Clock problems

Introduction to clock problems

A clock is a complete circle having 360 degrees. It is divided into 12 equal parts i.e. each part is 360/12 = 30°.

As the minute hand takes a complete round in one hour, it covers 360° in 60 minutes. In 1 minute it covers 360/60 = 6°/ minute.

Also, as the hour hand covers just one part out of the given 12 parts in one hour. This implies it covers 30° in 60 minutes i.e. ½° per minute.

This implies that the relative speed of the minute hand is 6 - ½ = 5 ½ degrees.
We will use the concept of relative speed and relative distance while solving problems on clocks.

Some facts about clock problems

- Every hour, both hands coincide once. In 12 hours, they will coincide 11 times. It happens due to only one such incident between 12 and 1'o clock.
- The hands are in the same straight line when they are coincident or opposite to each other.
- When the two hands are at a right angle, they are 15-minute spaces apart. In one hour, they will form two right angles and in 12 hours there are only 22 right angles. It happens due to right angles formed by the minute and hour hand at 3’o clock and 9'o clock.
- When the hands are in opposite directions, they are 30-minute spaces apart.

- If both the hour hand and minute hand move at their normal speeds, then both the hands meet after 65 $\frac{5}{11}$ minutes.

Types of clock problems

Type 1: Finding the time when the angle between the two hands is given.
When the angle between the hands is not perfect angles like 180°, 90°, or 270°, the solving of the questions becomes difficult and time-consuming at the same time. The logic below provides a trick to address problems involving angles of hands for other than standard aspects.

*T = 2/11 [H*30±A]*

Where:

1. T stands for the time at which the angle formed.
2. H stands for an hour, which is running.

(If the question is for the duration between 4 o'clock and 5 o'clock, it's the 4th hour which is running hence the value of H will be '4'.)

3. A stands for the angle at which the hands are at present.

(The value of A is provided in the question generally)

The clock is divided into two parts: 1st and 2nd half as shown above

If the time given in the question lies in the first half, then the positive sign is considered while evaluating the time else, then the negative sign is used.

Example: At what time between 3 and 4 o'clock, the hands make an angle of 10 degrees?
Solution: Given: H = 3 , A = 10
Since both three and four lies in the first half considered a positive sign.
T = 2/11 [H*30±A]
T = 2/11 [3*30+10]
T = 2/11 [90+10]
T = 2/11 [100]

T = 200/11
T =18 2/11

The answer indicates that the hands of a clock will make an angle of 10 between 3 and 4 o'clock at exactly 3:18:2/11 (3' o clock 18 minutes and 2/11 of minutes = 2/11*60 = 10.9 seconds)

Type 2: Finding the angle between the two hands at a given time.

Example:
The angle between the minute hand and the hour hand of a clock when the time is 4:20 is:
Answer: 10 degrees.
Solution: At 4:00, the hour hand was at 120 degrees.

Using the concept of relative distance, the minute hand will cover = $\frac{20 \times 11}{2}$ =110 degrees
The angle between the hour hand and the minute hand is = 120-110 = 10 degrees.

Type 3: Questions on clocks gaining/losing time.
If a watch indicates 9.20, when the correct time is 9.10, it is said to be 10 minutes too fast. And if it indicates 9.00, when the correct time is 9.10, it is said to be 10 minutes too slow.
Such kinds of problems appear in exams very often, when a clock runs faster or slower than the expected pace.

The clock is running fast: It is also referred to as gaining time i.e. when a normal clock covers 60 minutes, a faster clock will cover more than 60 minutes.

The clock is running slow: It is also referred to as losing time i.e. when a normal clock covers 60 minutes, a slower clock will cover less than 60 minutes.

Example: A watch gains 5 minutes in one hour and was set right at 8 AM. What time will it show at 8 PM on the same day?
Answer: 9 pm
Solution: A correct clock would have completed 12 hours by 8 pm. But the faster clock actually covers 5 min. extra in one hour. So, it will cover 12 x 5 = 60 minutes extra. Therefore, when the correct clock would show 8 pm, the faster clock will show 60 minutes extra i.e. 9 pm.