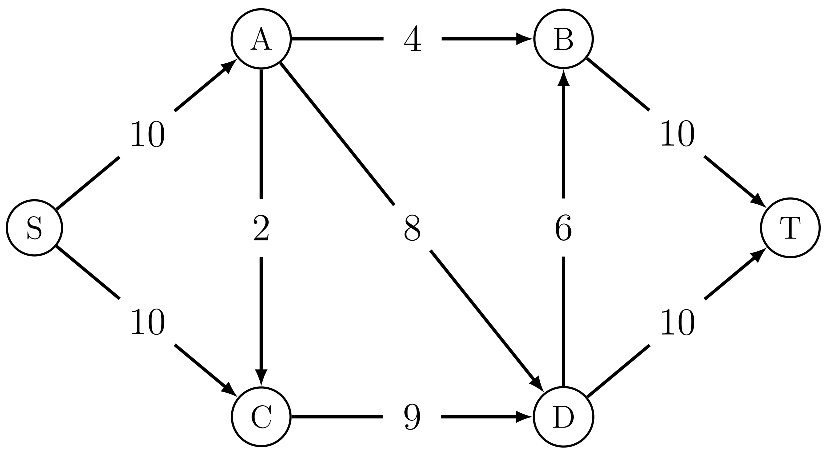
CSCI 270 - Spring 2023 - Discussion 8

1. Consider the following graph with edge capacities.



   (a) Run Ford-Fulkerson manually and confirm the residual graphs and the augmenting paths in each iteration.

   (b) Give a min-cut and confirm min-cut = maxflow.

2. Recall that the Ford-Fulkerson algorithm runs in $O(C|E|)$. Construct a flow network with $|E| \leq 10$ that requires at least 10^6 operations to complete.

3. There are n students in a class. We want to choose a subset of k students as a committee. There has to be m_1 number of freshmen, m_2 number of sophomores, m_3 number of juniors, and m_4 number of seniors in the committee. Each student is from one of k departments, where $k = m_1 + m_2 + m_3 + m_4$. Exactly one student from each department has to be chosen for the committee. We are given a list of students, their home departments, and their class (freshman, sophomore, junior, senior). Describe an efficient algorithm based on network flow techniques to select who should be on the committee such that the above constraints are all satisfied.

4. You want to assign n students to m project teams as evenly as possible. Each student has provided a list of projects he or she is willing to work on, and each project has

at least one interested student. Ideally, the students would be evenly spread between the projects, so that no project takes more than $\lceil \frac{n}{m} \rceil$ students. However, this will likely be impossible. Design a polynomial-time algorithm to find an assignment that minimizes the number of students assigned to each project team.

Hints

- Construct a graph in $\Theta(mn)$ time. You can leave some of the edge capacities undetermined.
- Assume that each project will take no more than n students. Is it possible that the value of the max-flow is lower than n?
- Assume that each project will take no more than $\lceil \frac{n}{m} \rceil$ students. Is it possible that the value of the max-flow is lower than n?
- Run Ford-Fulkerson algorithm at most $\Theta(\log n)$ times.

5. We define a most vital edge of a network as an edge whose deletion causes the largest decrease in the maximum s-t-flow value. Let f be an arbitrary maximum s-t-flow. Either prove the following claims or show through counterexamples that they are false:

   (a) A most vital edge is an edge e with the maximum value of c_e.

   (b) A most vital edge is an edge e with the maximum value of f_e.

   (c) A most vital edge is an edge e with the maximum value of f_e among edges belonging to some minimum cut.

   (d) An edge that does not belong to any minimum cut cannot be a most vital edge.

   (e) A network can contain only one most vital edge.