RUTGERS

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Financial Time Series Project Report

Modeling GDP with Exogenous Variables

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Objective:

The primary objective of this project is to analyze the explanatory powers of various economic indicators on real GDP of the United States. This analysis of the GDP has implications for economists around the world as many of the same indicators are published across countries. GDP is often used as a measure of comparison across countries. Additionally, GDP is used as a determining factor for a recession. The real GDP is recorded on a quarterly basis, which is less frequent than the exogenous indicators we used. Our initial expectation is that the monthly values for these independent variables will be able to accurately explain the real GDP for the quarter that the values are contained within. In order to solve this periodicity difference we will use a simple interpolation for the monthly GDP values. The factors we believe will accurately explain the real GDP which we are testing in this analysis are the Consumer Price Index (CPI), Producer Price Index (PPI), Purchasing Manager Index (PMI), unemployment rate, 10-year T-bond returns, retail loan rates, retail sales, housing sales, oil prices, S&P 500, NASDAQ, and VIX. These indicators measure various aspects of the economy and should have different levels of impact on GDP through time. It is likely that the explanatory power of these variables will change through time. We will run these regressions on the historical data using a sequence of 5 years windows in order to determine the change in the Beta's of the variables across time.

Data Gathering:

GDP as well as the factors we are analyzing all have different historical timeframes. The table below shows the different timeframes we were able to obtain for GDP and each of the factors we used in our analysis:

Variables	Date From	Date To	Data Type
Real GDP	March, 1949	December, 2021	Quarterly
CPI	March, 1949	December, 2021	Monthly
PPI	March, 1949	December, 2021	Monthly
PMI	March, 1949	December, 2021	Monthly
Brent Oil	July, 1988	December, 2021	Monthly
VIX Index	January, 1990	December, 2021	Monthly
NASDAQ	December, 1984	December, 2021	Monthly
S&P 500	January, 1950	December, 2021	Monthly
Retail Sales	January, 1992	December, 2021	Monthly
Housing Index	January, 1985	December, 2021	Monthly
Unemployment Rate	March, 1949	December, 2021	Monthly
Bank Prime Loan Rate	March, 1949	December, 2021	Monthly
10-Year-T-Bond Price Return	March, 1949	December, 2021	Monthly

Consumer Price Index (CPI): Even though GDP is already adjusted for inflation, we believe that CPI might still be significant in explaining GDP returns at different time periods.

Producer Price Index (PPI): Increasing producer prices serve as a leading indicator for inflation in that the increased costs will be passed down to consumers.

Purchasing Manager's Index (PMI): Since PMI represents the manufacturing sector, when PMI increases, it means that there is growth in the manufacturing sector, which in turn would positively impact GDP.

Brent Oil: Oil prices have a vital impact of GDP. Since for most countries import and export has a huge impact of GDP growth rate.

VIX: We believe that there is a negative correlation between GDP and VIX. During times of economic downfall and uncertainty VIX tends to rise and during good times (bull period) VIX tends to be low.

NASDAQ: As an equity market with an emphasis on the technology sector, if the NASDAQ increases, it generally reflects good economic conditions, and therefore it should have a positive relationship with GDP.

S&P500: The equity markets often reflect economic conditions broadly and will decrease in crisis periods and increase in growth periods. The same relationship is expected for GDP.

Retail Sales: The sales of retail products are an input into GDP. For this reason, it is likely that Retail sales will have a positive relationship with GDP.

Housing Index: Similarly to retail sales, housing index is also an input of GDP. Therefore, it also has a positive relationship with GDP.

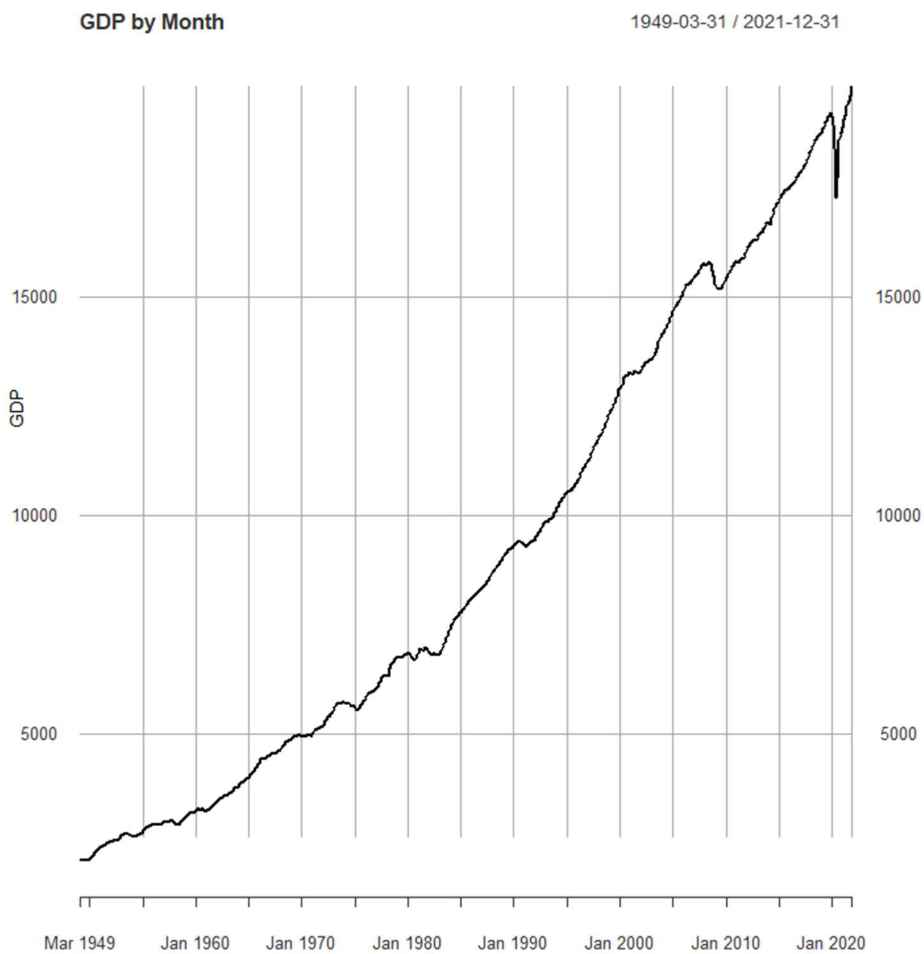
Unemployment: We believe that there will be a negative correlation between GDP and unemployment since as more people are unemployed, it generally reflects bad economic conditions, and therefore a decrease in GDP.

Prime Loan: Prime Loan Rate is the interest rate at which banks lend to customers. An increase in the bank prime loan rate will in turn decrease the purchasing power of people. This, in turn, will negatively affect GDP.

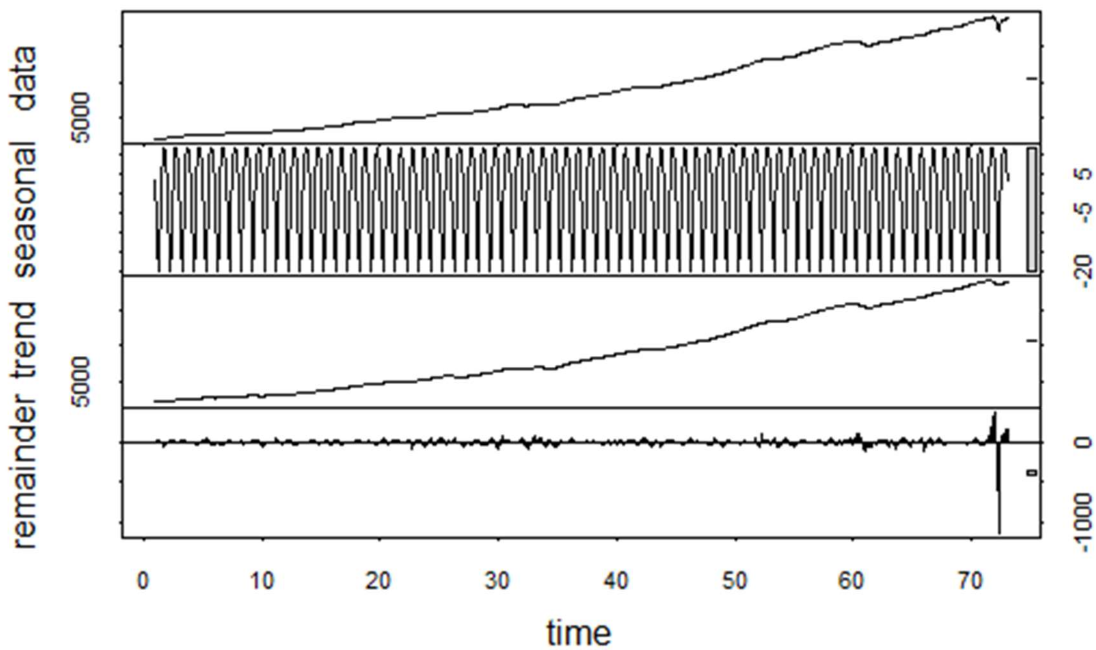
T-Bond (10y): The 10-year t-bond is expected to have a positive relationship with GDP because rates often increase in a strong economy and decrease in a bad economy as the FED implements monetary policy.

Pre-processing:

As mentioned previously, GDP data is at a quarterly level. However, all of the other variables are at a monthly level. For this reason, we needed to convert our GDP level data into monthly data. In order to do this, we used a straight-line interpolation for each quarterly GDP level. This gave us monthly GDP data that we could then use to do our time series analysis. After interpolating GDP, we obtained the following GDP by month:



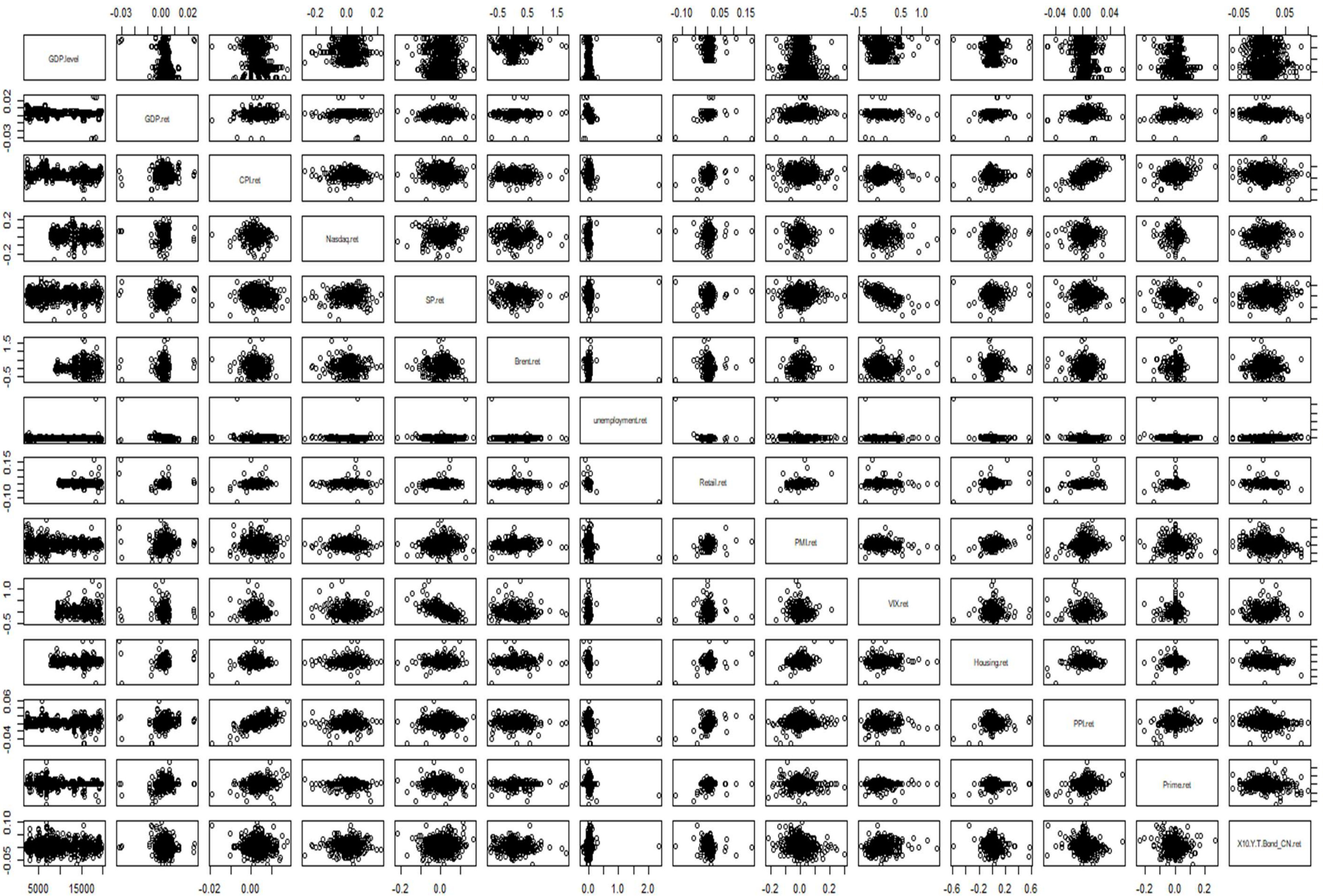
Next, we check our data for seasonality. As the plot of seasonal decomposition below shows, there is a seasonality component to GDP level. However, to correct for seasonality, we used GDP returns to perform time series analysis, which should be stationary. Therefore, there is no need to perform seasonal decomposition.



The next step was to convert all of our variables into returns in order to make our data stationary. In order to ensure our data was stationary, we proceeded to do an Augmented Dickey-Fuller test. As the table below shows, since the p-value of all of our variables are less than 0.05, we have enough evidence to reject the null hypothesis that they are not stationary, at significance level 0.05.

Variables	p_value
GDP.ADF	0.01
CPI.ADF	0.01
Nasdaq.ADF	0.01
SP.ADF	0.01
Brent.ADF	0.01
unemployment.ADF	0.01
Retail.ADF	0.01
PMI.ADF	0.01
VIX.ADF	0.01
Housing.ADF	0.01
PPI.ADF	0.01
Prime.ADF	0.01
X10.Y.T.Bond_CN.ADF	0.01

After checking for stationarity, we used a pairs plot to ensure there is no skewness. As the chart below shows, none of our variables are skewed except for unemployment.

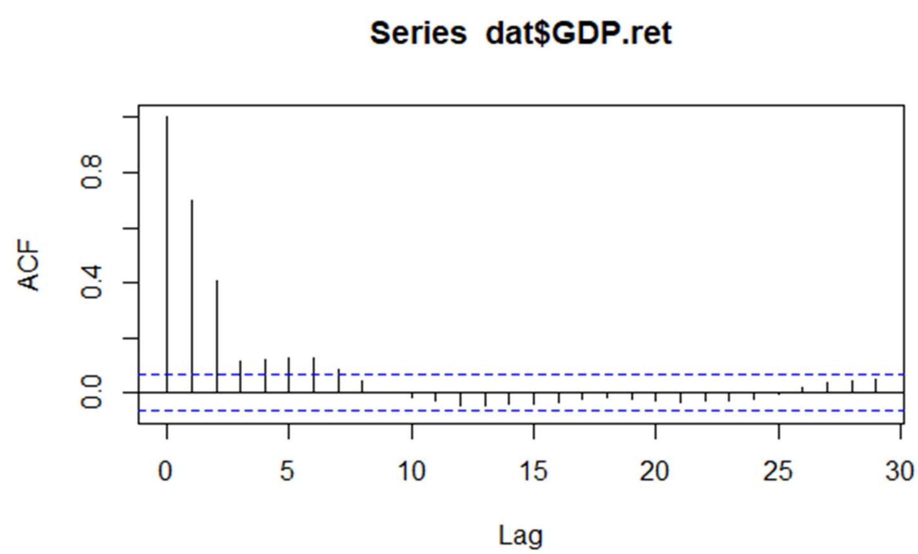

GDP.level
GDP.ret
CPI.ret
Nasdaq.ret
SP.ret
Brent.ret
unemployment.ret
Retail.ret
PMI.ret
VIX.ret
Housing.ret
PPI.ret
Prime.ret
X10.Y.T.Bond_CN.ret

Given the skewness of unemployment, we performed a log transformation to unemployment to remove the skewness.

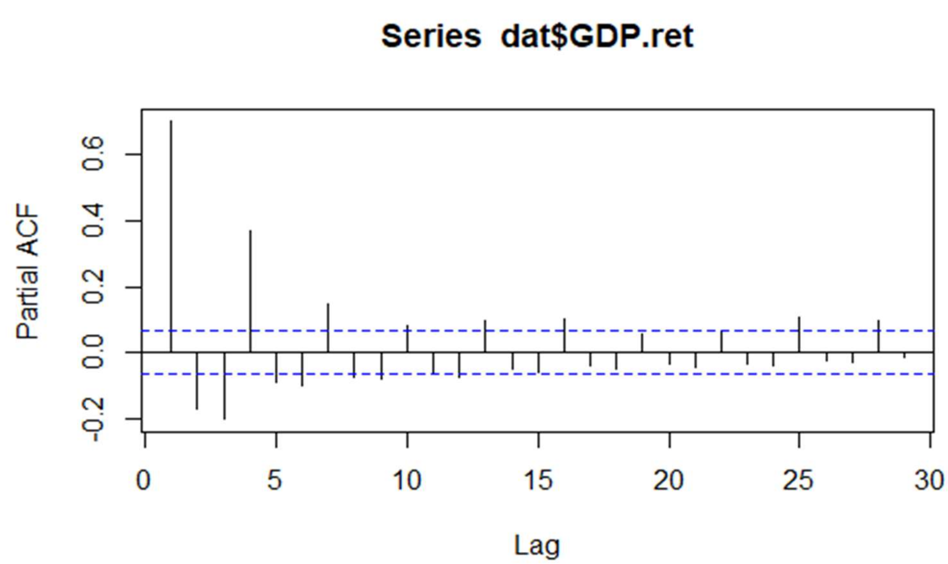
Modeling

In order to figure out which model to use, we first looked at the ACF and PACF plots.

ACF Plot:



PACF Plot:



As we can see in the ACF plot, the GDP returns have serial correlation up until lag 7. In addition, since the PACF plot tails off, this suggests that we should use an MA (7) process.

Based on the plots, we ran an MA (7) model:

```
Call:
arima(x = dat$GDP.ret, order = c(0, 0, 7), xreg = dat[c("CPI.ret", "SP.ret",
    "unemployment.ret", "PMI.ret", "PPI.ret", "Prime.ret", "X10.Y.T.Bond_CN.ret")])

Coefficients:
         ma1     ma2     ma3     ma4     ma5     ma6     ma7  intercept  CPI.ret  SP.ret  unemployment.ret  PMI.ret  PPI.ret  Prime.ret
      0.9847  0.9777  0.1042  0.0994  0.1047  0.0058  0.0010     0.0026  -0.0089  0.0016             6e-04   -6e-04   0.0078    -0.0011
s.e.  0.0352  0.0498  0.0576  0.0525  0.0561  0.0472  0.0362     0.0003   0.0093  0.0005             3e-04    6e-04   0.0034     0.0008
      X10.Y.T.Bond_CN.ret
                   0.0003
s.e.               0.0012

sigma^2 estimated as 5.103e-06:  log likelihood = 4030.55,  aic = -8029.1
```

Based on the significance of the MA t-stats, we can see that MA (3) to MA (7) are insignificant. Therefore, we ran an MA (2) model instead:

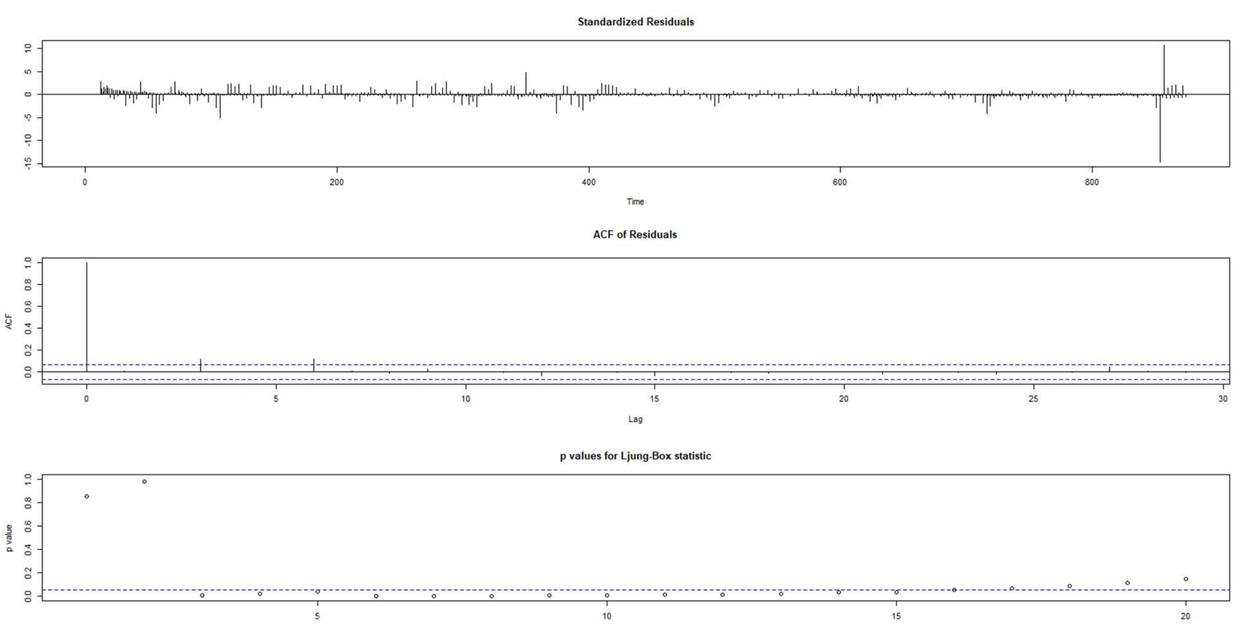
```
Call:
arima(x = dat$GDP.ret, order = c(0, 0, 2), xreg = dat[c("CPI.ret", "SP.ret",
    "unemployment.ret", "PMI.ret", "PPI.ret", "Prime.ret", "X10.Y.T.Bond_CN.ret")])

Coefficients:
         ma1     ma2  intercept  CPI.ret  SP.ret  unemployment.ret  PMI.ret  PPI.ret  Prime.ret  X10.Y.T.Bond_CN.ret
      0.9741  0.9676     0.0026  -0.0077  0.0015             7e-04   -6e-04   0.0072     -1e-03               0.0002
s.e.  0.0096  0.0128     0.0002   0.0091  0.0005             3e-04    6e-04   0.0034      8e-04               0.0011

sigma^2 estimated as 5.165e-06:  log likelihood = 4025.43,  aic = -8028.87
```

After running the MA (2), we did not have any insignificant MA components. We also noticed that in the MA (2) model, some of the exogenous variables became significant relative to the MA (7) model.

Next, we tested the adequacy of our model by checking the residuals using a Ljung-Box test. As we can see in the charts below, we have serial correlation in the residuals in almost all lags. Therefore, the model is not adequate.



Given the MA (2) model is inadequate, we used the auto-arima function to see which model it suggested as we knew we should also have an AR component as well.

The auto-arima function suggested we use an ARIMA (3,1,0) process:

```
> auto.arima(dat$GDP.ret)
Series: dat$GDP.ret
ARIMA(3,1,0)

Coefficients:
          ar1      ar2      ar3
      -0.0103  -0.0098  -0.4978
s.e.   0.0293   0.0293   0.0293

sigma^2 estimated as 6.752e-06:  log likelihood=3959.18
AIC=-7910.35   AICc=-7910.31   BIC=-7891.26
>
```

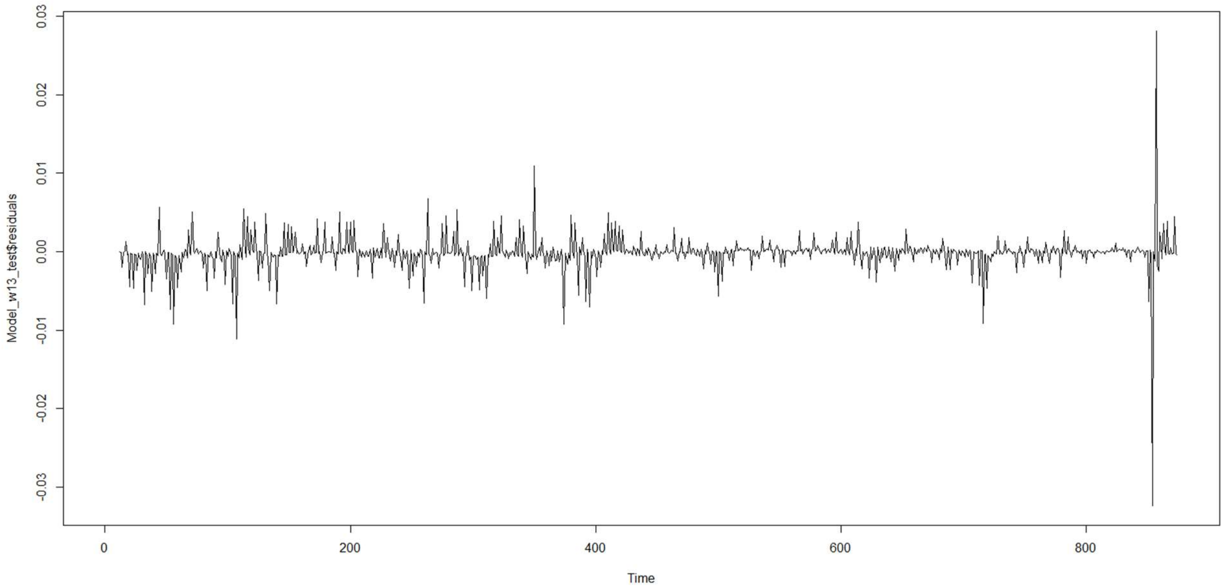
Using that as a starting point, we tried different MA components to see which one gave us the lowest AIC score with significant variables. Based on the results of our tests, we concluded that using ARIMAX (3,1,5) was the best model to use.

```
Call:
arima(x = dat$GDP.ret, order = c(3, 1, 5), xreg = dat[c("CPI.ret", "SP.ret",
    "unemployment.ret", "PMI.ret", "PPI.ret", "Prime.ret", "X10.Y.T.Bond_CN.ret")])

Coefficients:
          ar1      ar2     ar3     ma1     ma2      ma3      ma4      ma5  CPI.ret  SP.ret  unemployment.ret  PMI.ret  PPI.ret  Prime.ret
      -0.4687  -0.6378  0.1049  0.4581  0.6307  -0.9809  -0.4564  -0.6089  -0.0064  0.0017             6e-04   -7e-04   0.0075     -9e-04
s.e.   0.1472   0.1545  0.0400  0.1452  0.1578   0.0111   0.1409   0.1535   0.0103  0.0006             4e-04    5e-04   0.0036      8e-04
      X10.Y.T.Bond_CN.ret
                   0.0000
s.e.               0.0013

sigma^2 estimated as 5.064e-06:  log likelihood = 4028.1,  aic = -8024.2
```

We then checked our residual’s variance.



From the plot above, we can see unequal variance in the residuals; therefore, we ran a Dickey-Fuller test to check for stationarity.

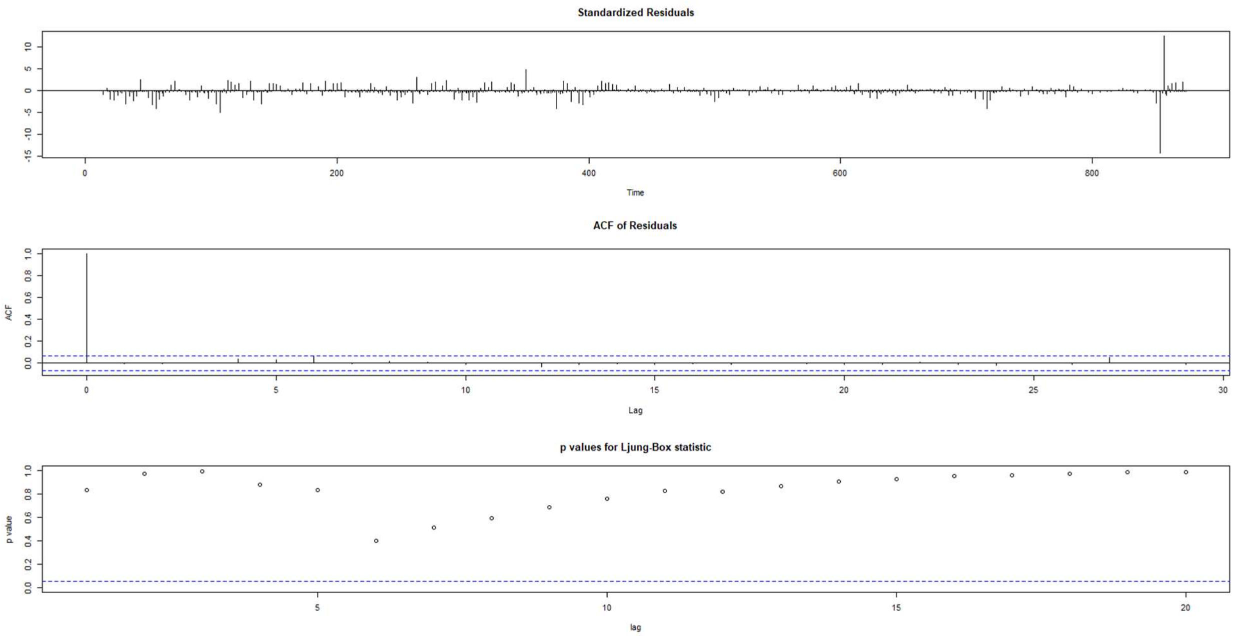
```
> adf.test(test)

        Augmented Dickey-Fuller Test

data:  test
Dickey-Fuller = -8.5589, Lag order = 9, p-value = 0.01
alternative hypothesis: stationary
```

Based on the results from the ADF test, the residuals are stationary at significance level 0.05.

We then double-checked TS-diagnostics again to ensure we have no autocorrelation at any lag:



Based on all the tests, we conclude that the best model is the ARIMAX (3,1,5).

Beta Analysis

The plot of residuals shows that the variance of the residuals fluctuates overtime. This suggests that the time series should be divided into separate time periods. This is potentially due to the development and change of the economy and markets over time. In order to test this theory, we have divided the date-range into multiple periods and re-run the analysis for each period independently (Model 1). With this analysis, the betas of the exogenous variables can better reflect the specific time periods. We have found that the significance of the variables changes across time.

Additionally, due to the increased availability of our dataset after 1985, we are able to build 2 models on top of model 1 to incorporate new regressors (Model 2 and Model 3). By comparing the coefficients of the same period across models, we can potentially identify some hidden relationships between regressors.

Model 1

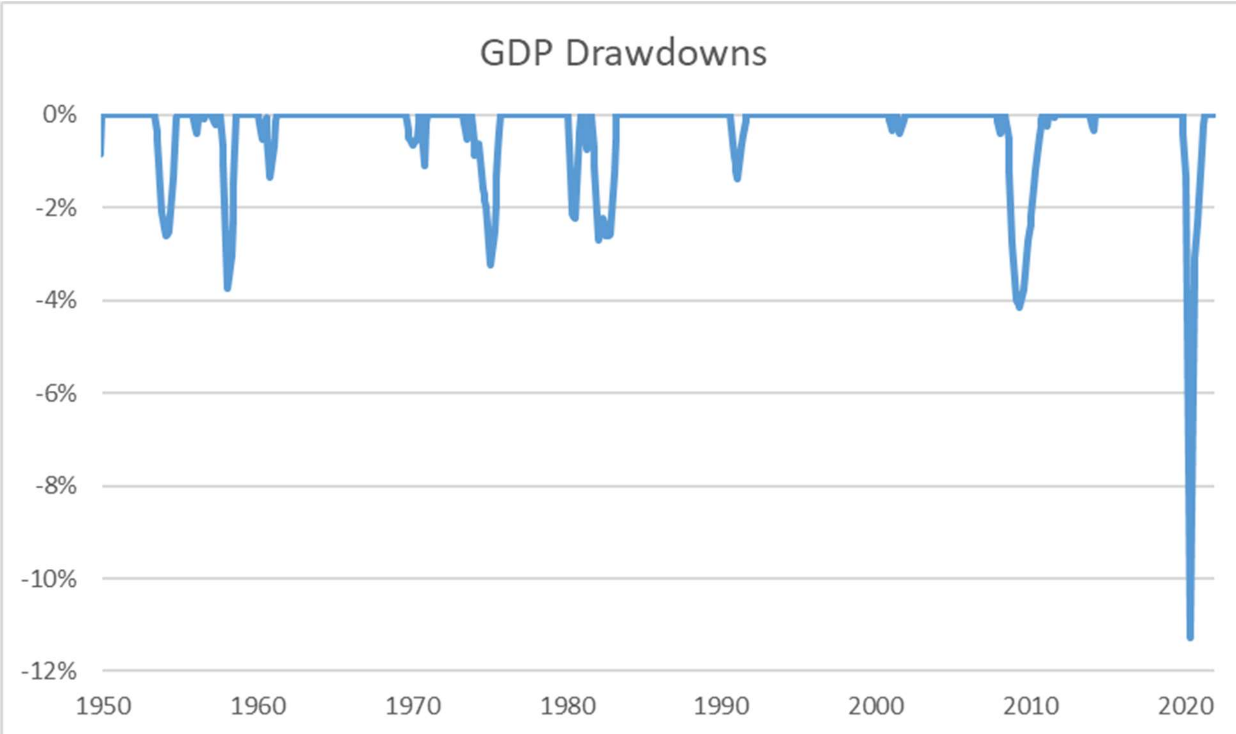
Table 2.1 shows whether the estimated parameter is significant or not for model 1.

Table 2.1

	CPI.ret	SP.ret	UEM.ret	PMI.ret	PPI.ret	Prime.ret	10B.ret
50-59	F	F	F	F	F	T	F
59-64	T	F	F	F	T	F	T
64-69	F	F	F	F	F	F	F
69-74	F	T	T	F	F	F	T
74-79	F	F	T	F	F	F	F
79-84	T	F	F	F	F	T	F
84-89	F	T	F	T	F	F	F
89-94	F	F	F	F	F	F	F
94-99	F	F	F	F	F	F	F
99-04	T	T	T	F	T	T	T
04-09	F	T	T	T	F	F	T
09-14	F	T	T	T	T	T	T
14-19	F	F	F	F	F	F	F

p-level: 0.05

Taking a look at the variables within a specific period, there are times where most of the variables are significant. For instance, from 2000 to 2015 many of the variables have significant betas. Specifically, from 2000 to 2005 the only insignificant regressor is PMI. Then from 2015 to 2020, the only insignificant regressor is CPI. This suggests that the economic indicators during this time had a strong relationship with the change in GDP. This is not the case for most time periods. Prior to 2000, most periods have three or fewer significant regressors. The 90s were a particular period of insignificant regressors. Not a single regressor has a significant beta during this period. These results may be suggesting that macroeconomic indicators are significant during periods where GDP has decreased. On the below GDP Drawdowns chart it can be noted that GDP does not draw down from 1965 to 1970, 1995-2000, or 2015 to 2020. These are three of the four periods where all the betas are insignificant. The remaining insignificant period is 1990 to 1995 where there was a drawdown at the beginning of the period.



We can also look at the specific variables across time to identify if their betas have changed behavior. There are three regressors that were significant for five of the twelve periods tested: S&P 500, unemployment, and 10-year t-bond. Unemployment is the most consistent regressor in regards to significance and direction of the beta. The betas are negative in all periods, regardless of significance. The next most consistent is the 10 year bond, which has a positive beta for four of its five significant periods. Then there is S&P 500, which has changed direction of the betas. Prior to 1990, the significant periods had negative betas. The remaining, more recent significant betas for S&P have been positive. The other regressors have been significant fewer times and with less consistency in the direction of the betas.

Model 2

Model 2 adds 4 new factors that were identified to have potential relationship with GDP rate of change (housing index returns, NASDAQ index return, Brent crude oil futures return, and VIX index return). The goal of this model is to 1) explore additional factors' relationship with GDP and how their relationship evolves over time and 2) provide clearer interpretations to model 1's factors by controlling for additional factors. Model 2 is specified to be identical to model 1(i.e. ARIMAX (3,1,5)) except for the 4 additional exogenous variables. In the similar manner as Model 1, Model 2 was estimated repeatedly on the subsets of 5 years, during the sample period 1994-2019.

Table 2.2

	CPI.ret	SP.ret	UEM.ret	PMI.ret	PPI.ret	Prime.ret	10B.ret	housing	Nasdaq.ret	brent.ret	VIX.ret
94-99	F	F	F	F	F	F	F	F	F	F	F
99-04	F	F	T	F	F	F	T	F	F	T	F
04-09	F	F	T	F	F	T	T	T	F	F	F
09-14	F	T	T	T	F	F	T	T	F	F	T
14-19	F	F	T	T	F	T	F	F	T	T	F

p-level: 0.05

As table 2.2 shows, during 1994-1999, none of the exogenous variables have statistically significant correlation with the GDP rate of change at 5% level. By comparing this to model 1's output, after controlling for the four variables, there is no change of significance in the original set of regressor.

During 1999-2004, Model2 shows unemployment rate, 10-year T-bond yield, and Brent crude oil futures return are significantly correlated with GDP rate of change. In contrast to model 1's output, CPI, S&P 500, PPI, and bank prime loan rate become insignificant after the additional set of regressors are controlled. This result might suggest that, during 99-04, CPI, S&P 500, PMI, PPI, and bank prime loan rate are correlated with GDP rate of change via one or more newly introduced regressors. In this case, it is likely to be the Brent crude oil futures return. Moreover, unemployment rate and 10-year T-bond yield are still significant factors.

During 2004-2009, Model2 shows unemployment rate, 10-year T-bond yield, bank prime loan rate, and housing index are significant factors. Notice that Brent crude oil futures return lost its explanatory power in this period. Compared to model 1, S&P 500 and PMI become insignificant after the additional set of regressors are controlled. One interesting observation is that the bank prime loan rate becomes significant after the additional factors are controlled. The reason might be the true effect of the bank prime loan rate is initially offset by one or more uncontrolled variables, and adding new variables "unmasks" the true correlation between bank prime loan rate and GDP.

During 2009-2014, Model2 shows unemployment rate, S&P, PMI, 10-year T-bond yield, VIX, and housing index are significant factors. Compared to model 1, bank prime loan rate and PPI become insignificant after the additional set of regressors are controlled.

During 2014-2019, Model2 shows that unemployment rate, PMI, bank prime loan rate, NASDAQ, 10 year T-bond yield, VIX, and Brent crude oil futures returns are significant factors. Notice that unemployment rate, PMI, and bank prime loan rate become significant compared to model 1. The possible explanation for this phenomena would be model 1 failed to uncover the true effect of these regressors' during this period since related factors are not controlled. Negative confounding effect appears in model 1.

By examining the model 2 over the whole sample period. One can observe the temporal changes of explanatory powers in the 11 factors. For instance, the correlation between unemployment rate and GDP rate of change has been consistently significant since 1999. Similarly, 10 year T-bond yield has been significantly associated with GDP from 1999-2014.

Model 3

Based on the similar logic, model 2 is further refined to allow for more factors. Specifically, the retail sales rate of change is taken into account. The same reason being some hidden relationships in model 2 might be discovered by controlling for retail sales. Identical model specification (ARIMAX (3,1,5)) is employed for model 3 as well. Due to the availability of the retail sale data, the scope of this model is limited to 1999-2019.

Table2.3

	CPI.ret	SP.ret	UEM.ret	PMI.ret	PPI.ret	Prime.ret	10B.ret	housing	Nasdaq.ret	brent.ret	VIX.ret	retail.ret
99-04	F	F	F	F	F	T	T	T	F	F	T	T
04-09	T	F	T	F	F	T	T	F	F	F	F	F
09-14	F	T	T	T	T	F	T	T	T	F	T	F
14-19	F	T	T	T	F	T	F	T	T	T	F	T

p-level: 0.05

The effect of introducing retail sales as the additional factors appeared in 1999-2004. Unemployment rate and Brent crude oil futures become insignificant when controlled for retail sales growth rate. The potential explanation might be that, during this particular period, unemployment rate and Brent crude oil futures are related to GDP through the channel of retail sales. Moreover, if retail sales rate is controlled, VIX, bank prime loan rate, and housing index become significant. This indicates that their relationship to GDP is offset by the uncontrolled variable.

During 2004-2009. Comparing the output of Model 2 and Model 3, the effect of CPI and bank prime loan rate becomes significant after the addition of retail sales. The effect of the housing index becomes insignificant.

During 2009-2014, the effect of PPI became significant, and effects of NASDAQ and Brent crude oil futures became significant after retail sales were introduced, compared to model 2 in the same period.

In the last sample period, S&P 500 and housing index became significant regressors of GDP rate of change as the additional regressor retail sales was introduced. Again, the inclusion of retail sales further reveal the true effect of S&P 500 and the housing index. Moreover, similar analysis can also be done across the sample period to identify temporal changes of explanatory powers in the 12 factors.

Conclusion

In this project, various time series analysis techniques were utilized to analyze macroeconomic factors and their relationship to the United States real GDP across time. Based on the nature of the data obtained, proper time series data exploration and pre-processing techniques like decomposition, stationarity transformation, skewness, and auto-correlation tests were used to ensure the model assumptions were met on the broadest possible level; ARIMA model with exogenous variables (ARIMAX) model was chosen to fulfill the research objective. Candidate model specifications were tested iteratively for model adequacy based on residual analysis.

Three final models with identical specifications were then fitted to the data sequentially. Model results allowed us to uncover the temporal changes in the relationship between a particular macroeconomic variable and GDP. Furthermore, some indirect, hidden channels of effects of variables were discovered statistically by comparing results across the 3 models, which provide direction for further economic research aiming to examine the logic behind these observations.

References

CPI: WRDS

Unemployment Rate: FRED

Brent Crude Oil Futures: Investing.com

S&P500: Yahoo finance

NASDAQ: Yahoo finance

VIX : Yahoo Finance

PMI: NASDAQ

Housing Market: https://www.nahb.org/news-and-economics/housing-economics/indices/housing-market-index

Retail sales: https://ycharts.com/indicators/us_retail_sales

PPI: U.S. Bureau of Labor Statistics - https://www.bls.gov/ppi/databases/

Bank Prime Loan Rate: FRED

10-Year T-Bond Price Return: WRDS