Q-3 $\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu)^2$

Let $x_1, x_2, x_3, \dots, x_n$ be n independent observations from a population with mean $\hat{\mu}$ & variance $\hat{\sigma}^2$.

$E(x_i) = \hat{\mu}$

$Var(x) = \hat{\sigma}^2$

$Var(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \hat{\sigma}^2 + \hat{\mu}^2$

$Var(\mu) = E(\mu^2) - [E(\mu)]^2$

$E(\mu^2) = \frac{\hat{\sigma}^2}{n} + \hat{\mu}^2$

$$E(\sigma^2) = \frac{1}{n-1} E\left[\sum (x_i^2 - 2x_i\mu + \mu^2)\right]$$

$$= \frac{1}{n-1} E\left[\sum x_i^2 - 2\mu \sum x_i + n\mu^2\right]$$

$$= \frac{1}{n-1} E\left[\sum x_i^2 - n\mu^2\right]$$

$$= \frac{1}{n-1} \sum E(x_i^2) - n\,E(\bar{w}^2)$$

$$= \frac{1}{n-1} \left[\sum (\hat{\sigma}^2 + \hat{w}^2) - n\left(\frac{\hat{\sigma}^2}{n} + \hat{w}^2 \right) \right]$$

$$= \frac{1}{n-1} \left[n(\hat{\sigma}^2 + \hat{w}^2) - \hat{\sigma}^2 - n\hat{w}^2 \right]$$

$$= \frac{1}{n-1} \left[(n-1)\,\hat{\sigma}^2 \right]$$

$$E(\sigma^2) = \hat{\sigma}^2$$

$$B(\sigma^2) = E(\sigma^2) - \hat{\sigma}^2$$

$$= \hat{\sigma}^2 - \hat{\sigma}^2$$

$$B(\sigma^2) = 0$$