CS215 Assignment 5
Estimation Transformations

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Question 2: Graph put in 'results' folder; 'q2.png'.

$X \sim U(0,1)$. $\quad y := \frac{-\log x}{\lambda}$. $\quad y = g(x) = \frac{-\log x}{\lambda}$.

$$\underline{g^{-1}(y) = e^{-\lambda y}}.$$

$$P(Y=y) = \cancel{P(X=g^{-1})} \; P(X = g^{-1}(y)) \left|\frac{d}{dy} g^{-1}(y)\right|$$

$$= 1 \times \left|\frac{d}{dy} e^{-\lambda y}\right| = \boxed{\lambda e^{-\lambda y} = P(Y=y)}$$

Analytical form. $\downarrow$ PDF of Y.

$$\boxed{p(y;\lambda) = \lambda e^{-\lambda y}}$$ Analytical form.

Posterior mean; $P(\lambda | \{y_i\}_1^N) = \frac{P(\{y_i\}_1^N | \lambda) \cdot P(\lambda)}{P(\{y_i\}_1^N)}$

$P(\lambda)$ is $\underline{\text{Gamma}(\lambda; \alpha, \beta)}$.

$$\equiv \prod_1^N \lambda e^{-\lambda y_i} \cdot \lambda^{\alpha-1} \exp(-\beta\lambda) \,/\, \text{constant.}$$

$$\equiv \lambda^{N+\alpha-1} \cdot \exp(-(\beta + \Sigma y_i)\lambda)$$

$$\equiv \text{Gamma}(\lambda; \alpha+N, \beta+\Sigma y_i)$$

Mean of $\text{Gamma}(\lambda;\alpha,\beta)$ is α/β.

$\therefore$ Mean of posterior is $\boxed{\frac{\alpha+N}{\beta+\Sigma y_i}} = \boxed{\frac{\alpha+N}{\beta+Y}}$ where $Y = \sum_1^N y_i$.

(Ans)

Interpretation: Relative error for finite N is less for posterior mean, than ML estimator.

(i) As N increase, posterior mean converges to ML estimate. & relative error decreases.

(ii) Posterior mean estimator is preferable because it has a lower relative error than that of ML estimator, for all values of N plotted.