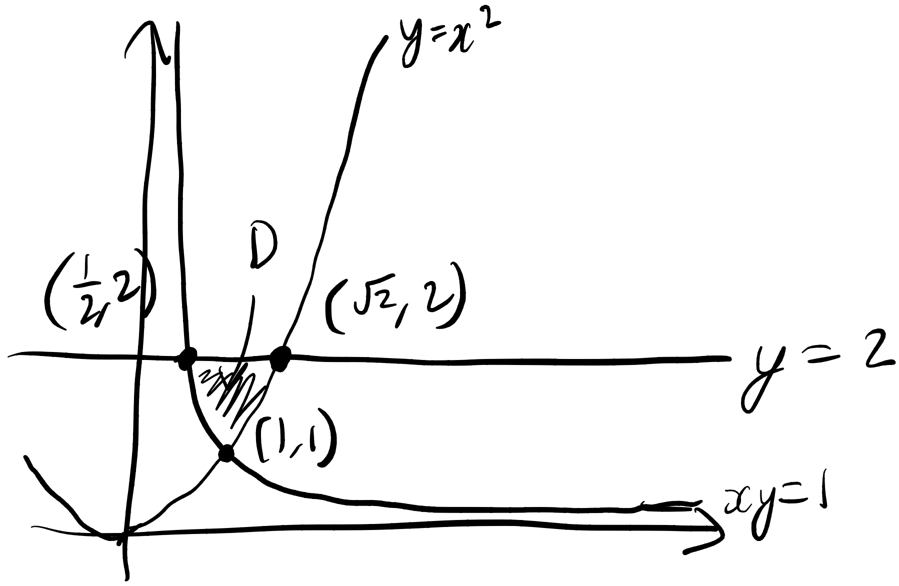
NEELU SARASWATIBHATLA (SRNS2)

Q6.



$$\text{LHS} = \iint_D x^2y\,dx\,dy = \int_{y=1}^{2}\int_{x=\frac{1}{y}}^{\sqrt{y}} x^2y\,dx\,dy$$

$$= \int_1^2 \frac{1}{3}y\left[x^3\right]_{\frac{1}{y}}^{\sqrt{y}}dy$$

$$= \frac{1}{3}\int_1^2 y\left(\sqrt{y}^3 - \frac{1}{y^3}\right)dy = \frac{1}{3}\int_1^2\left(y^{\frac{5}{2}} - y^{-2}\right)dy$$

$$= \frac{1}{3}\left[\frac{2}{7}y^{\frac{7}{2}} + \frac{1}{y}\right]_1^2 = \frac{1}{3}\left(\left(\frac{16\sqrt{2}}{7} + \frac{1}{2}\right) - \left(\frac{2}{7} + 1\right)\right)$$

$$= \frac{1}{3}\left(\frac{32\sqrt{2} + 7 - 4 - 14}{14}\right)$$

$$= \frac{32\sqrt{2} - 11}{14} = \text{RHS}$$

Q.E.D.

R6. (i) Let X be the number rolled on a single throw.

x	1	2	3	4	5	6
$P(X=x)$	$p/2$	p	p	p	p	$2p$

$$\frac{p}{2} + 4 \times p + 2p = 1$$

$$\frac{13}{2} p = 1$$

$$p = \frac{2}{13}$$

(ii) $$\langle x \rangle = 1\left(\frac{1}{13}\right) + (2+3+4+5)\left(\frac{2}{13}\right) + 6\left(\frac{4}{13}\right)$$

$$= \frac{1 + 28 + 24}{13}$$

$$= \frac{53}{13}$$

(iii) $$P(X > \langle x \rangle) = P\left(X > \frac{53}{13}\right) = P(X=5) + P(X=6)$$

$$= 3p = \frac{6}{13}$$

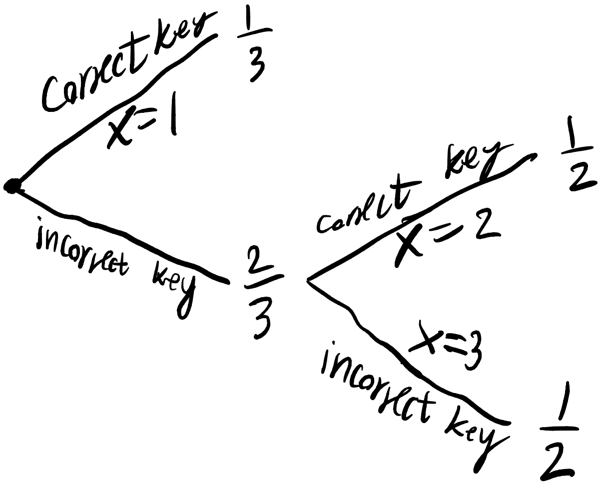
(iv) $$V(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = 1^2\left(\frac{1}{13}\right) + (2^2+3^2+4^2+5^2)\left(\frac{2}{13}\right) + 6^2\left(\frac{4}{13}\right) = \frac{253}{13}$$

$$\therefore \sigma^2 = V(X) = E(X^2) - (E(X))^2 = \frac{253}{13} - \left(\frac{53}{13}\right)^2$$

$$= \frac{480}{169}$$

R8. $\underline{X}$

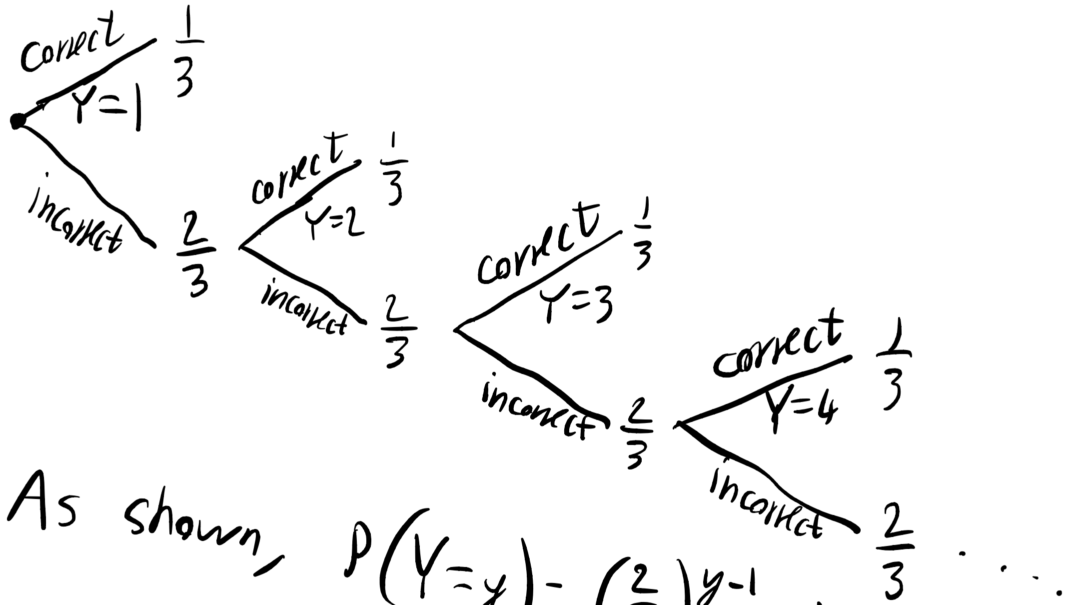


$\therefore$

x	1	2	3
$P(X=x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$$\langle X\rangle = (1+2+3)\left(\frac{1}{3}\right) = \underline{\underline{2}}$$

$\underline{Y}$



As shown, $P(Y=y) = \left(\frac{2}{3}\right)^{y-1}\left(\frac{1}{3}\right) = \frac{2^{y-1}}{3^y}$

$$\langle Y\rangle = \sum_{i=1}^{\infty} y\left(\frac{2^{y-1}}{3^y}\right) = \frac{1}{3}\left(1+2\left(\frac{2}{3}\right)+3\left(\frac{2^2}{3^2}\right)+4\left(\frac{2^3}{3^3}\right)+\dots\right)$$

$$= \frac{1}{3}\left(1-\frac{2}{3}\right)^{-2} = \frac{1}{3}\times\left(\frac{1}{3}\right)^{-2}$$

$$= \underline{\underline{3}}$$