#### Without momentum:

Choice of learning : Online learning - it is simple and requires minimal memory

With different learning rates the learning seems to be converging with different number of epochs and there is no observed pattern. That is, the number of epochs does not decrease with increasing learning rate (at least not strictly). Overall there seems to be a decrease in the number of epochs. The minimum of the error function seems to lie in a narrow valley and hence the gradient descent follows wide oscillations. The gradient descent oscillates back and forth resulting in larger number of epochs.

The initial weights and bias are initialized randomly and with different initial weights and bias, the number of epochs varies significantly. Learning was repeated with different initial weights.

#### With Momentum:

The number of epochs observed with momentum also does not follow a pattern. That is, it does not seem to increase or decrease with learning rate and gives different results with different learning rates. Theoretically using momentum helps avoid excessive oscillations of gradient descent and tends to follow one direction. But, from the test results it is clear that is not the case here. The combination of learning rates and momentum rate chosen here is not optimal.

However, increasing the learning rate beyond 0.5 (without momentum) reduces the number of epochs significantly.

#### Test results without using momentum:

Learning Rate	Number of epochs
0.05	67459
0.1	814628
0.15	31167
0.2	50322
0.25	13063
0.3	120775
0.35	43088
0.4	38863
0.45	30829
0.5	32588

#### Test results with momentum alpha = 0.9 :

Learning Rate	Number of epochs
0.05	22957
0.1	101532
0.15	83153
0.2	418794
0.25	11695
0.3	34178
0.35	97473
0.4	349479
0.45	87967
0.5	46217

```
import itertools
import numpy as np
import math
import random as rd
def initialize weights():
     weights = []
     for i in range(0,num units):
            weights.append([])
            for j in range(0,4):
                  rand = rd.random()
                  flag = rd.choice([0,1])
                  if(flag == 0):
                        rand = rand *(-1)
                  weights[i].append(rand)
      return weights
def initialize bias():
     weights = []
     for i in range(0,num units):
            rand = rd.random()
            flag = rd.choice([0,1])
            if(flag == 0):
                  rand = rand * (-1)
            weights.append(rand)
      return weights
def get_sigmoid(x,a):
      return 1 / (1 + math.exp((-1) * a * x))
```

```
def get_sigmoid_prime(x,a):
      sig = get sigmoid(x,a)
      return (a * sig * (1 - sig))
def get_v_of_n(x,w,b):
     v n = np.dot(x,w)
     v_n = b + v_n
      return v n
def desiredOutputs(lst):
      des output = []
     for i in lst:
            s = 0
            s = sum(i)
            if(s \% 2 == 0):
                  des_output.append(0)
            else:
                  des output.append(1)
      return des output
def activate(v_n):
     #print("In activate")
     y_n = []
     for i in v n:
            #print(i)
            y_n.append(get_sigmoid(i,1))
      return y_n
def get error(d n,y n):
      return (d_n - y_n)
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```
def get delta k(error out,v n out):
     return (error out * get sigmoid prime(v n out,1))
def
out weight update(w n,b n,delta w n,delta b n,learn rate,error out,v n
_out,y_n,alpha):
     w n plus one = []
     b n plus one = 0
     delta w n new = []
     delta b n new = 0
     for i in range(0,len(w n)):
           w n plus one.append(w n[i] + learn rate *
get delta k(error out, v n out) * y n[i] + delta w n[i])
           delta_w_n_new.append(alpha * (w_n_plus_one[i] - w_n[i]))
     b n plus one = b n + learn rate * get delta k(error out, v n out) *
1 + delta b n
     delta b n new = alpha * (b n plus one - b n)
     return (w n plus one,b n plus one,delta w n new,delta b n new)
def initializeWithZero(m,n):
     temp = []
     for i in range(0,m):
           temp.append([])
           for j in range(0,n):
                 temp[i].append(0)
     return temp
def
hidden weight update(w n,b n,delta w n,delta b n,learn rate,v n,x,delt
a k,alpha):
     w n plus one = []
     b n plus one = []
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delta w n new = initializeWithZero(num hidden units,len(w n[0]))
     delta b n new = []
     for i in range(0,num hidden units):
           w n plus one.append([])
           for j in range(0,len(w n[i])):
                 w n plus one[i].append(w n[i][i] + learn rate *
get sigmoid prime(v n[i],1) * delta k * w n[4][i] * x[i] + delta w n[i][j])
                 delta w n new[i][j] = alpha * (w n plus one[i][j] -
w_n[i][j])
           b n plus one.append(b n[i] + learn rate *
get sigmoid prime(v n[i],1) * delta k * w n[4][i] * 1 + delta b n[i])
           delta b n new.append(alpha * (b n plus one[i] - b n[i]))
     return(w n plus one,b n plus one,delta w n new,delta b n new)
def isDesiredError(error,stop_error):
     for i in error:
           if(i > stop error):
                 return False
     return True
def initializeWithOne(size):
     temp = []
     for i in range(0,size):
           temp.append(1)
     return temp
def update weights(weights,b,w n plus one,b n plus one,w,b1):
     for i in range(0,len(w_n_plus_one)):
           weights[4][i] = w n plus one[i]
     b[4] = b n plus one
     for i in range(0,num hidden units):
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for j in range(0,4):
                  weights[i][j] = w[i][j]
            b[i] = b1[i]
      return (weights,b)
def write to file(fd,learn rate,epochs,error):
      fd.write("Learning Rate: " + str(learn rate) + "\n")
     fd.write("Number of epochs : " + str(epochs) + "\n")
     fd.write("Error: " + str(error) + "\n")
def learn(train data,des output,learn rate,weights,b,fd,alpha):
      stop error = 0.05
      size = 16
      error = initializeWithOne(size)
      epochs = 0
      print("Learning Rate : " + str(learn_rate))
      print("alpha : " + str(alpha))
      #print(weights)
      while(not(isDesiredError(error,stop error))):
            delta b n out = 0
            delta w n out = [0,0,0,0]
            delta w n = initializeWithZero(num hidden units,4)
            delta b n = [0,0,0,0]
            #train data = rd.sample(train data,len(train data))
            #print(train data)
            for p in range(0,len(train data)):
                  #print("-----Training data : " + str(p))
                  v n = []
                  v n out = []
                  for i in range(0,num hidden units):
                       v n.append(get v of n(train data[p],weights[i],b[i]))
                  y n = activate(v n)
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#print(y n)
                 for i in range(num hidden units,num hidden units +
num out units):
                      v_n_out.append(get_v_of_n(y_n,weights[4],b[4]))
                 y n out = activate(v n out)
                 #print(y n out)
                 error out = get error(des output[p],y n out[0])
                 error[p] = abs(error out)
w n plus one,b n plus one,delta w n out,delta b n out =
out weight update(weights[4],b[4],delta w n out,delta b n out,learn rate
,error out,v n out[0],y n,alpha)
                 delta k = get delta k(error out,v n out[0])
                 w,b1,delta w n,delta b n =
hidden weight update(weights,b,delta w n,delta b n,learn rate,v n,train
_data[p],delta_k,alpha)
                 weights,b =
update weights(weights,b,w n plus one,b n plus one,w,b1)
           epochs = epochs + 1
           if(epochs \% 3000 == 0):
                 print(epochs)
           if(epochs \% 15000 == 0):
                 print("Error : " + str(error))
     write to file(fd,learn rate,epochs,error)
     # print(weights)
     # print(b)
def initializeWith(x,m,n):
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temp = []
      if(n == 0):
           for i in range(0,m):
                 temp.append(x[i])
      else:
           for i in range(0,m):
                  temp.append([])
                 for j in range(0,n):
                       temp[i].append(x[i][j])
      return temp
if name == ' main ':
      global num hidden units
      num hidden units = 4
      global num out units
      num out units = 1
      global num units
      num units = num out units + num hidden units
      n = 4
     lst = list(itertools.product([0, 1], repeat=n))
      des output = desiredOutputs(lst)
     #print(lst)
     #print(des output)
      global initial weights
      initial weights = initialize weights()
      global initial b
      initial b = initialize bias()
     #print(init b)
     fd = open("Results.txt",'w')
      learn rate = np.arange(0.05, 0.51, 0.05)
      alpha = 0
      print(learn rate)
```

```
fd.write("Initial Weights: " + str(initial weights))
fd.write("Initial bias : " + str(initial b))
for i in learn rate:
      init_weights = initializeWith(initial_weights,num_units,4)
      init b = initializeWith(initial b,num units,0)
      print("Initial Weights : ")
      print(init weights)
      print("Initial bias : ")
      print(init b)
      #learn(lst,des output,i,init weights,init b,fd)
      learn(lst,des output,i,init weights,init b,fd,alpha)
fd.close()
fd = open("Results momentum 3.txt",'w')
fd.write("Initial Weights: " + str(initial weights))
fd.write("Initial bias : " + str(initial b))
alpha = 0.9
for i in learn rate:
      init weights = initializeWith(initial weights,num units,4)
      init b = initializeWith(initial b,num units,0)
      learn(lst,des output,i,init weights,init b,fd,alpha)
fd.close()
```