Question 11.

We want to find the upper bound of the degree, i.e. under the circumstances when the maximum degree will occur.

The maximum degree of a node occurs when there is only one root node. A root node has to be the one with maximum degree as if one of its children has degree k, it has degree $k+1$, which is higher than k.

The greatest degree occurs when all nodes which aren't this single root (i.e. $n-1$ nodes) are all direct children of this root.

Let d be the degree of the tree.

From the Fibonacci shape theorem, the subtree rooted at this node has $\geq F_{d+2}$ nodes (as it has $n-1$ children).

$$F_{d+2} \geq \phi^d$$
$$F_{d+2} = n-1$$
$$\therefore n-1 \geq \phi^d$$
$$d \leq \log_\phi (n-1)$$
$$\therefore d = O(\log n) \text{ as required.}$$

□