Probability

Introduction to probability

Probability is one of the most important mathematical concepts that we use in our daily life.
Probability means the possibility of something. It is a mathematical tool that deals with the occurrence of random events. The value of the probability lies between 0 and 1.

The probability of an event is defined by the number of ways in which the event occurs divided by the number of outcomes in the sample space.

P(event) = n(E)/n(S)

Sample space: The sample space of an event is the set of all possible outcomes of that event.

For example:

1. You tossed a coin. Your sample space is head or tail.
   P(H) = 1/2.
2. You throw a dice. Your sample space {1,2,3,4,5,6}
   P(6) = 1/6.
3. England and India play a one-day match.
   In this case, 3 events will happen. 1. England wins 2. India wins 3. Match tie.
   P(tie) $\neq$ 1/3 because the possible outcomes of the India Vs England match is not the sample space in this situation.

Two things happen to form a sample space;
1. Exhaustive or complete list of all possible outcomes.
2. A list to become a sample space is that the outcome should be equally likely.

So, in India Vs England match, the tie is not an equally like outcome. Hence it is not in the sample space.

1st kind of questions based on coins:

Problem 1:
A coin tossed three times. What is the probability of a) All heads? b) Exactly two heads. c) Minimum two heads. d) At Least one head.
Solution:
List of the possible outcomes {HHH,HHT,HTH,THH,TTH,THT,HTT,TTT}
Total number of outcomes = 8. i.e. n(S) = 8.
a) All heads n(E) = 1.
P(All heads) = n(E)/n(S) = 1/8.

b) exactly two heads n(E) = 3
P(Exactly two heads) = 3/8.

c) minimum two heads n(E) = 4.
P(Minimum two heads) = 4/8.

d) P(At least 1 head) = 1 - P(not heads)
= 1 - P(all tails) = 1 - 1/8 = 7/8.

NOTE: 1. None event in probability is denoted by $\overline{E}$ or E' and P(E) + P($\overline{E}$) = 1.
2. The probability of all events in a sample space is 1.

2nd method: Without forming sample space
a) All heads.
If you do not want to form a sample space, you can define this in 3 events.
Event definition: All heads.
H&H&H i.e. $1/2 \times 1/2 \times 1/2 = 1/8$.

b) Exactly two heads.
Event definition: Exactly two heads.
H&H&T or H&T&H or T&H&H i.e. $1/2 \times 1/2 \times 1/2 + 1/2 \times 1/2 \times 1/2 + 1/2 \times 1/2 \times 1/2 =$ 1/8 + 1/8 + 1/8 = 3/8.

Biased coin question:

Problem 1:
A coin toss three times, what is the probability of getting 2 Heads and 1 Tail if the probability of a head is 0.6 and tail is 0.4?
Solution:
2 Heads and 1Tail.
Event definition: 2 Heads and 1 Tail
H&H&T or H&T&H or T&H&H i.e. $0.6\times 0.6 \times 0.4 + 0.6\times 0.6 \times 0.4 + 0.6\times 0.6 \times 0.4 = 3(0.6\times 0.6 \times 0.4) = 3\times 0.144 = 0.432$.

Probability-based on dice

The single dice situation is very simple. Normally we get in dice question type is the 2 dice situation. In such cases normally questions are asked on the sum of the dice.

In a 2 dice situation, you need to understand that there is a certain pattern for different numbers.
For example:

Sum 2 can happen in only 1 way.(i.e. 1,1)	←Sum 12 can happen in 1 way.
Sum 3 can happen in 2 ways.	←Sum 11 can happen in 2 ways.
Sum 4 can happen in 3 ways.	←Sum 10 can happen in 3 ways.
Sum 5 can happen in 4 ways.	←Sum 9 can happen in 4 ways.
Sum 6 can happen in 5 ways.	←Sum 8 can happen in 5 ways.
Sum 7 can happen in 6 ways.	

The pair which have same number of ways;

$2 \Leftrightarrow 12$ **sum of each pair = 14.(i.e. 2+12=14).**

$3 \Leftrightarrow 11$

$4 \Leftrightarrow 10$

$5 \Leftrightarrow 9$

$6 \Leftrightarrow 8$

Problem 1:

Two dice are thrown together. Find the probability of :

1. Getting a number greater than 10.
2. Getting a sum of 5.
3. Getting a sum is prime.
4. Getting a multiple of 3 or 4.

Solution:

1. Total number of possible outcome = 36
   Getting a number greater than 10 means we want 11 or 12.

Sum 11 can happen in 2 ways or Sum 12 can happen in 1 way.
Number of events of getting number greater than 10 = 2+1=3
There for probability of Getting a number greater than 10
P(E) = 3/36 = 1/12.

2. Total number of possible outcome = 36
   Getting a sum of 5:

Sum 5 can happen in 4 ways.
A number of events of getting a sum of 5 = 4.
There for probability of getting a sum of 5
P(E) = 4/36 = 1/9.

3. Total number of possible outcome = 36

Getting a sum is prime. In this case, we will go and search situations for sum2,sum3,sum5,sum7, and sum 11.
Sum 2 can happen in only 1 way.
Sum 3 can happen in 2 ways.
Sum 5 can happen in 4 ways.
Sum 7 can happen in 6 ways.
Sum 11 can happen in 2 ways.
Number of events of getting sum is prime = 1+2+4+6+2 =15
There for probability of getting a sum is prime
P(E) = 15/36 = 5/12.

4. Total number of possible outcome = 36

Getting a sum is multiple of 3or4. Multiple of 3 or 4 is 3,4,6,8,9,12
Sum 3 can happen in 2 ways.
Sum 4 can happen in 3 ways.
Sum 6 can happen in 5 ways.
Sum 8 can happen in 5 ways.

Sum 9 can happen in 4 ways.
Sum 12 can happen in 1 way.
Number of events of getting sum is multiple of 3or4 = 2+3+5+5+4+1 = 20
There for probability of getting a sum is multiple of 3or4
P(E) = 20/36 = 5/9.

Probability-based on cards

Some basic information about cards:

1. Pack of cards = 52
2. There are 4 suites in a pack of 52 cards.(**clubs,spades,diamonds,hearts)**
3. 13 cards in each of the 4 suits.
4. Each of 4 suits has an ace,2,3,4………,10, jack, queen, king.
5. Clubs and spades are in black color.
6. Diamonds and hearts are in red color.
7. Jack is at the same time in problems also referred to as Knave.
8. Jack, Queen, and King are face cards.

Problem 1:
A card is drawn from a pack of 52 cards. Find the probability:

1. A spade.
2. A king.
3. A Black card.
4. A king or a queen.
5. A face card.
6. A king or a spade.

Solution:

1. A total number of possible outcomes = 52.

The number of events of drawing a spade = 13.
Therefore the probability of a spade
P(E) = 13/52.

2. The total number of possible outcomes = 52.

The number of events of drawing a king = 4.
Therefore the probability of a king
P(E) = 4/52.

3. The total number of possible outcomes = 52.

The number of events of drawing a black card = 26.
Therefore the probability of a black card
P(E) = 26/52.

4. The total number of possible outcomes = 52.

The number of events of drawing a king or queen = 4+4=8.
Therefore the probability of a spade
P(E) = 8/52.

5. The total number of possible outcomes = 52.

The number of events of drawing a face card = 12.
Therefore the probability of a face card
P(E) = 12/52.

6. The total number of possible outcomes = 52.

A king or a spade: there are 4 kings(among 4 kings one king of spades) out of 52 cards and 13 cards of spades.

Number of events of drawing a king or a spade = 4+12 = 16
Therefore the probability of a king or a spade
P(E) = 16/52.

Problem 2:
Two cards are drawn at random **without replacement** from a pack of 52 cards. Find the probability of:

1. 1 queen and 1 king.
2. 1 red and 1 black.

Solution:

1. **1 queen and 1 king :**

The total number of possible outcomes = 52.
From a pack of 52 cards probability of queen = 4/52.
From a pack of 52 cards probability of king = 4/52.

1 queen and 1 king :
In this case, 1st is queen & 2nd is king or 1st is king and 2nd is the queen
Q&K or K&Q i.e. $4/52 \times 4/51 + 4/52 \times 4/51 = 8/(52 \times 51)$.
Therefore $P(1Q \& 1K) = 8/(52 \times 51)$.

2. **1 red and 1 black :**

A total number of possible outcomes = 52.
From a pack of 52 cards probability of red = 26/52.
From a pack of 52 cards probability of black = 26/52.

1 red and 1black :
In this case, 1st is red & 2nd is black or 1st is black and 2nd is red
R&B or B&R i.e. $26/52 \times 26/51 + 26/52 \times 26/51 = 52/(52 \times 51)$.
Therefore P(1Q & 1K) = 1/51.

Probability-based on balls from boxes

Problem 1:
A box contains 10 red, 5 blue, and 1 black. All the balls are identical and 1 ball drawn at random. What is the probability that :

1. The ball is red.
2. The ball is blue.
3. The ball is black.

Solution:
Total number of balls = 10+5+1=16. i.e. n(S) = 16.

1. **Ball is red:**

n(E) = number of ways of drawing red balls = 10.
Therefore probability of drawing red balls
p(E) = n(E)/n(S) = 10/16.

2. **Ball is blue:**

n(E) = number of ways of drawing blue balls = 4.
Therefore probability of drawing blue balls
p(E) = n(E)/n(S) = 5/16.

3. **Ball is black:**

n(E) = number of ways of drawing black balls = 1.
Therefore probability of drawing black balls
p(E) = n(E)/n(S) = 1/16.

One ball question is very simple, but the main question here draws two balls. In such cases, there are two kinds of questions.

1. Ball drawn with replacement.
2. Ball drawn without replacement.

Problem 2:
A box contains 10 red, 5 blue, and 1 black. All the balls are identical and 3 balls drawn at random one after the other with replacement. What is the probability that all 3 balls are red?

Solution:
Total number of balls = 10+5+1=16. i.e. n(S) = 16.
n(E) = number of ways of drawing red balls = 10.
Probability of a red ball = 10/16
Therefore the probability of drawing 3 red balls with replacement
1st red & 2nd red & 3rd red
$10/16 \times 10/16 \times 10/16$.

Draw 1 ball from 2 boxes or 3 boxes

Problem 1:
A box contains 10 red, 5 blue and 2 black and another box contains 5 red, 7 blue and 8 black. 1 ball drawn at random from any of the 2 boxes. Find the probability that the ball is black?

Solution:
Probability of black ball from 1st box = 2/16
Probability of black ball from 2nd box = 8/20.
Selection of 1st box = 1/2
Selection of 2nd box = 1/2
P(Ball is black) = 1st box & Black ball or 2nd box & Black ball
$= 1/2 \times 2/16 + 1/2 \times 8/20$
$= 1/16 + 8/40$.

Word-based question on probability

Problem 1:
What is the probability that there are 53 Sundays in a normal non-leap year?

Solution:
In a non-leap year = 365 days.
365 days have 52 complete weeks and 1 day.
The 365 days calendar will start on 1st January and 1st week will end on 7th January …….
And so on …. the 52nd week will end on 30th December.

For 53 Sundays in a non-leap year, the last day of the year 31th December has to be a Sunday, and the probability of 31st Dec being a sunday = 1/7.
Hence the answer = 1/7.

Problem 2:
What is the probability that there are 53 Sundays in a leap year?

Solution:
In a leap year = 366 days.
366 days have 52 complete weeks and 2 days.

The 52nd week would end on the 364th day of the year and that day would be 29th December.
For 53 Sundays in a non-leap year, the last 2 days of the year would be

1. Sunday or Monday
2. Saturday or Sunday
3. Monday or Tuesday
4. Tuesday or Wednesday
5. Wednesday or Thursday
6. Thursday or Friday and
7. Friday or Saturday

Last 2 days of the year out of 7 cases. Out of 7 cases, only 2 cases have Sundays in them.
Hence the probability of 53 Sundays in a leap year = 1/7.