Finance: 22:839:611

Sections: 30 and 31

Analysis of Fixed Income

Fall 2021

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Assignment 05

For this Assignment, we will fit the Vasicek one-factor model to the data as we discussed in class. The attached data file has 16 columns. The first three columns are the date. Ignore the 4^{th}. Columns 5 – 10 give you the rate on a constant maturity swap with maturities of 2-, 3-, 5-, 7-, and 10 years. We will not be using this data for this Assignment. Columns 11 – 16 give you the (par) rate or yield to maturities on constant maturity treasuries with maturities ranging from 3-months, 2-, 3-, 5-, 7-, and 10-years. We will use data in these last 8 columns to fit the one-factor Vasicek model.

1. Fit the one-factor Vasicek model to all 650 weeks of data by solving for the model parameters of $\alpha,\ \beta, and\ \sigma$.
    a. Note that the short-rate itself is unobservable.
    b. So, in fitting the model, we will also need to solve for the value of the short-rate itself. To do this, assume that the CMT(0.25) is fit exactly by the one-factor Vasicek model as discussed in class

2. Display a table of the model parameters $\alpha, \beta, and\ \sigma$

3. Graph the time-series for the short-rate. Compute the sample mean and standard-deviation of the short-rate. Compare the sample mean and standard-deviation to the one implied by the model parameters of $\alpha, \beta\text{?}, and\ \sigma$.

4. Analyze the properties of the actual CMT rates and the fitted model.

5. Assume that you have invested $100 in par bonds with maturities ranging from 1 – 30 years. Assume that the 10-year par bond is the only one that can be used to hedge these positions (supposedly because the 10-year par bond is very liquid). Solve for the hedging portfolio that results in net 0 duration. That is, how much do you need to invest in the 10-year par bonds to hedge the duration of a $100 position in par bonds with maturities ranging from 1 – 30 years. (29 different answers, one for each maturity, and ignoring the 10-year par bond which is our hedging instrument)

6. Recalculate problem (5) above. But now hedge against a one-basis shock to the short-rate. For example: Lets assume we are trying to hedge a $100 position in the 5-year par bond with a $X position in the 10-year par bond. Let's assume that the short-rate goes up or down by 1 basis point. Compute (given model parameters of $\alpha, \beta, and\ \sigma$) how does the 1-basis point change in the short-rate impact the price of the 5-year and the 10-yar par bond. Calculate the value of $X so that the net change in the price of your hedged portfolio is 0. Compare the hedged portfolios from problem (5) above to those in problem (6).