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ENGINEERING PHYSICS

Department of Science and Humanities

Class #32

- *Introduction to LASER*
- *Planck's expression for energy density*
- *Interaction of radiation with matter*

➤ *Suggested Reading*

1. *Lectures on Physics, Feynman, Leighton and Sands*
2. *Lasers - Principles and Applications, A.K.Ghatak and K. Thyagarajan*
3. *Learning material prepared by the Department of Physics*

➤ *Reference Videos*

1. <https://nptel.ac.in/courses/104/104/104104085/>

LASER is an acronym for

Light

Amplification by

Stimulated

Emission of

Radiation

- **Einstein 1917 – Mathematical expression for stimulated emission**
- **Theodore Maiman (1960) – First working laser (ruby laser)**

- Quanta of radiation emitted by the oscillators, $E = h\nu$
- Planck's expression for energy density of black body radiation at any frequency and temperature is

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(\exp^{\frac{h\nu}{kT}} - 1\right)}$$

Quantum transitions

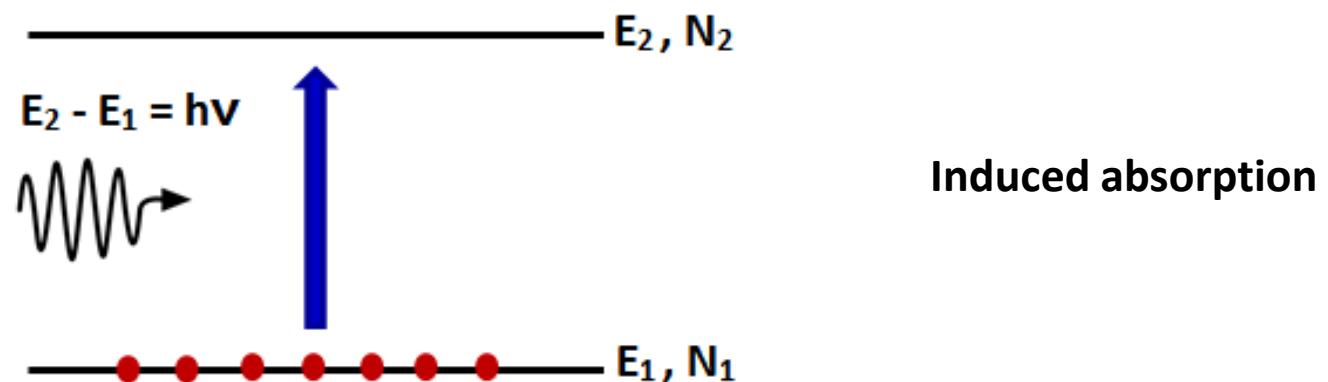
- Induced absorption (stimulated absorption)
- spontaneous emission
- Stimulated emission
- *To raise an electron from one energy level to another, “input energy” is required*
- *When falling from one energy level to another, there will be an “energy output” given by Planck’s law*
- $E = E_2 - E_1 = h\nu$, where E_2 is higher energy state
 E_1 is lower energy state

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Interaction of radiation with matter-Einstein's coefficients

1. Induced absorption (stimulated absorption)

When radiation is incident on an atom which is in its ground state of energy E_1 , it can absorb the photon of energy $h\nu$ and get raised to an excited state of energy E_2 , provided $h\nu = E_2 - E_1$. This process of absorbing energy from photons is called absorption of radiation.

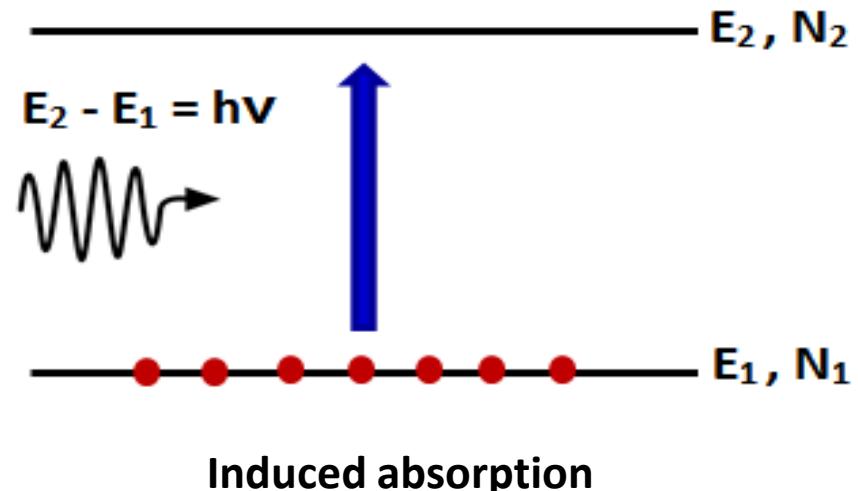


Rate of Induced Absorption

The rate of absorption is dependent on

- population of the ground state N_1 / lower energy state
- energy density of radiation $\rho(v)$ of the appropriate frequency ($E_2 - E_1 = hv$)
- The rate of induced absorption $R_{\text{ind abs}} = B_{12} * N_1 * \rho(v)$

where B_{12} is the Einstein's coefficient for induced absorption



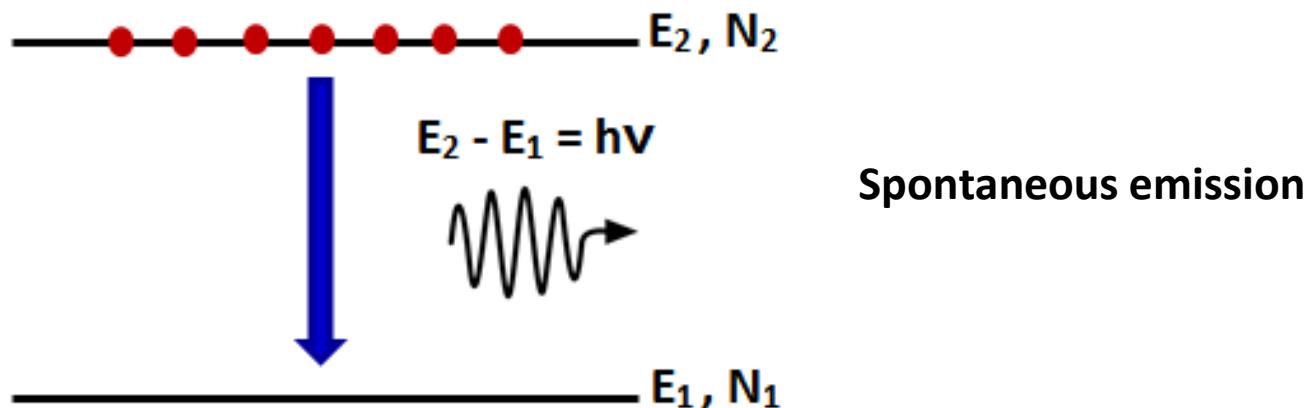
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Interaction of radiation with matter-Einstein's coefficients

2. Spontaneous emission

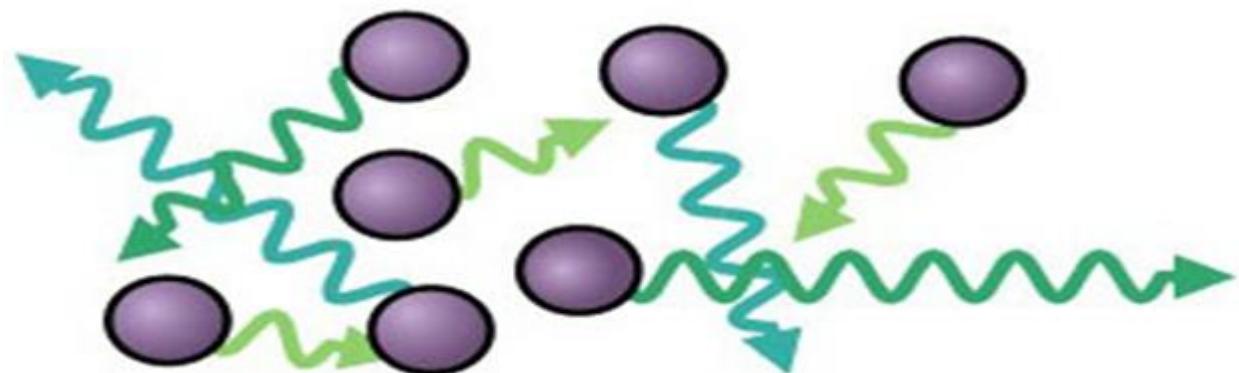
The process by which excited electrons emit photons while falling to the ground level or lower energy level without the aid of any external energy is called spontaneous emission

- The lifetime of electrons in the higher excited state is of the order of 10^{-9} s



Spontaneous Emission

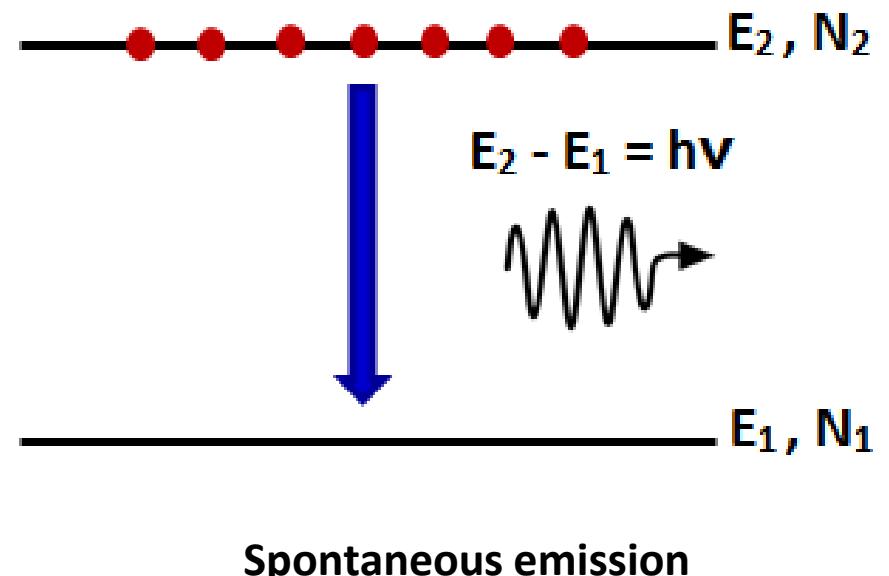
- Photons are emitted in random directions and hence they will not be in phase with each other. Therefore the light emitted is incoherent
- E.g. The light emitted from the Sun, candle flame, other ordinary sources such as tungsten filament, fluorescent tube lights is incoherent



Rate of Spontaneous Emission

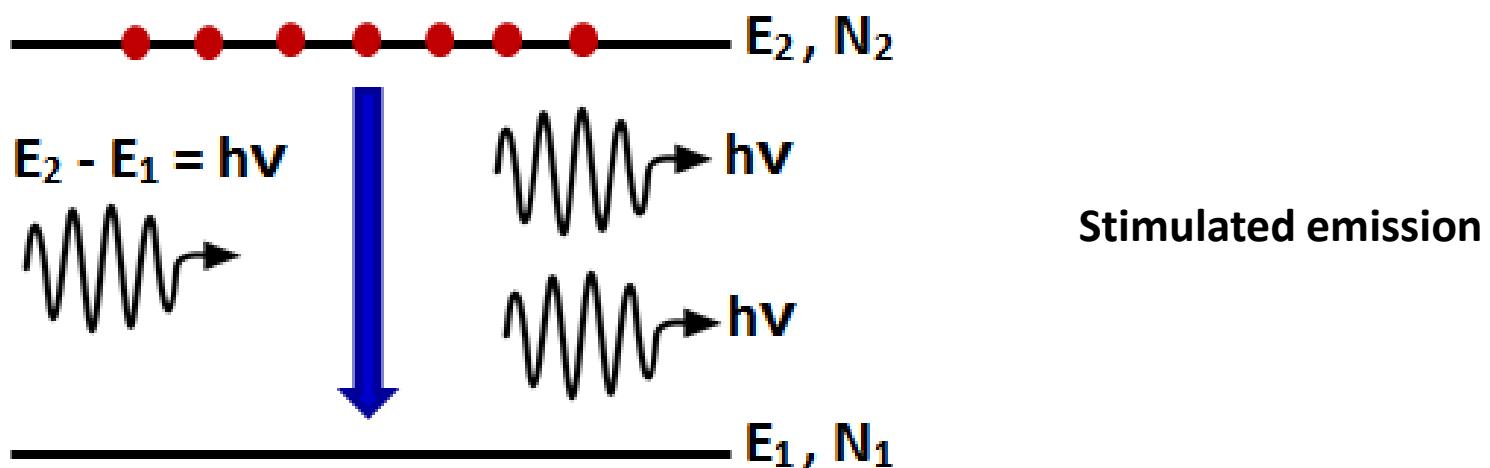
- The rate of spontaneous emission is dependent on the population of atoms in the excited state N_2 only
- The rate of spontaneous emission, $R_{\text{sp em}} = A_{21} * N_2$

Where A_{21} is the Einstein's coefficient for spontaneous emission



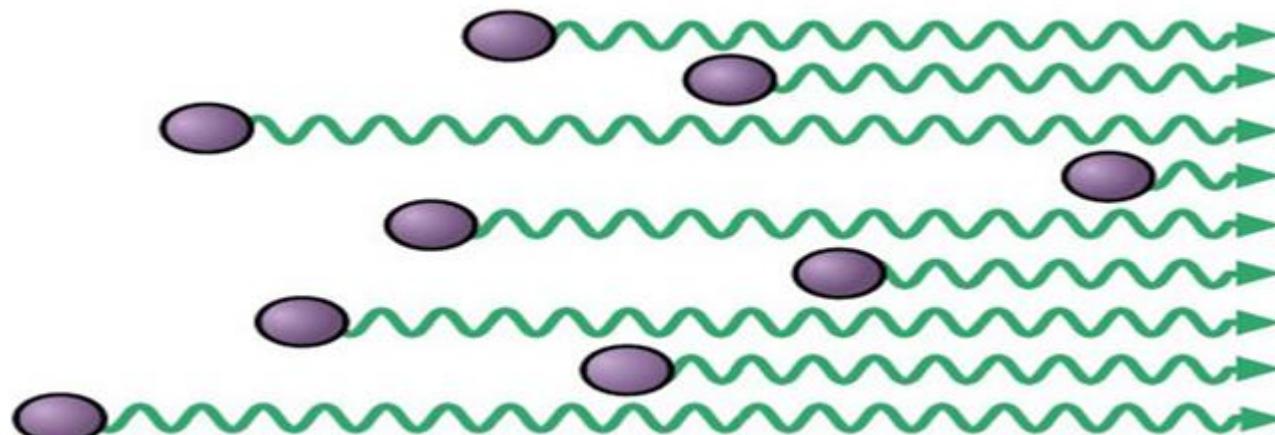
3. Stimulated emission

The process by which electrons in the excited state are stimulated under the influence of an external agency to emit photons while falling to the ground state or lower energy state is called stimulated emission



Stimulated Emission

- In stimulated emission process, each incident photon generates two photons
- The two photons are of same energy, same frequency, same phase, and travel in the same direction. Therefore the emitted light is coherent

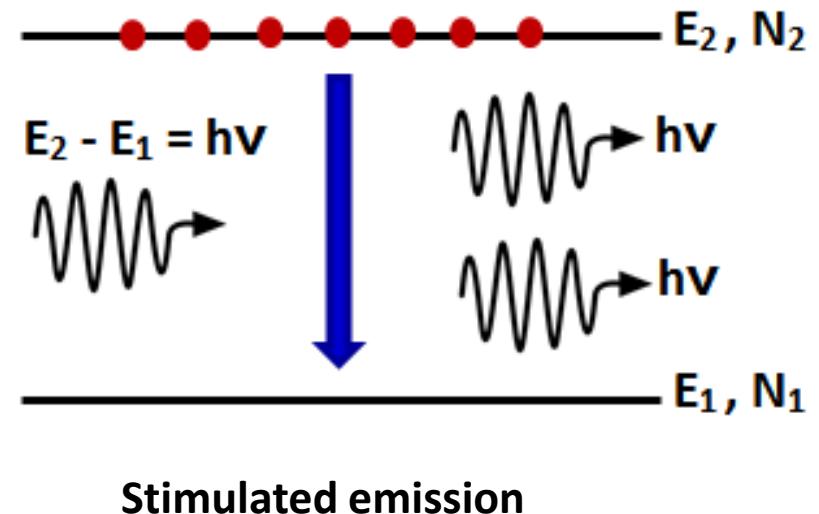


Rate of Stimulate Emission

The rate of stimulated emission is then dependent on the population of atoms in the excited state and the energy density of radiation

- The rate of stimulated emission, $R_{\text{stim}} = B_{21} * N_2 * \rho(v)$

where B_{21} is the Einstein's coefficient for stimulated emission



Interaction of radiation with matter

Quantum transitions

- Induced absorption (stimulated absorption)
- spontaneous emission
- Stimulated emission

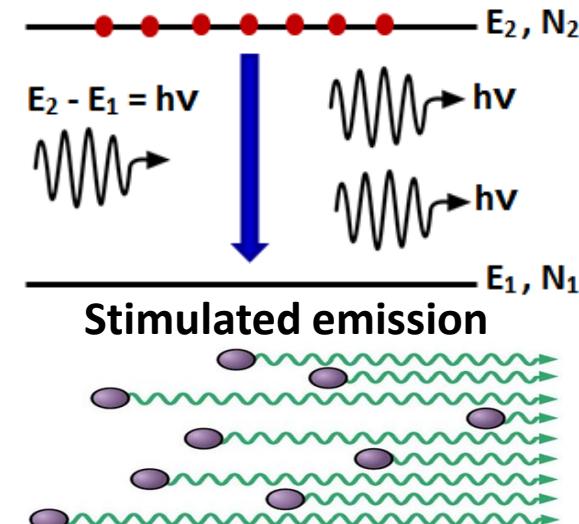
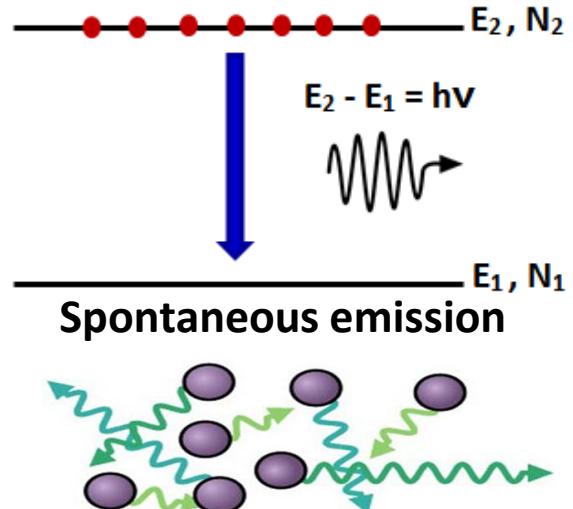
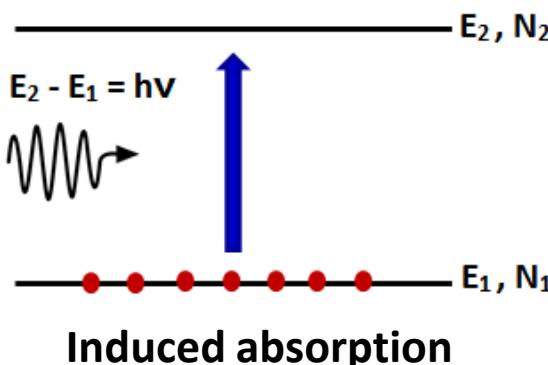
$$R_{\text{ind abs}} = B_{12} * N_1 * \rho(v)$$

$$R_{\text{sp em}} = A_{21} * N_2$$

$$R_{\text{stem}} = B_{21} * N_2 * \rho(v)$$

Population of the states N_1 & N_2

B_{12} A_{21} B_{21} - The Einstein's coefficients



Emitted light is incoherent - Photons are emitted in random directions and not in phase with each other (e.g. light from Sun, candle flame, other ordinary sources)

Emitted light is coherent-Photons are of same energy, same frequency, same phase, and travel in the same direction

The concepts which are correct are....

1. Laser stands for “Light amplification by systematic emission of radiation”
2. The lifetime of ground state is unlimited
3. Absorption process needs inducement energy
4. Spontaneous emission emits coherent light
5. The induced photon in stimulated emission has the same frequency, phase and plane of polarization as that of the stimulating photon

What are the probabilities that can change the photon flux when an em wave incidents on a system?

- (a) Spontaneous emission
- (b) Stimulated emission
- (c) Absorption

The total probability for an atom in state 2 to drop to level 1 is given by

Discuss the processes of interaction of radiation with matter.

Mathematically the stimulated emission is represented as,



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Class #33

- *Rate equations*
- *Expression for energy density of radiation*
- *Life time of electrons in the upper energy state*
- *Rate of stimulated emission to rate of spontaneous emission*

➤ *Suggested Reading*

1. *Lectures on Physics, Feynman, Leighton and Sands*
2. *Lasers - Principles and Applications, A.K.Ghatak and K. Thyagarajan*
3. *Learning material prepared by the Department of Physics*

➤ *Reference Videos*

1. <https://nptel.ac.in/courses/104/104/104104085/>

Rate equations

- The rate of induced absorption = $B_{12} * N_1 * \rho(v)$ -----(1)
- The rate of spontaneous emission = $A_{21} * N_2$ ----- (2)
- The rate of stimulated emission = $B_{21} * N_2 * \rho(v)$ ----- (3)

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission +
Rate of stimulated emission

Expression for energy density of radiation

- At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of
stimulated emission

$$B_{12} * N_1 * \rho(v) = A_{21} * N_2 + B_{21} * N_2 * \rho(v)$$

This gives, $\rho(v)(B_{12} * N_1 - B_{21} * N_2) = A_{21} * N_2$

$$\rho(v) = \frac{A_{21} * N_2}{(B_{12} * N_1 - B_{21} * N_2)} = \frac{A_{21}/B_{21}}{\left(\frac{B_{12} * N_1}{B_{21} * N_2} - 1\right)}$$

Expression for energy density of radiation

- By Maxwell Boltzmann distribution law, we have $\frac{N_2}{N_1} = \exp^{-\frac{hv}{kT}}$
- Substitution of this in the equation for energy density gives the expression for the energy density of radiation as

$$\rho(v) = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} \exp^{\frac{hv}{kT}} - 1\right)}$$

- Comparing this with the Planck's expression for energy density of radiation at any frequency and temperature ,

$$\rho(v) = \frac{8\pi hv^3}{c^3} \frac{1}{\left(\exp^{\frac{hv}{kT}} - 1\right)}$$

- Comparing the two equations term by term, we observe that

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

and $\frac{B_{12}}{B_{21}} = 1$

This implies that, $B_{12} = B_{21} = B$

- At thermal equilibrium the equation for energy density is,

$$\rho(\nu) = \frac{\frac{A}{B}}{\left(\exp^{\frac{h\nu}{kT}} - 1\right)}$$

$$\rho(\nu) = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} \exp^{\frac{h\nu}{kT}} - 1\right)}$$

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(\exp^{\frac{h\nu}{kT}} - 1\right)}$$

For a system to have a predominant stimulated photon emission, then the ratio of the rate of stimulated emission to the rate of spontaneous emission should be greater than 1

$$\frac{B * N_2 * \rho(v)}{A N_2} = \frac{\rho(v)}{\frac{A}{B}} = \frac{1}{\left(\exp^{\frac{hv}{kT}} - 1\right)} \approx \exp^{-\frac{hv}{kT}} = \frac{N_2}{N_1}$$

Generally, $\exp^{-\frac{hv}{kT}} < 1$, i.e., $N_2 < N_1$

So, $R_{\text{sp em}} > R_{\text{st em}}$

W.K.T, The rate of spontaneous emission, $R_{sp\ em} = A_{21} * N_2$

$$N_2 = N_2(0)e^{-A_{21}t}$$

$$A_{21} = \frac{1}{\tau}$$

where τ is the average life time of electrons in the upper energy state for spontaneous emission (of the order of 10^{-8} to 10^{-9} s)

Meta stable states

An atom in the excited state can have a life time in the excited state for longer periods of time of the order of milliseconds (10^{-2} to 10^{-3} s) - referred to as Meta stable states

Such excited atoms de-excite as stimulated emission – possible laser transition

The concepts which are correct are....

1. Rate of absorption process is proportional to population of ground state and life time
2. Rate of spontaneous emission is proportional to population of excited state
3. Life time of electrons in the upper energy state is about 10^2 seconds
4. The ratio of probabilities of spontaneous emission and stimulated emission is proportional to frequency



An emission system has two levels which gives rise to an emission wavelength of 546.1 nm. If the population of the lower state is 4×10^{22} at 600 K, estimate the population of the higher energy state.

Let N_1 and N_2 be the populations of energy states E_1 and E_2

According to the Maxwell Boltzmann distribution $\frac{N_1}{N_2} = \exp^{\frac{(E_2 - E_1)}{kT}}$

$$\text{Hence } N_2 = N_1 \exp^{\frac{-(E_2 - E_1)}{kT}} = N_1 \exp^{-\frac{hc}{\lambda kT}} = 3125$$

The ratio of population between the high energy states to the lower energy state is 5×10^{-19} at 400K. Find the emission wavelength between two states and the ratio A/B.

$$\text{Given } N_2/N_1 = 5 \times 10^{-19} \quad N_1/N_2 = 2 \times 10^{18} = e^{hv/kT}$$

$$\lambda = \frac{hc}{kT \cdot \ln\left(\frac{N_1}{N_2}\right)} = 854.56 \text{ nm}$$

$$\frac{A}{B} = \frac{8\pi h}{\lambda^3} = 2.667 \times 10^{-14}$$



The ratio of population of the upper excited state to the lower energy state in a system at 300K is found to be 1.2×10^{-19} . Find the wavelength of the radiation emitted and the energy density of radiation.

$$\frac{N_1}{N_2} = \exp \frac{(E_2 - E_1)}{kT} = \exp \frac{hc}{\lambda kT}$$

Wavelength $\lambda = 1.06 \times 10^{-6} \text{ m}$

$$\rho(v) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(\exp \frac{hv}{kT} - 1\right)} = 1.987 \times 10^{-33} \text{ Js/m}^3.$$

*An hypothetical atom has energy levels uniformly separated by 1.2 eV. Find the ratio of the no of atoms in the 7th excited state to that in the 5th excited state.
(Ans: 5.22×10^{-41})*

$$\frac{N_1}{N_2} = \exp \frac{(E_2 - E_1)}{kT} = \exp \frac{hc}{\lambda kT}$$

$$\frac{N_7}{N_5} = \exp -\frac{(E_2 - E_1)}{kT} = \exp -\frac{hc}{\lambda kT}$$



Calculate the population of the excited state at a temperature of 350 K, if the ground state has 10^{28} atoms and the transition corresponds to 700 nm radiation.

$$\frac{N_1}{N_2} = \exp \frac{(E_2 - E_1)}{kT} = \exp \frac{hc}{\lambda kT}$$

Wavelength $\lambda = 700 \times 10^{-9} \text{ m}$

$$N_2 = N_1 \exp \frac{-hc}{kT} = 282$$

The wavelength of emission is 600 Å and the life time is 10^{-6} s. Determine the coefficient for stimulated emission.

$$A_{21}/B_{21} = \frac{8\pi h\nu^3}{c^3}$$

$$A_{21} = \frac{1}{\tau}$$

Substitution gives, $B_{21} = 1.29 \times 10^{16}$

Arrive at an expression for the energy density of electromagnetic radiation in terms of Einstein's coefficients.

Show that at thermal equilibrium the ratio of the coefficient of spontaneous emission to the coefficient of stimulated emission is proportional to v^3 .



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➤ *Suggested Reading*

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 9.6*
2. *Learning material prepared by the Department of Physics*
3. *Optical Electronics, A. Yariv*

➤ *Reference Videos*

1. <https://ocw.mit.edu/resources/res-6-005-understanding-lasers-and-fiberoptics-spring-2008/laser-fundamentals-i/1>.

Class #35

- Condition for Laser action
- Population Inversion
- Two, Three and Four level systems
- Requisites of a Laser system

- According to Boltzmann's distribution function,

$$\frac{N_2}{N_1} = \exp^{-\frac{hv}{kT}} \ll 1$$

$$\frac{N_1}{N_2} = \exp^{\frac{(E_2 - E_1)}{kT}} \gg 1$$

- Under normal conditions of thermal equilibrium, $N_1 > N_2$

since $E_2 > E_1 \rightarrow$

Natural distribution of population

- For stimulated emission to dominate, $N_2 > N_1 \rightarrow$

Population Inversion

Population Inversion & Pumping

- For stimulated emission to dominate, population of atoms in the higher energy state must be greater than that in the lower energy state. i.e. $N_2 > N_1$ – ***This state is Population inversion***
- Population inversion can be achieved in matter which contain meta-stable (excited states with lifetime of an atom of the order of 10^{-3} seconds) states
- Whenever the state of population inversion is established, the system will be in non-equilibrium state

Pumping

The act of exciting atoms from lower energy state to a higher energy state by supplying energy from an external source is called **pumping**

Pumping methods

- Optical pumping
- Electric discharge
- Thermal
- Chemical means

In a two level laser system,

To establish population inversion, it requires $N_2 > N_1$

But from the MB distribution function we find that

$$\frac{N_2}{N_1} = \exp^{-\frac{hv}{kT}} < 1$$

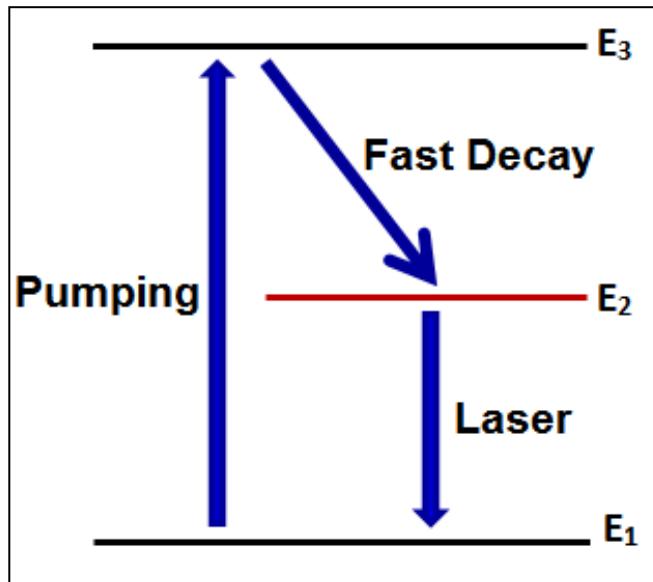
which implies that, $N_2 > N_1$

only if T is negative the ratio will be positive!

*$N_2 > N_1$ is impossible, therefore Laser emission is
not possible in a two level system*

Three Level System

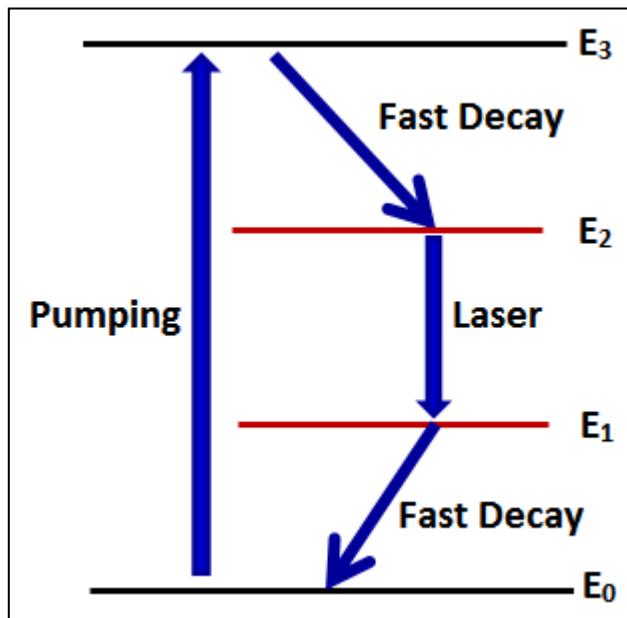
- The introduction of an intermediate level between the ground state and the upper excited state can result in **decoupling** the emission process and absorption process levels



Three Level Laser System

- Intermediate state is a metastable state
- Absorption is between E_1 and E_3
- Emission is between E_2 and E_1
- Laser emission is possible
- Pulsed Laser
- Ex. Ruby Laser

- A four level system can effectively decouple the absorption levels and the emission levels
-

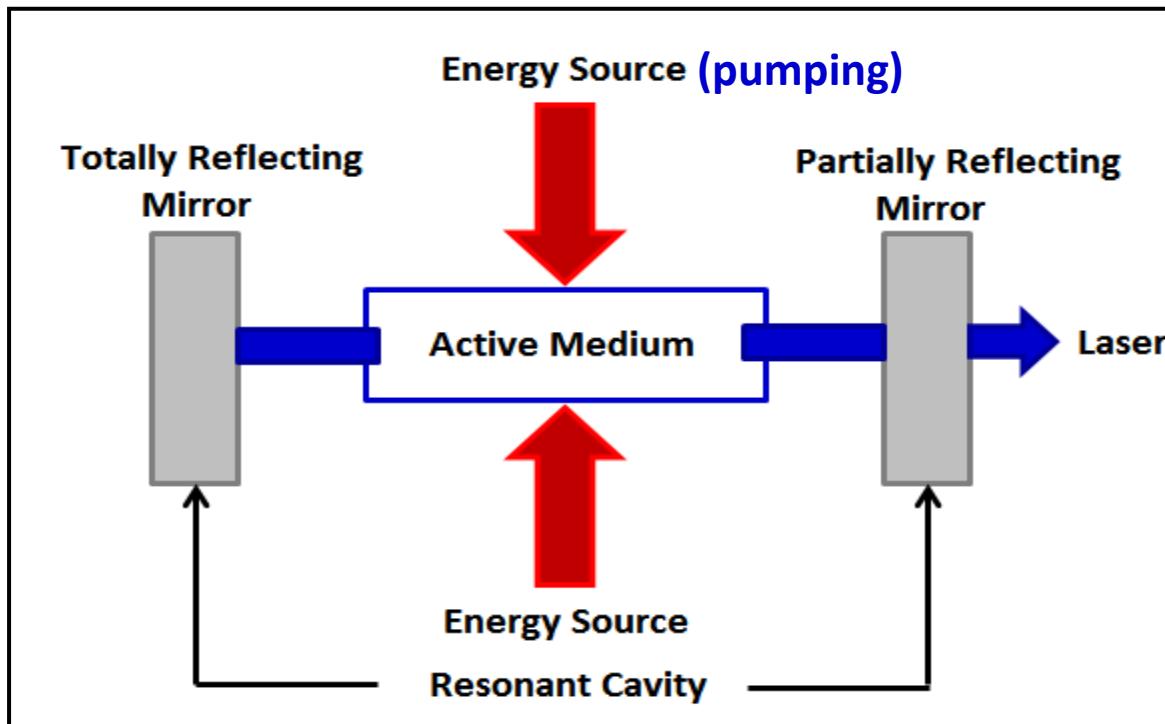


Four Level Laser System

- Absorption is between E_0 and E_3
- Emission is between E_2 and E_1
- Continuous Laser
- Ex. Helium-Neon Laser

Requisites of a Laser system

- **Active medium** → having metastable state which supports Population Inversion
- **Energy pump** → for pumping action, by which population inversion can be achieved
- **Resonating Cavity** → constitutes two parallel mirrors placed around the active medium



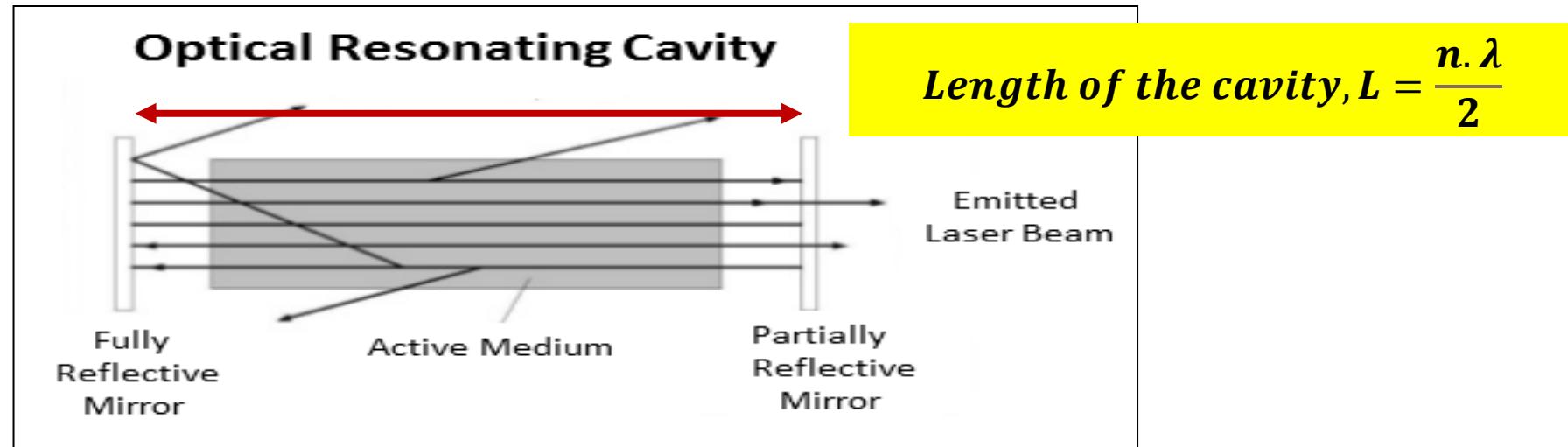
Examples of Active medium.

- GAS – CO₂ or Helium/Neon
- LIQUID – liquid dyes
- SOLID state - Ruby, Nd : YAG
- SEMICONDUCTORS -Gallium Arsenide, Indium Gallium Arsenide, or Gallium Nitride

Block diagram representing a Laser system

Optical Cavity or Optical Resonator

- The system with active medium between two parallel mirrors of high reflectivity is called **laser cavity or optical cavity**
- The mirrors reflect the stimulated emitted photons (travel perpendicular to the mirrors) to and fro leading to amplification - intensity of light builds up along the axis
- Photons that do not travel along the axis escape from the cavity



The concepts which are correct are....

- 1. Population inversion is an artificial situation**
- 2. Two level pumping scheme is feasible for laser action**
- 3. The three level systems produce light only in pulses**
- 4. Four level lasers do not operate in continuous wave mode**
- 5. In the absence of resonator cavity, there would be no amplification of light**

A laser emission from a certain laser has an output power of 10 milli watts. If the wavelength of the emission is 632.8nm, find the rate of emission of the stimulated photons.

Power of Laser $P = \frac{\text{Energy}}{\text{time}} = n \times h\nu = n \times h \times \frac{c}{\lambda}$ where n is the rate of stimulated emission.

$$n = \frac{P \times \lambda}{h \times c} = 3.18 \times 10^{16} \text{ per second}$$

A pulsed laser has a power of 1mW and lasts for 10 ns. If the no. of photons emitted per second is 3.491×10^7 , calculate the wavelength of the photons.

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{n \cdot h\nu}{t} = \frac{n \cdot hc}{\lambda \cdot t}$$

$$\text{Wavelength, } \lambda = \frac{n \cdot h \cdot c}{P \cdot t} = 693 \text{ nm}$$

If R_1 is the rate of stimulated emission and R_2 is the rate of spontaneous emission between two energy levels, show that
 $\lambda = hc / [kT \ln\{(R_2/R_1)+1\}]$.

$$\frac{Sp.\ Em.}{St.\ Em.} = \frac{R_2}{R_1} = e^{\frac{h\nu}{KT}} - 1$$

$$\frac{R_2}{R_1} + 1 = e^{\frac{h\nu}{KT}}$$

$$\ln\left\{\frac{R_2}{R_1} + 1\right\} = \frac{hc}{\lambda KT}$$

If $B_{10} = 2.7 \times 10^{19} m^3/W-s^3$ for a particular atom, find the life time of the 1 to 0 transition at 550nm

$$B_{10} = B_{01}$$

$$\frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$$

$$Life\ time, \tau = \frac{1}{A} = 370\ ns$$

What does Boltzmann equation signify? What is population inversion? How is it achieved?

Bring out the differences between three level and four level lasers. Why is a two level system not support laser action?

Elaborate on the requirements of a laser system.



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- Class #36

Gain and loss in laser systems

1. Gain in a cavity
2. Laser Comb
3. Line Broadening
4. Losses in the cavity

➤ *Suggested Reading*

1. Lasers: Fudamentals and Applications

K Thyagarajan, A Ghatak

2. Course material developed by the Department

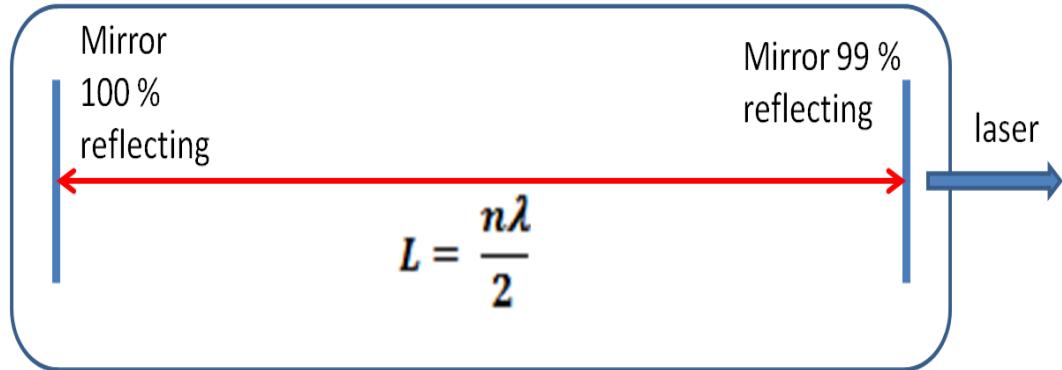
➤ *Reference Videos*

<https://ocw.mit.edu/resources/res-6-005-understanding-lasers-and-fiberoptics-spring-2008/laser-fundamentals-i/>

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Resonating Cavity

- **Consists of two mirrors of various geometries and coatings creating standing waves**



- **Because of the energy amplification due to stimulated emission**
- **The laser comes out of the partially reflecting mirror**
- **Photons travelling in directions not perpendicular to the mirrors are not amplified**

Resonating Cavity: Frequency Comb

$$n_1\lambda_1 = 2L$$

$$n_2\lambda_2 = 2L$$

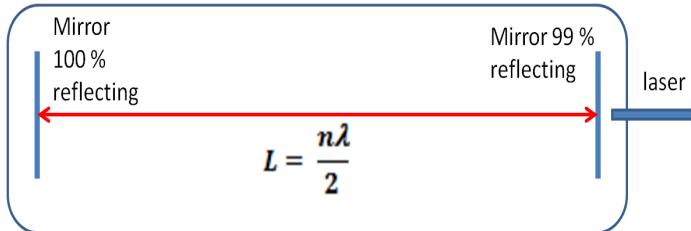
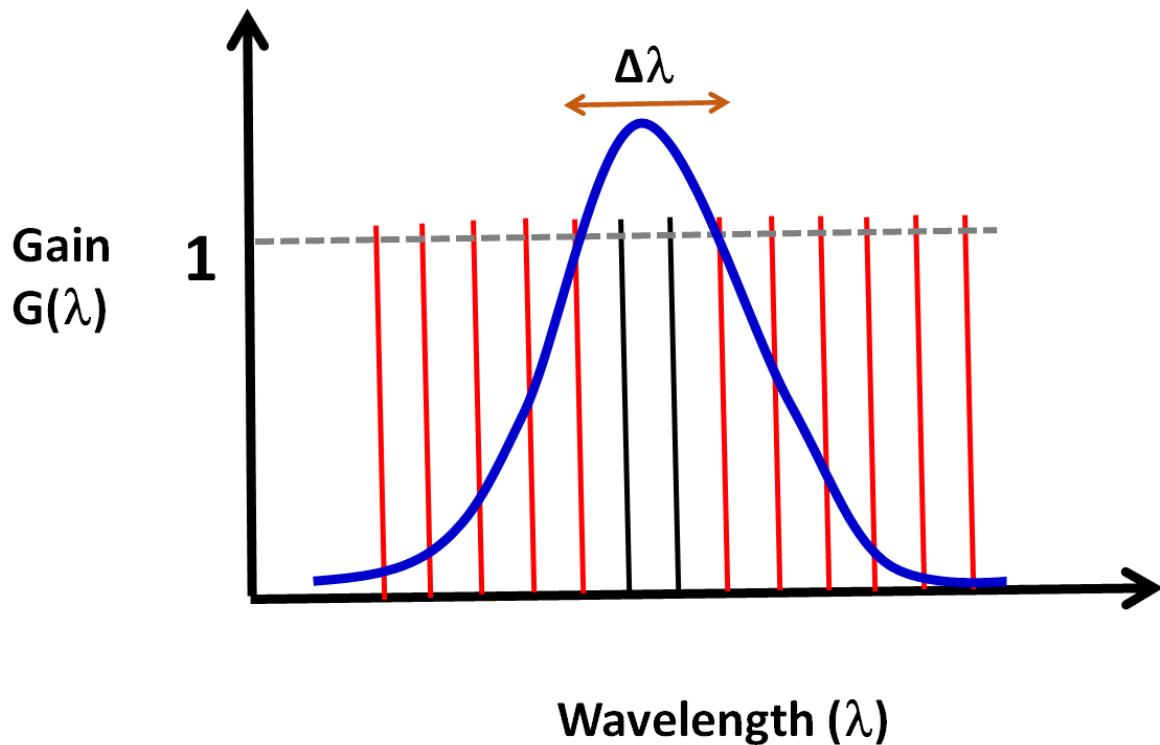
$$n_3\lambda_3 = 2L$$

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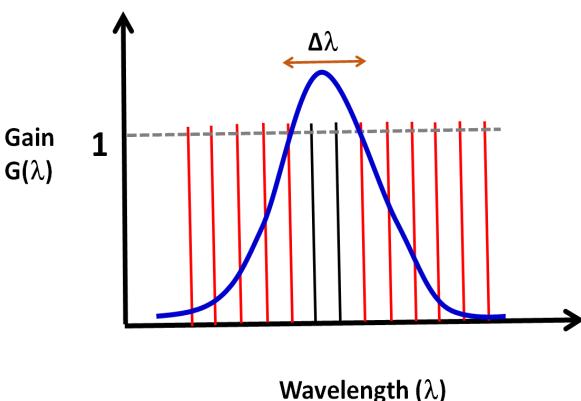
Gain is a function of wavelength



Frequency Comb : consists of a series of discrete equally spaced frequency lines corresponding to $n_1, n_2, n_3\dots$ etc. with respect to the laser cavity length.

The line width is narrow due to the fact that only those wavelengths for which the gain is above threshold will be amplified for output laser beam.

Resonating Cavity: Frequency Comb



Modes in a Laser

mode refers to a specific standing wave pattern that can exist in the **optical cavity** of the laser. These modes are defined by the conditions that light must meet to stay **in phase with itself** as it bounces back and forth inside the laser cavity.

Mode spacing: The frequency gap between adjacent longitudinal modes is: $\Delta f = c/2L$ (Cavity length, $L = \frac{n\lambda}{2}$). So, in a laser with a longer cavity, modes are more closely spaced.

A **frequency comb** is usually generated using an **ultrafast mode-locked laser**. This laser emits very short pulses of light (order of femtoseconds = 10^{-15} seconds). As time and frequency are related (via Fourier transform), these short pulses in the time domain translate to a broad set of evenly spaced frequencies in the frequency domain.

Losses in the cavity

1. Scattering(greater at shorter wavelengths)
2. Absorption in the beam path
3. Diffraction losses
4. Mirror losses



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Class #37

- Round trip gain in a laser medium
- Condition for round trip gain
- Amplified coherent light

➤ Suggested Reading

1. *Lectures on Physics, Feynman, Leighton and Sands*
2. *Lasers - Principles and Applications, A.K.Ghatak and K. Thyagarajan*
3. *Learning material prepared by the Department of Physics*

➤ Reference Videos

1. <https://nptel.ac.in/courses/104/104/104104085/>

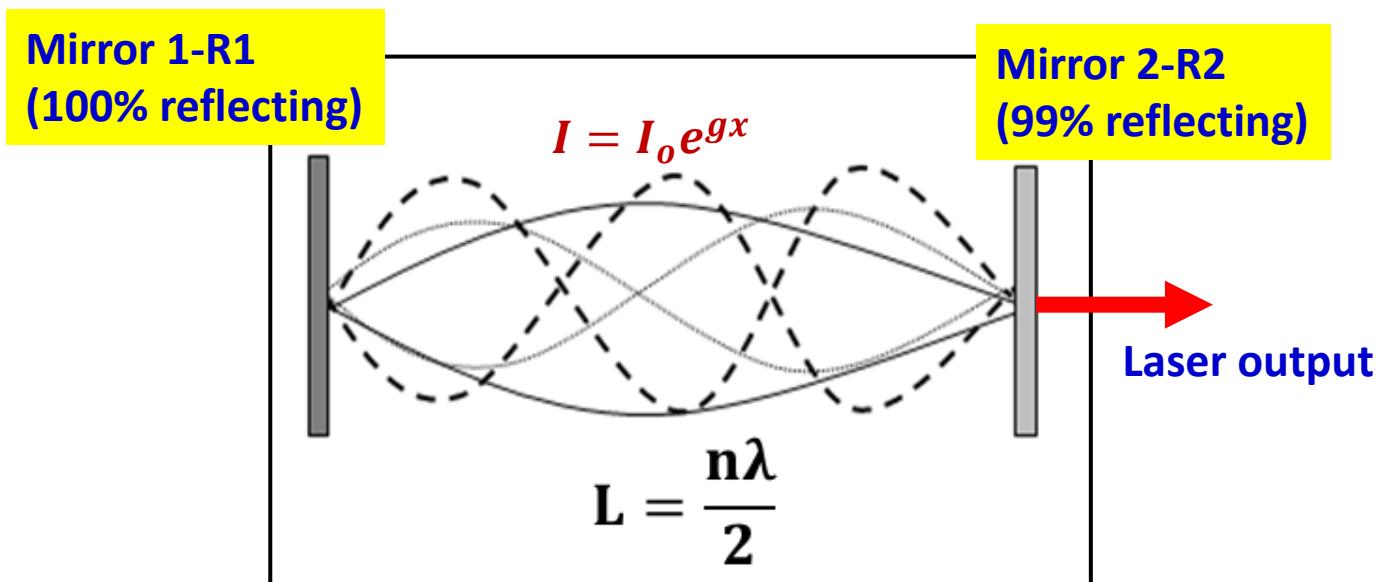
Resonating cavity (Optical cavity for light amplification - gain)



Resonator supports simultaneously several standing waves

$$\lambda = \frac{2L}{n}$$

The resonating cavity arrangement results in multiple travel of the stimulated emitted optical beam in the medium and beam intensity increases (gain) after few reflections



The gain of photons is given by the increasing intensity as $I = I_0 e^{gx}$

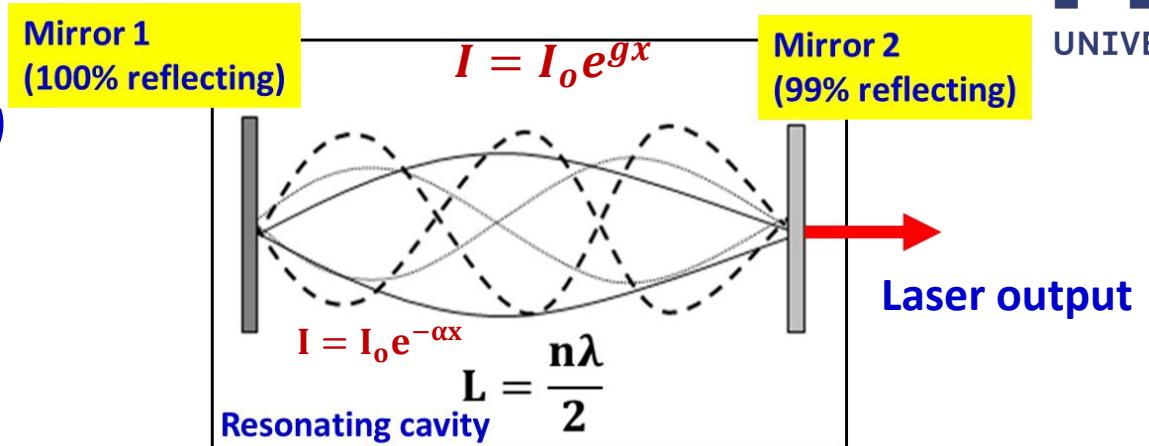
where g is the gain coefficient (m^{-1})

Light Amplification inside the resonant cavity

Resonating cavity: Loss Process and Round trip gain

Losses in the cavity

1. Scattering(greater at shorter wavelengths)
2. Absorption in the beam path
3. Diffraction losses
4. Mirror losses



The loss of photons is given by decreasing intensity as

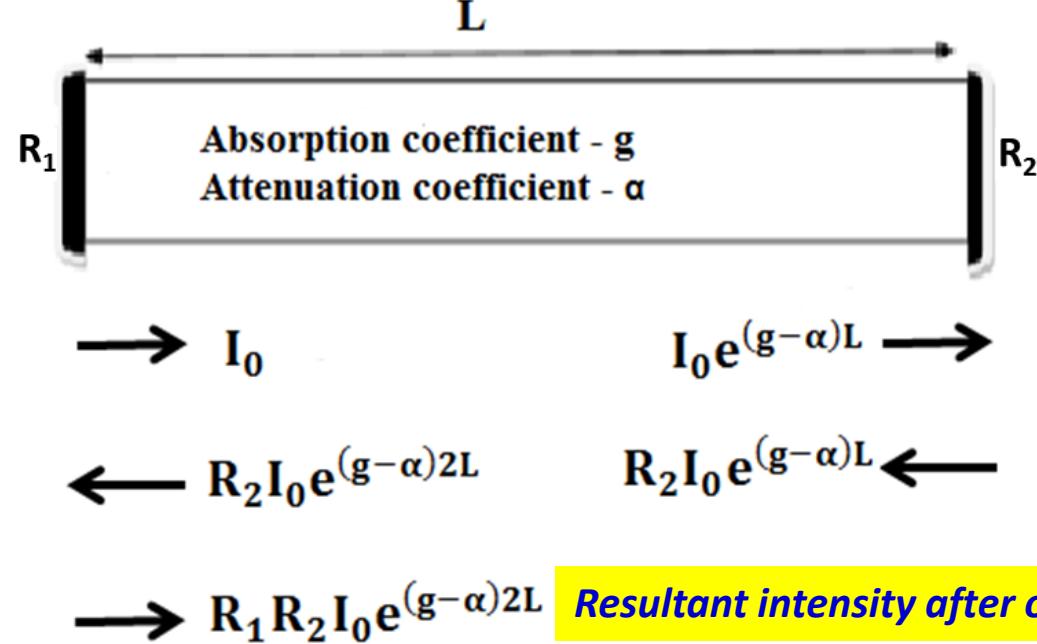
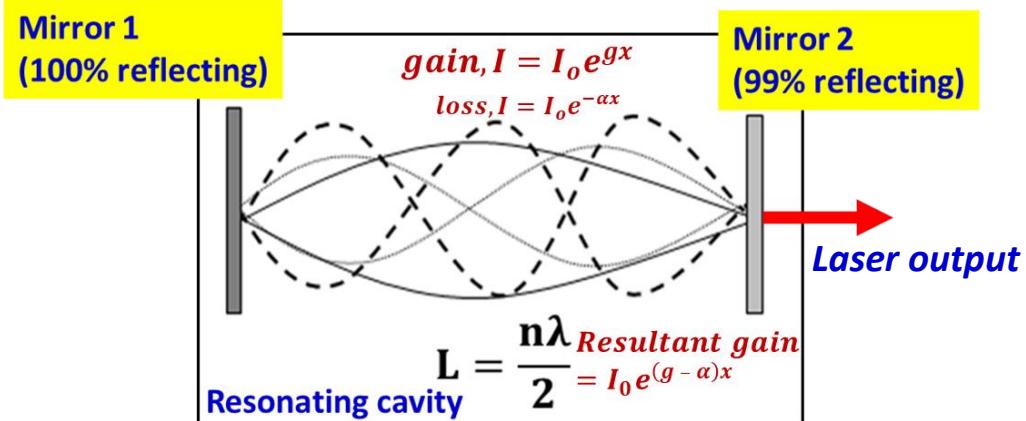
$$I = I_0 e^{-\alpha x}, \text{ where } \alpha \text{ is the loss coefficient (per meter)}$$

Round trip gain

If gain and losses are co-existing, then the resultant intensity for a distance x should be

$$I = I_0 R_1 R_2 e^{2(g - \alpha)x}, \text{ where } I_0 \text{ is the starting intensity of photons}$$

Round trip gain in a laser medium



$$\text{Amplification factor(gain)} = \frac{\text{Final intensity}}{\text{Initial intensity}}$$

$$\text{Amplification factor(gain)} = \frac{I_0 R_1 R_2 e^{(g - \alpha)2L}}{I_0}$$

In a round trip even if we achieve a gain compared to the loss over billions of such trips the total gain would be significant

R₁ and R₂ be the reflective coefficients of the mirrors
 L - the distance between the mirrors

Threshold condition, $R_1 R_2 e^{(g_{th} - \alpha)2L} = 1$

Threshold gain coefficient g_{th}



Threshold condition, $R_1 R_2 e^{(g_{th} - \alpha)2L} = 1$

Threshold Gain Coefficient (g_{th}) can be evaluated as,

$$e^{(g_{th} - \alpha)2L} = \frac{1}{R_1 R_2}$$

$$(g_{th} - \alpha)2L = \ln\left(\frac{1}{R_1 R_2}\right)$$

$$(g_{th} - \alpha) = \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

This implies that the gain of the system is dependent on the length of the cavity and the reflection coefficients of the two mirrors



$$\text{Threshold gain coefficient, } (g_{th}) = \alpha + \left\{ \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) \right\}$$

In another form as,

$$\text{Threshold gain coefficient, } (g_{th}) = \frac{1}{2L} (2\alpha L - [\ln R_1 R_2])$$

For amplification, $R_1 R_2 e^{(g - \alpha)2L} > 1$

The concepts which are correct are....

1. Attenuation coefficient, α includes all the distributed losses such as scattering and absorption occurring in the medium
2. For lasing, the initial gain must be less than the sum of the losses in the cavity
3. The threshold gain can be determined by considering the change in intensity of a beam of light undergoing a round trip within the resonator
4. The value of g must be less than g_{th} for laser oscillations to commence

Calculate the threshold gain factors of a laser which has a loss factor of 0.5 cm^{-1} if the configuration of the system is as follows

- (a) *A 50 cm tube with one mirror 99% reflecting and the output coupler 90% reflecting*
- (b) *A 20 cm tube with one mirror 99% reflecting and the output coupler 97% reflecting*

Comment on the results obtained.

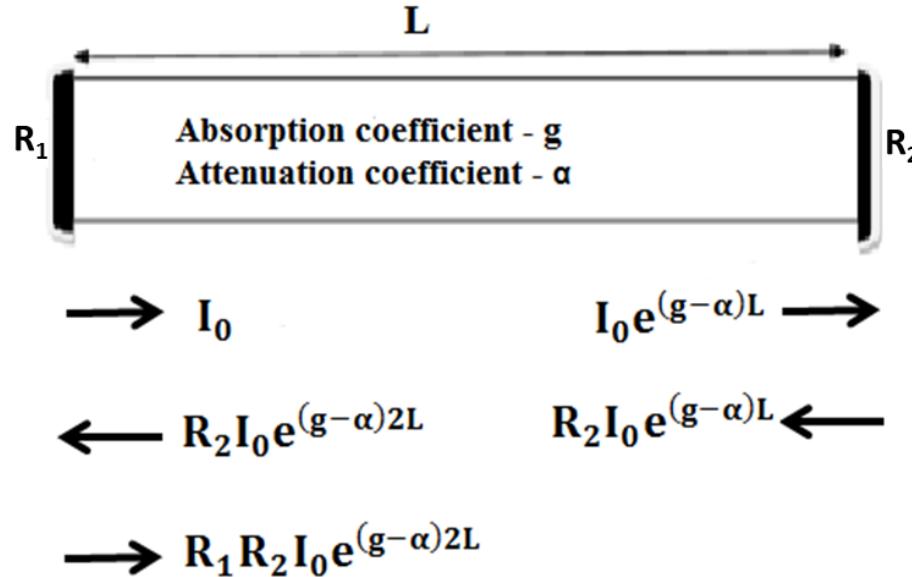
$$\text{Threshold gain coefficient, } (g_{th}) = \alpha + \left\{ \frac{1}{2L} \ln \left(\frac{1}{R_1 R_2} \right) \right\}$$

$$\text{Case1: } \ln \left(\frac{1}{R_1 R_2} \right) = .$$

More the threshold gain better the design of the optical cavity.

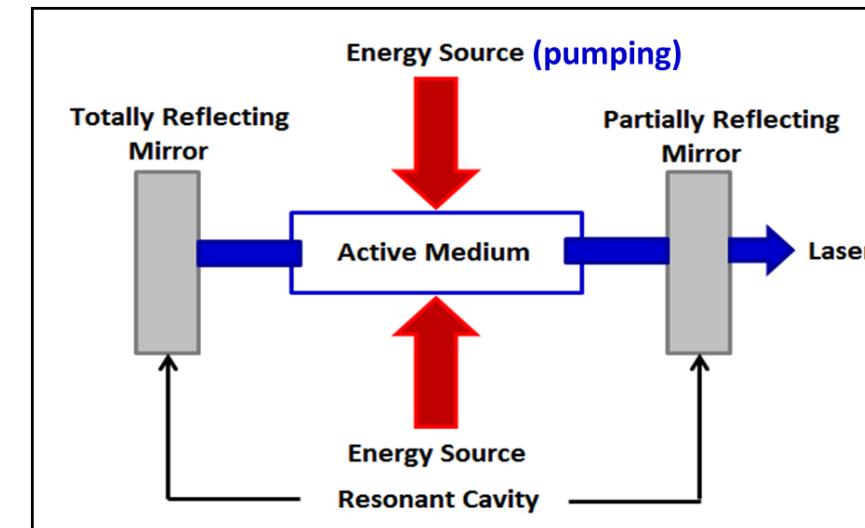


Discuss the concept of round trip gain and derive an expression for threshold gain.



$$\text{Threshold gain coefficient, } (g_{th}) = \alpha + \left\{ \frac{1}{2L} \ln \left(\frac{1}{R_1 R_2} \right) \right\}$$

Obtain an expression for threshold round trip gain in laser cavity and list the important requirements of a laser system.





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ENGINEERING PHYSICS

Department of Science and Humanities

➤ *Suggested Reading*

1. *Lectures on Physics, Feynman, Leighton and Sands*
2. *Lasers - Principles and Applications, A.K.Ghatak and K. Thyagarajan*
3. *Learning material prepared by the Department of Physics*

➤ *Reference Videos*

1. <https://nptel.ac.in/courses/104/104/104104085/>

Class #34

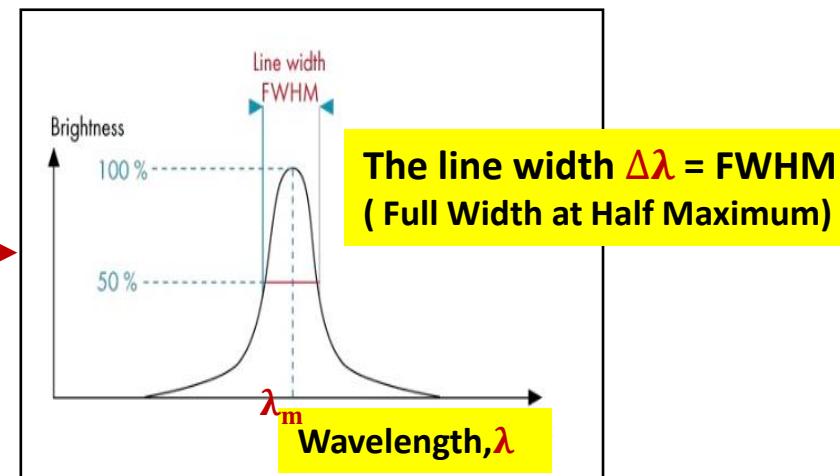
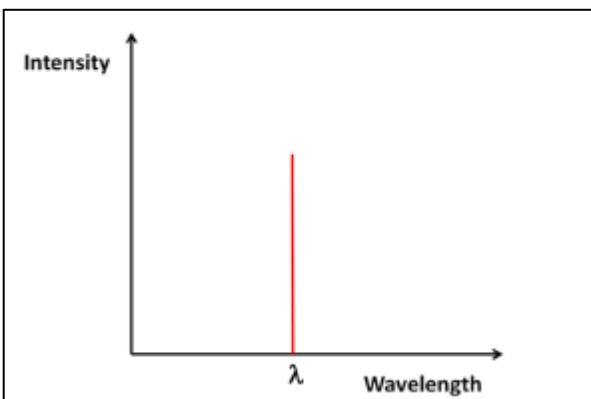
- **Monochromaticity**
- **Coherence**
- **Directionality**
- **High Intensity**

1. Monochromaticity (spectral line broadening)

Light from a laser typically comes from an atomic transition with a single precise wavelength. So the laser light expected to have single spectral colour

Laser spectral emission line have a finite width

However, the laser light is highly monochromatic but not truly monochromatic!



From the Doppler Effect of the moving atoms or molecules with diff. speeds at st. emission Line widths are also limited by the uncertainty principle - limits the accuracy of the energy (ΔE) of the photons emitted by electrons which spend times with a spread in time (Δt)

$$\Delta E = \Delta h\nu = h\Delta \left(\frac{c}{\lambda} \right) = \left| hc \cdot \frac{\Delta \lambda}{\lambda^2} \right|$$

$$\text{In terms of frequency, line width } \Delta\nu = \frac{c \cdot \Delta\lambda}{\lambda_m^2}$$

Generally LASER line widths are very small of the order of 10^{-6} Å as compared to 1 Å for ordinary monochromatic sources

2. Coherence

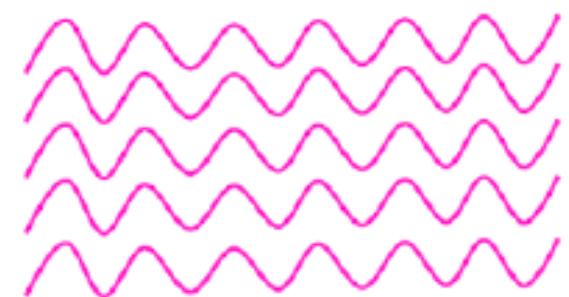
Temporal coherence & Spatial coherence

Inter-relationship of phases of a laser beam is **Coherence**

In the stimulated emission process, the emitted photons are "in phase" or have a definite phase relation to each other

i. **Temporal coherence:** Characteristic of a single beam

- If the electric fields of a laser are sampled at different times and if the samples exhibit a well defined phase correlation, then the laser is said to be temporally coherent
- The interference of a wave with a time delayed copy of itself will reveal the nature of temporal coherence
- Coherence time, $\tau_c = \frac{1}{\Delta\nu}$, where $\Delta\nu$ is the spread in the frequency (line width)
- Coherence length, $l_c = \tau_c \cdot c$, where c is the velocity of light

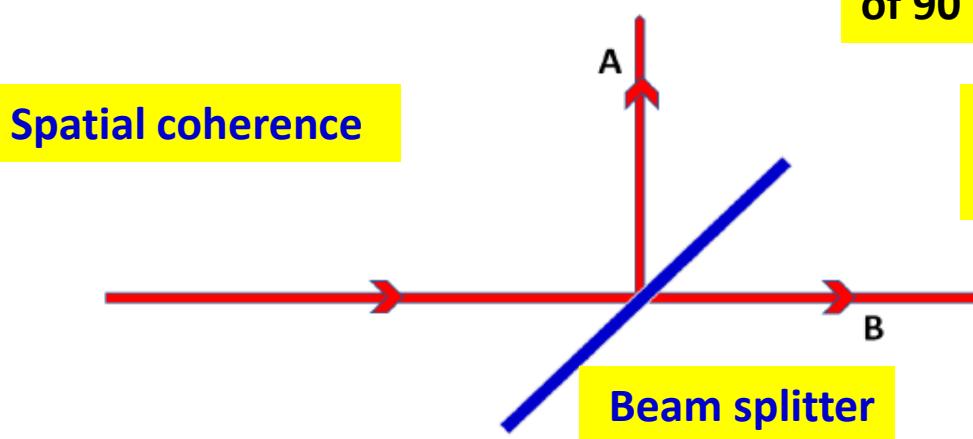


The time duration up to which a laser emission maintains its temporal coherence

The distance up to which a beam exhibits temporal coherence

ii. Spatial coherence: Relationship between two separate beams of light

- When electromagnetic fields at different spatial regions have a phase correlation (either zero or a constant phase difference), the beam is said to be spatially coherent



The E-fields at point A and B have a phase difference of 90° which remains constant over time

If the beams are made to interfere the resulting amplitude or intensity would remain unchanged in time

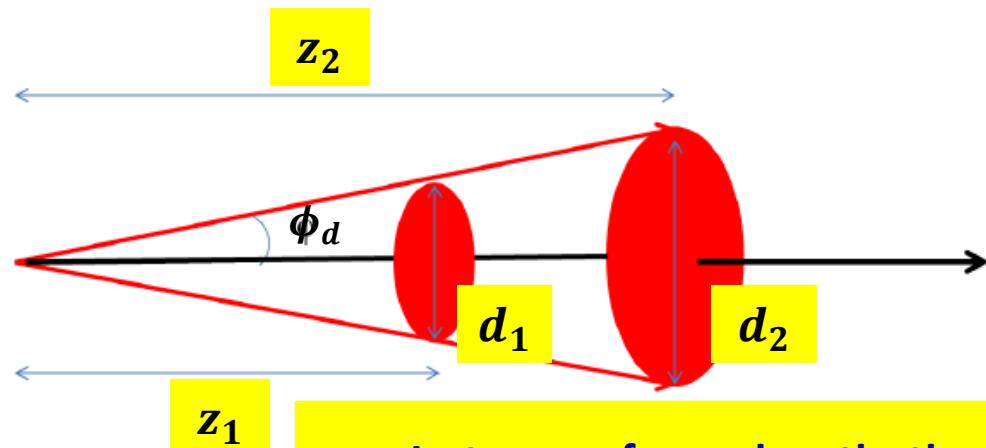
spatial coherence is described by the coherence width $l_w \approx \frac{\lambda}{\theta}$

3. Divergence (directionality): very low divergence

- Typically the divergence is of the order of mill radians (0.001°)

Divergence of a LASER beam is given by

$$\phi_d = \frac{d_2 - d_1}{z_2 - z_1}$$



In terms of wavelength, the divergence of a LASER beam is given

by $\theta = \frac{\lambda_0}{\pi\omega_0}$ where λ_0 is the wavelength, and ω_0 is the spot size.

4. Intensity

- The high intensity of a Laser arises out of the properties of Monochromaticity, Coherence and Low divergence

A lot of energy is concentrated in a small cross sectional area

High power IR lasers are used to cut metals

Applications in non linear optics: femto and atto second lasers, revolutionizing electronics and computer science

Light source	Light Power	Power density
Sun	10^{26} Watt	5×10^2 W/cm ²
100 W Filament-lamp	3 Watt	10^{-2} W/cm ²
He-Ne- Laser	1 mWatt	4×10^4 W/cm ²
CO ₂ Laser	60 Watt	5×10^6 W/cm ²
Pulsed Laser	1 GWatt	10^{14} W/cm ²

The concepts which are correct are....

1. A laser is coherent source because it contains uncoordinated waves of a particular wavelength
2. Laser beam can be used in interference because it is highly coherent
3. The directionality of a laser beam is measured by the divergence angle of the beam with the distance from the source

For an ordinary source, the coherence time $\tau_c = 10^{-10}$ second. Obtain the degree of non-monochromaticity for wavelength $\lambda_0 = 5400 \text{ \AA}$.

Given, $\tau_c = 10^{-10} \text{ s}$, $\lambda_0 = 5400 \text{ \AA}^0 = 5400 \times 10^{-10} \text{ m}$, To find $\Delta\lambda$

Coherence time, $\tau_c = \frac{1}{\Delta\nu}$ and $\Delta\nu = \frac{c \cdot \Delta\lambda}{\lambda^2}$

$$\Delta\lambda = \frac{\Delta\nu\lambda^2}{c} = 9.7 \times 10^{-12} \text{ m}$$

Calculate the coherence length of a laser beam for which the band width $\Delta\nu = 3000 \text{ Hz}$.

Given, $\Delta\nu = 3000 \text{ Hz}$, to find l_c

coherence length, $l_c = c \cdot \tau_c$

thus, $l_c = 100 \text{ km}$

Coherence time, $\tau_c = \frac{1}{\text{band width}} = \frac{1}{\Delta\nu} = 3.33 \times 10^{-4} \text{ s}$

The lifetime of transitions in a Na atoms emitting wavelength of 589.6nm is estimated to be 16.4ns. Calculate the Einstein's coefficients A and B. Calculate spectral broadening and the coherence length of radiations.

**Given, life time (relaxation time), τ or $\Delta t = 16.4 \text{ ns}$
and wavelength, $\lambda = 589.6 \text{ nm}$**

$$\text{Life time, } \tau = \Delta t = \frac{1}{A} = \frac{\lambda^3}{B \cdot 8\pi h}$$

$$\text{Einstein Coefficient, } A = \frac{1}{\tau} = 6.1 \times 10^7 \frac{m^3}{Ws^3}$$

$$\Delta\lambda = \frac{\lambda^2}{4\pi c \cdot \tau} = 5.6 \times 10^{-15} m$$

$$\Delta\nu = \frac{c \cdot \Delta\lambda}{\lambda^2} = 4.8 \times 10^6$$

$$B = \frac{\lambda^3}{\tau \cdot 8\pi h} = 7.5 \times 10^{20} \frac{m^3}{Ws^3}$$

$$\text{Coherence time, } \tau_c = \frac{1}{\Delta\nu} = 2.06 \times 10^{-7} s$$

$$\text{Coherence length, } l_c = c \cdot \tau_c = 62 m$$



Discuss the temporal and spatial coherence of a laser. Why is coherence an important property of a laser?

*What do you mean by line width and coherence length of a laser?
How are they related?*

Why a laser system is highly monochromatic but not truly monochromatic?



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ENGINEERING PHYSICS

Department of Science and Humanities

➤ *Suggested Reading*

1. *Optical Electronics, A. Yariv*
2. *Lasers - Principles and Applications, A.K.Ghatak and K. Thyagarajan*
3. *Learning material prepared by the Department of Physics*

➤ *Reference Videos*

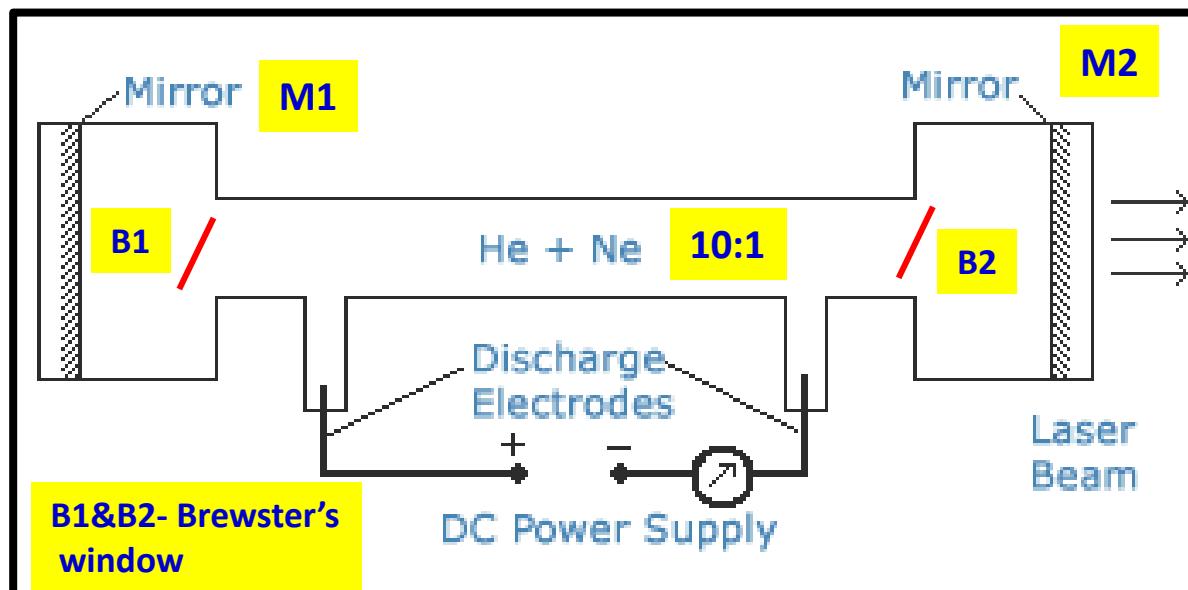
1. <https://nptel.ac.in/courses/104/104/104104085/>
2. <https://ocw.mit.edu/resources/res-6-005-understanding-lasers-and-fiberoptics-spring-2008/laser-fundamentals-i/>

Class #39

- **Atomic Laser: Helium-Neon Laser**
- **Gas Laser**
- **Construction**
- **Working -Energy level diagram**

Construction

- **Active medium:** He-Ne gas mixture in the ratio of 10:1
- **Energy pump:** Electrical discharge by a high voltage DC source or a RF source
- **Resonant cavity:** The cavity is evacuated glass/quartz tube of appropriate lengths and narrow with reflecting mirrors on both ends



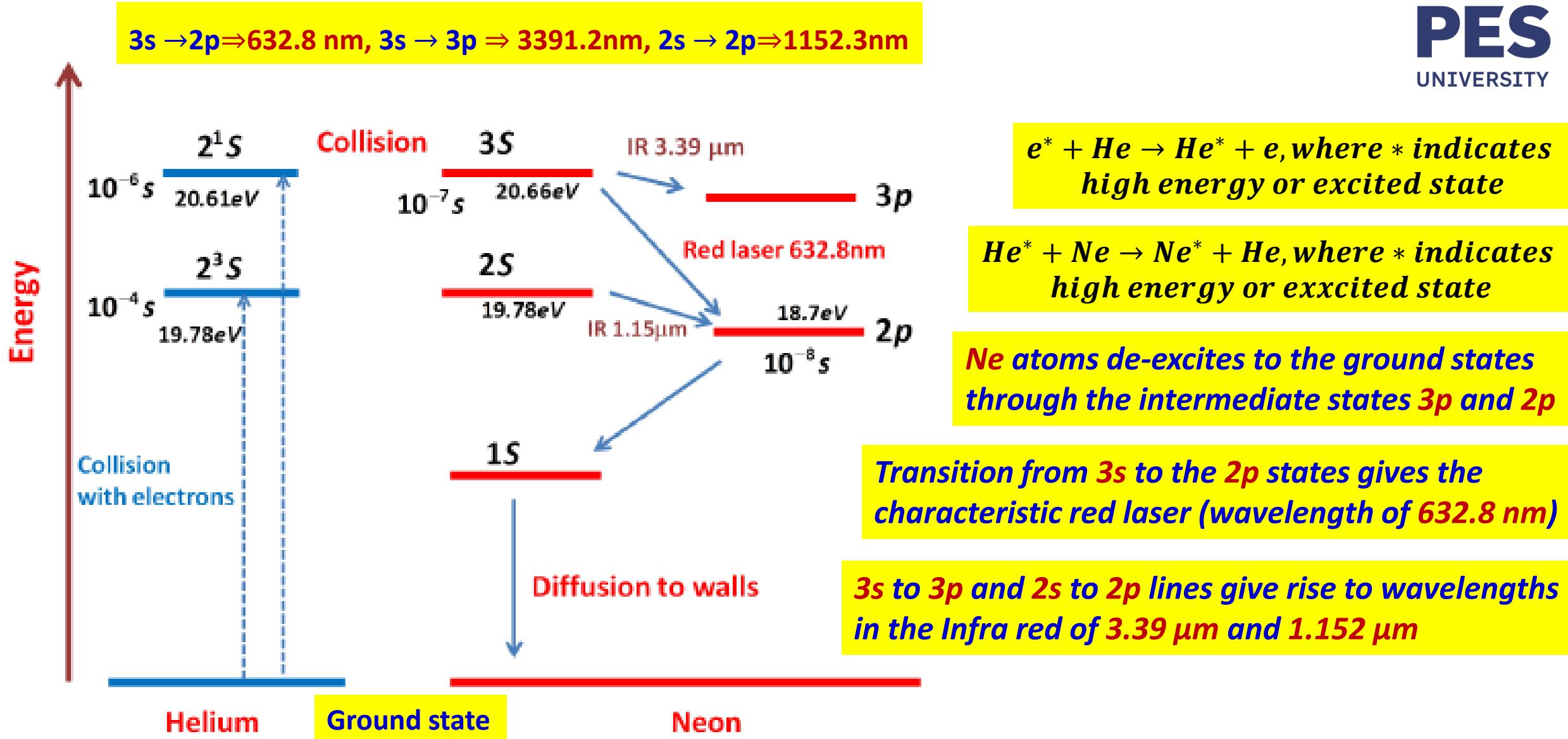
First gas laser

Continuous Four level laser

632.8 nm laser (visible)

Power ~ a few mW

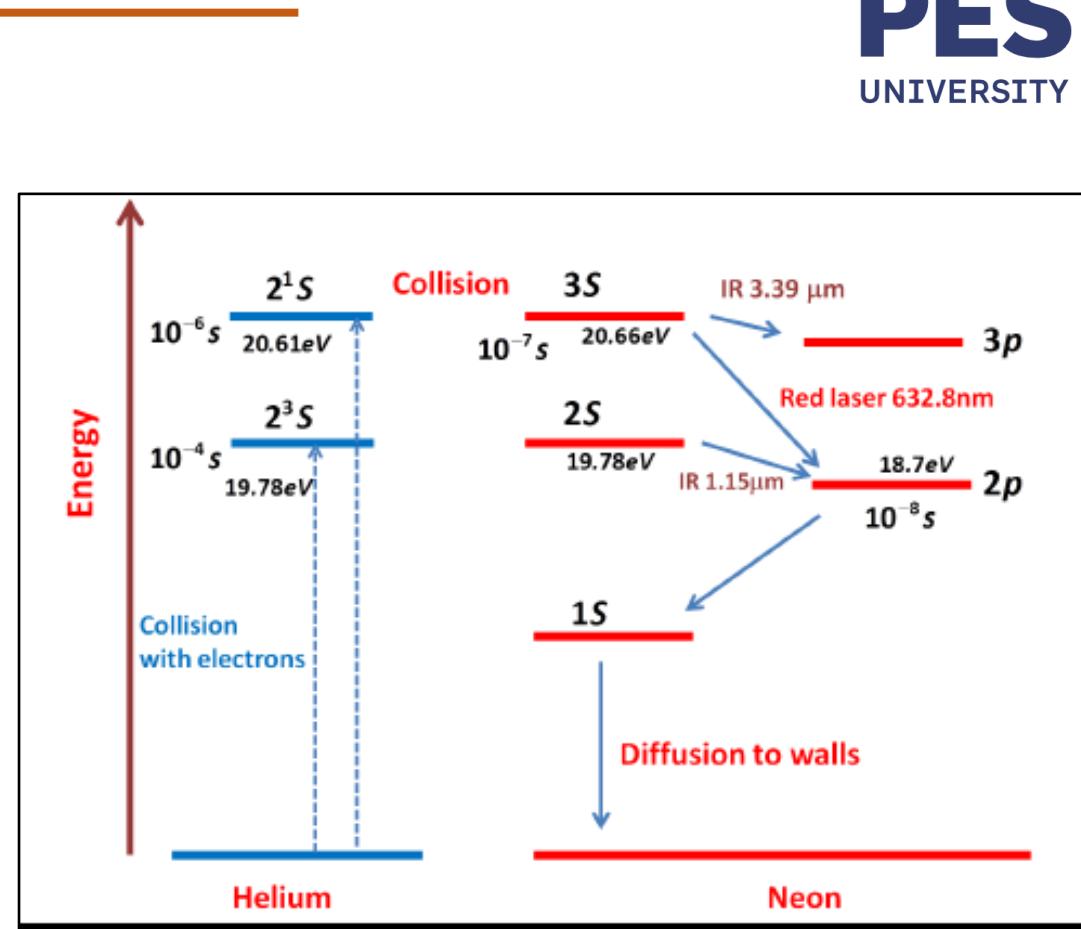
Helium-Neon Laser



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Helium-Neon Laser- Salient features

- **Uses four-level pumping scheme**
- **The active centres (lasing levels) are Ne atoms**
- **Absorption levels are in the He atoms**
- **Electric discharge is the pumping technique**
- **Resonant transfer of energy from He to Ne atoms upon collision: Population Inversion**
- **Lasing operates in continuous wave mode**
- **Low efficiency and low power output**
- **Tube diameter is narrow to ensure depopulation from 1s metastable state**



Discuss with appropriate energy level diagram how the He Ne system works as a continuous wave laser. Suggest an approximate distance between the mirrors for a characteristic neon red emission for practical He Ne laser.

With a neat energy level diagram, discuss how lasing action is achieved in atomic laser

Class #40

- **Molecular Laser: Carbon dioxide Laser**
- **CO_2 Molecule: Different modes of vibration**
- **Construction**
- **Working: Energy level diagram**

Molecular Laser-Carbon dioxide Laser : Modes of vibration

- CO_2 molecule - a linear symmetric molecule with carbon atom at the center and oxygen atoms on each side

CO_2 molecule has three modes of vibration:

1. Symmetric stretching
2. Asymmetric stretching
3. Bending mode

Quantized energies of the symmetric stretching are denoted as $(n00)$

Quantized energies of the asymmetric stretching are denoted as $(00n)$

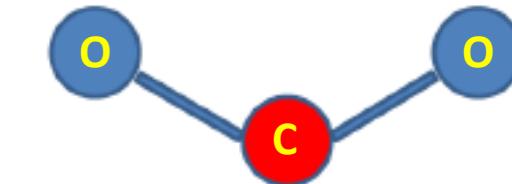
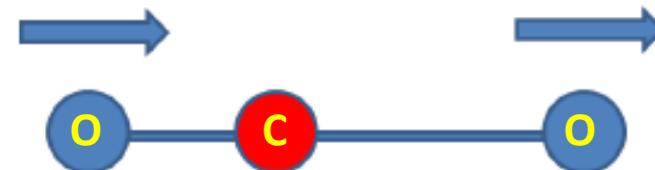
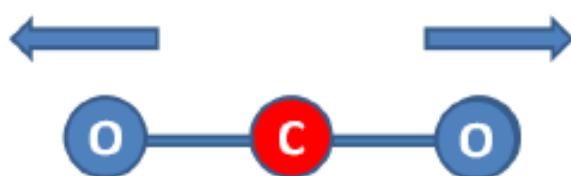
Quantized energies of the bending mode are denoted as $(0n0)$

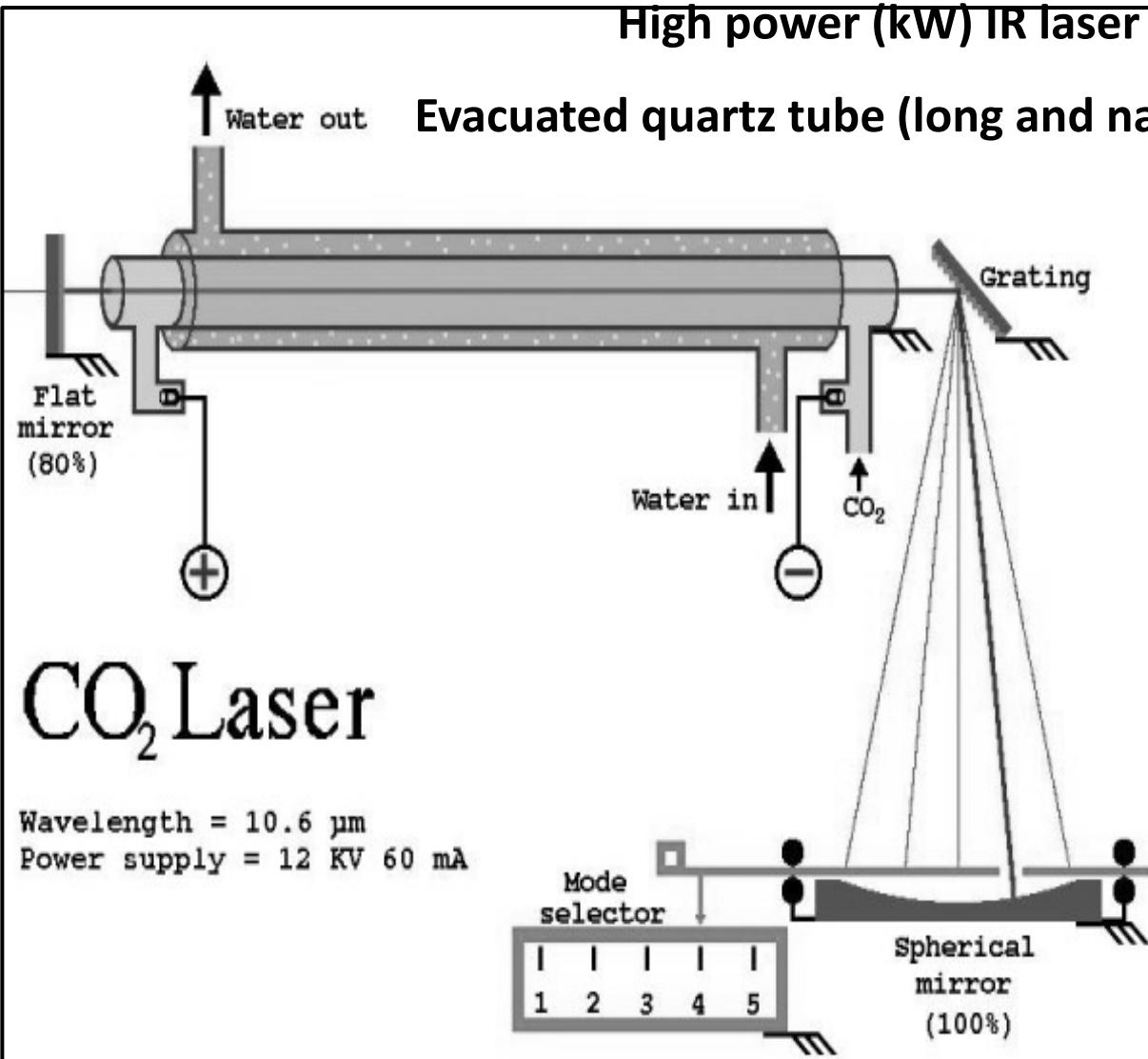
where n is a positive integer

1. Symmetric stretching:
Bond lengths always equal

2. Asymmetric stretching:
Bond lengths unequal

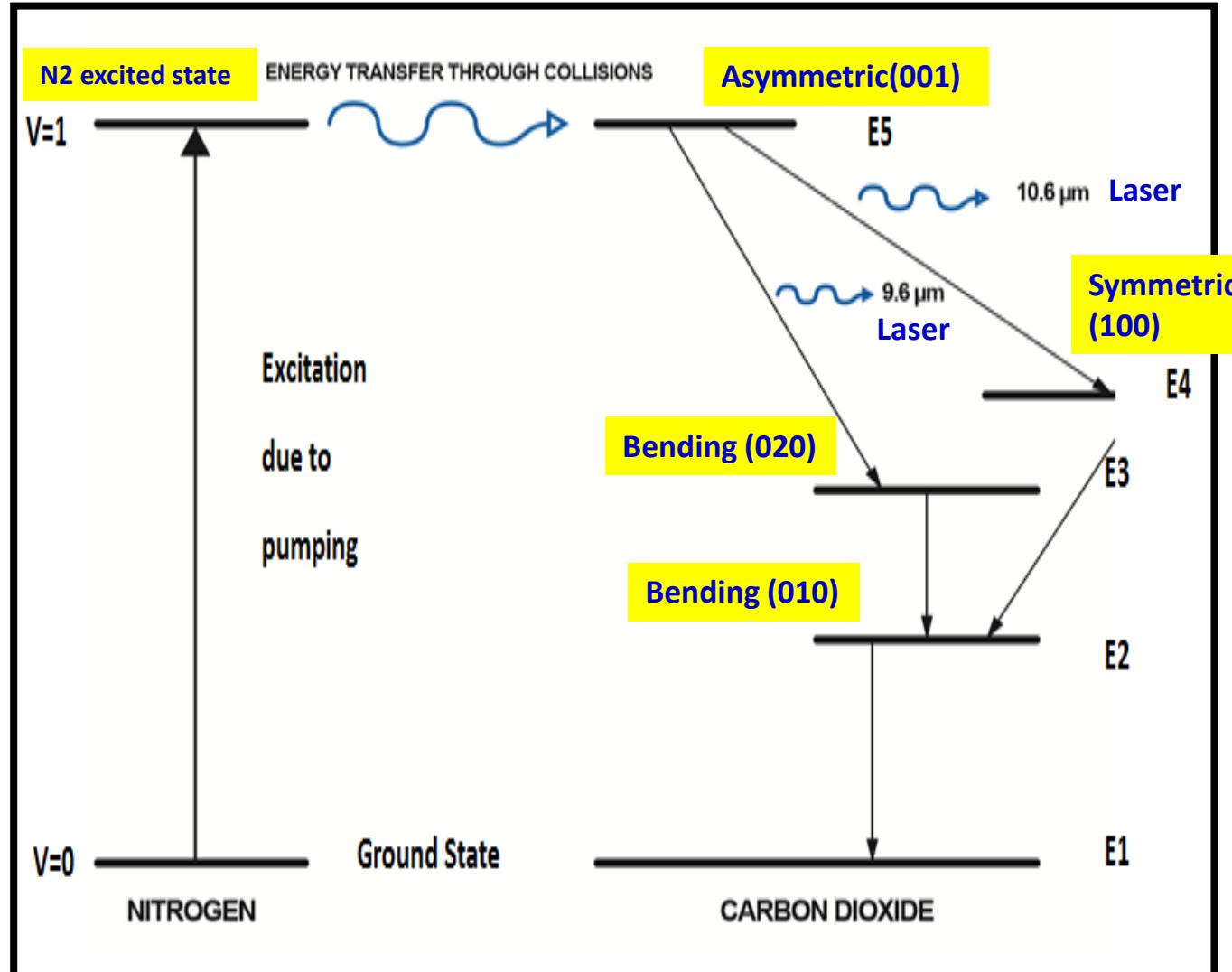
3. Bending mode





- **Active Medium:** A mixture of CO₂: N₂: He in the ratio of 1:2:8
- **Energy Pump:** DC or RF supply for electron discharge through the gas mixture
- **Optical cavity:**
 - A specialized optical resonator made of materials such as Ge, Zinc Selenide to avoid IR absorption
 - Cavity should have suitable infra red absorption coatings and effective cooling system

Working: Energy Level Diagram



Fast moving electrons from the discharge collide with N₂ molecules and excite them to their first excited state

These excited N₂ molecules then collide with CO₂ molecules and selectively excite them to the asymmetric 001 state, resulting population inversion

Transition from 001 (asymmetric) to 100 (symmetric) produce lasers of wavelength **10.6 μm (0.117eV)**

Transition from 001 (asymmetric) to 020 (bending) produce lasers of wavelength **9.6 μm (0.129 eV)**

Extensively used for welding, cutting ,drilling

- *Uses four-level pumping scheme - continuous wave mode*
- *The active centers are CO_2 molecules*
- *Helium (He) helps in the depopulation of lower levels, also as He has thermal conductivity to keep temperature of CO_2 low (to avoid population in the lower level by thermal excitation).*
- *Electrical discharge is the pumping agent*
- *Resonant transfer of energy from N_2 to CO_2 molecules atoms upon collision leads to population inversion*
- *High efficiency and high power output(several kilowatts)*
 - CO_2 laser operates with an efficiency of up to 30 %
 - The wavelength of CO_2 laser falls in a region where atmospheric attenuation is negligible. Hence, these lasers find applications in Optical radar systems

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Class #40..... Conceptual Questions

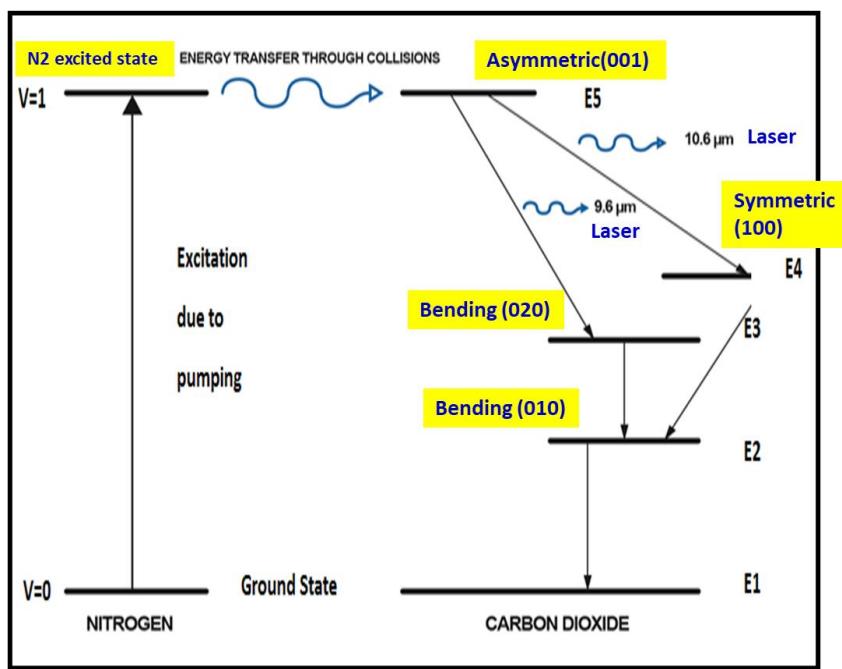
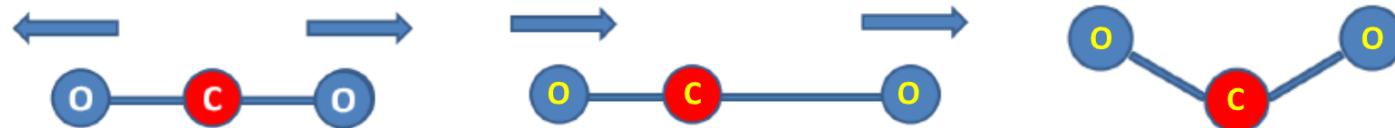
With a neat energy level diagram discuss how lasing action is achieved in molecular laser

Elaborating the concepts of vibrational modes and using the energy level diagram explain the transitions in a CO₂ laser system.

1. Symmetric stretching:
Bond lengths always equal

2. Asymmetric stretching:
Bond lengths unequal

3. Bending mode



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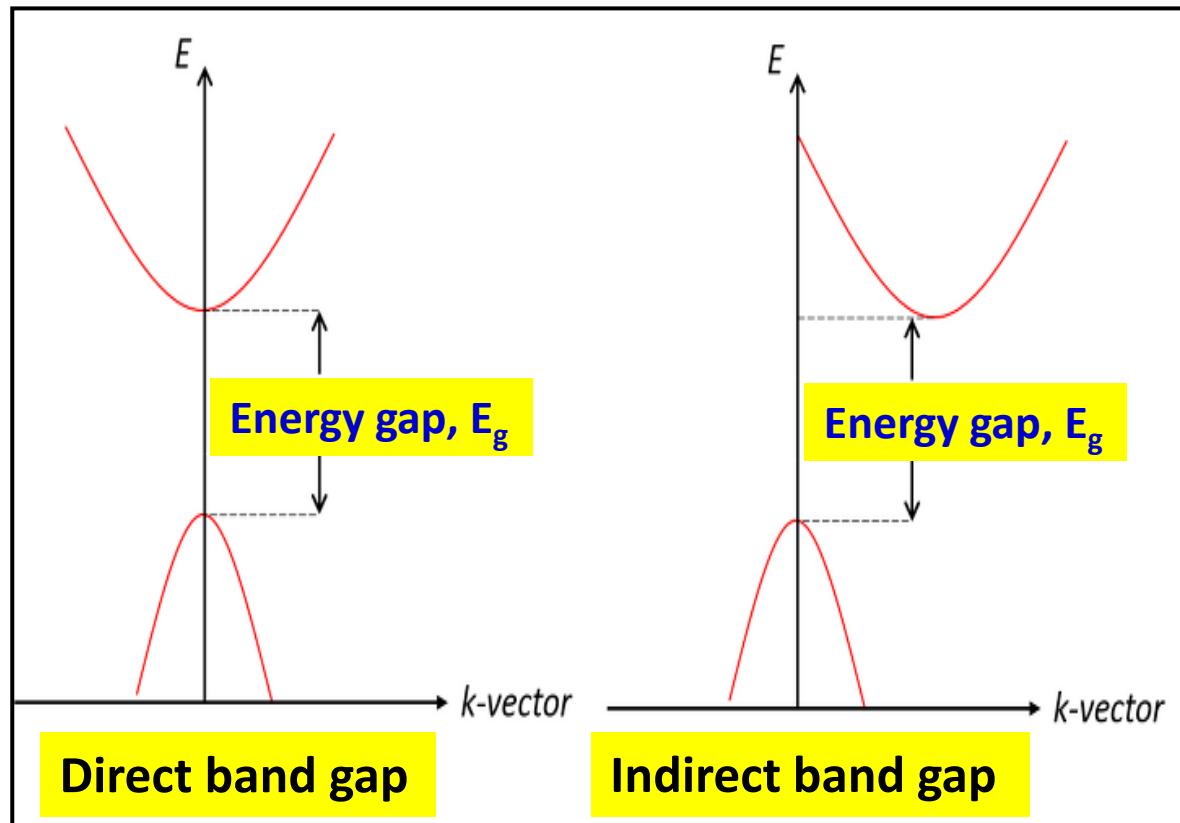
Semiconductor lasers

Class #41

- Semiconductor materials
- Direct and indirect band gap semiconductor
- Principle of LED

Direct and Indirect band gap semiconductors

Based on Kronig -Penney Model - E – k diagram



- **Direct band gap semiconductors**
 - Conduction band electron can recombine directly with a hole in the valence band
 - The recombination process leads to emission of light
 - Most of the compound semiconductors belong to this group
 - E.g.- GaAs, InP
- **Indirect band gap semiconductors**
 - Direct recombination of conduction band electron with a hole in the valence band is not possible (band structure is different)
 - The recombination process produces heat
 - E.g.- Si, Ge



- LED is a semiconductor diode that gives off light when it is forward biased
- LEDs are fabricated using compound semiconductors which have a direct band gap

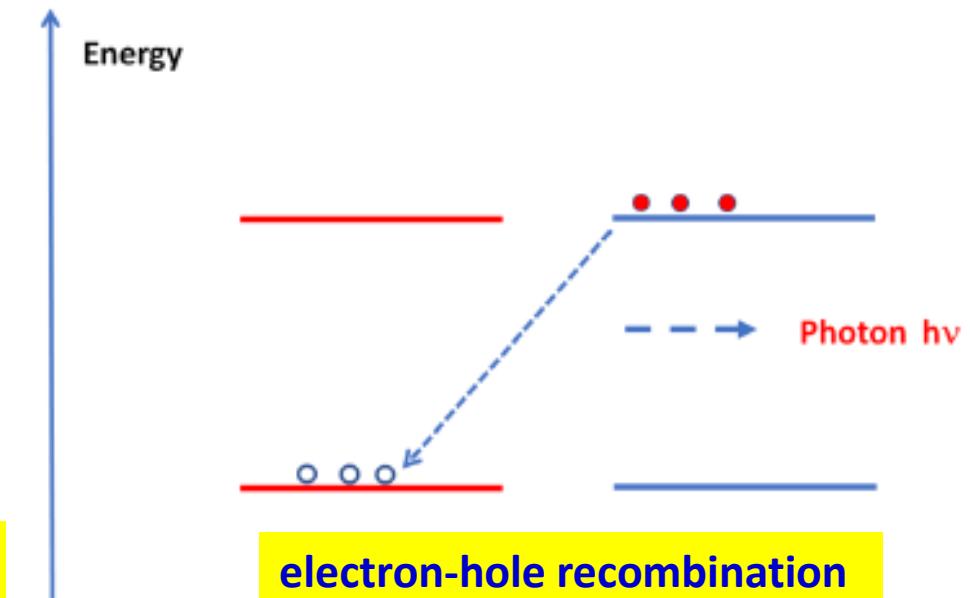
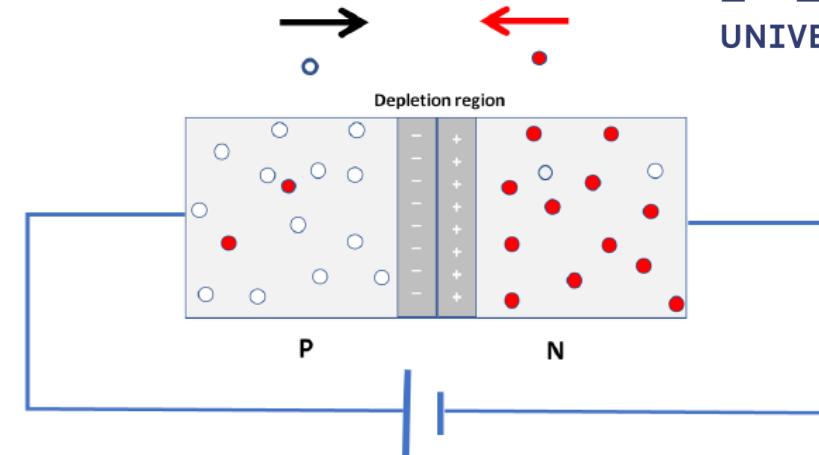
electron-hole recombination - energy E_g is given as photon

Principle: Injection electroluminescence

- The electron-hole recombination is the basic mechanism responsible for emission of light

$$\text{Wavelength of light is given by, } \lambda = \frac{hc}{E_g}$$

Semiconductors with suitable E_g emit light in the optical region



Construction

- **Active medium:** Heavily doped direct band gap semiconductors (e.g.: GaAs, InP, etc.- *Optically active electron-hole recombination*)
- **Energy Pump:** Electrical pumping (Forward biasing the p-n junction structure)
- **Laser Cavity:** Two opposite sides of diode laser are cleaved for emission of laser

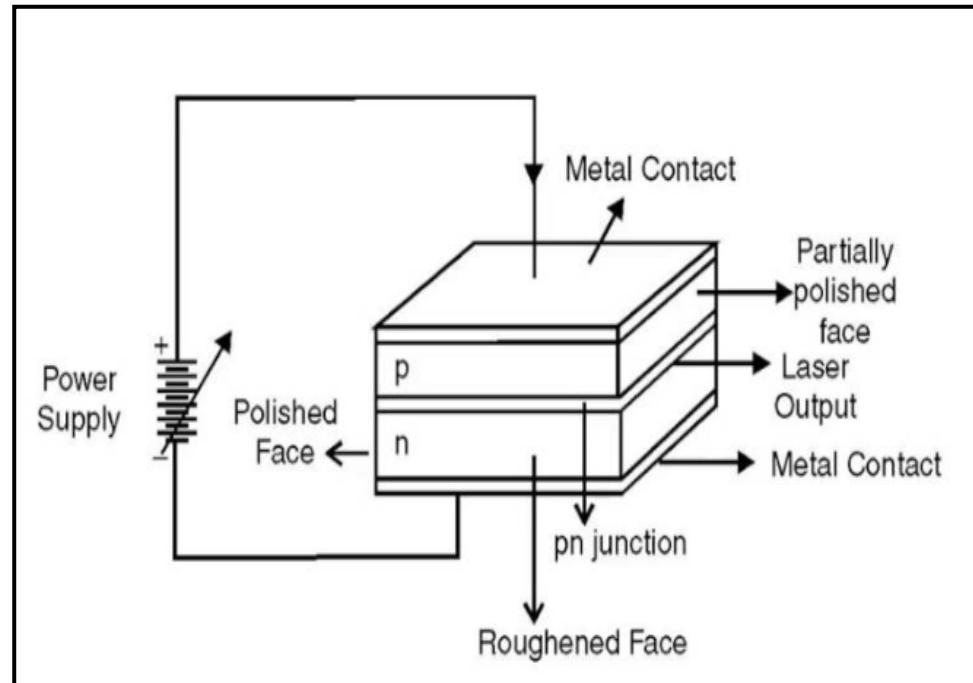
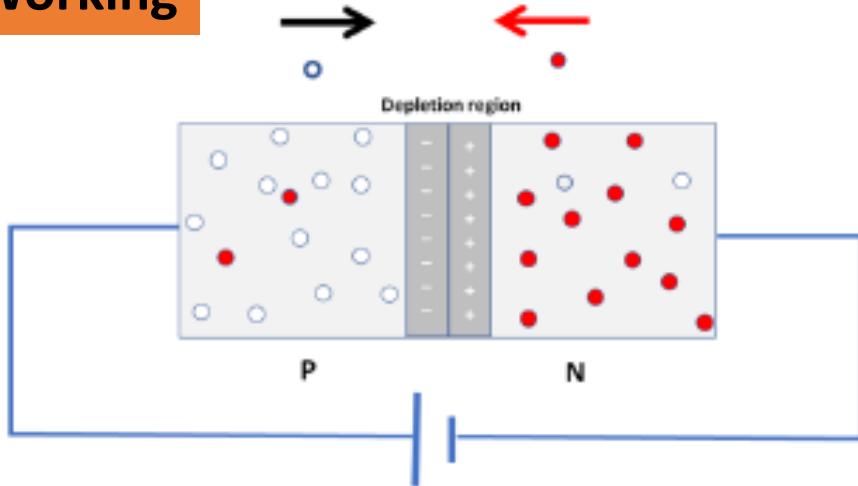


Image adapted from

[https://winnerscience.com/2011/04/26/
construction-of-semiconductor-laser/](https://winnerscience.com/2011/04/26/construction-of-semiconductor-laser/)

Homo junction Semiconductor Laser (Diode laser)-Energy band diagram

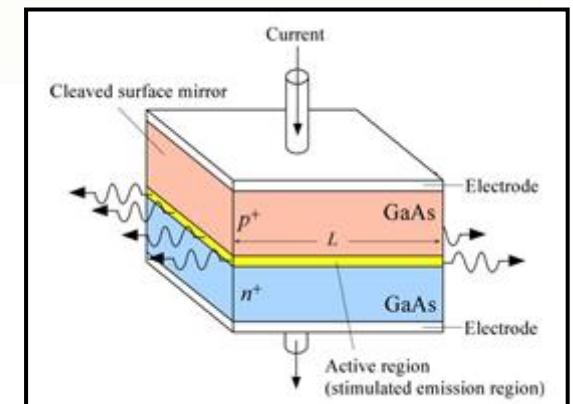
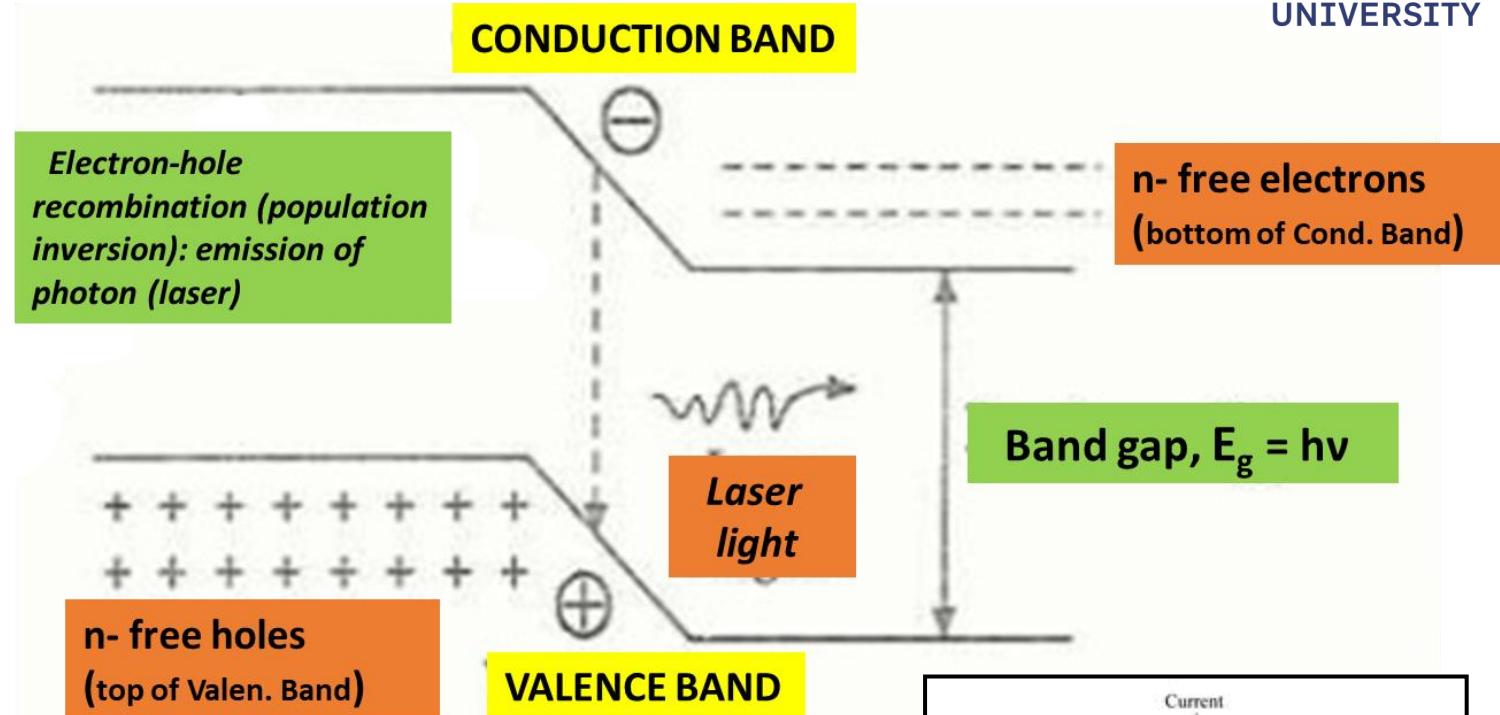
Working



Heavily doped PN junction under pumping (forward bias)

1. Pumping (Forward bias): Electrons and holes into the junction region in high concentrations

2. When diode current reaches a threshold value population inversion is attained

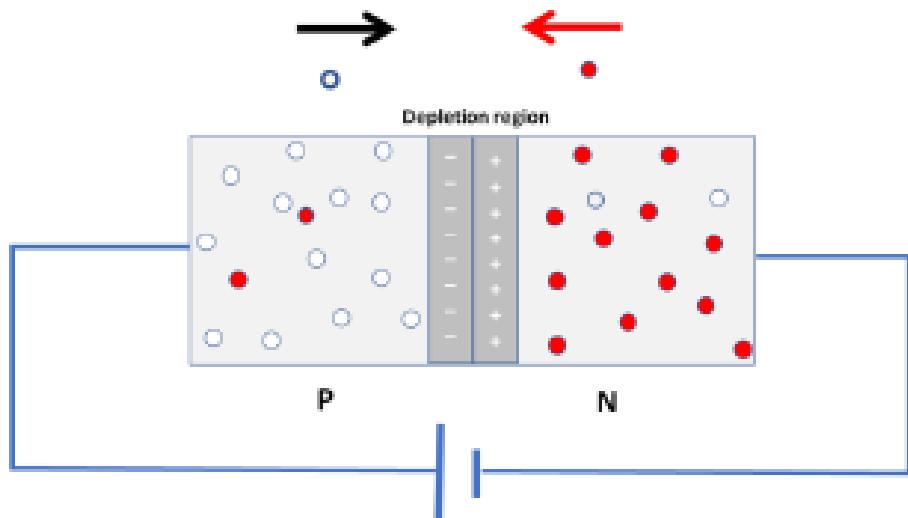


3. Recombination of electrons and holes in a narrow region (generation of coherent photons)

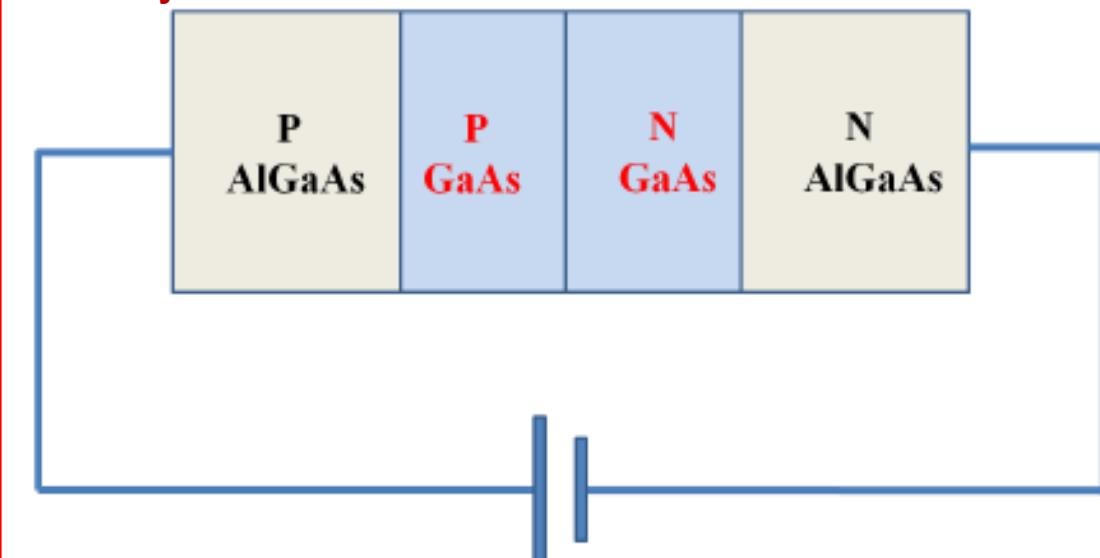
- *Cavity losses*
- *Requirement of very high forward current density (of the order of 10000 A/cm²)*
- *Not very efficient*

- *Principle of working: Similar to that of homojunction laser*
- *Junction structure: Between two different bandgap semiconductors (e.g.: GaAs and AlGaAs)*

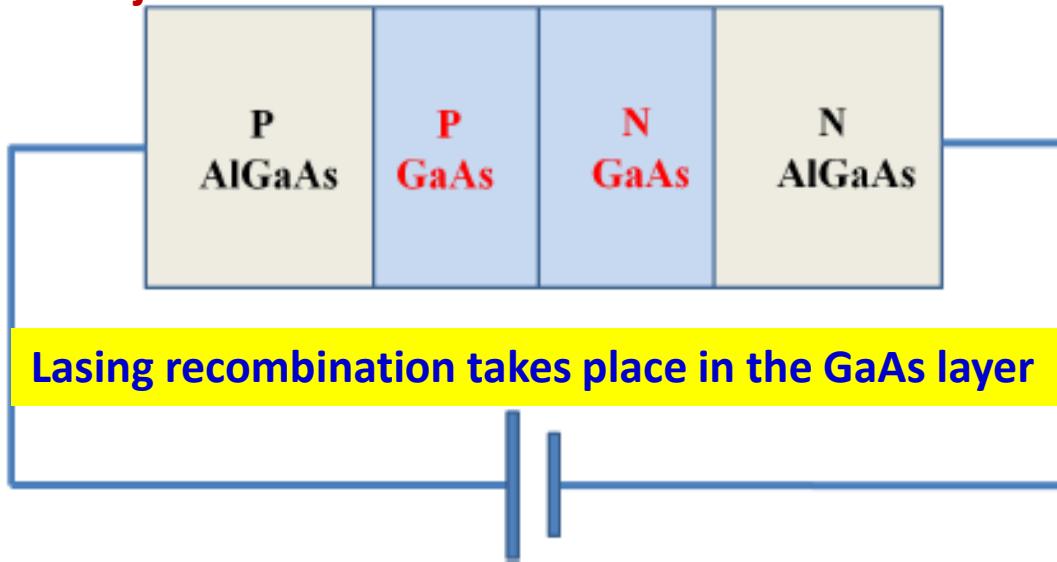
Homo-junction Semiconductor Laser



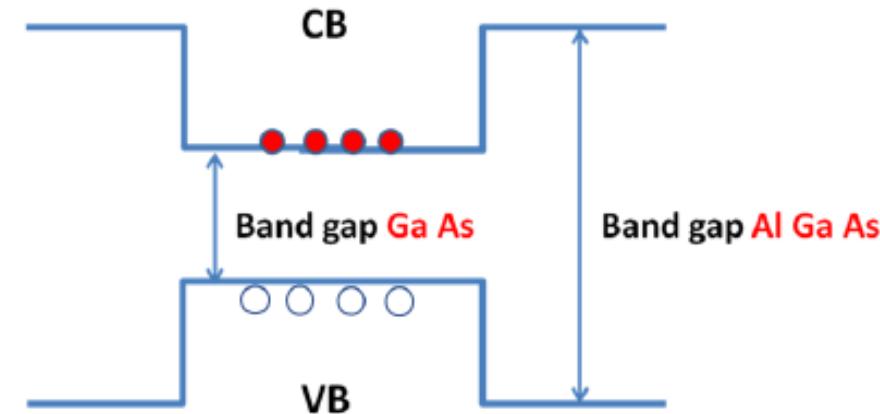
Heterojunction Semiconductor Laser



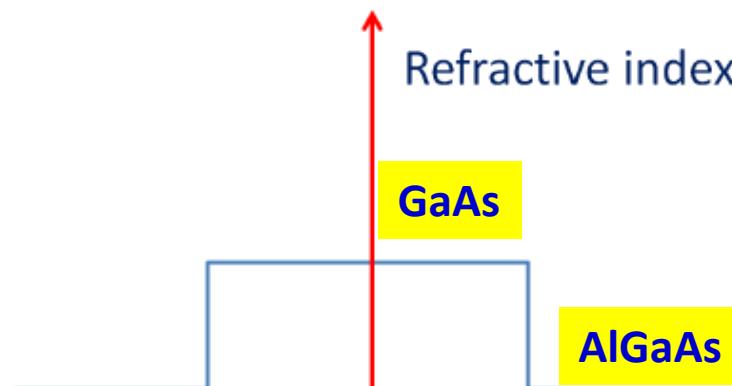
Heterojunction Semiconductor Laser



Heterojunction 'diode' provides-
High efficiency by
Charge confinement
Photon (light) Confinement
Works at low voltage



Charge confinement: The GaAs active layer has a lower band gap than the AlGaAs layers on either side



Photon Confinement: GaAs has higher Refractive Index than AlGaAs

- *Semiconductor diode lasers are of low cost*
- *Simple, compact and portable*
- *Diode lasers are remarkably small in size*
- *They operate at low powers*
- *Have high efficiency of the order of 40%*
- *Mass produced*

The band gap of GaAs is 1.42 eV. What is the wavelength of the laser beam emitted by a GaAs diode laser? To which region of the EM spectrum does it belong?

$$E_g = h\nu = \frac{hc}{\lambda} \text{ this gives, } \lambda = \frac{hc}{E_g}$$

$$E_g = 1.42 \text{ eV} = 1.42 \times 1.6 \times 10^{-19} \text{ J, on substitution, } \lambda = 8.75 \times 10^{-7} \text{ m}$$

WKT, visible spectrum from 400 to 700 nm

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Class #41..... Conceptual Questions

What is direct band gap semiconductor? Mention its significance as PN junction structures.



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ENGINEERING PHYSICS

Radhakrishnan S, Ph.D.

Department of Science and Humanities

➤ *Suggested Reading (e book)*

1. *Lasers - Fundamentals and Applications*

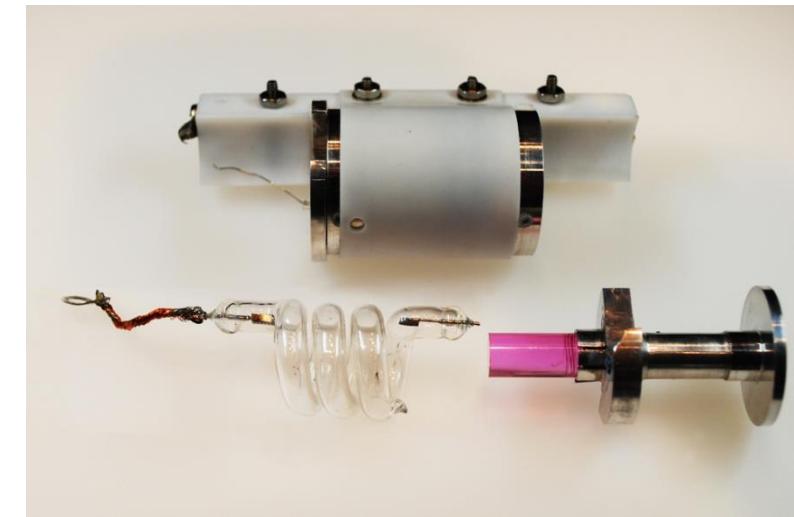
Thyagarajan K., Ghatak A Springer(2010)

Chapter 11.2

A solid state pulsed laser – Ruby LASER

*This is an example of a three level laser
(a pulsed laser)
and deliver very high energy*

*It's the first working laser by Theodore H.
Maiman at Hughes Research Laboratories in
1960*



Components of the first ruby laser

Active medium:

The active medium of the system is a Ruby crystal / Rod

- Ruby is Al_2O_3 doped with Cr^{3+} ions

Energy pump:

The energy pump for the Ruby Laser is a high power white light flash lamp capable of delivering energy densities up to 21 kJ m^{-2}

A capacitor is charged to high voltages and discharged through the lamp (like the photo flash). Typically, the discharge results in 675-770 J of energy

Resonant Cavity

A ruby rod of appropriate length with its ends highly polished and within the condition of $L = \frac{n\lambda}{2}$ of wavelength of the output light, and parallel to each other form the resonant cavity

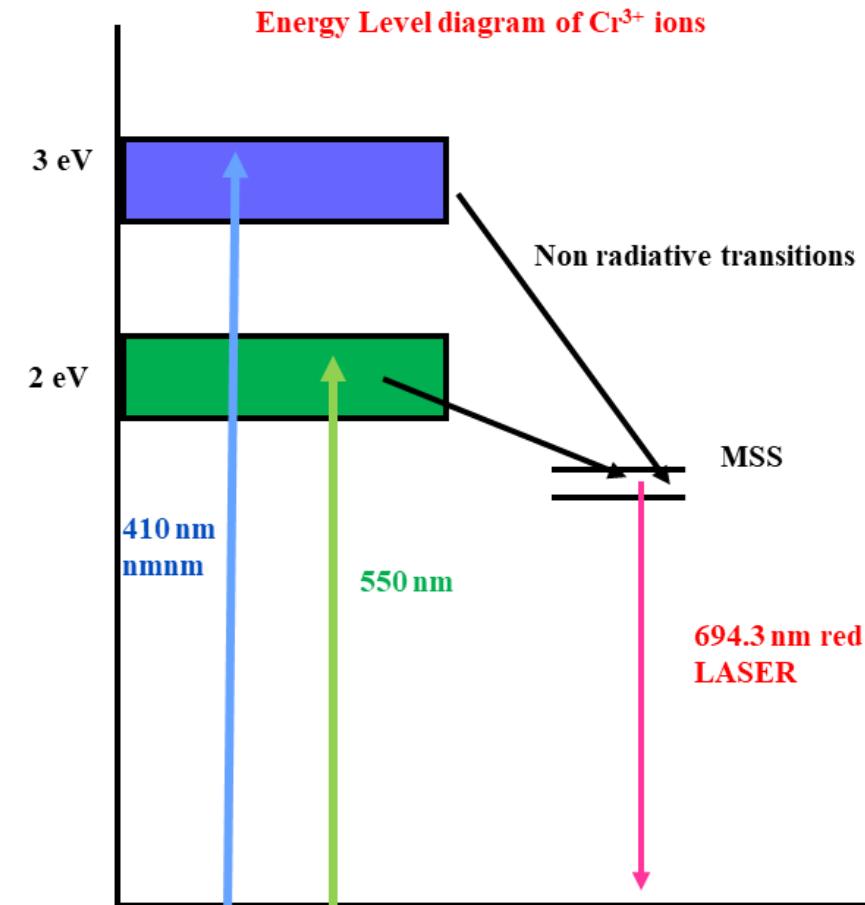
Silver coatings of the polished ends of the rod form the mirror arrangement

Antireflection coatings, external dielectric mirrors are also some of the design variations found

The ends cut and polished at Brewster's angle ($\tan \vartheta = n$, n is the refractive index)– polarized light

Energy level diagram

- The Cr^{3+} ions have absorption energy levels around 2 eV and 3 eV which are normal de-exciting states (lifetime of electrons in the order of 10^{-9}s)
- This corresponds to photons with 550 nm (green) and 410nm (blue) wavelengths as the absorption wavelengths



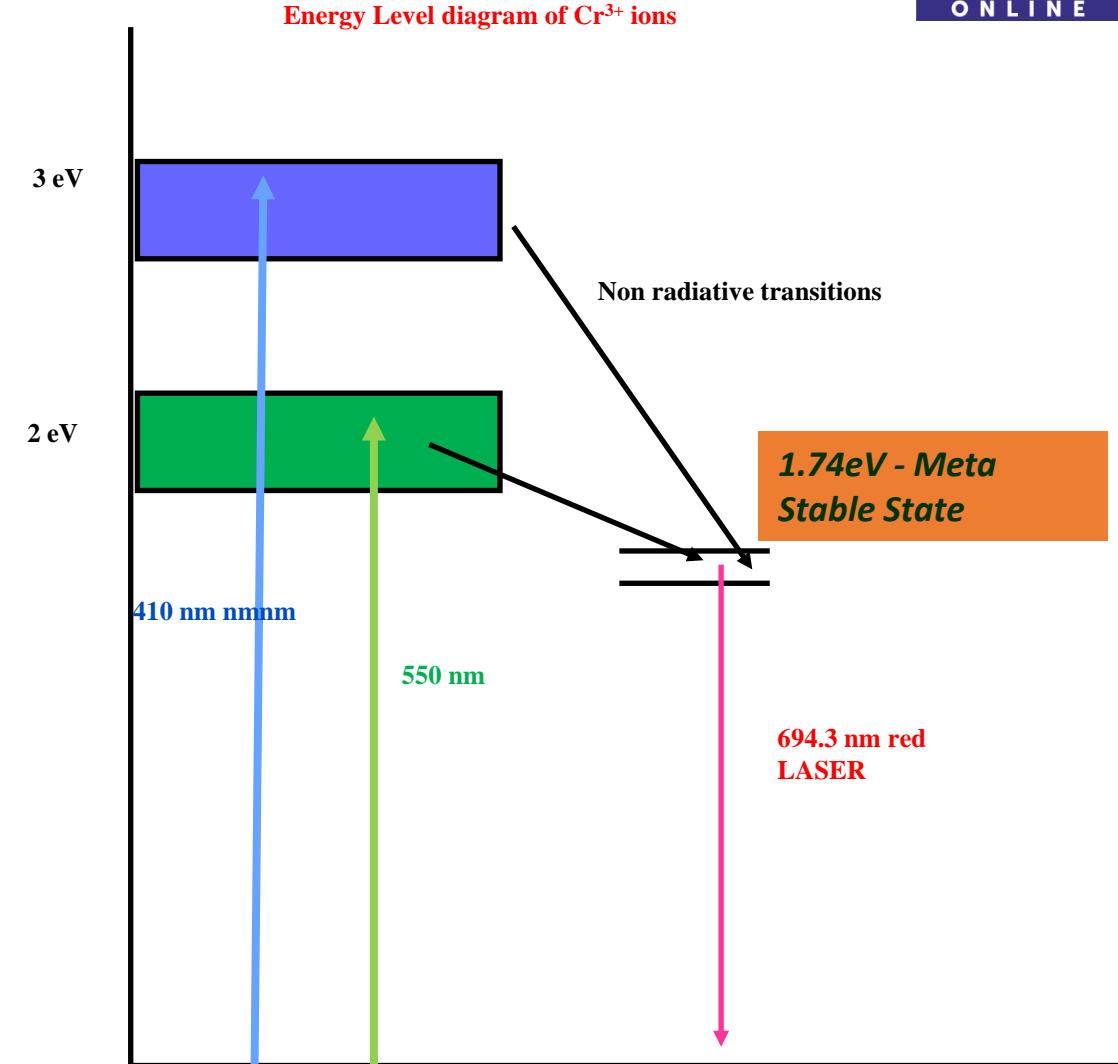
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Unit IV : Solid State LASER – Ruby LASER

One of the intermediate energy states of Cr^{3+} ions around 1.74eV is a Meta Stable State (lifetime of electrons 10^{-3} s)

The transition from the upper absorption energy levels to the intermediate state is through non radiative transitions.

For a short duration of time the population of the intermediate state > the population of the ground state - POPULATION INVERSION



Transitions from the meta stable intermediate state to the ground state (only partial decoupling) emit stimulated photons of leading to a LASER with a wavelength of 694.3 nm.

Typical ruby laser pulse widths are of the order of a millisecond

Role of Aluminum oxide crystal

Hosting the chromium ion and absorb the pump energy to excite the chromium ion through collisions



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Department of Science and Humanities

- Class #36

Gain and loss in laser systems

1. Gain in a cavity
2. Laser Comb
3. Line Broadening
4. Losses in the cavity

➤ *Suggested Reading*

1. Lasers: Fudamentals and Applications

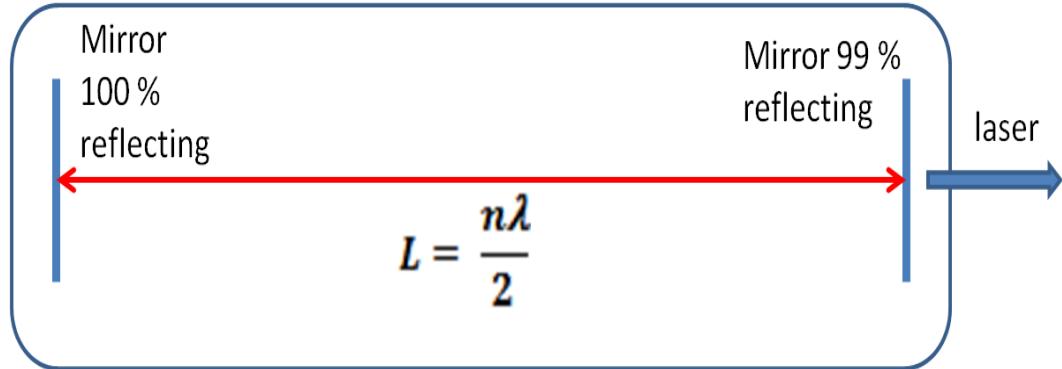
K Thyagarajan, A Ghatak

2. Course material developed by the Department

➤ *Reference Videos*

<https://ocw.mit.edu/resources/res-6-005-understanding-lasers-and-fiberoptics-spring-2008/laser-fundamentals-i/>

- **Consists of two mirrors of various geometries and coatings creating standing waves**



- **Because of the energy amplification due to stimulated emission**
- **The laser comes out of the partially reflecting mirror**
- **Photons travelling in directions not perpendicular to the mirrors are not amplified**

Resonating Cavity: Frequency Comb

$$n_1\lambda_1 = 2L$$

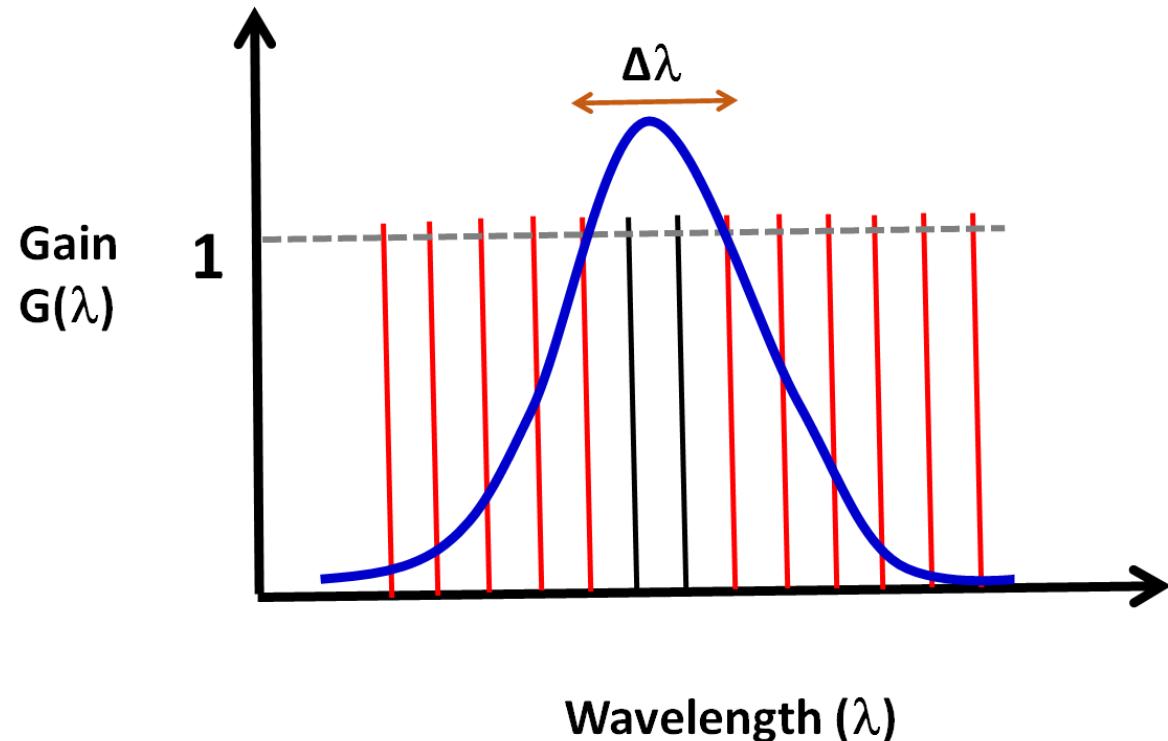
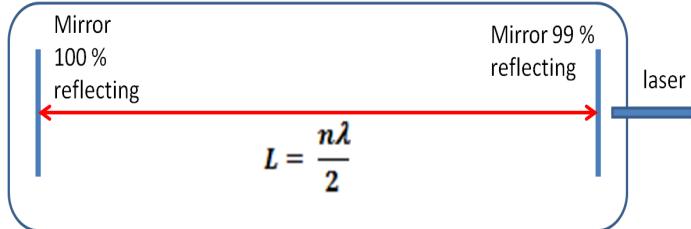
$$n_2\lambda_2 = 2L$$

$$n_3\lambda_3 = 2L$$

.

.

.



Gain is a function of wavelength

Frequency Comb : consists of a series of discrete equally spaced frequency lines corresponding to $n_1, n_2, n_3\dots$ etc. with respect to the laser cavity length.

The line width is narrow due to the fact that only those wavelengths for which the gain is above threshold will be amplified for output laser beam.

Losses in the cavity

1. Scattering(greater at shorter wavelengths)
2. Absorption in the beam path
3. Diffraction losses
4. Mirror losses

Class 49

- *Polarization in dielectrics*
- *Electric fields in a dielectric material*
- *Polarization mechanisms*

➤ *Suggested Reading*

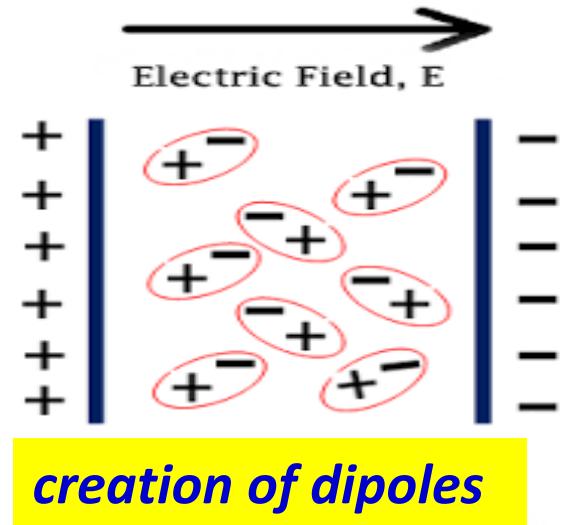
- 1. The Science and Engineering of Materials, Sixth Edition,
Chapter 19, Donald R. Askeland, Pradeep P. Fulay and Wendelin J.
Wright, 2010, Cengage Learning, Inc.*
- 2. Learning material prepared by the Department of Physics*

Dielectrics Materials

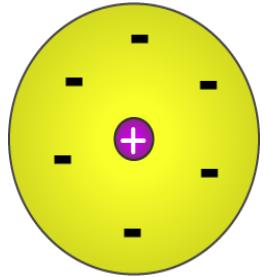
- All dielectrics are insulators, but not all insulators are dielectrics
- Dielectric materials respond to external electric fields and cause of separation of charges resulting the creation of dipoles
- The net dipole moment created per unit volume is called the Polarization

Dielectric constant (K or ϵ_r)

It is a measure of how susceptible the material is to the applied electric field

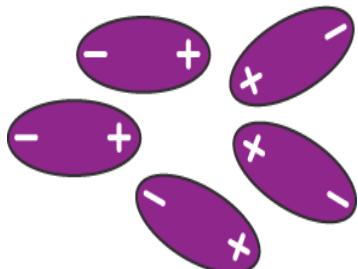


➤ Non-polar Molecule



- *centers of both positive as well as negative charges coincide*
- *molecules are symmetric in nature*
- *examples of non polar dielectrics are: methane , benzene etc.*

➤ Polar Molecule



- *centers of positive and negative charge will not coincide*
- *posses permanent dipole but due to randomness the net dipole moment is zero*
- *examples of the polar dielectrics is NH₃, HCl, water etc.*

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Polarization in dielectrics : Expression for polarization of a dielectric material



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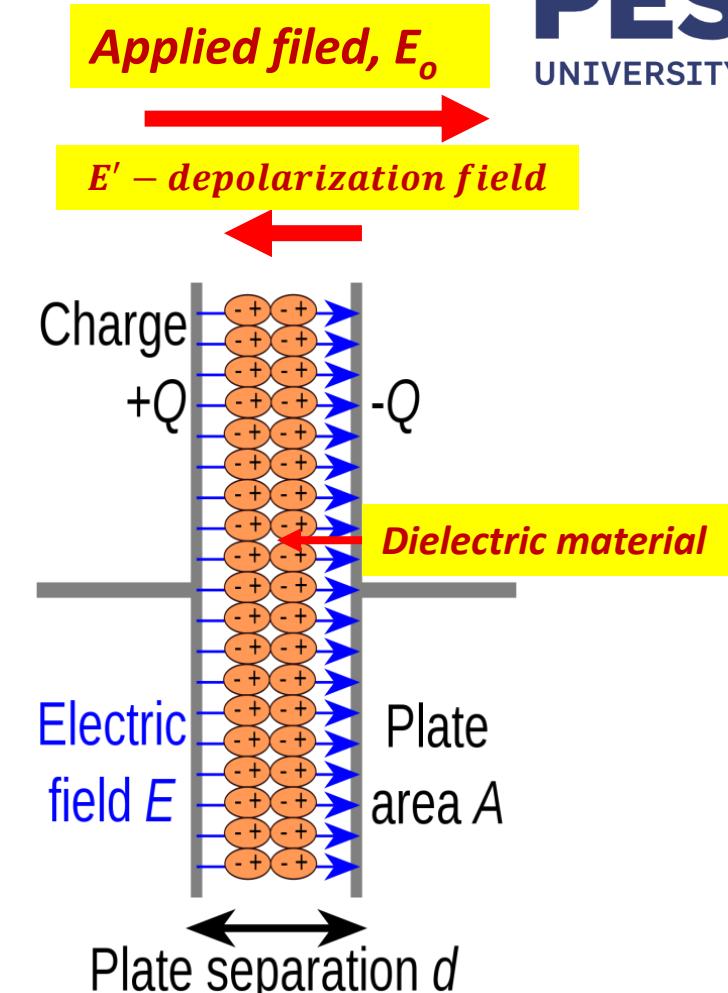
- Surface charge density on the plates due to application of electric field E_0 of the capacitor $\sigma = \epsilon_0 E_0$ (without dielectric material)
- If a dielectric material is placed between the plates of the capacitor, the surface density of charge due to polarization $\sigma_p = \epsilon_0 E' = \text{polarization, } P$
- The net electric field E between the plates is reduced by the dielectric constant ϵ_r of the material

$$E = \frac{E_0}{\epsilon_r} \text{ or } E_0 = \epsilon_r E$$

- The net electric field $E = E_0 - E' = \epsilon_r E - \frac{\sigma_p}{\epsilon_0}$

- Simplifying $\sigma_p = \epsilon_0 \epsilon_r E - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E$

- Hence the polarization in the material due to a net electric field is given by $P = \sigma_p = \epsilon_0 (\epsilon_r - 1) E$



Also, $P = \epsilon_0 \chi E$, where $\chi = (\epsilon_r - 1)$ or $\epsilon_r = 1 + \chi$ is the dielectric susceptibility of the material

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Polarization in dielectrics: Susceptibility, dielectric constant and atomic polarizability



- Net electric field $E = E_0 - E' = \epsilon_r E - \frac{\sigma_p}{\epsilon_0}$

- Polarization in the dielectric material $P = \sigma_p = \epsilon_0(\epsilon_r - 1)E$

Also, $P = \epsilon_0 \chi E$, where $\chi = (\epsilon_r - 1)$ or $\epsilon_r = 1 + \chi$
is the dielectric susceptibility of the material

Atomic polarizability, α_e

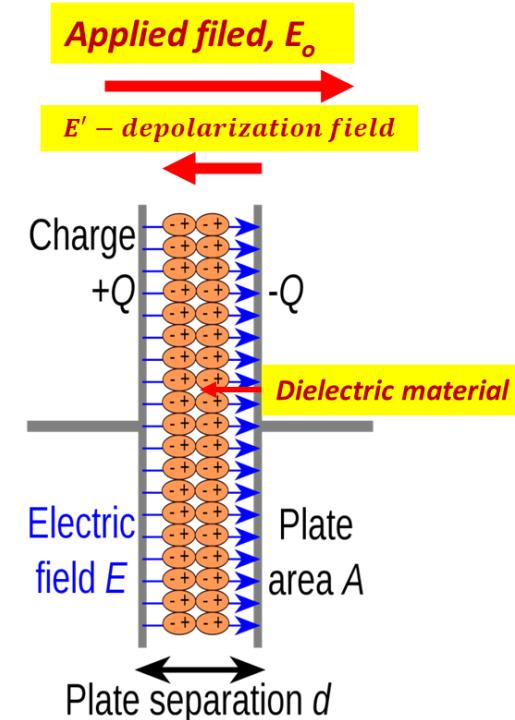
Relation between dielectric susceptibility with polarizability

For an atom, polarization is proportional to applied electric field, thus, $P \propto E$ or $P = \alpha_e E$, where ' α_e ' is called atomic polarizability (microscopic)

For N atoms per unit volume, polarization $P = \epsilon_0 \chi E = N \alpha_e E$

$$\chi = \frac{N \alpha_e}{\epsilon_0}$$

This relates macroscopic dielectric susceptibility with microscopic polarizability



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Components of electric fields (Local fields) in a dielectric material



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- For a polarized dielectric material, local electric field around a dipole inside the material experiences four components of electric field

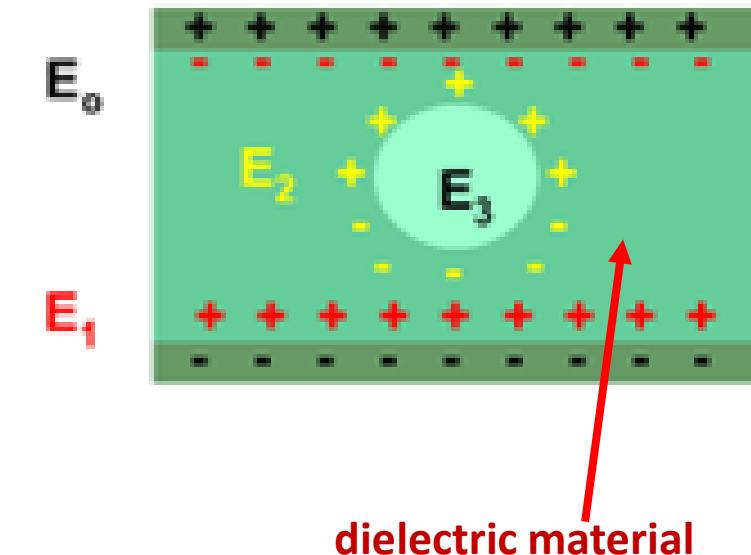
$$E_{loc} = E_0 + E_1 + E_2 + E_3$$

$E_0 \rightarrow$ External electric field

$E_1 \rightarrow$ Depolarization field

$E_2 \rightarrow$ Lorentz field on the surface of the spherical cavity

$E_3 \rightarrow$ Internal field due to other dipoles lying within the sphere



- *In the case of a dielectric with a cubic structure the effective field can be written*

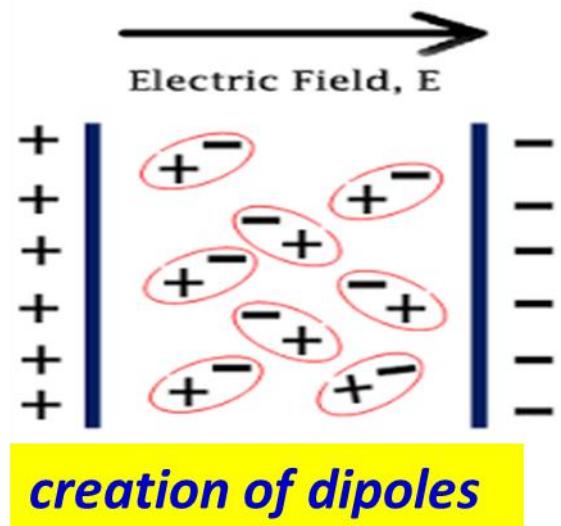
$$E_{in} = \frac{P}{3\epsilon_0}$$

$$E_{loc} = E + E_{in} = E + \frac{P}{3\epsilon_0}$$

$$\text{Polarization, } P = N\alpha_e E_{loc} = N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right) = \epsilon_0(\epsilon_r - 1)E$$

- Solving this we get $\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0}$
- *This is the Clausius Mosotti relation - Relation between the macroscopic dielectric constant to the microscopic polarisability of the material*

- In dielectric materials, separation of charges leads to polarization
- Polarization phenomena which occurs in dielectrics are
 1. Electronic polarization
 2. Ionic polarization
 3. Orientation or dipole polarization
 4. Space charge or interfacial polarization



➤ Electronic Polarization

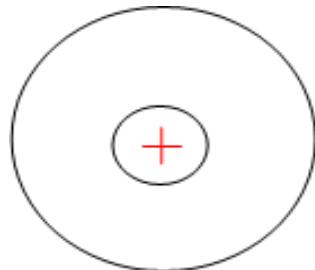
- When an atom is subjected to an electric field E, the centre of the negatively charged electron cloud no longer coincides with the positive nucleus and hence results in an induced dipole
- The polarization produced due to this induced dipole is called “Electronic polarization”

Electronic polarizability

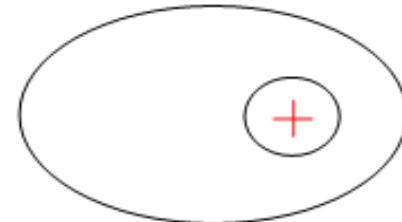
$$\alpha_e = 4\pi\epsilon_0 R^3$$

where R is the radius of the atom

Electronic polarizability proportional to radius of atom



When $E = 0$



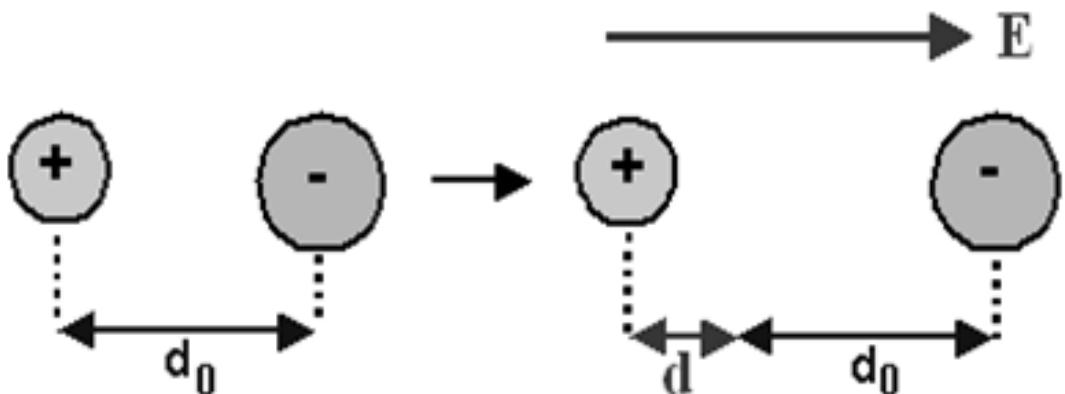
When electric field is applied

- The electronic polarization is temperature independent

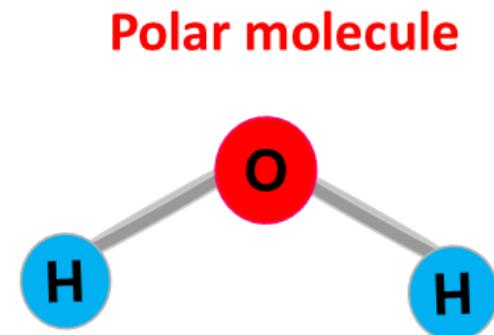
➤ Ionic Polarization

Relative displacement of ions in ionic crystals (ex: NaCl) in the presence of an external electric field results ionic polarization

- The ionic polarizability is independent of temperature
- Depends on the Young's modulus of the material



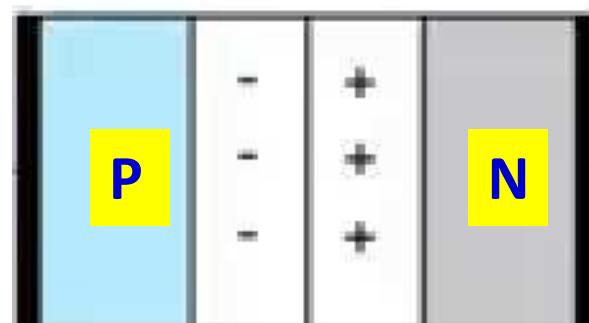
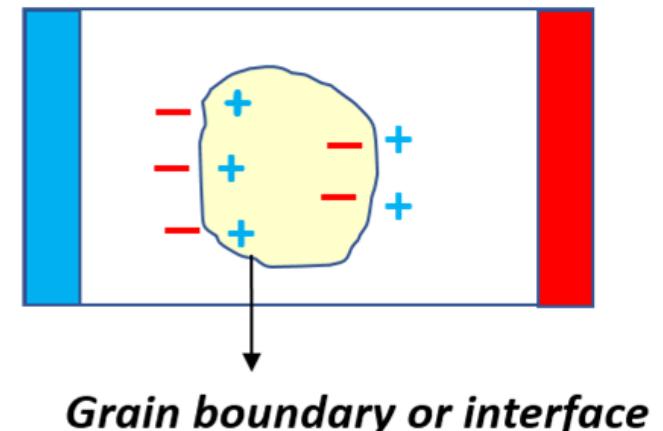
- Orientational polarization/Molecular polarization
- Polar molecules with permanent dipole moments show orientational polarization
- At normal temperatures, due to the random motion of the dipoles, net polarization is zero
- In an external field the dipoles align in the direction of the field and results in a net dipole moment for the material and is called orientational polarization



$$P = \frac{Np^2 E}{3kT}$$

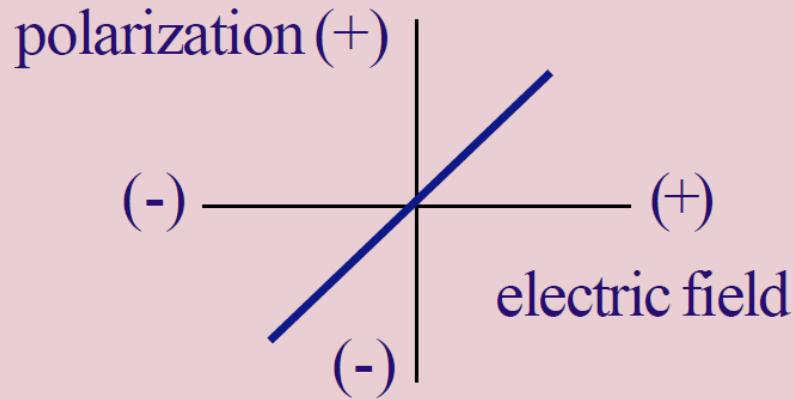
p induced dipole moment due to orientational polarization

- Space charge limited polarisation
- Due to accumulation of charges of the opposite polarities on either sides of the interface of two or more materials
- *Occurs at physical boundaries such as defects, impurities, grain or phase boundaries*
- P-N Junction structure with barrier potential is another example (modeled as junction capacitance)



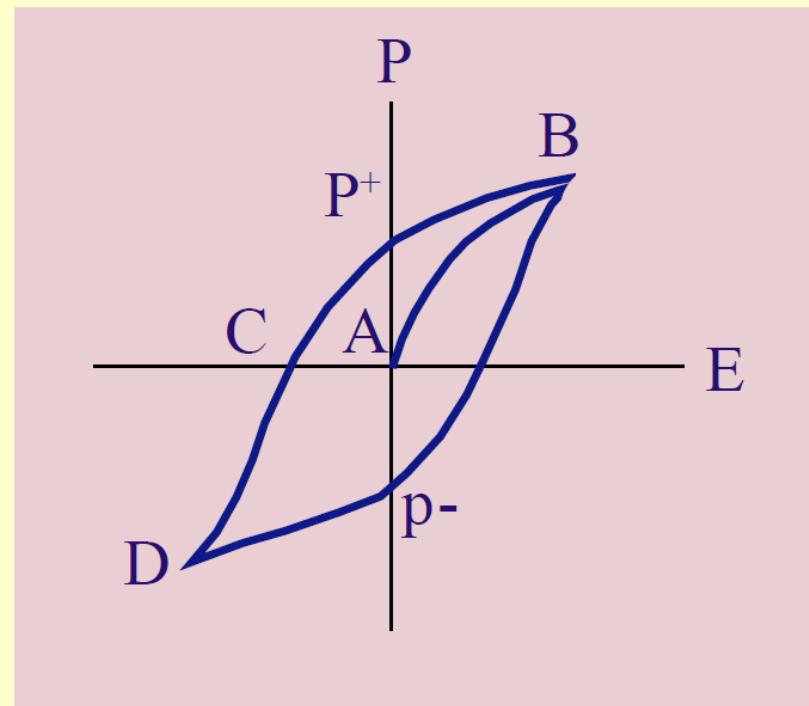
- The electronic polarisability depends on radius of the atom
- The ionic polarisability depends on Young's modulus of the materials.
- The orientation polarizability is independent of temperature
- The local electric field around a dipole depends on four components of electric field

- *Non-linear dielectric materials*
- *Piezo electric materials*
- *Pyro electric materials*
- *Ferro electric materials – hysteresis and application as memory materials*
- *Barium titanate structure-phase changes properties and applications*



Linear dielectrics

In linear dielectrics, polarization (P) is linearly related to E

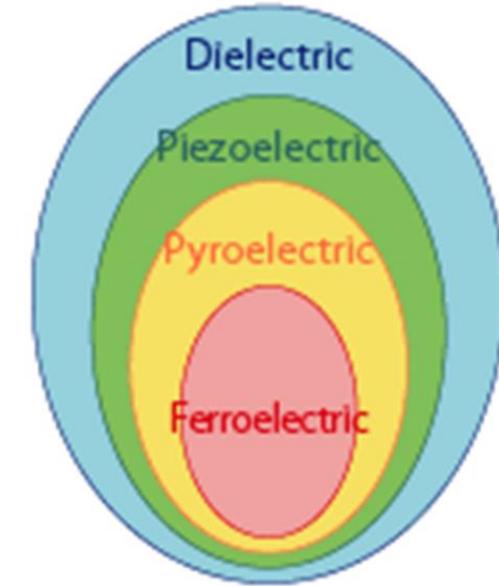
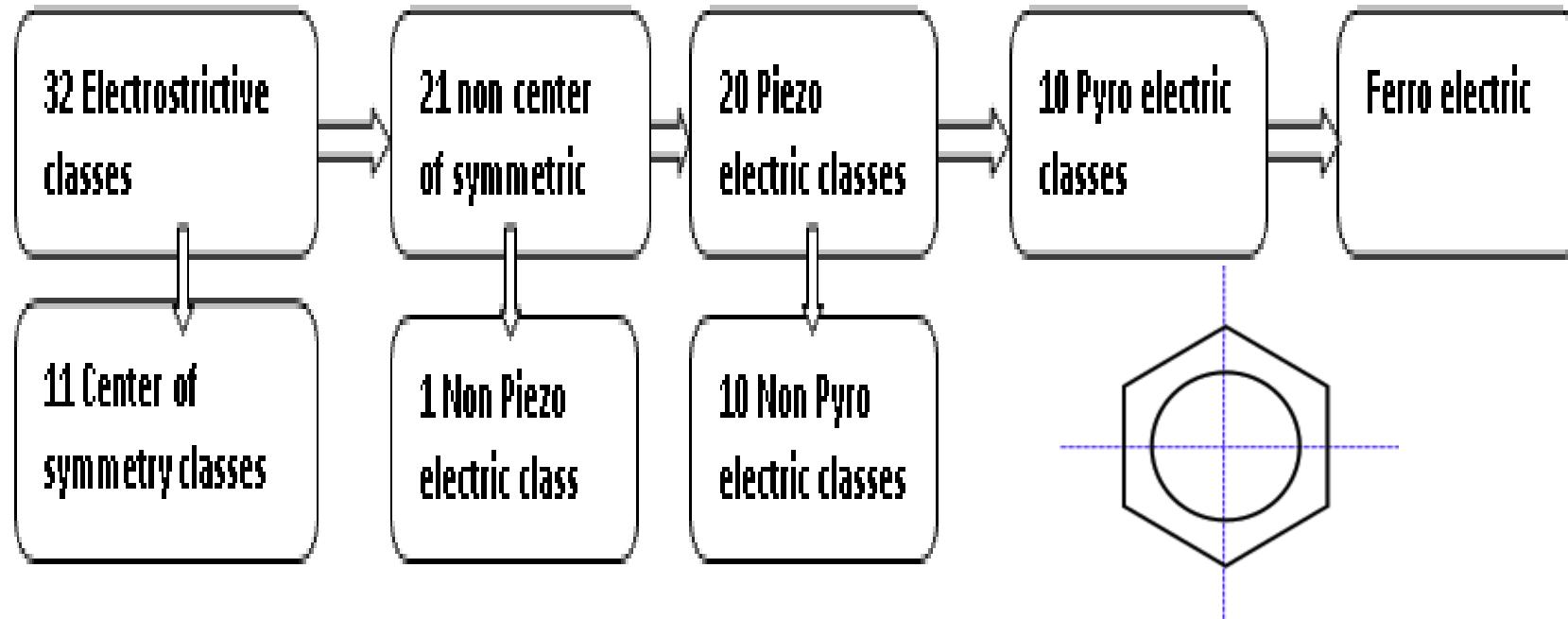


Nonlinear dielectrics

Nonlinear dielectrics : E and P are not linearly related

Non-Linear Dielectrics with non-centro symmetry (Electro strictive response – mechanical deformation with electric field) form a special class of materials with a wide range of applications

Classification



- Piezo electric materials
- Pyro electric materials
- Ferro electric materials

Piezoelectric materials : Piezoelectric effect

- Non-linear dielectric materials which convert mechanical energy to electrical energy (Piezoelectric Effect)
- Appearance of electric potential when pressure is applied
- Piezo electric materials also show invers piezo electric effect (electrical energy to mechanical energy) – very small effect (voltage of 10^3 V/cm produce only strain of 10^{-7} A°)
- Quartz is the most popular natural piezo material, another popular material is lead zirconate titanate (PZT)

The microscopic origin of piezoelectricity: displacement of ionic charges (polarization) within the crystal with applied strain

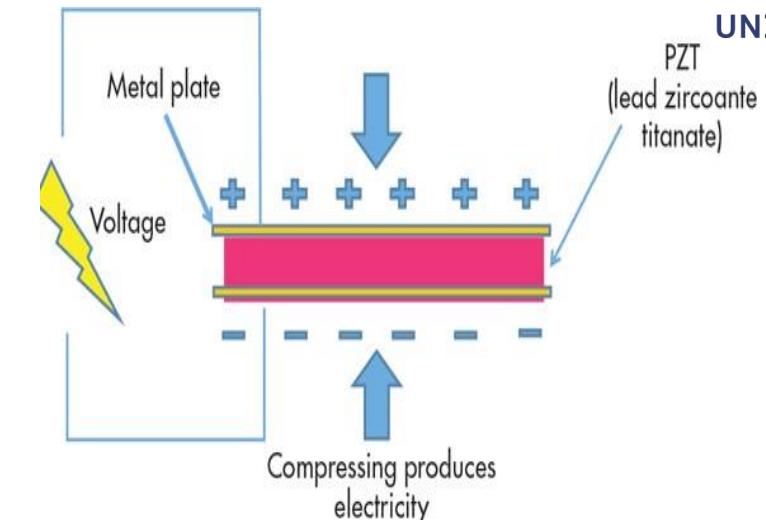
In the absence of strain, the distribution of the charges at their lattice sites is symmetric, so the internal electric field is zero

Applications:

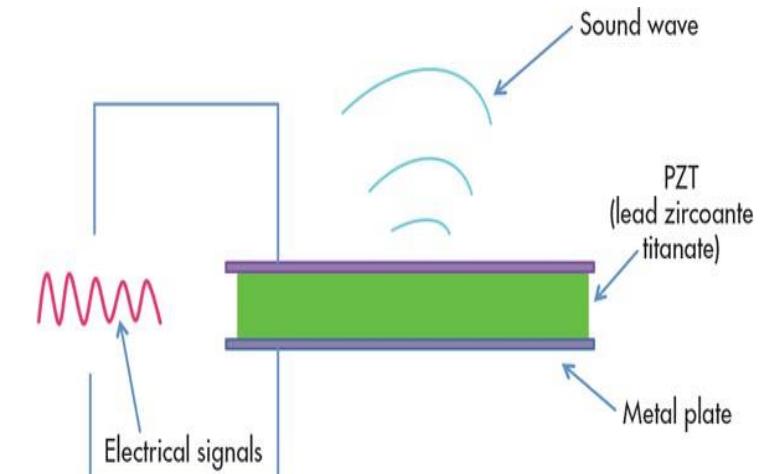
Important class of transducers, Sensors, Electric ignitors

Energy harvesting from impact

Direct piezoelectric effect



Inverse piezoelectric effect



- **Pyroelectricity is the ability to generate electrical potential when heated or cooled**
- **Pyroelectric materials are non-centrosymmetric non linear dielectrics**
- **The change in temperature modifies the positions of the atoms slightly within the crystal structure, such that the polarization of the material changes (gives rise to voltage)**
- **If the temperature is kept constant, voltage drops to zero (leakage current)**

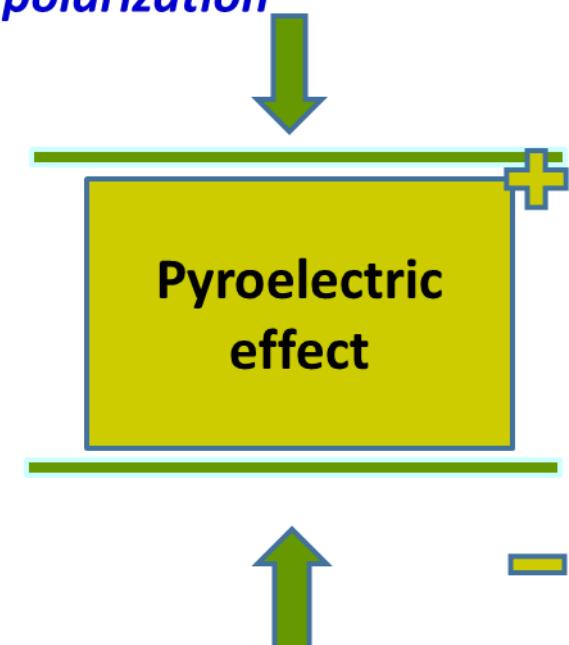
▪ **The pyroelectric coefficient** $P_i = \frac{\partial P_s}{\partial T}$

polarization vector P_s
with temperature T

Applications

- Products ranging from fire alarms to intruder detectors
- Energy harvesting
- Thermal imaging
- Radiometry
- Solar energy pyroelectric converter
- PIR remote-based thermometer
- PIR – based motion detectors

Change in temperature and polarization



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Ferroelectric materials: Ferroelectric Hysteresis and application as memory materials

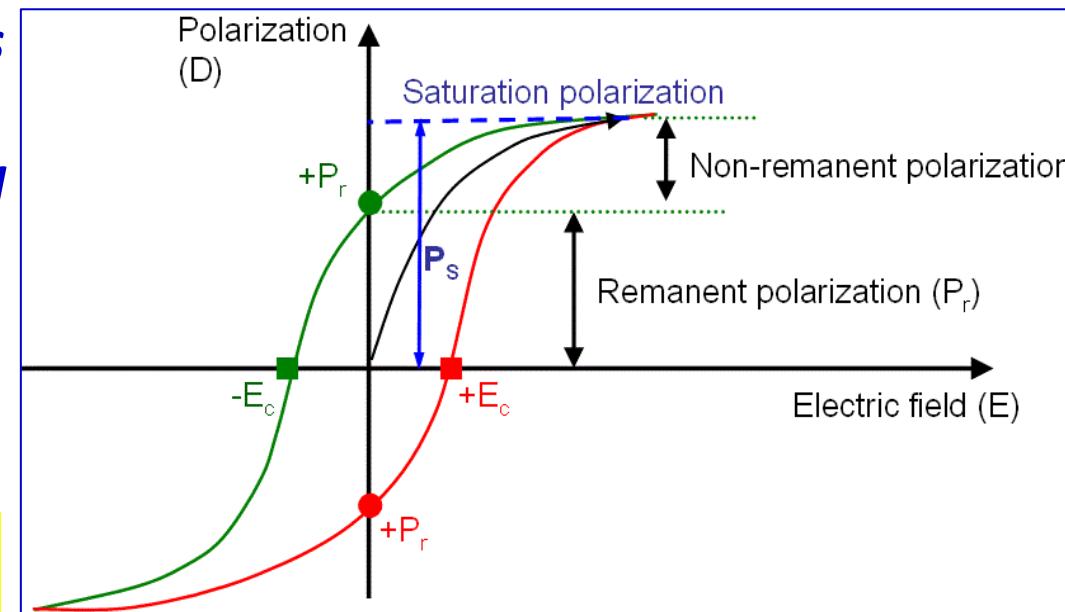
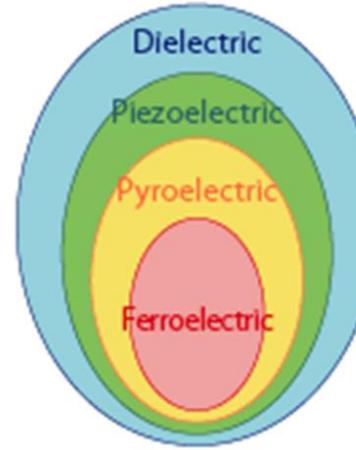


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- *Ferroelectrics are a class of non-centro symmetric crystals show non-linear polarization and subclass of the pyro electric / piezoelectric materials*
- *They show spontaneous polarization even in the absence of an electric field*
- *In the presence of electric field (E), ferroelectrics display a nonlinear response of polarization (P) and display hysteresis in the P v/s E variations (remember ferromagnetic hysteresis)*
- *P v/s E hysteresis has memory effect and used extensively in 'DRAMs' and 'SRAMs'*
- *$BaTiO_3$ is a classic example of a perovskite material*

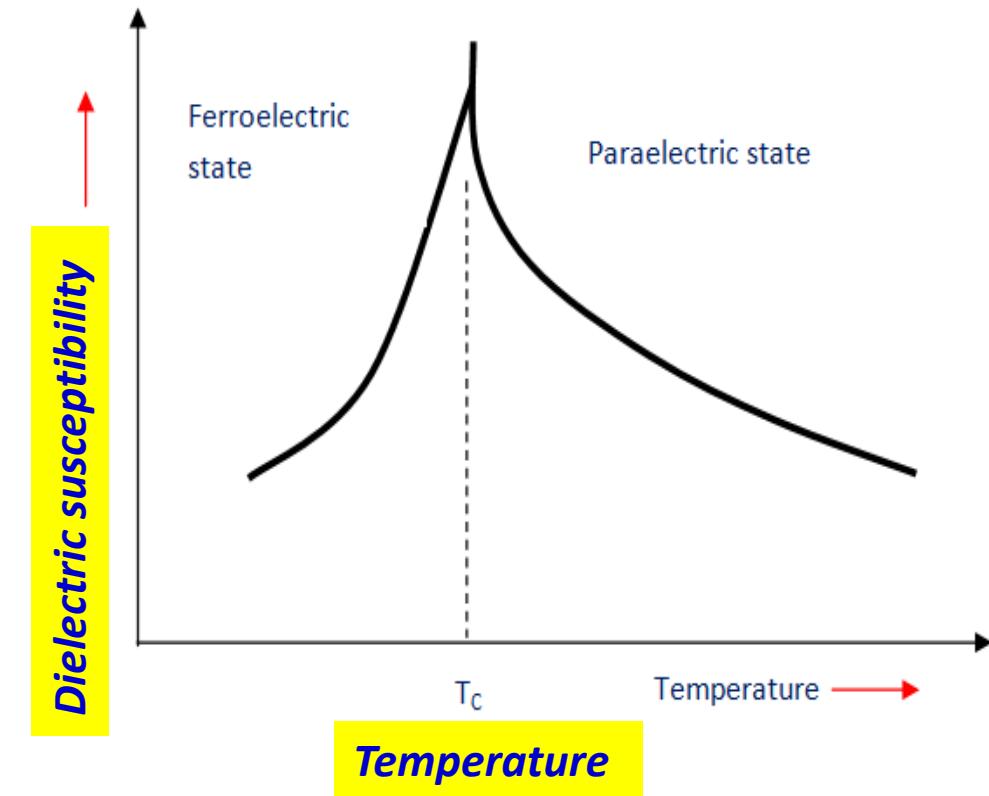
Applications of Ferroelectric materials

- Non volatile memory devices
- Photovoltaic devices
- Ferroelectric capacitors
- Ferroelectric liquid crystal display
- Sensors and actuators



Susceptibility (or dielectric constant) variation with temperature for Ferroelectric materials

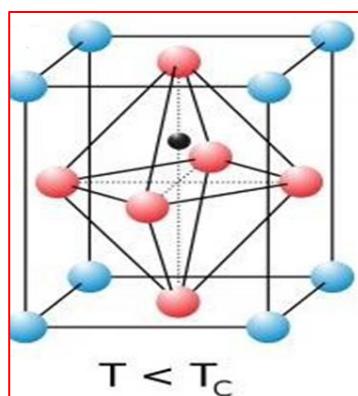
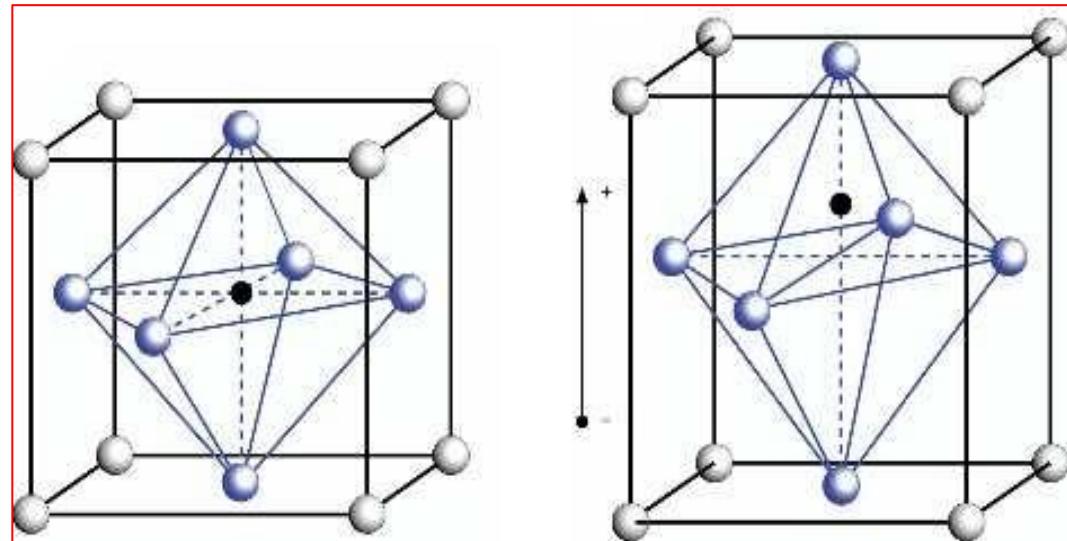
- The dielectric susceptibility ferro electric material is highly temperature dependent
- Below Curie temperature (T_c), the material shows spontaneous polarization
- At temperatures greater than T_c the material is para-electric



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Ferroelectric - BaTiO_3 : An Important Non-centro Symmetric Ferro-electric material (phase transition in barium titanate)

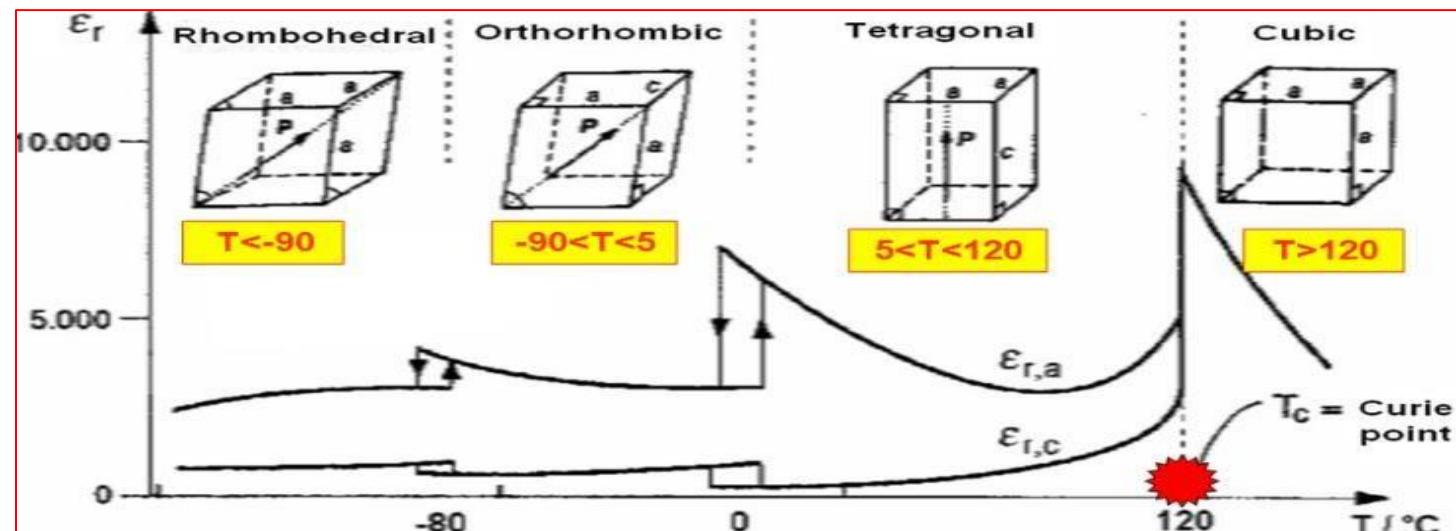
- BaTiO_3 is a classic example of non centro symmetric ferro-electric crystal
- It has wide operating temperature:
above 120°C , between 5°C to 120°C , below 5°C to -90°C and below -90°C
- Exhibits *phase transition*
Above 120°C (above Curie temperature) –
behave as **paraelectric (as linear dielectric)**
Below 120°C to 5°C behave as **ferroelectric**
Below 5°C **varying ferroelectric behavior**



Perovskite structure: ABO_3 Structure
(A = Barium (occupy corner),
B=Titanium (occupy center of the unit cell), Six Oxygen (occupy face center))

Ferroelectric - BaTiO₃: An Important Non-centro Symmetric Ferro-electric Material – Phase transition

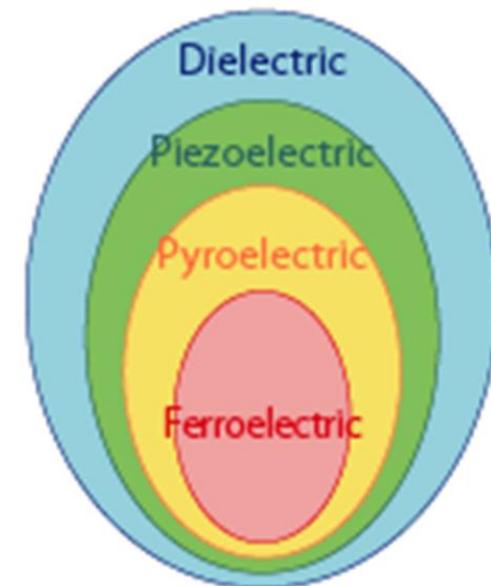
- The unit cell is cubic above the curie the temperature of about 120°C, and behaves as a paraelectric (para-electric phase material behaves as a linear dielectric)
- Between 5°C and 120°C the material is in the tetragonal phase : Ti ions are slightly displaced from the center of the crystal
- Below 5°C to -90°C the material has an orthogonal phase and below -90°C the material is in the orthorhombic phase
- In all these phases also the material exhibit ferroelectric behavior in varying proportions





- The piezoelectric materials used as a transducer
- Ferroelectric crystals are non-centro symmetric crystals
- Pyro-electric crystals are sensitive to applied pressure
- The structure of Barium Titante in between the temperature range -80 to 0° C is orthorhombic in nature
- Barium Titanite has Perovskite crystal structure
- BaTiO₃ is a good example of a centro symmetric crystal.

- All Piezoelectric materials are Ferroelectric
- The dielectric polarization in pyro electric materials depends on temperature
- Pyro electric crystals are centro symmetric crystals
- Transducer converts one form of energy into another
- In a pyroelectric material, the voltage developed due to change in temperature remains constant if the temperature is maintained constant



The concepts related to this class which are true are...

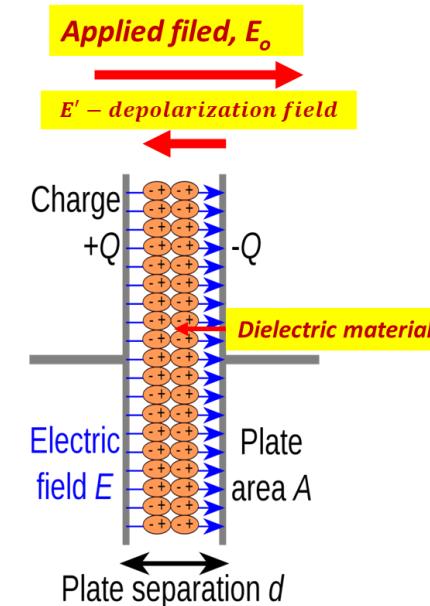
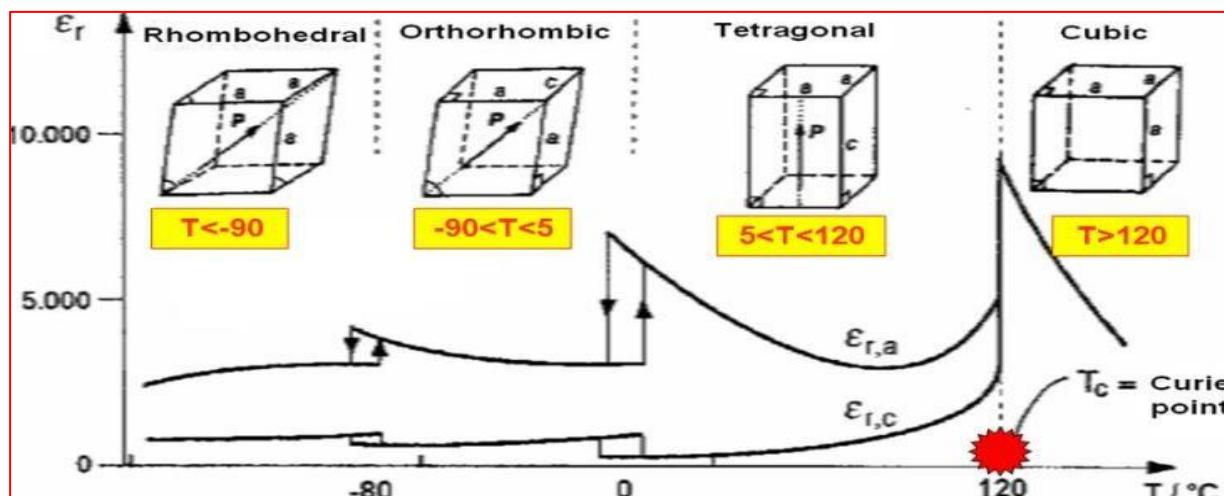
1. Spontaneous polarization exists in all dielectric materials
2. In ferroelectric materials, there exists more than one spontaneous polarization directions
3. At temperatures above Curie temperature, the material behaves as ferroelectric
4. Domain wall orientations are dictated by crystal symmetry
5. Remnant polarization in ferroelectric materials are used to store data

Derive an expression for polarization of a dielectric in an external electric field 'E'.

$$\text{The net electric field } E = E_0 - E' = \epsilon_r E - \frac{\sigma_p}{\epsilon_0}$$

- Simplifying $\sigma_p = \epsilon_0 \epsilon_r E - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E$

Discuss the phase transition of BaTiO₃ (Barium titanate) and explain the concept of Curie temperature.



- Exhibits phase transition

Above $120^\circ C$ (above Curie temperature)—behave as paraelectric (as linear dielectric)

Below $120^\circ C$ to $5^\circ C$ behave as ferroelectric

Below $5^\circ C$ varying ferroelectric behavior



What is meant by dielectric polarization? List the various kinds of polarization mechanisms that prevail in dielectric materials and how do these polarization mechanisms vary with temperature?

Write a note on piezoelectric and pyroelectric materials

- *Appearance of electric potential when pressure is applied*
- *Piezo electric materials also show invers piezo electric*

The microscopic origin of piezoelectricity: displacement of ionic charges (polarization) within the crystal with applied strain

- *Pyroelectricity is the ability to generate electrical potential when heated or cooled*
- *Pyroelectric materials are non-centrosymmetric non linear dielectrics*
- *The change in temperature modifies the positions of the atoms slightly within the crystal structure, such that the polarization of the material changes (gives rise to voltage)*

What are the components of electric fields that prevail in a dielectric and how do they affect the polarization of the material?

$$E_{loc} = E_0 + E_1 + E_2 + E_3$$

$E_0 \rightarrow$ External electric field

$E_1 \rightarrow$ Depolarization field

$E_2 \rightarrow$ Lorentz field on the surface of the spherical cavity

$E_3 \rightarrow$ Internal field due to other dipoles lying within the sphere

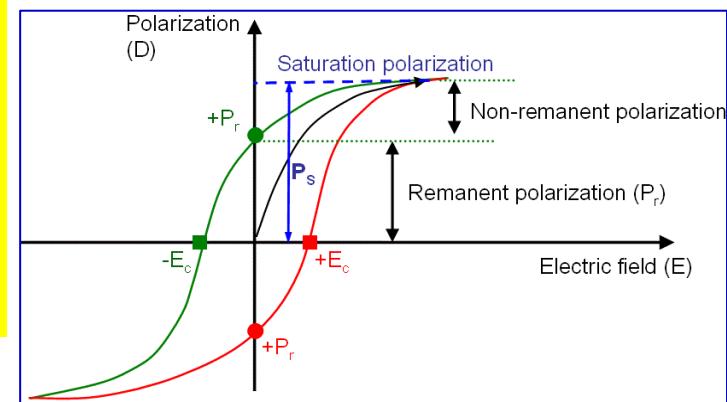
A solid has 5×10^{28} atoms per unit volume and a dielectric constant of

4. Estimate the polarizability.

$$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0}$$

Discuss ferroelectric materials with a suitable example and discuss the hysteresis in the polarization with varying field

- Ferroelectrics are a class of non-centro symmetric crystals show non-linear polarization and subclass of the pyro electric / piezoelectric materials
- They show spontaneous polarization even in the absence of an electric field
- In the presence of electric field (E), ferroelectrics display a nonlinear response of polarization (P) and display hysteresis in the P v/s E variations (remember ferromagnetic hysteresis)



Discuss the BaTiO₃ unit cell in the ferroelectric phase. What is the behavior of the system above Curie temperature.

- BaTiO₃ is a classic example of non centro symmetric ferro-electric crystal
- It has wide operating temperature:

above 120°C, between 5°C to 120°C, below 5°C to -90°C and below -90°C

- Exhibits phase transition

Above 120°C (above Curie temperature) – behave as paraelectric (as linear dielectric)

Below 120°C to 5°C behave as ferroelectric

Below 5°C varying ferroelectric behavior



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THANK YOU

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