For $\tan^{-1}\left(\frac{1}{239}\right)$, we want the smallest n such that

$$\frac{1}{2n-1}\left(\frac{1}{239}\right)^{2n-1} \geqslant \frac{1}{10^{13}-1}$$

Solving $\frac{1}{2n-1}\left(\frac{1}{239}\right)^{2n-1} = \frac{1}{10^{13}-1}$

$$(2n-1)(239)^{2n-1} = 10^{13}-1$$

Let $m = 2n-1$

$$m\,239^{m} = 10^{13}-1$$

If $10^{y} = 239^{x}$,

$y = x\log_{10}239$

$\approx 2.4x$

$\Rightarrow x \approx 0.42y$

$\Rightarrow 10^{13}-1 \approx 10^{13}$

$\approx 239^{5}$

$\rightarrow m\,239^{m} \approx 239^{5}$

$m = 3 = 2n-1 \Rightarrow n = 2$

$\therefore$ 9 terms from $4\tan^{-1}\left(\frac{1}{5}\right)$ + 2 terms from $\tan^{-1}\left(\frac{1}{239}\right)$

= <u>11 terms</u>

~~terms~~

$\tan^{-1}\left(\frac{1}{5}\right)$ gives you n_1 terms for 10^{-10}

$\tan^{-1}\left(\frac{1}{239}\right)$ n_2 for 10^{-10}

but both $\tan^{-1}\left(\frac{1}{5}\right)$, $\tan^{-1}\left(\frac{1}{239}\right)$ contribute to 10^{-10}

so it is <u>not</u> $n_1 + n_2$

<u>ex:</u> $n_1 = 1000$ $n_2 = 5$ ... $n \neq n_1 + n_2$