$\boxed{16.}$ $f = \ln(x^2+y^2)+z$

$$\nabla f = \left(\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}, 1 \right)$$

(i)

Cylinder: $\Phi = x^2+y^2 = 5^2$

$$\nabla \Phi = (2x, 2y, 0)$$

$$|\nabla \Phi| = 2\sqrt{x^2+y^2}$$

$$\underset{\sim}{\hat{n}} = \frac{\nabla \Phi}{|\nabla \Phi|}$$

$$\nabla f \cdot \underset{\sim}{\hat{n}} = \nabla f \cdot \frac{\nabla \Phi}{|\nabla \Phi|} = \frac{1}{2(x^2+y^2)^{3/2}} (4x^2+4y^2)$$

At $(3,-4,4)$,

$$\nabla f \cdot \underset{\sim}{\hat{n}} = \frac{4\times 9 + 4\times 16}{2(9+16)^{3/2}} = \frac{36+64}{250} = \frac{2}{5}$$

(ii) $\underset{\sim}{m} = (1,2,0)$

$$\underset{\sim}{\hat{m}} = \frac{1}{\sqrt{5}}(1,2,0)$$

$$\nabla f \cdot \underset{\sim}{\hat{m}} = \frac{2}{\sqrt{5}(x^2+y^2)}(x+2y) = \frac{2(x+2y)}{\sqrt{5}(x^2+y^2)}$$

At $(3,-4,4)$,

$$\nabla f \cdot \underset{\sim}{\hat{m}} = \frac{2(3-8)}{\sqrt{5}\sqrt{9+16}} = \frac{2(-5)}{\sqrt{5}(5)} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$\boxed{7.}$ $\Phi = xz + z^2 - xy^2$

$\nabla \Phi = (z - y^2, -2xy, x + 2z)$

At $(1,1,2)$,

normal $= \nabla \Phi = (1, -2, 5)$

Equation of plane:

$x - 2y + 5z = k$

Plane passes through $(1,1,2)$:

$1 - 2 + 10 = k$

$k = 9$

$\therefore x - 2y + 5z = 9$