \\ 0 & 6 & 0 & 1 \\ -2 & 0 & 3 & 2 \end{array}\right] \xrightarrow{???} \left[\begin{array}{ccc|c} 1 & -2 & -1 & 3 \\ 0 & 6 & 0 & 1 \\ 0 & -4 & 1 & 8 \end{array}\right]$$

make into a zero, use the row with the leading 1 to clear it out

(a) $2R_1 + R_3 \longrightarrow R_3$

(b) $3R_1 + R_3 \longrightarrow R_3$

(c) $3R_1 + R_2 \longrightarrow R_2$

(d) $-\frac{1}{2}R_3 \longrightarrow R_3$

(e) $2 + R_3 \longrightarrow R_3$

$$2 \cdot R_1 + R_3 \longrightarrow R_3$$

E

Multiple Choice 6. A line passes through the points $(-3, 2)$ and $\left(-3, \frac{5}{4}\right)$. Which of the following statements is <u>false</u>? (There is exactly one false statement).

(a) The line has x-intercept $(-3, 0)$. True

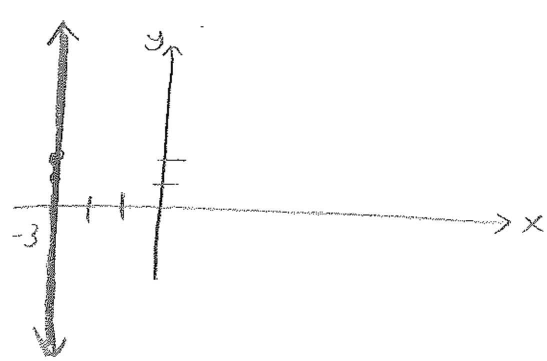
(b) $(-3, -24)$ is a point on the line. True

(c) The line has no y-intercept. True

(d) The equation of the line is $x = -3$. True

(e) The line has a slope of 0. FAISE, its slope is undefined

The line has points with x-coordinate equal to -3, so its equation is $X = -3$. It has an undefined slope, no y-intercept, points of the form $(-3, y)$ for any choice of y.



X= -3

C

Multiple Choice 7. Texas A&M has purchased new class shirts for the class of 2023 for the price of \$2,023. These shirts should last them for 4 years, after which we can assume a scrap value of \$23. Assume straight line depreciation. How much are the class shirts worth 2 years after they were purchased? Want: $V(2)$.

(a) \$500

(b) \$1523

(c) \$1023

(d) \$1000

(e) None of these.

Know: $(\overset{t_1}{0}, \overset{V_1}{2023})$ and $(\overset{t_2}{4}, \overset{V_2}{23})$ are two points on the linear depreciation graph.

Then $V(t) = \dfrac{23-2023}{4-0} t + 2023$

$V(t) = -500t + 2023$

$\Rightarrow V(2) = 1023$

The shirts are worth \$1023 after 2 years.

D

Multiple Choice 8. Let

$$M = \begin{bmatrix} 4 & -2 \\ 5 & -4 \\ x & 8 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}^{T} - 6 \begin{bmatrix} 3 & 4 \\ -2 & 0 \\ 6 & \frac{y+1}{3} \end{bmatrix}.$$

Find m_{32}.

(a) 8

(b) $2y - 22$

(c) $\frac{y}{3} - \frac{71}{3}$

(d) $2x - 2y - 26$

(e) m_{32} is not defined.

$$= \underbrace{\begin{bmatrix} 4 & -2 \\ 5 & -4 \\ x & 8 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}}_{2\times 2} - 6 \begin{bmatrix} 3 & 4 \\ -2 & 0 \\ 6 & \frac{y+1}{3} \end{bmatrix}$$

$$M = \underbrace{\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & x\cdot 2 + 8\cdot(-3) \end{bmatrix}}_{3\times 2} + \underbrace{\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & -6\left(\frac{y+1}{3}\right) \end{bmatrix}}_{3\times 2}$$

Don't need to compute the whole matrix to determine the entry we're asked for!

$$\Rightarrow m_{32} = 2x - 24 + \frac{-6y}{3} - \frac{6}{3}$$

$$= 2x - 24 - 2y - 2 = \boxed{2x - 2y - 26 = m_{32}}$$

A

Multiple Choice 9. A designer has a monthly fixed cost of \$10,000 for operation and a production cost of \$30 per design. She collects \$3,750 in revenue when she sells 50 designs. Find her profit when 223 designs are sold.

Want: $P(223)$. Need $P(x) = R(x) - (C(x))$.

(a) \$35

(b) -\$5,750

(c) \$20,035

(d) \$16,725

(e) None of these.

Well: $C(x) = 30x + 10\,000$

and $R(x) = p \cdot x$, with $R(50) = p \cdot 50 = 3750$
$\Rightarrow p = 75$.

So, $R(x) = 75x$.

Then $P(x) = R(x) - (C(x))$

$= 75x - (30x + 10\,000)$

$\Rightarrow P(x) = 45x - 10\,000$

So, $P(223) = 45 \cdot (223) - 10\,000 = 35$

Her profit from selling 223 designs is \$35.

A

Multiple Choice 10. Find the values of r, s and p that satisfy the equation below and then find the sum of the three values, $r + s + p$.

Need to simplify and identify r, s, p.

$$2\begin{bmatrix} 3 & -1 \\ 0 & r \end{bmatrix} + \begin{bmatrix} 1 & -1 & 7 \\ 0 & 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & -1 & p \\ 2 & 0 & s \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 29 & -14 \\ -9 & 3r \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 \\ 0 & 2r \end{bmatrix} + \begin{bmatrix} 1 & -1 & 7 \\ 0 & 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ p & s \end{bmatrix} = \begin{bmatrix} 29 & -14 \\ -9 & 3r \end{bmatrix}$$

$2 \times 3 \quad 3 \times 2$

(a) 5

(b) 7

(c) 11

(d) 3

(e) There is not enough information to determine the value of $r + s + p$.

$$\begin{bmatrix} 6 & -2 \\ 0 & 2r \end{bmatrix} + \begin{bmatrix} 1+1+7p & 2+0+7s \\ 0-3-2p & 0+0-2s \end{bmatrix} = \begin{bmatrix} 29 & -14 \\ -9 & 3r \end{bmatrix}$$

$$\begin{bmatrix} 8+7p & 7s \\ -3-2p & 2r-2s \end{bmatrix} = \begin{bmatrix} 29 & -14 \\ -9 & 3r \end{bmatrix}$$

Comparing entries, we need:

$$\begin{cases} 8+7p = 29 \Rightarrow 7p = 21 \Rightarrow p = 3 \\ 7s = -14 \Rightarrow s = -2 \\ -3-2p = -9 \Rightarrow -2p = -6 \Rightarrow p = 3 \\ 2r - 2s = 3r \Rightarrow -r = 2s \Rightarrow -r = 2(-2) \Rightarrow -r = -4 \Rightarrow r = 4. \end{cases}$$

$\Rightarrow$ so $r + s + p = 4 + (-2) + 3 = 5$.