Alligation

Introduction to Alligation

The concept of alligation is closely related to the weighted average.

Alligations is a visual approach to solve weighted averages, involving the mixing of two groups.

For example:

Two varieties of rice at 50 per kg and 80 per kg are mixed together in the ratio 3 : 7. Find the average price of the resulting mixture.

Solution : By using weighted average formula; Aw = (n1A1 +n2A2) / (n1+n2)

$$\text{Average price} = (3\times 50 + 7\times 80) / (3 + 7)$$
$$= 710 / 10$$
$$= 70.$$

The weighted average approach is slightly slower than if we see the same situation through alligations. Alligations are a faster approach.

The mathematical formula for alligation:

In the case of a situation where just two groups are being mixed, we can write weighted average formula:

$$A_w = (n_1A_1 + n_2A_2) / (n_1 + n_2)$$

Here, we have 2 groups with averages A1, A2 and having n1 and n2 elements respectively.

Rewriting this equation we get:

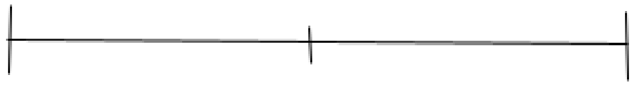
$$(n_1 + n_2) A_w = n_1A_1 + n_2A_2$$
$$n_1(A_w - A_1) = n_2 (A_2 - A_w) \text{ or}$$
$$n_1/n_2 = (A_2 - A_w)/(A_w - A_1) \text{ .... The alligation equation.}$$

As a convenient convention, we take $A_1 < A_2$. Then, by the principal of averages, we get $A_1 < A_w < A_2$.

Situations in alligation problems

Situation 1: When A1, A2, n1, and n2 are known and Aw is unknown.

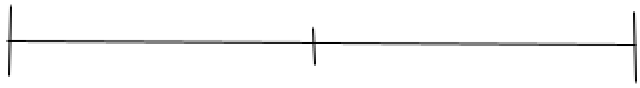
A1 Aw A2



n1 n2

For example:

50 Aw 80

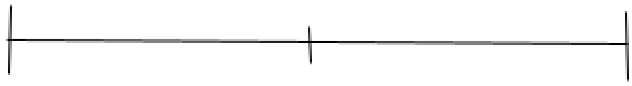


3 7

Since the total distance = (80 - 50) = 30. If we split 30 into 3:7, the value of 3 parts and 7parts are 9 and 21 respectively.
Thus the distance between Aw and 50 is corresponding to n2 (i.e. 7) and 7 parts are equal to 21.
I.e. Aw - 50 = 21 ≫ Aw = 71.

Situation 2: When A1, A2, and Aw are known and n1: n2 is unknown.

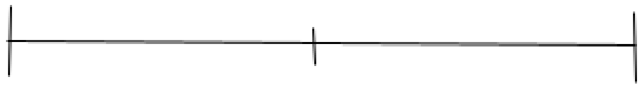
A1 Aw A2



n1 n2

For example:

70 80 84

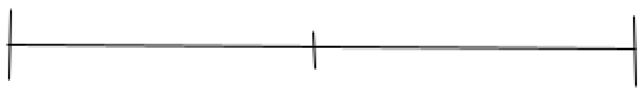


n1 n2

By using alligation equation,
n1/n2 = (A2 – Aw)/(Aw – A1)
n1:n2 = 4:10 or 2:5.

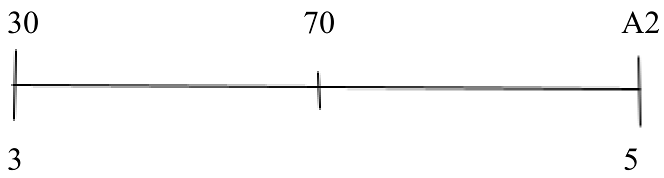
Situation 3: When A1, Aw, and n1:n2 are known and A2 is unknown.

A1 Aw A2



n1 n2

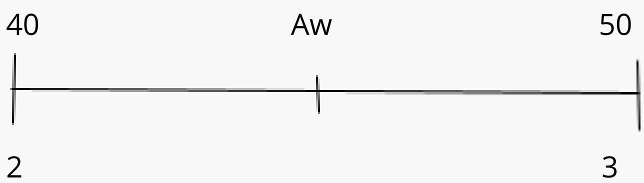
For example:



By using alligation equation,
n1/n2 = (A2 – Aw)/(Aw – A1)
3/5 = (A2 - 70)/(70-30)
A2 = 94.

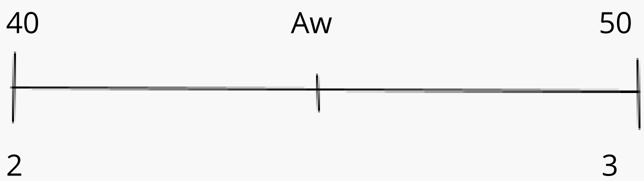
Problems where we can use alligation-1

Example 1:
Two varieties of rice at 40 per kg and 50 per kg are mixed together in the ratio 2 : 3. Find the average price of the resulting mixture.
Solution :



Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2 (i.e. 3) and 3 parts are equal to 6.
I.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average price of the resulting mixture is at 46 per kg.

Example 2:
A man has driven a car at 40kmph and 50kmph. He has driven for 2 hours and 3 hours respectively. Find the average speed of a car?
Solution :



Here, Aw is the average speed of the car.

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average speed of the car is 46kmph.

These two questions are on the surface different from each other, the first one was talking about average price and the other is talking about the average speed, But structurally both are the same.
Equation in 1st question ;

Average price = (n1A1 +n2A2) / (n1 + n2).

Here, n1= 2kg, n2= 3kg , A1= 40per kg, A2= 50 per kg.
So,

Average price = (2*40 + 3*50)/(2+3)

Equation in 2nd question ;

Average speed = (t1S1 +t2S2) / (t1 + t2).

Here t1= 2hr, t2= 3hr , S1= 40kmph, S2= 50kmph.
So,

Average speed= (2*40 + 3*50)/(2+3)

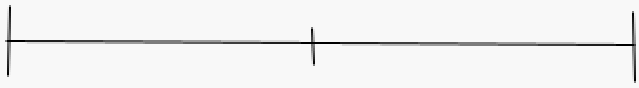
By looking at these two equations you will observe that these both are the same, only difference is in variables.

Problems where we can use alligation-2

Example 1:
We have two mixtures of milk and water, the 1st mixture contains 40% milk & 60% water and the 2nd mixture contains 50% milk & 50% water. These two mixtures are mixed in ratio 2:3, then find the % of milk in the mixture?

Solution : Using milk %

40	Aw (% of milk)	50



2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.

i.e. Aw-40 = 6

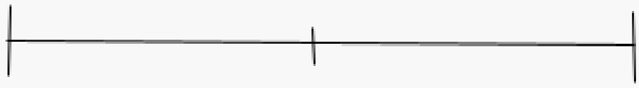
≫ Aw (% of milk) = 46%.

Another way to solve this question is by using water %

The 1st mixture has 60% water and the 2nd mixture has 50% water.

According to convention, we need A1< Aw < A2 and the ratio of 1st mixture to 2nd mixture is 2:3, this will be inverted here because we have to flip the % here to make it according to the given convention.

50 Aw (% of water) 60



3 2

Since the total distance = (60 - 50) = 10. If we split 10 into 3:2, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 50 is corresponding to n2(i.e. 2) and 2 parts are equal to 4.

i.e. Aw-50 = 4

≫ Aw (% of water) = 54%.

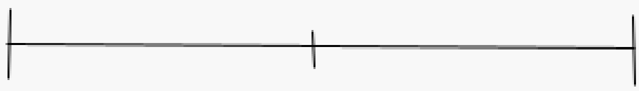
Thus; % of milk = 100 - 54 = 46%.

Example 2:

Anjali mixes 2 alloys of gold and copper in ratio 2:3. The 1st alloy contains 40% gold and the 2nd alloy contains 50% gold. Find the gold % in the mixture?

Solution :

40 Aw (% of gold) 50



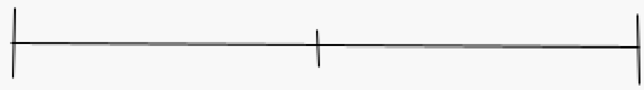
2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw (% of gold) = 46%.
Another way to solve this question is by using copper %.

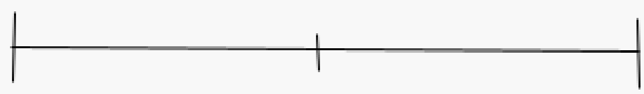
Problems where we can use alligation-3

Example 1:
A shopkeeper sold chairs and tables. The ratio of the cost price of chair and table is 1:2. He sold chairs at 30% profit and tables at 60% profit. What is the average % profit?
Solution :
30 Aw(average % profit) 60



1 2
Since the total distance = (60 - 30) = 30. If we split 30 into 1: 2, the value of 1 part and 2 parts are 10 and 20 respectively.
Thus the distance between Aw and 30 is corresponding to n2(i.e. 2) and 2 parts are equal to 20.
I.e. Aw-30 = 20
≫ Aw (average % profit) = 50%.

Example 2:
A shopkeeper sold chairs and tables. He sold tables at 20% profit and chairs at 30% loss. Thereby he made no profit or no loss in the transaction. What is the cost price ratio of table to chair?
Solution :
Chair **Table**
-30 0 20

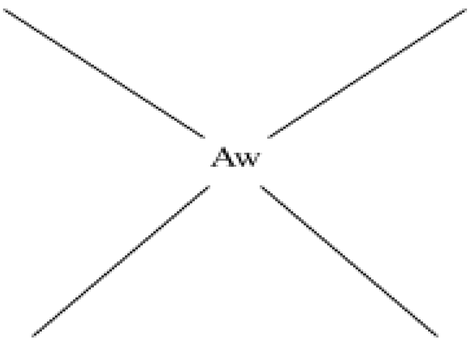


n1 n2
n1/n2 = (A2 – Aw)/(Aw – A1)
Here A1= -30, A2= 20, Aw= 0.
n1/n2 = (20 – 0)/(0 – (-30))

n1/n2 = 20/30 i.e. n1:n2 = 2:3.
Thus, table to chair cost price ratio = 3:2.

Cross diagram approach

A1 A2

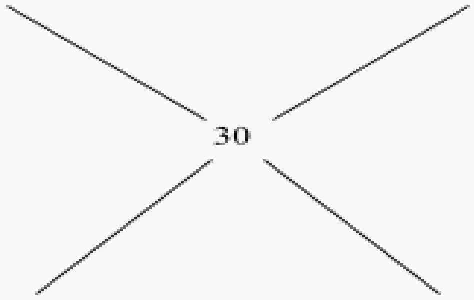


A2-Aw	:	Aw-A1
n1	:	n2

Note: That the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.
As we have seen, there are five variables in the alligation equation.
The three averages A1, A2, and Aw. and the two weights n1 and n2.

Example 1:
On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find
(a) The ratio of students in the classes
(b) The number of students in the first class if the second class had 30 students.
Solution :

25 40



A2-Aw : Aw-A1

10 : 5

(a) The ratio of students in class is 10:5 i.e 2:1.

(b) If the ratio is 2: 1 and the second class has 30 students, then the first class will have 60 students.