Time and work

Introduction to time and work

Work is defined as something which has an effect or outcome. The basic concept of Time and Work is similar to that across all Arithmetic topics, i.e. the concept of Proportionality.

Method for solving time and work

1. Fraction method :

Let the total work = 1unit.
A can finish the work in 12 days and B can finish the work in 15 days.
A's per day work = 1/12 unit.
B's per day work = 1/15 unit.

In time & work the basic equation is;
Rate of work×Time = work done

Rate of work = 1/12 + 1/15 = 9/60 unit
9/60×t = 1. Therefore t = 60/9 = 6.66 days.
Time is reciprocal of rate of work.
It is a very combusive method. One advantage of this method is in the last step you just take the reciprocal of the value you got.

2. Percentage method :

Let the total work = 100%
A can finish the work in 12 days and B can finish the work in 15 days.
A's per day work = 1/12 i.e. 8.33%
B's per day work = 1/15 i.e. 6.66%

Rate of work×Time = work done
Rate of work = 8.33 + 6.66 = 15%
15×t = 100. Therefore t = 100/15 = 6.66%.
It is a better method than fraction, but this method has only the problem of decimal work.

For example; A can finish the work in 5 days and B can finish the work in 9 days.
A's per day work = 1/5 i.e. 20%
B's per day work = 1/9 i.e. 11.11%
Rate of work = 20+11.11 = 31.11%. So in this case numbers are not supporting you.

3. LCM method :

A can finish the work in 12 days and B can finish the work in 15 days.
Assume total work be the LCM of 12 &15.
LCM(12,15) = 60.
A's per day work = 60/12 = 5 unit.
B's per day work = 60/15 = 4 unit.
One day total work = 5+4 = 9unit.
Total time required = total work / per day work
= 60/9 = 6.33 days.
This is the better method to work upon by avoiding the use of decimal work

Types of problems in time and work

People come and go type problem :

Example: A can do a piece of work in 10 days. B can also do the same work in 12 days and C can do the same work in 15 days. A & B start the work and work for 2 days and then B leave and after 1 more day, C joins A to complete the work. In how many days will the work be completed?

Solution :

Total work = LCM(10,12,15) = 60 units.
A's per day work = 60/10 = 6 units.
B's per day work = 60/12 = 5 units.
C's per day work = 60/15 = 4 units.
A+B per day work = 6+5 = 11 units.
Work in 2 days = 11×2 = 22 units.

On the 3rd day, A is working alone and B left.
3rd work = 6 units.
Total work in 3 days = 22+6 = 28 units.
So; work left = 60-28 = 32 units. This work has to be done by A & C.
A+C per day work = 6+4 = 10 units. Therefore remaining work 32 units will take 32/10 = 3.2 days more.
Hence total days required = 3 + 3.2 = 6.2 days.

Pipe & Cistern Problem :

Example: 2 pipes A & B are filling a tank. A can fill it in 12 hours and B can fill it in 15 hours. How much time will they take to fill an empty tank?

Solution :

A can fill the tank in 12 hours and B can fill the tank in15 hours.

Assume the total capacity of the tank be the LCM of 12 &15.

LCM(12,15) = 60 L.

A's per hour filling = 60/12 = 5 L.

B's per hour filling = 60/15 = 4 L.

In one hour total filling = 5+4 = 9 L.

Total time required = total capacity / per hour filling

= 60/9 = 6.33 hours.

Time and work (man-days) :

Here we will discuss that the work is measured in terms of man-day or man-hours.

Let 20 men work on a project for 8 days. Work done can be measured in such a case, as multiplication of 20×8 and units used here man-days. i.e 20×8 = 160 man-days.

We use the concept of work equivalence in such situation means;

20 men working for 8 days is the same as 10 men working for 16 days is same as 1 man working for 160 days i.e $20\times8 \equiv 10\times16 \equiv 1\times160$.

Example: A certain number of people can complete a piece of work in 55 days. If there were 6 more men added, the work could get done in 11 days less.What is the number of men initially?

Solution :

Assume in the starting there is x number of men.

Total work is done by x men = $x \times 55$ man-days.

6 men more join & work is done in 55 - 11 = 44 days.

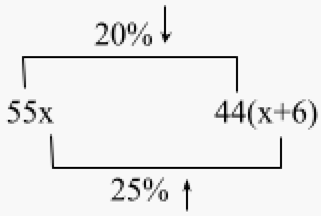
So; according to work equivalence ;

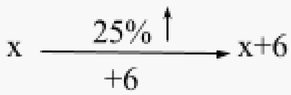
$x \times 55 = (x+6)\times 44$

$55x = 44x + 264 \Rightarrow x = 24$ men.

We can do this question by-product constancy also.

The numerical component of the product is going down by 20% and the other component going up by 25%.





+6 present 25% increase on x. 25% is 6 and 100% is 6/25×100 = 24.
Hence the number of men = 24 men.

Example: 10 men working 6 hours a day can complete work in 18 days. In how many hours a day should 15 men work for 12 days. So that they can complete double the work?
Solution :
Original work = $10 \times 18 \times 6$ man-days.
New work = $10 \times 18 \times 6$×2
Let x hours per day 15 men take.
According to work equivalence;
$10 \times 18 \times 6$×2 = $15 \times 12 \times x$
Therefore x = 12 hr/day

Time and work (man-days)-2 :

Example: A contractor undertakes to complete a job in 100 days and employs 200 men to complete the work. After 50 days he finds that only 40% of the work is completed. To complete the work in time how many men should he hire?
Solution :
Work to be done in 50 days = 200×50 =10000 man-days
10000 man-days are only 40% of the work.
Remaining work = 100 - 40 = 60%
40% work = 10000 man-days
60% work = (10000/40)×60 = 15000 man-days.

You have only 50 more days left. Let n be the number of men required to complete the work.
Therefore; $50 \times n = 15000$ and $n = 300$ men.
Hence; 300 - 200 = 100 men need to hire.

The Specific Case of Building a Wall :

Building of a wall of a certain length, breadth, and height.
In such cases, the following formula applies:
$\frac{M1 \times D1 \times T1}{M2 \times D2 \times T2} = \frac{L1 \times B1 \times H1}{L2 \times B2 \times H2}$
where L, B, and H are respectively the length, breadth, and height of the wall to be built, while m, t, and d are respectively the number of men, the amount of time per day, and the number of days. Further, suffix 1 is for the first work situation, while suffix 2 is for the second work situation.

Example: 12 men working 8 hours a day can completely build a wall of length 12ft, breadth 40 ft, and height 4ft in 10 days. How many days will 10 men working 6 hours a day require to build a wall of length 24ft, breadth 60ft, and height of 2ft?
Solution :
Using formula;
$\frac{M1 \times D1 \times T1}{M2 \times D2 \times T2} = \frac{L1 \times B1 \times H1}{L2 \times B2 \times H2}$

Here, L1 is 12ft	L2 is 24ft
B1 is 40ft	B2 is 60ft
H1 is 4ft	H2 is 2ft
while M1 is 12 men	M2 is 10 men
D1 is 10 days	D2 is unknown
and T1 is 8 hours a day	T2 is 6 hours a day

$\frac{12 \times 10 \times 8}{10 \times D2 \times 6} = \frac{12 \times 40 \times 4}{24 \times 60 \times 2}$
16/D2 = 2/3 ,   D2 = 24 days

Men, Women & Children :

Example: 20 women can do work in 16 days while 16 men can do it in 15 days. What is the ratio of the capacity of a man and a woman?
Solution :

Total work to be done = 20 × 16 = 320 woman-days.
or total work to be done = 16 × 15 = 240 man-days.
Since, the work is the same, we can equate 240 man-days = 320 woman-days.
Hence, 3 man-days = 4 woman-days or 1 man-day = 1.33 woman-days.
Assume total work = 12 unit
1 man-day work rate = 4 units.
1 woman-day work rate = 3 units.
Therefore the work rate of man to woman = 4:3.
The answer is not 3:4, the answer is 4:3 because 3 man-days doing the same work as 4 woman-days. So; the work rate of a man must be higher than the work rate of a woman.