

# Logarithms

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## Introduction To Logs:

Questions based on this chapter are not so frequent in aptitude exams. You will find some questions based on logs, to solve those questions you have to learn some basic formulae.

### Definition of “log”:

Let 'a' be a positive real number and  $a^b = c$ . then 'b' is called the logarithm of 'c' to the base 'a' and written as  $\log_a c$  and vice versa, if  $\log_a c = b$ , then  $a^b = c$ .

NOTE: Log of a negative base is not defined.

$\log_a c = b$  is possible if and only if  $a > 0$  and  $c > 0$ .

### Formulae for log:

1.  $\log_b a + \log_b c = \log_b (a \times c)$
2.  $\log_b a - \log_b c = \log_b \frac{a}{c}$
3.  $\log_a 1 = 0$  for all  $a > 0$
4.  $\log_a a = 1$  for all  $a > 0$
5.  $\log_c a^b = b \log_c a$

### Base change rule:

Till now all the formulae are in logarithm with the same base. However, there are a lot of situations in Logarithm problems where you have to operate on logs having different bases. Those situations are:

1.  $\log_y x = \log_z x / \log_z y$
2.  $\log_y x = \log_x x / \log_x y = 1 / \log_x y$
3.  $\log_{(y^z)} x = (1/z) \log_y x$

### Problem 1:

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$\log_3 x = \log_{12} y = a$ , where  $x, y$  are real positive numbers. If  $G$  is the geometric mean of  $x$  and  $y$ . What is the value of  $\log_6 G$ ?

**Solution:**

From the statement,  $\log_3 x = \log_{12} y = a$ , we have

$$\log_3 x = a \quad \text{and} \quad \log_{12} y = a$$

By definition of the log;

$$\log_3 x = a, x = 3^a \quad \text{and} \quad \log_{12} y = a, y = 12^a$$

$G$  is the geometric mean of  $x$  and  $y$ . So,  $G = \sqrt{xy}$

$$G = \sqrt{3^a \cdot 12^a} = \sqrt{36^a} = 6^a$$

$$\text{Now; } \log_6 G = \log_6 6^a = a \log_6 6 = a$$

Hence,  $\log_6 G = a$ .