1. a) $E[X] = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6}$

b) $E[X] = \frac{1}{36}(2) + \frac{2}{36}(3) + \frac{3}{36}(4) + \frac{4}{36}(5)$

$+ \frac{6}{36}(7) + \frac{5}{36}(8) + \frac{4}{36}(9) + \frac{3}{36}(10)$

$+ \frac{2}{36}(11) + \frac{1}{36}(12) = \frac{252}{36}$

c) k slots, so $\frac{k}{37}$ prob of winning

$E[X] = \frac{k}{37}\left(\frac{36}{k}\right) + \cancel{\frac{37-k}{37}(0)} - 1$

$= \frac{36}{37} - 1$

d) $\frac{1}{6}(1+2+3+4+5) + \frac{1}{6}\left(6 + \frac{1}{6}(1+2+3+4+5+6)\right)$

$= \frac{24.5}{6}$

e) Recursive, so

$R = \frac{1}{6}(1+2+3+4+5) + \frac{1}{6}(6+R)$

$= \frac{21}{6} + \frac{R}{6} \Rightarrow 4.2$

3. a) $\int_0^1 (x-.5)^2 dx = \int_0^1 x^2 - x + .25 dx$

$= \frac{1}{3}x^3 - \frac{1}{2}x^2 + .25x \Big|_0^1 = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{1}{12}$

b) $V[X] = \int (x - E[X])^2 f(x)\, dx$

$= \int (x^2 - 2xE[X] + E[X]^2) f(x)\, dx$

$= \int x^2 f(x)\, dx - \int 2x f(x) E[X]\, dx + \int E[X]^2 f(x)\, dx$

$= E[x^2] - 2E[x]^2 + E[x]^2$

$= E[x^2] - E[x]^2$

c) $V[a+bX] = \int (a + bX - E[a+bX])^2 f(x) dx$

$= \int (a + bX - a - bE[x])^2 f(x)\, dx$

$= \int b^2 (X - E[X])^2 f(x) dx$

$= b^2 \int (X - E[X])^2 f(x) dx$

$= b^2 V[X]$ ✓

d) $F_x\left(\frac{y-a}{b}\right) = \int_{-\infty}^{\infty} \frac{1}{2x} \exp\left\{ -\frac{1}{2}\left(\frac{(y-a)(b-\mu}{\sigma}\right)^2\right.$

form of norm dist $\Rightarrow \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{y-\mu'}{\sigma'}\right)^2\right\} dx$

4. a) $cov(X, y) = \int_y \int_x (x - E[X])(y - E[y]) f_x(x)$
$F_y(y)\, dx\, dy$

$= \int_y \left[\int_x x f_x(x) dx - E[x] \right]$
$y - (E[y]))\ F_y(y)\, dy$

$= \int_y [0]\,(y - E[y])\, F_y(y)\, dy$

$= 0$

b) Simulation in code file

5. a) $E[\varepsilon] = \int (x - E[x]) f(x) dx$

$= \int x f(x) dx - E[x]$
$= E(x)$

$= 0$

b) $V(\varepsilon) = \int (x - E[x] - E[x - E[x]])^2 f(x)dx$

$= \int (x - E[x] - 0)^2 f(x) dx$

$= V[x]$

c) $E[x] = E[E[x]] + E[\sigma_x \varepsilon]$

$= E[x] + \sigma_x E[\varepsilon]$

$= E[x] + \sigma_x 0$

$= E[x]$

d) $V[x] = \int E[x] + \sigma_x \varepsilon - E[E[x] + \sigma_x \varepsilon]^2 f(x)$

$= \int (E[x] + \sigma_x \varepsilon - E[x] - \sigma_x E[\varepsilon])^2 f(x)dx$

$= \int (\sigma_x \varepsilon - \sigma_x E[\varepsilon])^2 f(x) dx$

$= \int \sigma_x^2 (\varepsilon - E[\varepsilon])^2 f(x) dx$

$= \sigma_x^2 \int (\varepsilon - E[\varepsilon])^2 f(x) dx$

$= \sigma_x^2 V[\varepsilon] = \sigma_x^2$

6. $E[f_{x,h}(x)] = \frac{F(x+h) - F(x-h)}{2h}$

So $$\frac{F(x) + h(f(x) + \cancel{\frac{1}{2}h^2 f'(x)} + O(h^3)) - F(x) + h(f(x)) - \cancel{\frac{1}{2}h^2 f'(x)} - O(h)^3}{2h}$$

$\Rightarrow 2hf(x) + \cancel{O(h^3)} + hf(x) \cancel{- O(h^3)}$

$\Rightarrow \underline{f(x) + O(h^2)}$