(i) $\tan^{-1}(1) = \frac{\pi}{4}$

$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ WHY WE WANT π

To calculate π to 10 d.p., $\frac{\pi}{4}$ needs to be calculated to 12 d.p. (as $\frac{\pi}{4} = 0.25\pi$)

WHY

$\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots\right]$

The first $\frac{1}{n}$ to more than 12 d.p. is $\frac{1}{10^{13}}$

$\therefore$ We need terms up to $\frac{1}{10^{13}-1}$

$\therefore$ We want the smallest n such that $\frac{1}{2n-1} \geq \frac{1}{10^{13}-1}$

$2n-1 = 10^{13}-1$

$2n = 10^{13}$

$n = 5\times10^{12}$ terms

(ii) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$

$= \frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B}$

$= \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\sin B\cos A}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}$

$= \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan\left(\tan^{-1}(A)+\tan^{-1}(B)\right) = \frac{\tan\left(\tan^{-1}(A)\right) + \tan\left(\tan^{-1}(B)\right)}{1 - \tan\left(\tan^{-1}(A)\right)\tan\left(\tan^{-1}(B)\right)} = \frac{A+B}{1-AB}$

$\tan^{-1}(A)+\tan^{-1}(B) = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$