$\boxed{11.}$

$$dU = TdS - pdV$$

$$H = U + pV$$

$$dH = dU + pdV + Vdp$$

$$= (TdS - pdV) + pdV + Vdp$$

$$= TdS + Vdp$$

Since dH is an exact differential,

$$\left(\frac{\partial V}{\partial S}\right)_p = \left(\frac{\partial T}{\partial p}\right)_S \quad \text{as required}$$

$\blacksquare$

$$dU = TdS - pdV$$

$$= T\left(\left(\frac{\partial S}{\partial p}\right)_V dp + \left(\frac{\partial S}{\partial V}\right)_p dV\right) - pdV$$

$$= T\left(\frac{\partial S}{\partial p}\right)_V dp + \left(T\left(\frac{\partial S}{\partial V}\right)_p - p\right) dV$$

$$\therefore \left(\frac{\partial U}{\partial p}\right)_V = T\left(\frac{\partial S}{\partial p}\right)_V \quad \text{and} \quad \left(\frac{\partial U}{\partial V}\right)_p = T\left(\frac{\partial S}{\partial V}\right)_p - p$$

$$\frac{\partial^2 U}{\partial V \partial p} = \frac{\partial^2 U}{\partial p \partial V}$$

$$\therefore \left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial S}{\partial p}\right)_V + \cancel{T\left(\frac{\partial^2 S}{\partial V \partial p}\right)} = \left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial S}{\partial V}\right)_p + \cancel{T\left(\frac{\partial^2 S}{\partial p \partial V}\right)} - 1$$

$$\therefore \left(\frac{\partial S}{\partial V}\right)_p \left(\frac{\partial T}{\partial p}\right)_V - \left(\frac{\partial S}{\partial p}\right)_V \left(\frac{\partial T}{\partial V}\right)_p = 1$$

as required

$\blacksquare$

12.

$$dU = TdS - pdV$$

$$dG = Vdp - SdT$$

$$dG = \left(\frac{\partial G}{\partial p}\right)_T dp + \left(\frac{\partial G}{\partial T}\right)_p dT$$

$$\therefore \left(\frac{\partial G}{\partial p}\right)_T = V \quad \text{and} \quad \left(\frac{\partial G}{\partial T}\right)_p = -S$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$G = pV + g(T) \qquad\qquad G = -TS + h(p)$$

$\therefore G = pV - TS + k$, where k is a constant

$$dG = \left(\frac{\partial G}{\partial p}\right)_T dp + \left(\frac{\partial G}{\partial T}\right)_p dT$$

$$\left(\frac{\partial G}{\partial p}\right)_T = V \quad \text{and} \quad \left(\frac{\partial G}{\partial T}\right)_p = -S$$

$$\frac{\partial^2 G}{\partial T \partial p} = \left(\frac{\partial V}{\partial T}\right)_p \quad \text{and} \quad \frac{\partial^2 G}{\partial p \partial T} = -\left(\frac{\partial S}{\partial p}\right)_T$$

$$\frac{\partial^2 G}{\partial T \partial p} = \frac{\partial^2 G}{\partial p \partial T}$$

$$\therefore \left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$$

$$\therefore \left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p \quad \text{as required}$$

∎