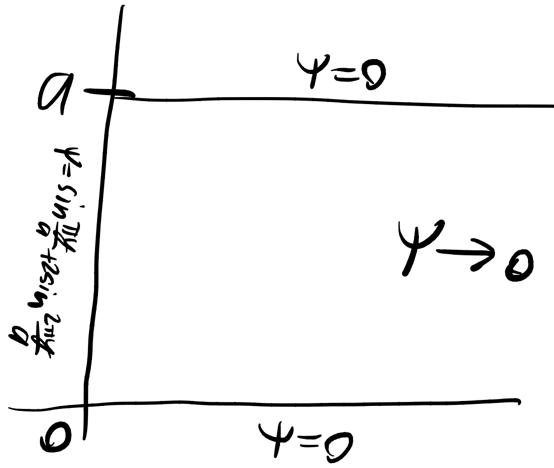
1.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$



Let $\psi(x,y) = X(x)Y(y)$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{d^2 X}{dx^2} Y \qquad \frac{\partial^2 \psi}{\partial y^2} = \frac{d^2 Y}{dy^2} X$$

$$\therefore Y\frac{d^2 X}{dx^2} + X\frac{d^2 Y}{dy^2} = 0$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} = -\frac{1}{Y}\frac{d^2 Y}{dy^2} = \lambda$$

$$\frac{d^2 X}{dx^2} - \lambda X = 0, \qquad \frac{d^2 Y}{dy^2} + \lambda Y = 0$$

Choose $\lambda > 0$ and set $\lambda = m^2$

$$X_m(x) = A_m e^{mx} + B_m e^{-mx}, \quad Y_m(y) = C_m \sin(my) + D_m \cos(my)$$

$$\psi(x,0) = \psi(x,a) = 0 \Rightarrow X(x)Y(0) = X(x)Y(a) = 0$$

$Y(0) = 0 \Rightarrow D_m = 0$

$Y(a) = 0 \Rightarrow$ allowed values of $m = \frac{n\pi}{a}$, $n \in \mathbb{N}$

(absorbed C_n into A_n and B_n)

$$\therefore \psi_n(x,y) = \left(A_n e^{\frac{n\pi}{a}x} + B_n e^{-\frac{n\pi}{a}x}\right)\sin\left(\frac{n\pi}{a}y\right)$$

$$\lim_{x\to\infty} \psi_n(x,y) = A_n e^{\frac{n\pi}{a}x}\sin\left(\frac{n\pi}{a}y\right) = 0$$

$\Rightarrow A_n = 0$ as this needs to be true for all y

$$\therefore \psi_n(x,y) = B_n e^{-\frac{n\pi}{a}x}\sin\left(\frac{n\pi}{a}y\right)$$

$$\therefore \Psi(x,y) = \sum_{n=1}^{\infty} B_n e^{-\frac{n\pi}{a}x} \sin\left(\frac{n\pi}{a}y\right)$$

$$\Psi(0,y) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{a}y\right) = \sin\left(\frac{\pi}{a}y\right) + 2\sin\left(\frac{2\pi}{a}y\right)$$

$$\Rightarrow B_n = 1 \quad , \quad n \in \{1,2\}$$

$$\therefore \Psi(x,y) = \sum_{n=1}^{2} e^{-\frac{n\pi}{a}x} \sin\left(\frac{n\pi}{a}y\right)$$

$$\therefore \Psi(x,y) = e^{-\frac{\pi}{a}x} \sin\left(\frac{\pi}{a}y\right) + e^{-\frac{2\pi}{a}x} \sin\left(\frac{2\pi}{a}y\right)$$