Unit-2 class-8

1. Find $L^{-1}\left[s\log\left(\frac{s+4}{s-4}\right)\right]$

Ans : $f(t)=\frac{2[4t\cosh 4t-\sinh 4t]}{t^2}$

$$L^{-1}\left[s\log\left(\frac{s+4}{s-4}\right)\right]$$

$$F_1(s) = s\log(s+4) - s\log(s-4)$$

$$\frac{dF_1(s)}{ds} = \frac{s}{s+4} + \log(s+4) - \frac{s}{s-4} - \log(s-4)$$

$$L^{-1}\left[\frac{dF_1(s)}{ds}\right] = L^{-1}\left[\frac{\cancel{s^2}-4s-\cancel{s^2}-4s}{s^2-16} + \log(s+4) - \log(s-4)\right]$$

$$-tf(t) = -8\times\cosh 4t + L^{-1}\left[\log(s+4)-\log(s+4)\right]$$

$$F_2(s) = \log(s+4) - \log(s-4)$$

$$\frac{d}{ds}F_2(s) = \frac{1}{s+4} - \frac{1}{s-4} \Rightarrow L^{-1}\left[-\frac{d}{ds}F_2(s)\right] = -e^{-4t} + e^{4t}$$

$$tf_2(t) = -e^{-4t} + e^{4t} \Rightarrow f_2(t) = \frac{2}{t}\sinh 4t$$

$$-tf(t) = -8\cosh 4t + \frac{2\sinh 4t}{t}$$

$$f(t) = \frac{8\cosh 4t}{t} - \frac{2\sinh 4t}{t^2} = \frac{2}{t^2}\left(4t\cosh 4t - \sinh 4t\right)$$

2. Find $L^{-1}\left[\cot^{-1}\left(\frac{s}{2}\right)\right]$

Ans : $f(t)=\frac{\sin 2t}{t}$

$$L^{-1}\left[\cot^{-1}\left(\frac{s}{2}\right)\right]$$

$$F(s) = \cot^{-1}\left(\frac{s}{2}\right)$$

$$\frac{dF(s)}{ds} = \frac{-1}{1+\left(\frac{s}{2}\right)^2} \times \frac{1}{2} = \frac{-2}{s^2+4}$$

$$L^{-1}\left[\frac{dF(s)}{ds}\right] = L^{-1}\left[\frac{-2}{s^2+4}\right]$$

$$-tf(t) = -\sin 2t$$

$$f(t) = \frac{\sin 2t}{t}$$

3. Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$

Ans : $f(t)=\frac{e^{-bt}-e^{-at}}{t}$

$$F(s) = \log\left(\frac{s+a}{s+b}\right) = \log(s+a) - \log(s+b)$$

$$L^{-1}\left[\frac{d}{ds}F(s)\right] = L^{-1}\left[\frac{1}{s+a} - \frac{1}{s+b}\right]$$

$$L^{-1}\left[-\frac{d}{ds}F(s)\right] = L^{-1}\left[\frac{1}{s+b} - \frac{1}{s+a}\right]$$

$$tf(t) = e^{-bt} - e^{-at}$$

$$f(t) = \frac{e^{-bt} - e^{-at}}{t}$$

4. Find $L^{-1}\left[\log\left(1-\frac{a^2}{s^2}\right)\right]$

Ans : $f(t)=\frac{2(1-\cosh at)}{t}$

$$F(s) = \log\left(1-\frac{a^2}{s^2}\right) = \log\left(\frac{s^2-a^2}{s^2}\right) = \log(s^2-a^2) - \log s^2$$

$$\frac{dF(s)}{ds} = \frac{2s}{s^2-a^2} - \frac{2}{s}$$

$$-\frac{dF(s)}{ds} = \frac{2}{s} - \frac{2s}{s^2-a^2}$$

$$L^{-1}\left[-\frac{dF(s)}{ds}\right] = L^{-1}\left[\frac{2}{s} - \frac{2s}{s^2-a^2}\right]$$

$$tf(t) = 2 - 2\cosh at$$

$$f(t) = \frac{2(1-\cosh at)}{t}$$

5. Find $L^{-1}\left[\log\frac{(s^2+4)}{s(s+4)(s-4)}\right]$

Ans : $f(t)=\frac{1+2(\cosh 4t-\cos 2t)}{t}$

$$F(s) = \log(s^2+4) - \log s - \log(s+4) - \log(s-4)$$

$$-\frac{dF(s)}{ds} = \frac{-2s}{s^2+4} + \frac{1}{s} + \frac{1}{s+4} + \frac{1}{s-4}$$

$$L^{-1}\left[-\frac{dF(s)}{ds}\right] = L^{-1}\left[\frac{1}{s} + \frac{1}{s+4} + \frac{1}{s-4} - \frac{2s}{s^2+4}\right]$$

$$tf(t) = 1 + e^{-4t} + e^{4t} - 2\cos 2t$$

$$f(t) = \frac{1 + 2(\cosh 4t - \cos 2t)}{t}$$