

Natural Sciences Tripos, Part IA
Mathematical Methods II, Course B
Answers to Example Sheet 1
Ordinary Differential Equations

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Lent Term 2021

Please communicate any errors to `smc1@cam.ac.uk`.

Skills section

- S1. (a) $\frac{4}{5}(1+x)^{5/4} + c$
(b) $\ln(1+x) - (1+x)^{-1} + c$
(c) $\frac{1}{2}(3+x^2)^2 + c$
(d) $-\cos(x^2) + c$
(e) $\ln(\sin x) + c$
(f) $2x \sin x + (2-x^2) \cos x + c$
(g) $\frac{1}{3} \cos^3 x - \cos x + c$
(h) $2 \ln(2-x) - \ln(1-x) + c$
(i) $\arctan x + c$
(j) $2(1-x)^{1/2} \cos(1-x)^{1/2} - 2 \sin(1-x)^{1/2} + c$
- S3. (a) $y = (c - \frac{3}{4}x^4)^{1/3} - 1$
(b) $y^{-3} e^y = cx^4$
- S4. (a) $y = 2 + c e^{-x^2}$
(b) $y = \frac{1}{2}x^3 + cx^3 e^{1/x^2}$

Standard questions

5. $y = (c - \frac{3}{4}x^4)^{1/3} - x - 1$
6. (a) $y = (c e^{-4x} - x + \frac{1}{4})^{-1/4}$ [substitute $z = y^{-4}$]
(b) $y = (c e^x - \sin x)^{-1}$ [substitute $z = y^{-1}$]
7. $y^2 + 2xy + 2x^2 = c \exp\{4 \arctan[(y/x) + 1]\}$
8. (a) $y = x \arcsin(cx)$
(b) $y = \exp[x \pm (x^2 + c)^{1/2}]$ [substitute $z = \ln y - x$]
(c) $y = (cx^{-2} - \frac{1}{2}x^2 - \frac{2}{3}x)^{1/2}$
9. (a) $y = e^{3x} - e^{2x}$
(b) $y = n^{-1} \sin nx$ [or $y = x$ if $n = 0$]
(c) $y = \frac{1}{\sqrt{3}} e^{-x} \sin(\sqrt{3}x)$
(d) $y = 2 - 2 \cos 3x + \frac{1}{3} \sin 3x$
(e) $y = \frac{1}{12} e^{5x} + \frac{2}{3} e^{2x} - \frac{3}{4} e^x$
(f) $y = (A + Bx) e^x$
(g) $y = e^{2x} + (A + Bx + \frac{1}{2}x^2) e^x$
10. (a) $q = (\sqrt{2} + 1)(Q/2) \exp[-(2 + \sqrt{2})t/4RC] - (\sqrt{2} - 1)(Q/2) \exp[-(2 - \sqrt{2})t/4RC]$
(b) $q = Q(1 - t/2RC) \exp(-t/2RC)$
(c) $q = Q \exp(-t/2RC)[\cos(t/2RC) - \sin(t/2RC)]$
13. (a) $x = 2 e^{at}$, $y = (1 + 2bt) e^{at}$