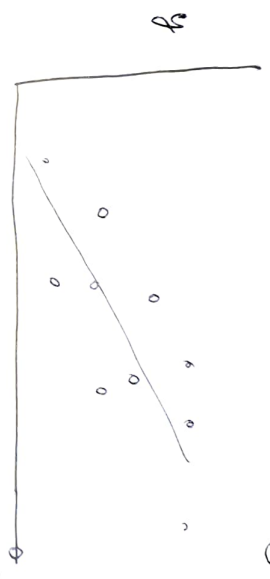1) Dot product ↓

$$\begin{bmatrix} a & b & c & \cdots \end{bmatrix} \quad \begin{bmatrix} p \\ q \\ r \\ \vdots \end{bmatrix} = \begin{matrix} (a \times p) + (b \times q) \\ + (c \times r) \\ + \cdots \end{matrix}$$

$1 \times n$

$$\begin{bmatrix} a \\ b \\ c \\ \vdots \end{bmatrix}$$

$y = ax + b$

$y = 10x + 2$

$y = 10 \times 100 + 2$

$= 1002$

2) Length

$$\sqrt{x^T x} \qquad = \sqrt{x \cdot x}$$

$$= x$$

$$\begin{bmatrix} \\ \end{bmatrix}$$

3) Distance between two vector x, y

$$|x - y| = \sqrt{(x-y)^T (x-y)}$$

$$\sqrt{x_1^2 + x_2^2 + x_3^2 + \cdots}$$
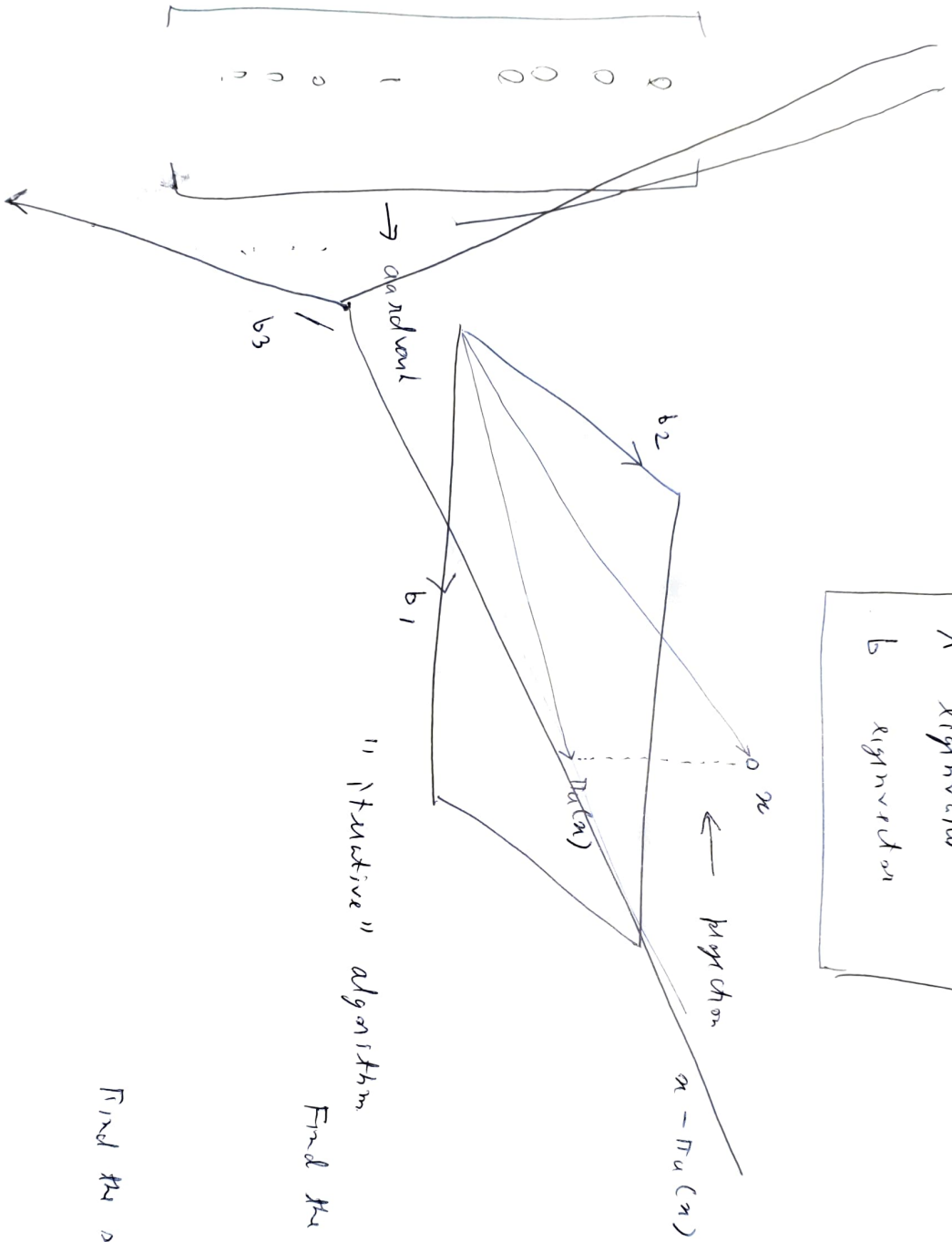
4) inner product

$$\langle x, y \rangle$$

5) Length

$$\|x\| = \sqrt{\langle x, x \rangle}$$

λ   eigenvalue
b   eigenvector

b_2

b_3

clarshant

b_1

π

projection

$\pi_u(x)$

x

$x - \pi_u(x)$

"Intuitive" algorithm

Find the first principal component (eigenvector)

$\underline{50\%}$

Find the second principal component (eigenvector)

30%

$b_1, \quad b_2$

basis

$\langle b_1, b_2 \rangle = 0$

$\|b_1\| = 1$

$\|b_2\| = 1.$

$x - \pi_u(x)$

$\pi_u(x)$

$U \longrightarrow \qquad b$
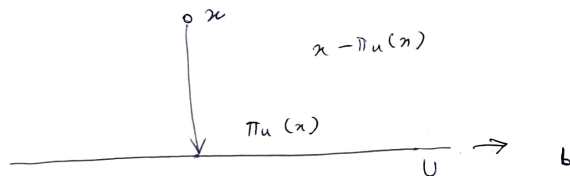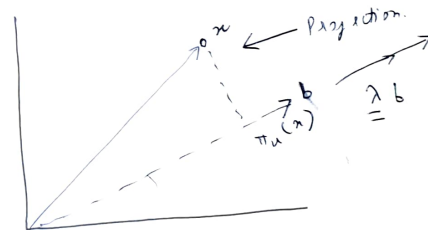
$\min \|x - \pi_u(x)\|$

$\langle x - \pi_u(x), b \rangle = 0$

$\pi_u(x) = \underline{\lambda} b$

$\langle b, x - \pi_u(x) \rangle = 0$

$\langle b, x - \lambda b \rangle = 0$

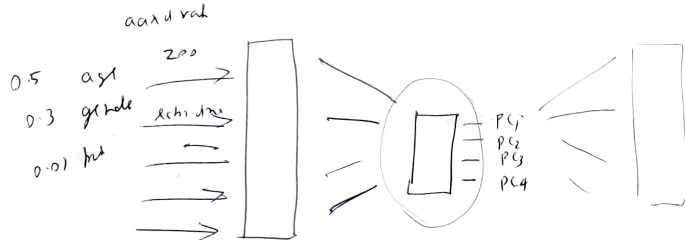$\langle b, x \rangle - \lambda \langle b, b \rangle = 0$

$\lambda = \dfrac{\langle x, b \rangle}{\langle b, b \rangle} = \boxed{\lambda = \dfrac{b^T x}{\|b\|^2}}$

λ eigenvalue

b is eigenvector

Projection

λb

CONCEPT

aardvark

0.5 ası   zoo
0.3 grade   echidna
0.01 het

PC1
PC2
PC3
PC4

linear

$$\phi_1 x_1 + \phi_2 x_2 + \ldots\ldots$$

0
0
0
1
0
0
0
0
9

→ aardvark

16,000

GPT-3

0.01
0.09
0.99

0.01
0.8
0.

sb2333@cam.ac.uk.

→

aardvah and an echidna
in a zoo and they are weird!