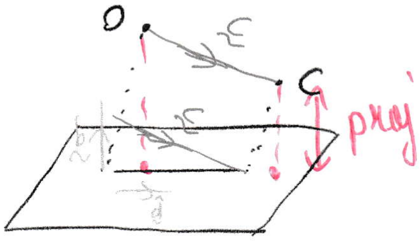
5. The plane $b\cdot(r-a)=0$ has the following properties:

- b is a normal vector to the plane.
- There is a point on the plane whose position vector is a.

(i)



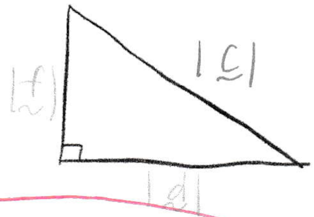
We need to find a unit vector $\hat{d}$ that is the direction vector of the projection of $\overrightarrow{OC}$ on the plane. We can find the length of this projection using $\overrightarrow{OC}\cdot\hat{d}$, i.e. $c\cdot\hat{d}$.

We can do this by finding a vector f such that $c+f$ is in the same direction as d, as this would mean $d=c+f$, so $\hat{d}=\frac{d}{|d|}$. Moving c to the plane as shown in the diagram, f is in the same direction as the normal b, as $\hat{d}$ is on the plane so f must be perpendicular to the plane.

definition of projection

$\therefore f=-(c\cdot\hat{b})\hat{b}$

$\vec{b}\times\vec{c}$



$$\therefore d=c+f$$
$$=c-(c\cdot\hat{b})\hat{b}$$

use how $|d|$ give $-|c|^2$

$$|f|=|(c\cdot\hat{b})\hat{b}|$$
$$=c\cdot\hat{b}$$

OH!!!

$$\therefore |d|=|f|^2-|c|^2$$
$$=c\cdot\hat{b}-|c|^2$$

use $\vec{c}\times\vec{b}$

$$\therefore \hat{d}=\frac{d}{|d|}=\cancel{\frac{c-(c\cdot\hat{b})\hat{b}}{c\cdot\hat{b}-|c|^2}}$$

$$\therefore \text{Length of projection}=c\cdot\hat{d}$$
$$=c\cdot\left(\frac{c-(c\cdot\hat{b})\hat{b}}{c\cdot\hat{b}-|c|^2}\right)$$

$\Rightarrow \frac{(\vec{c}\times\vec{b})}{|b|}\times\frac{\vec{b}}{|b|}$

$$=\frac{1}{c\cdot\hat{b}-|c|^2}\left(c\cdot c-(c\cdot\hat{b})(c\cdot\hat{b})\right)$$
$$=\underline{\underline{\frac{|c|^2-(c\cdot\hat{b})^2}{c\cdot\hat{b}-|c|^2}}}\quad\left(\text{where } \hat{b}=\frac{b}{|b|}\right)$$