CSCI 270 - Spring 2023 - HW10

Due April 12, 2023

Problem 1 (25pts)

Consider the partial satisfiability problem, denoted as 3-Sat(α). We are given a collection of k clauses, each of which contains exactly three literals, and we are asked to determine whether there is an assignment of true/false values to the literals such that at least αk clauses will be true. Note that 3-Sat(1) is exactly the 3-SAT problem from lecture.

Prove that 3-Sat(15/16) is **NP**-complete.

Hint: If x, y, and z are literals, there are eight possible clauses containing them: $(x \vee y \vee z), (!x \vee y \vee z), (x \vee !y \vee z), (x \vee y \vee !z), (!x \vee !y \vee z), (!x \vee y \vee !z), (x \vee !y \vee !z), (!x \vee !y \vee !z)$

Problem 2

[25 pts.] Consider a graph $G = (V, E)$ and two integers k, m.

2a

A **k-clique** is a subset of nodes $u_i \in G, i = 1, .., k$ such that there is an edge connecting every pair of distinct vertices u_i, u_j. In other words, the **k-clique** is a complete sub-graph of G. Prove that finding a clique of size k is NP-Complete. [15 pts.]

2b

The **Dense Subgraph** problem is to find a subset V' of V, whose size is at most k and are connected by at least m edges. Prove that the **Dense Subgraph** problem is NP-Complete. [10 pts.]

Problem 3 (25 pts)

Consider a modified SAT problem, SAT' in which given a CNF formula having m clauses and n variables $\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_\mathrm{n}$, the output is YES if there is an assignment to the variables such that exactly $m - 2$ clauses are satisfied, and NO otherwise. Prove that SAT' is NP-Complete.

Problem 4 (25 pts)

Show that Vertex Cover is still NP-complete even when all vertices in the graph are restricted to have even degree.

Practice Problems

Problem 5 (25 pts)

(Kleinberg and Tardos, Chapter 8, Exercise 5)

Consider a set $A = \{a_1, \ldots, a_n\}$ and a collection $B_1, B_2, \ldots, B_m$ of subsets of A (i.e., $B_i \subseteq A$ for each i).

We say that a set $H \subseteq A$ is a *hitting set* for the collection $B_1, B_2, \ldots, B_m$ if H contains at least one element from each B_i—that is, if $H \cap B_i$ is not empty for each i (so H "hits" all the sets B_i).

We now define the *Hitting Set Problem* as follows. We are given a set $A = \{a_1, \ldots, a_n\}$, a collection $B_1, B_2, \ldots, B_m$ of subsets of A, and a number k. We are asked: Is there a hitting set $H \subseteq A$ for $B_1, B_2, \ldots, B_m$ so that the size of H is at most k?

Prove that Hitting Set is NP-complete.