NEELU SARASWATIBHATLA (SRNS2)

1. (i) A logic variable can take one of 2 values:
   - true/1/high
   - false/0/low

   (ii) A logic gate takes one or more inputs and performs a logic function on them, returning an output.

   (iii) A logic function takes one or more boolean inputs and returns a boolean output.

2. NOT ← name

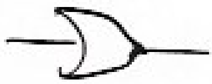
← gate

boolean algebraic expression

a	$\bar{a}$
0	1
1	0

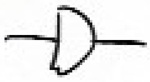
truth table

OR



a	b	$a+b$
0	0	0
0	1	1
1	0	1
1	1	1

AND



a	b	$a.b$
0	0	0
0	1	0
1	0	0
1	1	1

4 more:

NOR

a	b	$\overline{a+b}$
0	0	1
0	1	0
1	0	0
1	1	0

NAND

a	b	$\overline{a.b}$
0	0	1
0	1	1
1	0	1
1	1	0

XOR

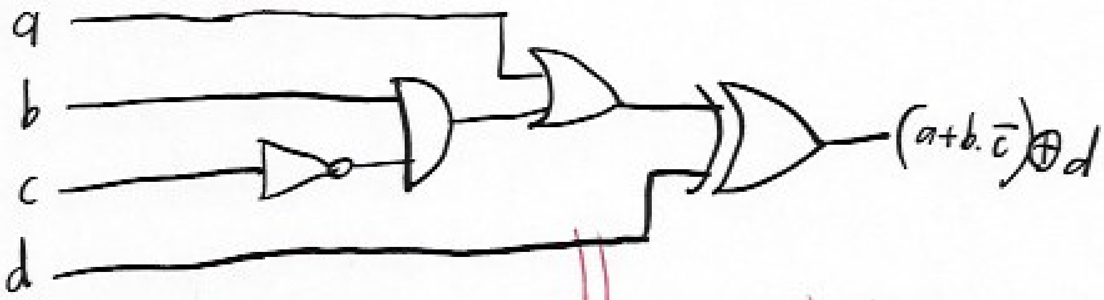
a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

XNOR

a	b	$\overline{a \oplus b}$
0	0	1
0	1	0
1	0	0
1	1	1

3. A logic function with more than 3 logic gates does more than 3 simple operations.

Example: $(a+b.\bar{c})\oplus d$



Check again

a	b	c	d	$[a+b.\bar{c}]\oplus d$	
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	0	0	0
0	0	1	1	1	1
0	1	0	0	1	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	1	0
1	1	1	1	1	1

$\cancel{(a+b.\bar{c})}\oplus d$

$\cancel{=(a\oplus d)+(b.\bar{c})\oplus d)}$

NO

$a=0, b=1, c=1$

$(a+b)\oplus c = 0$

$(a\oplus c)+(b\oplus c) = 1$

4. 1. (a)

a	b	$\bar{a}$	$\overline{\bar{a}.b}$	$\bar{b}$	$\overline{a.\bar{b}}$	$x\left[\overline{(\overline{\bar{a}.b}).(\overline{a.\bar{b}})}\right]$
0	0	1	1	1	1	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	1	0	1	0

(b)

a	b	$\bar{a}$	$\overline{\bar{a}+b}$	$\bar{b}$	$\overline{a+\bar{b}}$	$x\left[\overline{(\overline{\bar{a}+b})+(\overline{a+\bar{b}})}\right]$
0	0	1	0	1	0	1
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	1	0	0	0	0	1

2. (a) $LHS = a.b.c + a.b.\bar{c} = (a.b).(c+\bar{c}) = (a.b).1 = a.b = RHS$

(b) $LHS = a.(\bar{a}+b) = a.\bar{a} + a.b = 0 + a.b = a.b = RHS$

(c) $LHS = a.b + \bar{a}.c = (a.b.c + a.b.\bar{c} + \bar{a}.b.c + \bar{a}.\bar{b}.c) + a.b.c + \bar{a}.b.c$

$= a.b.(c+\bar{c}) + \bar{a}.c.(b+\bar{b}) + b.c(a+\bar{a})$

$= (a.b + \bar{a}.c + b.c) + a.\bar{a}$

$= a.(\bar{a}+b) + c.(\bar{a}+b)$

$= (a+c).(\bar{a}+b) = RHS$

3

ⓓ $LHS = (a+c).(a+d).(b+c).(b+d)$

$= \big((a+c).(a+d)\big).\big((b+c).(b+d)\big)$

$= \big(a.a+a.c+a.d+c.d\big).\big(b.b+b.c+b.d+c.d\big)$

$= \big(a+a.c+a.d+c.d\big).\big(b+b.c+b.d+c.d\big)$

$= (a+c.d)(b+c.d)$

$= a.b+a.c.d+b.c.d+c.d$

$= a.b+c.d = RHS$ V

3. $Z = \overline{(\bar{a}+a.b.c)} + b$ $\Rightarrow \overset{\text{(deMorgan)}}{(\bar{a}+abc)\cdot\bar{b}} = \bar{a}\bar{b} + a\overset{0}{b\bar{b}}c =$

$= \bar{a}\bar{b} = \overline{a+b}$

$= \bar{\bar{a}}.\overline{(a.b.c)}+b$

$= a.(\bar{a}+\bar{b}+\bar{c})+b$

$= a.\bar{a}+a.\bar{b}+a.\bar{c}+b$

$= a.\bar{b}+a.\bar{c}+b$

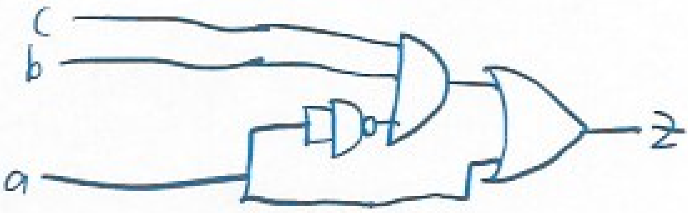
$= a.\bar{b}.c+a.\bar{b}.\bar{c}+a.b.\bar{c}+\cancel{a.\bar{b}.\bar{c}}+a.b.c+\cancel{a.b.c}+\bar{a}.b.c+\cancel{\bar{a}.b.c}$? ? ?

$= a.\bar{b}.c+a.\bar{b}.\bar{c}+a.b.\bar{c}+a.b.c+\bar{a}.b.c$

$= a.\bar{b}.(c+\bar{c})+a.b(c+\bar{c})+\bar{a}.b.c$

$= a.\bar{b}+a.b+\bar{a}.b.c$

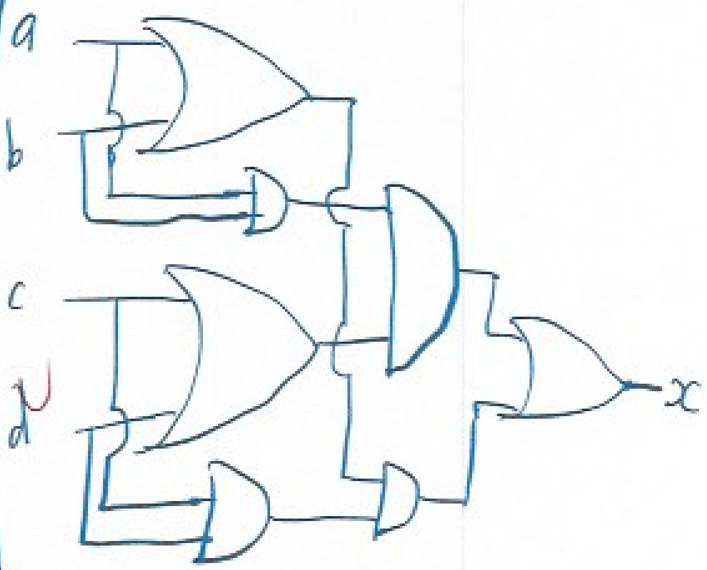
$= \underline{\underline{a+\bar{a}.b.c}}$ –



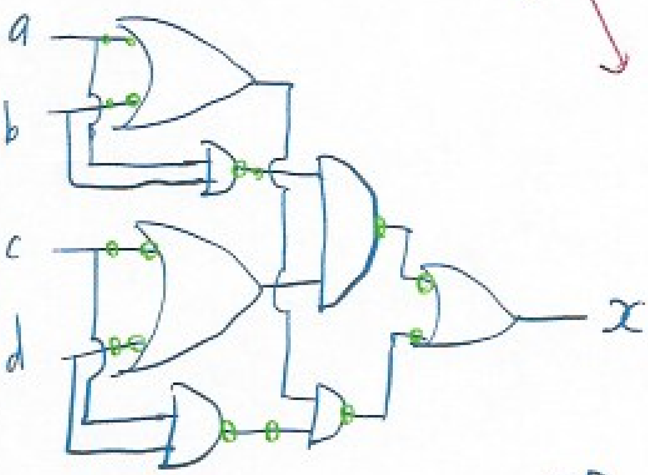
4.

ab \ cd	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	0	1	1	1
10	0	0	1	0

$x = a\cdot b\cdot d + a\cdot b\cdot c$
√ $\quad + b\cdot c\cdot d + a\cdot c\cdot d$
$= a\cdot b(c+d) + c\cdot d\,(a+b)$

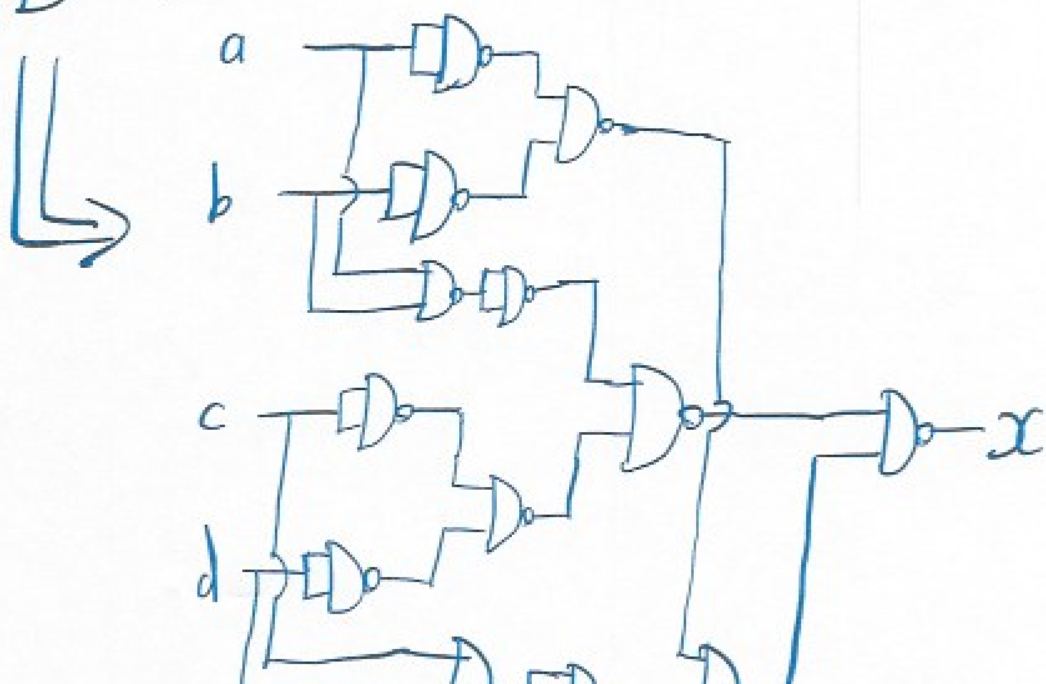


5. (a)



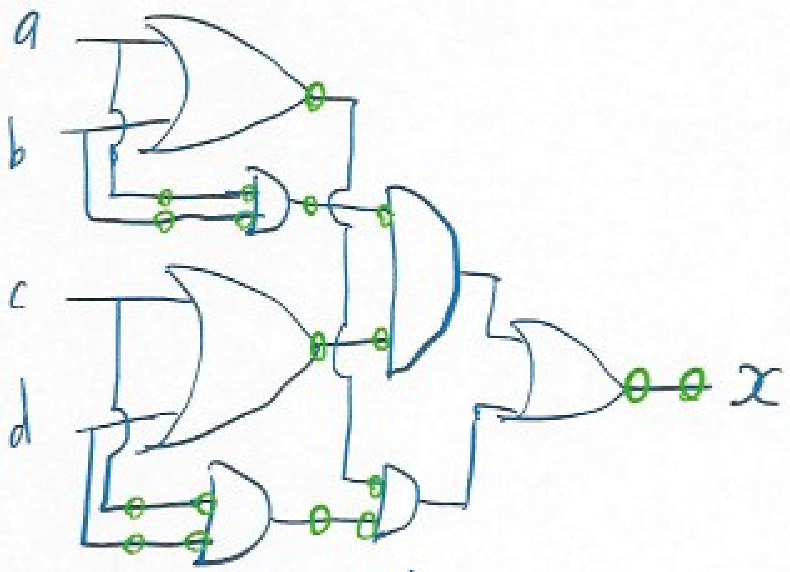
$\overline{\overline{abd}\cdot\overline{abc}\cdot\overline{bcd}\cdot\overline{acd}}$

V

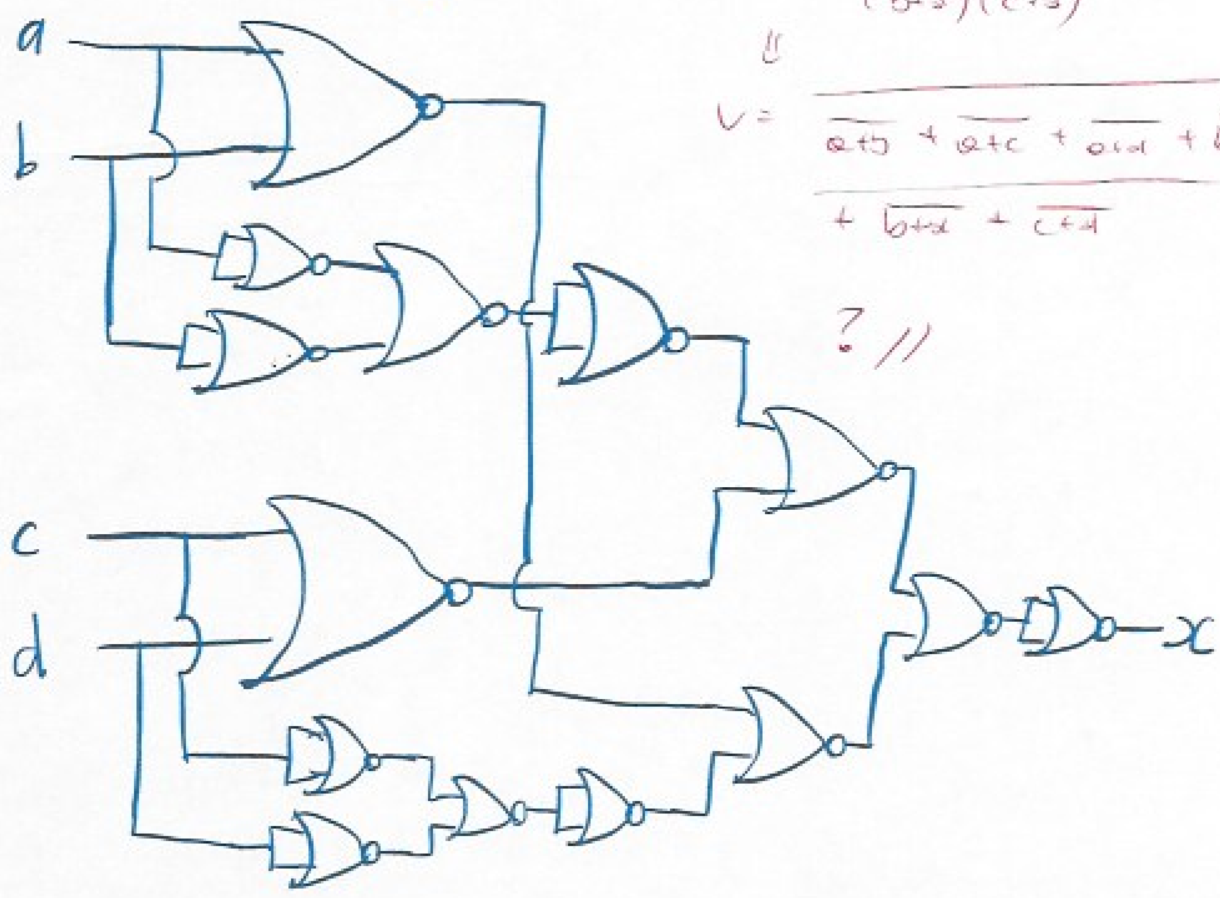


5

(b)



$\Downarrow$



$v = (a+b)(a+c)(a+d)(b+c)(b+d)(c+d)$

$\Downarrow$

$v = \overline{\overline{a+b} + \overline{a+c} + \overline{a+d} + \overline{b+c} + \overline{b+d} + \overline{c+d}}$

? //

6. $f = \bar{a}.\bar{d} + \bar{b}.\bar{c} + \bar{a}.b.\bar{c}.d$

don't care: $a.\bar{b}.c.\bar{d}$

ab \ cd	00	01	11	10
00	1	1		1
01	1	1		1
11				
10	1	1		X

$\overline{a}\overline{c} + \overline{b}\overline{c} + \overline{a}\overline{d}$

$f = \cancel{ab + abc}$

$\bar{f} = cd + ab$

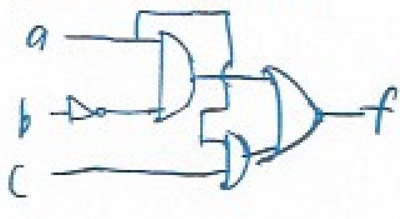
$f = \bar{b}.\bar{d} + a.\bar{b}.\bar{c} + \bar{a}.b.\bar{c}.d$

$\bar{f} = ab + b.c + c.d + b.\bar{d} + \bar{a}.\bar{b}.d$

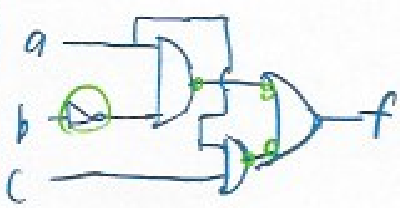
7. $f = a.\bar{b}.\bar{c} + a.\bar{b}.c + a.b.c$

a \ bc	00	01	11	10
0				
1	1	1	1	

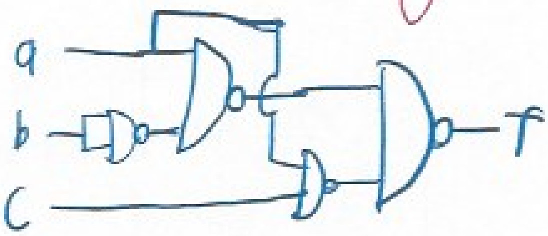
$f = a.\bar{b} + a.c$ ✓



(a)



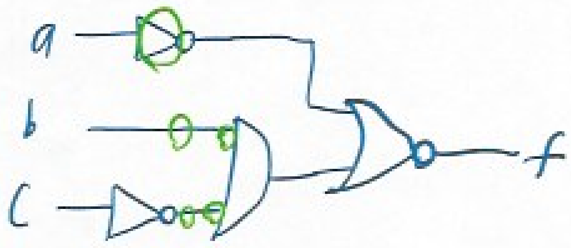
$\Rightarrow$



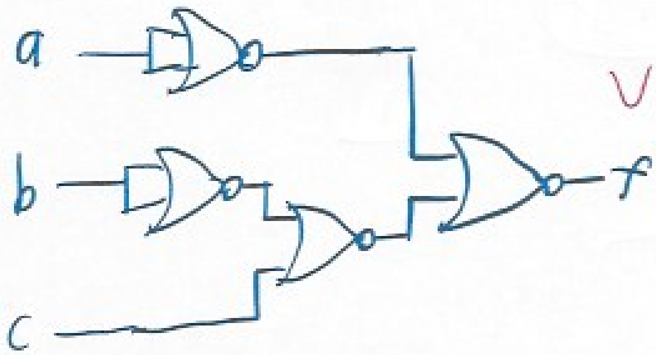
✓

(b) $\bar{f} = \bar{a} + b.\bar{c}$ $\quad f = \bar{\bar{f}}$

∴



$\Rightarrow$



5.

CD \ AB	00	01	11	10
00			1	
01		1	1	
11		1	1	1
10			1	1

(i) $f = A.B.\bar{C}.\bar{D} + \bar{A}.B.\bar{C}.D + A.B.\bar{C}.D$
$+ \bar{A}.B.C.D + A.B.C.D + A.\bar{B}.C.D$
$+ A.B.C.\bar{D} + A.\bar{B}.C.\bar{D}$

(ii) $f = A.B + B.D + A.C$

CD \ AB	00	01	11	10
00			1	1
01		1	1	
11		1	1	
10			1	

(i) $f = A.B.\bar{C}.\bar{D} + A.\bar{B}.\bar{C}.\bar{D} + \bar{A}.B.\bar{C}.D$
$+ A.B.\bar{C}.D + \bar{A}.B.C.D + A.B.C.D$
$+ A.B.C.\bar{D}$

(ii) $f = A.B + B.D + A.\bar{C}.\bar{D}$

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CD \ AB	00	01	11	10
00			1	1
01	1	1		1
11	1			1
10	1			

(i) $f = A\cdot B\cdot\bar{C}\cdot\bar{D} + A\cdot\bar{B}\cdot\bar{C}\cdot\bar{D} + \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot D + \bar{A}\cdot B\cdot\bar{C}\cdot D + A\cdot\bar{B}\cdot\bar{C}\cdot D + \bar{A}\cdot\bar{B}\cdot C\cdot D + A\cdot\bar{B}\cdot C\cdot D + \bar{A}\cdot\bar{B}\cdot C\cdot\bar{D}$

(ii) $f = A\cdot\bar{C}\cdot\bar{D} + \bar{A}\cdot\bar{C}\cdot D + \bar{B}\cdot D + \bar{A}\cdot\bar{B}\cdot C$

$= \bar{C}(A\cdot\bar{D} + \bar{A}\cdot D) + \bar{B}(D + \bar{A}\cdot C)$

$= (A \oplus D)\cdot\bar{C} + \bar{B}(D + \bar{A}\cdot C)$

CD \ AB	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

(i) $f = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D} + A\cdot\bar{B}\cdot\bar{C}\cdot\bar{D} + \bar{A}\cdot B\cdot\bar{C}\cdot D + A\cdot B\cdot\bar{C}\cdot D + \bar{A}\cdot B\cdot C\cdot D + A\cdot B\cdot C\cdot D + \bar{A}\cdot\bar{B}\cdot C\cdot\bar{D} + A\cdot\bar{B}\cdot C\cdot\bar{D}$

(ii) $f = B\cdot D + \bar{B}\cdot\bar{D}$

(iii)

CD \ AB	00	01	11	10
00			0	
01	0		0	0
11	0	0	0	0
10			0	

already covered

$\bar{f} = A\cdot B + C\cdot D + \bar{B}\cdot D + \cancel{A\cdot D}$

$\bar{f} = \overline{(\bar{A}+\bar{B})\cdot(\bar{C}+\bar{D})\cdot(B+\bar{D})\cdot\cancel{(\bar{A}+\bar{D})}}$

$f = (\bar{A}+\bar{B})\cdot(\bar{C}+\bar{D})\cdot(B+\bar{D})\cdot\cancel{(\bar{A}+\bar{D})}$

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6. $F(A,B,C,D) = \Sigma m(0,2,5,6,7,8,10,12,13,14,15)$

Implication Table

~~0000~~ ~~0010~~ ~~0101~~ ~~0110~~

~~0111~~ ~~1000~~ ~~1010~~ ~~1100~~

~~1101~~ ~~1110~~ ~~1111~~

	Column 1	Column 2	Column 3	
0	0000 ✓	00–0 *	–11– *	ok
		–000 *	11–– *	ok
2	0010 ✓	011– ✓	–1–1	
5	0101 *	1–10 *	1––0	
8	1000 ✓	110– ✓	––10	
6	0110 ✓	111– ✓	–0–0	
10	1010 ✓			
12	1100 ✓			
7	0111 ✓			
13	1101 ✓			
14	1110 ✓			
15	1111 ✓			

ANS: $\bar{B}\bar{D} + BD + C\bar{D} + A\bar{D}$

Prime Implicant Chart

		0	2	5	6	7	8	10	12	13	14	15
8,10	0,2 (00–0)	X	⊗									
2,6,10,14	0,8 (–000)	X					⊗					
7,13,15	5 (0101)			⊗								
8,12	10,14 (1–10)							⊗			X	
v	6,7,14,15 (–11–)				⊗	X					X	X
v	12,13,14,15 (11––)								⊗	X	X	X

$\therefore F(A,B,C,D) = \bar{A}\cdot\bar{B}\cdot\bar{D} + \bar{B}\cdot\bar{C}\cdot\bar{D} + \bar{A}\cdot B\cdot\bar{C}\cdot D + A\cdot C\cdot\bar{D} + B\cdot C + A\cdot B$

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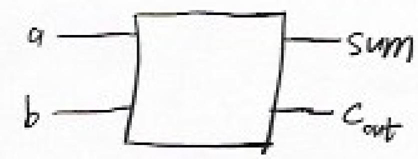
7. A half adder takes 2 input bits (a and b), and outputs a sum bit and a carry out bit. A full adder takes a carry in bit in addition to a and b, and outputs a sum bit and a carry out bit.

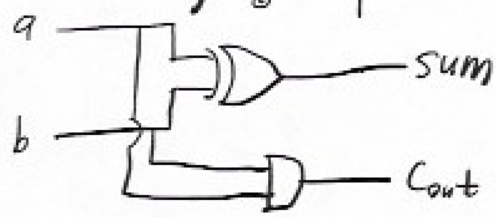
Half adder



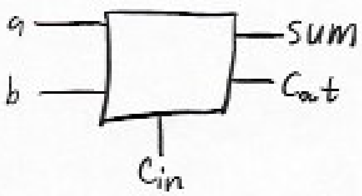
a	b	sum	C_{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$C_{out} = a.b$

$Sum = a \oplus b$

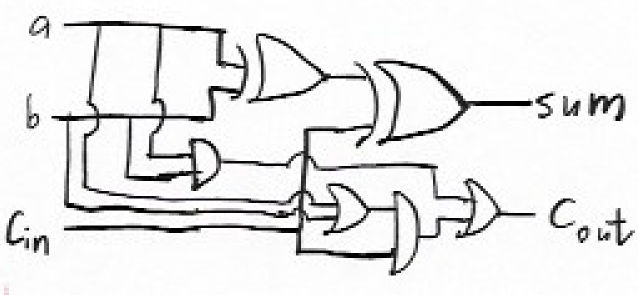


Full adder



a	b	C_{in}	sum	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$sum = \bar{C}_{in}(a \oplus b) + C_{in}\left(\overline{\bar{a}.\bar{b} + a.b}\right)$$
$$= \bar{C}_{in}(a \oplus b) + C_{in}\left(\overline{(a+b)\cdot(\bar{a}+\bar{b})}\right)$$
$$= \bar{C}_{in}(a \oplus b) + C_{in}\left(\overline{a.\bar{a} + a.\bar{b} + \bar{a}.b + b.\bar{b}}\right)$$
$$= \bar{C}_{in}(a \oplus b) + C_{in}\left(\overline{\bar{a}.b + a.\bar{b}}\right)$$
$$= C_{in}(\overline{a \oplus b}) + \bar{C}_{in}(a \oplus b)$$
$$= a \oplus b \oplus C_{in}$$



C_{out}

C_{in} \ ab	00	01	11	10
0			1	
1		1	1	1

$C_{out} = a.b + b.C_{in} + a.C_{in}$

$= a.b + C_{in}(a+b)$