Natural Sciences Tripos, Part IA
Mathematical Methods II, Course B
Answers to Example Sheet 3

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Skills section

S1. (a) (yz, xz, xy)

(b) $(1,1,1)\,(x+y+z)^{-2}\exp[-1/(x+y+z)]$

(c) $-2(\alpha^2 x, \beta^2 y, \gamma^2 z)\exp(-\alpha^2x^2-\beta^2y^2-\gamma^2z^2)$

(d) $(x,y,z)\,r^{-1}$

(e) $-(x,y,z)\,r^{-3}$

(f) $(x,y,z)\,r^{-1}F'(r)$

S2. (a) $\boldsymbol{x} = (1+\cos t, 1+\sin t, 0), \quad 0 \le t < 2\pi$

(b) $\boldsymbol{x} = (-1+2t, t, t), \quad 0 \le t \le 1$

(c)

$$\boldsymbol{x} = \begin{cases} (1-t, t, 0), & 0 \le t \le 1, \\ (0, 2-t, t-1), & 1 \le t \le 2, \\ (t-2, 0, 3-t), & 2 \le t \le 3 \end{cases}$$

S3. (i) $3x^2+3y^2+6z$

(ii) $(2y, 2z-2x, -3)$

S4. (i) 3, $\mathbf{0}$

(ii) $\boldsymbol{a}\cdot\boldsymbol{b}$, $\boldsymbol{b}\times\boldsymbol{a}$

(iii) 0, $2\boldsymbol{a}$

(iv) 0, $\mathbf{0}$

S5. (a) $2\pi^2$ if $m = 0$, otherwise 0

(b) 0 if $m = 0$, otherwise $-2\pi/m$

(c) $a(\mathrm{e}^a - 1)/(a^2 + 4m^2\pi^2)$

(d) $-2m\pi(\mathrm{e}^a - 1)/(a^2 + 4m^2\pi^2)$

(e) $3\pi/4$ if $m = \pm 1$, $\pi/4$ if $m = \pm 3$, otherwise 0

(f) 0

Standard questions

Scalar and vector fields

6.
$$\boldsymbol{\nabla} f = \left(\frac{2x}{x^2 + y^2}, \frac{2y}{x^2 + y^2}, 1 \right)$$

(i) $2/5$

(ii) $-2\sqrt{5}/25$

7. $(1, -2, 5)$ (unnormalized), $x - 2y + 5z = 9$

8. $6x + 3y - z = 6$, southwest

9. $\pi a^4/4$

10. $\sin y$

11. -2π, 0

12. $\partial P/\partial y = \partial Q/\partial x$

(a) No. 3/2, 4/3

(b) Yes. $f = \mathrm{e}^{xy} + x^2 + xy$. $1 + \mathrm{e}$, $1 + \mathrm{e}$

13. 2, 2. $f = x^4 z + x^2 + yz^2 - y^2$

14. $\pi\,\mathrm{e}^{-2t}$, $2\pi\,\mathrm{e}^{-2t}$

15. (i) $(4/5)\pi a^5(\alpha + \beta + \gamma)$

(ii) $(3/2)\pi a^4 h(\alpha + \beta) + 2\pi a^2 h^3 \gamma$

16. (i) 17/2

(ii) $6 + b/2$

19. 2π

20. -2π

Fourier series

23. T

24. $\sin 2\theta,\ \frac{1}{2} + \frac{1}{2}\cos 2\theta,\ \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$

25.
$$\frac{l}{2} - \frac{4l}{\pi^2}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^2}\cos[(2m+1)\pi x/l]$$

26. (i)
$$\frac{\sinh\pi}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}(\cos nx - n\sin nx)\right]$$

$\cosh\pi$

(ii)
$$\frac{\sinh\pi}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}\cos nx\right]$$
$$-\frac{2\sinh\pi}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n n}{n^2+1}\sin nx$$

27.
$$\frac{\pi a_0^2}{2} + \pi\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

28. (a) $b_n = 0$
(b) $a_n = 0$
(c) $a_{2m+1} = b_{2m} = 0$
(d) $a_{2m} = b_{2m+1} = 0$
(e) $a_{4m+1} = a_{4m+2} = a_{4m+3} = b_{4m+1} = b_{4m+2} = b_{4m+3} = 0$
(f) $b_{4m} = 0$; $a_{4m+1} = b_{4m+1}$; $a_{4m+2} = 0$; $a_{4m+3} = -b_{4m+3}$
(g) $a_n = b_n = 0$ except a_0 (arbitrary)
(h) $a_{4m+1} = a_{4m+2} = a_{4m+3} = b_n = 0$

29*.
$$c_n = \begin{cases}\pi^2/3, & n = 0,\\ 2(-1)^n/n^2, & n \neq 0\end{cases}$$
$$\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$$