HCF and LCM

Introduction to HCF and LCM

HCF:

The greatest number which divides each of the two or more numbers is called **HCF or Highest Common Factor**. It is also called the **Greatest Common Measure(GCM)** and **Greatest Common Divisor(GCD)**.

LCM:

Least Common Multiple(LCM) is a method to find the smallest common multiple between any two or more numbers. A common multiple is a number which is a multiple of two or more numbers.

How to find HCF

1. **By Prime factorization method:**
   i) Write down the ***prime factors*** of the given numbers.
   ii) Write down the ***prime factors*** which are common to both.
   iii) And products of the common factors will give you HCF of the numbers.

Example: Find the HCF of 150 & 375.
Answer: 75
Solution:
Step 1: Write down the prime factors of the given numbers.
$150 = 2 \times 3 \times 5 \times 5$
$375 = 3 \times 5 \times 5 \times 5$
Step 2: Write down the prime factors which are common to 150 & 375.
3,5 & 5.
Step 3: Products of the common factors are $3 \times 5 \times 5$
Hence, HCF = 75.

Note: To find HCF of more than 2 numbers
Let us take three numbers a,b & c.
To find their HCF, what you need to do is, first find out the prime factors of each of the numbers.
Say,

$$a = 2^3 \times 3^4 \times 5^1 \times 11^2$$
$$b = 2^5 \times 3^5 \times 5^2 \times 7^3$$
$$c = 2^6 \times 3^4 \times 5^3 \times 7^2$$

HCF (a,b,c) →All common prime factors with their ***lowest*** available power.
Thus, HCF of a,b,c will be

$$HCF=2^3 \times 3^4 \times 5^1$$

2. **By division method:**

If we were given two numbers, then

- First, divide the large number by a small number.
- If the remainder is left, then divide the first divisor by remainder.
- If the remainder divides the first divisor completely, then it is the HCF or highest common factor of the given two numbers.
- If the remainder does not divide the first divisor completely, then repeat the steps.

Example: What is the HCF of 120 and 100.
Answer: 20
Solution: Divide 120 by 100.
120/100 → 1 and remainder is 20
Now, divide the first divisor 100 by the first remainder 20
100/20 → 5 and remainder is 0.
Therefore, 20 is the HCF of 120 and 100.

3. **By shortcut method:**

When you talk about the common factor of two numbers X & Y. then the common factor has to leave the same remainder "zero". Which means
Let two numbers X & X+12, the only numbers that will have the possibility of leaving the same remainder zero would be factors of 12.

$$1 \times 12$$
$$2 \times 6$$
$$3 \times 4$$

All the common factors of these two numbers would come in the factors of 12, they can't come from any outer range. And hence, if all the common factors of X & X+12 are inside the factors of 12, So the HCF of X & X+12 would also come from the factors of 12. Which means HCF of X & X+12, can only be one of (1,2,3,4,6 & 12) these numbers.

Example: Find the HCF of 38 & 50?
Answer: 2
Solution:50-38=12, factor of 12 are 12,6,4,3,2&1.

12 →Does Not divide 38, so this is not HCF of these two numbers.
6→Does Not divide 38, so this is not HCF of these two numbers.
4 →Does Not divide 38, so this is not HCF of these two numbers.
3 →Does Not divide 38, so this is not HCF of these two numbers.
2 →Divide 38, so this is the HCF of these two numbers.
Then it is obvious it will divide 38+12 and hence HCF is 2.

Note: To find HCF of more than 2 number by shortcut method

Let us consider the numbers are x, x+12, y, z.
For finding the HCF of these numbers, take the differences between the numbers. Here, many differences are possible, but you have to choose the smallest difference between any pair of these numbers.
Write the factors of that number and the HCF of all these numbers would be from the factor list.

Sometimes you might want to go for prime number difference instead of the smallest difference,
For example, suppose the numbers are 44,56 & 93.

So, 56 - 44=12
93 - 56=37
93 - 44=49

Here, a better difference to take here is 37 because 37 is a Prime number, then the factors of 37 are either 1 or 37. So, HCF, in this case, is either 1 or 37. 37 does not divide any number, so, the HCF=1.

Example: A nursery has 363,429 and 693 plants respectively of 3 distinct varieties. It is desired to
place these plants in straight rows of plants of 1 variety only so that the number of rows required is the minimum. What is the size of each row and how many rows would be required?

Answer: 45 rows
Solution: The size of each row would be the HCF of 363, 429, and 693.
Difference between 363 and 429 =66.
Factors of 66 are 66, 33, 22, 11, 6, 3, 2, 1.
66 need not be checked as it is even and 363 is odd. 33 divides 363, hence would automatically divide 429 and also divides 693.
Hence, 33 plants are the correct answer for the size of each row.
For the number of rows that would be required = Minimum number of rows required = 363/33 + 429/33 + 693/33 = 11 + 13 + 21 = 45 rows.

How to find LCM

1. **By prime factorization:**

Step1: Find the prime factor of two numbers a & b.
Step2: Write down all the prime factors that appear at least once in the numbers a & b.
Step3: Write all the prime factors with their highest power.
Step4: Products of all the prime factors with their highest power will give you LCM of a & b.

Example: Let's have two numbers 12 & 80.
Answer: 240
Solution: **Step1:** List the prime factors
$12=2 \times 2 \times 3$
$80=2 \times 2 \times 2 \times 2 \times 5$
Step2: Write down all the prime factors that appear, at least once in the numbers: 2,3,5.
Step3: Write all the prime factors with their highest power: $2^4 \times 3^1 \times 5^1$
Step4: The LCM = $2^4 \times 3^1 \times 5^1$
= 240.

2. **By shortcut method:**

As you saw LCM is the product of the highest power of all the prime factors, but that process would be very tedious, especially when the numbers are small.
When the numbers are small the logic of LCM builds around the ***Co-prime numbers***.
Co-prime Number: Two numbers are Co-prime to each other when they have no common factor among each other.

For example: (6, 13), (7, 11), (9, 19) etc.

Three numbers are Co-prime to each other when pairwise, each pair is Co-prime.
For example: Three numbers be a,b and c are Co-prime when,
a,b are Co-prime,
a,c are Co-prime,
& b,c are Co-prime.
All three pairs should be Coprime to each other, only then, a, b and c will be Co-prime.

NOTE: When a & b is Co-prime then the HCF should be 1.

Some important points about the Co-prime numbers:

(i) Two consecutive natural numbers are always co-prime (Example 5, 6; 82, 83; 749, 750 etc.)
(ii) Two consecutive odd numbers are always co-prime (Examples: 7, 9; 51, 53; 513, 515 etc.)
(iii) Two prime numbers are always co-prime (Examples: 13, 17; 53, 71 and so on)
(iv) One prime number and another composite number (such that the composite number is not a
multiple of the prime number) are always co-prime (Examples: 17, 38; 23, 49 and so on, but note
that 17 and 51 are not co-prime, as 51 is a multiple of 17)

Shortcut for LCM:
Step1: When the numbers are co-prime, then LCM is simply their product.
So, 7, 9 and 11 are co-prime, The LCM is $7 \times 9 \times 11$.

Step2: What to do when you have a mix of prime and Co-prime.

NOTE: (i). LCM has to be the multiple of HCF.
(ii). For any two numbers a & b, Product of two numbers $(a \times b) = LCM \times HCF$
(this formula is valid for two numbers)

Example: LCM of four numbers 42, 44, 18, 25.
Answer: 69300
Solution:(i) If you see any co-prime put them down in your LCM. Here you can see 18 & 25 are Co-prime (and 25,42 ; 25,44 are also Co-prime).
(ii) LCM of these numbers starts with $18 \times 25 \times$.... And
(iii) Now the logic of LCM should contain all the other numbers from the given numbers.
(iv) Out of the LCM, you should be able to construct 42 and 44 also.
(v) The factor of 42= $2 \times 3 \times 7$. Inside 18 you have 2 & 3, But you don't have 7 in 25 and 18. To construct 42, you should have a 7 in your LCM. (LCM=$18 \times 25 \times 7$....)
(vi) The factor of 44= $2 \times 2 \times 11$. Inside 18, you have one 2, but there is no 11 and other 2 in this LCM; so, to construct 44 you need to introduce 2 & 11 into the LCM.
So, LCM will be = $18 \times 25 \times 7 \times 2 \times 11$.

HCF and LCM of fraction

HCF of a Fraction:

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

LCM of a Fraction:

$$\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Example: LCM & HCF of 1/2 ,5/7 and 8/11 are:
Answer: LCM= 40/1
HCF= 1/(2 x 7x 11)

Solution:LCM = LCM(1,5,8) / HCF(2,7,11)
HCF = HCF(1,5,8) / LCM(2,7,11)
So,
LCM= 40/1
HCF= 1/(2 × 7 × 11)

Example: Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

a) 4
b) 10
c) 15
d) 16

Answer: d) 16
Solution: L.C.M. of 2, 4, 6, 8, 10, 12 is 120.
So, the bells will toll together after every 120 seconds(2 minutes).
In 30 minutes, they will toll together (30/2)+1 16 times.