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Assignment - 1

① $f(x) = \begin{cases} x & -\pi < x < 0 \\ -x & 0 < x < \pi \end{cases}$

Obtain fourier series coff a_0 using Euler's ~~fnc~~ formula for periodic fnc.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} x \cdot dx + \int_{0}^{\pi} (-x) \cdot dx \right]$$

$$= \frac{1}{\pi} \left[\left[\frac{x^2}{2} \right]_{-\pi}^{0} - \left[\frac{x^2}{2} \right]_{0}^{\pi} \right]$$

$$= \frac{1}{\pi} \left[\left[0 - \frac{\pi^2}{2} \right] - \left[\frac{\pi^2}{2} - 0 \right] \right]$$

$$= \frac{1}{\pi} \left[-\frac{\pi^2}{2} - \frac{\pi^2}{2} \right]$$

$$= \frac{1}{\cancel{\pi}} \left[\frac{-\cancel{2}\pi^{\cancel{2}}}{\cancel{2}} \right]$$

$$= -\pi$$

2. Obtain the Fourier co-eff using Euler's function.

$$f(x) = \begin{cases} 0 & -\pi < x < -\pi/2 \\ 1 & -\pi/2 < x < \pi/2 \\ 0 & \pi/2 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 1 \cdot dx$$

$$= \frac{1}{\pi}\left[x\right]_{-\pi/2}^{\pi/2} = \frac{1}{\pi}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{1}{\pi}\left[\frac{2\pi}{2}\right]$$

$$a_0 = 1$$

3. Obtain the fourier co-eff b_1 using Euler's formula

$$I = \begin{cases} I_0 \sin\theta & \text{for } 0 < \theta \le \pi \\ 0 & \text{for } \pi < \theta \le 2\pi \end{cases}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} f(x) \sin nx$$

$$b_1 = \frac{1}{\pi}\int_0^{\pi} I_0 \sin\theta \sin x$$

$$= \frac{I_0}{\pi}\int_0^{\pi} \sin^2\theta \cdot d\theta = \frac{I_0}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} = \frac{I_0}{2\pi}\int_0^{\pi} 1-\cos 2\theta$$

$$= \frac{I_0}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{I_0}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 - 0\right]$$

$$= \frac{I_0}{2\pi}\left[\pi\right] = b_1 = \frac{I_0}{2}$$

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Assignment - 2

Q1. Obtain the fourier series expansion of $f(x) = |\cos x|$ $(-\pi, \pi)$.

$$f(-x) = |\cos(-x)|$$
$$= \cos x$$

even function

$\therefore b_n = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \longrightarrow (1).$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

0 to $\pi/2 = \cos x$
$\pi/2$ to $\pi = -\cos x$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} |\cos x|\, dx$$
$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx\right]$$
$$= \frac{2}{\pi}\left[\sin x \Big|_0^{\pi/2} - \sin x \Big|_{\pi/2}^{\pi}\right]$$
$$a_0 = 4/\pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$$
$$= \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\, dx$$
$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \cos nx\, dx + \int_{\pi/2}^{\pi} -\cos x \cos nx\, dx\right]$$
$$= \frac{2}{\pi}\left[\frac{1}{2}\int_0^{\pi/2} [\cos(n+1)x + \cos(n-1)x]\, dx - \int_{\pi/2}^{\pi} \cos(n+1)x + \cos(n-1)x]\, dx\right]$$
$$= \frac{2}{\pi}\left[\frac{1}{2}\left(\frac{\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{(n-1)}\Big|_0^{\pi/2}\right) - \left[\frac{\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{n-1}\right]\Big|_{\pi/2}^{\pi}\right.$$

$$= \frac{1}{\pi}\left[\frac{\sin(n+1)\pi/2}{n+1} - \cancelto{0}{\frac{\sin(0)}{n+1}} + \frac{\sin(n-1)\pi/2}{n-1} - \cancelto{0}{\frac{\sin 0}{n-1}}\right.$$

$$\left. - \cancelto{0}{\frac{\sin(n+1)\pi}{n+1}} + \frac{\sin(n+1)\pi/2}{n+1} - \cancelto{0}{\frac{\sin(n-1)\pi}{n-1}} + \frac{\sin(n-1)\pi/2}{(n-1)}\right]$$

$$= \frac{1}{\pi}\left[\frac{2\sin(n+1)\pi/2}{n+1} + \frac{2\sin(n-1)\pi/2}{n-1}\right]$$

$$= \frac{2}{\pi}\left[\frac{\sin(n+1)\pi/2}{n+1} + \frac{\sin(n-1)\pi/2}{n-1}\right]$$

$\sin(90+\theta)$ $\qquad$ $\sin(90-\theta)$

$$= \frac{2}{\pi}\left[\frac{\sin\left(\pi/2 + n\pi/2\right)}{\cancel{x\pi}\, n+1} - \frac{\sin\left(\pi/2 - n\pi/2\right)}{n-1}\right]$$

$$= \frac{2}{\pi}\left[\frac{\cos n\pi/2}{n+1} - \frac{\cos n\pi/2}{n-1}\right]$$

$$= \frac{2}{\pi}\cos n\pi/2\left[\frac{n-1-n-1}{n^2-1}\right]$$

$$= \frac{-4}{\pi(n^2-1)}\cos n\pi/2$$

Here $\boxed{n \neq 1}$

$a_1 = 0$

$$|\cos x| = \frac{2}{\pi} - \sum_{n=2}^{\infty} \frac{4\cos n\pi/2}{\pi(n^2-1)}\cos nx.$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=2}^{\infty} \frac{\cos n\pi/2}{(n^2-1)}\cos nx$$

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Assignment-02

2. Find the Fourier series of $f(x) = |x|$ in $(-1,1)$

$(-1, 1)$

$(-l, l) \Rightarrow l = 1$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{1}\int_0^1 x\,dx.$$

$$= 2\left.\frac{x^2}{2}\right|_0^1$$

$$\boxed{a_0 = 1}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx.$$

$b_n = 0$ {even func}.

$$= 2\int_0^1 x\cos n\pi x\,dx.$$

$$= 2\left[x\frac{\sin n\pi x}{n\pi} - \left(-\frac{\cos n\pi x}{n^2\pi^2}\right)\right]_0^1 dx.$$

$$= \frac{2}{n^2\pi^2}\Big[\cos n\pi x\Big]_0^1$$

$$= \frac{2}{n^2\pi^2}\left[\cos n\pi - 1\right]$$

$$= \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_1^\infty a_n\cos\frac{n\pi x}{l}$$

$$\boxed{f(x) = \frac{1}{2} + \sum_1^\infty \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]\cos n\pi x}$$

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Unit4

Assignment-2 | Q.3

Q: Find the fourier series of $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$

Ans: We observed that,

$f(-x) = f(x)$; so, even function.

↳ b_n doesn't exist

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{2}\int_0^1 (1-x)\,dx = \frac{1}{2}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx = \frac{2}{2}\int_0^1 f(x)\cos\frac{n\pi x}{2}$$

$$= \int_0^1 (1-x)\cos\frac{n\pi x}{2} = \left[\frac{2(1-x)\sin\left(\frac{n\pi x}{2}\right)}{n\pi} - \frac{2\cdot 2(-1)\cos\left(\frac{n\pi x}{2}\right)}{n\pi\,(-n\pi)}\right]_0^1$$

$$= \left[\frac{-4\cos\left(\frac{n\pi x}{2}\right)}{n^2\pi^2}\right]_0^1 = \left[\frac{-4\cos\left(\frac{n\pi}{2}\right)}{n^2\pi^2} + \frac{4}{n^2\pi^2}\right]$$

$$a_n = 4\left[\frac{1-\cos\left(\frac{n\pi}{2}\right)}{n^2\pi^2}\right] = \frac{4}{n^2\pi^2}\left(1-\cos\frac{n\pi}{2}\right)$$

$$f(x) = \frac{a_0}{2} + \sum a_n\cos\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{1}{4} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(1-\cos\left(\frac{n\pi}{2}\right)\right)\cdot\cos\left(\frac{n\pi x}{2}\right)$$

Ans.

Unit - 5

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Assignment - 3

Question - 1

Obtain the Fourier series for the following
$f(x) = e^x$ in $(-l, l)$

$\Rightarrow$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l} \quad \text{--- ①}$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx.$$

$$= \frac{1}{l}\int_{-l}^{l} e^x\,dx.$$

$$= \frac{1}{l}\left[e^x\right]_{-l}^{l} = \frac{1}{l}\left[e^l - e^{-l}\right] = \frac{2\sinh l}{l} \quad \left[\because \frac{e^l - e^{-l}}{2} = \sinh l\right]$$

$$\boxed{a_0 = \frac{2\sinh l}{l}}$$

$$a_n = \frac{1}{l}\int_{-l}^{l} e^x \cos\frac{n\pi x}{l}\,dx$$

formula :-

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$$

here :- $a = 1$, $b = \frac{n\pi}{l}$.

$$a_n = \frac{1}{l}\left[\frac{e^x}{1+\left(\frac{n\pi}{l}\right)^2}\left[\cos\frac{n\pi x}{l} + \frac{n\pi}{l}\sin\frac{n\pi x}{l}\right]\right]_{-l}^{l}$$

$$a_n = \frac{1}{l}\left[\frac{e^x l^2}{l^2+n^2\pi^2}\left[\cos\frac{n\pi x}{l} + \frac{n\pi}{l}\sin\frac{n\pi x}{l}\right]\right]_{-l}^{l}$$

$$a_n = \frac{1}{l}\left[\frac{e^l l^2}{l^2+n^2\pi^2}\left[\cos n\pi + \frac{n\pi}{l}\cancel{\sin n\pi}\right] - \frac{e^{-l} l^2}{l^2+n^2\pi^2}\left[\cos(-n\pi) + \cancel{\frac{n\pi}{l}\sin(-n\pi)}\right]\right]$$

$$a_n = \frac{1}{l}\left[\frac{e^l l^2}{l^2+n^2\pi^2}(-1)^n - \frac{e^{-l} l^2}{l^2+n^2\pi^2}(-1)^n\right].$$

$$a_n = \frac{l(-1)^n}{l^2+n^2\pi^2}(e^l - e^{-l}) \qquad \left[\because e^l - e^{-l} = 2\sinh l\right].$$

$$\boxed{a_n = \frac{l(-1)^n(2\sinh l)}{l^2+n^2\pi^2}}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} e^x \sin\frac{n\pi x}{l}\,dx.$$

formula:-

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$$

here $a=1$ $b=\frac{n\pi}{l}$

~~$b_n = \frac{1}{l}$~~

$$b_n = \frac{1}{l}\int_{-l}^{l} e^x \sin\frac{n\pi x}{l} = \frac{1}{l}\left[\frac{e^x}{1^2+\left(\frac{n\pi}{l}\right)^2}\left[\sin\frac{n\pi x}{l} - \frac{n\pi}{l}\cos\frac{n\pi x}{l}\right]\right]_{-l}^{l}$$

$$b_n = \frac{1}{l}\left[\frac{e^l l^2}{l^2+n^2\pi^2}\left[\cancel{\sin n\pi} - \frac{n\pi}{l}\cos n\pi\right] - \frac{e^{-l} l^2}{l^2+n^2\pi^2}\left[\cancel{\sin(-n\pi)} - \frac{n\pi}{l}\cos(-n\pi)\right]\right]$$

$$b_n = \frac{1}{l}\left[\frac{e^l l^2}{l^2+n^2\pi^2}\left[-\frac{n\pi}{l}(-1)^n\right] - \frac{e^{-l} l^2}{l^2+n^2\pi^2}\left[-\frac{n\pi}{l}(-1)^n\right]\right]$$

$$b_n = \frac{1}{l}\left[\frac{-l^2(-1)^n}{l^2+n^2\pi^2} \times \frac{n\pi}{l}\left[e^{l}-e^{-l}\right]\right].$$

$$\boxed{b_n = \frac{-(-1)^n n\pi}{l^2+n^2\pi^2}\, 2\sinh l}$$

Substituting a_0, a_n, b_n in (1).

$$f(x) = \frac{\sinh l}{l} + \sum_{n=1}^{\infty} \frac{2l(-1)^n(\sinh l)}{l^2+n^2\pi^2}\cos\frac{n\pi x}{l}$$

$$- \frac{(-1)^n n\pi\,(2\sinh l)}{l^2+n^2\pi^2}\sin\frac{n\pi x}{l}$$

$$f(x) = \frac{\sinh l}{l} + 2\sinh l \sum_{n=1}^{\infty} \frac{l(-1)^n}{l^2+n^2\pi^2}\cos\frac{n\pi x}{l} - \frac{(-1)^n n\pi}{l^2+n^2\pi^2}\sin\frac{n\pi x}{l}$$

$$f(x) = \frac{\sinh l}{l} + 2\sinh l \sum_{n=1}^{\infty} \frac{(-1)^n}{l^2+n^2\pi^2}\left[l\cos\frac{n\pi x}{l} - n\pi\sin\frac{n\pi x}{l}\right]$$

ASS 3

A4

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(2) $f(x) = \begin{cases} 0 & \text{in } (-\pi, 0) \\ \sin x & \text{in } (0, \pi) \end{cases}$ deduce $\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{1.5} + \frac{1}{5.7} \cdots$

$\Rightarrow \quad a_n = \frac{1}{\pi} \int_0^{\pi} \sin x \cos nx \, dx$

$\frac{1}{\pi} \int_0^{\pi} \frac{1}{2} \frac{\sin (n+1)x}{(n+1)} - \frac{\sin (n-1)x}{n-1}$

$\frac{1}{2\pi} \left[-\frac{\cos (n+1)x}{n+1} \right]_0^{\pi} + \left[\frac{\cos (n-1)x}{(n-1)} \right]_0^{\pi}$

$\frac{1}{2\pi} \left[-\frac{((-1)^{n+1}-1)}{n+1} + \frac{(-1)^{n+1}+1}{n-1} \right]$

$a_n = -\left(\frac{1 + \cos n\pi}{\pi (1 - n^2)} \right)$

$\frac{1}{2\pi} \left[\frac{(-(-1)^{n+1}+1)n-1 + (n+1)((-1)^{n+1}+1)}{n^2-1} \right]$

$\frac{1}{2\pi} \left[\frac{-n(-1)^{n+1} + (-1)^{n+1} - (+n-1) + n(-1)^{n+1} + (-1)^{n+1}(+n+1)}{n^2-1} \right]$

$\frac{1}{2\pi} \left[\frac{2(-1)^{n+1} - 2}{n^2-1} \right]$

$\frac{2}{2\pi} \left[\frac{(-1)^{n+1} - 1}{n^2-1} \right]$

$\frac{1(-1)}{\pi} \left[\frac{(-1)^n + 1}{n^2-1} \right]$

$a_n = -\frac{1}{\pi} \left[\frac{1 + (-1)^n}{n^2-1} \right]$

$a_0 = +\frac{2}{\pi}$

$b_n = 0 \quad , \quad b_1 = 1/2$

$a_0 = \frac{1}{\pi} \int_0^{\pi} \sin x \, dx$

$= \frac{1}{\pi} \left[\cos x \right]_0^{\pi} = \frac{+2}{\pi}$

$$f(x) = \frac{+1}{\pi} + \sum_{n=1}^{\infty} \frac{+1}{\pi(n^2-1)} \left[1+(-1)^n\right] \cos nx + \frac{1}{2}\sin x$$

$$f(x) = \frac{+1}{\pi} + \sum_{n=2}^{\infty} \frac{+1}{\pi(n+1)(n-1)} \left[1+(-1)^n\right] \cos nx + \frac{1}{2}\sin x]$$

Put $x = \pi/2$

$$\sin\frac{\pi}{2} = \frac{+1}{\pi} - \frac{2}{\pi}\left[\frac{\cos 2\pi/2}{1\cdot 3} + \frac{\cos 4\pi/2}{3\cdot 5} + \frac{\cos 6\pi/2}{5\cdot 7} \cdots\right] + \frac{1}{2}$$

$$1 = \frac{1}{\pi} - \frac{2}{\pi}\left[\frac{-1}{1\cdot 3} + \frac{1}{3\cdot 5} - \frac{1}{5\cdot 7} \cdots\right] + \frac{1}{2}$$

$$\left(1 - \frac{1}{2} - \frac{1}{\pi}\right) = \frac{2}{\pi}\left(\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots\right)$$

$$\left(\frac{\pi-2}{2\pi}\right) \times \frac{\pi}{2} = \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots$$

$$\boxed{\frac{\pi-2}{4} = \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots}$$

NOTE:

※ In This particular problem

$$a_n = -\left[\frac{1+\cos n\pi}{\pi(1-n^2)}\right] \quad n \neq 1, \text{ So find } a_1$$

$$\boxed{a_1 = 0}$$

$b_n = 0$. for $n \neq 1$, so find b_1

$$\boxed{b_1 = 1/2}$$

Assignment 3

Q-3 $f(x) = x\cos x$ in $(-\pi, \pi)$

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→ ~~given~~ function is odd function $\left\{ f(-x) = -x\cos x = -f(x) \Rightarrow \text{odd function} \right\}$

$\therefore a_0 = 0$

$a_n = 0$

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin nx}$$

$$\boxed{b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x\left[\cos x \sin(nx)\right]dx \qquad \sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$b_n = \frac{\cancel{2}}{\pi}\cdot\frac{1}{\cancel{2}}\int_0^{\pi} x\left[\sin(n+1)x\,dx\right] + \int_0^{\pi} x\left[\sin(n-1)\right]dx$$

$$b_n = \frac{1}{\pi}\left[x\left(\frac{-\cos(n+1)x}{(n+1)}\right) - \left(\frac{-1}{n+1}\right)\frac{\sin(n+1)x}{(n+1)}\right]_0^{\pi} + \left[x\,\frac{(-\cos(n-1)x)}{(n-1)} - \left(\frac{-1}{n-1}\right)\frac{\sin(n-1)x}{(n-1)}\right]_0^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-1}{n+1}x\cos(n+1)x + \frac{1}{(n+1)^2}\cancel{\sin(n+1)x}^{\,0}\right]_0^{\pi} + \left[-\frac{x\cos(n-1)x}{(n-1)} + \frac{1}{(n-1)^2}\cancel{\sin(n-1)x}^{\,0}\right]_0^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-1}{n+1}\left[\pi\cos(n+1)\pi - 0\right] - \frac{1}{n-1}\left[\pi\cos(n-1)\pi - 0\right]\right]$$

$$b_n = \frac{\cancel{\pi}}{\cancel{\pi}}\left[\frac{-(-1)^{n+1}}{n+1} - \frac{(-1)^{n-1}}{n-1}\right]$$

$$b_n = \frac{(-1)^n}{n+1} + \frac{(-1)^n}{n-1}$$

$$= (-1)^n\left[\frac{1}{n+1} + \frac{1}{n-1}\right]$$

$$= (-1)^n\left[\frac{n-1+n+1}{n^2-1}\right]$$

$$b_n = \frac{(-1)^n(2n)}{n^2-1}$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{(-1)^n\,2n}{n^2-1}\sin(nx)$$

Assignment - 04

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1) Find the Fourier series of $f(x) = \begin{cases} -1 & -\pi < t < -\pi/2 \\ 0 & -\pi/2 < t < \pi/2 \\ 1 & \pi/2 < t < \pi \end{cases}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{-\pi/2} -1\, dx + \int_{-\pi/2}^{\pi/2} 0\, dx + \int_{\pi/2}^{\pi} 1\, dx$$

$$= \frac{1}{\pi}\left[[-x]_{-\pi}^{-\pi/2} + [x]_{\pi/2}^{\pi} \right]$$

$$= \frac{1}{\pi}\left[-\left[-\frac{\pi}{2}+\pi\right] + (\pi - \pi/2) \right]$$

$$= \frac{1}{\pi}\left[-\pi/2 + \pi/2 \right] = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{-\pi/2} (-1)\cos nx\, dx + \int_{\pi/2}^{\pi} (1)\cos nx\, dx.$$

$$= \frac{1}{\pi}\left[-\left[\frac{\sin nx}{n}\right]_{-\pi}^{-\pi/2} + \left(\frac{\sin nx}{n}\right)_{\pi/2}^{\pi} \right]$$

$$= \frac{1}{\pi}\left\{ \frac{\sin n\pi/2}{n} - 0 + 0 - \frac{\sin n\pi/2}{n} \right\}$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{-\pi/2} (-1)\sin nx\, dx + \int_{\pi/2}^{\pi} (1)\sin nx\, dx.$$

$$= \frac{1}{\pi}\left\{ -\left[\frac{-\cos nx}{n}\right]_{-\pi}^{-\pi/2} + \left[-\frac{\cos nx}{n}\right]_{\pi/2}^{\pi} \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{\cos n\pi/2}{n} - \frac{(-1)^n}{n} - \frac{(-1)^n}{n} + \frac{\cos n\pi/2}{n} \right\}$$

$$= \frac{2}{\pi n}\left[\cos n\pi/2 - (-1)^n \right]$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[\cos\frac{n\pi}{2} - (-1)^n\right]\sin nx$$

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\cos n\pi/2 - (-1)^n\right]\sin nx$$

$x = \pi/2$

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\cos n\pi/2 - (-1)^n\right]\sin n\pi/2$$

$$\cancel{\frac{\pi}{2} = 1 + \frac{1}{3}}$$

$$f(x) = \frac{2}{\pi}\left\{1 + \sin\frac{3\pi}{2} + \sin\frac{5\pi}{2} + \cdots\right\}$$

$$f(x) = \frac{2}{\pi}\left[\sin x - \sin 2x + \frac{\sin 3x}{3} - \cdots\right]$$

on expanding the series.

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Question no 2 , Assignment 4

2) $f(x) = |\sin x| \quad -\pi < x < \pi$

$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} |\sin x| dx$ — $|\sin x|$ is a even function. a_0, a_n > exists, $b_n = 0$

$a_0 = \frac{2}{\pi} \int_0^{\pi} \sin x \, dx$

$a_0 = \frac{2}{\pi} [-\cos x]_0^{\pi}$

$a_0 = \frac{2}{\pi} [-\cos \pi - \cos 0]$

$= \frac{2}{\pi} [2]$

$a_0 = \frac{4}{\pi}$

$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$

$= \frac{2}{\pi} \int_0^{\pi} \sin x \cos nx \, dx$

$= \frac{1}{\pi} \int_0^{\pi} (\sin(n+1)x - \sin(n-1)x) dx$ — $\boxed{\because 2\sin A \cos B = \sin(B+A) - \sin(B-A)}$

$= \frac{1}{\pi} \int_0^{\pi} \sin(n+1)x dx - \int_0^{\pi} \sin(n-1)x \, dx$

$= -\frac{1}{\pi} \left[\frac{\cos(n+1)x}{n+1} \right]_0^{\pi} + \frac{1}{\pi} \left[\frac{\cos(n-1)x}{n-1} \right]_0^{\pi}$ — $\because \boxed{\cos n\pi = (-1)^n}$

$= \frac{-1}{\pi} [(-1)^{n+1} - 1] \left[\frac{1}{n+1} - \frac{1}{n-1} \right] = \frac{2[(-1)^{n+1} - 1]}{\pi(n^2-1)}$

$$a_n = \frac{2[(-1)^{n+1} - 1]}{\pi(n^2-1)}$$

where a_n does not exist for $n = 1$.

$a_n = 0$, when $n =$ odd

$a_n = \dfrac{-4}{\pi(n^2-1)}$, when $n =$ even

$$f(x) = \frac{a_0}{2} + \sum_{n=2}^{\infty} \frac{-4}{\pi} \frac{\cos nx}{n^2-1}$$

$$|\sin x| = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=2}^{\infty} \frac{\cos nx}{n^2-1}$$

$$|\sin x| = \frac{2}{\pi} - \frac{4}{\pi}\left[\frac{\cos 2x}{3} + \frac{\cos 4x}{15} + \cdots\right]$$

$=$

MATHEMATICS

Assignment No 05 : Q1

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Section - C ; Sem - 2nd

* Half-range Fourier Series

* obtain the Fourier Series expansion of x^2 as a cosine series in $(0,\pi)$.

Ans: Half range Series - Cosine Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx))$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{\pi}\left(\frac{\pi^3}{3}\right)$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x).\cos(nx).dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 \cos(nx)\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2\sin(nx)}{n} - \frac{2x}{n}\left(\frac{-\cos(nx)}{n}\right) + \left(\frac{-2}{n^2}\right)\left(\frac{\sin(nx)}{n}\right)\right]_0^{\pi}$$

$$\therefore \boxed{a_n = \frac{4(-1)^n}{n^2}}$$

$$\boxed{\therefore f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}\cos(nx)\right)}$$

$$\boxed{f(x) = \frac{\pi^2}{3} - 4\left[\cos x - \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x + \frac{1}{4^2}\cos 4x + \dots\right]}$$

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Assignment - 6

2) Obtain the half range cosine series for the function $f(x) = x$ in the interval $0 < x < 2$.

half range cosine series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

$l = 2$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{2}\int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2 = 2$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx = \int_0^2 x\cos\left(\frac{n\pi x}{2}\right)dx$$

$$\cancel{= \left[\frac{x\sin}{n\pi}\left(\frac{n\pi x}{2}\right) + \frac{\cos(n\pi x/2)}{(n\pi)^2}\right]_0^2}$$

$$\cancel{= \frac{\cos(n\pi)}{(n\pi)^2} - \frac{1}{(n\pi)^2} = \frac{1}{(n\pi)^2}\{(-1)^n - 1\}}$$

$$\cancel{f(x) = 1 + \sum_{n=1}^{\infty} \frac{1}{(n\pi)^2}\{(-1)^n - 1\}\cos\frac{n\pi x}{2}}$$

$$= \int_0^2 x \cdot \cos\left(\frac{n\pi x}{2}\right)dx = \left[x \cdot \frac{\sin\frac{n\pi x}{2}}{\frac{n\pi}{2}} - (1)\cdot\frac{2}{n\pi}\cdot\frac{-\cos\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)}\right]_0^2$$

$$= \left[\frac{2x}{n\pi}\sin\frac{n\pi x}{2} + \frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2}\right]_0^2$$

$$= \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$$

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ASSIGNMENT - 6

Q1. Define $f: T \to \mathbb{R}$ by $f(x) = x^2$ for $-\pi \leq x \leq \pi$ use Parseval's theorem to deduce that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

$f(x) = f(-x)$

$\therefore$ x^2 is an even function

$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$, $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$ $b_n = 0$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x).dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2.dx$$

$$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = \frac{2}{\pi}\left[\frac{\pi^3}{3}\right]$$

$$a_0 = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx .dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 . \cos nx . dx$$

$$= \frac{2}{\pi}\left[\cancel{x^2 . \left(\frac{\sin nx}{n}\right)}^{0} - 2x\left(-\frac{\cos nx}{n^2}\right) + \cancel{2 \times \frac{1}{n^2}\left(-\frac{\sin nx}{n}\right)}^{0}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[2x . \left(\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\cancel{\pi}}\left[\frac{2\cancel{\pi}}{n^2}(\cos n\pi - \cancel{\cos} 0)\right]$$

$$a_n = \frac{4}{n^2}(-1)^n.$$

Fourier Series is

$$x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}(-1)^n \cos nx\right)$$

By Parseval's identity for even functions

$$2\int_0^{\pi} [f(x)]^2 \, dx = \pi\left[\frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2\right]$$

$$\rightarrow 2\int_0^{\pi} [x^2]^2 \, dx = \pi\left[\frac{1}{2}\left(\frac{2\pi^2}{3}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}(-1)^n\right)^2\right]$$

$$2\int_0^{\pi} x^4 \, dx = \pi\left[\frac{4\pi^4}{18} + \sum_{n=1}^{\infty}\frac{16}{n^4}\right]$$

$$2\left[\frac{x^5}{5}\right]_0^{\pi} = \frac{4\pi^5}{18} + 16\pi\sum_{n=1}^{\infty}\frac{1}{n^4}.$$

$$2\left[\frac{\pi^5}{5}\right] - \frac{\cancel{4}^{2}\pi^5}{\cancel{18}_{9}} = 16\pi\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\frac{18\pi^5 - 10\pi^5}{45} = 16\pi\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\frac{\cancel{8}^{1}\pi^5}{45\times\cancel{16}_{2}\cancel{\pi}} = \sum_{n=1}^{\infty}\frac{1}{n^4}.$$

$$\frac{\pi^4}{90} = \sum_{n=1}^{\infty}\frac{1}{n^4}.$$

Assignment

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If $\frac{a_0}{2} + \sum a_n \cos \frac{n\pi x}{l}$ is half range cosine of $f(x)$ period of $2l$ in $(0,l)$, then show that mean square value of $f(x)$ in $(0,l)$ is $\frac{1}{2}\left\{\frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2\right\}$ Use this result to evaluate $1^{-4} + 3^{-4} + 5^{-4} + \ldots$ from the half range cosine series of function $f(x)$ of period 4 defined in $(0,2)$ by

$$f(x) = \begin{cases} \pi x & 0 < x < 1 \\ \pi(2-x) & 1 < x < 2 \end{cases}$$

$\rightarrow$ $(0,2)$ $\therefore l = 2.$

by using parsevals identity

$$2\int_0^l f(x)^2 \cdot dx = l\left[\frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2\right]$$

Refer Scanned copy of Solution (next ans) and use parseval identity to find $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}.$

$$a_0 = \frac{2}{l}\int_0^l f(x)\cdot dx$$

$$= \frac{2}{2}\int_0^2 f(x)\cdot dx$$

$$= \int_0^1 f(x)\cdot dx + \int_1^2 f(x)\cdot dx$$

$$= \pi\int_0^1 x\cdot dx + \pi\int_1^2 (2-x)\cdot dx.$$

$$= \pi\left[\frac{x^2}{2}\right]_0^1 + \pi\left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{\pi}{2} + \pi\left[4 - \frac{4}{2} - 2 + \frac{1}{2}\right] = \pi$$

$$1 \qquad 2^2 \qquad 3^2 \qquad 4^2 \qquad 5^2 \qquad \cdots \qquad 12$$

Example 10.10. *Obtain Fourier series for the function*

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$$

(Rohtak, 2013 ; V.T.U., 2011 ; Bhopal, 2008)

Deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots \infty = \dfrac{\pi^2}{8}$.

Solution. The required series is of the form

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$$

Then

$$a_0 = \int_0^1 \pi x dx + \int_1^2 \pi(2-x)dx = \pi \left| \frac{x^2}{2} \right|_0^1 + \pi \left| 2x - \frac{x^2}{2} \right|_1^2 = \pi\left(\frac{1}{2}\right) + \pi\left\{(4-2) - \left(2 - \frac{1}{2}\right)\right\} = \pi$$

$$a_n = \int_0^1 \pi x \cos n\pi x dx + \int_1^2 \pi(2-x) \cos n\pi x dx$$

$$= \left| \pi x \cdot \frac{\sin n\pi x}{n\pi} - \pi\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) \right|_0^1 + \left| \pi(2-x)\frac{\sin n\pi x}{n\pi} - (-\pi)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) \right|_1^2$$

$$= \left(\frac{\cos n\pi}{n^2\pi} - \frac{1}{n^2\pi^2}\right) - \left(\frac{\cos 2n\pi}{n^2\pi} - \frac{\cos n\pi}{n^2\pi}\right) = \frac{2}{n^2\pi}[(-1)^n - 1]$$

$$= 0 \text{ when } n \text{ is even} ; -\frac{4}{n^2\pi} \text{ when } n \text{ is odd.}$$

$$b_n = \int_0^1 \pi x \sin n\pi x \, dx + \int_1^2 \pi(2-x) \sin n\pi x \, dx$$

$$= \left| \pi x \left(-\frac{\cos n\pi x}{n\pi}\right) - \pi\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right|_0^1 + \left| \pi(2-x)\left(\frac{-\cos n\pi x}{n\pi}\right) - (-\pi)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right|_1^2$$

$$= \left(-\frac{\cos n\pi}{n}\right) + \left(\frac{\cos n\pi}{n}\right) = 0$$

Hence $$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \ldots \infty\right)$$

Putting $x = 2$, $$0 = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos 2\pi}{1^2} + \frac{\cos 6\pi}{3^2} + \frac{\cos 10\pi}{5^2} + \ldots \infty\right)$$

Whence $$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \infty = \frac{\pi^2}{8}.$$

Example 10.11. *Find the Fourier series for*

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Assignment - 7

Q1) Compute the first two harmonic of the fourier series of $f(x)$ given the following table:

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

interval : $0 \le x \le 2\pi$

x	y	$\cos x$	$\cos 2x$	$y \cos x$	$y \cos 2x$
0	1	1	1	1	1
$\pi/3$	1.4	0.5	-0.5	0.7	-0.7
$2\pi/3$	1.9	-0.5	-0.5	-0.95	-0.95
π	1.7	-1	1	-1.7	1.7
$4\pi/3$	1.5	-0.5	-0.5	-0.75	-0.75
$5\pi/3$	1.2	0.5	-0.5	0.6	-0.6
				-1.1	-0.3

x	y	$\sin x$	$\sin 2x$	$y \sin x$	$y \sin 2x$
0	1	0	0	0	0
$\pi/3$	1.4	0.866	0.866	1.2124	1.2124
$2\pi/3$	1.9	0.866	-0.866	1.6454	-1.6454
π	1.7	0	0	0	0
$4\pi/3$	1.5	-0.866	0.866	-1.299	1.299
$5\pi/3$	1.2	-0.866	-0.866	-1.0392	-1.0392
				0.5196	-0.1732

$$a_1 = \frac{2}{N} \Sigma y \cos x = \frac{2}{6}(-1.1) = -0.367$$

$$a_2 = \frac{2}{N} \Sigma y \cos 2x = \frac{2}{6}(-0.3) = -0.1$$

$$b_1 = \frac{2}{N} \Sigma y \sin x = \frac{2}{6}(0.5196) = 0.1732$$

$$b_2 = \frac{2}{N} \Sigma y \sin 2x = \frac{2}{6}(-0.1732) = -0.0577$$

First two harmonics are $a_1 \cos x + b_1 \sin x$ & $a_2 \cos 2x + b_2 \sin 2x$

$-0.367 \cos x + 0.1732 \sin x$ & $-0.1 \cos 2x - 0.0577 \sin 2x$

ASSIGNMENT - 7

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Q 2

The turning moment T on the crankshaft of a steam engine for the crack angle θ degrees is given as follows. Expand T in a series up to first harmonics.

θ	y(T)	$\cos\frac{n\pi x}{l}$	$y(T)\cdot\cos\frac{n\pi x}{l}$	$\sin\frac{n\pi x}{l}$	$y(T)\sin\frac{n\pi x}{l}$
0	0	1	0	0	0
15	2.7	0.866	2.3382	0.5	1.35
30	5.2	0.5	2.6	0.866	4.5032
45	7.0	0	0	1	7
60	8.1	-0.5	-4.05	0.866	7.0146
75	8.3	-0.866	-7.1878	0.5	4.15
90	7.9	-1	-7.9	0	0
105	6.8	-0.866	-5.8888	-0.5	-3.4
120	5.5	-0.5	-2.75	-0.866	-4.763
135	4.1	0	0	-1	-4.1
150	2.8	0.5	1.3	-0.866	-2.4248 -2.2516
165	1.2	0.866	1.0392	-0.5	-0.6
~~180~~	59.4		-20.4992		8.73011 8.9032

$$a_0 = \frac{59.4}{6} = 9.9$$

N = 12

2l = 180

l = 90

$$a_1 = \frac{2}{N}\sum Y\cos\left(\frac{n\pi x}{l}\right) = \frac{2}{12}(-20.4992) = \cancel{-3.3998}\ -3.4165$$

$$b_1 = \frac{2}{N}\sum Y\sin\left(\frac{n\pi x}{l}\right) = \frac{2}{12}(\cancel{8.73011}\ 8.9032) = \cancel{1.45501}\ 1.4839$$

First harmonic up to (include constant term also)

$$\frac{a_0}{2} + a_1\cos x + b_1\sin x = 4.95 - 3.4165\cos\theta + 1.4839\sin\theta$$

~~4.95 - 3.3998 cos x + 1.45501 sin x~~

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Q The following values of y gives the displacement in inches of a certain machine part for the rotation x of the flywheel. Expand y in terms of Fourier Series:

x	0	π/6	2π/6	3π/6	4π/6	5π/6
y	0	0.2	14.4	17.8	17.3	11.7

$a_0 = \frac{2}{N} \Sigma y = \frac{2}{6}(\Sigma y) = \underline{\underline{20.4667}}$

$a_n = \frac{2}{N} \Sigma y \cos nx$

$a_1 = \frac{2}{6} \Sigma y \cos x = \frac{1}{3} \Sigma y \cos x = \underline{\underline{-3.8030}}$

$b_n = \frac{2}{N} \Sigma y \sin nx$

$b_1 = \frac{2}{6} \Sigma y \sin x = \frac{1}{3} \Sigma y \sin x = 17.0676.$

Fourier Series

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$f(x) = \frac{20.4667}{2} + -3.8030 \cos x + \cdots + 17.0676 \sin x + \cdots$

$f(x) = 10.2335 + -3.8030 \cos x + \cdots + 17.0676 \sin x + \cdots$

NOTE:

Actual Question is

x	0	π/6	2π/6	3π/6	4π/6	5π/6
y	0	9.2	14.4	17.8	17.3	11.7

Then we get the answer given in Pesu portal.

Assignment-8

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Q2. Compute the first two harmonics of the fourier series for $f(x)$ from the following:

x	30	60	90	120	150	180	210	240	270	300	330	360
y	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

$$a_0 = \frac{2}{N} \Sigma y$$

$$= \frac{2}{12}(2.34+3.01+3.68+4.15+3.69+2.20+0.83+0.51+0.88+1.09+1.19+1.64)$$

$$= \frac{1}{6}(25.21) = 4.202$$

x	y	$\cos x$	$y \cos x$	$\cos 2x$	$y \cos 2x$
30	2.34	0.866	2.026	0.5	1.17
60	3.01	0.5	1.505	-0.5	-1.505
90	3.68	0	0	-1	-3.68
120	4.15	-0.5	-2.075	-0.5	-2.075
150	3.69	-0.866	-3.195	0.5	1.845
180	2.20	-1	-2.20	1	2.20
210	0.83	-0.866	-0.718	0.5	0.415
240	0.51	-0.5	-0.255	-0.5	-0.255
270	0.88	0	0	-1	-0.88
300	1.09	0.5	0.545	-0.5	-0.545
330	1.19	0.866	1.030	0.5	0.595
360	1.64	1	1.64	1	1.64

$\Sigma y\cos x = -1.68$ $\quad\quad$ $\Sigma y\cos 2x = -1.068$

$a_1 = \frac{2}{N}\Sigma y\cos x = \frac{1}{6}(-1.68) = -0.280$

$a_2 = \frac{2}{N}\Sigma y\cos 2x = \frac{1}{6}(-1.068) = -0.178$

x	y	$\sin x$	$y\sin x$	$\sin 2x$	$y\sin 2x$
30	2.34	0.5	1.17	0.866	2.026
60	3.01	0.866	2.606	0.866	2.6066
90	3.68	1	3.68	0	0
120	4.15	0.866	3.59	-0.866	-3.186
150	3.69	0.5	1.845	-0.866	-3.195
180	2.20	0	0	0	0
210	0.83	-0.5	-0.415	0.866	0.718
240	0.51	-0.866	-0.44	0.866	0.441
270	0.88	-1	-0.88	0	0
300	1.09	-0.866	-0.943	-0.866	-0.943
330	1.19	-0.5	-0.595	-0.866	-1.030
360	1.64	0	0	0	0

$\Sigma y\sin x = 9.708$ $\quad\quad$ $\Sigma y\sin 2x = -2.97$

$b_1 = \frac{2}{N}\Sigma y\sin x = \frac{1}{6}(9.708) = 1.618$

$b_2 = \frac{2}{N}\Sigma y\sin 2x = \frac{1}{6}(-2.97) = -0.495$

NOTE: Pesu portal answer is wrong.

$\therefore$ fourier series $= \frac{a_0}{2} + \Sigma a_n\cos nx + b_n\sin nx$

$= \frac{4.202}{2} + a_1\cos x + b_1\sin x + a_2\cos 2x + b_2\sin 2x$

$= 2.101 - 0.280\cos x + 1.618\sin x - 0.178\cos 2x - 0.495\sin 2x$

Assignment - 9

Devi Krishikesh Reddy

1) Find the complex Fourier series of $f(x) = x$, $-\pi < x < \pi$ integer.

Solution

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx} \quad ; \quad C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx$$

$$\boxed{f(x) = x}$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\, e^{-inx}\, dx$$

$$\frac{1}{2\pi}\left[x\,\frac{e^{-inx}}{-in}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} (x)' \left(\int e^{-inx} dx\right) dx$$

$$\frac{1}{2\pi}\left[x\,\frac{e^{-inx}}{-in}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{e^{-inx}}{-in}\, dx$$

$$\frac{1}{2\pi}\left[x\,\frac{e^{-inx}}{-in}\right]_{-\pi}^{\pi} - \left[\frac{e^{-inx}}{(-in)^2}\right]_{-\pi}^{\pi}$$

$$\quad (1) \qquad\qquad\qquad (2)$$

Solving (2)

$$\left[\frac{e^{-in\pi} - e^{in\pi}}{i*(i*n^2)}\right] = 2\left[-\frac{\sin(n\pi)}{(i*n^2)}\right]$$

$\hookrightarrow 0$ as $\sin(n\pi) = 0$

Solving (1)

$$\left[\frac{\pi e^{-in\pi}}{-in} - \frac{-\pi e^{+in\pi}}{-in}\right] = (2)*\left(\frac{\pi}{-in}\right)\left[\frac{e^{-in\pi} + e^{in\pi}}{2}\right]$$

$\hookrightarrow \cos n\pi$

$$= \frac{2*\pi}{i*(-n)} * (-1)^n$$

$\hookrightarrow$ Substitute value of (1) back.

$$\frac{1}{\cancel{2\pi}} \times \frac{\cancel{2\pi}}{(-n)} \times \frac{(-1)^n}{(i)} = \frac{(-1)^n}{(i)(-n)} = C_n$$

Substitute value of C_n back in $f(x)$.

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{i(-n)} e^{inx} \qquad \left[\frac{1}{-n} \Rightarrow \frac{-1}{n} \Rightarrow \frac{i \times i}{n} \right]$$

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{i \times \cancel{i}}{\cancel{i}} \frac{(-1)^n}{+n} e^{inx}$$

$$f(x) = i \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n} e^{inx}$$

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D. BHANU PRAKASH

UNIT-4

ASSIGNMENT-9

SNO: 2

Find the complex form of the fourier series of the $f(x) = e^x$, in $-\pi \leq x \leq \pi$.

Soln $f(x) = e^x \quad -\pi < x < \pi$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$$

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cdot e^{-inx} dx$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^x \cdot e^{-inx} dx$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{x(1-in)} dx$$

$$= \frac{1}{2\pi} \left[\frac{e^{x(1-in)}}{1-in} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi(1-in)} \left[e^{\pi(1-in)} - e^{-\pi(1-in)} \right]$$

$$= \frac{1}{2\pi(1-in)} \left[e^{\pi} (e^{-in\pi}) - e^{-\pi} (e^{in\pi}) \right]$$

$e^{-in\pi} = \cos n\pi = (-1)^n$

$e^{in\pi} = \cos n\pi = (-1)^n$.

$$= \frac{(-1)^n}{2\pi(1-in)} \left[\frac{e^{\pi} - e^{-\pi}}{\cancel{2}} \right] \times \cancel{2}$$

$$= \frac{(-1)^n \cdot \sinh \pi}{\pi(1-in)}$$

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n \cdot \sinh \pi}{(1-in)\pi} \cdot e^{inx}$$

$$= \frac{\sinh \pi}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n (1+in)}{1+n^2} (e^{inx})$$

Assignment - 10 PES2UG22CS173
Dhanush - H

1) Find the complex form of the Fourier series of $f(x) = \cosh 3x + \sinh 3x$ in $(-3,3)$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{l}}$$

$$C_n = \frac{1}{2l} \int_{-l}^{l} f(x) e^{\frac{-in\pi x}{l}} dx$$

$l = 3$

$$C_n = \frac{1}{6} \int_{-3}^{3} (\cosh 3x + \sinh 3x) e^{\frac{-in\pi x}{l}} dx$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$= \frac{1}{12} \int_{-3}^{3} \left(e^{3x} + \cancel{e^{-3x}} + e^{3x} - \cancel{e^{-3x}}\right) e^{\frac{-in\pi x}{3}} dx$$

$$= \frac{1}{12} \int_{-3}^{3} 2e^{3x} \cdot e^{\frac{-in\pi x}{3}} dx$$

$$= \frac{1}{6} \int_{-3}^{3} e^{\left(3 - \frac{in\pi}{3}\right)x} dx$$

$$= \frac{1}{6\left[3 - \frac{in\pi}{3}\right]} \left[e^{\left[3 - \frac{in\pi}{3}\right]x}\right]_{-3}^{3}$$

$$= \frac{1}{6\left[3 - \frac{in\pi}{3}\right]} \left[e^{[9 - in\pi]} - e^{-[9 - in\pi]}\right]$$

$$e^{inx} = \underbrace{\cos n\pi}_{(-1)^n} + i \underbrace{\sin n\pi}_{0}$$

$$e^{-inx} = \underbrace{\cos n\pi}_{(-1)^n} - i \underbrace{\sin n\pi}_{0}$$

$$\frac{1}{6\left[3-\frac{in\pi}{3}\right]}\left[e^{9}e^{-in\pi}-e^{-9}e^{in\pi}\right]$$

$$\frac{1}{6\left[3-\frac{in\pi}{3}\right]}\left[(-1)^n\left(e^{9}-e^{-9}\right)\right]$$

$$= \frac{1}{\cancel{6}_3\left[3-\frac{in\pi}{3}\right]}2\sinh 9\,(-1)^n$$

$$= \frac{(-1)^n}{9-in\pi}\sinh 9$$

$$C_n = \frac{(-1)^n(9+n\pi)\sinh 9}{81+(n\pi)^2}$$

$$f(x) = \sum_{-\infty}^{\infty}(-1)^n\frac{9+n\pi}{81+(n\pi)^2}\sinh 9\,e^{\frac{in\pi x}{3}}$$

$$= \sinh 9\sum_{-\infty}^{\infty}\frac{(-1)^n(9+n\pi)}{81+(n\pi)^2}e^{\frac{in\pi x}{3}}$$

Assignment 10: Q2

Dhanush M
PES2UG22CS174

Q) Find the complex form of the fourier series of the periodic function $f(x) = \begin{cases} -k & \text{in } (-\pi, 0) \\ k & \text{in } (0, \pi) \end{cases}$

solution:

C_n is given by $C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx}\, dx$

$$= C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx}\, dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -k e^{-inx}\, dx + \int_{0}^{\pi} k e^{-inx}\, dx\right]$$

$$= \frac{k}{2\pi}\left[\int_{0}^{\pi} e^{-inx}\, dx - \int_{-\pi}^{0} e^{-inx}\, dx\right]$$

$$= \frac{k}{2\pi}\left[\frac{e^{-inx}}{-in}\Big|_{0}^{\pi} - \frac{e^{-inx}}{-in}\Big|_{-\pi}^{0}\right]$$

$$= \frac{k}{2\pi}\left[\frac{e^{-in\pi}-1}{-in} - \frac{(1-e^{in\pi})}{-in}\right]$$

$$= \frac{k}{-in2\pi}\left[(e^{in\pi}+e^{-in\pi}) - 2\right]$$

$$= \frac{k}{-in2\pi}\left[2\cos n\pi - 2\right]$$

$$= \frac{\cancel{2}k}{-in\cancel{2}\pi}\left[\cos n\pi - 1\right]$$

$$= \frac{k}{i\pi}\left[\frac{1-\cos n\pi}{n}\right]$$

$$C_n = \frac{k}{i\pi}\left[\frac{1-(-1)^n}{n}\right]$$

$$f(x) = \frac{k}{i\pi}\sum_{n=-\infty}^{\infty} \frac{1-(-1)^n}{n} \cdot e^{inx}$$

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Question Bank question 1

Q1. Expand $f(x) = 1 - x^2$ as a fourier series in the interval $-1 < x < 1$ $\quad (-l < x < l)$

$2l = 2 \quad l = 1$

$f(x) = 1 - x^2$ is an even function $\therefore b_n = 0$

$$a_0 = \frac{1}{l} \int_{-l}^{l} f(x)\, dx$$

$$= \frac{1}{1} \int_{-1}^{1} 1 - x^2\, dx$$

$$= \frac{4}{3} \text{ (calc)}$$

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l}\, dx$$

$$= \frac{1}{1} \int_{-1}^{1} (1 - x^2) \cos \frac{n\pi x}{1}\, dx$$

$$= \left[\cancel{\frac{(1 - x^2) \sin n\pi x}{n\pi}} - \frac{(-2x)}{n\pi} \frac{(-\cos n\pi x)}{n\pi} + \cancel{\frac{(-2)(-\sin n\pi x)}{n^2\pi^2 (n\pi)}} \right]_{-1}^{1}$$

$$= \left[\frac{-2(-1)^n}{n^2\pi^2} - \frac{2(-1)^n}{n^2\pi^2} \right]$$

$$= -\frac{4(-1)^n}{n^2\pi^2} = \frac{4(-1)^{n+1}}{n^2\pi^2}$$

$$1 - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

$$1 - x^2 = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2\pi^2} \cos \frac{n\pi x}{1}$$

$$1 - x^2 = \frac{2}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos n\pi x$$

Mathematics

Question Bank

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C - section

Q2) Find the half range Fourier Series - cosine series for the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$. Hence deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$.

Soln-

$f(x) = x(\pi - x)$ over $(0, \pi)$

Half range Fourier cosine series of $f(x)$ is given by -

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

where, $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x(\pi - x)\,dx = \frac{2}{\pi}\left[\frac{x^2\pi}{2} - \frac{x^3}{3}\right]_0^{\pi}$

$$a_0 = \frac{2}{\pi}\left[\frac{\pi^3}{2} - \frac{\pi^3}{3}\right] = \frac{2}{\pi}\left[\frac{\pi^3}{6}\right] = \frac{\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi}(x(\pi - x))\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(x\pi - x^2)\frac{\sin nx}{n} - (\pi - 2x)\left(\frac{1}{n}\right)\left(-\frac{\cos nx}{n}\right) + (-2)\left(-\frac{1}{n^2}\right)\frac{\sin nx}{n}\right)\Bigg]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{(\pi - 2x)}{n^2}\cos nx\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi n^2}\left[-\pi(-1)^n - \pi(1)\right]$$

$$a_n = \frac{-2}{n^2}\left(1 + (-1)^n\right)$$

$$\therefore\ f(x) = \frac{\pi^2}{6} + \sum_{n=1}^{\infty} \frac{-2}{n^2}\left(1 + (-1)^n\right)\cos nx$$

$$x(\pi - x) = \frac{\pi^2}{6} - 2\sum_{n=1}^{\infty} \frac{1}{n^2}\left[1 + (-1)^n\right] \cos nx$$

Put $x = \frac{\pi}{2}$,

$$\frac{\pi}{2}\left(\frac{\pi}{2}\right) = \frac{\pi^2}{6} - 2\sum_{n=1}^{\infty} \frac{1}{n^2}\left[1 + (-1)^n\right] \cos n\pi/2$$

$$\frac{\pi^2}{4} - \frac{\pi^2}{6} = -2\sum_{n=1}^{\infty} \frac{1}{n^2}\left[1 + (-1)^n\right] \cos n\pi/2$$

$$\frac{2\pi^2}{24} = -2\sum_{n=1}^{\infty} \frac{1}{n^2}\left(1 + (-1)^n\right) \cos n\pi/2$$

$$\frac{\pi^2}{24} = \sum_{n=1}^{\infty} \frac{1}{n^2}\left((-1)^{n+1} - 1\right) \cos n\pi/2 \quad //$$

on expanding the series.

$$\frac{\pi^2}{24} = 2\sum_{n=1}^{\infty} \frac{1}{n^2} \quad \text{for } n \text{ is even.}$$

$$\Rightarrow \frac{\pi^2}{48} = \sum \frac{1}{n^2} \quad \text{if } n \text{ is even.}$$

NOTE:

$$a_n = \begin{cases} -\frac{4}{n^2} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

Question bank - Q3)

Expand $f(x) = \begin{cases} x & \text{in } 0 < x < \pi/2 \\ \pi - x & \text{in } \pi/2 < x < \pi \end{cases}$ in half range Fourier cosine series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \, dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi/2} x + \int_{\pi/2}^{\pi} \pi - x$$

$$= \frac{2}{\pi}\left(\left.\frac{x^2}{2}\right]_0^{\pi/2} + \left.\pi x\right]_{\pi/2}^{\pi} - \left.\frac{x^2}{2}\right]_{\frac{\pi}{2}}^{\pi}\right)$$

$$= \frac{2}{\pi}\left(\frac{\pi^2}{8} + \cancel{\pi^2} - \cancel{\frac{\pi^2}{2}} - \cancel{\frac{\pi^2}{2}} + \frac{\pi^2}{8}\right)$$

$$= \frac{2}{\pi}\left(\frac{\pi^2}{4}\right) = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx \, dx = \frac{2}{\pi}\left\{\int_0^{\pi/2} x\cos nx \; + \int_{\pi/2}^{\pi} (\pi - x)\cos nx \cdot dx\right\}$$

$$= \frac{2}{\pi}\left\{\left[x\frac{\sin nx}{n} - \frac{1}{n}\left(-\frac{\cos nx}{n}\right)\right]_0^{\pi/2}\right\} + \left\{\left[\frac{(\pi - x)\sin nx}{n} - \left(\frac{(-1)}{n}\left(\frac{-\cos nx}{n}\right)\right)\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{\frac{\pi}{2}\sin\frac{\pi}{2}n}{n} + \frac{1}{n^2}\cos n\frac{\pi}{2} - \left(\frac{1}{n^2}\right)\right\} + \left\{-\frac{\cos n\pi}{n^2} - \left(\frac{\pi}{2}\frac{\sin n\pi/2}{n} - \frac{\cos n\pi/2}{n^2}\right)\right\}$$

$$= \frac{2}{\pi}\left(\frac{2}{n^2}\cos\frac{n\pi}{2} - \frac{1}{n^2}\left((-1)^n + 1\right)\right)$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \cancel{2}\,\frac{2}{\pi n^2}\left(\cos\frac{n\pi}{2} - (-1)^n - 1\right)\cos nx$$

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Question Bank.

Find the half range SINE series

Q4) $f(x) = \begin{cases} \frac{1}{4} - x, & \text{in } \left(0, \frac{1}{2}\right) \\ x - \frac{3}{4}, & \text{in } \left(\frac{1}{2}, 1\right) \end{cases}$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$b_n = \frac{2}{l} \int_0^l f(x) \cdot \sin \frac{n\pi x}{l} \cdot dx \qquad l = 1$$

$$b_n = 2 \cdot \int_0^{1/2} \left(\frac{1}{4} - x\right) \sin n\pi x \cdot dx + 2 \int_{1/2}^{1} \left(x - \frac{3}{4}\right) \sin n\pi x \cdot dx$$

$$= 2\left[\left(\frac{1}{4} - x\right)\left[\frac{-\cos n\pi x}{n\pi}\right] - \left(\frac{-1}{n\pi}\right)(-1)\left[\frac{\sin n\pi x}{n\pi}\right]\right]_0^{1/2}$$

$$+ 2\left[\left(x - \frac{3}{4}\right)\left[\frac{-\cos n\pi x}{n\pi}\right] - (1)\left(\frac{-1}{n\pi}\right)\frac{\sin n\pi x}{n\pi}\right]_{1/2}^{1}$$

$$= 2\left[\left[\frac{1}{4} - \frac{1}{2}\right] \cdot \left(\frac{-\cos n\pi/2}{n\pi}\right) - \left(\frac{1}{n\pi}\right)\frac{\sin n\pi/2}{n\pi}\right]$$

$$- 2\left[\left(\frac{1}{4} - 0\right)\left(\frac{-\cos n\pi(0)}{n\pi}\right) - \left(\frac{1}{n\pi}\right)\left[\frac{\cancel{\sin n\pi(0)}}{n\pi}\right]\right] \quad 0.$$

$$+ 2\left[\left(1 - \frac{3}{4}\right)\left[\frac{-\cos n\pi}{n\pi}\right] + \left(\frac{1}{n\pi}\right)\frac{\cancel{\sin n\pi}}{n\pi}\right] - \quad 0$$

$$2\left[\left(\frac{1}{2} - \frac{3}{4}\right)\left[\frac{-\cos n\pi/2}{n\pi}\right] + \left(\frac{1}{n\pi}\right)\frac{\sin n\pi/2}{n\pi}\right]$$

$$= 2\left[\cancel{\frac{1}{4}\cdot\frac{\cos n\pi/2}{n\pi}} - \frac{1}{(n\pi)^2}\sin n\pi/2 + \frac{1}{4}\cdot\frac{1}{n\pi}\right.$$

$$\left. + \frac{1}{4}\left(\frac{-1}{n\pi}\right)(-1)^n - \cancel{\frac{1}{4}\cdot\frac{\cos n\pi/2}{n\pi}} - \frac{1}{(n\pi)^2}\sin n\pi/2\right]$$

$$= 2\left[\frac{1}{4n\pi}\left(1-(-1)^n\right) - \frac{2}{(n\pi)^2}\sin n\pi/2\right]$$

$$= \frac{1}{2n\pi}\left(1-(-1)^n\right) - \frac{4}{(n\pi)^2}\sin n\pi/2$$

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{2n\pi}\left(1-(-1)^n\right) - \frac{4}{(n\pi)^2}\sin n\pi/2 \,.\, \sin n\pi x.$$

Question Bank - 5Q

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Q) Find the Half range sine series for $f(x) = (x-1)^2$ in the interval $0 \le x \le 1$

Sol- $b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

here $l = 1$

$\therefore b_n = \frac{2}{1} \int_0^1 (x-1)^2 \sin\left(\frac{n\pi x}{1}\right) dx$

$$= 2\left[\frac{(x-1)^2}{n\pi}(-\cos(n\pi x)) - \frac{2(x-1)}{(n\pi)^2}(-\sin(n\pi x)) + \frac{2}{(n\pi)^3}(\cos(n\pi x))\right]_0^1$$

$$= 2\left[0 - 0 + \frac{2}{(n\pi)^3}(\cos(n\pi)) - \frac{1}{n\pi}(-\cos(0)) + 0 - \frac{2}{(n\pi)^3}\cos(0)\right]$$

$$= 2\left[\frac{1}{n\pi}(\cos(0)) + \frac{2}{(n\pi)^3}\left\{\cos(n\pi) - \cos(0)\right\}\right]$$

$$= \frac{2}{n\pi}\left[1 + \frac{2}{(n\pi)^2}\left\{(-1)^n - 1\right\}\right]$$

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CH-4

QUESTION BANK

QUESTION - 7

Find fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $0 < x < 2\pi$.

Deduce $\sum \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$

$$f(x) = \left(\frac{\pi - x}{2}\right)^2 = \frac{x^2 - 2x\pi + \pi^2}{4}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos Nx + b_N \sin Nx.$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x) \qquad a_0 = \frac{1}{\pi}\int_0^{2\pi} \frac{x^2 - 2x\pi + \pi^2}{4} \qquad a_0 = \frac{1}{4\pi}\left[\frac{x^3}{3} - x^2\pi + x\pi^2\right]_0^{\pi}$$

$$a_0 = \frac{1}{4\pi}\left[\frac{\pi^3}{3} - 4\pi^3 + 2\pi^3\right] \qquad a_0 = \frac{\pi^3}{6} \times \frac{4}{4\pi} \qquad a_0 = \frac{\pi^2}{6}$$

$$a_N = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx \qquad a_0 = \frac{1}{4\pi}\int (x^2 - 2x\pi + \pi^2)\cos Nx$$

$$a_N = \frac{1}{4\pi}\left[\frac{(x^2 - 2x\pi + \pi^2)}{N}\cancel{\sin Nx}^{\,0} - \frac{(2x - 2\pi)\cos nx}{-N^2} + \frac{2\cancel{\sin Nx}^{\,0}}{N^3}\right]_0^{2\pi}$$

$$a_N = \frac{1}{4\pi}\left[\frac{4\pi \cos(2n\pi)}{N^2}\right] \qquad a_n = \frac{1}{N^2}$$

$$b_N = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx \qquad b_n = \frac{1}{4\pi}\int_0^{2\pi}(x^2 - 2x\pi + \pi^2)\sin nx$$

$$b_n = \frac{1}{4\pi}\left[\frac{(x^2 - 2\pi x + \pi^2)\cos nx}{-n} - \frac{(2x - 2\pi)\sin Nx}{-n^2} + \frac{2\cos nx}{-n^3}\right]_0^{2\pi}$$

$$b_n = -\frac{1}{4\pi}\left[\frac{4\cancel{\pi^2} - \cancel{4\pi^2} + \pi^2}{n} + \frac{2}{n^3} - \frac{\pi^2}{n} - \frac{2}{N^3}\right] \qquad b_N = 0$$

$$\boxed{\therefore f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{N^2}\cos Nx}$$

put $x = \pi$

$$\left(\frac{\pi - \pi}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2}$$

$$-\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\frac{\pi^2}{12} = \frac{1}{-1} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$$

$$\boxed{\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}}$$

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Question Bank :

Q9. Find the complex Fourier series for the function $f(x) = \begin{cases} 0, & 0<x<l \\ 1, & l<x<2l \end{cases}$

Soln: complex form of fourier series for $f(x)$ is

$$f(x) = \sum_{n=-\infty}^{\infty} C_n \, e^{\frac{in\pi x}{l}} \longrightarrow (1)$$

$$C_n = \frac{1}{2l} \int_0^{2l} f(x) \cdot e^{\frac{-in\pi x}{l}} \cdot dx$$

$$\therefore \; C_n = \frac{1}{2l} \left\{ \cancel{\int_0^l 0 \cdot dx}^{\,0} + \int_l^{2l} e^{\frac{-in\pi x}{l}} \cdot dx \right\}$$

$$\therefore \; C_n = \frac{-1}{2in\pi} \left[e^{-2in\pi} - e^{-in\pi} \right]$$

$$= \frac{-1}{2in\pi} \left[\cos 2n\pi - i\sin 2n\pi - (\cos n\pi - i\sin n\pi) \right].$$

$$\therefore \; C_n = \frac{-1}{2in\pi} \left[1 - (-1)^n \right] \qquad (\text{except } n=0)$$

at $n=0$, $C_n = \frac{1}{2}$ and for all even values of n, $C_n = 0$

Therefore, for $n = \pm 1, \pm 3, \pm 5 \cdots$, using eqn (1) we get

$$f(x) = \cdots + \frac{1}{3i\pi} e^{\frac{-3i\pi x}{l}} + \frac{1}{i\pi} e^{\frac{-i\pi x}{l}} + \frac{1}{2}$$

$$- \frac{1}{i\pi} e^{\frac{i\pi x}{l}} - \frac{1}{3i\pi} e^{\frac{3i\pi x}{l}} - \cdots$$

$$= \frac{1}{2} + \frac{1}{i\pi}\left(e^{\frac{-i\pi x}{l}} - e^{\frac{i\pi x}{l}}\right) + \frac{1}{3i\pi}\left(e^{\frac{-3i\pi x}{l}} - e^{\frac{3i\pi x}{l}}\right) + \dots$$

$$= \frac{1}{2} + \frac{1}{i\pi} \sum_{n=-\infty}^{\infty} \frac{1}{(2n-1)} \left[e^{-\left[\frac{(2n-1)\pi}{l}\right]ix} - e^{\left[\frac{(2n-1)\pi}{l}\right]ix} \right]$$

$$= \frac{1}{2} - \frac{1}{i\pi} \sum_{n=-\infty}^{\infty} \frac{1}{(2n-1)} \left[e^{\left(\frac{(2n-1)\pi}{l}\right)ix} - e^{\left(\frac{(2n-1)\pi}{l}\right)ix} \right]$$

Also, we can write the answer as.

$$f(x) = \frac{1}{2} + \frac{i}{2\pi} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1-(-1)^n}{n} \cdot e^{\frac{in\pi x}{l}}.$$

UNIT - 4

ASSIGNMENT

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Question Bank Q10:

Find the Fourier series ~~for~~ up to ~~the~~ first harmonic for $f(x)$ given by the following table.

x	0	60	120	180	240	300	360
$f(x)$	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Soln

Period = 2π.

$$\boxed{f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x}$$

$$a_0 = \frac{2}{N} \Sigma y$$

$$a_n = \frac{2}{N} \Sigma y \cos nx \qquad a_1 = \frac{2}{N} \Sigma y \cos x$$

$$b_n = \frac{2}{N} \Sigma y \sin nx \qquad b_1 = \frac{2}{N} \Sigma y \sin x$$

$$a_0 = \frac{2}{N} \Sigma y$$

$$= \frac{2}{6}(7.9 + 7.2 + 3.6 + 0.5 + 0.9 + 6.8 + 7.9)$$

$$= 8.9667$$

x	y	$\cos x$	$\sin x$	$y\cos x$	$y\sin x$.
0	7.9	1	0	7.9	0
60	7.2	0.5	0.866	3.6	6.2352
120	3.6	-0.5	0.866	-1.8	3.1176
180	0.5	-1	0	-0.5	0
240	0.9	-0.5	-0.866	-0.45	-0.7794
300	6.8	0.5	-0.866	3.4	-5.8888
Total				12.15	2.6847.

$$a_1 = \frac{2}{6}(12.15)$$

$$= \underline{\underline{4.05}}$$

$$b_1 = \frac{2}{6}(2.6847)$$

$$= \underline{\underline{0.8949}}$$

$$f(x) = \frac{8.9667}{2} + 4.05 \cos x + 0.8949 \sin x$$

Q.B - 11th Question

DISHANTH.K
PES2UG22CS18[illegible]

11. Find Fourier Series for the function $f(x)$

$$= \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$$

Fourier series is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l}$$

$$a_0 = \frac{1}{l} \int_c^{c+2l} f(x)\,dx$$

$$a_n = \frac{1}{l} \int_c^{c+2l} f(x) \cos \frac{n\pi x}{l}\,dx$$

$$b_n = \frac{1}{l} \int_c^{c+2l} f(x) \sin \frac{n\pi x}{l}\,dx$$

where, $2l = 2$
so, $l = 1$

$$a_0 = \frac{1}{l} \int_c^{c+2l} f(x)\,dx$$

$$= \frac{1}{1} \int_0^2 f(x)\,dx$$

$$= \int_0^1 \pi x\,dx + \int_1^2 \pi(2-x)\,dx$$

$$= \pi\left[\frac{x^2}{2}\right]_0^1 + \pi\left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \pi\left[\frac{1}{2} + 4 - 2 - 2 + \frac{1}{2}\right]$$

$$a_0 = \pi$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cdot\cos\frac{n\pi x}{l}\,dx$$

$$= 1\int_0^2 f(x)\,\cos\frac{n\pi x}{l}\,dx$$

$$= \int_0^1 \pi x\cdot\cos\frac{n\pi x}{l} + \int_1^2 \pi(2-x)\cdot\cos\frac{n\pi x}{l}$$

$$= \pi\left[\frac{x\cdot\sin n\pi x}{n\pi} + \frac{1}{n\pi}\left[\frac{\cos n\pi x}{n\pi}\right]\right]_0^1 + \pi\left[(2-x)\cdot\frac{\sin n\pi x}{n\pi} - \frac{\cos n\pi x}{n^2\pi^2}\right]_1^2$$

$$= \pi\left[\frac{(-1)^n}{n^2\pi^2} - \frac{1}{n^2\pi^2} - \frac{(-1)^{2n}}{n^2\pi^2} + \frac{(-1)^n}{n^2\pi^2}\right] \qquad \left[(-1)^{2n} = (-1)^n = 1\right]$$

$$= \frac{1}{n^2\pi}\left[2(-1)^n - 2\right]$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\cdot\sin\frac{n\pi x}{l}\,dx$$

$$= 1\int_0^2 f(x)\cdot\sin\frac{n\pi x}{l}\,dx$$

$$= \int_0^1 \pi x\cdot\sin n\pi x\,dx + \int_1^2 \pi(2-x)\cdot\sin n\pi x\,dx$$

$$= \pi\left[\left[-x\cdot\frac{\cos n\pi x}{n\pi} + \frac{1}{n\pi}\left(\frac{\sin n\pi x}{n\pi}\right)\right]_0^1 + \left[-(2-x)\frac{\cos n\pi x}{n\pi} + \frac{1}{n\pi}\left(\frac{\sin n\pi x}{n\pi}\right)\right]_1^2\right]$$

$$= \pi\left[-\frac{(-1)^n}{n\pi} + \frac{(-1)^n}{n\pi}\right]$$

$b_n = 0$

Fourier series is,

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left[(-1)^n - 1\right] \cdot \cos n\pi x$$

Prashtant Dayal

PES2UG22CS187

Unit 4 Question Bank

Q12 Develop $f(x) = e^{-x}$ in Fourier Series in $(-3,3)$.

$\therefore l = 3$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$= \frac{1}{3}\int_{-3}^{3} e^{-x}\,dx$$

$$= \frac{1}{3}\left[-e^{-x}\right]_{-3}^{3}$$

$$= \frac{1}{3}\left(-e^{3} + e^{-3}\right)$$

$$a_0 = \frac{2\sinh 3}{3} \qquad \left[\sinh x = \frac{e^{x} - e^{-x}}{2}\right]$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{3}\int_{-3}^{3} e^{-x}\cos\frac{n\pi x}{3}\,dx$$

$$= \frac{1}{3}\left[\frac{e^{-x}}{1^2+\left(\frac{n\pi}{3}\right)^2}\left[-\cos\frac{n\pi x}{3} + \frac{n\pi}{3}\sin\frac{n\pi x}{3}\right]\right]_{-3}^{3}$$

$$\cancel{= \frac{e^{-x}}{3(1+n\pi)} \quad \frac{e^{-x}}{3\left(1+\frac{n\pi^2}{9}\right)}}$$

$$= \frac{1}{3}\left[\frac{e^{-3}}{1+\left(\frac{n\pi}{3}\right)^2}\left(-\cos n\pi + \frac{n\pi}{3}\sin n\pi\right) - \frac{e^{3}}{1+\left(\frac{n\pi}{3}\right)^2}\left\{-\cos(-n\pi) + \frac{n\pi}{3}\sin(-n\pi)\right\}\right]$$

$$a_n = \frac{1}{3}\left[\frac{e^{-3}}{1+\frac{n^2\pi^2}{9}}\{-(-1)^n\} - \frac{e^{3}}{1+\frac{n^2\pi^2}{9}}\{-(-1)^n\}\right]$$

$$= \frac{1}{3\left(\frac{9+n^2\pi^2}{9}\right)}\left[-e^{-3}(-1)^n + e^{3}(-1)^n\right]$$

$$= \cancel{\frac{1}{2\pi}}\ \frac{(-1)^n}{\frac{9+n^2\pi^2}{3}}\left(e^3 - e^{-3}\right)$$

$$= \frac{3(-1)^n}{9+n^2\pi^2}\, 2\sinh 3$$

$$a_n = \frac{6(-1)^n \sinh 3}{9+n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{3}\int_{-3}^{3} e^{-x}\sin\frac{n\pi x}{3}\,dx$$

$$= \frac{1}{3}\left[\frac{e^{-x}}{1^2+\frac{n^2\pi^2}{3^2}}\left((-1)\sin\frac{n\pi x}{3} - \frac{n\pi}{3}\cos\frac{n\pi x}{3}\right)\right]_{-3}^{3}$$

$$= \frac{1}{3}\left[\frac{e^{-3}}{1+\frac{n^2\pi^2}{9}}\left(-\cancelto{0}{\sin n\pi} - \frac{n\pi}{3}\cos n\pi\right) - \frac{e^{3}}{1+\frac{n^2\pi^2}{9}}\left(-\cancelto{0}{\sin(-n\pi)} - \frac{n\pi}{3}\cos(-n\pi)\right)\right]$$

$$= \frac{1}{3\left(\frac{9+n^2\pi^2}{9}\right)}\left[\frac{n\pi}{3}e^{-3}\{-(-1)^n\} - e^{3}\frac{n\pi}{3}\{-(-1)^n\}\right]$$

$$= \frac{3}{9+n^2\pi^2}(-1)^n\left(e^3 - e^{-3}\right)\frac{n\pi}{3}$$

$$b_n = \frac{3(-1)^n}{9+n^2\pi^2} \, 2\sinh 3 \, \frac{n\pi}{3}$$

$$b_n = \frac{2n\pi(-1)^n \sinh 3}{9+n^2\pi^2}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}$$

$$= \frac{\sinh 3}{3} + \sum_{n=1}^{\infty} \frac{6(-1)^n \sinh 3}{9+n^2\pi^2} \cos\frac{n\pi x}{3} + \frac{2n\pi(-1)^n}{9+n^2\pi^2}$$

$$\sinh 3 \sin\frac{n\pi x}{3}$$

$$f(x) = \sinh 3 \left[\frac{1}{3} + \frac{2(-1)^n}{9+n^2\pi^2} \sum_{n=1}^{\infty} 3\cos\left(\frac{n\pi x}{3}\right) + n\pi \sin\left(\frac{n\pi x}{3}\right) \right]$$

Drupad. H. Patel

PES2UG22CS188

13. Expand $f(x) = \begin{cases} 0 & \text{for } -2 < x < 0 \\ 1 & \text{for } 0 < x < 2 \end{cases}$ in fourier series

$f(x)$ is an even function.

$(-l, l)$

$$a_0 = \frac{1}{l}\int_{-2}^{2} f(x)\,dx$$

$$= \frac{1}{l}\int_{0}^{2} f(x)\,dx$$

$$= \frac{1}{2}[x]_0^2$$

$$a_0 = 1$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{2}\int_0^2 \cos\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{2}\left[\frac{\sin n\pi/2}{n\pi/2}\right]_0^2$$

$$a_n = 0$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{2}\int_0^2 \sin\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{2}\left[-\frac{\cos\frac{n\pi x}{2}}{n\pi/2}\right]_0^2$$

$$= \frac{1}{2}\left[\frac{-(-1)^n}{n\pi/2} + \frac{1}{n\pi/2}\right]$$

$$= \frac{1}{2}\left[\frac{1-(-1)^n}{n\pi/2}\right]$$

$$= \frac{1}{2}\times\frac{2}{n\pi}\left[1-(-1)^n\right]$$

$$b_n = \frac{1}{n\pi}\left[1-(-1)^n\right]$$

DURGADEEPA·P·
PES2UG22CS189

QB

14) Find the Fourier series for the function $f(x) = x - x^2$ in $-1 \leq x \leq 1$

$(-1, 1)$ $(-l, l)$ $\Rightarrow l = 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$= \frac{1}{1}\int_{-1}^{1} x - x^2\,dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \left[\frac{1}{2} - \frac{1}{3}\right] - \left(\frac{1}{2} + \frac{1}{3}\right)$$

$$= \frac{1}{6} - \frac{5}{6}$$

$$= \frac{-4}{6} = \frac{-2}{3}$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{1}\int_{-1}^{1}(x - x^2)\cos n\pi x\,dx$$

$$= \left[\cancel{(x-x^2)\cdot\frac{\sin n\pi x}{n\pi}} - \frac{(1-2x)}{n\pi}\cdot\frac{-\cos n\pi x}{n\pi} + \cancel{\frac{-(-2)}{n^2\pi^2}\cdot\frac{\sin n\pi x}{n\pi}}\right]_{-1}^{1}$$

$$= \cancel{\frac{-(-2)}{}}\left[\frac{1-2x}{n^2\pi^2}\cos n\pi x\right]_{-1}^{1}$$

$$= \frac{1}{n^2\pi^2}\left[(1-2)(-1)^n - 3(-1)^n\right]$$

$$= \frac{-1(-1)^n - 3(-1)^n}{n^2\pi^2}$$

$$= \frac{-4(-1)^n}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \int_{-1}^{1}(x-x^2)\cdot\sin n\pi x\,dx$$

$$= \left[x-x^2\cdot\frac{\cos n\pi x}{n\pi} + \cancel{\frac{(1-2x)}{n\pi}\frac{\sin n\pi x}{n\pi}}^{\,0} + \frac{\pm 2}{n^2\pi^2}\frac{\cos n\pi x}{n\pi}\right]_{-1}^{1}$$

$$= \frac{2(-1)^n}{n^3\pi^3} - \left\{\frac{-2-(-1)^n}{n\pi} + \frac{2(-1)^n}{n^3\pi^3}\right\}$$

$$= \frac{2(-1)^n}{n^3\pi^3} - \frac{2(-1)^n}{n\pi} - \frac{2(-1)^n}{n^3\pi^3}$$

$$= \frac{2(-1)^{n+1}}{n\pi}$$

~~Fourier series~~: $\cancel{x-x^2} = \frac{\left(\frac{-2}{3}\right)}{2} + \sum_{n=1}^{\infty}\frac{-4(-1)^n}{n^2\pi^2}\cos n\pi x + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n\pi}\sin n\pi x$

~~Put x~~ =

ASSIGNMENT - 4

Earnest Crasto
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Q15. Find the fourier series of $f(x) = e^x$ in the interval $0 < x < 2\pi$

Period = 2π

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^x\,dx = \frac{1}{\pi}[e^x]_0^{2\pi} = \frac{1}{\pi}[e^{2\pi} - 1]$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^x \cos(nx)\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^x}{1^2+n^2}((1)\cos(nx) + n\sin(nx))\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi}}{1+n^2}(\overset{(1)}{\cos(2\pi n)} + n\cancel{\sin(2n\pi)}^{0}) - \frac{e^0}{1+n^2}(\overset{(1)}{\cos(0)} + n\cancel{\sin(0)}^{0})\right]$$

$$= \frac{1(e^{2\pi}-1)}{\pi(1+n^2)}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^x \sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^x}{1+n^2}(\sin(nx) - n\cos(nx))\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi}}{1+n^2}(\cancel{\sin(2n\pi)}^{0} - n\cancel{\cos(2n\pi)}^{(1)}) - \frac{e^0}{1+n^2}(\cancel{\sin(0)}^{0} - n\overset{(1)}{\cos(0)})\right]$$

$$= \frac{1}{\pi}\frac{(-ne^{2\pi} + n)}{1+n^2}$$

$$= \frac{-n}{\pi}\frac{(e^{2\pi}-1)}{1+n^2}$$

$\therefore$ Fourier series of $f(x)$

$$\Rightarrow e^x = \frac{(e^{2\pi}-1)}{2\pi} + \sum_{n=1}^{\infty} \frac{(e^{2\pi}-1)}{(1+n^2)}\frac{\cos(nx)}{\pi} + \left(\frac{-n}{\pi}\right)\left(\frac{e^{2\pi}-1}{1+n^2}\right)$$

Question Bank

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Q16) Find the fourier series for the function $f(x)=|\sin x|$ in $-\pi<x<\pi$

Solution: $f(x)=|\sin x|$ is an even function.

thus $\boxed{b_n=0}$

$$a_0=\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$=\frac{2}{\pi}\int_0^{\pi}\sin x\,dx=\frac{2}{\pi}\left[-\cos x\right]_0^{\pi}=\frac{2}{\pi}(2)=\frac{4}{\pi}$$

$$\boxed{a_0=\frac{4}{\pi}}$$

$$a_n=\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$=\frac{2}{\pi}\int_0^{\pi}\sin x\cos nx\,dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\sin(1+n)x+\sin(1-n)x$$

$$=\frac{1}{\pi}\left[\frac{-\cos(1+n)x}{(1+n)}-\frac{\cos(1-n)x}{1-n}\right]_0^{\pi}$$

$$=\frac{1}{\pi}\left[\frac{-(-1)^{1+n}}{(1+n)}-\frac{(-1)^{1-n}}{1-n}+\frac{1}{1+n}-\frac{1}{1-n}\right]$$

$$=\frac{2}{\pi}\left[\frac{1+(-1)^n}{1-n^2}\right]$$

$$\boxed{a_n=\frac{2}{\pi}\left[\frac{1+(-1)^n}{1-n^2}\right]}$$

$$f(x)=\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{2}{\pi}\left(\frac{1+(-1)^n}{1-n^2}\right)\cos nx$$

Fardeen Akeel Shaikh PES2UG22CS192

Question Bank (Maths Assignment)

Q. 17) Find the Fourier Series for the function $f(x) = x^2$ in $-1 \leq x \leq 1$.

Ans $f(x) = x^2$, $f(-x) = (-x)^2 = x^2$

$\therefore f(-x) = f(x)$. $\therefore$ The function is even. $(b_n = 0)$.

$$a_0 = 2\int_0^1 x^2\,dx.$$

$$= 2\left[\frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{3}\right] = \boxed{\frac{2}{3}}.$$

$$a_n = 2\int_0^1 x^2 \cos n\pi x\,dx.$$

$$= 2\left[\frac{x^2 \sin n\pi x}{n\pi} + \frac{2x}{n^2\pi^2}\cos n\pi x + \frac{2}{n^3\pi^3}\sin n\pi x\right]_0^1$$

$$= 2\left[\frac{2(-1)^n}{n^2\pi^2} - 0\right] = \boxed{\frac{4(-1)^n}{n^2\pi^2}},$$

$\therefore$ The Fourier Series,

$$\boxed{f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2\pi^2}\cos n\pi x}$$

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PES2UG22CS193
Question Bank
Question 18

Q] $f(x) = x\sin x$; interval $0<x<2\pi$

$F(-x) = -x\sin(-x)$

$= x\sin x$

$= F(x)$

$\therefore$ Function is even

$\therefore b_n = 0$

$f(-x) = f(x) \rightarrow$ even fn

for the case $(-\pi, \pi)$.

This function is neither odd nor even in $(0, 2\pi)$.

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx}$$

$$\boxed{\begin{aligned} a_n &= \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx \\ a_0 &= \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \end{aligned}}$$

Find b_n and prove $b_n = 0$.

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x\sin x \cdot dx$$

$$= \frac{1}{\pi}\left(-x\cos x - \sin x\right)_0^{2\pi}$$

$$= \frac{1}{\pi}(-2\pi) = -2$$

$$\boxed{a_0 = -2}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x\sin x\cos nx\,dx.$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\left(\sin(n+1)x + \sin(n-1)x\right)dx$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} x\sin(n+1)x + \int_0^{2\pi} x\sin(n-1)x\,dx\right]$$

$$= \frac{1}{2\pi}\left[\left[-\frac{x\cos(n+1)x}{n+1} - \cancel{\left(\frac{-1}{n+1}\right)\frac{\sin(n+1)x}{(n+1)}}\right]_0^{2\pi} + \left[-\frac{x\cos(n-1)x}{(n-1)} - \cancel{\left(\frac{-1}{n-1}\right)\left(\frac{\sin(n-1)x}{n-1}\right)}\right]_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[-\frac{2\pi\cos(n+1)2\pi}{n+1} - \frac{2\pi\cos(n-1)2\pi}{(n-1)}\right] = \frac{1}{2\pi}\left[-\frac{4n\pi}{n^2-1}\right] = \frac{-2n}{n^2-1}$$

$$\boxed{a_n = \frac{-2n}{n^2-1}}$$

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} x\sin x \cos nx \, dx \Big|_{n=1}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\sin 2x \, dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\sin 2x \, dx$$

$$= \frac{1}{2\pi}\left[-\frac{x\cos 2x}{2} - \left(\frac{-1}{2}\right)\frac{\sin 2x}{2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[-\frac{2\pi(1)}{2}\right]$$

$$= -1/2$$

$$\boxed{a_1 = -1/2}$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} x\sin x \sin nx \, dx \Big|_{n=1}$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\sin^2 x \, dx.$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} x - \int_0^{2\pi} x\cos 2x\right]$$

$$= \frac{1}{2\pi}\left[\left[\frac{x^2}{2}\right]_0^{2\pi} - \left[\cancelto{0}{\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}}\right]_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{4\pi^2}{2}\right]$$

$$= \pi$$

$$\boxed{b_1 = \pi}$$

PES2UG22CS194
G Nishruth Chowdary

19) obtain a Fourier series to represent the following periodic function.

$$f(x) = \begin{cases} 0 & 0 < x < \pi \\ 1 & \pi < x < 2\pi \end{cases}$$

→ $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx.$$

$$a_0 = \frac{1}{\pi} \int_{\pi}^{2\pi} 1 \cdot dx.$$

$$= \frac{1}{\pi} (2\pi - \pi)$$

$$= \frac{1}{\pi} (\pi) = 1.$$

$$\boxed{a_0 = 1}$$

$$a_n = \frac{1}{\pi} \int_{\pi}^{2\pi} \cos nx \cdot dx.$$

$$= \frac{1}{\pi} \left[\frac{\sin nx}{n} \right]_{\pi}^{2\pi}$$

$$\boxed{a_n = 0.}$$

$$b_n = \frac{1}{\pi} \int_{\pi}^{2\pi} \sin nx \, dx$$

$$= \frac{1}{\pi} \left(-\frac{\cos nx}{n} \right)_{\pi}^{2\pi}$$

$$= \frac{-1}{\pi n} \left(\cos 2n\pi - \cos n\pi \right)$$

$$= \frac{-1}{\pi n} \left(1 - (-1)^n \right)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-1}{\pi n} (1 - (-1)^n) \sin nx.$$

MATH ASSIGNMENT

G. Spandana

PES2UG22CS195

Question Bank 20

→ Find the Fourier Series for the $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$

Sol:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{-1}\right]_0^{2\pi}$$

$$= \frac{-1}{\pi}\left[e^{-2\pi} - 1\right]$$

$$a_0 = \frac{1}{\pi}\left[1 - e^{-2\pi}\right]$$

$$\boxed{\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\cos nx\,dx$$

$a = -1$, $b = n$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{1+n^2}(-\cos nx + n\cancel{\sin nx})\right]_0^{2\pi}$$

$$= \frac{1}{\pi} \times \frac{1}{1+n^2}\left[-e^{-2\pi}(1) + e^0(1)\right]$$

$$= \frac{1}{\pi} \times \frac{1}{1+n^2}\left[-e^{-2\pi} + 1\right] \Rightarrow a_n = \frac{1}{\pi}\left[\frac{1-e^{-2\pi}}{1+n^2}\right].$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx \qquad \begin{pmatrix} a=-1 \\ b=n \end{pmatrix}$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{-x}\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{1+n^2}\left(-\overset{0}{\cancel{\sin nx}} - n\cos nx\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\times\frac{1}{1+n^2}\left[-n\left(e^{-2\pi}\right)(1) - n\left(-e^{0}\right)(1)\right]$$

$$= \frac{1}{\pi}\times\frac{n}{1+n^2}\left[-e^{-2\pi}+1\right]$$

$$b_n = \frac{n}{\pi}\left[\frac{1-e^{-2\pi}}{1+n^2}\right].$$

$$f(x) = \frac{1}{2\pi}\left[1-e^{-2\pi}\right] + \sum_{n=1}^{\infty} \frac{1}{\pi}\left[\frac{1-e^{-2\pi}}{1+n^2}\right]\cos nx + \frac{n}{\pi}\left[\frac{1-e^{-2\pi}}{1+n^2}\right]\sin nx.$$

MATH ASSIGNMENT

GAGAN GURUPRASHD PHADKE

PES2UG22CS196

QB

Q21) $f(x) = e^{-ax}$ in the interval $-\pi < x < \pi$

A Fourier series $\rightarrow f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\,dx = \frac{1}{\pi}\left[\frac{e^{-ax}}{a}\right]_{-\pi}^{\pi} = \frac{1}{a\pi}\left(e^{-a\pi} - e^{a\pi}\right)$$

$$a_0 = \frac{2\sinh a\pi}{a\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\cos nx\,dx = \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}\left(-a\cos nx + n\sin nx\right)\right]_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\cdot\frac{1}{a^2+n^2}\left[-e^{-a\pi}\cdot a\cdot\cos n\pi + e^{a\pi}\cdot a\cdot\cos n(-\pi)\right]$$

$$a_n = \frac{1}{\pi(a^2+n^2)}\left[-ae^{-a\pi}(-1)^n + ae^{a\pi}(-1)^n\right] = \frac{a(-1)^n}{\pi(a^2+n^2)}\cdot 2\sinh a\pi$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\sin nx\,dx = \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}\left(-a\sin nx - n\cos nx\right)\right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi}\cdot\frac{1}{a^2+n^2}\left[-e^{-a\pi}\cdot n\cos n\pi + e^{a\pi}\cdot n\cos n(-\pi)\right]$$

$$b_n = \frac{1}{\pi(a^2+n^2)}\left[-ne^{-a\pi}(-1)^n + ne^{a\pi}(-1)^n\right] = \frac{n(-1)^n}{\pi(a^2+n^2)}\cdot 2\sinh a\pi$$

$$\therefore f(x) = \frac{\sinh a\pi}{a\pi} + \sum_{n=1}^{\infty} \frac{2a(-1)^n \sinh a\pi}{\pi(n^2+a^2)}\cos nx + \frac{2n(-1)^n \sinh a\pi}{\pi(n^2+a^2)}\sin nx$$

Math Assignment

PES2UG22CS197

~Gagan S L

Question bank - 22 Q

Express $f(x) = \cos(wx)$ in $-\pi < x < \pi$ as a Fourier series, where w is a constant.

As $f(-x) = f(x)$ => It is an even function.

$\therefore b_n = 0$ for the series between the interval $-\pi < x < \pi$

~~ae~~ => $f(x) = \dfrac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \cos nx$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} \cos wx \, dx = \frac{2}{\pi}\left[\frac{\sin wx}{w}\right]_0^{\pi} = \frac{2}{\pi w}\sin w\pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \cos nx \cdot \cos wx \, dx \cancel{= \frac{2}{\pi}\int_0}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\pi}\int_0^{\pi}\left[\cos(nx + wx) + \cos(nx - wx)\right]dx$$

$\because \cos x \cdot \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$

$$= \frac{1}{\pi}\left[\frac{\sin(nx+wx)}{n+w} + \frac{\sin(nx - wx)}{n-w}\right]_0^{\pi}$$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$, $\sin(x-y) = \sin x \cos y - \cos x \sin y$

$$= \frac{1}{\pi}\left[\frac{\sin(n\pi + \omega\pi)}{\omega + n} + \frac{\sin(n\pi - \omega\pi)}{n-\omega}\right]$$

$\sin n\pi = 0$, $\cos n\pi = (-1)^n$

$$= \frac{1}{\pi}\left[\frac{\cancel{\sin n\pi \cos \omega\pi} + \cos n\pi \sin \omega\pi}{\omega + n} + \frac{\cancel{\sin n\pi \cos \omega\pi} - \cos n\pi \sin \omega\pi}{n-\omega}\right]$$

$$= \frac{1}{\pi}\left[(-1)^n \sin \omega\pi \left(\frac{1}{\omega + n} + \frac{(-1)}{n-\omega}\right)\right]$$

$$= \frac{(-1)^n \sin \omega\pi}{\pi} \cdot \frac{-2\omega}{n^2 - \omega^2} = \frac{(-1)^{n+1} \sin(\omega\pi) \cdot 2\omega}{\pi(n^2 - \omega^2)}$$

$$f(x) = \frac{\sin \omega\pi}{\omega\pi} + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\omega \sin \omega\pi}{\pi(n^2 - \omega^2)} \cos nx$$

Gagana Sindhu P
PES2UG22CS198

Question Bank

23. Obtain the fourier series for the function

$$f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

→ $$f(-x) = \begin{cases} -(-x) & -\pi < -x < 0 \\ -x & 0 < -x < \pi \end{cases}$$

$$f(-x) = \begin{cases} x & \pi > x > 0 \\ -x & 0 > x > -\pi \end{cases}$$

$\therefore f(-x) = f(x)$; Fourier series is given by $\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx}$

$f(x)$ is an even function $\Rightarrow b_n = 0$.

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = \pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - 1\left(\frac{1}{n}\right)\left(-\frac{\cos nx}{n}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\cancelto{0}{\frac{\pi \sin n\pi}{n}} + \frac{\cos n\pi}{n^2} - \left(0 + \frac{\cos 0}{n^2}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = \frac{2}{\pi n^2}\left(-1 + (-1)^n\right)$$

$$\boxed{a_n = \frac{2}{\pi n^2}\left[-1 + (-1)^n\right]}$$

$$\therefore\ f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left[-1 + (-1)^n\right] \cos nx + 0$$

$$\boxed{f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left[-1 + (-1)^n\right] \cos nx}$$

Name - Ganesh R. Halembre
SRN - PES2UG22CS199
Sem - II Sec - C

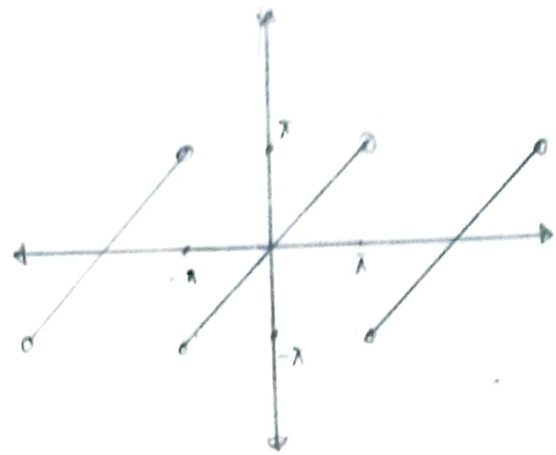
MATHS ASSIGNMENT

UNIT - 4 FOURIER SERIES

QUESTION BANK

Q.2(i) Find the fourier series of $f(x) = x$ in interval $-\pi < x < \pi$. Draw its Graph

The function $f(x) = x$ graph:



$f(x) = x$

$f(-x) = -x = -f(x)$

$\therefore$ The Given function is odd

Fourier Cosine function

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = 0$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx = 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx = \frac{2}{\pi}\int_{0}^{\pi} x\sin nx$

$= \frac{2}{\pi}\left[-\frac{x\cos nx}{n} - \frac{(-1)\overset{0}{\cancel{\sin nx}}}{n^2}\right]_0^{\pi}$ $\quad \because \sin n\pi = 0$ & $\sin 0 = 0$

$= \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n} - (0)\right]$

$= \frac{2}{\cancel{\pi}}\left[\frac{-\cancel{\pi}(-1)^n}{n}\right]$

$= -\frac{2}{n}(-1)^n$

$= \frac{2}{n}(-1)^{n+1}$

$\therefore$ The fourier ~~tran~~ series of $f(x) = x$ is given by

$f(x) = \sum_{n=1}^{\infty} f(n)\sin nx = \sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin nx$

Assignment - 4

Name :- Garvit Agarwal

PES2UG22CS200

Section - C

(Q1) Express $f(x) = x$ as a H.H - range sine series in $0 < x < 2$

$\Rightarrow$ $f(x) = x$ $\quad 0 < x < 2$

$l = 2$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right)$$

$$b_n = \frac{2}{2}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)dx = \int_0^2 x\sin\left(\frac{n\pi x}{2}\right)dx$$

$$= \left[x\left(-\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\right) - \left(-\frac{4}{n^2\pi^2}\right)\sin\left(\frac{n\pi x}{2}\right)\right]_0^2$$

$$= \left\{-\frac{4}{n\pi}(-1)^n - 0\right\} - [0 - 0]$$

$$b_n = -\frac{4}{n\pi}(-1)^n$$

$$\therefore f(x) = -\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi x}{2}\right)$$

$$= \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi}{2}x\right)$$

Gauthami . S
PES2UG22CS201
C - Section

Question bank:

26) Q.

Express $f(x) = x$ as a Half Range cosine Series in $0 < x < 2$

Sol: Half Range cosine Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n x \pi}{l}$$

$(0, 2)$

$(0, l)$

$l = 2$

$l = 2$

$f(x) = x$

$$a_0 = \frac{2}{l} \int_0^l f(x) dx$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l}$$

$$a_0 = \frac{2}{2} \int_0^2 x\, dx$$

$$= \frac{\cancel{2}}{2} \left[\frac{x^2}{\cancel{2}}\right]_0^2$$

$$= \frac{1}{2} [4 - 0] = 2$$

$$a_n = \frac{\cancel{2}}{\cancel{2}} \int_0^2 x \cos \frac{n\pi x}{2} dx .$$

$$= \left[x\left[\frac{\sin \frac{n\pi x}{2}}{n\pi/2}\right] - \left[\frac{2}{n\pi}\right]\left[\frac{-\cos \frac{n\pi x}{2}}{n\pi/2}\right]\right]_0^2$$

$$= \left[\frac{2x}{n\pi} \sin \frac{n\pi x}{2} + \frac{4}{n^2\pi^2} \cos \frac{n\pi x}{2}\right]_0^2$$

$$= \left[0 + \frac{4}{n^2\pi^2}(-1)^n\right] - \left[\frac{4}{n^2\pi^2}(1)\right]$$

$$a_n = \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

$$= \frac{2}{2} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right] \cos \frac{n\pi x}{2}$$

$$= 1 + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{-1}{n^2}\left[1 + (-1)^{n+1}\right] \cos \frac{n\pi x}{2}$$

$$= 1 - \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2}\left[1 + (-1)^{n+1}\right] \cos \frac{n\pi x}{2}$$

$$= 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{2} \quad \text{if } n \text{ is odd.}$$

(OR) $$= 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos \frac{(2n-1)\pi x}{2}$$

GOWRI. M. B
PES2UG22CS202
section: C

27. express $f(x) = x^2$ as a half range cosine series in $0 < x < \pi$

~~Ans~~

Soln:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2}{3\pi}\left[\pi^3 - 0\right] = \boxed{\frac{2}{3}\pi^2 = a_0}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\,dx$$

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} - \frac{2x}{n^2}\cdot(-\cos nx) + 2\left(\frac{-1}{n^3}\right)\sin nx\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[x^2\frac{\cancel{\sin \pi x}^{0}}{n} + \frac{2\pi}{n^2}\cos \pi x - \frac{2}{n^3}\cancel{\sin \pi x}^{0}\right] - [\;0\;]$$

$$= \frac{2}{\pi}\left[\frac{2\pi}{n^2}(-1)^n\right]$$

$$\boxed{a_n = \frac{4}{n^2}\left[(-1)^n\right]}$$

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SRN: PES2UG22CS203

Engineering Mathematics-II
Assignment-3

Question Bank

28) Find the fourier series of the function $f(x) = x^2$ in $-l \leq x \leq l$.

→ $f(x) = x^2$

$f(-x) = -x^2 = x^2$

$\therefore f(x) = f(-x)$.

$\therefore$ It is an even function

Hence $b_n = 0$.

The fourier series of $f(x)$ over $(-l, l)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

where $a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx = \frac{2}{l}\int_0^l f(x)\,dx$.

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x) \cdot \cos\frac{n\pi x}{l} = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l}\int_0^l x^2\,dx$$

$$= \frac{2}{l}\left[\frac{x^3}{3}\right]_0^l$$

$$= \frac{2\cancel{l^3}l^2}{3\cancel{l}} = \frac{2l^2}{3}$$

$$a_n = \frac{2}{l}\int_0^l x^2 \cos\frac{n\pi x}{l}$$

$$= \frac{2}{l}\left[x^2\left(\cancel{\frac{\sin\frac{n\pi x}{l}}{\frac{n\pi}{l}}}\right)^{\to 0} - (2x)\left(\frac{l}{n\pi}\right)\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) + (2)\left(\frac{1}{n^2\pi^2}\right)\cancel{\left(\frac{\sin n\pi x}{n\pi}\right)}^{\to 0}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{l^2}{n^2\pi^2}\left(2x\cos n\pi x\right)_0^l\right]$$

$$= \frac{4l^2}{n^2\pi^2 l}\left(l\cdot(-1)^n - 0(1)\right)$$

$$= \frac{4\cancel{l}(-1)^n l^2}{n^2\pi^2\cancel{l}} = \frac{4(-1)^n l^2}{n^2\pi^2}$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$$= \frac{\frac{2l^2}{3}}{2} + \sum_{n=1}^{\infty} \frac{4(-1)^n l^2}{n^2\pi^2} \cos\frac{n\pi x}{l}.$$

$$= \frac{\cancel{2} l^2}{\cancel{6}\,3} + \sum_{n=1}^{\infty} \frac{4(-1)^n l^2}{n^2\pi^2} \cos\frac{n\pi x}{l}.$$

$$f(x) = \frac{l^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n l^2}{n^2\pi^2} \cos\frac{n\pi x}{l}.$$

Question Bank

29. Find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ using Fourier series.
[Assum $f(x) = x^2$ in the interval $(-\pi, \pi)$]

Soln

$$\left.\begin{aligned} f(x) &= x^2 \\ f(-x) &= x^2 \end{aligned}\right\} \text{Even function}$$

$$\therefore \; b_n = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx.$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x^2\, dx$$

$$= \frac{2}{\pi} \left[\frac{x^3}{3} \right]_0^{\pi}$$

$$a_0 = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x^2 \cos nx\, dx$$

$$= \frac{2}{\pi} \left[x^2 \frac{\sin nx}{n} - 2x\left(\frac{1}{n}\right)\left(-\frac{\cos nx}{n}\right) + 2\left(\frac{-1}{n^2}\right) \frac{\sin nx}{n} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[2x \frac{\cos nx}{n^2} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{2\pi}{n^2} \cos n\pi - 0 \right]$$

$$a_n = \frac{4(-1)^n}{n^2}$$

$$f(x) = \frac{2\pi^2}{3\times 2} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nx$$

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

Put $x = \pi$

$$\pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi \qquad \because \cos n\pi = (-1)^n$$

$$\pi^2 - \frac{\pi^2}{3} = 4\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{n^2}$$

$$\frac{\cancel{2}\pi^2}{3\times \cancel{4}\,2} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2}$$

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

//

Question Bank => Q => 30

G. Visweswaran
PES2UG22CS205

30) $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx].$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} \sqrt{2\sin^2 x/2} \Rightarrow \frac{\sqrt{2}}{\pi} : \left[\frac{-\cos x/2}{1/2}\right]_0^{2\pi} \Rightarrow \frac{\sqrt{2}}{\pi}[2+2] \Rightarrow \frac{4\sqrt{2}}{\pi}.$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \sqrt{2}\sin x/2 \cos nx \cdot dx$$

$$= \frac{\sqrt{2}}{\pi}\int_0^{2\pi} \sin\left(\frac{x}{2}+nx\right) + \sin(x/2 - nx)\cdot dx = \frac{\sqrt{2}}{\pi}\int_0^{2\pi} \sin x(1/2+n) + \sin x\,(1/2-n)$$

$$= \frac{\sqrt{2}}{\pi}\left[\left[\frac{-\cos x(1/2+n)}{1/2+n}\right]_0^{2\pi} + \left[\frac{-\cos x\,(1/2-n)}{1/2-n}\right]_0^{2\pi}\right]$$

$$= \frac{\sqrt{2}}{\pi}\left[\frac{-\cos 2\pi(1/2+n)+1}{1/2+n}\right] + \left[\frac{-\cos x\, 2\pi(1/2-n)+1}{1/2-n}\right]$$

$$a_n = \frac{1}{\sqrt{2}\pi}\left[\frac{4}{2n+1} - \frac{4}{2n-1}\right] \Rightarrow \frac{1}{\sqrt{2}\pi}\left[\frac{-8}{4n^2-1}\right] \Rightarrow \frac{-4\sqrt{2}}{\pi(4n^2-1)}$$

$$\boxed{a_n = \frac{-4\sqrt{2}}{\pi(4n^2-1)}}$$

$\Rightarrow b_n = 0$. (even function). $\Rightarrow$ $f(x)$ is even if $f(2\pi - x) = f(x)$.
for fn defined in $(0, 2\pi)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$f(x) = \frac{2\sqrt{2}}{\pi} + \sum_{n=1}^{\infty} \frac{-4\sqrt{2}}{\pi(4n^2-1)}\cdot \cos nx$$

Put $x = 0$

$$0 = \frac{2\sqrt{2}}{\pi} + \sum_{n=1}^{\infty} \frac{-4\sqrt{2}}{\pi(4n^2-1)}\cdot(1) \Rightarrow \frac{+2\sqrt{2}}{\pi} = \frac{+4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$$

$$\boxed{1/2 = \sum_{n=1}^{\infty} \frac{1}{4n^2-1}}$$

Bollineni Keerthi
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Question Bank Q. 31

Find the half range Fourier cosine series for the function $f(x) = x - x^2$ in $0 < x < 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$0 < x < 1$

$(0, l)$

$l = 1$

$$a_0 = \frac{2}{l} \int_0^l f(x)\, dx$$

$$= \frac{2}{1} \int_0^1 x - x^2 \, dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$a_0 = 2\left[\frac{1}{2} - \frac{1}{3} - 0\right] = 2\left[\frac{3-2}{6}\right] = \frac{1}{3}$$

$$a_n = \frac{2}{1} \int_0^1 (x - x^2) \cos n\pi x \, dx$$

$$= 2\left[x - x^2\left(\frac{\sin n\pi x}{n\pi}\right) - (1-2x)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) + (-2)\left(-\frac{\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$$

$$= 2\left[0 - (-1)\left(\frac{-(-1)^n}{n^2\pi^2}\right) - 2(0) - \left[0 - \left(\frac{-1}{n^2\pi^2}\right) + 0\right]\right]$$

$$= 2\left[-\frac{(-1)^n}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right]$$

$$a_n = \frac{-2}{n^2\pi^2}\left[(-1)^n + 1\right]$$

$$\therefore a_0 = \frac{1}{3}, \quad a_n = \frac{-2}{n^2\pi^2}\left[(-1)^n + 1\right].$$

Question Bank

PES2UG22CS137
C. R. VARSHA

32) Obtain a cosine series for $f(x) = \begin{cases} 1 & \text{for } 0<x<1 \\ x & \text{for } 1<x<2 \end{cases}$

$\rightarrow$ Cosine series $= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$

$(0,2) \Rightarrow (0,l) \quad \therefore \quad l = 2$

$$a_0 = \frac{2}{l} \int_0^l f(x)\,dx$$

$$= \frac{\cancel{2}}{\cancel{2}} \int_0^2 f(x)\,dx = \int_0^1 1\,dx + \int_1^2 x\,dx$$

$$= [x]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = 1 + \frac{4}{2} - \frac{1}{2} = 1 + \frac{3}{2}$$

$$\boxed{a_0 = \frac{5}{2}}$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos\frac{n\pi x}{l}\,dx$$

$$= \frac{\cancel{2}}{\cancel{2}} \int_0^2 f(x) \cos\frac{n\pi x}{2}\,dx$$

$$= \int_0^1 \cos\frac{n\pi x}{2}\,dx + \int_1^2 x \cos\frac{n\pi x}{2}\,dx$$

$$= \left[\sin\frac{n\pi x}{2}\left(\frac{2}{n\pi}\right)\right]_0^1 + \left[x \sin\frac{n\pi x}{2}\left(\frac{2}{n\pi}\right) - \left(-\cos\frac{n\pi x}{2}\right)\left(\frac{4}{n^2\pi^2}\right)\right]_1^2$$

$$= \cancel{\frac{2}{n\pi} \sin\frac{n\pi}{2}} + \left[\frac{4}{n^2\pi^2}\cos n\pi - \cancel{\frac{2}{n\pi}\sin\frac{n\pi}{2}} - \frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}\right]$$

$$\boxed{a_n = \frac{4}{n^2\pi^2}\left[\cos n\pi - \cos\frac{n\pi}{2}\right]}$$

$$f(x) = \frac{5/2}{2} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}\left[\cos n\pi - \cos\frac{n\pi}{2}\right] \cos\frac{n\pi x}{2}$$

$$= \frac{5}{4} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2}\left[\cos n\pi - \cos\frac{n\pi}{2}\right] \cos\frac{n\pi x}{2}$$

$$= \frac{5}{4} + \frac{4}{\pi^2}\left[(-1-0)\cos\frac{\pi x}{2} + \frac{1}{4}(1+1)\cos\frac{2\pi x}{2} + \dots\right]$$

$$\therefore \quad f(x) = \frac{5}{4} - \frac{4}{\pi^2}\left[\cos\left(\frac{\pi x}{2}\right) - \frac{1}{2}\cos\left(\frac{2\pi x}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi x}{2}\right) + \dots\right]$$

Question - Bank

Q. No: 33

CHAITANYA. N
PES2UG22CS138

Find the fourier Series for the function $f(x) = 2x - x^2$ in $0 < x < 2$.

$\Rightarrow$ Given interval is of the type $(0, 2l)$;

$$\Rightarrow 2l = 2$$

$$\therefore \boxed{l = 1}$$

Fourier Series :- $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)$

(i) $a_0 = \frac{1}{l}\int_0^2 f(x)\,dx$

$$= \int_0^2 (2x - x^2)\,dx = \left[2 \cdot \frac{x^2}{2} - \frac{x^3}{3}\right]_0^2 = 2^2 - \frac{2^3}{3} = \frac{4}{3}$$

(ii) $a_n = \frac{1}{l}\int_0^2 f(x)\cos n\pi x = \int_0^2 (2x - x^2)\cos n\pi x$.

$$\Rightarrow \left[\cancel{(2x-x^2)\frac{\sin n\pi x}{n\pi}}^{\,0} - (2-2x)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) + \cancel{(-2)\left(-\frac{\sin n\pi x}{n^3\pi^3}\right)}^{\,0}\right]_0^2$$

$$\Rightarrow \left[(2-2x)\frac{\cos n\pi x}{n^2\pi^2}\right]_0^2 = \frac{1}{n^2\pi^2}\left[(2-4)\cos 2n\pi - (2)\right]$$

$$\Rightarrow \frac{-4(-1)^{2n}}{n^2\pi^2} = \frac{-4}{n^2\pi^2}$$

(iii) $b_n = \frac{1}{l}\int_0^2 f(x)\sin n\pi x = \int_0^2 (2x - x^2)\sin n\pi x$

$$= \left[(2x-x^2)\left(-\frac{\cos n\pi x}{n\pi}\right) - \cancel{(2-2x)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)}^{\,0} + (-2)\frac{\cos n\pi x}{n^3\pi^3}\right]_0^2$$

$$\Rightarrow \left[(-2)\frac{\cos 2n\pi}{n^3\pi^3} + \frac{(2)}{n^3\pi^3}\right] = \frac{(-2)(-1)^{2n} + 2}{n^3\pi^3} = 0$$

$$f(x) = 2x - x^2 = \frac{2}{3} + \sum_{n=1}^{\infty}\left(\frac{-4}{n^2\pi^2}\right)\cos n\pi x + 0$$

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SRN : PES2UG22CS141

★) Question bank:

34) Find the Fourier series up to first harmonic for f(x) given by the following table.

x	0	1	2	3	4	5	6
$f(x)$	9	18	24	28	26	20	9

Sol:

$$2l = 6$$
$$l = 3$$
$$N = 6$$

$$a_0 = \frac{2}{N}\Sigma y \qquad a_n = \frac{2}{N}\Sigma y \cos\frac{n\pi x}{l}$$

$$a_0 = \frac{1}{3}(125) \qquad a_1 = \frac{1}{3}\Sigma y \cos\frac{\pi x}{3}$$

$$a_0 = 41.66 \qquad b_n = \frac{2}{N}\Sigma y \sin\frac{n\pi x}{l}$$

$$= \frac{1}{3}\Sigma y \sin\frac{\pi x}{3}$$

x	y	$y\cos\frac{\pi x}{3}$	$y\sin\frac{\pi x}{3}$
0	9	9	0
1	18	9	15.588
2	24	-12	20.784
3	28	-28	0
4	26	-13	-22.51
5	20	10	-17.32
Total :		-25	-3.458

$$a_1 = \frac{1}{3} \Sigma y \cos \frac{\pi x}{3}$$

$$= \frac{1}{3}(-25)$$

$$= \underline{\underline{-8.333}}$$

$$b_1 = \frac{1}{3} \Sigma y \sin \frac{\pi x}{3}$$

$$= \frac{1}{3}(-3.458)$$

$$= \underline{\underline{-1.1547}}$$

$$f(x) = \frac{41.66}{2} + [(-8.33]\cos x + (-1.15)\sin x]$$

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PES2UG22CS142

Question Bank

35) Find the Fourier series upto harmonic for f(x) given by:-

x°	0	60	120	180	240	300
f(x)	0.8	0.6	0.4	0.7	0.9	1.1.

$N = 6$

$2l = 360$

$l = 180$

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x)$$

$$a_0 = \frac{2}{N}\sum y \qquad a_n = \frac{2}{N}\sum y \cos nx \qquad b_n = \frac{2}{N}\sum y \sin nx.$$

x°	y	$\cos\frac{\pi x}{180}$	$\sin\frac{\pi x}{180}$	$y\cos\frac{\pi x}{180}$	$y\sin\frac{\pi x}{180}$
0	0.8	1	0	0.8	0
60	0.6	0.5	0.86	0.3	0.51
120	0.4	-0.5	0.86	-0.2	0.34
180	0.7	-1	0	-0.7	0
240	0.9	-0.5	-0.86	-0.45	~~-0.77~~
300	1.1	0.5	-0.86	0.55	-0.94
	$\Sigma = 4.5$			$\Sigma y\cos\frac{\pi x}{180} = 0.3$	$\Sigma y\sin\frac{\pi x}{180} = -0.8$

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x)$$

$$a_0 = \frac{2}{N} \sum y = \frac{2}{6}(4.5) = 1.5$$

$$a_1 = \frac{2}{N} \sum y \cos x = \frac{2}{6}(0.3) = 0.1$$

$$b_1 = \frac{2}{N} \sum y \sin x = \frac{2}{6}(-0.86) = -0.286 = -0.29$$

$$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x$$

$$= \frac{1.5}{2} + (0.10)\cos x + (-0.29)\sin x$$

$$= 0.75 + 0.10 \cos x + (-0.29) \sin x.$$

CHARAN. V

PES2UG22CS143

Assignment

3) Find the fourier series upto first harmonic for $f(x)$ given by the table.

$x°/x$	$f(x)/y$	$\cos\frac{\pi x}{l}$	$y\cos\frac{\pi x}{l}$	$\sin\frac{\pi x}{l}$	$y\sin\frac{\pi x}{l}$
0	7.9	1	7.9	0	0
60	7.2	0.5	3.6	0.8660	6.2352
120	3.6	-0.5	-1.8	0.8660	3.1176
180	0.5	-1	-0.5	0	0
240	0.9	-0.5	-0.45	-0.8660	-0.7794
300	6.8	0.5	3.4	-0.8660	-5.8888
~~360~~	~~7.9~~ $\Sigma y = 26.9$		$\Sigma \cos\frac{\pi x}{l} = 12.15$		$\Sigma y \sin\frac{\pi x}{l} = 2.6846$

Period = 360

$2l = 360$

$\boxed{l = 180}$

$\boxed{N = 6}$

$a_0 = \frac{2}{N} \Sigma y$

$= \frac{2}{6} \Sigma 26.9$

$\boxed{a_0 = 8.9667}$

$$a_1 = \frac{2}{6} \Sigma y \cdot \cos \frac{\pi x}{l}$$

$$= \frac{1}{3} \times 12.15$$

$$\boxed{a_1 = 4.05}$$

$$b_1 = \frac{2}{6} \Sigma y \cdot \sin \frac{\pi x}{l}$$

$$= \frac{1}{3} \times 2.6846$$

$$\boxed{b_1 = 0.8948}$$

Fourier series of I harmonic :

$$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x$$

$$f(x) = \frac{8.9667}{2} + 4.05 \cos x + 0.8948 \sin x$$

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QUESTION BANK

37. The turning moment T units of a crank shaft of a steam engine are given for a series of values of the crank angle θ degrees.

θ	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Find the first 3 terms of the sine series

$\theta°$	T (y)	$\sin\theta$	$y\sin\theta$	$\sin 2\theta$	$y\sin 2\theta$	$\sin 3\theta$	$y\sin 3\theta$
0	0	0	0	0	0	0	0
30	5224	0.5	2612	0.866	4523.984	1	5224
60	8097	0.866	7012.002	0.866	7012.002	0	0
90	7850	1	7850	0	0	−1	−7850
120	5499	0.866	4762.134	−0.866	−4762.134	0	0
150	2626	0.5	1313	−0.866	−2274.116	1	2626
$\Sigma y = 29296$		$\Sigma y\sin\theta = 23549.136$		$\Sigma y\sin 2\theta = 4500$		$\Sigma y\sin 3\theta = 0$	

$$T = b_1 \sin\theta + b_2 \sin 2\theta + b_3 \sin 3\theta + \dots$$

$$b_1 = \frac{2}{6}\Sigma y\sin\theta = \frac{23549.136}{3} = 7849.712$$

$$b_2 = \frac{2}{6}\Sigma y\sin 2\theta = \frac{4500}{3} = 1500$$

$$b_3 = \frac{2}{6}\Sigma y\sin 3\theta = \frac{0}{3} = 0$$

$$T = 7849.7\sin\theta + 1500\sin 2\theta + 0$$

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QB-1, Q.No-39:

In the range $(-2,2)$, $f(x)$ is defined by the relation:

$$f(x) = \begin{cases} 0, & -2 < x < 0 \\ a, & 0 < x < 2 \end{cases}$$

Expand $f(x)$ in fourier series:

solution:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}$$

$$l = 2.$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\left[\int_{-2}^{0} 0\,dx + \int_{0}^{2} a\,dx\right]$$

$$a_0 = \frac{1}{2}\left[2a - 0\right] = \underline{a}.$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$a_n = \frac{1}{2}\left[\int_{-2}^{0} 0\cos\frac{n\pi x}{2}\,dx + \int_{0}^{2} a\cos\frac{n\pi x}{2}\,dx\right]$$

$$a_n = \frac{a}{2}\left(\frac{\sin\frac{n\pi x}{2}}{n\pi/2}\right)_0^2$$

$$\underline{a_n = 0}.$$

$$b_n = \frac{1}{l}\int_{l}^{l} f(x) \sin\frac{n\pi x}{l}\, dx$$

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x) \sin\frac{n\pi x}{2}\, dx$$

$$b_n = \frac{1}{2}\left[\int_{-2}^{0} 0 \sin\frac{n\pi x}{2}\, dx + \int_{0}^{2} a \sin\frac{n\pi x}{2}\, dx\right]$$

$$b_n = \frac{1}{2}\left[\frac{-a\cos\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right]_0^2$$

$$\underline{\underline{b_n = \frac{a}{n\pi}\left(1-(-1)^n\right)}}$$

$$f(x) = \frac{a}{2} + \sum_{n=1}^{\infty} \frac{a}{n\pi}\left(1-(-1)^n\right)\sin\frac{n\pi x}{2}$$

$$\underline{\underline{f(x) = a\left[\frac{1}{2} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\left(1-(-1)^n\right)\sin\frac{n\pi x}{2}\right]}}$$

UNIT-04
Q- 40

CHETHAN M
PES2UG22CS147

Find the Fourier Series for the function $f(x) = 2x - x^2$ in $0 < x < 3$

$3 = 2l \quad \therefore l = 3/2$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$= \frac{2}{3}\int_0^3 2x - x^3 = 0$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{3}\int_0^3 (2x-x^2)\cos\left(\frac{n2\pi x}{3}\right)dx$$

$$\frac{2}{3}\left[\frac{(2x-x^2)\sin\left(\frac{n2\pi x}{3}\right)}{\left(\frac{n2\pi}{3}\right)} - \frac{(2-2x)\left(-\cos\left(\frac{n2\pi x}{3}\right)\right)}{\left(\frac{n2\pi}{3}\right)^2} - \frac{2\left(-\sin\left(\frac{n2\pi x}{3}\right)\right)}{\left(\frac{n2\pi}{3}\right)^3}\right]_0^3$$

$$\frac{2}{3}\left[\left[0 - \frac{4}{\left(\frac{n2\pi}{3}\right)^2} - 0\right] - \left[0 + \frac{2}{\left(\frac{n2\pi}{3}\right)^2} + 0\right]\right] = \left(\frac{2}{3}\right)\frac{-6}{\left(\frac{n2\pi}{3}\right)^2} = \frac{-9}{n^2\pi^2}$$

$$\boxed{a_n = \frac{-9}{n^2\pi^2}}$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{3}\int_0^3 (2x-x^2)\sin\left(\frac{n2\pi x}{3}\right)dx$$

$$\frac{2}{3}\left[\frac{(2x-x^2)\left(-\cos\left(\frac{n2\pi x}{3}\right)\right)}{\left(\frac{n2\pi}{x}\right)} - \frac{(2-2x)\left(-\sin\left(\frac{n2\pi x}{3}\right)\right)}{\left(\frac{n2\pi}{x}\right)^2} + \frac{(-2)\cos\left(\frac{n2\pi x}{3}\right)}{\left(\frac{n2\pi}{x}\right)^2}\right]_0^3$$

$$\frac{2}{3}\left[\left[\frac{3}{\left(\frac{n2\pi}{3}\right)} - 0 - \frac{2}{\left(\frac{n2\pi}{x}\right)^2}\right] - \left[\frac{-2}{\left(\frac{n2\pi}{x}\right)^2}\right]\right] = \frac{2}{3}\times\frac{9}{n2\pi} = \frac{3}{\pi n}$$

$$\boxed{b_n = \frac{3}{\pi n}}$$

Ad.

$$f(x) = \frac{a_0}{2} + \sum_{n=0}^{n=\infty} f(x)\cos(nx) + \sum_{n=0}^{n=\infty} f(x)\sin(nx)$$

$$\boxed{9x - x^2 = 0 + \sum_{n=0}^{n=\infty} \frac{-9}{n^2\pi^2}\cos(nx) + \sum_{n=0}^{n=\infty} \frac{3}{\pi n}\sin(nx)}$$

CHETHAN N

PES2UG22CS148

Q_{B} 41) Find the half range Cosine series for

$$f(x) = \begin{cases} c & 0<x<a \\ 0 & a<x<l \end{cases} \qquad l=a$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{l}\int_0^a c\,dx$$

$$= \frac{2c}{l}[x]_0^a$$

$$a_0 = \frac{2ca}{l}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^a c\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2c}{l}\int_0^a \cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2c}{l}\left[\frac{\sin\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right]_0^a$$

$$= \frac{2c}{l}\left[\frac{\cancel{l}\sin n\pi x}{\cancel{l}n\pi}\right]_0^a$$

$$a_n = \frac{2c}{n\pi}\left(\frac{\sin n\pi a}{l}\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{l}$$

$$f(x) = \frac{ca}{l} + \sum_{n=1}^{\infty}\frac{2c}{n\pi}\left(\frac{\sin n\pi a}{l}\right)\frac{\cos n\pi x}{l}$$

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Question Bank

NAME: CHINMAYI C H

SRN: PES2UG22CS149

42] Find the value of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ using Fourier Series

Given $f(x) = \begin{cases} -1+x, & -\pi < x < 0 \\ 1+x & 0 < x < \pi \end{cases}$

Sol^n $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$

$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$ $\qquad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\,dx$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\,dx$

$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{\pi} \int_{-\pi}^{\pi} (1+x)\,dx$

$= \frac{1}{\pi} \left[x + \frac{x^2}{2} \right]_{-\pi}^{\pi}$

$= \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+x) \cos nx\,dx$

$= \frac{1}{\pi} \left[\cancel{(1+x)\frac{(\sin nx)}{n}}^{\,0} - \left(\frac{1}{n}\right)\left[-\frac{\cos nx}{n} \right] \right]_{-\pi}^{\pi}$

$= \frac{1}{\pi} \left[\frac{\cos nx}{n^2} \right]_{-\pi}^{\pi}$

$= \frac{1}{\pi} \left[\cancel{\frac{(-1)^n}{n^2}} - \cancel{\frac{(-1)^n}{n^2}} \right]$

$= \underline{\underline{0}}$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\,dx$

$= \frac{1}{\pi} \int_{-\pi}^{\pi} (1+x) \sin nx\,dx$

$$\Rightarrow \frac{1}{\pi}\left[(1+x)\left(-\frac{\cos nx}{n}\right) - (1)\left[-\cancel{\frac{\sin nx}{n^2}}^{0}\right]\right]_{-\pi}^{\pi}$$

$$\Rightarrow \frac{1}{\pi}\left[(1+\pi)\left[-\frac{(-1)^n}{n}\right] - (1-\pi)\left[-\frac{(-1)^n}{n}\right]\right]$$

$$\Rightarrow \frac{1}{\pi}\left[-\frac{(-1)^n}{n}\left[\cancel{1}+\pi-\cancel{1}+\pi\right]\right]$$

$$= \frac{2\cancel{\pi}\left[-(-1)^n\right]}{\cancel{\pi}\, n}$$

$$b_n = \frac{2(-1)^{n+1}}{n}$$

$$\boxed{f(x) = \frac{\cancel{2}^{1}}{\cancel{2}} + \sum_{n=1}^{\infty} 0 + \frac{2(-1)^{n+1}}{n}\sin nx.}$$

$$1+x = 1 + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin nx.$$

put $x = \pi/2$

$$1+\pi/2 = 1 + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin n\pi/2.$$

$$1+\pi/2 = 1 + 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin n\pi/2$$

$$1+\pi/2 = 1 + 2\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\cdots\right]$$

$$\frac{\frac{2+\pi}{2} - 1}{2} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\cdots$$

$$\boxed{\pi/4 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\cdots\cdots}$$

UNIT - 4

Question Bank

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PES2UG22CS150

Q43:- Find the Fourier series of the function $f(x) = \begin{cases} -1, & -3<x<0 \\ 1, & 0<x<3 \end{cases}$

$f(x)$ lies in $(-3,3)$, the given function is odd $\Rightarrow a_0 = a_n = 0$

$\Rightarrow l = 3$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l} \right]$$

$$a_0 = \frac{1}{3}\int_{-3}^{3} f(x)\,dx$$

$$= \frac{1}{3}\left[\int_{-3}^{0} (-1)\,dx + \int_{0}^{3} (1)\,dx \right]$$

$$= \frac{1}{3}\left[[-x]_{-3}^{0} + [x]_{0}^{3} \right]$$

$$= \frac{1}{3}\left[-3+3 \right] = 0$$

$$a_n = \frac{1}{3}\int_{-3}^{3} f(x)\cdot \cos\frac{n\pi x}{3}\,dx$$

$$= \frac{1}{3}\left[\int_{-3}^{0} (-1)\cos\frac{n\pi x}{3}\,dx + \int_{0}^{3} (1)\cos\frac{n\pi x}{3}\,dx \right]$$

$$= \frac{1}{3}\left[(-1)\left(\cancel{\frac{\sin\left(\frac{n\pi x}{3}\right)}{\left(\frac{n\pi}{3}\right)}} \right)_{-3}^{0} + \left[\cancel{\frac{\sin\left(\frac{n\pi x}{3}\right)}{\left(\frac{n\pi}{3}\right)}} \right]_{0}^{3} \right]$$

$$= \frac{1}{3}(0) \Rightarrow a_n = 0$$

$$b_n = \frac{1}{3}\int_{-3}^{3} f(x)\sin\frac{n\pi x}{3}\,dx$$

$$= \frac{1}{3}\left[\int_{-3}^{0}(-1)\sin\frac{n\pi x}{3}\,dx + \int_{0}^{3}\sin\frac{n\pi x}{3}\,dx\right]$$

$$= \frac{1}{3}\left[\left[\frac{\cos\left(\frac{n\pi x}{3}\right)}{(n\pi/3)}\right]_{-3}^{0} - \left[\frac{\cos\left(\frac{n\pi x}{3}\right)}{(n\pi/3)}\right]_{0}^{3}\right]$$

$$= \frac{1}{3}\left[\frac{\cos(0)}{n\pi/3} - \frac{\cos n\pi}{n\pi/3} - \left[\frac{\cos n\pi}{n\pi/3} - \frac{\cos(0)}{n\pi/3}\right]\right]$$

$$= \frac{1}{3}\left[\frac{3}{n\pi} - \frac{3\cos n\pi}{n\pi} - \frac{(\cos n\pi)3}{n\pi} + \frac{3}{n\pi}\right]$$

$$= \frac{1}{3}\left[\frac{6}{n\pi} - \frac{6\cos n\pi}{n\pi}\right]$$

$$= \frac{1}{\cancel{3}} \times \frac{\cancel{6}\,2}{n\pi}\left[1-(-1)^n\right]\sin\frac{n\pi x}{3} = f(x)$$

$$= \frac{2}{\pi}\,\frac{1}{n}\left[1-(-1)^n\right]\sin\frac{n\pi x}{3} = f(x)$$

put $n=(2n-1)$

$$\Rightarrow f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1(1+1)}{(2n-1)}\sin(2n-1)\pi/3$$

$$f(x) = \frac{4}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\sin(2n-1)\frac{\pi}{3}x$$

(OR) Since it is an odd fn, $a_0 = 0$, $a_n = 0$.

$$b_n = \frac{2}{l}\int_0^l f(x) \cdot \sin\frac{n\pi x}{l}\, dx$$

$$= \frac{2}{3}\int_0^3 1 \cdot \sin\frac{n\pi x}{3}\, dx$$

$$= \frac{2}{3}\left[\frac{-\cos\frac{n\pi x}{3}}{\frac{n\pi}{3}}\right]_0^3$$

$$\Rightarrow \frac{2}{\cancel{3}} \times \frac{-\cancel{3}}{n\pi}\left[\cos n\pi - \cos 0\right]$$

$$= \frac{-2}{n\pi}\left[(-1)^n - 1\right]$$

$$b_n = \frac{2}{n\pi}\left[1 - (-1)^n\right]$$

$$\therefore b_n = \frac{4}{n\pi} \text{ if } n \text{ is odd}$$

$$= 0 \text{ if } n \text{ is even.}$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\frac{n\pi x}{3} \quad \text{for } n \text{ is odd}$$

$$f(x) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{4}{(2n-1)} \sin\frac{(2n-1)\pi x}{3}.$$

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PES2UG22CS151

Question Bank

44. Find the Fourier coefficients and Fourier series of the square-wave function f defined by

$$f(x) = \begin{cases} 0 & \text{if } -\pi \le x < 0 \\ 1 & \text{if } 0 \le x < \pi \end{cases} \quad \text{and } f(x+2\pi) = f(x).$$

The Fourier expansion of $f(x)$ over $(-\pi, \pi)$ is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} (0 \cdot dx) + \int_{0}^{\pi} 1 \cdot dx \right]$$

$$= \frac{1}{\pi} [x]_0^{\pi}$$

$$= \frac{\pi}{\pi} = \underline{\underline{1}}$$

calculation of a_n

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos nx\, dx$$

$$= \frac{1}{\pi} \left\{ \int_{-\pi}^{0} 0 \cdot \cos nx\, dx + \int_{0}^{\pi} 1 \cdot \cos nx\, dx \right\}$$

$$= \frac{1}{\pi} \left[\frac{\sin nx}{n} \right]_0^{\pi}$$

$$= \underline{\underline{0}}$$

calculation of b_n

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin nx\, dx$$

$$= \frac{1}{\pi} \left\{ \int_{-\pi}^{0} 0 \cdot \sin nx\, dx + \int_{0}^{\pi} 1 \cdot \sin nx\, dx \right\}$$

$$= \frac{1}{\pi} \left[\frac{-\cos nx}{n} \right]_0^{\pi}$$

$$= \frac{1}{\pi n} \left[-(-1)^n + 1 \right]$$

$$= \frac{1 - (-1)^n}{n\pi}$$

Put $n = 2k-1$

$k = \frac{n+1}{2}$ when $n = 1, n = \infty$

$k = 1, k = \infty$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)^{2k-1}}{(2k-1)\pi} \sin(2k-1)\pi x$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)^{2k}(-1)^{-1}}{(2k-1)\pi} \sin(2k-1)\pi x$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)}{(2k-1)\pi} \sin(2k-1)\pi x$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{(2k-1)\pi} \sin(2k-1)\pi x$$

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Question Bank

45) Find Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$ Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\cdot dx = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)\cdot dx$$

$$= \frac{1}{2\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{2\pi}$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\cdot dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{\pi - x}{2}\cos nx\cdot dx$$

$$= \frac{1}{2\pi}\left[(\pi - x)\frac{\sin nx}{n} - (-1)\left(\frac{-\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{\pi - x}{2}.\cancel{dx}\sin nx\cdot dx$$

$$= \frac{1}{2\pi}\left[(\pi - x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(\frac{-1}{n}\right)\right] \times \frac{\sin nx}{n}\Bigg]_0^{2\pi}$$

$$b_n = \frac{1}{2\pi}\left[\frac{\pi}{n} + \frac{\pi}{n}\right]$$

$$= \frac{1}{n}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos x\cdot dx + \sum_{n=1}^{\infty} b_n\sin x\cdot dx$$

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} \frac{1}{n} \sin nx$$

$$= \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots$$

Put $x = \frac{\pi}{2}$

$$\frac{\pi - x}{2} = \frac{\pi - \frac{\pi}{2}}{2} = \frac{\pi}{4}$$

$$\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2}$$

$$\frac{\pi}{4} = \sin\frac{\pi}{2} + \frac{1}{2}\sin\frac{2\pi}{2} + \sin\frac{3\pi}{2} \times \frac{1}{3} + \sin\frac{4\pi}{2} \times \frac{1}{4} + \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

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QB 46. Find the complex form of Fourier series for the function $f(t) = \sin t$ in $(0, \pi)$

$(0, 2l)$ $\quad l = \pi/2$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/\pi/2}$$

$$c_n = \frac{1}{\pi} \int_0^{\pi} \sin x \, e^{-i2nx} dx$$

$$c_n = \left[\frac{e^{-i2nx}}{(1-4n^2)\pi} [\cancel{-i2n \sin x}^{\,0} - \cos x] \Big|_0^{\pi} \right]$$

$$c_n = \frac{-e^{-2in\pi}(-1)}{(1-4n^2)\pi} + \frac{1}{(1-4n^2)\pi}$$

$$c_n = \frac{1 + e^{-2in\pi}}{(1-4n^2)\pi} = \frac{2}{(1-4n^2)\pi}$$

$$e^{-i2\pi n} = \cos 2\pi n - i \sin 2\pi n = (-1)^{2n} - 0 = 1$$

$$\sin x = \sum_{-\infty}^{\infty} \frac{2}{(1-4n^2)\pi} e^{i2nx}$$

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Question Bank

~~48~~

47) Find the Fourier Series of $f(x) = x(2\pi - x)$ in $0 < x < 2\pi$.

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x(2\pi - x)$$

$$= \frac{1}{\pi}\int_0^{2\pi} 2\pi x - x^2$$

$$= \frac{1}{\pi}\int_0^{2\pi} \pi x^2 - \frac{x^3}{3}$$

$$= 4\pi^2 - \frac{8\pi^2}{3}$$

$$a_0 = \frac{4\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x(2\pi - x)\cos nx\, dx.$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} 2\pi x\cos nx - x^2\cos nx$$

$$= \frac{1}{\pi}\left[\frac{2\pi x \overset{0}{\cancel{\sin(nx)}}}{n} - \frac{2\pi(-\cos nx)}{n^2}\right]_0^{2\pi}$$

$$-\left[\frac{x^2 \overset{0}{\cancel{\sin x}}}{n} - \frac{2x(-\cos nx)}{n^2} - 2\frac{(-\overset{0}{\cancel{\sin nx}})}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{2\pi}{n^2}[1-1] - \frac{4\pi}{n^2} - 0\right]$$

$$= \frac{1}{\cancel{\pi}}\left[-\frac{4\cancel{\pi}}{n^2}\right]$$

$$a_n = \frac{-4}{n^2}$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_1^{\infty} a_n \cos nx$$

$$= \frac{4\pi^2}{6} + \sum_1^{\infty} \frac{-4}{n^2}\cos nx.$$

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Unit III

Question Bank | Q.48 |

Q. Find the Fourier Half range a) cosine series b) sine series

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \end{cases}$$

Ans: $a_0 = \frac{2}{2}\int_0^2 f(x)\,dx = \int_0^1 x + \int_1^2 2-x = \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$

$$a_0 = \frac{1}{2} + \frac{1}{2} = 1$$

$$a_n = \frac{2}{2}\int_0^2 f(x)\cos\frac{n\pi x}{2} = \int_0^1 x\cos\frac{n\pi x}{2} + \int_1^2 (2-x)\cos\frac{n\pi x}{2}$$

$$= \left[\frac{2x\sin\left(\frac{n\pi x}{2}\right)}{n\pi} + \frac{4\cos\frac{n\pi x}{2}}{n^2\pi^2}\right]_0^1 + \left[\frac{(2-x)\sin\left(\frac{n\pi x}{2}\right)}{n\pi} - \frac{4\cos\frac{n\pi x}{2}}{n^2\pi^2}\right]_1^2$$

$$= \left(\frac{4\cos\frac{n\pi}{2}}{n^2\pi^2} - \frac{4}{n^2\pi^2}\right) + \left(-\frac{4\cos n\pi x}{n^2\pi^2} + \frac{4\cos\frac{n\pi}{2}}{n^2\pi^2}\right)$$

$$\therefore a_n = \frac{8\cos\frac{n\pi}{2}}{n^2\pi^2} - \frac{4(1+(-1)^n)}{n^2\pi^2}$$

$$b_n = \frac{2}{2}\int_0^2 f(x)\sin\frac{n\pi x}{2} = \int_0^1 x\sin\frac{n\pi x}{2} + \int_1^2 (2-x)\sin\frac{n\pi x}{2}$$

$$= \left[\frac{2x\cos\frac{n\pi x}{2}}{-n\pi} + \frac{4\sin\frac{n\pi x}{2}}{n^2\pi^2}\right]_0^1 + \left[\frac{2(2-x)\cos\frac{n\pi x}{2}}{-n\pi} - \frac{4\sin\left(\frac{n\pi x}{2}\right)}{n^2\pi^2}\right]_1^2$$

$$= \left(\frac{2\cos\frac{n\pi}{2}}{-n\pi} + \frac{4\sin\frac{n\pi}{2}}{n^2\pi^2}\right) + \left(-\frac{2\cos\frac{n\pi}{2}}{-n\pi} + \frac{4\sin\frac{n\pi}{2}}{n^2\pi^2}\right)$$

$$\therefore b_n = \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}$$

sin series: $\sum_{n=1}^{\infty} \frac{8}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{2}\right)$

cos series: $\frac{1}{2} + \sum_{n=1}^{\infty}\left\{\frac{8\cos\frac{n\pi}{2}}{n^2\pi^2} - \frac{4[1+(-1)^n]}{n^2\pi^2}\right\}\cos\frac{n\pi x}{2}$

Ans