$\boxed{8.}$ $\left(p+\frac{a}{V^2}\right)(V-b)=RT$

$$pV-pb+aV^{-1}-bV^{-2}=RT$$

$$\left(\frac{\partial p}{\partial V}\right)_T V+p-b\left(\frac{\partial p}{\partial V}\right)_T-aV^{-2}+2bV^{-3}=0$$

$$\left(\frac{\partial p}{\partial V}\right)_T=\frac{aV^{-2}-2bV^{-3}-p}{V-b}$$

$$p\left(\frac{\partial V}{\partial T}\right)_p-a\left(\frac{\partial V}{\partial T}\right)_p V^{-2}+2b\left(\frac{\partial V}{\partial T}\right)_p V^{-3}=R$$

$$\left(\frac{\partial v}{\partial T}\right)_p=\frac{R}{p-aV^{-2}+2bV^{-3}}=-\frac{R}{aV^{-2}-2bV^{-3}-p}$$

$$V-b=R\left(\frac{\partial T}{\partial p}\right)_V \Rightarrow \left(\frac{\partial T}{\partial p}\right)_V=\frac{V-b}{R}$$

$$\left(\frac{\partial p}{\partial V}\right)_T\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_V=\left(\frac{\cancel{aV^{-2}-2bV^{-3}-p}}{\cancel{V-b}}\right)\left(-\frac{\cancel{R}}{\cancel{aV^{-2}-2bV^{-3}-p}}\right)\left(\frac{\cancel{V-b}}{\cancel{R}}\right)$$

$$=-1$$

$\boxed{9.}$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1} \begin{pmatrix} u \\ v \end{pmatrix}$$

Let $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$\det(A) = \cos^2\theta + \sin^2\theta = 1$$

$$A^{-1} = \frac{\mathrm{adj}(A)}{\det(A)} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$x(u,v) = u\cos\theta + v\sin\theta$$
$$y(u,v) = -u\sin\theta + v\cos\theta$$

$$\frac{\partial x}{\partial u} = \cos\theta \qquad \frac{\partial x}{\partial v} = \sin\theta$$

$$\frac{\partial y}{\partial u} = -\sin\theta \qquad \frac{\partial y}{\partial v} = \cos\theta$$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$$

$$= \frac{\partial f}{\partial x}\cos\theta - \frac{\partial f}{\partial y}\sin\theta$$

$$\frac{\partial^2 f}{\partial u^2} = \cos\theta \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial x}\right) - \sin\theta \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial y}\right)$$

$$= \cos\theta \frac{\partial^2 f}{\partial x^2}\frac{\partial x}{\partial u} - \sin\theta \frac{\partial^2 f}{\partial y^2}\frac{\partial y}{\partial u}$$

$$= \cos^2\theta \frac{\partial^2 f}{\partial x^2} + \sin^2\theta \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$$

$$= \frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta$$

$$\frac{\partial^2 f}{\partial v^2} = \sin\theta \frac{\partial^2 f}{\partial x^2}\frac{\partial x}{\partial v} + \cos\theta \frac{\partial^2 f}{\partial y^2}\frac{\partial y}{\partial v}$$

$$= \sin^2\theta \frac{\partial^2 f}{\partial x^2} + \cos^2\theta \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} = \cos^2\theta \frac{\partial^2 f}{\partial x^2} + \sin^2\theta \frac{\partial^2 f}{\partial y^2} + \sin^2\theta \frac{\partial^2 f}{\partial x^2} + \cos^2\theta \frac{\partial^2 f}{\partial y^2}$$

$$= (\cos^2\theta + \sin^2\theta)\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\therefore \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} \qquad \blacksquare$$