UNIT - 1

1) $E = BC$

let $E_x = E_0 \sin(kz - wt)$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}\left(-\frac{\partial E_x}{\partial z}\right) + \hat{k}\left(-\frac{\partial E}{\partial y}\right)$$

$$= \hat{j}\frac{\partial E_x}{\partial z} - \hat{k}\frac{\partial E_x}{\partial y}$$

$$= \hat{j}\frac{\partial}{\partial z}(E_0 \sin(kz - wt)) - \hat{k}\frac{\partial}{\partial y}\cancel{(E_0 \sin(kz - wt))}^{0}$$

$$= \hat{j}\,(kE_0 \sin(kz - wt))$$

$$-\frac{\partial B}{\partial t} = E_0 k \sin(kz - wt)$$

$$\cancel{-}B = \cancel{-}E_0 \frac{k}{w}\cos(kz - wt)$$

$$B = \frac{E_0}{c}\cos(kz - wt) \qquad c = \frac{w}{k}$$

$$B = \frac{E_0}{c}$$

2) MAXWELL EQUATIONS

i) wave Eqn for E-Field

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial B}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E}^2 = -\frac{\partial}{\partial t}\left(\mu_0\hat{j} + \mu_0\varepsilon_0\frac{\partial E}{\partial t}\right)$$

$$\vec{\nabla}\cancel{\left(\frac{\rho}{\varepsilon_0}\right)}^{0} - \vec{\nabla}^2\vec{E}^2 = -\frac{\partial}{\partial t}\left(\cancel{\mu_0\hat{j}}^{0} + \mu_0\varepsilon_0\frac{\partial E}{\partial t}\right)$$

$$\cancel{+}\nabla^2\vec{E} = \cancel{+}\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}}$$

ii) Wave Eqn for B-Field

$$\nabla\times\vec{B} = \mu_0\hat{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{B}}^{0}) - \nabla^2\vec{B}^2 = -\frac{\partial}{\partial t}\left(\cancel{\mu_0\hat{j}}^{0} + \mu_0\varepsilon_0\frac{\partial B}{\partial t}\right)$$

$$-\nabla^2\cdot\vec{B} = -\mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$$

$$\boxed{\nabla^2\cdot\vec{B} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}}$$

2) **AVERAGE ENERGY** & POYNTING VECTOR

Energy in inductor,

$$E_m = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{N^2\mu_0 A}{l}\right)\left(\frac{B^2 l^2}{\mu_0^2 N^2}\right) = \frac{1}{2}\frac{B^2 A l}{\varepsilon_0}$$

per unit volume,

$$E_M = \frac{1}{2}\frac{B^2}{\varepsilon_0}$$

Energy in capacitor,

$$E_e = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(E^2 d^2) = \frac{1}{2}\epsilon_0 (Ad) E^2$$

per unit volume,

$$E_e = \frac{1}{2}\epsilon_0 E^2$$

Total energy / unit volume,

$$E = E_e + E_m$$

$$= \frac{1}{2}\frac{B^2}{\mu_0} + \frac{1}{2}\epsilon_0 E^2$$

$$= \frac{1}{2}\left(\frac{E^2}{c^2}\right)\frac{1}{\mu_0} + \frac{1}{2}\epsilon_0 E^2 \qquad B = E/c$$

$$= \frac{1}{2}\frac{\mu_0 \epsilon_0 E^2}{\cancel{\mu_0}} + \frac{1}{2}\epsilon_0 E^2 \qquad c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\boxed{E = \epsilon_0 E^2}$$

Poynting vector:

$$\text{flux} = \frac{E \times cAt}{At} = E \times c = \epsilon_0 E^2 c$$

$$= c\,\epsilon_0 E (Bc) = c^2 \epsilon_0 EB$$

$$\therefore \boxed{\vec{S} = c^2 \epsilon_0 \vec{E} \times \vec{B}}$$

4) RAYLEIGH - JEANS LAW

dN b/w ν and $\nu + \delta\nu$

$$dN = \frac{8\pi}{c^3} \nu^3 \, d\nu$$

dN per unit volume $= \frac{8\pi}{c^3} \nu^2 \, d\nu$

Avg. energy of oscillators : (Maxwell Boltzmann law)

$$\langle E \rangle = K_B T$$

Energy density :

$$\rho(\nu)\, d\nu = \langle E \rangle \cdot dN = \boxed{\frac{8\pi}{c^3} \nu^2 d\nu \; K_B T}$$

5) PLANCK'S RADIATION LAW

energy of oscillators are multiples of frequency times a constant. ie. $E = nh\nu$

Avg. energy of oscillators :

$$\langle E \rangle = \frac{h\nu}{e^{h\nu/KT} - 1}$$

Energy density :

$$\langle E \rangle \cdot dN = \boxed{\frac{8\pi h\nu^3}{c^3} \quad \frac{1}{e^{h\nu/KT} - 1}}$$

6) COMPTON SHIFT

Rest mass of particle : $E = m_0 c^2$

Total energy of particle : $E = \sqrt{p^2c^2 + m_0^2c^4}$

Momentum conservation along incident direction :

$$p_i + 0 = p_f \cos\theta + p_e \cos\theta$$

Momentum conservation in $\perp^r$ direction :

$$0 = p_f \sin\theta - p_e \sin\phi$$

conservation of momentum before & after collision :

$$p_e^2 = p_i^2 + p_f^2 - 2p_ip_f \cos\theta \longrightarrow (1)$$

conservation of energy before and after collision :

$$p_ic + m_oc^2 = p_fc + \sqrt{p_e^2c^2 + m_o^2c^4}$$

$$p_e^2 = p_i^2 + p_f^2 - 2p_ip_f + 2m_oc(p_i - p_f) \longrightarrow (2)$$

compare (1) and (2)

$$-2p_ip_f + 2m_oc(p_i - p_f) = 2p_ip_f\cos\theta$$

put $p_i = \frac{h}{\lambda_i}$, $p_f = \frac{h}{\lambda_f}$

$$\boxed{\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_oc}(1 - \cos\theta)}$$

7) ~~DE-BROGLIE WAVE~~ HIESENBERG UNCERTAINTY PRINCIPLE

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$x \propto \frac{1}{h}$$

$$\Delta k \propto \frac{1}{\Delta x}$$

$$\Delta k \cdot \Delta x \sim \frac{1}{2}$$

$$\Delta\left(\frac{2\pi}{\lambda}\right) \cdot \Delta x \sim \frac{1}{2}$$

$$\Delta\left(\frac{2\pi}{h}\right) p \cdot \Delta x \sim \frac{1}{2}$$

$$\left(\frac{2\pi}{h}\right) \Delta p \cdot \Delta x \sim \frac{1}{2}$$

$$\Delta p \cdot \Delta x \sim \left(\frac{h}{2\pi}\right) \frac{1}{2}$$

$$\boxed{\Delta p \cdot \Delta x \geq \frac{\hbar}{2}}$$

8) PHASE AND GROUP VELOCITY

$$V_p = \frac{w}{k} \quad \text{and} \quad V_g = \frac{dw}{dk}$$

$$V_g = \frac{d}{dk}(V_p \cdot k)$$

$$= k\frac{dV_p}{dk} + V_p\frac{dk}{dk}$$

$$= k\frac{dV_p}{dk} + V_p$$

$$= k \cdot \frac{dV_p}{d\lambda} \cdot \frac{d\lambda}{dk} + V_p$$

$$\frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2}$$

$$= \frac{\cancel{2\pi}}{\cancel{\lambda}} \left(\frac{-\lambda^{\cancel{2}}}{\cancel{2\pi}} \right) \frac{dV_p}{d\lambda} + V_p$$

$$\boxed{V_g = V_p - \lambda \frac{dV_p}{d\lambda}}$$

$$k = \frac{2\pi}{\lambda} = \left(\frac{2\pi}{h}\right) p = \frac{p}{\hbar} \quad \Rightarrow \quad p = \hbar k$$

$$w = 2\pi v = \frac{2\pi E}{h} = \frac{E}{\hbar} \quad \Rightarrow \quad E = \hbar w$$

$$V_g = \frac{dw}{dk} = \frac{d(E/\hbar)}{d(p/\hbar)} = \frac{dE}{dp}$$

Energy:

$$E = KE + PE = \frac{p^2}{2m} + V(x)$$

$$\frac{dE}{dp} = \frac{2p}{2m} + 0$$

$$\frac{dE}{dp} = \frac{p}{m}$$

$$\frac{dE}{dp} = \frac{mv}{m}$$

$$\frac{dE}{dp} = v$$

$$\boxed{V_g = \frac{dE}{dp} = v}$$

9) UNCERTAINTY PRINCIPLES

position momentum uncertainty

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

energy time uncertainty

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

circular motion uncertainty

$$\Delta \theta \cdot \Delta L \geq \frac{\hbar}{2}$$

→ SCHRÖDINGER'S TIME DEPENDENT WAVE EQN (1D)

General form of wave fn. in 1D.

$$\Psi(x,t) = e^{i/\hbar (px - Et)}$$

Total energy of system,

$$E = KE + V$$

This eqn. remains invarient when multiplied with $\Psi(x,t)$

$$E\Psi(x,t) = KE\Psi(x,t) + V\Psi(x,t)$$

eigen value eqn.
$\hat{G}\Psi = G\Psi$

$$\hat{E}\Psi(x,t) = \widehat{KE}\,\Psi(x,t) + V\Psi(x,t)$$

rewritten in terms of operators.

$$i\hbar \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi$$

$$\boxed{\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + i\hbar\frac{d\Psi}{dt} - V\Psi = 0}$$

- momentum operator

$$\hat{p} = \left\{-i\hbar \frac{\partial}{\partial x}\right\}$$

- KE operator

$$\widehat{KE} = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right\}$$

- total energy operator

$$\hat{E} = \left\{i\hbar \frac{\partial}{\partial t}\right\}$$

→ SCHRÖDINGER'S TIME INDEPENDENT WAVE EQN

for steady state system → time independent

$$\psi(x,t) = e^{i/\hbar (px - Et)}$$

$$\psi(x,t) = Ae^{i/\hbar (px)} \cdot Ae^{-i/\hbar (Et)}$$

$$\psi(x,t) = \psi(x) \cdot \phi(t)$$

↳ time dependent component

↳ space dependent component

substituting $\psi(x,t) = \psi(x) \cdot \phi(t)$ in time dependent eqn.

$$\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + i\hbar \frac{d\psi}{dt} - V\psi = 0$$

$$\frac{\hbar^2}{2m} \frac{d^2\psi(x) \cdot \phi(t)}{dx^2} + i\hbar \frac{d\psi(x) \cdot \phi(t)}{dt} - V\psi(x) \cdot \phi(t) = 0$$

$$\frac{\hbar^2}{2m} \frac{d^2\psi(x) \cdot \phi(t)}{dx^2} + E\psi(x) \cdot \phi(t) - V\psi(x) \cdot \phi(t) = 0$$

$$\left\{ \frac{\hbar^2}{2m} \cdot \frac{d^2\psi(x)}{dx^2} + E\psi(x) - V\psi(x) \right\} * \phi(t) = 0$$

product is zero implies either one is zero.

$$\boxed{\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + E\psi(x) - V\psi(x) = 0}$$

$$\boxed{\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi(x) = 0}$$

PHYSICS UNIT - 2

→ FREE PARTICLE

free particle → no force

$$F = 0 \Rightarrow -\frac{dV}{dx} = 0$$

$$V = 0 \quad \text{or} \quad V = \text{constant}$$

General schrodinger wave eqn.

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi(x) = 0$$

when $V = 0$,

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi(x) = 0$$

$$\boxed{\frac{d^2\psi(x)}{dx^2} + k^2\psi = 0}$$

where $k^2 = \dfrac{2mE}{\hbar^2}$

General soln. is of the form

$$\psi = Ae^{ikx} + Be^{-ikx}$$

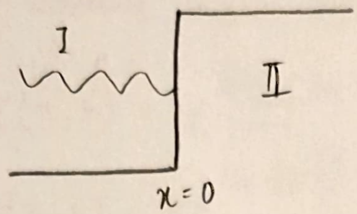
Ae^{ikx} → moving in +ve x direction

Be^{-ikx} → moving in -ve x direction

Energy of wave.

$$E = \frac{\hbar^2 k^2}{2m}$$

→ STEP POTENTIAL



1) **case 1 :** $E > V_0$

SWE in Region I : $x < 0$; $V = 0$

$$\frac{d^2\psi_I(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi_I(x) = 0$$

$V = 0$:

$$\frac{d^2\psi_I(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi_I(x) = 0$$

$$\frac{d^2\psi_I(x)}{dx^2} + k_I^2\psi_I(x) = 0 \qquad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

general soln :

$$\boxed{\psi_I = Ae^{ik_1x} + Be^{-ik_1x}}$$

Ae^{ik_1x} → incident wave

Be^{-ik_1x} → reflected wave

SWE in Region II : $x > 0$; $V = V_0 < E$, $(E - V_0)$

$$\frac{d^2\psi_{II}(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi_{II}(x) = 0$$

$$\frac{d^2\psi_{II}(x)}{dx^2} + k_{II}^2\psi_{II}(x) = 0 \qquad k_{II} = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

General soln:

$$\psi_{II}(x) = De^{ik_{II}x} + Fe^{-ik_{II}x}$$

Beyond $x=0$, no wave disruption, $Fe^{-ik_{II}x} = 0$

$$\boxed{\psi_{II}(x) = De^{ik_{II}x}}$$

$De^{ik_{II}x} \rightarrow$ transmitted wave

Reflection coefficient:

$$\boxed{R = \left(\frac{k_I - k_{II}}{k_I + k_{II}}\right)^2}$$

$$\boxed{R + T = 1}$$

Transmission coefficient:

$$\boxed{T = \frac{4k_I k_{II}}{(k_I + k_{II})^2}}$$

2) case II: $E < V_0$

Region I: $x<0$; $V=0$

SWE:

$$\frac{d^2\psi_I(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi_I(x) = 0$$

$$\frac{d^2\psi_I(x)}{dx^2} + k_I^2\psi_I(x) = 0 \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}}$$

General soln:

$$\boxed{\psi_I = Ae^{ik_I x} + Be^{-ik_I x}}$$

Region II : $x > 0$; $V = V_0 > E$ $(E - V_0) \to -ve$

SWE :

$$\frac{d^2\psi_{II}(x)}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi_{II}(x) = 0$$

$$\frac{d^2\psi_{II}(x)}{dx^2} - \alpha^2\psi_{II}(x) = 0 \qquad \alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

General soln :

$$\psi_{II}(x) = Fe^{\alpha x} + Ge^{-\alpha x}$$

$\psi_{II}(x)$ should be finite for all values of x.

$F = 0 \Rightarrow$

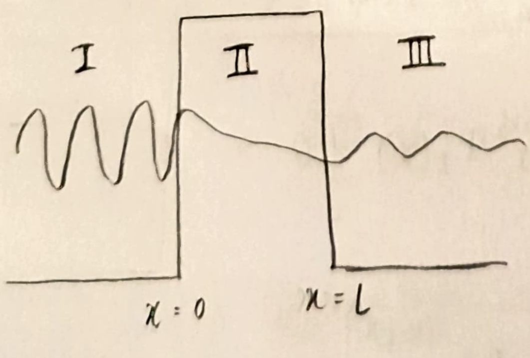
$$\boxed{\psi_{II}(x) = Ge^{-\alpha x}}$$

↳ exponentially decaying function

Penetration Depth :

$$\Delta x = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(V_0 - E)}}$$

→ FINITE POTENTIAL BARRIER



Region I : $V = 0$ for $x < 0$

Region II : $V = V_0$ for $0 < x < L$ ⟵ width of barrier

Region III : $V = 0$ for $x > L$

- **Region I : $x < 0$; $V = 0$**

$$\frac{d^2\psi_1(x)}{dx^2} + \frac{2m}{\hbar^2} E\psi_I(x) = 0$$

$$\frac{d^2\psi_I(x)}{dx^2} + k_I^2\psi_I(x) = 0 \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}}$$

General soln:

$$\boxed{\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}}$$

- **Region II : $x > 0$; $V = V_0 > E$** ; $(E - V_0) \rightarrow -ve$

$$\frac{d^2\psi_{II}(x)}{dx^2} - \frac{2m}{\hbar^2}(E - V_0)\psi_{II}(x) = 0$$

$$\frac{d^2\psi_{II}(x)}{dx^2} - \alpha^2\psi_{II}(x) = 0 \qquad \alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

General solution:

$$\boxed{\psi_{II}(x) = De^{-\alpha x}}$$

↳ exponentially decaying function

- **Region III : $x > L$; $V = 0$**

$$\frac{d^2\psi_{III}(x)}{dx^2} + \frac{2m}{\hbar^2} E\psi_{III}(x) = 0$$

$$\frac{d^2\psi_{III}(x)}{dx^2} + k_{III}^2\psi_{III}(x) = 0 \qquad k_{III} = \sqrt{\frac{2mE}{\hbar^2}}$$

General soln :

$$\boxed{\psi_{III}(x) = Ge^{ik_{III}x}}$$

Transmitted coefficient :

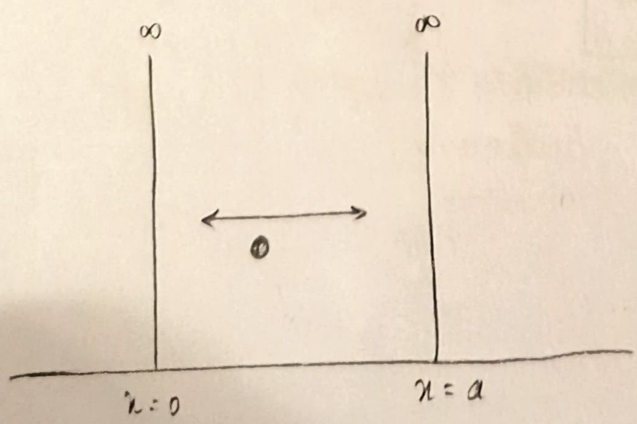
$$\boxed{T \approx e^{-2\alpha L}} \qquad \alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

→ **Applications of Barrier Tunneling**

- Alpha Decay (He_2^4)

alpha particles emitted from nucleus with energy of order 8 MeV .

→ **INFINITE POTENTIAL BARRIER**



Particle has zero probability of being outside $x=0$ & $x=a$

$V=0$ for $0<x<a$

$V=\infty$ at $x=0$ and $x=a$

SWE :

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}E\,\psi(x) = 0$$

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi = 0 \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

General soln :

$$\boxed{\psi(x) = A\sin(kx) + B\cos(kx)}$$

Boundary conditions :

$\psi = 0$ & $d\psi = 0$ at $x=0$ and $x=a$

- First boundary : $x=0$

$$\psi(x=0) = 0$$

$$B = 0$$

$$\psi(x) = A\sin(kx) + B\cos(kx) = A\sin(kx)$$

- second boundary : $x=a$

$$\psi(x=a) = A\sin(k \times a) = 0$$

possible only if $(ka) = n\pi$ or $k = \frac{n\pi}{a}$

$\psi(x) = A\sin(kx)$

$\psi(x) = A\sin\left(\frac{n\pi}{a}x\right)$

Eigen Energy Equation $\boxed{E = \frac{n^2h^2}{8ma^2}}$

NORMALIZATION

$\int \psi^* \psi \, dx = 1$

$\int_0^a \left[A\sin\left(\frac{n\pi}{a}x\right)\right]^2 dx = 1$

$\frac{A^2}{2}\int_0^a \left[1-\cos\left(\frac{2n\pi}{a}x\right)\right] dx = 1$ $\quad \sin^2 x = \frac{1-\cos 2x}{2}$

$\frac{A^2}{2}[a-0] = 1$

$A = \sqrt{\frac{2}{a}}$

$\therefore$ Eigen wave Function:

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}$$

General soln:

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}}\begin{cases}\sin\left(\frac{n\pi}{a}x\right) & n=2,4,6\ldots \\ \cos\left(\frac{n\pi}{a}x\right) & n=1,3,5\ldots\end{cases}}$$

$\rightarrow$ odd parity

$\rightarrow$ even parity

→ EIGEN FUNCTIONS

$$E = \frac{n^2 h^2}{8ma^2}$$

- $n = 1$

$$E_1 = \frac{h^2}{8ma^2} = E_0 \qquad \psi_1(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{\pi}{a}x\right)$$

- $n = 2$

$$E_2 = \frac{4h^2}{8ma^2} = 4E_0 \qquad \psi_2(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi}{a}x\right)$$

- $n = 3$

$$E_3 = \frac{9h^2}{8ma^2} = 9E_0 \qquad \psi_3(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{3\pi}{a}x\right)$$

- $n = 4$

$$E_4 = \frac{16h^2}{8ma^2} = 16E_0 \qquad \psi_4(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{4\pi}{a}x\right)$$

→ PROBABILITY DENSITIES :

- $$|\psi_1(x)|^2 = \frac{2}{a} \cos^2\left(\frac{\pi}{a}x\right)$$

$$|\psi_2(x)|^2 = \frac{2}{a} \sin^2\left(\frac{2\pi}{a}x\right)$$

$$|\psi_3(x)|^2 = \frac{2}{a} \cos^2\left(\frac{3\pi}{a}x\right)$$

$$|\psi_4(x)|^2 = \frac{2}{a} \sin^2\left(\frac{4\pi}{a}x\right)$$

→ PARTICLES IN 2D POTENTIAL

SWE in x direction:

$$\frac{d^2\Psi}{dx^2} + k_x^2\Psi = 0$$

SWE in y direction:

$$\frac{d^2\Psi}{dy^2} + k_y^2\Psi = 0$$

Eigen function of system:

$$\Psi_{n_x n_y} = \Psi_{n_x} \times \Psi_{n_y}$$

- First allowed state, ~~m~~ $n_x = n_y = 1$

$$\Psi_{11} = \frac{2}{a}\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}y\right)$$

- $n_x = 2,\ n_y = 1$

$$\Psi_{21} = \frac{2}{a}\cancel{\cos}\sin\left(\frac{2\pi}{a}x\right)\cos\left(\frac{\pi}{a}y\right)$$

- $n_x = 1,\ n_y = 2$

$$\Psi_{12} = \frac{2}{a}\cos\left(\frac{\pi}{a}x\right)\sin\left(\frac{2\pi}{a}y\right)$$

Energy eigen values:

$$E_{n_x n_y} = \frac{h^2}{8ma^2}\left(n_x^2 + n_y^2\right)$$

- $E_{11} = 2E_0$
- $E_{21} = E_{12} = 5E_0$

Diff. combination of quantum numbers, same energy value, wave functions diff. ⟶ DEGENERATE

in general, in 2D system,

when $n_x = n_y$ → single energy state

when $n_x \neq n_y$ → degeneracy factor of 2

→ PARTICLES IN 3D POTENTIAL

$$\Psi_{n_x n_y n_z} = \Psi_{n_x} \times \Psi_{n_y} \times \Psi_{n_z}$$

- $n_x = 1, n_y = 1, n_z = 1$

$$\Psi_{111} = \left(\frac{2}{a}\right)^{3/2} \cos\left(\frac{\pi}{a}x\right) \cos\left(\frac{\pi}{a}y\right) \cos\left(\frac{\pi}{a}z\right)$$

- $n_x = 2, n_y = 1, n_z = 1$

$$\Psi_{211} = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{2\pi}{a}x\right) \cos\left(\frac{\pi}{a}y\right) \cos\left(\frac{\pi}{a}z\right)$$

- $n_x = 1, n_y = 2, n_z = 1$

$$\Psi_{121} = \left(\frac{2}{a}\right)^{3/2} \cos\left(\frac{\pi}{a}x\right) \sin\left(\frac{2\pi}{a}y\right) \cos\left(\frac{\pi}{a}z\right)$$

- $n_x = 1, n_y = 1, n_z = 2$

$$\Psi_{112} = \left(\frac{2}{a}\right)^{3/2} \cos\left(\frac{\pi}{a}x\right) \cos\left(\frac{\pi}{a}y\right) \sin\left(\frac{2\pi}{a}z\right)$$

Energy :

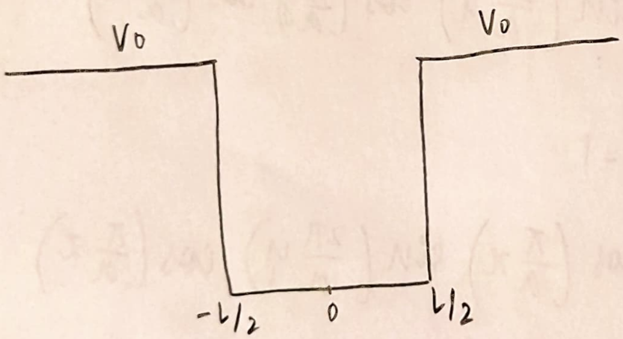
$$E_n = \frac{h^2}{8ma^2}(n_x^2 + n_y^2 + n_z^2)$$

- $E_{111} = 3E_0$
- $E_{211} = 6E_0$
- $E_{121} = 6E_0$
- $E_{112} = 6E_0$

$\left.\begin{matrix} E_{211} \\ E_{121} \\ E_{112} \end{matrix}\right\}$ triply degenerate

- when $n_x = n_y = n_z$ → no degeneracy (singleton)
- when 2 are equal → degeneracy factor 3
- when 3 are unequal → degeneracy factor 6

→ PARTICLE IN 1D POTENTIAL WELL



Region I : $V = V_0$ for $x < -L/2$

Region II : $V = 0$ for $-L/2 < x < L/2$

Region III : $V = V_0$ for $x > L/2$

$$E_{infinite} = \frac{n^2h^2}{8ma^2}$$

$$E_{finite} = \frac{n^2h^2}{8m(a+2\Delta x)^2}$$

$$\boxed{E_{finite} < E_{infinite}}$$

→ HARMONIC OSCILLATOR

Mechanical energy oscillates b/w PE and KE.

$$F = ma = m\frac{d^2x}{dt^2}$$

Restoring force,

$$F_r = -kx$$

$$w = \sqrt{\frac{k}{m}} \; ; \; k = mw^2$$

$$F_r = -kx = -\frac{dV}{dx}$$

P.E. of system.

$$V(x) = \frac{1}{2}kx^2$$

Eigen function of Harmonic oscillator:

$$\Psi_n(x) = N H_n(X) e^{-1/2(x)^2}$$

$N \rightarrow$ normalization constant

$H_n(X) \rightarrow$ Hermite polynomials

UNIT - 3

1) ELECTRICAL CONDUCTIVITY

$$I = neAV_d$$

$$J = \frac{I}{A} = neV_d$$

$$\sigma = \frac{J}{E} = \frac{neV_d}{E} = \frac{ne \cdot e\cancel{E}\tau}{m\cancel{E}} = \frac{ne^2\tau}{m}$$

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$

$$\boxed{\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}}$$

2) FERMI - FACTOR

$$\boxed{f(E) = \frac{1}{e^{\frac{E-E_f}{KT}} + 1}}$$

- at 0K,

  $E < E_f \Rightarrow f(E) = 1$

  $E > E_f \Rightarrow f(E) = 0$

- $T > 0K$,

  $E = E_f \Rightarrow f(E) = 1/2$

  $E < E_f \Rightarrow f(E) = 0.5$ to 1

  $E > E_f \Rightarrow f(E) = 0.5$ to 0

3> DENSITY OF STATES

$$\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + E\psi(x) = 0$$

$$\frac{\hbar^2}{2m} \frac{d^2\psi(y)}{dy^2} + E\psi(y) = 0$$

$$\frac{\hbar^2}{2m} \frac{d^2\psi(z)}{dz^2} + E\psi(z) = 0$$

Eigen energy values :

$$E_{nx} = \frac{h^2 n_x^2}{8mL^2}$$

$$E_{ny} = \frac{h^2 n_y^2}{8mL^2}$$

$$E_{nz} = \frac{h^2 n_z^2}{8mL^2}$$

Total energy :

$$E_n = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2)$$

$$E_n = \frac{h^2}{8mL^2} R^2$$

volume of shell of thickness dR

$$= \frac{\pi R^2 dR}{2}$$

$$E_n = E_0 R^2$$

$$dR = \frac{dE}{2(E_n E_0)^{1/2}}$$

no. of energy states b/w E and $E+dE$

$$\frac{\pi R^2 dR}{2} = \frac{\pi}{4} \frac{E_n}{E_0} \frac{dE}{(E_n E_0)^{1/2}}$$

$$= \frac{\pi}{4} \frac{En^{1/2}}{E_0^{3/2}} dE$$

$$g(E)dE = \frac{\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{3/2} En^{1/2} dE$$

per unit volume,

$$\boxed{g(E)dE = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{3/2} E^{1/2} dE}$$

4> FERMI ~~AVERAGE~~ ENERGY

Density of occupied states:

$$N(E) = g(E) * Fd$$

free e^- conc.

$$n = \int_0^{E_f} g(E) * Fd \, dE$$

$$= \int_0^{E_f} g(E) \, dE$$

$$= \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{3/2} \int_0^{E_f} E^{1/2} dE$$

$$\boxed{n = \frac{\pi}{3} \left(\frac{8m}{h^2} \right)^{3/2} E_f^{3/2}}$$

$$\boxed{E_f = \left(\frac{3}{\pi} \right)^{2/3} \left(\frac{h^2}{8m} \right) n^{2/3}}$$

5) AVERAGE ENERGY

$$\text{avg. energy} = \frac{\text{total energy of } e^- \text{ in all energy states}}{\text{total no. of } e^-}$$

$$= \frac{\int_0^{E_f} g(E) * E * F_d \, dE}{\int_0^{E_f} g(E) * F_d \, dE}$$

$$= \frac{\cancel{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2}} \int_0^{E_f} E^{1/2} dE * E}{\cancel{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2}} \int_0^{E_f} E^{1/2} dE}$$

$$E = \frac{3}{5} E_f$$

$$\boxed{E = 0.6 \, E_f}$$

6) WIEDEMANN- FRANZ LAW

Thermal conductivity.

$$K = \frac{1}{3} \frac{CV}{V} v \cdot L$$

$$= \frac{1}{3} \cdot \frac{1}{V} \cdot \frac{\pi^2}{2} N_{eff} \frac{K_B^2 T}{E_f} v_F \cdot v_F \cdot \tau$$

$n = N/V$

$$= \frac{\pi^2}{6} n_{eff} \frac{K_B^2 T}{E_f} v_F^2 \tau$$

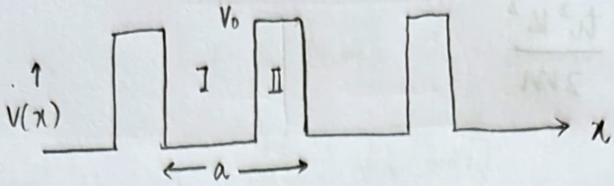
$E_f = \frac{1}{2} m v_f^2$

$$= \frac{\pi^2}{3} n_{eff} \frac{K_B^2 T}{m} \tau$$

$\sigma = \frac{n_{eff} e^2 \tau}{m}$

$$\boxed{\frac{K}{\sigma} = \frac{\pi^2}{3e^2} K_B^2 T}$$

7> KP MODEL



$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2} E\psi_1 = 0$$

$$\frac{d^2\psi_2}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi_2 = 0$$

put $a^2 = \frac{2m}{\hbar^2}(V_0 - E)$ and $k^2 = \frac{2mE}{\hbar^2}$

$$\frac{d^2\psi_1}{dx^2} + k^2\psi_1 = 0$$

$$\frac{d^2\psi_2}{dx^2} - a^2\psi_2 = 0$$

$$\psi(x) = e^{ikx} \cdot V_k(x)$$

solution exists if:

$$\cos(ka) = \frac{maV_0c}{\hbar^2}\frac{\sin(ka)}{ka} + \cos(ka)$$

$$\frac{maV_0c}{\hbar^2} \rightarrow P$$

- when $P \rightarrow \infty$, $k = \frac{n\pi}{a}$ $\Rightarrow$ $\boxed{E = \frac{\hbar^2\pi^2n^2}{2ma^2}}$

- when $P \rightarrow 0$, $k = k$ $\Rightarrow$ $\boxed{E = \frac{\hbar^2k^2}{2m}}$

8> EFFECTIVE MASS

Energy eqn. $E = \frac{\hbar^2 k^2}{2m}$

Diff. twice wrt. k.

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m^*}$$

$$\boxed{m^* = \left[\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right]^{-1}}$$

9> MAGNETIC MOMENT

current, $I = -\frac{e}{T} = \frac{-e\omega}{2\pi} = \frac{-ev}{2\pi r}$

$$\mu = IA$$

$$= \frac{-ev}{2\cancel{\pi}\cancel{r}} \cdot \cancel{\pi} r^{\cancel{2}}$$

$$= -\frac{1}{2} evr$$

$$= -\frac{1}{2} evr\left(\frac{m}{m}\right)$$

$$= \frac{-m_e evr}{2m_e}$$

$$\boxed{\mu = \frac{-eL}{2m_e}}$$

$mvr = L$

- orbital magnetic moment

$$L = \sqrt{l(l+1)}\,\hbar$$

$$\mu = \frac{e\hbar}{2m}\sqrt{l(l+1)}$$

$$\boxed{\mu = \mu_B \sqrt{l(l+1)}}$$

- spin magnetic moment

$$s = \pm\sqrt{s(s+1)}\,\frac{\hbar}{2}$$

$$\boxed{\mu = \frac{e\hbar}{m}\sqrt{s(s+1)}}$$

net magnetic moment :

$$\boxed{\mu = g_L \frac{e\hbar}{2m}}$$

10> LARMOR PRECESSION

$$\tau = \frac{dL}{dt}$$

$$= \mu \times B$$

$$= \mu B \sin\theta$$

$$= -\nu L B \sin\theta$$

$$= \frac{-eB}{2m} L \sin\theta$$

$\mu = -\nu L$

$\nu = \frac{e}{2m}$

$$\boxed{\tau = \omega_L \cdot L \sin\theta}$$

$\omega_L \rightarrow$ precessional frequency

11> CURIE WIESS LAW

$$\chi = \frac{M}{H} = \frac{C}{T}$$

$$\frac{M}{H_0 + \lambda M} = \frac{C}{T}$$

$$C(H_0 + \lambda M) = MT$$

$$CH_0 + C\lambda M = MT$$

$$CH = M(T - C\lambda)$$

$$\frac{M}{H} = \frac{C}{T - C\lambda}$$

$$\boxed{\chi = \frac{C}{T - T_c}}$$

$T_c \rightarrow$ curie temp.

UNIT - 4

1) EINSTEIN'S EQN

At thermal equilibrium,

Rate of abs. = Rate of R_{sp} + Rate of R_{st}

$$B_{12} \cdot N_1 \cdot \rho(\nu) = A_{21} \cdot N_2 + B_{21} \cdot N_2 \cdot \rho(\nu)$$

$$\rho(\nu)\left(B_{12} \cdot N_1 - B_{21} \cdot N_2\right) = A_{21} \cdot N_2$$

$$\rho(\nu) = \frac{A_{21} \cdot N_2}{B_{12} \cdot N_1 - B_{21} \cdot N_2}$$

$\div\ B_{21} N_2$

$$\rho(\nu) = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1}$$

$$\frac{N_2}{N_1} = e^{\Delta E/kT}$$

$$\boxed{\rho(\nu) = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} e^{\Delta E/kT} - 1\right)}}$$

from Planck's eqn,

$$\rho(\nu) = \frac{8\pi h\nu^3/c^3}{e^{\Delta E/kT} - 1}$$

comparing,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}$$

$$\frac{B_{12}}{B_{21}} = 1 \Rightarrow B_{12} = B_{21} = B$$

$$\boxed{\rho(\nu) = \frac{A/B}{e^{\Delta E/KT} - 1}}$$

2> **ROUND TRIP GAIN**

gain in photons, $I = I_0 e^{gx}$
loss in photons, $I = I_0 e^{-\alpha x}$

Resultant intensity at distance L

$$I = I_0 R_1 R_2 e^{(g-\alpha)2L}$$

$$\text{Gain} = \frac{\text{final intensity}}{\text{initial intensity}}$$

$$\text{Gain} = \frac{\cancel{I_0} R_1 R_2 e^{(g-\alpha)2L}}{\cancel{I_0}}$$

$$\text{Gain} = R_1 R_2 e^{(g-\alpha)2L}$$

consider gain = 1

$$1 = R_1 R_2 e^{(g-\alpha)2L}$$

$$\frac{1}{R_1 R_2} = e^{(g-\alpha)2L}$$

$$\ln\left(\frac{1}{R_1 R_2}\right) = (g-\alpha)2L$$

$$\boxed{g = \alpha + \frac{1}{2l} \ln\left(\frac{1}{R_1 R_2}\right)}$$

3> POLARIZATION

surface charge density (without dielectric)

$$\sigma = \varepsilon_0 E_0$$

with dielectric

$$(P \Rightarrow) \sigma_p = \varepsilon_0 E'$$

net E b/w plates

$$E = \frac{E_0}{\varepsilon_r}$$

$$E_0 = \varepsilon_r E$$

Net Electric field :

$$E = E_0 - E'$$

$$E = \varepsilon_r E - \frac{\sigma_p}{\varepsilon_0}$$

$$\sigma_p = \varepsilon_0 \varepsilon_r E - \varepsilon_0 E$$

$$\sigma_p = \varepsilon_0 (\varepsilon_r - 1) E$$

$$P = \varepsilon_0 (\varepsilon_r - 1) E$$

$$\boxed{P = \varepsilon_0 \chi E}$$ $\chi \rightarrow$ dielectric susceptibility

also, $\boxed{P = N \alpha E}$

$N \rightarrow$ atoms / volume
$\alpha \rightarrow$ polarizability

$$N \alpha E = \varepsilon_0 \chi E$$

$$\boxed{\chi = \frac{N\alpha}{\varepsilon_0}}$$

4) CLAUSIUS-MOSOTTI RELATION

$$E_{loc} = E + E_{in}$$

$$= E + \frac{P}{3\varepsilon_0}$$

$$P = N\alpha_e E_{loc}$$

$$= N\alpha_e \left(E + \frac{P}{3\varepsilon_0}\right)$$

$$= \varepsilon_0(\varepsilon_r - 1)E$$

solving,

$$\boxed{\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}}$$