CS 215 ASSIGNMENT

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Question - 5

Frobenius norm: $||A||_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} |a_{ij}|^2}$ where A is a $m \times n$ matrix

Given 4 eigenvectors & mean (vector) of the data

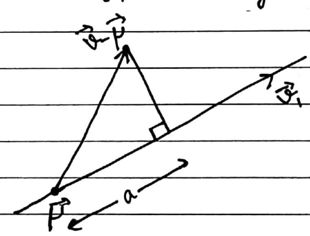
Note that $||A-B||_F = ||A'-B'||_F$ where A & B are $m\times n$ matrices and A' & B' are the corresponding concatenated column vectors

$\Rightarrow$ Frobenius norm of vector $\equiv$ magnitude of vector

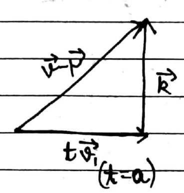
We want to express

$\vec{v}' = (a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3 + d\vec{v}_4) + \vec{\mu}$

where $\{v_i\}$ are given 4 eigenvectors & $\vec{v}'$ is closest representation of $\vec{v}$



$a \equiv$ projection of $\vec{v}-\vec{\mu}$ along $\vec{v}_1$



$\vec{k} = \vec{v} - \vec{\mu} - t\,\vec{v}_1$

$\langle \vec{k}, t\vec{v}_1 \rangle = 0$ (orthogonal)

$\Rightarrow \langle \vec{v} - \vec{\mu} - t\vec{v}_1, t\vec{v}_1 \rangle = 0$

$\Rightarrow \langle \vec{v} - \vec{\mu} - t\vec{v}_1, \vec{v}_1 \rangle = 0$

$\Rightarrow \langle \vec{v} - \vec{\mu}, \vec{v}_1 \rangle = t \langle \vec{v}_1, \vec{v}_1 \rangle = t$ (v_1 is normalized)

$\therefore a = t = \langle \vec{v} - \vec{\mu}, \vec{v}_1 \rangle$

$= \mathrm{dot}(\bar{v} - \bar{\mu}, \bar{v}_1)$

So closest representation of $\vec{v}$ is

$$\vec{v}' - \mu = \langle \vec{v}-\vec{\mu}, \vec{v}_1 \rangle \vec{v}_1 + \langle \vec{v}-\vec{\mu}, \vec{v}_2 \rangle \vec{v}_2 + \langle \vec{v}-\vec{\mu}, \vec{v}_3 \rangle \vec{v}_3 + \langle \vec{v}-\vec{\mu}, \vec{v}_4 \rangle \vec{v}_4$$

Generation of random samples:

$D = 19200$ (dimension of vector)

$N = 4$ (no. of vectors)

Using spectral theorem (SVD, C is symmetric)

$$\underbrace{C}_{D\times D} = \underbrace{U}_{D\times N}\,\underbrace{S}_{N\times N}\,\underbrace{U^T}_{N\times D} = \underbrace{A}_{D\times N}\,\underbrace{A^T}_{N\times D} \quad \text{——} (1)$$

We are decomposing C into $D\times N$ matrix because we are assuming that only N eigenvectors are responsible for generating the data & the rest are effectively ignored.

$$\underbrace{S}_{\text{diagonal of eigenvalues}} = W\underbrace{W^T}_{\text{diagonal of } \sqrt{\text{eigenvalues}}} \quad \text{——} (2)$$

Note that columns of U are eigenvectors of C and diagonal of S is eigenvalues of C. By spectral theorem.

$$S = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix}$$

$$W = \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 & 0 \\ 0 & 0 & \sqrt{\lambda_3} & 0 \\ 0 & 0 & 0 & \sqrt{\lambda_4} \end{bmatrix}$$

So A can be UW as $AA^T = UWW^TU^T$ (from (1) & (2))

We can now generate random samples as $X = AY + \mu = UWY + \mu$

where $Y \sim G(0,1)$, size$(Y) = N\times 1$

Here UW & μ are known.