For $\tan^{-1}(\frac{1}{3})$, we want the smallest n such that

$$\left(\frac{1}{2n-1}\right)\left(\frac{1}{3}\right)^{2n-1} \geqslant \frac{1}{10^{13}-1}$$

Solving $\left(\frac{1}{2n-1}\right)\left(\frac{1}{3}\right)^{2n-1} = \frac{1}{10^{13}-1}$

$$(2n-1)(3^{2n-1}) = 10^{13}-1$$

Let $m = 2n-1$

$$m3^m = 10^{13}-1$$

If $3^x = 10^y$,

$x = y\log_3 10$

$\approx 2.1y$

$\Rightarrow 10^{13}-1 \approx 10^{13}$

$\approx 3^{27}$

$$m3^m \approx 3^{27}$$

If $m = 21$,

$21 \times 3^{21} = 7 \times 3^{22} < 3^{27}$

If $m = 25$,

$25 \times 3^{25} > 3^{27}$

If $m = 23$,

$23 \times 3^{23} < 3^{27}$

$\therefore m = 23 = 2n-1 \Rightarrow n = 12$

$\Rightarrow 12$ terms of $\tan^{-1}(\frac{1}{3})$

you need to calculate the 10^{-10} from the SUM then deduce n. of course 2 terms per n.

$\therefore 20$ terms of $\tan^{-1}(\frac{1}{2}) + 12$ terms of $\tan^{-1}(\frac{1}{3})$

$= 32$ terms