6. $f = \ln(x^2+y^2) + z$

$\nabla f = \left(\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}, 1 \right)$

(i)

Cylinder: $\Phi = x^2+y^2 = 5^2$

$\nabla \Phi = (2x, 2y, 0)$

$|\nabla \Phi| = 2\sqrt{x^2+y^2}$

$\hat{n} = \frac{\nabla \Phi}{|\nabla \Phi|}$

little drawing please...just to visualise the problem

$\nabla f \cdot \hat{n} = \nabla f \cdot \frac{\nabla \Phi}{|\nabla \Phi|} = \frac{1}{2(x^2+y^2)^{3/2}} (4x^2+4y^2)$

At $(3, -4, 4)$,

$\nabla f \cdot \hat{n} = \frac{4 \times 9 + 4 \times 16}{2(9+16)^{3/2}} = \frac{36+64}{250} = \frac{2}{5}$ ok

(ii) $m = (1, 2, 0)$

$\hat{m} = \frac{1}{\sqrt{5}}(1, 2, 0)$

$\nabla f \cdot \hat{m} = \frac{2}{\sqrt{5}(x^2+y^2)} (x+2y) = \frac{2(x+2y)}{\sqrt{5}(x^2+y^2)}$ ok

At $(3, -4, 4)$,

$\nabla f \cdot \hat{m} = \frac{2(3-8)}{\sqrt{5}\sqrt{9+16}} = \frac{2(-5)}{\sqrt{5}(5)} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

why do you have the sqrt here

7. $\Phi = xz + z^2 - xy^2$

$\nabla\Phi = (z - y^2, -2xy, x + 2z)$

At $(1, 1, 2)$,

normal $= \nabla\Phi = (1, -2, 5)$

Equation of plane:

$x - 2y + 5z = k$

Plane passes through $(1, 1, 2)$:

$1 - 2 + 10 = k$

$k = 9$

$\therefore x - 2y + 5z = 9$

ok