Permutation and combination

Introduction to permutation and combination

Permutation and combination are all about counting and arrangements made from a certain group of data. You have a counting situation that requires formulas. If count is small you do not require formulas but if the count is large you require formulas for counting.
For example:
If you have to count 1 to 10, you can easily do this, but if you have to count up to 10255 it will require formulas.

Permutation: In mathematics, permutation relates to the act of arranging all the things of a set into some sequence or order.

Combination: Combinations can be defined as the number of ways in which 'r' things at a time can be selected from amongst 'n' things available for selection.

This chapter gives you counting situations that are mapped to the use of certain formulas and you have to know which formula is used in which situation.

Every P & C question will always end with asking you to "Find the numbers of ways?' doing something. Whenever you identify that the question is a P & C question, you 1st ask yourself if it is a selection question, distribution question, or it is an arrangement question then you go with an appropriate formula.

This chapter splits into 3 parts:
1. Selection **2.** Distribution **3.** Arrangement

Selection:

Selection can be defined as the number of ways in which r things at a time can be selected from amongst n things available for selection.
Let say select two people for 4 people A,B,C,D and count the number of different ways i which one can make the selection.

Count physically;
1st selection is AB, 2nd selection is AC, 3rd selection is AD, 4th selection is BC, 5th selection is BD, 6th selection is CD.
Hence the number of possible selections = 6.
But if you have to select 8 people from the 16 people. You can not physically count the number of selections because there are so many possible cases which are not possible to visualize. Hence in order to handle this situation you need the **nCr** formula.

This formula tells us if you have 'n' "distinct" objects from them select 'r' objects and you want to count the number of selections.

Thus, **nCr = n! /[r! (n–r)!]** ; where n ≥ r.

Formulae For Selection

Already we have discussed two formulae for selection,

1. **nCr =** $\frac{n!}{r!(n-r)!}$
2. **nCr = nC(n-r)**
3. **Total number of selections of zero or more things out of n different things**

   nCo + nC1 + nC2 + ... + nCn

   nCo + nC1 + nC2 + ... + nCn = 2^n

Questions on selection

Problem 1:

In a room there are 8 men, and 6 women and a handshake is held between 1 man and 1 woman.What is the number of handshakes?

Solution:

To visualize this take a small case, 3 men A,B,C and 2 women D, E in a room and they start handshake with each other.

Then, 1. A handshake with D.
2. A handshake with E.
3. B handshake with D.
4. B handshake with E.
5. C handshake with D.
6. C handshake with E.

Hence the total number of handshakes is 6.

This is similar to selecting a man and a woman. Number of handshake = 3C1×2C1 = 3 ×2 = 6.

Hence, in the given questions selecting a man out of 8 men and a woman out of 6 women, then the number of handshake will be 8C1×6C1 = 8×6 = 48.

Problem 2:

In a room there are a certain number of people and everybody handshake with each other. It was found that the number of handshakes was 153. Find the number of people in the room?

Solution:

Let's say in the room there are n people and everybody handshake with each other.

Total number of handshake = nC2 = 153

$$n \times (n-1)/2 = 153$$

$$n^2 - n - 306 = 0$$

Therefore n = 18,-17 but the number of people can not be -ve. So, n = 18 people.

Type 1: Question involving pre selection

Problem 1:

In a cricket team there are 16 players and select 11 players such that the captain is always selected. Find the total number of selections?

Solution:

Here given that the captain always be selected (i.e. preselected) now you have to select only 10 players from 15 players.
Therefore, selection of 10 from 15 = 15C10.

Type 2: Constraint based selection

Problem 1:

Out of 6 men and 4 women and you have to select a committee of 3 with at least one woman. In how many different ways can it be done?

Solution:

You have committee with at least 1 woman are,
1 women and 2 men or 2 women and 1 man or 3 women and no man
4C1×6C2 + 4C2×6C1 + 4C3×6C0

2nd method:

Committee of all men subtracted from total number of committee i.e. 10C3 - 6C3

From 10 people if you want to draw a committee of 3, will be 10C3.

If divide 10 people into 6 men and 4 women and you have to make committee of 3 and do not given any constraint in case you decide to do this problem using how many men and how many women then you have to write all possible committee i.e.

3 men & no woman or 2 men & 1 woman or 1 man & 2 women or no man & 3 women i.e.
6C3×4C0 + 6C2×4C1 + 6C1×4C2 + 6C0×4C3

Distribution of identical objects

Distribution can happen of identical objects or distinct objects.

Number of ways of distributing n identical things among r persons when each person may get any number of things = (n + r – 1) C(r–1)

Problem 1:

If you have 4 identical objects to give between two friends X & Y. What are the number of distributions?

Solution:

	X	**Y**
1st distribution	4	0
2nd distribution	3	1
3rd distribution	2	2
4th distribution	1	3
5th distribution	0	4

Therefore total number of distributions = 5

According to formula;

Here n = 4 and r = 2

So, the total number of distributions = (4+2-1)C(2-1) = 5C1 = 5.

Problem 2:

If x+y+z = 20 and x,y,z are whole numbers. How many solutions does x+y+z = 20 have?

Solution:

x+y+z = 20 is the same as distributing 20 objects between x,y and z.

Here n = 20 and r = 3.

So, the total number of solutions = (20+3-1)C(3-1) = 22C2 = 231 .

If x,y,z are natural numbers, in this case this formula does not work directly because in this case zero is not allowed.

Formulae For Arrangement

1. MNP Rule

It tells us if you have 3 tasks to do and there are M ways of doing the first thing, N ways of doing the second thing and P ways of doing the third thing then there will be M × N × P ways of doing all the three things together.

This formula is used to do problems on arrangements and also used for distribution of distinct objects.

Problem 1:

Shubham wants to go from Mumbai to Pune and Pune to Delhi and Delhi to Kolkata. There are 6 trains from Mumbai to Pune, 5 trains from Pune to Delhi and 8 trains from Delhi to kolkata. Find the total number of ways of travelling?

Solution:

$$\text{Mumbai} \xrightarrow[6\,T]{} \text{Pune} \xrightarrow[5\,T]{} \text{Delhi} \xrightarrow[8\,T]{} \text{Kolkata}$$

So, total number of ways of travelling = 6× 5 × 8= 240.

2. r! Formula

If you have 'r' distinct things and you want to place them in 'r' places, then the total number of ways = **r!**

Problem 1:

6 people ABCDEF and you want to sit them on 6 chairs. Find the total number of ways of sitting?

Solution:

The 1st chair can be filled by 6 people.
The 2nd chair can be filled by 5 people.
The 3rd chair can be filled by 4 people.
The 4th chair can be filled by 3 people.
The 5th chair can be filled by 2 people.

The 6th chair can be filled by 1 person.
So the total number of ways = 6! = $6 \times 5 \times 4 \times 3 \times 2 \times 1$= 720.
r! Nothing but the MNP rule used for ‘r’ distinct objects in ‘r’ places.

3. r! modified for arrangement of identical objects

Number of arrangements of ‘n’ things out of which P1 are alike and are of one type, P2 are alike and are of a second type and P3 are alike and are of a third type and the rest are all different
= **n!/ P1! P2! P3!**

For example:
AAA BB CCC and you want to be placed in 8 places.
AAA are 3 alike things, BB are two alike things and CCC are three alike things.

So, total number of ways = 8!/3!$\times$ 2! $\times$ 3!

4. nPr formula

nPr = number of arrangements of ‘n’ distinct things taken r at a time.
nPr = n!/ (n – r)!; n ≥ r

For example:
Six people ABCDEF arrange in 3 places = 6P3 = 6!/3! = 120.
Similar situation is getting handled using the MNP rule. So, according to MNP rule, 6 people arranging in 3 places = $6 \times 5 \times 4$ = 120

The Relationship Between Permutation & Combination:

When we look at the formulae for Permutations and Combinations and compare the two we see
that,
nPr = r! × nCr
i.e. The arrangement of r things out of n is nothing but the selection of r things out of n followed by the arrangement of the r selected things amongst themselves.

Generic Questions On Arrangements

Problem 1:
In how many ways 7 people A,B,C,D,E,F,G are arranged in a straight line in 7 places such that A is always in the middle?

Solution:
Middle place is fixed by A and the remaining 6 places are filled by 6 people.
So, total number of ways = 6!.

Problem 2:
In how many ways 7 people A,B,C,D,E,F,G are arranged in 7 places such that no two of A,B,C are together?

Solution:
A,B,C in 3 places is 3! And D,E,F,G in 4 places is 4!
Total number of ways = $3! \times 4!$.

Problem 3:
In how many ways 7 people A,B,C,D,E,F,G are placed in 7 places such that A & B are together?

Solution:
A&B are together. So, A&B counted as one person and 5 people separately, effectively there are 6 people.
Arrangement of 6 people is 6! And arrangement of AB = 2!.
Therefore total number of ways = $6! \times 2!$.

Questions On Word Formation

Type 1: Word formation question
Problem 1:
How many words can be formed with the word PATNA, LUCKNOW and JAIPUR which have
1. No restrictions.
2. Total number of new words
3. Start with the first letter.
4. Start and end with vowels.

Solution:
PATNA
1. Total number of letters - P,T,N occurs once while A occurs twice.
So, the total number of words that can be formed = 5!/2! = 60

2. Total number of new words = 60 - 1 = 59.

3. We can arrange only 4 letters (as place of P is restricted) in which A occurs twice.
So, the total number of words that can be formed = 4!/2!

4. In the word PATNA in which we have 2 vowels(A,A).
So, the total number of words that start with A and end with A = 3!

LUCKNOW
1. Total number of distinct letters = 7.
So, the total number of words that can be formed = 7!

2. Total number of new words = 7! - 1.

3. We can arrange only 6 letters (as place of L is restricted)
So, the total number of words that can be formed = 6!

4. In the word LUCKNOW in which we have 2 vowels(U,O). Arrangement of two vowel = 2!
So, the total number of words that can be formed = 2!×5!

JAIPUR

1. Total number of distinct letters = 6.
So, the total number of words that can be formed = 6!

2. Total number of new words = 6! - 1.

3. We can arrange only 5 letters (as place of J is restricted)
So, the total number of words that can be formed = 5!

4. In the word JAIPUR in which we have 3 vowels(A,I,U). We have to select 2 vowels and arrange them amongst 1st and last place = 3C2×2!and also arrange 3 consonants and 1 vowel = 4!
So, the total number of words that can be formed = 3C2×2!×4!.

Type 2: Dictionary position question
Problem 1:
What is the dictionary position of the word RUPAJI that can be formed by letters of the word JAIPUR?

Solution:
1st arrange all the letters of the word JAIPUR in alphabetically order for reference.
A-I-J-P-R-U
Number of words starting with A = 5!
Number of words starting with I = 5!
Number of words starting with J = 5!
Number of words starting with P = 5!
Number of words starting with R = 5!
Number of words starting with U = 5!

You are looking for the word RUPAJI. In this word letter 'U' will come only after the letter 'R'. so, the words starting with letter 'U' are not considered. RUPAJI one of the word inside words start with letter 'R'

Before the words start with the letter 'R' we have words = 5! + 5! +5! + 5! = 480 words.

Words start with the letter 'R'

Number of words starting with RA = 4!

Number of words starting with RI = 4!
Number of words starting with RJ = 4 !
Number of words starting with RP = 4!
Number of words starting with RU = 4!

RUPAJI one of the word inside the words start with letters ‘RU’
Before the words start with the letters ‘RU’ we have words = 480 + 4! + 4! + 4! + 4! = 480 + 96 = 576 words.

Words start with the letter ‘RU’
Number of words starting with RUA = 3!
Number of words starting with RUI = 3!
Number of words starting with RUJ = 3!
Number of words starting with RUP = 3!

RUPAJI one of the word inside the words start with letters ‘RUP’

Before the words start with the letters ‘RUP’ we have words = 480 + 96 + 18 = 594 words.
Remaining lettres A,I,J six words can be form from A,I,J

AIJ,AJI,IAJ,IJA,JAI,JIA. So out of six the 2nd word AJI will complete the word RUPAJI
Therefore the position of the word RUPAJI = 594 + 2 = 596.

Questions On Number Formation

Forming numbers with and without replacement:
Problem 1:
How many 4 digit numbers can be formed by using digit 1,2,3,4,5,6 and 7 with replacement of digit allowed?

Solution:
To forming a 4 digit number with replacement;

1st place can be filled with any of the 7 digits.
2nd place can be filled with any of the 7 digits.
3rd place can be filled with any of the 7 digits.
4th place can be filled with any of the 7 digits.
Therefore total number of ways = $7 \times 7 \times 7 \times 7 = 7^4$

Limit based question:
Problem 1:
How many 4 digit numbers can be formed by using digit 0,1,2,3,4 such that the numbers are not greater than 4000?

Solution:
In this question we can think that numbers are not greater than 4000. So, numbers are starting with digit 1,2 and 3. First place cannot be filled with zero because it makes 4 digit numbers in 3 digit numbers.

Numbers starting with 1
1st place can be filled with 1 digit i.e 1.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits
So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.

Numbers starting with 2
1st place can be filled with 1 digit i.e. 2.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits
So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.

Numbers starting with 3
1st place can be filled with 1 digit i.e. 3.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits

So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.
And number 4000 itself will get counted.
Therefore total 4 digit numbers = 125+125+125+1 = 376.

NOTE : When in number formation nothing is mentioned about weather repetition allowed or not, in that case default is repetition allowed.

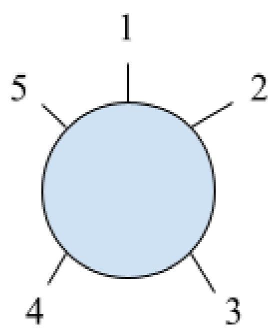
Circular Arrangements

In this chapter you just need to understand a couple of things. On a circle every position is the same, unlike straight lines every position is different.

1. **Number of ways of placing 'r' distinct objects on 'r' places is equal to (r-1)!**
2. **If there is a reference point on a circle no need to do minus 1.**

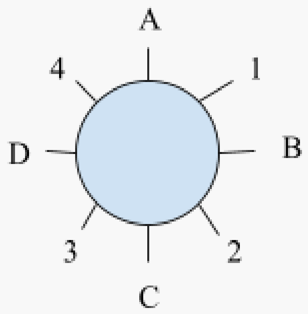
For example:
How many ways of arranging 5 people on seats in a circular table (seat 1 is a reference point)?

Solution:



Seat 1 is a reference point. So, the number of arrangements = 5!

Problem 1:
In how many ways 4 Indian and 4 European sit in alternate places around a circle?



Solution:

Let say 4 Indian sit in A,B,C,D places around a circle. Now you have a circle with a reference point.

Number of ways of arranging 4 Indian = (4-1)! = 6 and Number of ways of arranging 4 European = 4! = 24

Therefore total number of ways = 6 $\times 24 = 144$.

3. **'N' objects arrange around a circle where clockwise is equal to anticlockwise, then the number of arrangements = (n-1)!/2**