CSCI 270
HW2

1. Arrange these functions under the O notation using only = (equivalent) or $\subset$ (strict subset of):
    a) $2^{\log n}$
    b) 2^{3n}
    c) $n^{n \log n}$
    d) $\log n$
    e) $n\log(n^2)$
    f) n^{n^2}
    g) $\log(\log(n^n))$

    All logs are base 2. (10 pts)

2. Given functions f_1, f_2, g_1, g_2 such that $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$. For each of the following statements, decide whether it is true or false and briefly explain why. (12 pts)

    a) $f_1(n)/f_2(n) = O(g_1(n)/g_2(n))$
    b) $f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$
    c) $f_1(n)^2 = O(g_1(n)^2)$
    d) $\log_2(f_1(n)) = O(\log_2(g_1(n)))$

3. Given an undirected graph G with n nodes and m edges, design an O(m+n) algorithm to detect whether G contains a cycle. Your algorithm should output a cycle if there is one. (12 pts)

4. Solve Kleinberg and Tardos, Chapter 2, Exercise 6. (14 pts)

5. What Mathematicians often keep track of a statistic called their Erdős Number, after the great 20th century mathematician. Paul Erdős himself has a number of zero. Anyone who wrote a mathematical paper with him has a number of one, anyone who wrote a paper with someone who wrote a paper with him has a number of two, and so forth and so on. Supposing that we have a database of all mathematical papers ever written along with their authors: (6 pts)
    a. Explain how to represent this data as a graph.
    b. Explain how we would compute the Erdős number for a particular researcher.
    c. Explain how we would determine all researchers with Erdős number at most two.

6. Given a DAG, give a linear-time algorithm to determine if there is a simple path that visits all vertices. (8 pts)

Ungraded Problems

1. What is the worst-case performance of the procedure below?

```
c = 0
i = n
while i>1 do
        for j = 1 to i do
                c = c+1
        end for
        i = floor(i/2)
end while
return c
```

   Provide a brief explanation for your answer.