Neel Kawithar
UIN: 011793294

Q. 4

→ Using the given matrix chain <12, 9, 11, 14, 16, 13>

$A \times B \times C \times D \times E$

(1)

12	9	9	11	11	14	14	16	16	13
d_0		d_1		d_2		d_3		d_4	d_5

(2) $$c[i,j] = \min_{i \le k < j} \{ c[i,k] + c[k+1,j] + d_{i-1} \cdot d_k \cdot d_j \}$$

c	i→ 1	2	3	4	5
1	0	1188	2898	5130	6678
j 2		0	1386	3402	5274
↓ 3			0	2464	4752
4				0	2912
5					0

k=1 $c[1,2] = d_0 \cdot d_1 \cdot d_2 = 12 \cdot 9 \cdot 11 = 1188$

(3) $c[2,3] = d_1 \cdot d_2 \cdot d_3 = 9 \cdot 11 \cdot 14 = 1386$

$c[3,4] = d_2 \cdot d_3 \cdot d_4 = 11 \cdot 14 \cdot 16 = 2464$

$c[4,5] = d_3 \cdot d_4 \cdot d_5 = 14 \cdot 16 \cdot 13 = 2912$

k	1	2	3	4	5
1	0	1	1	1	1
2		0	2	3	4
3			0	3	4
4				0	4
5					0

(4) $$c[1,3] = \min_{1 \le k < 3} \begin{cases} k=1: c[1,1] + c[2,3] + d_0 d_1 d_3 \\ \quad 0 + 1386 + 12 \cdot 9 \cdot 14 = 2898 \\ k=2: c[1,2] + c[3,3] + d_0 d_2 d_3 \\ \quad 1188 + 0 + 12 \cdot 11 \cdot 14 = 3036 \end{cases}$$

$$c[2,4] = \min_{2 \le k < 4} \begin{cases} k=2: c[2,2] + c[3,4] + d_1 \cdot d_2 \cdot d_4 \\ \quad 0 + 2464 + 9 \cdot 11 \cdot 16 = 4048 \\ k=3: c[2,3] + c[4,4] + d_1 \cdot d_3 \cdot d_4 \\ \quad 1386 + 0 + 9 \cdot 14 \cdot 16 = 3402 \end{cases}$$

$$c[3,5] = \min_{3 \le k < 5} \begin{cases} k=3: c[3,3] + c[4,5] + d_2 \cdot d_3 \cdot d_5 \\ \quad 0 + 2912 + 11 \cdot 14 \cdot 13 = 4914 \\ k=4: c[3,4] + c[5,5] + d_2 \cdot d_4 \cdot d_5 \\ \quad 2464 + 0 + 11 \cdot 16 \cdot 13 = 4752 \end{cases}$$

⑤ $c[1,4] = \min_{1 \le k < 4}$

$k=1$: $c[1,1] + c[2,4] + d_0 \cdot d_1 \cdot d_4$
$0 + 3402 + 12 \cdot 9 \cdot 16 = 5130$

$k=2$: $c[1,2] + c[3,4] + d_0 \cdot d_2 \cdot d_4$
$1188 + 2464 + 12 \cdot 11 \cdot 16 = 5764$

$k=3$: $c[1,3] + c[4,4] + d_0 \cdot d_3 \cdot d_4$
$2898 + 0 + 12 \cdot 14 \cdot 16 = 5586$

$c[2,5] = \min_{2 \le k < 5}$

$k=2$: $c[2,2] + c[3,5] + d_1 \cdot d_2 \cdot d_5$
$0 + 4752 + 9 \cdot 11 \cdot 13 = 6039$

$k=3$: $c[2,3] + c[4,5] + d_1 \cdot d_3 \cdot d_5$
$1386 + 2912 + 9 \cdot 14 \cdot 13 = 5936$

$k=4$: $c[2,4] + c[5,5] + d_1 \cdot d_4 \cdot d_5$
$3402 + 0 + 9 \cdot 16 \cdot 13 = 5274$

⑥ $c[1,5] = \min_{1 \le k < 5}$

$k=1$: $c[1,1] + c[2,5] + d_0 \cdot d_1 \cdot d_5$
$0 + 5274 + 12 \cdot 9 \cdot 13 = 6678$

$k=2$: $c[1,2] + c[3,5] + d_0 \cdot d_2 \cdot d_5$
$1188 + 4752 + 12 \cdot 11 \cdot 13 = 7656$

$k=3$: $c[1,3] + c[4,5] + d_0 \cdot d_3 \cdot d_5$
$2898 + 2912 + 12 \cdot 14 \cdot 13 = 7994$

$k=4$: $c[1,4] + c[5,5] + d_0 \cdot d_4 \cdot d_5$
$5130 + 0 + 12 \cdot 16 \cdot 13 = 7626$

⑦ Hence, optimal parenthesization is:

$$(A(((BC)D)E))$$

Also, optimal cost is: 6678