1. The probability density function of a random variable X is

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

Find $P(X < 4)$; $P(X \geq 5)$; $P(3 < X \leq 6)$. What will be the minimum value of k so that $P(X \leq 2) > 0.3$.

A.

$\Sigma P(X) = 1 \Rightarrow K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$49K = 1 \Rightarrow K = 1/49$

So, $P(X<4) = K + 3K + 5K + 7K = 16K = \frac{16}{49}$

$P(X \geq 5) = 11K + 13K = 24K = \frac{24}{49}$

$P(3 < X \leq 6) = 9K + 11K + 13K = 33K = \frac{33}{49}$

And, $P(X \leq 2) = K + 3K + 5K = 9K$

$P(X \leq 2) > 0.3$

$9K > 0.3$

$K > \frac{0.3}{9}$

$K > \frac{1}{30}$

Min. value of $K = 1/30$

2. A coin is tossed three times. Let X denote the number of heads showing up. Find the distribution of X. Also, find its mean and variance.

A.

0 Heads $\Rightarrow \frac{1}{2}(T) \times \frac{1}{2}(T) \times \frac{1}{2}(T)$

1 Head $\Rightarrow \left(\frac{1}{2}(H) \times \frac{1}{2}(T) \times \frac{1}{2}(T)\right) + \left(\frac{1}{2}(T) \times \frac{1}{2}(H) \times \frac{1}{2}(T)\right) + \left(\frac{1}{2}(T) \times \frac{1}{2}(T) \times \frac{1}{2}(H)\right)$

2 Heads $\Rightarrow \left(\frac{1}{2}(H) \times \frac{1}{2}(H) \times \frac{1}{2}(T)\right) + \left(\frac{1}{2}(H) \times \frac{1}{2}(T) \times \frac{1}{2}(H)\right) + \left(\frac{1}{2}(T) \times \frac{1}{2}(H) \times \frac{1}{2}(H)\right)$

3 Heads $\Rightarrow \frac{1}{2}(H) \times \frac{1}{2}(H) \times \frac{1}{2}(H)$

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

$\mu = \Sigma x_i p_i = \frac{3}{2}$

$\sigma^2 = \Sigma x_i^2 p_i - \mu^2$

$= 3 - \frac{9}{4} = \frac{3}{4}$

(Use Calculator)

(For 991-CW ⇒ Statistics > 2 Variable)