$\boxed{\text{Q8.}}$ Proof by induction

Base case: $u = w = v$

Trivially, $\underset{0}{\underset{\parallel}{d(w,v)}} = \underset{0}{\underset{\parallel}{d(u,w)}} + \underset{0}{\underset{\parallel}{c(w \to v)}} = 0$

Inductive Step

CASE 1: $\nexists$ path $u \to \dots \to v$, so

$\exists$ path $u \to \dots \to x \to w$ [$u = x = w$ is a possibility]

with $d(u,w) \leq d(u,x) + c(x \to w)$

$\wedge \nexists$ edge $w \to v$

$c(w \to v) = \infty$

$d(u,v) = \infty$

$\infty \leq \infty$

$\therefore d(u,v) \leq d(u,w) + c(w \to v)$

CASE 2: $\exists$ path $u \to \dots \to v$, so

$\exists$ path $u \to \dots \to x \to w$

with $d(u,w) \leq d(u,x) + c(x \to w)$

$\wedge \exists$ edge $w \to v$

There is therefore definitely a path from u to v through w.

$\therefore$ If the previously known value of $d(u,v) \geq d(u,w) + c(w \to v)$ then we now have a $d(u,v) = d(u,w) + c(w \to v)$. If it was less then obviously $d(u,v) \leq d(u,w) + c(w \to v)$ and nothing changes. In the future if a better path is found then $d(u,v) < d(u,w) + c(w \to v)$. If not, then nothing changes. ∎