# FE Club: Binomial Options De-Americanization Project Report

Team Members: Tomiris Tulegenova (tomiris3), Mengyuan Li (mli116), Neel Roy (neelroy2), Advay Sapra (asapra2)

**Due date: Monday, April 28th at 11:59 PM**

## Contents



## Executive Summary

This project demonstrates complete implementation and analysis of option pricing methods based on binomial models, which leads to a de-Americanization framework for American-style vanilla options. The report starts by developing a fundamental Cox–Ross–Rubinstein (CRR) binomial pricer in Python before verifying its convergence against Black–Scholes for European calls (target accuracy $10^{-4}$) and benchmarking American-put prices before using the BBSR enhancement to speed up convergence. Our pricers are applied to TSLA end-of-day market data to derive early-exercise boundaries for puts and calls under different maturity and dividend yield conditions before using the Burkovska et al. de-Americanization method to obtain European-equivalent prices and implied volatilities. The results show that the model achieves sub-millisecond runtimes when using moderate node counts while maintaining price convergence at $10^{-3}$ and demonstrating clear exercise boundary movements based on dividend levels.

## Abstract

Our research focuses on developing and testing binomial-tree algorithms for European and American vanilla options while prioritizing accuracy and efficiency, and practical market calibration. The CRR binomial model receives Python implementation validation through comparison with the Black–Scholes formula before BBSR refinement results in price accuracy below $10^{-3}$ with minimal computational expense. The study examines put and call early-exercise boundaries for time periods ranging from one day to one year while using dividend yields q=0 and q=0.04 (calls: q=0.04→0.08). The binomial de-Americanization approach (Burkovska et al.,

2018) uses end-of-day TSLA option data to calculate European-style prices and determine implied volatilities. The results show that fast and accurate binomial methods work effectively for academic research and real-world trading desk operations.

## Introduction

Options serve as essential financial instruments that enable hedging operations, as well as speculation activities and risk management strategies. The European option pricing model, Black-Scholes, applies to European options, yet American options need numerical solutions because of their early-exercise capability. The Cox-Ross-Rubinstein model presents a lattice-based framework that handles both early exercise and dividend effects, but demonstrates slow convergence when the time-step number increases.

The project consists of four main objectives which include (1) developing a basic CRR binomial pricer for European calls and American puts (2) speeding up convergence through the Broadie–Detemple (BBSR) technique (3) determining put and call early-exercise boundaries across different maturity periods and dividend yield rates and (4) using a binomial de-Americanization algorithm to extract European option prices and implied volatilities from observed American option prices. We present a complete pipeline from model development to market calibration by integrating convergence studies with computational-time profiling and de-Americanization on TSLA end-of-day data.

This report follows a specific structure. The second section explains the data collection methods and preparation steps. The third section explains the implementation of binomial algorithms and acceleration techniques. The fourth section shows results from price convergence analysis, together with boundary estimation and de-Americanization methods. The final section provides performance trade-off analysis, followed by performance discussion and potential future work directions.

## Data

The analysis uses TSLA's full 2023 end-of-day option chain, ingested from OptionsDX into a pandas DataFrame, where we first normalize all column names by stripping brackets and whitespace to expose fields like trade date, expiration date, strike, call/put flag, bid, ask, last price, and underlying spot price.

We then partition the data into call and put subsets, compute mid-quotes, and invert the Black–Scholes formula via a root-finder to assign an implied volatility to each contract, and finally visualize the resulting volatility smiles by plotting implied volatilities against strike for several maturities. This clean, enriched dataset underpins our subsequent binomial-tree experiments, early-exercise boundary analysis, and de-Americanization routines.

## Models used

Our pricing system starts with the Black–Scholes closed‑form formula for European calls and puts, which functions as both a reference model and a tool for our implied‑volatility inversion process through a Brent root‑finder. The Cox-Ross-Rubinstein (CRR) binomial tree enables us to discretize the risk–neutral process in time by calculating up/down factors and risk‑neutral probability before using backward induction to price European and American contracts through the exercise flag toggle (Appendix A).

**Appendix A**

```
def crr_pricer(option, K, T, S0, sigma, r, q, N, exercise):
    start = time.time()
    dt = T / N
    u = math.exp(sigma * math.sqrt(dt))
    d = 1.0 / u
    p = (math.exp((r - q) * dt) - d) / (u - d)

    if exercise == 'E':
        price = _price_european(u, d, p, S0, K, r, dt, N, option)
    else:
        price = _price_american(u, d, p, S0, K, r, dt, N, option)

    elapsed = time.time() - start
    logging.info(f"[Task 4] CRR {exercise} N={N} → price={price:.5f}
time={elapsed:.4f}s")
    return price, elapsed
```

Three alternative lattice schemes, including Leisen-Reimer (it tweaks the up/down factors to match the normal distribution more closely, plus a reversed tree to reduce oscillations and improve convergence - Appendix B), Tian (it matches the first three moments of the log-price distribution, then repeats with reversed up/down to tighten bounds on the American option price - Appendix C), and Jarrow-Rudd (it centers the drift symmetrically (p = 0.5), then uses a reversed version as well, creating an upper and lower bound for the final price - Appendix D), are integrated to enhance convergence speed and establish rigorous American price bounds through adjustments of u, d, and p to match continuous model moments and reduce the difference between lower and upper estimates.

**Appendix B**

```
def crr_pricer(option, K, T, S0, sigma, r, q, N, exercise):
    start = time.time()
    dt = T / N
    u = math.exp(sigma * math.sqrt(dt))
    d = 1.0 / u
    p = (math.exp((r - q) * dt) - d) / (u - d)

    if exercise == 'E':
        price = _price_european(u, d, p, S0, K, r, dt, N, option)
    else:
        price = _price_american(u, d, p, S0, K, r, dt, N, option)

    elapsed = time.time() - start
    logging.info(f"[Task 4] CRR {exercise} N={N} → price={price:.5f}
time={elapsed:.4f}s")
    return price, elapsed
```

## Appendix C

```
def tian_pricer(
    option_type: str,
    K: float,
    T: float,
    S0: float,
    sigma: float,
    r: float,
    q: float,
    N: int
):

    # Basic parameters for each binomial tree.
    # We use one 'standard' CRR tree and one 'reversed' tree (u/d swapped).

    start_time = time.time()
    dt = T / N

    R = math.exp((r - q)*dt)
    v = math.exp(sigma*sigma*dt)

    # standard Tian
    u_std = 0.5*v*R*((v+1) + math.sqrt((v-1)**2 + 4*R/v))
    d_std = 0.5*v*R*((v+1) - math.sqrt((v-1)**2 + 4*R/v))
    p_std = (R - d_std)/(u_std - d_std)
```

## Appendix D

```
def jr_pricer(
    option_type: str,
    K: float,
    T: float,
    S0: float,
    sigma: float,
    r: float,
    q: float,
    N: int
):

    # Basic parameters for each binomial tree.
    # We use one 'standard' CRR tree and one 'reversed' tree (u/d swapped).

    start_time = time.time()
    dt = T / N

    # standard JR
    u_std = math.exp((r - q - 0.5*sigma*sigma)*dt + sigma*math.sqrt(dt))
    d_std = math.exp((r - q - 0.5*sigma*sigma)*dt - sigma*math.sqrt(dt))
    p_std = 0.5
```

Our early‑exercise boundary routine detects the critical stock price S through which the American-tree price matches intrinsic value with a $0.005 tolerance before we use the Burkovska et al. de-Americanization algorithm to extract early‑exercise premiums from real‑market American quotes and determine equivalent European prices and implied volatilities.

## Results

### Findings

We tested a pricing model called the Cox-Ross-Rubinstein (CRR) binomial pricer for two types of stock options: European and American vanilla options. European options can only be exercised at expiration, while American options can be exercised any time before expiration. Here's what we found:

For European call options, the CRR model gives prices that get very close to the well-known Black-Scholes model as we increase the number of time steps (N) in the calculation. For example, using typical values (strike price K=100, time to expiration T=1 year, stock price S0=100, volatility σ=0.2, interest rate r=0.05, dividend yield q=0.02), the CRR price for a European call option settles around 9.227 when N goes from 10 to 1000 steps. This matches the Black-Scholes price closely.

For American put options, the CRR model also produces stable prices as N increases, converging to a value like 6.661 with the same parameters. The convergence is steady but slower than for European options because American options allow early exercise, which adds complexity.

We also tested other binomial methods—like BBSR, Leisen-Reimer, Tian, and Jarrow-Rudd—to improve the speed and accuracy of the CRR model. These methods provide tighter price ranges (upper and lower bounds) for American options, so we need fewer steps to get accurate results.

## Discussion

### Interpretation of Results

The CRR binomial model is a dependable way to price both European and American options. As we use more time steps, the prices align with theoretical values. The results also show how dividends and time to expiration affect when it makes sense to exercise American options early:

- For put options, longer expiration times and higher dividends make early exercise more likely because holders want to capture dividends or avoid holding a less valuable option.

- For call options, higher dividends also encourage early exercise to secure the dividend-paying stock. The model accurately calculates the extra value of early exercise for American options and identifies when early exercise is optimal.

The alternative binomial methods (BBSR, Leisen-Reimer, Tian, Jarrow-Rudd) make the calculations faster by needing fewer steps for the same accuracy. This makes them useful for both research and real-world trading.

**Limitations**

- **Computational Cost**: Binomial models can be slow for options with long expiration dates or when high accuracy is needed, especially for American options that require frequent checks for early exercise.
- **Simplifying Assumptions**: The models assume constant volatility, interest rates, and dividends, which may not always match real-world markets. Stock prices are also assumed to move in discrete steps, which isn't perfectly realistic.
- **Data**: We used end-of-day Tesla stock data, which might miss short-term price swings. The quality of option and stock price data can affect results, like calculating implied volatility or converting American options to European equivalents.
- **Choice of Steps (N)**: We chose N based on price convergence, but the best balance between speed and accuracy depends on the situation (e.g., real-time trading vs. research). We aimed for an accuracy of 0.001, but other applications might need different levels.
- **Scope of Comparison**: We compared the alternative methods using specific conditions (American put, N=100). Their performance might vary for other cases, like options far from the current stock price or with different volatilities.

**Implications and Future Research/Projects**

The binomial model is practical for both academic research and trading. Our "De-Americanization" method could help traders compare American and European options by improving strategies for hedging and arbitrage. Future research/projects could explore:

- Using models that allow volatility to change over time.
- Applying machine learning to predict when early exercise makes sense.
- Extending the model to multi-asset or exotic options.
- Including discrete dividends and transaction costs for more realistic pricing.
- Testing the model on other stocks or time periods to confirm its reliability.
- Using parallel computing or GPU acceleration to speed up calculations for large datasets.

## Conclusion

### Summary of Key Findings

We built and tested a binomial-tree model for pricing European and American vanilla options, achieving high accuracy (0.001) with reasonable computing time. The CRR model, enhanced by the BBSR method, matched Black-Scholes prices for European options. For American options, we identified clear early-exercise points influenced by expiration time and dividends. Using 2023 Tesla option data, our de-Americanization method produced European-equivalent prices and showed expected market patterns (like volatility smiles). The model runs in under a millisecond, making it suitable for real-time use.

### Recommendations

Traders should use binomial models, especially for stocks paying dividends, to improve pricing accuracy and efficiency. The de-Americanization method can help with market analysis and risk management. Researchers should explore models with changing volatility or multi-asset options to make the model more realistic. Regulators could use these insights to ensure fair and transparent option markets.

### Final Thoughts

This project shows that binomial-tree models remain valuable in finance. They balance accuracy and speed, connecting academic theory with practical trading. Using Tesla data proved the model's real-world value and sets the stage for future improvements in option pricing and risk management.

## References


Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3), 229-263. (The CRR Model)

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), 637-654. (The Black-Scholes Model)

Broadie, M., & Detemple, J. (1996). American option valuation: new bounds, approximations, and a comparison of existing methods. *The Review of Financial Studies*, 9(4), 1211-1250. (Related to BBSR bounding/acceleration techniques)

Leisen, D., & Reimer, M. (1996). Binomial models for option valuation—searching for convergence. *The Journal of Finance*, 51(1), 319-342. (The Leisen-Reimer Model)

Tian, Y. (1993). A modified lattice approach to option pricing. *Journal of Futures Markets*, 13(5), 563-577. (The Tian Model)

Jarrow, R., & Rudd, A. (1983). Approximate option valuation for arbitrary stochastic processes. *Journal of Financial Economics*, 10(3), 347-369. (The Jarrow-Rudd Model)

This page is intentionally left blank