Question 4.

(i) As determined in Q2, the cost of inc() is $2m$ − # of 1s in A.

$c = 2m - k$ where k = # of 1s in A after increment.

$\Delta\Phi = k - O(\log_2 m)$

$c + \Delta\Phi = 2m - k + k - O(\log_2 m) = O(m)$

$\therefore$ amortised cost $= O(m)$.

(ii) Let Φ = #roots + 2 × #losers

In a Fibonacci heap, push() just adds a new root to the root list.

$\therefore c = O(1)$

$\Delta\Phi = 1$ as # roots increases by 1 but #losers doesn't change.

$\therefore$ amortised cost $= c + \Delta\Phi = O(1) + 1 = O(1)$

□