Ratio, proportions, variation 2

Introduction to Ratio, proportion, variation

We have already discussed the theory of this chapter and did some problems based on that, now we will go further with some standard problems.

Problems On Ratio-1:

Problem 1:
Divide rupees 252 amongst A, B, and C such that 1/3rd of what A gets is equal to 1/5th of what B gets is equal to 1/4th of what C gets. How much A, B, and C will get individually?

Solution :
Here 2 equations are formed;
A + B + C = 252 ....(1)
A/3 = B/5 = C/4 ....(2)

Let's say, eq (2) is equal to k.
A/3 = B/5 = C/4 = k , A = 3k , B = 5k , C = 4k
Put value of A,B and C in eq(1) we get;
3k+5k+4k = 252, 12k = 252 and k = 252/12 = 21.
Hence the numbers are; A = 3×21= 63, B = 5×21= 110 and C =4×21= 84.

Problem 2:
Divide rupees 517 amongst A, B, and C such that 1/3rd of what A gets is equal to 2/5th of what B gets is equal to 3/7th of what C gets. How much A, B ,and C will get individually?

Solution :
A + B + C = 517 ....(1)
A/3 = B2/5 = C3/7 ....(2).
You can get a direct ratio from eq(2).
A:B:C = 3:5/2:7/3

Whenever you have a ratio that itself has its component in the fractions, you should multiply the ratio by the denominator LCM.

LCM (2,3) = 6. Multiply A:B: C by 6 you will get a proper ratio.
So; A:B:C = 18:15:14
So, Sum of the component of ratio = 18+15+14 = 47.
47≡517 that means the multiplier would be 11.

Hence the numbers are; A = 18×11= 198, B = 15×11= 165 and C = 14×11= 154.

Problems On Ratio-2:

Problem 1:
Anjali has 2 mixtures of milk and water. One mixture has milk to water in ratio 3:8 and 2nd mixture has milk to water in ratio 2:7. She mixes equal quantities of these mixtures. What is the ratio of milk to water in the final mixture?
Solution :

Mixture 1	Mixture 2
M:W = 3:8	M:W = 2:7

Take the LCM of 3+8 = 11 and 2+7 = 9. LCM(11,9) = 99. Take 99L for both mixtures because mix equal quantities of mixture.

Mixture 1	Mixture 2
99L	99L
M:W = 3:8	M:W = 2:7

Using multiplier logic;

3+8 = 11≡99	2+7 = 9≡99
11 being 99 so; multiplier would be 9.	9 being 99 so; multiplier would be 11.
Hence M = 3×9 = 27 & W = 8×9 = 72.	Hence M = 2×11 = 22 & W =7×11= 77

Thus; total milk = 27+ 22 = 49 and total water = 72 + 77 = 149.
Hence the final mixture has milk to water ratio = 49:149.

Problem 2:
Shubham has 2 mixtures of milk and water. One mixture has milk to water in ratio 3:8 and the 2nd mixture has milk to water in ratio 2:7. He is mixing these mixtures in 2:3. What is the ratio of milk to water in the final mixture?

Solution :

Mixture 1 Mixture 2

M:W = 3:8 M:W = 2:7

Take the LCM of 3+8 = 11 and 2+7 = 9. LCM(11,9) = 99. Here you can not take 99L for each mixture because the question is not talking about equal quantities.

Mixture 1 = 99L and mixture2 = 99L, To make both the mixture in 2:3. Then;

Mixture1 = 99×2 = 198L and Mixture2 = 99×3 = 297L

Mixture 1	**Mixture 2**
198L	297L
M:W = 3:8	M:W = 2:7

Using multiplier logic;

3+8 = 11≡198 2+7 = 9≡297

11 being 198 so; multiplier would be 18. 9 being 297 so; multiplier would be 33.

Hence M =3×18= 54 & W =8×18=144 Hence M = 2×33 = 66 & W =7× 33=231

Thus; total milk = 54 + 66 = 120 and total water = 144 + 231 = 375.

Hence the final mixture has milk to water ratio = 120:375.

Problems On Ratio-3:

Problem 1:

The income of P & Q is in ratio 1:2 and expenditure of P & Q is in ratio 1:3. If each saves 500 of their income. Find the P's income.

Solution :

Lets P's income = x and Q's income = 2x.

P's expenditure = y and Q's expenditure = 3y.

And we know;

Saving = Income - Expenditure

Saving for P; x - y = 500 ....(1)

And Saving for Q; 2x - 3y = 500 ....(2)

Solving eq(1) and (2) we get;

x = 1000 and y = 500.

Hence P's income = 1000.

Problem 2:
Rupees 232 is to be divided among 150 girls and boys, such that each girl gets Rs 1 and each boy gets Rs 2. Find the number of boys and girls.

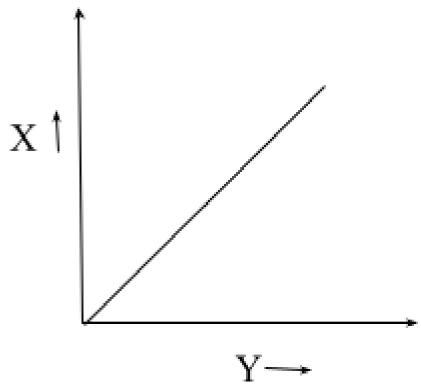
Solution :
Let the number of girls = G and number of boys = B
G + B = 150 ....(1)
G + 2B = 232 ....(2)
Solving eq (1) and (2) we get;
G = 68 & B = 82.w

Variation and its 3 types :

Variation is an important concept in mathematics. To understand variation first you need to understand 3 kinds of variation.

1. **Direct variation :**
   x varies directly as y or x is directly proportional to y.
   Mathematically; **x**α**y**.

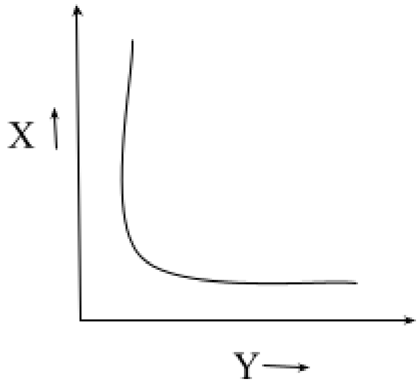
(a) Logical implication: When x increases y increases. And if x decreasing y also decreases
(b) Calculation implication: If x increases by 20%, y will also increase by 20%.
(c) Ratio : If x is increasing by 1/5 then y will also increase by 1/5.
(d) Graphical implications: The following graph is representative of this situation.



(e) Equation implication: The ratio x/y is constant i.e x = ky (where k is a constant)

2. **Inverse variation :**
   X is inversely proportional to y or x varies inversely as y or product of x and y is constant.
   Mathematically; **x**α**1/y**.

(a) Logical implication: When x increases y decreases and vice versa.
(b) Percentage implication: If x increases by 25% then y decreases by 20%.
(c) Ratio implication: If x increases by 1/4 then y decreases by 1/5.
(d) Graphical implications: The following graph is representative of this situation.



(d) Equation implication: The product x × y is constant.

3. **Joint variation :**
   If x varies jointly as y & z or **xα(y$\times$z) $\Rightarrow$x = k(y$\times$z).** Or if x varies as y when z is constant and x varies as z when y is constant.
   Mathematically; **xα(y$\times$z)**

Problem 1:
Given that, x directly varies with y and x is 18 when y is 7. Find x when y is 21?

Solution :
x directly varies with y i.e xα**y** **or** x = ky ....(1)
Replace x and y with their respective values. So; eq (1) becomes
18 = k×7 ⇒k = 18/7.
When y =21 the value of x is;
from(1); x = 18/7×21 = 54.

Problem 2:
The duration of a railway journey varies as the distance and inversely as the velocity, while velocity varies as the square root of quantity of the coal used and inversely as the number of carriages in the train. In the journey of 50 km in half an hour with 18 carriages, 100 kg of coal is required. How much coal will consume in a journey of 42km in 28 minutes with 16 carriages?

Solution :

There are 5 variables.

Assume duration = T, distance = D, velocity = V, quantity of coal = Qc and No. of carriage = N.

According to question;

T $\alpha \frac{D}{V}$ ....(1) and V = $\frac{\sqrt{Qc}}{N}$ ....(2)

From (2) put value of V in (1);

T $\alpha \frac{D\times N}{\sqrt{Qc}}$ or T = $\frac{k\times D\times N}{\sqrt{Qc}}$ ....(3)

Put value of T = 30min, D = 50km, N = 18 and Qc = 100 kg in (3);

30 = $\frac{k\times 50\times 18}{\sqrt{100}} \Rightarrow$ k = 1/3.

Now from eq (3);

T = $\frac{1}{3}\frac{D\times N}{\sqrt{Qc}}$ ....(4)

Therefor for the T = 28min, D = 42km, N = 16 ; the value of coal required is,

28 = $\frac{1}{3}\frac{42\times 16}{\sqrt{Qc}} \Rightarrow \sqrt{Qc}$ = 8; Hence Qc = 64 kg.