Percentages

Introduction to Percentages

The basic definition of percentage is essentially out of 100. The percentage is derived from the French word 'cent'. The meaning of 'cent' in French is 100.
The percentage is used to compare data and numbers.
For example:
(a) If there are 5 (A, B, C, D, E) students who have taken the 12th board exam from five different boards. The percentages they get is a defined thing i.e. comparison between 5 diverse students in 5 diverse boards.

A	B	C	D	E
86%	92%	94%	78%	52%

By seeing the percentage of these students we can compare which student is better.

(b) GDP defines how the world is doing in terms of Global world economies. GDP compares different countries' economies in terms of their percentage.

Mathematically;
Any ratio if you multiply by 100, it gives you its percentage value. The percentage is denoted by the sign "%".

Why when the ratio is multiplied by 100, gives you a percentage value? You can see that from the unitary method.
Unitary method: It is a method which talks about a situation where two variables are moving linearly w.r.t. each other.

Example: You bought 10 bananas for 30 rupees then, how many rupees will you need to buy 15 bananas?
Solution: let x rupees you will need to buy 15 bananas.
10 bananas = 30Rs
15 bananas = x Rs
Cross multiply and equate;
$10*x = 15\times30$
$x = 45$.
So, 45 rupees is the amount that you will need to buy 15 bananas.

Example: You scored 10 out of 20 in a quiz and you want to put it in % then, how much out of 100 did you score?
Solution:

10 out of 20.

x out of 100.

So, by unitary method;

$20\times x = 10\times100$

$x = (10/20) \times100$

$x = 50\%$

NOTE: Any fraction multiplied by 100 gives its percentage value.

Concept of percentage change

Percentage always happens when you go from one number to the next number.
Basic structure of percentage change will always be in the situation, where you are talking about the difference between two numbers.
Let say we have number x becoming y. The percentage change between x to y.

$$x \longrightarrow y$$

Formula for percentage change:

Percentage change = (change/original value)*100

1st you have to identify which number is the original number that depends on which direction you are looking at percentage change. So, percentage change is always a **directional input**.
If x changes to y the percentage change going from x to y, will be having x as the original value.

$$x \longrightarrow y$$

If y changes to x. So, in this situation the percentage change will have to be seen from y to x and will be having y as the original value.

$y \longrightarrow x$

For example;
If you have two numbers 20 & 40. So, going from 20 to 40.

$20 \longrightarrow 40$

Here change = 40 - 20 = 20, and original value = 20.
% change = (20/20)×100 = 100%
The change is +ve. So, % change is increasing by 100%.

20 to 40 have different % change than coming from 40 to 20.

$20 \longleftarrow 40$

Here change = 20 - 40 = -20, and original value = 40.
% change = (-20/40)×100 = -50%
So, % change is decreasing by 50%.

NOTE: 1. In percentage change, there should be two numbers.
2. You need to understand which number is the original number.

People make a very common mistake in the % change calculation.
In the question given that 50 to 75, instead of this they calculated 75 to 50. Because the language of % change can get complex sometimes, where language structures are used especially in DI.

Percentage change graphics

It is an important concept in percentage change and important for chapters like interest, profit, and loss, etc. As the name suggests, percentage change graphics means the graphical method of doing the percentage change.

Basics of percentage change:

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1. 100% of a number is a number itself.
2. 10% of a number is a shift of 1 decimal point on the number towards left.
3. 1% of a number is a shift of 2 decimal points on the number towards left.
4. 0.1% of a number is a shift of 3 decimal points on the number towards left and so on….

For example:
Let say a number N=52123.
100% of the number N is 52123
10% of the number N is 5212.3

1% of the number N is 521.23
0.1% of the number Nis 52.123

PCG has two structures:

Structure 1:
Given the starting value and the ending value. You have to calculate:
1. Absolute change (below the arrow).
2. % change(above the arrow).

$$S \xrightarrow[\text{Absolute change}]{\text{\% change}} E$$

Example:
Let us say, 40 changing into 52.

$$40 \longrightarrow 52$$

Absolute change = 52 - 40 = +12. Absolute change is +ve that means an increase in % change.
10% of the number 40 is 4, and the number 12, is 3 times the number 4 which means that the percentage increases by 30% (3 * 10% = 30%).

$$40 \xrightarrow[+12]{30\% \uparrow} 50$$

Structure 2:
1. Starting value is given to you,
2. Percentage change is given to you.
3. Absolute change you need to calculate.
4. And calculating the ending value.

Example1:
There is a number 40 that has to be increased by 30%.

$$40 \xrightarrow{30\% \uparrow}$$

Solution:
We were doing this problem by the unitary method.
40 is 100%

x is 130%
Cross multiply and equate;
x = (40×130)/100.
Rather than this, a much easy calculation is done through percentage change graphics.

$$40 \xrightarrow{30\% \uparrow}$$

10% of 40 is 4. 30% increase means adding 4, 3 times. 4+4+4 = 12 i.e adding 12 in 40 so the ans is 52.

$$40 \xrightarrow[4+4+4=12]{30\% \uparrow} 52$$

Example2:
The number 37 has to be increased by 13%.

$$37 \xrightarrow{13\% \uparrow}$$

Solution: In this question, you have to build up 13% by;
10% of 37 is 3.7
1% of 37 is 0.37
1% of 37 is 0.37
1% of 37 is 0.37
So, 13% of 37 is 4.81(3.7+0.37+0.37+0.37 = 4.81), adding 4.18 in 37. So, the answer is 41.81

$$37 \xrightarrow[+4.81]{13\% \uparrow} 41.81$$

PCG applied to percentage change:

The 1st structure under which you can use the percentage change in quantitative aptitude is product change situation.

Example1:
Let say a product $x \times y$. x is increased by 20% and y is increased by 30%. You want to find out what is the % change in the product?
Solution:
x would become x(1+ (20/100)) = $x \times 1.2$
y would become y(1+ (30/100)) = $y \times 1.3$
So, in product; $x \times 1.2 \times y \times 1.3$= 1.56xy. This means, 56% change.

Same question can be done by PCG. If you assume your original product to be 100. And this product will go through two changes, 20% increase in x and 30% increase in y. You have to put two arrows,
One for ‘x’ and other for ‘y’.

$$100 \xrightarrow[\text{‘x’}]{20\%} \quad \xrightarrow[\text{‘y’}]{30\%}$$

If x increases by 20% the product also increases by 20% and then if y increases by 30% the product also increases by 30%.

$$100 \xrightarrow[+20]{20\%} 120 \xrightarrow[+36]{30\%} 156$$

i.e. 56% increase in the product.

Problems on percentage change:

Area and volume-based problem:

Problem 1:
The length of a rectangle goes up by 30% and the breadth of the rectangle comes down by 10%. What is the percentage change in area?
Solution:
Area = $l \times b$and now it becomes a product change situation.
Assume the original area = 100. Makes two arrows one for length and other is for breadth.

$$100 \xrightarrow[\text{‘l’}]{30\% \uparrow} \quad \xrightarrow[\text{‘b’}]{10\% \downarrow}$$

$$100 \xrightarrow[+30]{30\% \uparrow} 130 \xrightarrow[-13]{10\% \downarrow} 117$$

Hence 17% is the increase in the area of the rectangle.

Problem 2:
The length of a rectangle is decreased by 20% and the breadth of the rectangle is increased by 23%. What is the percentage change in area?
Solution:
Area = $l \times b$and now it becomes a product change situation.
Assume the original area = 100. Makes two arrows one for length and the other is for breadth.

$$100 \xrightarrow[\text{‘b’}]{23\% \uparrow} \quad \xrightarrow[\text{‘l’}]{20\% \downarrow}$$

For easy calculation, we put breadth on the 1st arrow and length on the 2nd arrow.

$$100 \xrightarrow[+23]{23\% \uparrow} 123 \xrightarrow[-24.6]{20\% \downarrow} 98.4$$

Hence, 1.6% is the decrease in the area of the rectangle.

We can do the same problem with the help of the following formula;
Percentage change = (a + b + ab/100)

Let us say, x increases by 20% and y increases by 10%. Then the percentage change;
Percentage change = 20 + 10 + (20×10)/100
= 32%.

But rather than this PCG is a more easy way to solve this problem.

One other problem to this formula, if $x \times y \times z$situation occurs then the formula can not make a change 3 components of the of product. PCG is always better for these problems.

Expenditure and revenue problem:

Problem 1:
The price of a commodity has gone up by 20% and a person reduces its consumption by 10%. What is the % change in the expenditure?
Solution :
Price ×consumption = expenditure.
Assume the original expenditure = 100. Makes two arrows one for price and the other is for consumption.

$$100 \xrightarrow[\text{'P'}]{20\% \uparrow} \quad \xrightarrow[\text{'C'}]{10\% \downarrow}$$

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-12]{10\% \downarrow} 108$$

Hence, 8% is the increase in the expenditure of the commodity.

Problem 2:
A shopkeeper selling chairs, reduces the price of chairs by 20% due to which he gets an increment of 60% in the sale. What is the percentage change in the revenue?
Solution :
Price ×sale = revenue.
Assume the original revenue = 100. Makes two arrows one for price and other is for sale.

$$100 \xrightarrow[\text{'P'}]{20\% \downarrow} \quad \xrightarrow[\text{'S'}]{60\% \uparrow}$$

$$100 \xrightarrow[-20]{20\% \downarrow} 80 \xrightarrow[+48]{60\% \uparrow} 128$$

Hence, 28% is the increment in the revenue.

PCG applied to product constancy:

Product constancy is after the series of changes, you need to come back to the original value. Product constancy is applied in a lot of questions directly.

$$100 \longrightarrow \quad \longrightarrow \quad \longrightarrow 100$$

Problem 1:
Price of a commodity has gone up by 25% and the consumption is reduced such that the expenditure remains constant.
Solution :
Price ×consumption = expenditure.
Let 100 be the original expenditure after two change one on price and other on consumption, the expenditure should be back at 100.
After a 25% increment in price, expenditure becomes 125. So, 125 should be reduced by 25 to keep expenditure constant i.e consumption reduced by 20%.

$$100 \xrightarrow[+25]{25\%\uparrow} 125 \xrightarrow[-25]{20\%\downarrow} 100$$

25% increase in price is offset by a 20% decrease in consumption to keep expenditure constant.

Problem 2:
The length of a cuboid has increased by 20%, the breadth has increased by 50%. How much should you reduce the height to keep the volume constant?
Solution :
Volume = $l \times b \times h$
After 20% and 50% increment in length and breadth respectively, the volume becomes 180. So, 180 should be reduced by 80 to keep volume constant i.e height dropped by 44.44%.
Drop in height = (80/180)×100
= (4/9)×100 = 44.44%

$$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[+60]{50\%\uparrow} 180 \xrightarrow[-80]{44.4\%\downarrow} 100$$

PCG applied on successive percentage change :

Successive percentage change use of PCG is structurally very similar to product change use of PCG. One small difference is that in product change we have seen that the arrows are interchangeable w.r.t. each other but in successive percentage change use of PCG we can not interchange the arrows because sometimes we need intermediate value, if we interchange the arrows then we do not get the exact intermediate value. You can understand that difference through some examples/problems.

Problem 1:
Population of the town goes up by 20% in 1st year, comes down by 10% in 2nd year and goes up by 5% in 3rd year. What is the % change in population after 3 years ?

Solution :
Let the population of the town is 100. Population after one year becomes 120 with an increase of 20%. Population after 2 year will become 108 and after the 3rd year the population will become 113.4.

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-12]{10\% \downarrow} 108 \xrightarrow[+5.4]{5\% \uparrow} 113.4$$

% change in the population after 3 years is 13.4%. But the intermediate value is important, if anyone asks what is the % change in population after 2 years.
If you interchange the arrows e.g 10% is placed on the last arrow.

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[+6]{5\% \uparrow} 126 \xrightarrow[-12.6]{10\% \downarrow} 113.4$$

Final value does not make a difference. But after two year the population value is wrong. If the question is built on intermediate value then you will go wrong if you do not keep the arrow constant as they are, that is the only difference in this.

A to B to A problems (compare two numbers) :

Very often we face a situation, where we compare two numbers, say A and B. In such cases, if we are given % change from A to B, then the reverse relationship can be determined by using PCG in the same way as the product constancy.

Problem 1:
A’s salary is 25% more than B’s salary. By what percent is B’s salary less than A’s salary?

Solution :

Let B’s salary = 100.

$$100(B) \xrightarrow[+25]{25\%\uparrow} 125(A) \xrightarrow[-25]{20\%\downarrow} 100(B)$$

A drop of 25 on 125 gives a 20% drop.

Hence B’s salary is 20% less than A’s.

NOTE: Product constancy table is also useful for this situation.

Problem 2:

B gets 20% more marks than A and C gets 50% more marks than B, then how much % less than C does A get?

Solution :

Lets A’s marks = 100.

$$\underset{\text{‘A’}}{100} \xrightarrow[+20]{20\%\uparrow} \underset{\text{‘B’}}{120} \xrightarrow[+60]{50\%\uparrow} \underset{\text{‘C’}}{180}$$

Coming back from C to A, a drop of 80 on 180 i.e 80/180 = 4/9. The fraction 4/9 is equivalent to 44.44%. Hence, A gets 44.44% marks less than C.