

Ex. 35

$$f(n) = f\left(\frac{n}{2}\right) + k$$

$$\text{Let } n = 2^m$$

$$f(2^m) = f\left(\frac{2^m}{2}\right) + k$$

$$f(2^m) = f(2^{m-1}) + k$$

$$= f(2^{m-2}) + k + k$$

$$= f(2^{m-3}) + k + k + k$$

$$= \dots$$

$$= f(2^{m-p}) + pk$$

$$\text{set } p = m$$

$$f(2^m) = f(2^{m-m}) + mk$$

$$= f(1) + mk$$

$$= mk + q, \quad q \in \Theta(n)$$

$$f(n) = mk + q$$

$$f(n) = k \lg n + q$$

$$\therefore f(n) \in \Theta(\lg n)$$