ENGINEERING MATHEMATICS - I

Random variables and Probability Distributions

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ENGINEERING MATHEMATICS - I

UNIT 2 : Random Variables and Probability Distributions

Session : 4

Sub Topic : Bernoulli Distribution

BERNOULLI DISTRIBUTION

Suppose that a trial, or an experiment, whose outcome can be classified as either a success or a failure is performed. If we let $X = 1$ when the outcome is a success and $X = 0$ when the outcome is a failure, then the probability mass function of X is given by

$$P(X = 0) = p(0) = 1 - p,$$

$$P(X = 1) = p(1) = p$$

Where p, with $0 \le p \le 1$, is the constant probability that the trial is a success.

The random variable X is said to be a Bernoulli random variable

BERNOULLI DISTRIBUTION

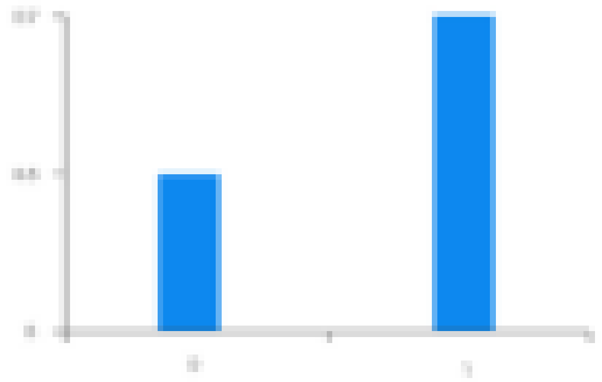


Figure: Pmf of Bern(0.7) random variable

Remarks

- Bernoulli distribution often serves as a starting point for more complex distributions. For example: Bernoulli process lays a foundation for binomial distribution, geometric distribution, negative binomial distribution, all of which play a crucial role in advanced probability.
- Bernoulli distribution is a discrete probability distribution.
- Bernoulli distribution describes the probability of achieving a success or a failure.

Remarks

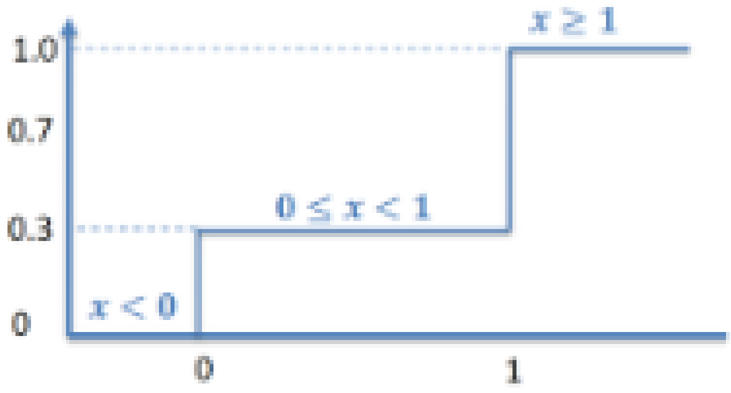
- A Bernoulli trial is an even that has only 2 possible outcomes (success or failure). For example: A coin will land heads or tails.
- In experiments and clinical trials, the Bernoulli distribution sometimes is used to model a single individual experiencing an event like death, disease or disease exposure. The model is an excellent indicator of the probability a person has the event in question.

CDF of a Bernoulli Distribution

The CDF of a Bernoulli random variable is given by

$$F_X(x) = \begin{cases} 0, & x < 0 \\ 1 - p, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

The cdf may be represented graphically as



CDF of a Bern(0.7) random variable

Conditions under which a Bernoulli Distribution is defined

1. Only two possible outcomes are possible.
2. Each of the outcomes has a fixed probability p of occurring.
3. Trials are entirely independent of each other.

Example

Example: An archer can hit the target with probability 0.6. What is the probability that he misses the target ?

Solution: Probability (Archer hits the target) = P(success) = 0.6. Hence, probability(Archer misses the target) = P(failure) = 1- 0.6 = 0.4

Expectation of a Bernoulli Random variable

Mean or Expectation of a Discrete random variable is defined as :

$$\sum_{n=0}^{\infty} x_n P[X = x_n]$$

That is, the expectation of a discrete random variable is the sum of the products of all outcomes and their corresponding probabilities. Thus, for a Bernoulli random variable, we have

$$\begin{aligned} E[X] &= 0 \times P[X = 0] + 1 \times P[X = 1] \\ &= 0 \times (1 - p) + 1 \times (p) \\ &= p \end{aligned}$$

Variance of a Bernoulli Random variable

The variance of a discrete random variable is, by definition,

$$Var(X) = E[X^2] - \{E[X]\}^2$$

Where $E[X^2] = \sum_{0}^{1} x^2 P(X = x) = 1.p = p$. Let $q = 1 - p$. This gives,

$$Var(X) = p - p^2 = p(1 - p) = pq$$

Remarks

- If the experiment is a deterministic experiment with $p = 0$, i.e the outcome is impossible. Hence the variance must be zero as can be verified from the expression for variance.
- If the experiment is a deterministic experiment with outcome $p = 1$, then the outcome is certain. Hence the variance is again 0.
- Sum of independent Bernoulli random variables is a Binomial random variable. This will be proved and discussed in the lecture entitled "Binomial distributions"

Higher Moments of a Bernoulli Random variable

The r^{th} moment of a Bernoulli random variable is given by

$$E[X^r] = \sum_{0}^{1} x^r P(X = x)$$

$$= 0 \times (1 - p) + 1 \times p$$

$$= p = E[X]$$

Example Let X be a Bernoulli random variable with parameter $P = \frac{1}{2}$.Find the tenth moment of X.

Solution: By definition, $E[X^r] = E[X]$ for a Bernoulli random variable. Hence the tenth moment of X is $E[X^{10}] = E[X] = p = \frac{1}{2}$.

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Thank You