Averages

Introduction to Averages

An average is a number that measures the central tendency of a set of numbers. In other words, it is an estimate where the centre point of the set of numbers lies. Average is also known as the mean.

Another meaning of average is, the average is that single number, that can replace each of the given numbers present in the set with the average number and still get the same total.

For Example:

The average of 5 numbers 11, 14, 17, 18, and 20 is:

Average = $(11 + 14 + 17 + 18 + 20)/5$ = 80/5=16

This means that if you replace all the 5 numbers with 16 (average number), even then the sum will be 80, there would be no change in the total.

How to find the average

1. **Formula**

$$\text{Average} = \frac{\text{Sum of the numbers}}{\text{Number of numbers}}$$

In mathematics, the average is equal to the sum of the set of numbers divided by the numbers of values in the sets

2. **Assumed Average Approach**

We already know that Average is that one number that can replace each of the numbers in a group of numbers and still keep the same total.

By using this concept the assumed average approach is a bypass for getting the average of the numbers.

Let us say 6, 10, 7 & 5 are the 4 numbers. So, Average is;

Average = (6 + 10 + 7 + 5)/4 = 7
7 can replace all the 4 numbers.
If you see the deviation between the numbers and their average (between left column and the right column), the direction should be: left column - right column

Left column	Right column	Deviation
6	7	- 1
10	7	+3
7	7	0
5	7	- 2

The net sum of all these deviations is 0 (-1+3+0-2 = 0). This means the average value is correct.

The following are some steps to calculate the correct average from the assumed average:
Step1. You have to assume an average.
Step2. Calculate how much the given numbers deviate from the assumed average.
Step3. Calculate the sum of all the deviations (i.e. Total deviation).
Step4. Calculate the average deviation with the help of the following formula :

$$\text{Average Deviation} = \frac{\text{Total Deviation}}{\text{Number of numbers}}$$

Step5. Now, the correct average will be equal to the sum of the assumed average and average deviation. i.e.
Correct average = Assumed average + Average Deviation.

Example: Let 37, 75, 83, 94 & 46 are 5 numbers. You don't know the average and you want to find out the average for these numbers without doing the sum of these numbers.
Answer: 67

Solution: **Step1.** For this example, assume an average of letting us say, 60.
Step2. Deviation calculation
60 to 37 there is a deviation of -23.
60 to 75 there is a deviation of +15.
60 to 83 there is a deviation of +23.

60 to 94 there is a deviation of +34.
60 to 46 there is a deviation of -14.

Step3. Total deviation = -23+15+23+34-14 = 35.
Step4. Average deviation = 35/5 = 7.
Step5. Correct average = 60+7 = 67.

You can assume any value of average, but the assumed value should be nearly equal to the one of the given value for simple calculation.

In the above example, let us say you assume the average to be 70 instead of 60.
Step1. Assumed average = 70.
Step2. Deviation calculation

70 to 37 there is a deviation of -33.
70 to 75 there is a deviation of +5.
70 to 83 there is a deviation of +13.
70 to 94 there is a deviation of +24.
70 to 46 there is a deviation of -24.

Step3. Total deviation = -33+5+13+24-24 = -15.
Step4. Average deviation = -15/5 = -3.
Step5. Correct average = 70+(-3) = 67.
So you can see that the answer will be the same irrespective of what average you take.

The benefit of the assumed average method is that it is much faster in the case when numbers are bigger and they are clustered (for example,a group of numbers between the range of 300 to 400), then your calculation is much faster than what you normally do.

NOTE : 1. Average of first n natural numbers = (n+1)/2
2. Average of first n even numbers = n+1
3. Average of first n odd numbers = n

Standard rules in average problems

1. **Standard Language In Average**

Every chapter has standard language inside it. You can also observe that there is some standard language inside the Average chapter.

You can understand the standard language on average with the help of some examples. So, here we understand the standard language with the help of the following examples:

Example 1: **Statement :** The average of 5 numbers is 12.
Explanation: When you see this statement two reactions come to mind. The 1st one is that, $5 \times 12 = 60$. and 2nd is that, add 12 five times i.e. 12 + 12 + 12 + 12 + 12 = 60.
So, there are two approaches to tackle this statement.

Example 2: **Statement 1:** The average age of 24 students and principal is 15.
Solution: When you look at the statement you realize that there are 25 people with an average of 15. Your reaction is $25 \times 15 = 375$, that means the total age of 25 people is 375.

2. **Standard Situation In Averages**

Situation 1:

This chapter is about identifying those standard situations that are generally asked in exams with the help of some examples. In the above 3 examples (example No. 2, 3 & 4) one thing is common that one new number is entering into the group.
In example 2. A Group of 24 students and principal added to it.
In example 3. Group of 9 innings and added 10th innings to it.
In example 4. It has an 11 monthly income and added 12th-month income into it.
Here the situation is entering a new number.

Example: Let us say you got 5 numbers with an average of 12 and 6th number entered and the average of all 6 numbers becomes 15. What is the 6th number?
Answer:
Solution: There are two ways of solving this type of question.

The 1st way;

6th number = Total of 6 numbers - Total of 5 numbers

$= 6 \times 15 - 5 \times 12 = 30$

The 2nd way;

The addition of a 6th number increases the average by 3.

12 + 3 = 15
12 + 3 = 15
12 + 3 = 15
12 + 3 = 15
12 + 3 = 15

The +3 appearing 5 times is due to the 6th number, which is able to maintain the average of 15 first, and then 'give 3' to each of the first 5.
Hence, the 6th number in this case = **maintain + contribute**

$= 15+3 \times 5 = 30$

Standard situation 2:
The 2nd standard situation is about what happens if more than one number enters. This situation can also be explained with the help of examples:

Example: Let us say 8 numbers with an average of 10. Two new numbers enter due to that the average becomes 13. What is the average value of these two numbers?
Answer: 25
Solution: There are two approaches to solve this question;

1. Total difference approach:
   Total of two numbers = $10\times 13 - 8 \times 10 = 50$
   Thus, the average of two numbers = 50/2 = 25.

2. 2nd approach;
   The addition of 2 numbers, increases the average by 3.

Average of 8 numbers	Average after 2 number entry
10	13
10	13
10	13
…	…
…	…
8 times…	10times…

Average of two number = **maintain + average contribution**

= 13 + (3×8)/2

= 13 + 24/2 = 25

The contribution 24 has to be brought by these two together.
You can understand this situation like when you and your friend go to a hotel and you are going to be paid equally. Bill comes out of 24, then you will divide the bill into 2. So, each individual will pay 12.

Example: After 120 innings batsman has an average of 55. And he realizes that he is going to play 180 innings more and he wants an average of 100 runs per inning. So what should be the average of the remaining 180 innings?
Answer: 130
Solution: Average increases by 45runs.

Average in first 120 innings	Average after 300 innings
55	100
55	100
55	100
…	100
…	…
120 times…	300 times…

180 new innings maintain the average 100 and make the average contribution in 120 innings.
Average of remaining 180 innings = maintain + average contribution

$$= 100 + (45\times120)/180$$
$$= 130$$

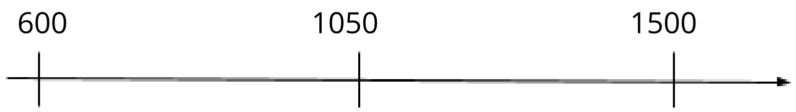
Standard situation 3:
The 3rd standard situation that you will see in the average chapter is replacement of a number.

Example: A set of 5 numbers with an average of 13 and one number is replaced. Average is increased by 4. The outgoing number is 32, then find the replaced number?
Answer: 52
Solution: In this situation, there is an outgoing number and an incoming number and the average changes by 4 for 5 numbers. The difference in the total = (5×17) - (5×13) = 20.
Incoming number how much larger = (change in average) ×(number of numbers)
= 4 ×5 = 20
If the average increases then it is obvious that the incoming number is larger.
Incoming number - outgoing number = difference in total
Incoming number = 20 + 32 = 52.

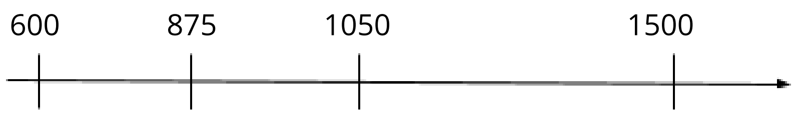
Concept of weighted average

The concept of a weighted average can be understood with the help of an example.
Suppose I had to buy a T-shirt and jeans and let us say that the average cost of a T-shirt was 600, while that of jeans was 1500.
In such a case, the average cost of a T-shirt and jeans would be given by (600 + 1500)/2 = 1050.
This can be observed on the number line as:

(midpoint) = answer.



From the figure it is observed that the average occurs at the midpoint of the two numbers.
Now, let us try to modify the situation:
Suppose I had to buy 3 T-shirts and 1jeans. In such a case I would end up spending (600 + 600 + 600 +1500) = 3300 in buying a total of 4 items. So,
Average = 3300/4 = 825. Clearly, the average has shifted.
On the number line :



It is clearly visible that the average has shifted towards 600 (which was the cost price of the T-shirts, the larger purchased item.)
In a way, this shift is similar to the way a two-pan weighing balance shifts when the weights are put on it. The balance shifts towards the pan containing the larger weight.
Similarly, in this case, the correct average (875) is closer to 600 than it is to 1500. Since, this is very similar to the system of weights, we call this a weighted average situation.

Formula for weighted average:
Let say, we have k groups with averages A1, A2 ... Ak and having n1, n2 ... nk elements then the weighted average is;

$$A_w = \frac{n1A1+n2A2+n3A3+\ldots\ldots+nkAk}{n1+n2+n3+\ldots\ldots+nk}$$

Situations involving weighted average

Situation 1: Purchasing two kinds or k varieties of something and mixing them together, to form composite.

Example: Suppose I purchase 30Rs/kg rice and 70Rs/kg rice in the ratio 2:3. What is the average price of rice?

Solution : Average price = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30 , A2 = 70 , n1 = 2 , n2 = 3

Average price = $(2\times 30+3\times70)/(2 + 3)$

$= 270/5 = 54$Rs/kg.

Situation 2:

Example: Let's say you drive a car 30km/hr and 70km/hr and drive it for 2hr and 3 hr respectively. Find the average speed?

Solution : Average speed = (total distance)/(total time)

$= (2\times 30+3\times70)/(2 + 3)$

$= 270/5 = 54$ km/hr.

Situation 1 and 2 are the same but the story is different.

Situation 3:

Example: Let say you invest 2 lac and give 30% return. Investment of 3 lac rupees, give70% return. What is the average % return?

Solution : Average % return = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30% , A2 = 70% , n1 = 2 Lac , n2 = 3 Lac

Average % return = $(2\times 30+3\times70)/(2 + 3)$

$= 270/5 = 54$ %.

Situation 4:

Example: There are two sections, in section 1 there are 20 students who scored 30 marks on an average in exam, while in section 2 there are 30 students who scored 70 marks on an average in exam. What is the average marks of both the sections?

Solution : Ratio of the quantities 20:30 = 2:3

So, Average marks = $(2\times 30+3\times70)/(2 + 3)$

= 270/5 = 54.

This situation can be modified into Boys and Girls in a class with ratio 2:3 and Boys average marks is 30 and Girls average marks is 70. So what are the average marks of the class?
Average marks of the class will be 54.

Situation 5: Alloys and Mixture

Example: Let say two water and milk solutions of 2L and 3L, In one solution milk is 30% and other solution milk is 70% respectively. Mix both the solutions then what is the % of milk in the mixture?
Solution : % of milk in the mixture = (2× 30+3×70)/(2 + 3)
= 270/5 = 54%.
Instead of water milk solution, we can take gold and copper alloy, 2kg gold and copper alloy with 30% of gold & 3kg gold and copper alloy with 70% of gold. If both the alloys are mixed and a new alloy is formed, then what is the % of gold in the new alloy?
Solution : % of gold in the new alloy = (2× 30+3×70)/(2 + 3)
= 270/5 = 54%.

These are some important situations that are used in weighted averages.