

CS 270 Exam 1 Sample 1

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Problem 1. True/False Questions

Answer:

- True. A cyclic directed graph cannot have a topological ordering.
- False. BFS can be used to find the shortest path on unweighted graphs.
- False. The same graph can have multiple topological orderings if sibling nodes aren't ordered.
- True. All MST algorithms are greedy because they always select an edge with a specific criteria to create the overall MST.
- False. A strongly connected graph means that there exists a path from all nodes to every other nodes, but since edges are directed, no bidirectionality means going the reverse direction can take much longer.
- True. The shortest path will be the same because edges are directed.
- True. The shortest path will be the same because edges are directed.
- False. Consider the following counter example. $M = (m_1, m_2)$ and $W = (w_1, w_2)$. If m_1 and m_2 both prefer w_1 over w_2 **and** w_1 and w_2 both prefer m_1 over m_2 , then the following matching will result $\{(m_1, w_1), (m_2, w_2)\}$, where a man and a woman are each other's last preference and get matched.
- True. After every removal of the largest element, we need to trickle down which takes $\log n$ time, therefore $k \log n > c * 1$ for some c .
- True. A strongly connected (directed) graph must have a cycle because if the paths from u to v and from v to u exist and are disjoint, then their union will create a cycle.

- True. The heaviest edge weight is removed until the graph becomes unconnected or acyclic in the Reverse Delete algorithm to create the MST.
- False. The height of a complete binary tree with n nodes is $O(\log n)$.
- True. Dijkstra's will work on any non-negative cost edged graph.

Problem 2. Multiple Choice Questions

Answer:

1. (d). Finding an arbitrary element has a worst case $O(n)$ in a binary heap because there does not exist any ordering between left and right subtrees, and there is only an ordering between parent and children nodes.
2. (a). Insertion/Bubble/Selection all run in $O(n^2)$, but they are lower bounded by n^2 as well since it does not do all n iterations of work at each iteration. Rather it does some amount of work up to n at each iteration.
3. (c). Men end up with their best valid partner that they have not proposed to before.
4. (a) & (c). $\log n + n \log n$ and $n + \log(n^n)$ are both $\Theta(n \log n)$
5. (a) & (b). $n \log n$ and n^2 are bigger than n , but less than n^3 .
6. (e). The runtime of Dijkstra's algorithm using a binary heap is $O(|E| \log |V|)$.