A Review And Analysis of Graph Coloring Techniques

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***Abstract*—In graph coloring problem (GCP), a graph has been colored such that a color is assigned to each vertex were neighboring vertices should have different colors. Graph coloring is anomaly of graph labelling. Graph coloring includes vertex coloring, face coloring and edge coloring. Graph coloring is used in many research areas and it has many applications like travelling sales mans problem, resource networking,etc. This paper discusses the different works carried out in graph coloring which includes the publications from the late 90s to the recent ones.The techniques include greedy and heuristic methods,neural networks genetic algorithm, ant colony systems, watermarking techniques, swarm intelligence,using MPI and oepn MP distributed and parallel methodologies. Each of these algorithms have different specialties. We have compared the algorithms in each technique using some constraints. The constraints that we have considered for comparison include execution time, number of colors, change in size of dataset and complexity. Each technique is analyzed. The applications solved using graph coloring are also discussed.**

***Index Terms*—Graph, Graph coloring**

I. INTRODUCTION

Suppose we want to group persons in n commission of k individuals such that to give their point of view on some topics. Based on the importance, the topics are assigned to more than one commission. Care should be taken that they don't have more than i persons in common for each commission.

Let G=(V,E) be the graph used to model this problem, in which each vertex conforms to a commission and if the corresponding commissions share at least one topic, there is an edge between a pair of vertices.There are different types of graphs like simple graph, null graph, complete graph,planar graph each of having different characteristics. If two vertices share an edge then they are adjacent to each other.Vertices which comes in boundary will refers to boundary vertices and vertices which are not boundary is known as interior vertices.Path which is not having vertices in boundary is known as internal path. Colors will be assigned to each vertex where colors refers to persons. The problem is to find the minimal number of colors used to assign each vertex such that neighboring vertices don't share same color.[1]. some times numbers 1,2..etc refers to colors which is used to color the graph. Graph coloring can also be performed in digital form.

In other word, Graph coloring is an anomaly of graph labelling. Graph coloring includes vertex coloring, face coloring and edge coloring. Vertex coloring in simple words is the assigning of colors to vertices where no two adjacent vertices have the same color.Capability of a technology can be accessed using these constraints. The least number of colors used for coloring of graph is called chromatic number.A graph is k-colorable, when it can be allot a (proper) k-coloring and if its chromatic number is just k,then it is k-chromatic. Edge coloring assigns color to each edge such that no two neighbouring edge share the same color and face coloring is allotting of colors to each face where no two faces that are having same boundary share the same color.

In various research areas like data mining, image segmentation, clustering, image capturing, networking..etc graph coloring is used. The most vital theory of graph coloring is also used in resource allocation and scheduling. The application of graph coloring are travelling sales mans problem, database design concepts and resource networking [2]. This leads to the evolution of new algorithms and new thesis that can be used in enormous applications. On the requirement basis various coloring techniques are available. Other applications of graph coloring includes making time table, sudoku, register allocation, bipartite graph, map coloring etc. The major part of data visualization is color coding.

The purpose of this paper is, to study the different existing techniques used for solving the graph coloring problem. The techniques include greedy and heuristic methods,neural networks genetic algorithm, ant colony systems, watermarking techniques, swarm intelligence,using MPI and open MP distributed and parallel methodologies.

II. ANALYSIS OF GRAPH COLORING METHODOLOGIES

Greedy and Heuristic Methodologies

Greedy algorithm is an algorithm model that follows the method in which chooses an optimal solution in each stage. For some mathematical problems it produces good solutions and it basically follows two properties. Even if it reduces the number of number of computational steps, it is not guaranteed that it produces an optimal solution. There are different greedy algorithms available for graph coloring and the guarantee which is given by the basic algorithm is that they won't use any more number of colors than the number of vertices.

Scott H. Cameron [3] suggests solution for graph coloring problem by set covering problem. The set covering problem is that, they require minimal number of channels which satisfy the constraint of frequency assignment of a collection of channels. This problem is related with graph coloring and the solution is found by initially generating a binary array which will permit the hypothesis to be tested. In graph coloring, the minimal number of colors which required for coloring a graph is called chromatic number. In result, the problem of establishing the number of channels wanted to provide frequency support with co channel constraint is demonstrated using graph coloring problem.In [4] vertex coloring is the coloring of nodes in a graph like no two neighbouring nodes share the same color. The algorithm presented is for pseudo two coloring of a general graph. Topological via minimization (TVM) and constrained via minimization (CVM) are the two basic approaches to via minimization. Pseudo coloring technique is applied to CVM. The Algorithm uses the relationship between odd cycles and comparability graphs. Comparability graphs have the property that their edges can be directed. The coloring heuristic uses an edge orientation technique to take the advantage of information about the odd cycles in a graph and thereby produces a more accurate pseudo 2- coloring, which need a violating graph edge for all odd cycle. This method takes the advantage of odd cycle membership of nodes in graph.

Another solution is by using energy function. Energy function provides a broad procedure for deriving analytic algorithm for different variants of problem in independent manner[15]. Three different variants of GCP considered are the minimum coloring problem, the spanning sub graph coloring problem, and the induced sub graph coloring problem. Optimization is based on multiple restart quasi-Hopfield networks where the problem cognition is imbed in the energy function. Multi valued neural network is proposed.

Coloring of edges using minimal number of colors by heuristic algorithm is considered. Simple undirected graphs are considered. Edge coloring problems are transformed to node coloring problems. For the edge-coloring application, each color assigned by a heuristic should be as near to correct as possible. Blind, local and global are the three classifications in heuristics. Six algorithms for edge coloring considered are Heuristic cyclic, random, linE, ant, antN, symC.Two fitness functions are used in coloring[16]. The solution was represented in the EA as the list of all edge colors in the order of the edge numbering. Mutation and crossover are the two operators used.In paper [18], the method suggests that,by giving changes to the sizes of constant weight codes and asymmetric codes, a new genetic and heuristic algorithm is developed. Edges can be developed in two separate ways. Tabu search is a local search method. Evolutionary algorithms constitute a large family of parallel algorithms. Greedy algorithm is used along with genetic algorithm. Sequential method and permutation methods are the two methods used in searching for colors.

Evolutionary approach for graph coloring method[20] works on artificial string represents coloring solution. The evolutionary algorithm is a algorithm which keeps a population of artificial chromosomes. The proposed algorithm (EA) incorporates a transcription structure transcriptase that contains information about vertices and edges of the underlying graph. A conservative-mutation, regressive-mutation, and a crossover operator are the three EA operators used. EA can handle wide range of graphs since they are

TABLE I
COMPARISON BETWEEN GREEDY ALGORITHMS

Method	Year	Advantages	Disadvantages
[3]	1977	Minimum number of channels, Both upper bound and lower bound is utilized.	All constraints. are not satisfied
[4]	1995	pseudo coloring technique is used.	Layer violation is not possible.
[15]	2002	Simple Clean structure, Easily Parallelizable	Dealing with big data is difficult, Slow.
[16]	2003	Fast in nature	Finding minimum number not possible in all cases
[18]	1998	Lower bound is improved, Based on tabu search	Accuracy is less
[20]	1999	Applied to random normal graphs	Complex in nature
[21]	2004	Depending upon input size meta heuristics can be choosen	Big data is not possible

independent of specifications.Andrew Lim and Fan Wang[21] suggests an Extension of classical algorithm like meta heuristic. This method focuses on building robust coloring using fixed number of colors.It also considers the possibility of correcting those coloring where both vertices of an missing edge share the same color Partition based encoding is used in search space. TabuVec and TabuList are the two tabu lists used. Different sizes of input are required.

We have discussed about the algorithms that comes under greedy methodologies.Table 1 shows a comparison of these techniques. Some of those algorithms have less execution time , less accuracy and less complexity. In [21] we can see that depending up on the input size we can choose different meta heuristic available. The execution time and complexity depends up on the heuristic. Among greedy algorithms discussed, [21] is satisfied some of the constraint. But the problem arise in case of big data.

Neural Networks

Neural network is type of artificial intelligence which try to follow the way in which a brain works. There are processing elements and neural network mainly works by creating a connection between these elements rather than using zeros and ones as in digital networks.

Neural network is used for solving graph coloring problem. In order to reduce search space add potts neurons, graph coloring problem is converted to a maximum weight stable set problem [5]. A d-coloring is possible for all graphs. A new graph is generated and then adds d vertices in order to get a minimal coloring. Critical point is the mapping of the problem on minimizing a energy function. Each cluster of neurons represents one out of n choices and vector groups are formed from neurons. The problem of symmetry of the solution space is overcome by giving each color a different input bias. The result shows that the potts neurons can corporate well with traditional spin glasses.

David W. Gassen and Jo Dale Carothers, proposes a neural network model which minimizes the number of colors used in graph coloring problem[6]. The goal is to include register minimization. The graph in which each node stands for a program variable and an edge is drawn between two variables if they impede with each other is known as interference graph. The advanced algorithm of neural network is used. The network is based on binary neuron model. The network changes from initial random state to minimal energy state by selecting a proper energy function. The results show that by increasing the number of variables and probability of interference, the number of colors wanted will also increases.Dynamical hysteresis neural network is analyzed in [22]. Phase difference between each neuron is the information. Oscillatory hysteresis neural network is applied to solve GCP. System exhibits an in-phase synchronization when the cross-connection coefficient denotes an excitatory connection.Q'tron neural network model is used for solving graph coloring problem and it is known as energy system is proposed by Tai-Wen Yue,Zou Zhong Lee[26]. By making free the local minima they attained a goal directed search.The model is developed by placing all the feasible coloring scheme in global minima. Suppose K colors are used for solving the problem, then by decreasing the values of k goal can be achieved. The network is settle down only if the goal is reachable. The proposed models experimental results is compared with DSATUR algorithm.

TABLE II
COMPARISON OF NEURAL NETWORK ALGORITHMS

Method	Year	Advantages	Disadvantages
[5]	-	Lesser search space	More running time for big data
[6]	1993	Lesser number of color is used	Accuracy is less
[22]	2003	Simple algorithm, 2 coloring is possible	k colorable graph is not possible
[26]	2007	Optimized solution can be obtained	Not compiler specific

Table 2 show the comparison of these algorithms. Even though[26] is having computational complexity, Optimal solution can be obtained using this algorithm. Other algorithms have less accuracy and more run time. But those algorithms uses lesser number of colors.

Watermarking and online Techniques

The process of hiding digital information is known as watermarking. Embedding,attack and divison are the three steps in watermarking.

Two watermarking techniques are used for analyzing graph coloring problem is used in [7]. Credibility and upward are the two most important principles for watermarking technique. The first technique forces vertices to be labelled by adding different edges between some well chosen pair of vertices. The other technique select one maximal independent set from the initial graph and then mark each of the set by definitely one color. The graphs from real-life model and the DIMACS confront graphs, color several set of random graphs. With the same reach of runtime, watermark graphs can be colored by no overhead.The problem of graph coloring is considered online by Sundar Vishwanathan [8]. One vertex of the graph is presented at a time. As soon as the vertex is presented, online algorithm should assign a color to a vertex. The objective of and online algorithm is to color the graph efficiently. In order to mend on the color of each vertex, the algorithm takes polynomial amount of time. Graph and the arrangement of the vertices is determined by the assailant . It is fixed for the rest and presents at a time one vertex. The residue set is the set which contain the remaining

TABLE III
COMPARISON BETWEEN WATERMARKING AND ONLINE TECHNIQUES

Method	Year	Advantages	Disadvantages
[7]	1998	No overhead	Complex in nature
[8]	1990	Efficient in nature	Not work well in general graphs

vertices which is not colored.In residue set, each vertex will be adjacent to some of the vertices in greedy class. Before moving to next vertex, color the current vertex within exponential amount of time. Smallest admissible color is assigned to each vertex by greedy algorithm. This technique doesnt work well in general graphs. From table 3 we can that [7] is having less overhead but complex in nature and [8] is efficient in nature.

Distributed and Parallel methodologies

This field studies about distributed systems. In distributed systems, messages will be passed through network in-order to have communication between computers. Another specialty is that the problem will be separated in to different tasks and each system will be assigned different tasks. The tasks will be executed in parallel and the computational time can be reduced.

Parallel algorithm for coloring graph [9] is a simple and effectual parallel algorithm for coloring of nodes in a graph. The algorithm described is suitable for implementing in parallel machines that communicate by passing messages whether they are connection machines. The central feature of the algorithm is that by using bias variable, one can trade off the measure the performance of the algorithm. The basic computing element is processing element (PE). Each vertex is associated with a PE. The algorithm contains logical steps which are completed in synchronicity by all active PE's. Algorithm for parallel machines is used when parallel machine has atleast nodes to color. This algorithm is trivially modified for machines with fewer processors than the nodes to color which is known as algorithm for non massively parallel machines.

In a distributed setting, a generalized coloring of weighted graph in a shared setting is considered with positive real valued interference couplings[10]. Proposed system minimizes weight of edges. Finding a sub graph with maximum weight subset of edges to be colored, avoids getting stumped in a not too good local optimum. By solving maximum colorable sub graph problem, minimum weight coloring is searched. Investigate problem in two flavors. First, agents know only the planar graph neighbors. Second, the metric for maximality is the sum of the values of all edges in the sub graph.The method proposed by Marek Kubale, ukasz Kuszner[23] follows a distributed coloring algorithm for graph coloring.

TABLE IV
COMPARISON BETWEEN DISTRIBUTED ALGORITHMS

Method	Year	Advantages	Disadvantages
[9]	1998	Simple, Effective in nature.	Used only when least number of nodes, Run time is more.
[10]	2010	Weight of edge is minimized, noise strategy is used.	Becomes complex for big data.
[23]	2002	Solved using distributed network, useful for bounded degree graphs.	Less accurate.
[29]	2011	Used for large graphs, Lesser conflicts.	Probability of minimization of number of color is less.
[32]	2017	Balanced coloring, used for big data.	If degree is more, then the coloring becomes harder.
[34]	2015	Incorporated with hadoop, used for big data, lesser runtime.	Not optimal in nature.

The problem is solved in a distributed network. Vertices and edges refers to models and links. There is no shared memory for processors and each processor know its own unique identifier. System is con-temporized in rounds. Some parameters like degree, random value and first legal color is send to each neighboring vertices initially. Then compare the parameters received and chose the vertex which is having higher priority. if the vertex is larger priority then keep the legal color which is initially assigned to it and terminate. Otherwise update the list and continue. The different facts got after the analysis of the algorithm are one vertex will be colored in each round and in each round one new color will be assigned to any of the vertex and these facts are proved. This method is more useful for bounded degree graphs.

A graph coloring framework by Ahmet Erdem Saryucey, Erik Saule, and Umit V. C atalyurek [29], is developed also in a distributed manner. This is applicable when the graph is very large. They have mainly concentrated in two techniques. Visit of two different vertex order in one technique and second one concentrates on the use of recoloring in a distributed environment. We can use one node when the graph is small enough. On comparing both greedy and distributed, distributed environment uses more number of colors and the coloring is done in multiple rounds. Conflicts arise while coloring can be reduced by using super steps. Both asynchronous and synchronous modes are possible. In addition with the above parameters, color selection strategy also affects the framework. In recoloring algorithm, those vertices which is in same color class will be colored in same super step. Processor allocation is in round robin manner.

Algorithms for Balanced Graph Colorings with Applications

in Parallel Computing[32] mainly aims on attaining balance coloring in parallel computing. Distance coloring and partial balance 2 coloring are the two variants considered. Three different guided balance algorithms are proposed and parallelism those algorithms. ab initio approach is used for balance coloring and the other used guided approach. Guided approach used scheduled and unscheduled moves.Inorder to manipulate large amount of data Hadoop is used in [34]. Graph coloring is incorporated in to hadoop. In this paper they have compared four parallel algorithms. Bulk synchronous parallel model is used over hadoop and it consist of supersteps.. Local Maxima First, Local Minima-Maxima First, Local Largest Degree First, Local Smallest-Largest Degree First are the algorithms considered. Experimental results give that different algorithm is better in different constraints.

Table 4 gives the comparison between distributed algorithms. One of the specialty of distributed algorithm is that, they are having lesser execution time on comparing with other methods. Big data is handled in [32]. One of the problem distributed algorithm face is the accuracy.

Genetic Algorithm

Evolutionary ideas like natural selection and genetics is followed in genetic algorithms. On comparing with conventional methods genetic algorithm methods are more robust.

A new bi-objective model is used for graph coloring[11]. The algorithm applys effective crossover and simple mutation operators as genetic operators. To solve the new Bi- objective model a new genetic algorithm is intended consisting of selection, crossover and mutation operators. Each gene of an individual represents a node in the graph. Crossover technique should preserve building blocks. It can effectively look in to the search space near parents taken part in the crossover operation. Pareto ranking is used in the selection operator. Simulation results shows that the algorithm is effective.The robustness of the genetic algorithm is used to solve the problem by Raja Marappan, Gopalakrishnan Sethumadhavan[12]. Innovative single parent conflict gene crossover and conflict gene mutation are used as operators. These operators contract the search space and minimize the generations. During each generation, it applies fitness proportionate selection method for selecting better gene. The random generated operators determine the operators. In order to get optimal solution, worst gene sequence is replaced with better gene sequence. Proposed genetic algorithm reduces computational complexity.

Hybrid genetic algorithm is used for describing GCP[19]. Each solution is represented by isomorphic genotypes. Metric function is defined between genotypes. Behavior of geno types is controlled by phenotypic distance. GAs employs crossovers as a main search operator which supports their global search power. For coloring problem, a new crossover technique called Harmonic Crossover is proposed. In the design of GA to the GCP, encoding solutions as strings allows the application of one-point (lX), two-point (2X) and uniform (UX) crossovers. When a suitable distance between two solutions is defined, this method is applicable.Multi-seperation technique is followed in [25] and an multi-seperation model is developed . Steps followed is in the form of an instance and finally correct scheme will be obtained. The model consist of electrode plate, temperature controller chamber and the core device.The problem is changed to satisfy ability (SAT) problem and then the MS based coloring algorithm is implemented in that. The experimental results show that this method is having not only lesser time but also lesser DNA codes.

TABLE V
COMPARISON BETWEEN GENETIC ALGORITHMS

Method	Year	Advantages	Disadvantages
[11]	2010	Effective in nature.	Complex for big data.
[12]	2013	Computational complexity is less, optimal solution.	More run time.
[19]	1999	More applicable when distance is defined.	Complex in nature.
[25]	2008	Lesser execution time,lesser number of codes.	Expensive for implementation.
[33]	2016	Works well in minimal population, less complex.	Complex for big data.
[35]	2015	Execution time is less.	Complexity is more.

Aim is to obtain a minimal chromatic number and it is achieved by using genetic divide and conquer method is used in [33]. The vertex set id divide in to different subsets and then solved. The new algorithm is proposed using Single Parent Conflict Gene crossover (SPCGX) conflict edge mutation operators. Stochastic convergence is also analyzed based on certain constraints. Experimental results show that, the method is work well even in minimal population. This devised method reduces complexity as compared with the basic genetic algorithm.Based on Compute Unified Device Architecture (CUDA) programming, parallel genetic algorithm is formulated to solve coloring problem[35]. The operators are designed in parallel which is in threads. This algorithm is generated from sequential genetic algorithm. The only difference on comparing with the other technique is that execution is parallel and the time is less. Rest of the steps are same as in general genetic algorithm. Table 5 shows the comparison of the genetic algorithms. Complexity and big data handling is tough in some cases of genetic algorithms.

Ant Colony System

Ant colony is a meta heuristic method in which, for a given optimization problem, artificial ants as a set of software agents search for a good solution.nThe ant colony system algorithm is inspired from by the pasturing behavior of ants.

TABLE VI
COMPARISON BETWEEN ANT COLONY ALGORITHMS

Method	Year	Advantages	Disadvantages
[13]	2003	Few execution time.	Complex for big data.
[14]	2005	Fast.	Not parallel in nature.

It is basically a population based approach.In modified ant colony system for coloring graphs[13], ANTCOL algorithm is used to solve the coloring problem and task of each ant is to color the graph in a constructive way. ANTrandom is used as a constructive method. Basically ants visits edges and it will apply the local updating rule,they change the amount of pheromone trail. After all ants complete the tour global updating rule is applied. XRLF(Extended RLF) apply randomize to RLF, that used in general search.Construction strategy and improvement strategy are the two strategies of ant colony system. The method iteratively set up the solution in construction strategy is used in [14]. Tabu search is used in improvement strategy. Constructive methods used are RLF and DSATUR. Tabu search moves from a coloring to another. Based on the used constructive method, heuristic information is defined. Ant colony system is quite fast along with these strategies. Table 6 shows the comparison between the ant colony algorithms. Even though they are having lesser execution time and faster, complexity is more.

Particle Swarm Optimization

This method have many similarities with genetic method. The particles which is known as potential solutions will fly through the problem space.But it wont have any crossover or mutation operators as there in genetic algorithm. This method is easy is implement and it is having only a few number of parameters to adjust.

Artificial Bee Colony (ABC) algorithm[17] is used for solving GCP. Sequence of nodes is generated for the given graph. Coloring algorithm is implemented on the given graph. It is for coloring of the graph and it is based on the order of the previous nodes. Vector components are mapped with a series of integer numbers. Recursive largest first (RLF) heuristic algorithm is used for color assignment. It colors with least amount of numbers. With different densities the algorithm proposed is tested on randomly rated graphs.The technique used in [28] is based on ants AS-GCP and bees artificial colonies ABC-GCP. Ants is based on Greedy Partitioning Crossover and the other one is based on mutation operators,improved version of GPX(Greedy Partitioning Crossover) and temperature. Efficiency of basic swarm intelligence method is improved in this technique. Parameter is initially set in the method and then the algorithm is compared with 15 different algorithms. AS-GCP is mainly based on fitness functions and the crossover operator is combined with mutual search operator and it is implemented using basic GPX. In ABC-GCP initialization operator have a fundamental role because it has the fundamental role

TABLE VII
COMPARISON BETWEEN SWARM INTELLIGENCE ALGORITHMS

Method	Year	Advantages	Disadvantages
[17]	2012	Minimal number of colors is used.	Less accurate.
[28]	2005	More efficient.	Complex in nature.
[36]	2015	Accurate,simple in nature.	Complex for big data.

and it uses improved version of GPX. Takuya Aoki, Claus Aranha, Hitoshi Kanoh[36] suggests a solution based on hamming distance. Particle Swarm Optimization (PSO) is a population based search algorithm. In the search space number of individuals named particles are placed. In this graph 3 coloring problem is considered. Red, Green and Blue are the colors which they considered. Hamming distance is that which is used to find the distance between particles. Transition probabilities is used to find the internal relationship between the particles. Operation of transition probability is changed at last and a new transition probability is formulated. Tabel 7 shows the comparison between the particle swarm optimization algorithms. From that we can see that [36] is simple and accurate in nature.

Graph Coloring Solved Using Libraries, Open MP and MPI

For shared memory devices, open MP is used for programming and for distributed memory MPI is used. This can also be used for solving graph coloring. Libraries are also available.

Koala graph coloring library[27] is developed for graph coloring. The library is coded by using c++ and each of them can be used separately. It is an open source library. Greedy algorithms and exact coloring algorithms are considered. They provide a set of operators. All the implementation details are hidden. On difference with other models they have considered triple tuples. Dynamicgraph interface is implemented. Procedural graphs is the solitary feature of koala library. Hybrid graph coloring algorithm is formulated in [30] and it is implemented in multi-core cluster. They have used both open MP and MPI by adjusting the number of OpenMP threads per MPI process in hybrid environment. The algorithm is implemented in a distributed platform known as zoltan. There will be an edge for each MPI process. Other mechanisms are same as in [29]. There must be frequent communication between MPI processors for reducing the conflicts and the communication is done through dedicated messages. These parallelism can be obtained by using openMP. Some of the disadvantages of [29] is overcomed in this method.

Applications Solved Using Graph Coloring

Graph coloring have various applications as already mentioned in introduction. One the application of graph coloring is time table making. CCTP refers to college course timetable

problem.They have considered both problems like feasible and nice timetable making. In [24] this problem is solved using combined genetic algorithm and ant colony optimization.By using these methods the factors of time tabling problem like compactness and balancing can be attained. All the constraints of time table problem is fitted to a graph coloring model. The overall algorithm consist of three modules. Initial module consist of creation of graph and that graph will be given as an input to the next module which is soft computing module. Coloration is performed in that module by two soft methods. In the final module the timetable is prepared from the colorated graph. Mohanad Mohammed Abdulkareem,Labeeb Mohsin Abdullah proposes a matrix based graph coloring algorithm for assignment and ressignment[31]. To identify the cells in a network physical cell identity(PCI) is used. Matrix based coloring algorithm is formulated to solve this problem. which is a modified version of the basic matrix algorithm. Incidence matrix is used in this algorithm where rows represents vertices and columns represents edges.

III. Conclusion

In graph coloring problem (GCP), a graph has been colored such that a color is assigned to each vertex were neighboring vertices should have different colors. Chromatic number is the minimal number of colors used for coloring the graph. The different techniques used for solving the coloring problem includes greedy and distributed methods, neural networks, genetic algorithm, ant colony systems, watermarking techniques, swarm intelligence, particle swarm intelligence are discussed. These techniques are compared based on certain constraints. From the analysis we can see that big data can be handled easily by distributed algorithms and having lesser execution time. Greedy algorithm is having less complexity as compared with genetic algorithm. Libraries can be chosen as per the requirement and the implementation details are hidden from the user.

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