Ratio, proportions, variation

Introduction to Ratio, proportion, variation

The ratio is a method to compare quantities. When you compare the quantities the first thing that comes to mind is that the quantities should be in the same unit.

Example: 20kmph and 30kmph are the two quantities which are in the same unit.
So,
Ratio = 20/30 = 2/3
= 2:3.
If quantities are in different units, then they can't be compared.
For example:
20 km and 18Rs/kg are the two quantities in different units. So, these two quantities can't be compared.

Proportion basically equates to two or more ratios. When two ratios are equal, the four quantities composing them are said to be proportional. Thus if a/b = c/d, then a, b, c, d are proportional.
The proportion can be written as;
a:b::c:d, that means a is to b as c is to d. Also, it can be written as a:b = c:d.
NOTE: The terms a and d are called the extremes while the terms b and c are called the means.

- ➢ If four quantities are in proportion then the product of extremes and product of means are equal.
  Let a,b,c and d are in proportion. Then ; $a \times d = b \times c$ i.e, ad = bc.
- ➢ Sometimes the mean proportion is the same.
  Let say a:b::b:c is referred to as a continued proportion. Thus, the product of extremes is equal to the product of means.

$a \times c = b \times b$ i.e $b^2 = ac$ or we can say that $b = \sqrt{ac}$. So, b is called a geometric mean
between a & c.

NOTE: Mean proportion is always the geometric mean of extremes.

Example: Let us say 2:3::a:33. What is the value of a?
Solution: the product of extremes = the product of means
$2 \times 33 = 3 \times a$
a = 22.

Some properties of ratio and proportion

Ratio:

1. If we multiply the numerator and the denominator of the ratio by the same number, the ratio does not change.
   Thus, multiplying 'm' by both numerator and denominator of the same ratio gives,
   a/b = ma/mb
   For example :
   For Ratio = 3/4
   Multiply the numerator and the denominator by 6 i.e 3/4 = $(3 \times 6)/(4 \times 6)$= 18/24
   Here 3/4 is the **lowest/basic form** of a ratio. This lowest/basic form gives an infinite number of ratio values.
   For example :
   3/4=6/8=15/20=18/24=.....................so on.

NOTE: In the lowest form of ratio the numerator and the denominator are always coprime numbers.

2. If we divide the numerator and the denominator of a ratio by the same number, then the ratio does not change. Thus;
   Dividing 'd', by both numerator and denominator or ratio a/b gives,
   a/b = $(a \div d)/b \div d)$

3. Dividing one ratio by another ratio can be expressed as a new ratio.
   Let the 2 ratios be 'a/b' and 'c/d'. Therefore,
   (a/b)÷(c/d) OR
   a/b:c/d = ad/bc
   For example:
   2/3:4/5 = $(2 \times 5)/(4 \times 3)$
   = 10/12.

4. The multiplication of two ratios a/b and c/d gives:
   a/b×c/d = ac/bd.

5. If a/b = c/d = e/f = k then;
   (a+c+e)/(b+d+f) = k.
   For example : 2/3 = 4/6 = 10/15 = 200/300 = k then,
   (2+4+10+200) / (3+6+15+300) = 216/324 = 2/3.

6. When numbers are added in both numerator and denominator to maintain equality, then the numbers should have the same ratio as that of the original ratio in which we are adding.
   Let say ratio = 400/800
   $400/800 = (400+2)/(800+4)$ i.e $a/b = (a + c)/(b + d)$ if and only if $c/d = a/b$.

7. In a ratio, if we add two numbers such that their ratio is larger than the original ratio, then the final ratio becomes larger.
   Let say a ratio = 400/800.
   $(400+5)/(800+7)$.Here, ratio 5/7 is larger than the original ratio($400/800 = 1/2$).
   i.e $c/d > a/b$ then $(a + c)/(b + d) > a/b$
   i.e. $(400+5)/(800+7) > 400/800$
   In case you add a smaller ratio than your final ratio will be less than the original ratio.
   Let say a ratio = 400/800.
   $(400+3)/(800+7)$. Here, the ratio of 3/7 is smaller than the original ratio.
   i.e. $c/d < a/b$ then $(a + c)/(b + d) < a/b$
   i.e. $(400+3)/(800+7) < 400/800$

8. If, some ratio is in fractional form, then to convert it into an integral ratio, multiply all fractions by LCM of their denominators.
   For example:
   1/2: 3/5: 7/6 to convert this ratio into integral ratio, multiply all the fractions by LCM of their denominators (2,5&6). LCM(2,5,6) = 30.
   i.e $30/2 : (3\times30)/5 : (7\times30)/6 = 15:18:35$.

Proportions:

1. **Invertendo:** If $a/b = c/d$ then $b/a = d/c$
2. **Alternando:** If $a/b = c/d$, then $a/c = b/d$
3. **Componendo:** If $a/b = c/d$, then $(a+b)/b = (c+d)/d$.
4. **Dividendo:** If $a/b = c/d$, then $(a-b)/b = (c-d)/d$.
5. **Componendo and Dividendo:** If $a/b = c/d$, then $(a + b)/(a - b) = (c + d)/(c - d)$

Chain Ratio

Chain ratio is a ratio in which one to next, next to the next, and next to next ratios are given. Let say A: B, B: C, and C:D are chain ratios given and convert these ratios into A:B: C:D.
For example :
A:B = 3:5, B:C = 7:8 then, convert chain ratios into a single ratio A:B:C.
Here B is a common element in both the ratios. To equate 5 & 7, take LCM of 5 & 7.
LCM(5,7) = 35. To make common element 35. Multiply the ratios A: B and B: C by 7 and 5 respectively. Thus, A: B will become 21:35, and B: C will become 35:40. B is the same in both cases.
Hence A: B: C is 21:35:40.

Example: If there are 4 and 5 ratios in this case the LCM process will become tedious.
Let us say, A:B = 3:5, B:C = 7:8 and C:D = 9:13. Find A:B:C:D?
Solution :
We have already calculated A: B: C is 21:35:40 and we have C:D is 9:13. C is a common element in both the ratio. To equate 40 and 9, take LCM of 40 & 9.
LCM(40,9) = 360. To make common element 360. Multiply the ratio A: B: C and C:D by 9 and 40 respectively. Thus; A: B: C will become 189:315:360 and C:D will become 360:520. C is the same in both cases.
Hence A:B:C:D is 189:315:360:520.
If D: E is also there this will become even longer to do because you will have to take LCM 3 times.

Methods to solve chain ratio problems

Bypass method:

There is a bypass to this without doing LCM to convert it into a single ratio.
Let us say A: B is N1:D1, B: C is N2:D2, C:D is N3:D3, and D: E is N4:D4. Find A:B: C:D: E.
The value of A would correspond to the multiplication of all numerators. So, A would be N1N2N3N4.
The value of B would be D1N2N3N4.
The value of C would be D1D2N3N4.
The value of D would be D1D2D3N4.
And the value of E would be D1D2D3D4.

A B C D E

N1N2N3N4 : D1N2N3N4 : D1D2N3N4 : D1D2D3N4 : D1D2D3ND

Example: A: B is 3:5, B: C is 7:8, and C:D is 9:13. Find A:B: C:D.
Solution: A B C D
N1N2N3: D1N2N3: D1D2N3: D1D2D3
A B C D
$3 \times 7 \times 9 : 5 \times 7 \times 9 : 5 \times 8 \times 9 : 5 \times 8 \times 13$
A B C D
189 : 315 : 360 : 520

Example: There are three sections A, B, and C in a school. Section A & B have a student ratio of 5: 7. Section B & C have a student ratio of 8: 11. The number of students in section C is 154. What is the total no of students in all sections?
Solution: Given A: B is 5:7 and B: C is 8:11. A:B: C will be;
A B C
$5 \times 8 : 7 \times 8 : 7 \times 11$
A: B: C is 40: 56: 77.
The number of students in section C is 154.
Assume A= 40x, B = 56x and C=77x.
We have C = 154. Thus; 77x = 154, x = 2.
Students in section A = 40×2 = 80. Students in section B = 56×2 = 112.
Total number of students in all sections = 80 + 112 + 154 = 346.

Multiplier logic

It is an important construct of thinking in a ratio situation.
In the last topic, we had a question about 3 sections in a class. In that, we had a ratio 40: 56: 77. And the number of students in section C was 154.
We assumed 3 numbers were 40x,56x, and 77x.
We had C = 154. Thus; 77x = 154,
x = 2. **Here x = 2 is a multiplier.**
Students in section A = 40×2 = 80. Students in section B = 56×2 = 112.
Total number of students in all sections = 80 + 112 + 154 = 346.

1st way in which a multiplier could be communicated to you:
Sometimes this multiplier will be communicated to you by giving you an individual value of one of the given numbers.
Let us say 3 children have toys in the ratio 3:4:9. The child with the largest number of toys is 36 toys.
i.e 9 is 36, Which means a multiplier of 4.
Hence, the number of toys with each child will be 3×4 = 12, 4×4 = 16 and 9×4 = 36.

2nd way in which a multiplier could be communicated to you:
Let us say the salary of three people is 5:7:13 and the total is 225.
The total ratio 5: 7: 13 is 25. And the total in the actual number running parallel to the given ratio is 225. i.e 25 is 225, which means a multiplier of 9.
Hence the numbers are $5\times9 = 45$, $7\times9 = 63$ and $13\times9 = 117$.

3rd way in which a multiplier could be communicated to you:
If a ratio of 5: 7: 13 is given. If the difference between the smaller two numbers is 18.
Difference between smaller two numbers = 7-5 = 2. So, 2 is 18, which means a multiplier of 9.
Hence the numbers are $5\times9 = 45$, $7\times9 = 63$ and $13\times9 = 117$.