ENGINEERING MATHEMATICS - II

Random variables and probability distributions

Random variables and probability distributions

- Exponential Distribution

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Exponential Distribution

- The exponential distribution is a continuous distribution that is sometimes used to model the time that elapses before an event occurs. Such a time is often called a waiting time.

- The exponential distribution is sometimes used to model the lifetime of a component.

- In addition, there is a close connection between the exponential distribution and the Poisson distribution.

Exponential Distribution

- The probability density function of the exponential distribution involves a parameter which is a positive quantity constant λ whose value determines the density function's location and shape.

- The probability density function of the exponential distribution with parameter $\lambda > 0$ is $f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$

- If X is a random variable whose distribution is exponential with parameter $\lambda > 0$, then $X \sim Exp(\lambda)$.

Exponential Distribution

- The cumulative distribution function of the exponential distribution is easy to compute.
- For $x \leq 0, F(x) = P(X \leq x) = 0$.
- For $x > 0$, the cumulative distribution function is

$$F(x) = P(X \leq x) = \int_0^x \lambda e^{-\lambda t}\, dt = 1 - e^{-\lambda x}.$$

Mean and variance of exponential random variable

- The mean and variance of an exponential random variable can be computed by using integration by parts.

- If $X \sim Exp(\lambda)$, then $\mu_X = \frac{1}{\lambda}$ and ${\sigma^2}_X = \frac{1}{\lambda^2}$

Problems:

If $X \sim Exp(2)$, then find μ_X; ${\sigma^2}_X$; and $P(X \leq 1)$.

$$\mu_X = \frac{1}{2} = 0.5;\ {\sigma^2}_X = \frac{1}{4} = 0.25;\ P(X \leq 1) = \int_0^1 e^{-2t}\,dt = 1 - e^{-2} = 0.865.$$

Problems:

In a certain town, the duration of a shower is exponentially distributed with the mean 5 minutes. What is the probability that shower will last for

a) less than 10 minutes b) 10 minutes or more; c) between 10 and 12 minutes.

$\mu_X = \frac{1}{5} = \lambda$ and x be the duration of the shower.

a) P(less than 10 minutes)=$P(x<10)$=0.8646.

b) P(10 min or more)=$P(x >=10)$= 0.1353.

c) P(between 10 and 12)=$P(10<x<12)$=0.0446.

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