NEELU SARASWATIBHATLA (SRNS2)

F13. $\cos(n\theta) + i\sin(n\theta) = (\cos\theta + i\sin\theta)^n$

-1.2358

$$= \sum_{k=0}^{n} \binom{n}{k} \cos^{n-k}\theta \sin^k\theta \, i^k$$

$$\cos(n\theta) = Re\left(\sum_{k=0}^{n} \binom{n}{k} \cos^{n-k}\theta \sin^k\theta \, i^k\right)$$

$$= \sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k} \cos^{n-2k}\theta \sin^{2k}\theta (-1)^k$$

$$\sin(n\theta) = Im\left(\sum_{k=0}^{n} \binom{n}{k} \cos^{n-k}\theta \sin^k\theta \, i^k\right)$$

$$= \sum_{k=0}^{\lfloor (n-1)/2 \rfloor} \binom{n}{2k+1} \cos^{n-2k-1} \sin^{2k+1} (-1)^k$$

may not be the easiest way

what about

$\cos n\theta = Re\, e^{in\theta}$

$= Re\,(e^{i\theta})^n$

??

$$\boxed{\begin{aligned} \sum_{k=1}^{n} \cos(k\theta) &= \sum_{m=1}^{n} \sum_{k=0}^{\lfloor m/2 \rfloor} \binom{m}{2k} \cos^{m-2k}\theta \sin^{2k}\theta (-1)^k \\ \sum_{k=1}^{n} \sin(k\theta) &= \sum_{m=1}^{n} \sum_{k=0}^{\lfloor (m-1)/2 \rfloor} \binom{m}{2k+1} \cos^{m-2k-1}\theta \sin^{2k+1}\theta (-1)^k \end{aligned}}$$

(a) $$\sum_{n=1}^{5} \cos(n\theta) = \sum_{m=1}^{5} \sum_{k=0}^{\lfloor m/2 \rfloor} \binom{m}{2k} \cos^{m-2k}\theta \sin^{2k}\theta (-1)^k$$

$$= \sum_{k=0}^{0} \binom{1}{2k} \cos^{1-2k}\theta \sin^{2k}\theta (-1)^k + \sum_{k=0}^{1} \binom{2}{2k} \cos^{2-2k}\theta \sin^{2k}\theta (-1)^k$$

$$+ \sum_{k=0}^{1} \binom{3}{2k} \cos^{3-2k}\theta \sin^{2k}\theta (-1)^k + \sum_{k=0}^{2} \binom{4}{2k} \cos^{4-2k}\theta \sin^{2k}\theta (-1)^k$$

$$+ \sum_{k=0}^{2} \binom{5}{2k} \cos^{5-2k}\theta \sin^{2k}\theta (-1)^k$$