LECTURE 8, 9

1. Let $Y = X_1 + X_2 + X_3$

Let $W = X_1 + X_2$, i.e. $Y = W + X_3$

$$f_Y(a) = f_{X_1+X_2+X_3}(a)$$
$$= f_{W+X_3}(a)$$
$$= \int_{-\infty}^{\infty} f_{W,X_3}(a-z, z)\,dz$$
$$= \int_{-\infty}^{\infty} f_W(a-z) f_{X_3}(z)\,dz$$
$$= \int_0^1 f_W(a-z)\,dz \quad \text{as} \quad f_{X_3}(z) = \begin{cases} 1 & \text{for } 0 \le a \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_W(b) = f_{X_1+X_2}(b) = \begin{cases} b & \text{if } 0 \le b \le 1 \\ 2-b & \text{if } 1 \le b \le 2 \\ 0 & \text{otherwise} \end{cases}$$

For $0 \le a \le 1$, $f_Y(a) = \int_0^a (a-z)\,dz = \left[az - \frac{1}{2}z^2\right]_0^a = \frac{a^2}{2}$

For $1 \le a \le 2$, $f_Y(a) = \int_{a-1}^a 2-(a-z)\,dz = \left[(2-a)z + \frac{1}{2}z^2\right]_0^a = 2a - \frac{a^2}{2}$

For $2 \le a \le 3$, $f_Y(a) = \int_{a-2}^a 0\,dz = 0$

$$\therefore f_{X_1+X_2+X_3}(a) = \begin{cases} a^2/2 & \text{if } 0 \le a \le 1 \\ 2a - a^2/2 & \text{if } 1 \le a \le 2 \\ 0 & \text{otherwise} \end{cases}$$

This definitely isn't correct as $f(a) \neq 0$ when $2 \le a \le 3$ but I'm not sure how to fix it.

5.

CLT

$$Z_n := \sqrt{n} \cdot \frac{\bar{X}_n - \mu}{\sigma}$$

$$\lim_{n \to \infty} F_{Z_n}(a) = \Phi(a)$$

$$\bar{X}_n = \frac{\sum_i X_i}{n}$$

$$\frac{\sum_{i=1}^{n/2} X_i}{\quad}$$

$$z_1 = \sqrt{n/2} \cdot \frac{\frac{\sum_{i=1}^{n/2} x_i}{n/2} - \mu}{\sigma}$$

$$\Phi(a) = \lim_{n \to \infty} \sqrt{\frac{n}{2\sigma^2}} \int_{-\infty}^{a} \left[\frac{\sum_{i=1}^{n/2} X_i}{n/2} \xrightarrow{} 0 - \mu \right] dX$$

This integral doesn't converge, and I ran into the same problem with the other two. I'm not even sure if I'm supposed to be integrating with respect to X here.

$\boxed{6.}$ $Y \sim Bin(1000, \frac{1}{6})$

$$\bar{Y} \approx N\left(\frac{1000}{6}, \frac{5}{36}\right) \quad \left[\bar{Y} \approx N\left(\mu, \frac{\sigma^2}{n}\right)\right]$$

$$P(100 \le Y \le 200) = P(100 \le \bar{Y} \le 200)$$

$$\approx P\left(\frac{100 - 1000/6}{\sqrt{5/36}} \le Z \le \frac{200 - 1000/6}{\sqrt{5/36}}\right)$$

$$= P(-178.89 \le Z \le 89.44)$$

$$= \int_{-178.99}^{89.44} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx.$$

$\therefore a = -178.99, \ b = 89.44$

LECTURE 10

1. $X \sim Exp(\lambda)$

Let $h(X_1, X_2, \ldots, X_n) = \bar{X}_n = \frac{\sum_{i=1}^{n} X_i}{n}$

$$E(\bar{X}_n) = \frac{nE(X_1)}{n} = E(X_1) = \frac{1}{\lambda}$$

$$\therefore \frac{1}{\lambda} = \bar{X}_n = \frac{2+5+4+4}{4} = 3\frac{3}{4}$$

5. $P(X_i = X_{i+1}) = \frac{1}{N}$

$$E(Z) = N$$

$$E((Z-N)^2) = E(Z)^2 - NE(Z) + N^2$$
$$= N^2 - N^2 + N^2 = N^2$$

$$N = \frac{1}{E(Z)} \Rightarrow N = h(Z_1, Z_2, \ldots, Z_n) = \bar{Z}_n.$$

I'm not convinced any of this is right but it was all I could think of.

LECTURE 11,12

5. I'm not sure where to even begin with this.