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Question-4 Suppose a greedy algorithm is used to return the smallest number of coins for a total of n cents, using coin denominations of 1, 2, and 4 cents. The algorithm selects one coin at a time, dispenses this coin, decreases n by the denomination of this coin, and proceeds in this fashion until n reaches zero. The greedy choice consists in always selecting a coin with the largest possible value. The required coins of given denominations are always available. Prove that this greedy algorithm always returns an optimal result.

1. Given:

- a. In the problem, it is stated that the greedy algorithm always selects a coin with the largest possible value.
- b. The number of 1-cent coins will be one because if number of 1-cent coins is 2, it can be replaced by one 2-cent coin. If number of 1-cent coins are 4 it can be replaced by one 4-cent coin. This proves maximum number of 1-cent coins in the solution by the greedy algorithm will be one.
- c. Similar to above proof, it can be shown that maximum number of 2-cent coins in the solution by the greedy algorithm will be one.

This shows the greedy algorithm uses maximum of one 1-cent coin and one 2-cent coin

2. Proof of optimality

- a. The greedy algorithm uses maximum of one 1-cent coin and one 2-cent coin. So, the coin with maximum value (that is 4-cent coin) is always the maximum number of coins in the optimal solution.
- b. To make a certain value of money, using the maximum value coin gives least number of coins.
- c. This proves, the greedy algorithm provides the optimal solution, that is the least number of coins because we are choosing maximum value coin for making most of the given value of amount, n .