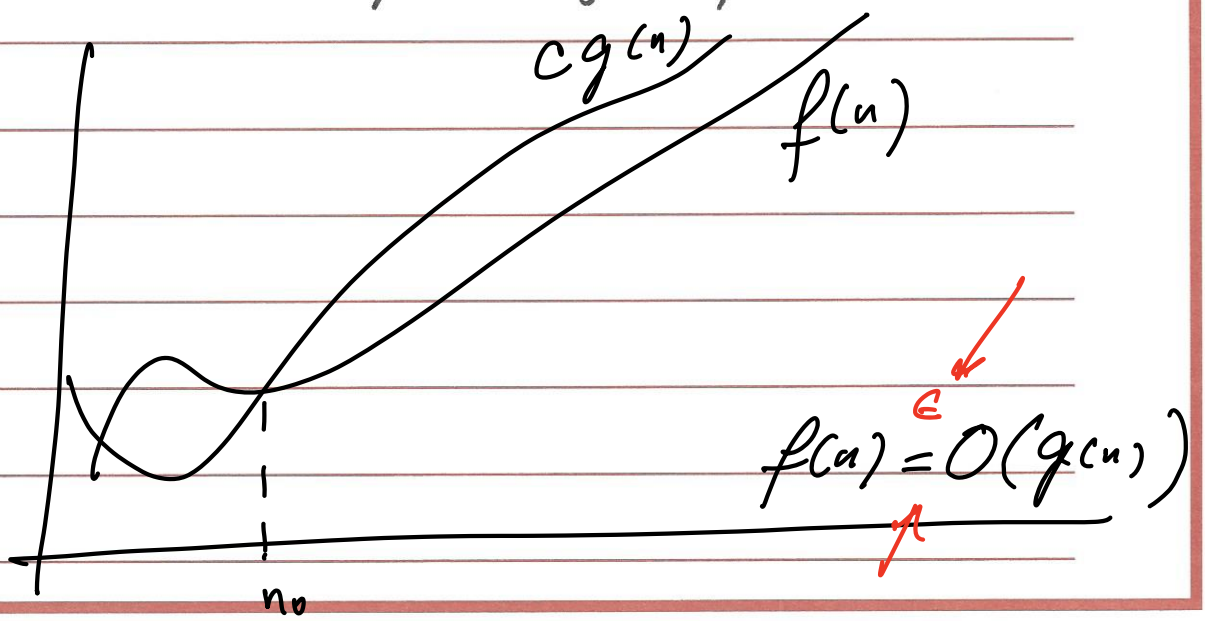
Review of the Asymptotic Notations

Formally, $O(g(n)) = \{ f(n) \mid$ there exist positive constants c and n_0 such that $0 \leq f(n) \leq c\,g(n)$ for all $n \geq n_0 \}$

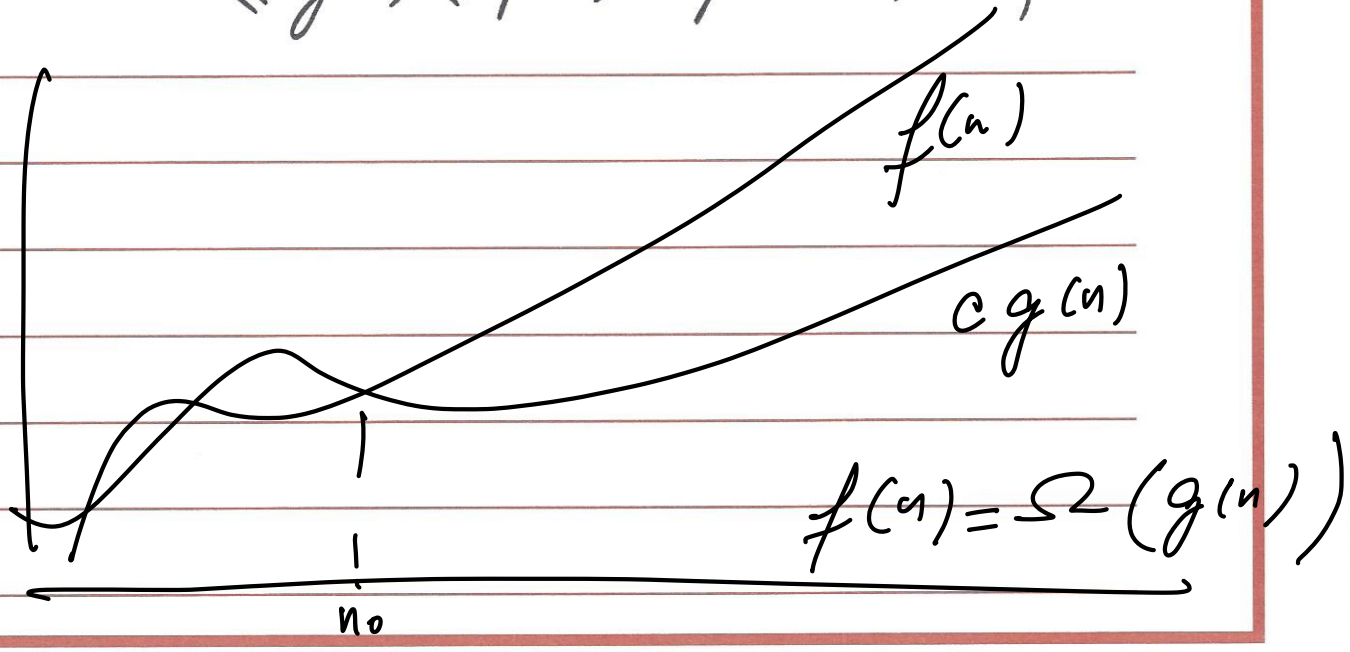


T — Any quadratic function is $O(n^2)$

T — Any linear " " $O(n^2)$

F — Any cubic " " $O(n^2)$

$\Omega(g(n)) = \{ f(n) \mid$ there exist positive constants c and n_0 such that $0 \leq cg(n) \leq f(n)$ for $n \geq n_0 \}$

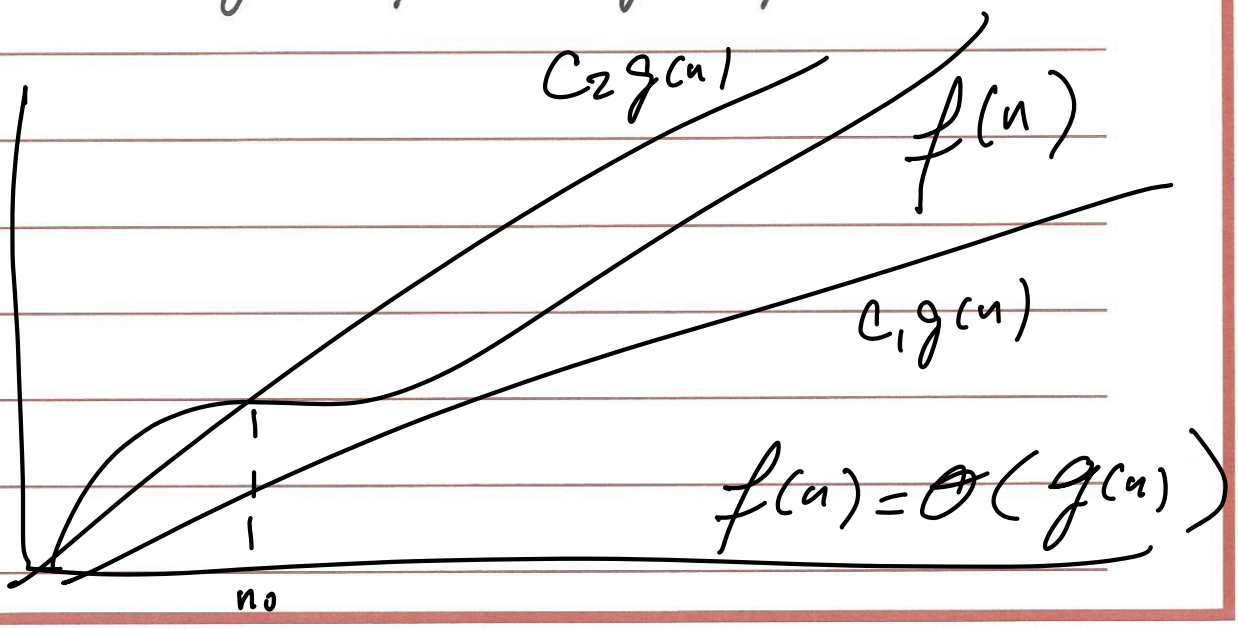


T Any quadratic function is $\Omega(n^2)$

F Any linear " " $\Omega(n^2)$

T Any cubic " " $\Omega(n^2)$

$\Theta(g(n)) = \{ f(n) \mid$ there exist positive constants c_1, c_2, and n_0 such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0 \}$



- Any quadratic func. is $\Theta(n^2)$
- Any linear " " $\Theta(n^2)$
- Any cubic " " $\Theta(n^2)$

	Worst Case	Best Case
linear Search	$O(n), \Theta(n), \Omega(n)$	$O(1), \Omega(1), \Theta(1)$
Binary "	$O(\lg n), \Theta(\lg n),$	$O(1), \Omega(1), \Theta(1)$
Insertion Sort	$O(n^2), \Theta(n^2)$	$O(n), \Theta(n), \Omega(n), \Omega(1)$
merge Sort	$O(n \lg n), \Theta(n \lg n)$	$O(n \lg n), \Theta(n \lg n)$

Worst Case performance

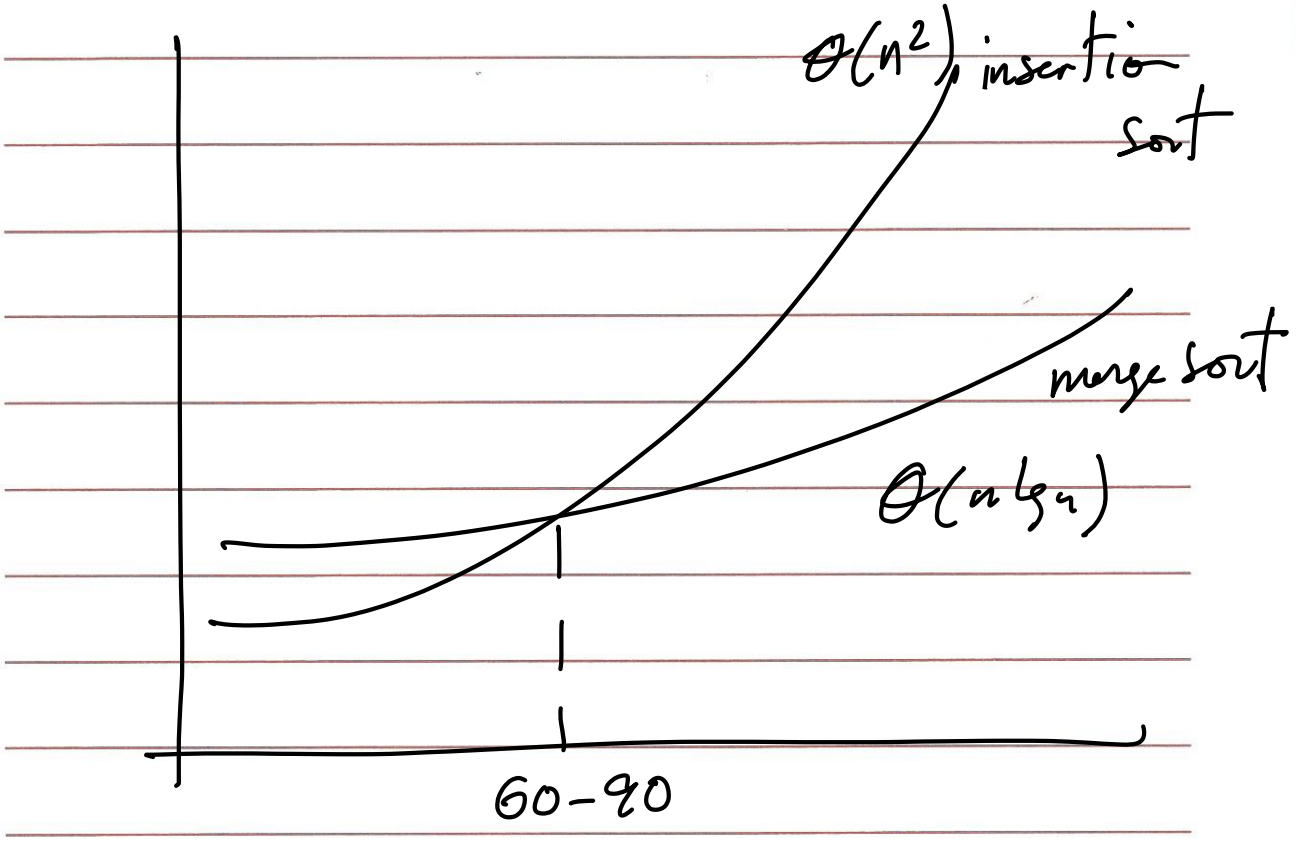
Algorithm A: $\Theta(4^n n^3 \lg n)$

Algorithm B: $\Theta(3^n n^8 (\lg n)^2)$

1- Exponential comp. — fastest growing

2- polynomial "

3- Logarithmic " — Slowest growing



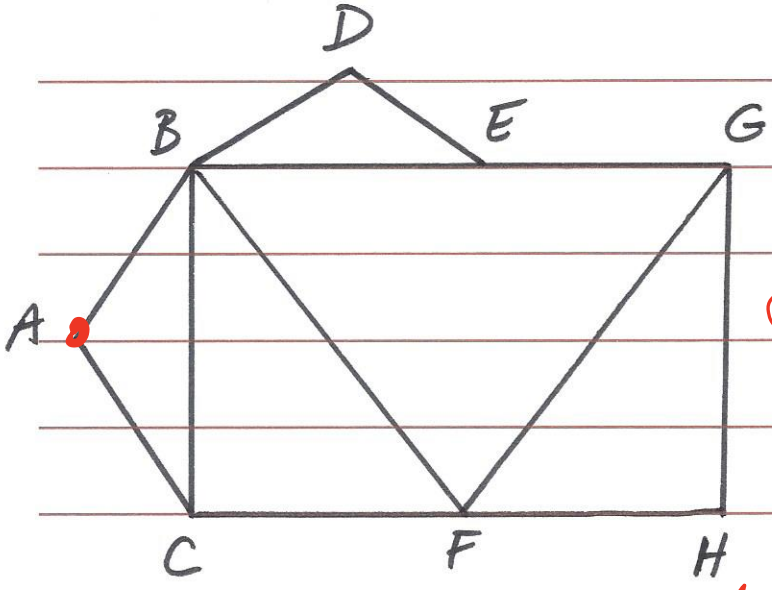
Review of BFS & DFS

Q: What are we searching for?

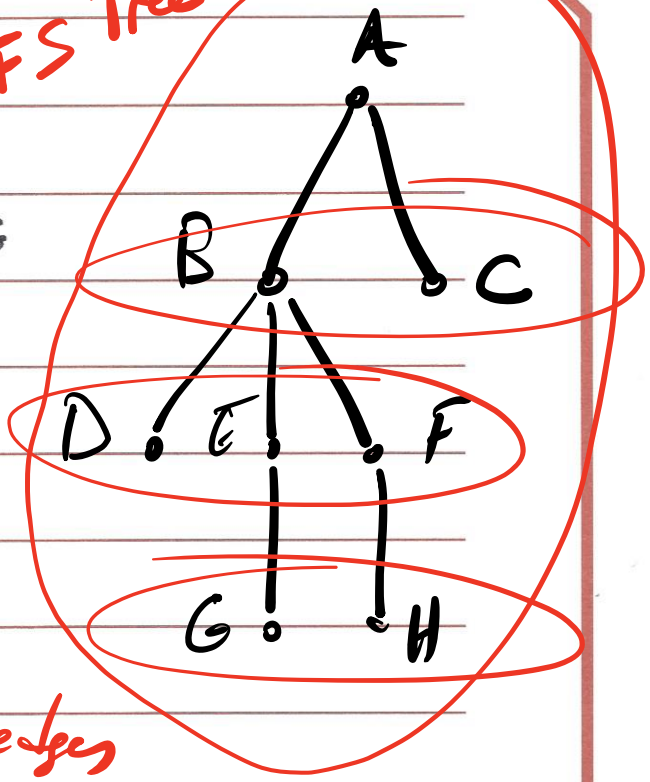
- Find out if there is a path from A to B.

- Find all nodes that can be reached from A.

BFS



BFS Tree

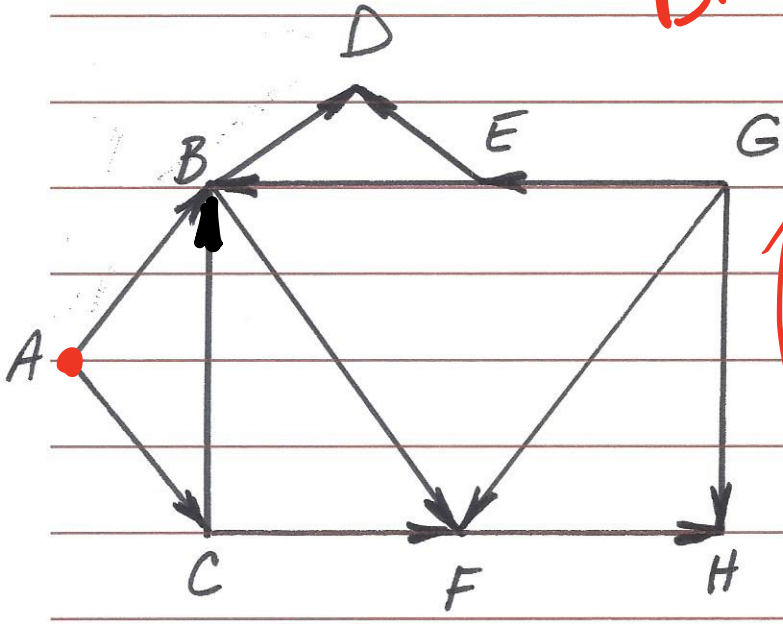


$O(m+n)$

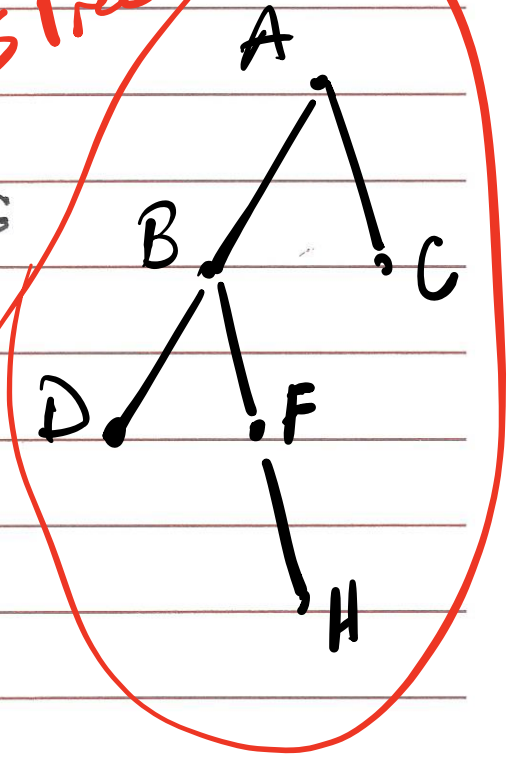
m = no. of edges

n = no of nodes

DFS

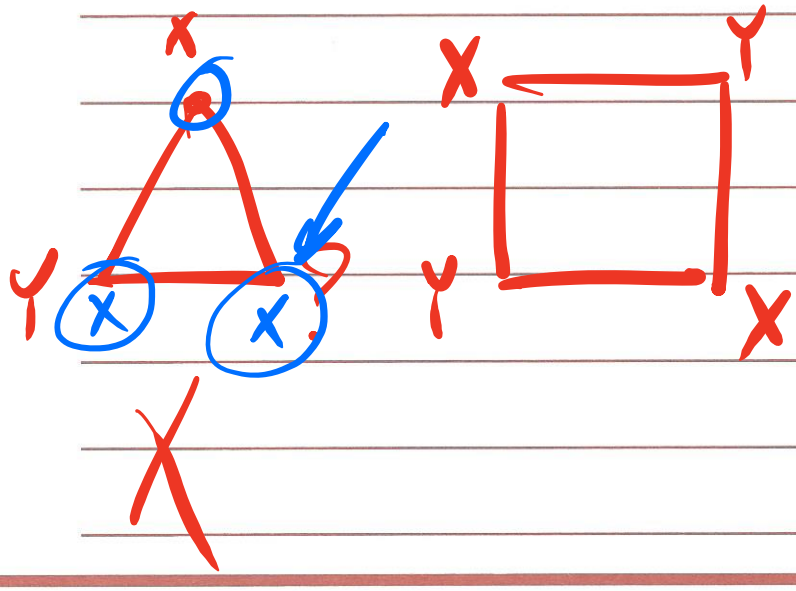


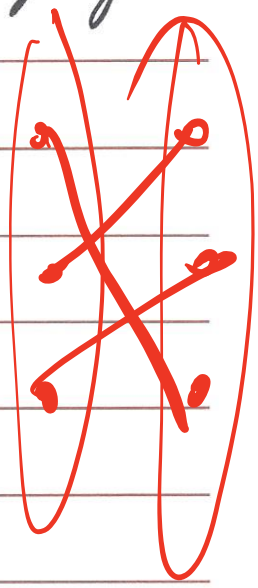
DFS Tree

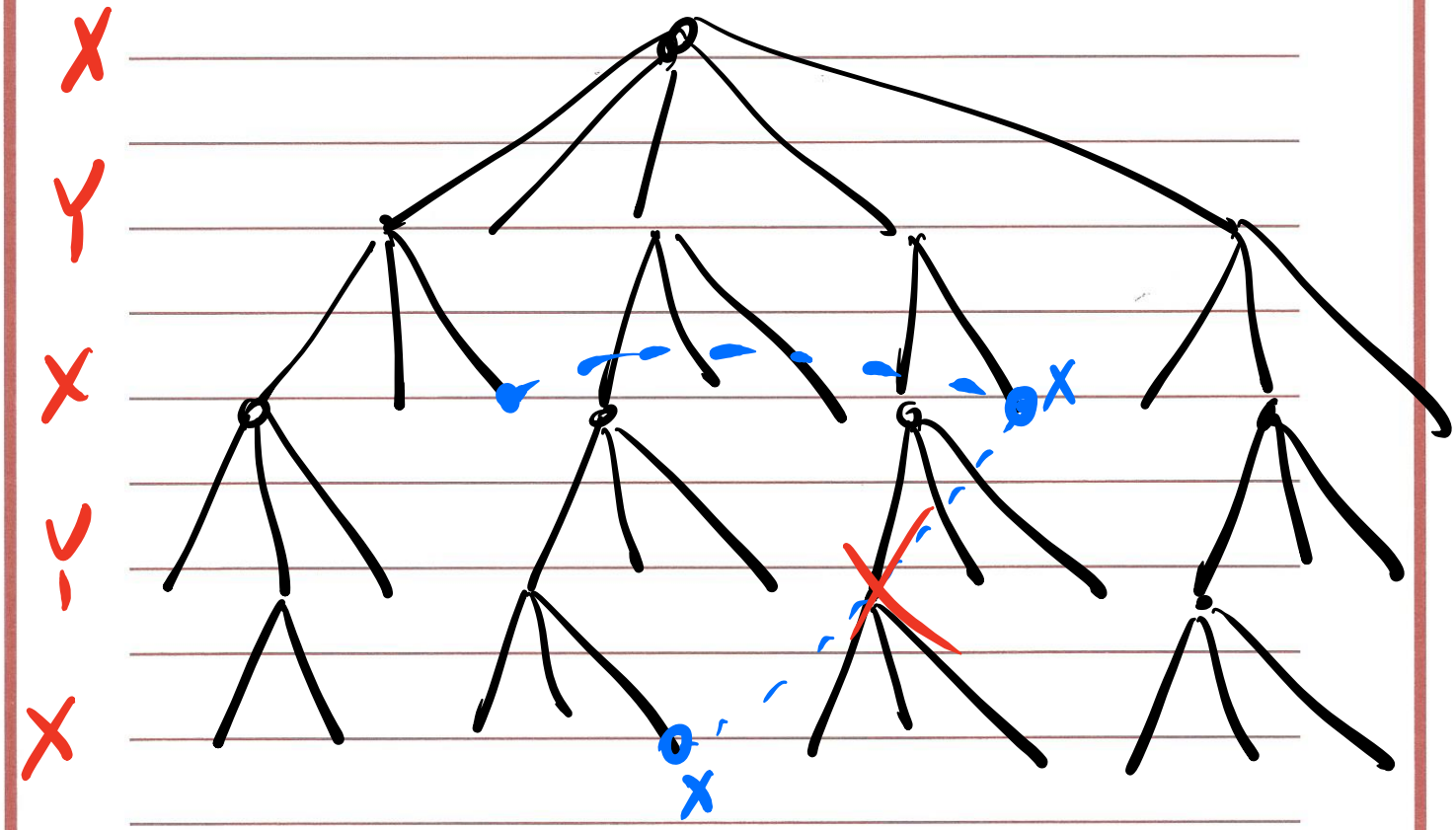


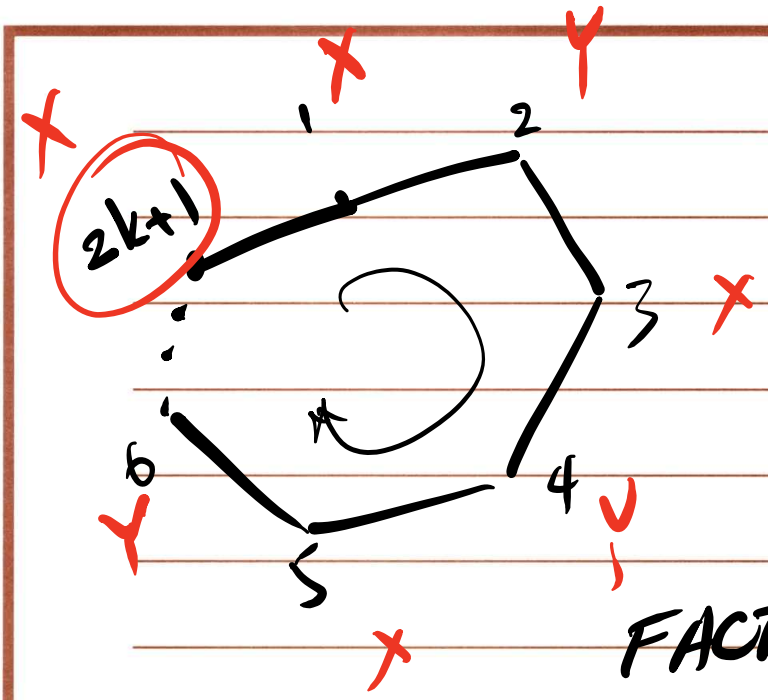
$O(m+n)$

Q: How do you determine if a graph is bipartite?

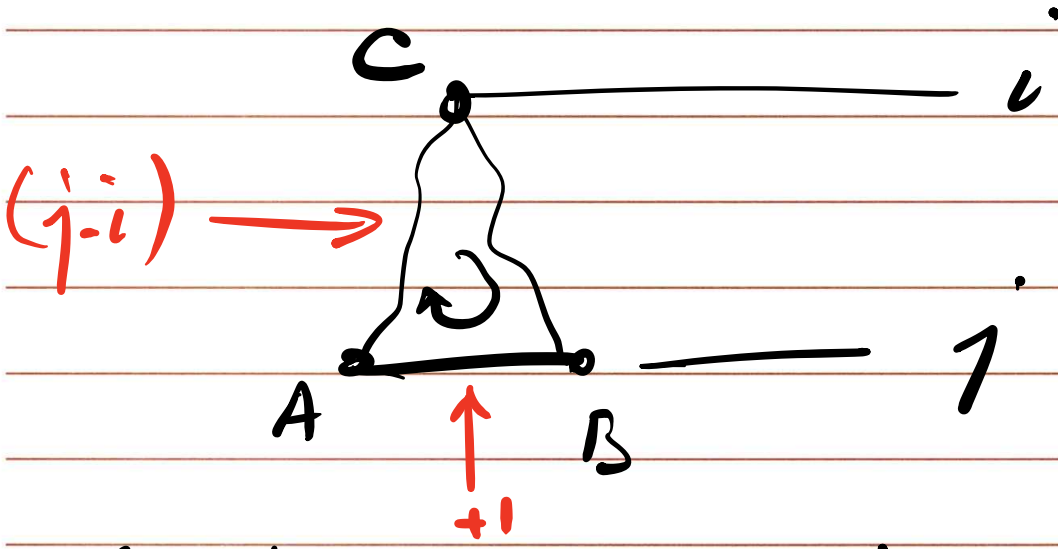








FACT: If a graph G is bipartite then it cannot contain an odd cycle.



length of cycle = $2*(j-i)+1$

odd!

Solution:

$O(m+n)$ — Run BFS starting from any node, say S. Label each node Red or Blue depending on whether they appear at an odd or even level on the BFS tree.

$O(m)$ — Then, go through all edges and examine the labels at the two ends of the edge. If all edges have a Red end and a Blue end, then the graph is bipartite.

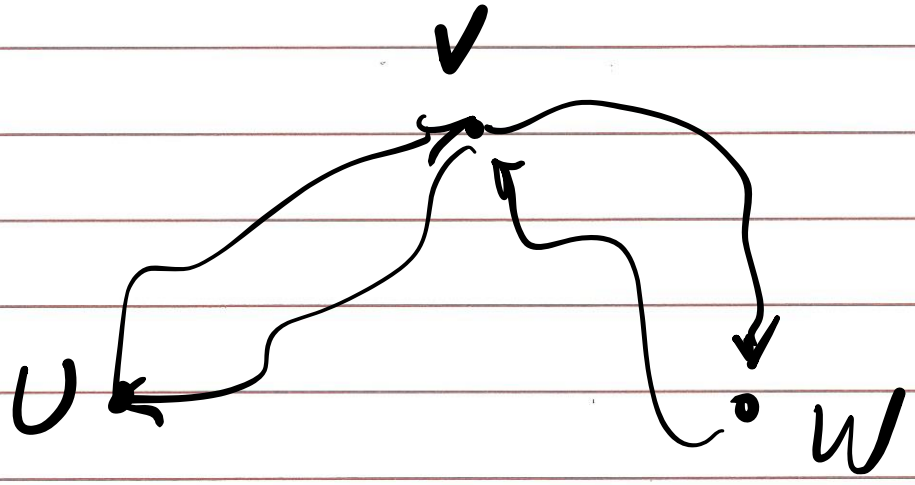
Otherwise, the graph is not bipartite.

overall Complexity = $O(m+n)$

Def. A directed graph is strongly connected if there is a path from any point to any other point in the graph.

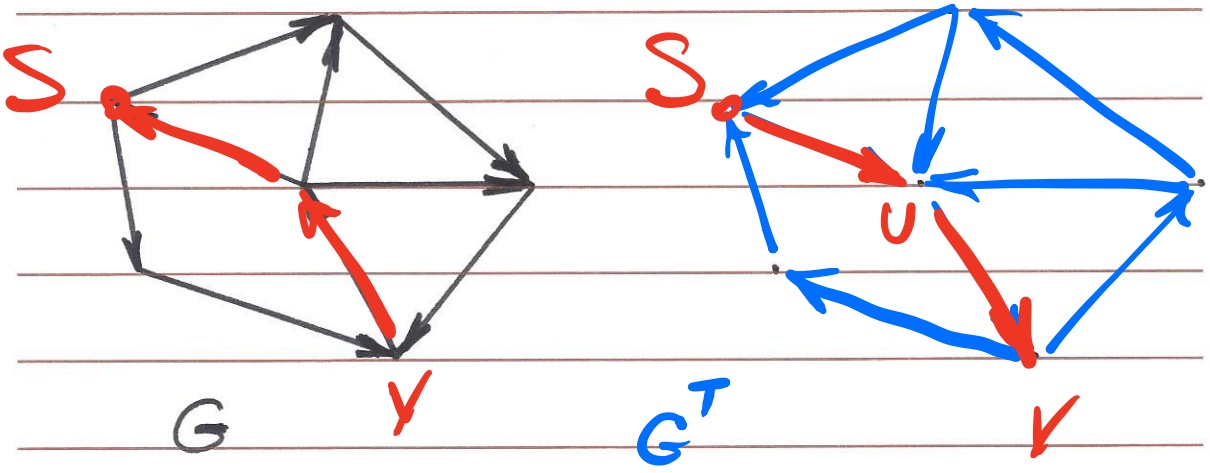
Q: How do you know if a given directed graph is strongly connected?

Brute force: Run BFS/DFS from every node $\rightarrow O(n(m+n))$

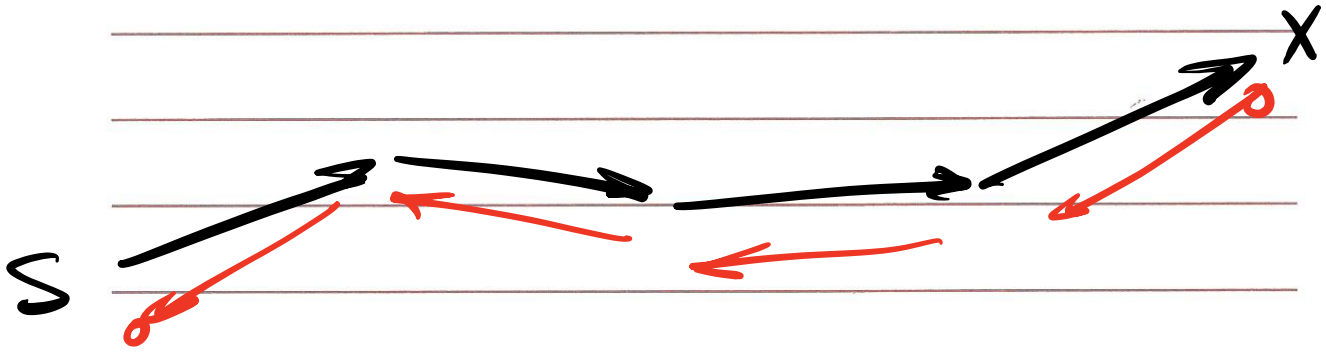


u & w are mutually reachable

Transpose of a directed graph



Mutually Reachable Nodes



Solution:

1. Use BFS or DFS to find all nodes reachable from s (an arbitrary node) in G. If some nodes are not reachable from s, stop. The graph is not strongly connected.

O(m+n)

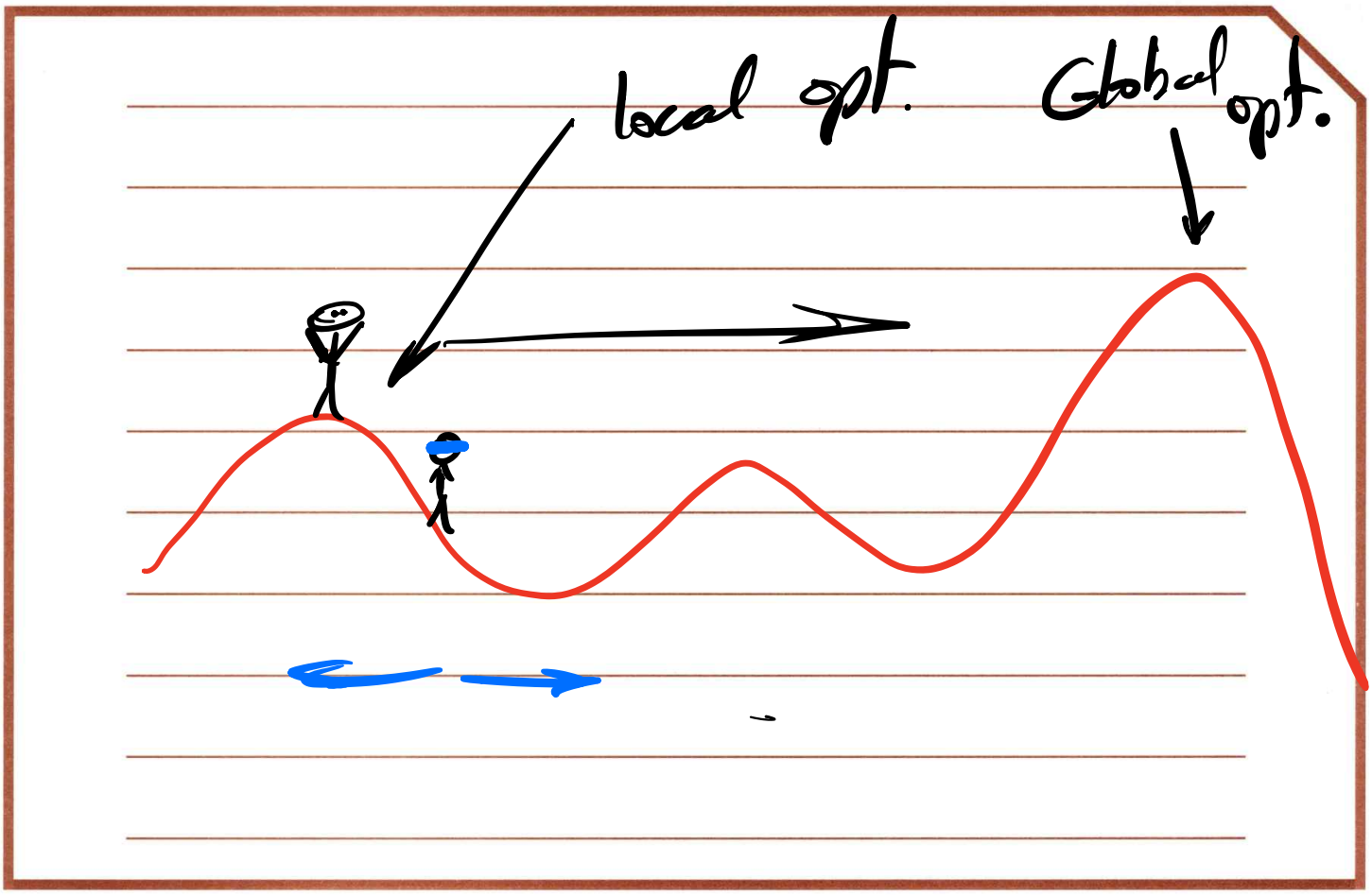
Otherwise, continue with step 2.

O(m) 2. Create G^T (Transpose of G)

O(m+n) 3. Use BFS or DFS to find all nodes reachable from s in G^T. If some nodes are not reachable from s, then the graph is not strongly connected.

Otherwise, the graph is strongly connected.

overall performance = O(m+n)


local opt.
Global opt.