ENGINEERING MECHANICS - STATICS

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ENGINEERING MECHANICS

Unit-2

Resultants

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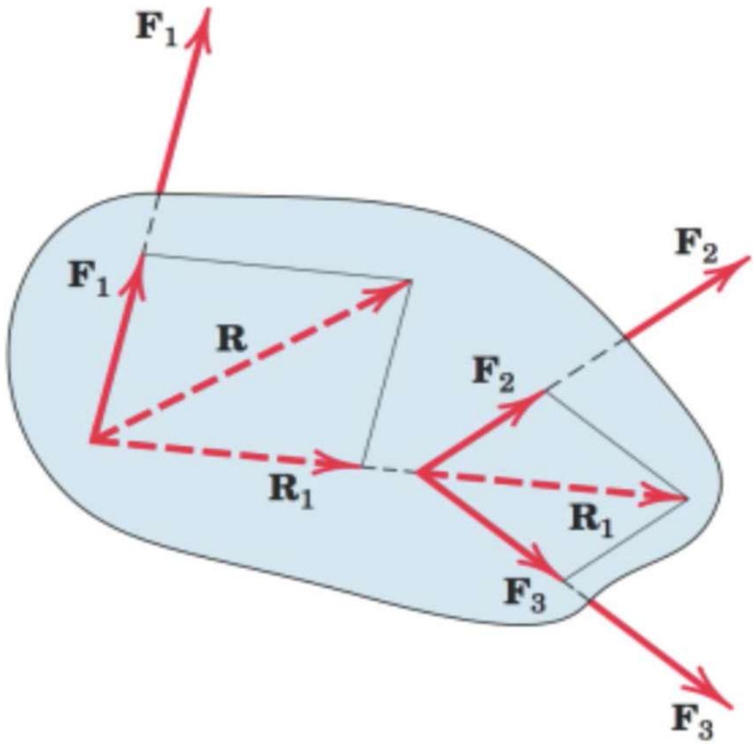
Resultant: The resultant of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.



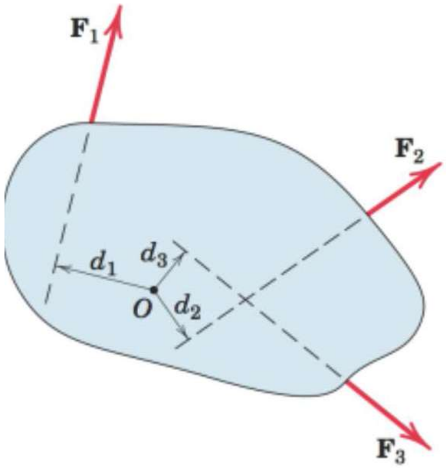
The most common type of force system occurs when the forces all act in a single plane, say, the x-y plane, as illustrated by the system of three forces F1, F2, and F3 in Fig.

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

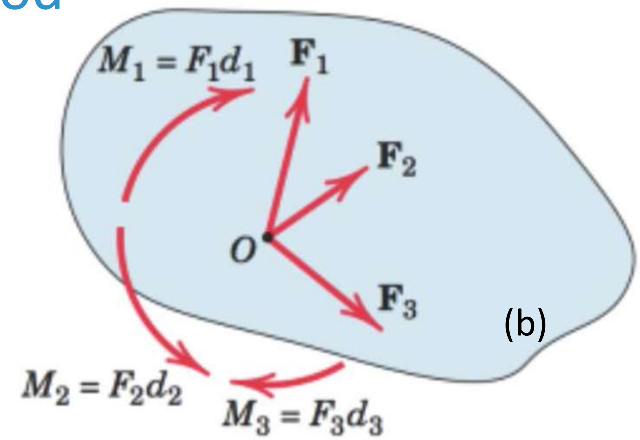
$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \tag{2/9}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

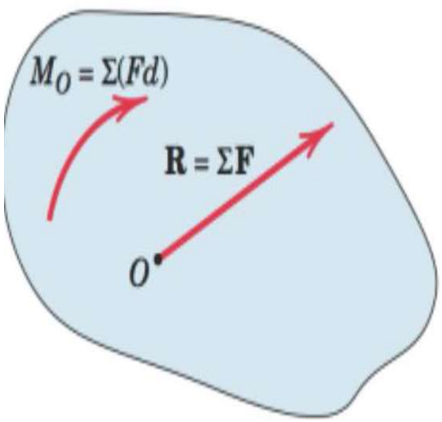
Algebraic Method



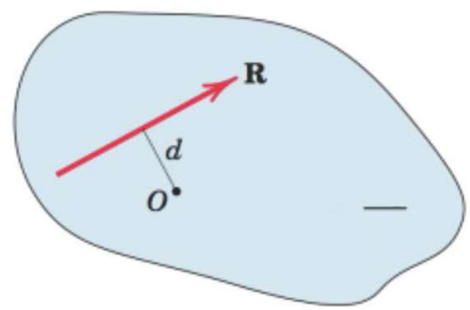
(a)



(b)



(c)



(d)

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Fig (a) and (b), where M11 M2, and M3 are the couples resulting from the transfer of forces F1, F2, and F3 from their respective original lines of action to lines of action through point 0.
2. Add all forces at 0 to form the resultant force R, and add all couples to form the resultant couple M0. We now have the single force couple system, as shown in Fig. (c).

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

3. In Fig. (d), find the line of action of R by requiring R to have a moment of M0 about point 0. Note that the force systems of Fig. (a) and (d) are equivalent, and that ∑Fd in Fig. (a) is equal to Rd in Fig. (d).

This process is summarized in equation form by

$$\mathbf{R} = \sum \mathbf{F}$$

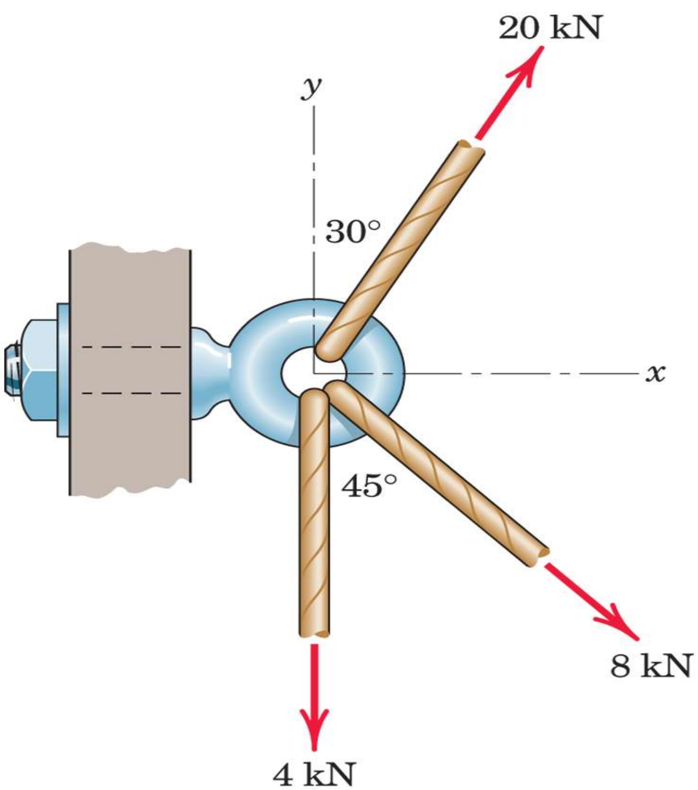
$$\mathbf{M0} = \sum \mathbf{M} = \sum (\mathbf{Fd})$$

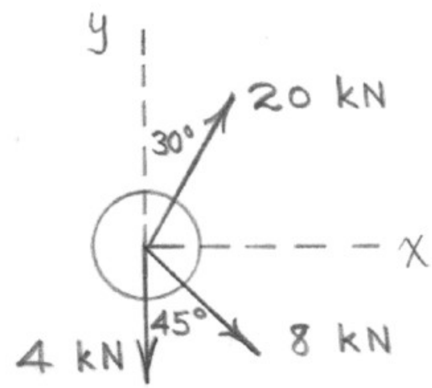
$$\mathbf{Rd} = \mathbf{M0}$$

ENGINEERING MECHANICS

Resultants - Numerical

2/79) Determine the resultant R of the three tension forces acting on the eye bolt. Find the magnitude of R and the angle θ which R makes with the positive x-axis.





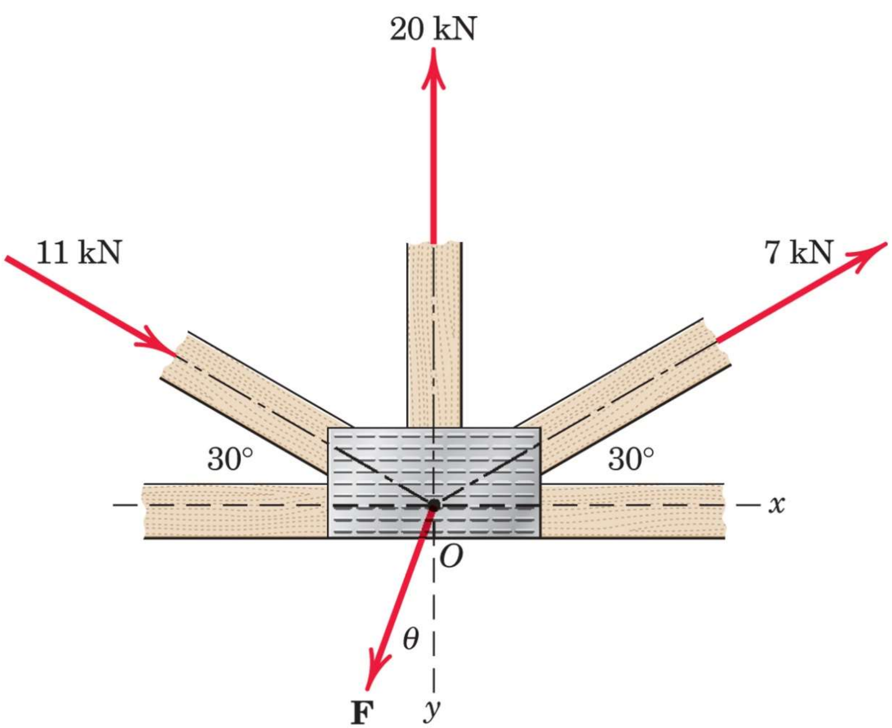
$R_x = \Sigma F_x = 20 \sin 30^\circ + 8 \sin 45^\circ = 15.66 \text{ kN}$

$R_y = \Sigma F_y = 20 \cos 30^\circ - 8 \cos 45^\circ - 4 = 7.66 \text{ kN}$

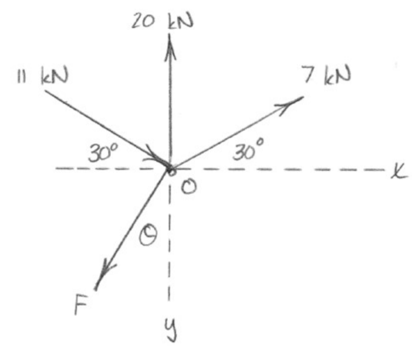
$R = \sqrt{R_x^2 + R_y^2} = \underline{17.43 \text{ kN}}$

$\theta_x = \tan^{-1} (R_y / R_x) = \underline{26.1^\circ}$

2/80) Determine the force magnitude F and direction θ (measured clockwise from the positive y-axis) that will cause the resultant R of the four applied forces to be directed to the right with a magnitude of 9 kN.



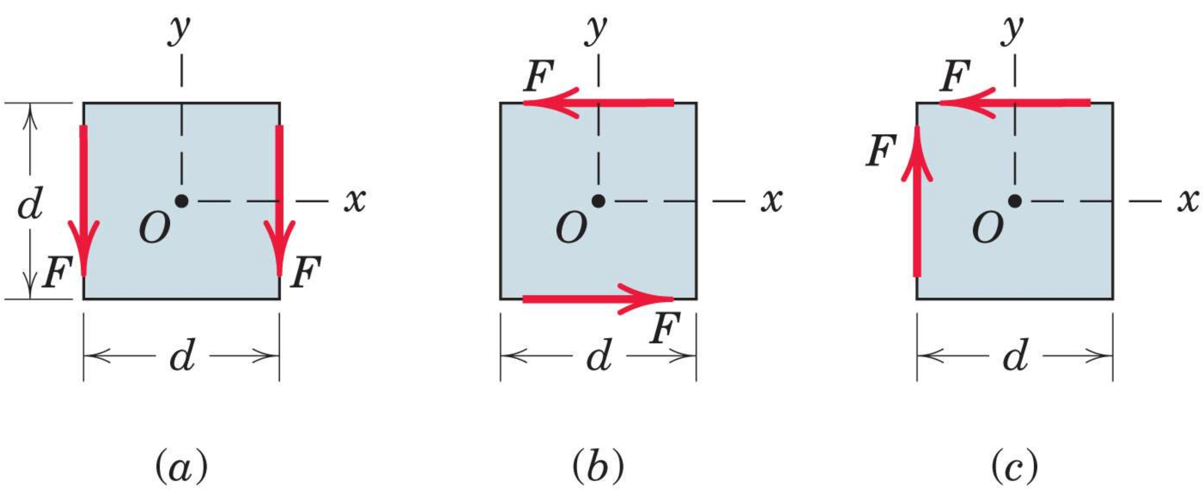
R= 9 kN, RIGHTWARD



$$\begin{cases} R_x = 9 = 11\cos 30^\circ + 7\cos 30^\circ - F\sin\theta \\ R_y = 0 = 11\sin 30^\circ - 7\sin 30^\circ + F\cos\theta - 20 \end{cases}$$

SOLVING... $\underline{F = 19.17\text{ kN}}$ AND $\underline{\theta = 20.1^\circ}$

2/82) Determine the equivalent force-couple system at the center 0 for each of the three cases of forces being applied along the edges of a square plate of side d.

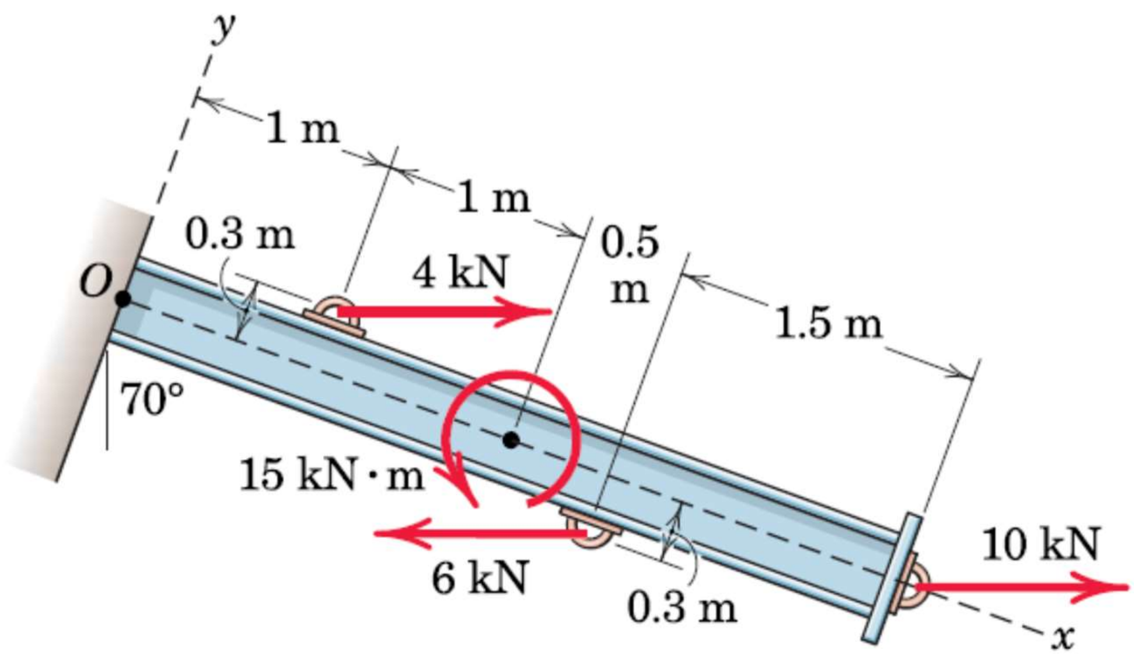


2/82 (a) $\underline{R} = -2F\underline{j}$, $\underline{M}_o = \underline{0}$

(b) $\underline{R} = \underline{0}$, $\underline{M}_o = Fd\underline{k}$ (+$\underline{k}$ is out)

(c) $\underline{R} = -F\underline{i} + F\underline{j}$, $\underline{M}_o = \underline{0}$

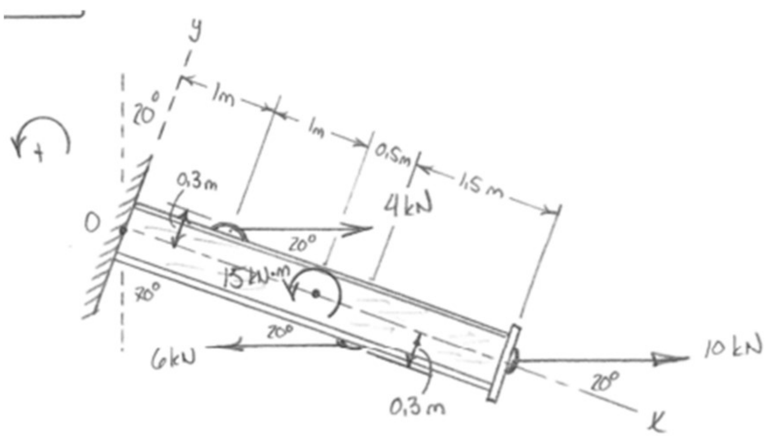
2/81) Replace the three horizontal forces and applied couple with an equivalent force-couple system at O by specifying the resultant R and couple Mo. Next, determine the equation for the line of action of the stand-alone resultant and force R.



Resultants - Numerical

$R = 10 + 4 - 6 \longrightarrow R = 8\ kN$

$\underline{R} = 8\cos 20^\circ \underline{i} + 8\sin 20^\circ \underline{j} \longrightarrow \underline{R} = 7.52\ \underline{i} + 2.74\ \underline{j}\ kN$



$M_o = 15 + 4\sin 20^\circ(1) - 6\sin 20^\circ(2) + 10\sin 20^\circ(4) - 4\cos 20^\circ(0.3) - 6\cos 20^\circ(0.3)$

$\therefore\ M_o = 22.1\ kN \cdot m\ \text{CCW}$

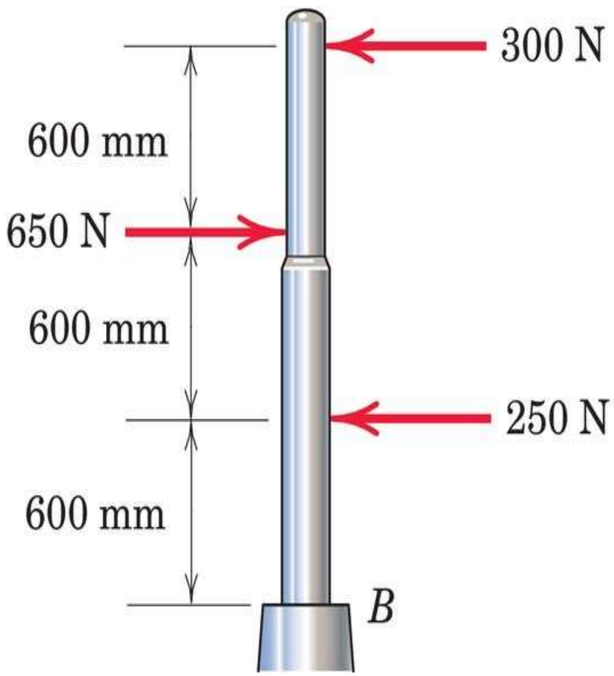
• Line-of-Action:

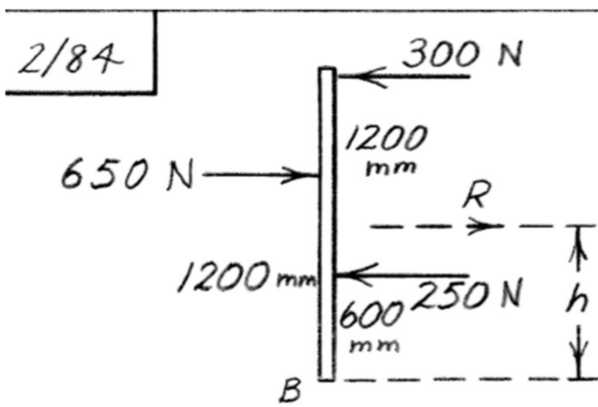
$\underline{r} \times \underline{R} = M_o \longrightarrow (x\underline{i} + y\underline{j}) \times (7.52\underline{i} + 2.74\underline{j}) = 22.1\ \underline{k}$

$\underline{k}: \quad 2.74x - 7.52y = 22.1$

$\therefore \quad y = 0.364x - 2.94\ (m)$

2/84) Determine the height h above the base B at which the resultant of the three forces acts.





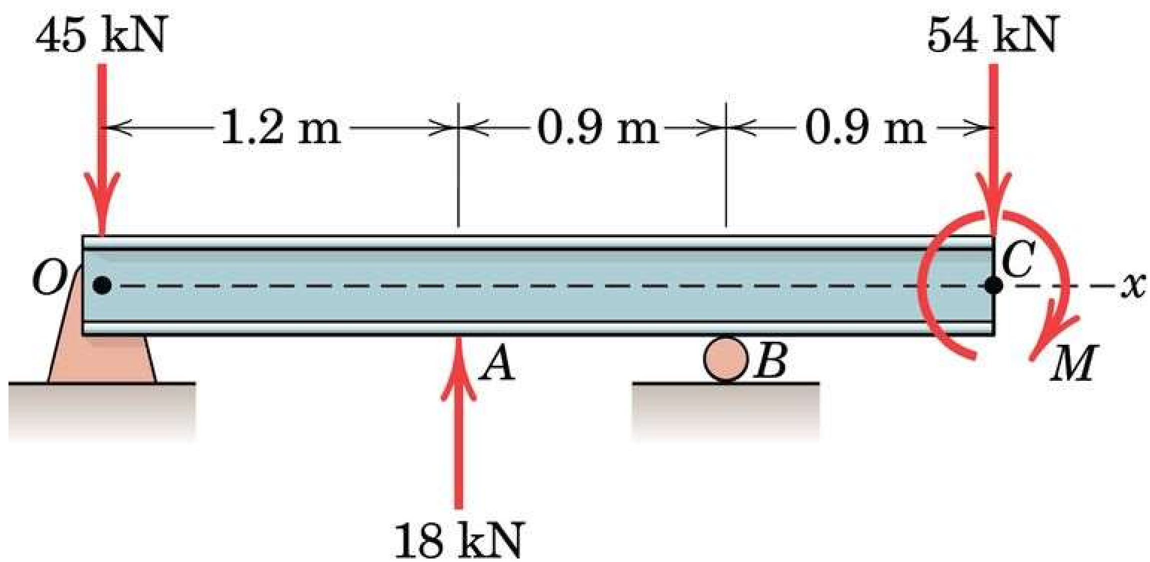
$R = \Sigma F = 650 - 250 - 300$
$= 100 \text{ N}$

$Rh = \Sigma M_B :$

$100h = 650(1.2) - 300(1.8) - 250(0.6)$

$\underline{h = 0.9 \text{ m}}$

2/86) If the resultant of the loads shown passes through point B, determine the equivalent force-couple system at 0.



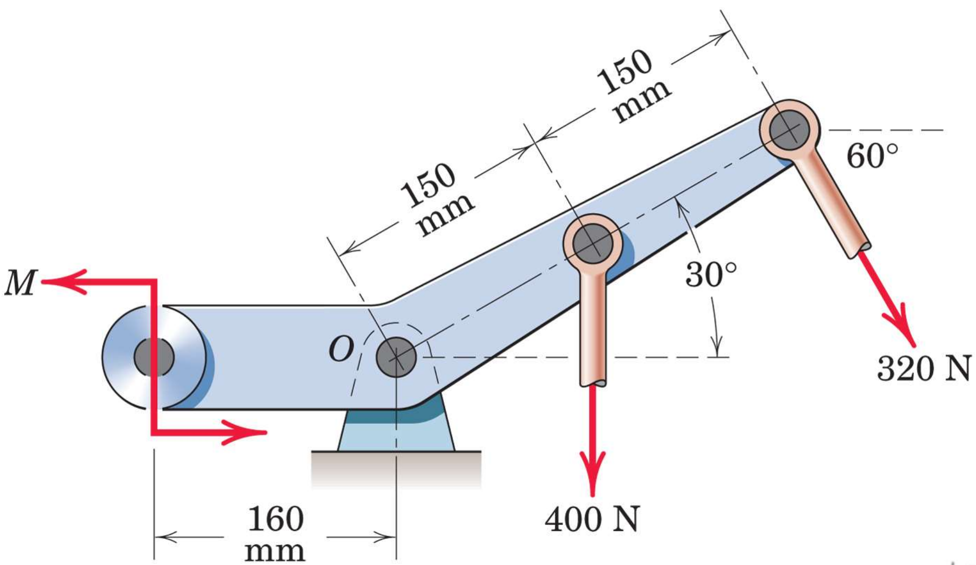
$R = 81$ kN DOWN

$\Sigma M_B = 0: \quad 45(2.1) - 18(0.9) - 0.9(54) - M = 0$

so... $M = 29.7$ kN·m CW

$M_O = 18(1.2) - 54(3) - 29.7 = -170.1$ so... $M_O = 170.1$ kN·m CW

2/87) If the resultant of the two forces and couple M passes through point 0, determine M.

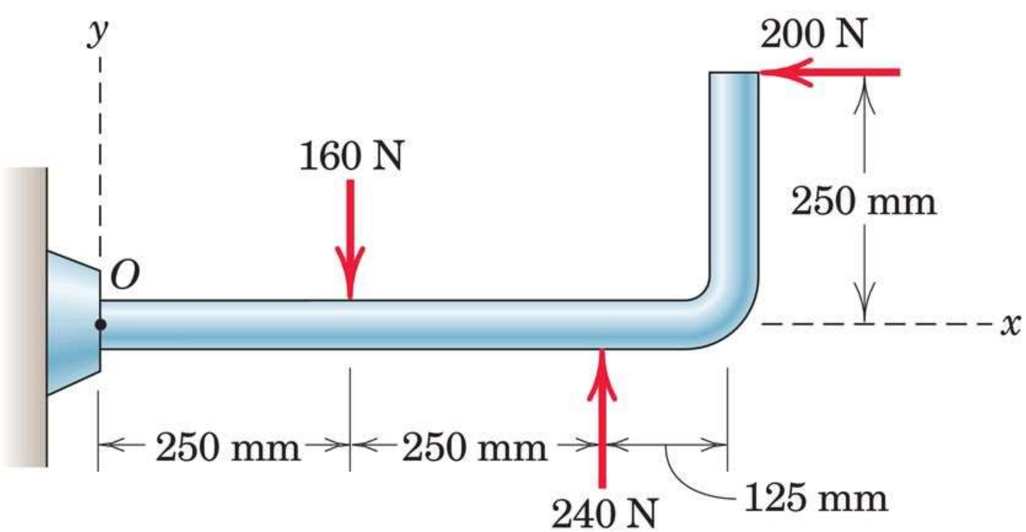


2/87 $M_o = 0$, so

$M - 400(0.150 \cos 30°) - 320(0.300) = 0$

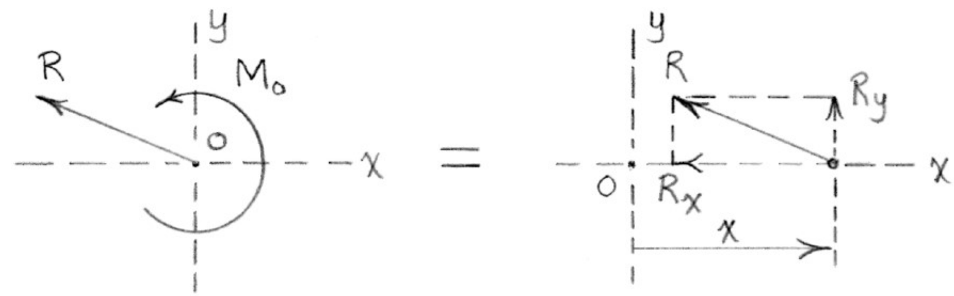
$M = 148.0$ N·m

2/89) Replace the three forces acting on the bent pipe by a single equivalent force R. Specify the distance x from point 0 to the point on the x-axis through which the line of action of R passes.



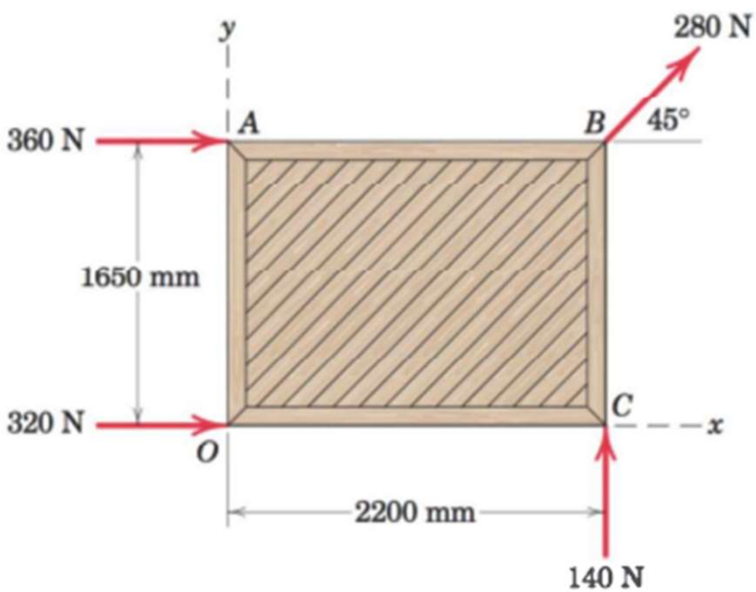
$$\underline{R} = -200\underline{i} + 80\underline{j} \text{ N}$$

$$\circlearrowleft + \; M_o = -160(0.25) + 240(0.50) + 200(0.25) = 130 \text{ N}\cdot\text{m}$$



$$R_y x = M_o, \quad x = \frac{130}{80} = \underline{1.625 \text{ m} \quad (\text{off pipe})}$$

2/90) Four people are attempting to move a stage plate from across the floor. If they exert the horizontal forces shown, determine (a) the equivalent force-couple system at O and (b) the points on the x- and y-axes through which the line of action of the single resultant force R passes.



$\underline{R} = (360 + 320 + 280\cos 45°)\,\underline{i} + (140 + 280\sin 45°)\,\underline{j}$

$\underline{R} = 878\,\underline{i} + 338\,\underline{j}$ N

$M_O = 2.2(140 + 280\sin 45°) - 1.650(360 + 280\cos 45°) = -177.1$ N·m

$M_O = 177.1$ N·m CW

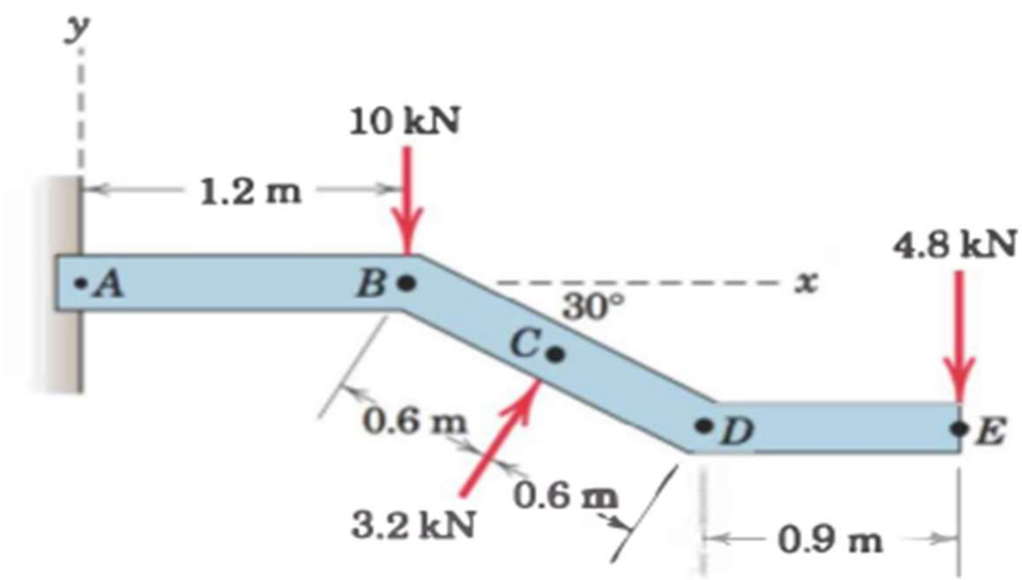
FOR CW MOMENT ABOUT O, POSITIVE R_x IS PLACED ABOVE O.

$R_x y = M_O \rightarrow 878y = 177.1 \rightarrow$ $y = 0.202$ m OR 202 mm ABOVE O

FOR CW MOMENT ABOUT O, POSITIVE R_y IS PLACED LEFT OF O.

$R_y x = M_O \rightarrow 338x = 177.1 \rightarrow$ $x = 0.524$ m OR 524 mm LEFT OF O

2/91) Replace the three forces which act on the bent bar by a force-couple system at the support point A. Then determine the x-intercept of the line of action of the stand-alone resultant force R.



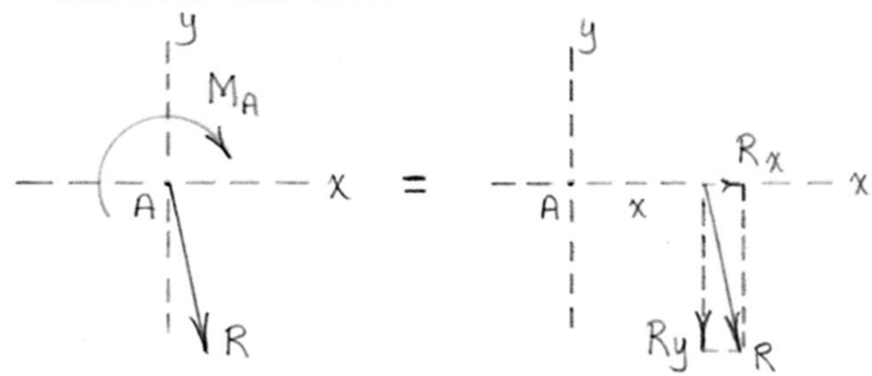
2/91 | Equivalent force-couple system at A:

$$\underline{R} = -10\underline{j} - 4.8\underline{j} + 3.2(\sin 30°\,\underline{i} + \cos 30°\,\underline{j})$$

$$= \underline{1.6\underline{i} - 12.03\underline{j}\ \text{kN}}$$

$$M_A = 10(1.2) + 4.8(1.2 + 1.2\cos 30° + 0.9) - 3.2\sin 30°(0.6\sin 30°) - 3.2\cos 30°(1.2 + 0.6\cos 30°)$$

$$= \underline{21.8\ \text{kN}\cdot\text{m}\ \ CW}$$

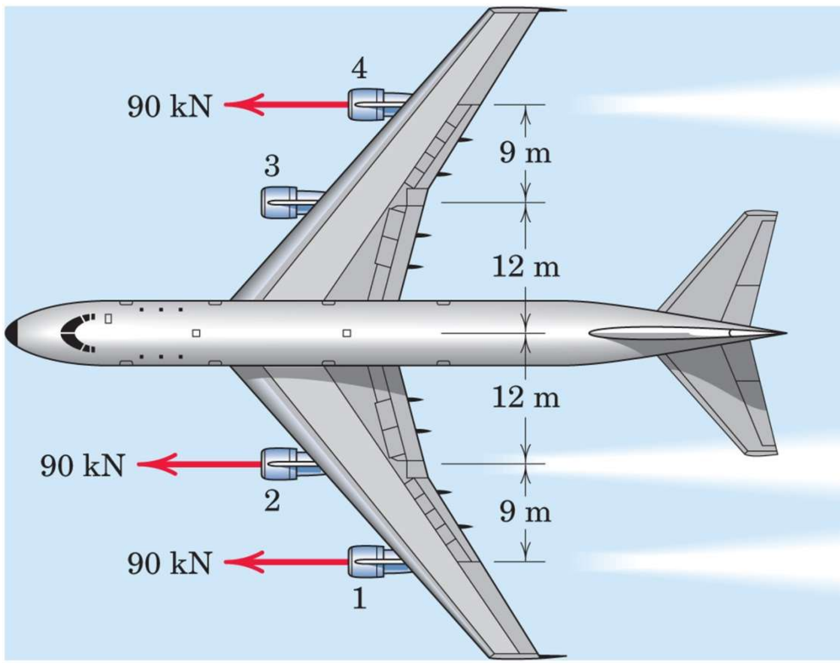


Condition : $x|R_y| = M_A$

$$x = \frac{21.8}{12.03} = \underline{1.814\ \text{m}}$$

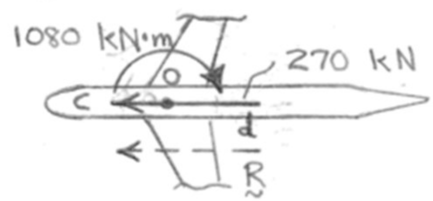
ENGINEERING MECHANICS
Resultants - Numerical

2/93) A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two dimensional problem.



2/93 Force - Couple system at point O

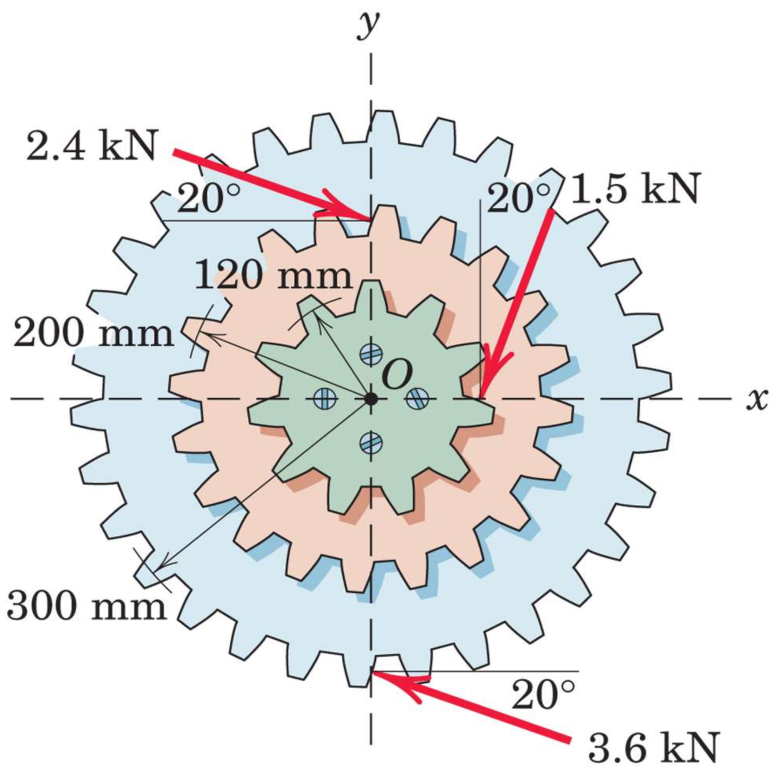
$$\begin{cases} R = 3(90) = \underline{270 \text{ kN}} \ (\leftarrow) \\ +\circlearrowleft M_O = 12(90) = 1080 \text{ kN}\cdot\text{m} \end{cases}$$

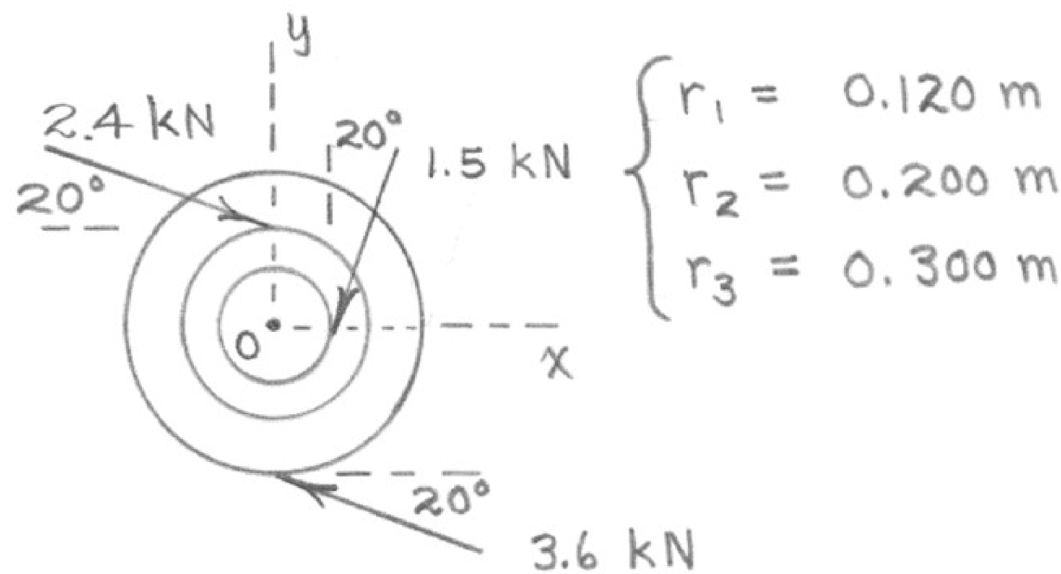


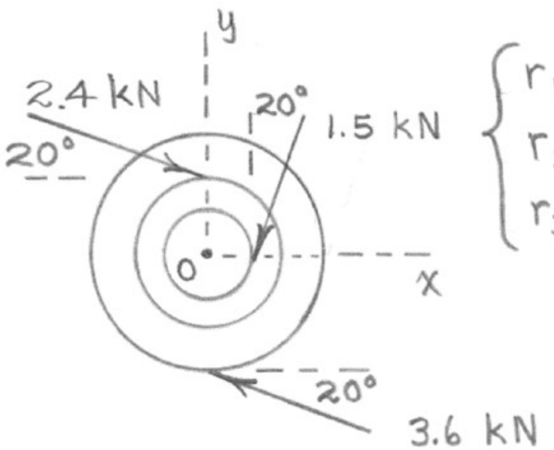
$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= \underline{4 \text{ m}}$$

2/94) Determine the x- and y-axis intercepts of the line of action of the resultant of the three loads applied to the gearset.







$$\begin{cases} r_1 = 0.120 \text{ m} \\ r_2 = 0.200 \text{ m} \\ r_3 = 0.300 \text{ m} \end{cases}$$

$$\underline{R} = \Sigma \underline{F} = 2.4(c20^\circ \underline{i} - s20^\circ \underline{j}) + 1.5(-s20^\circ \underline{i} - c20^\circ \underline{j})$$

$$+ 3.6(-c20^\circ \underline{i} + s20^\circ \underline{j}) = -1.641\underline{i} - 0.999\underline{j} \text{ kN}$$

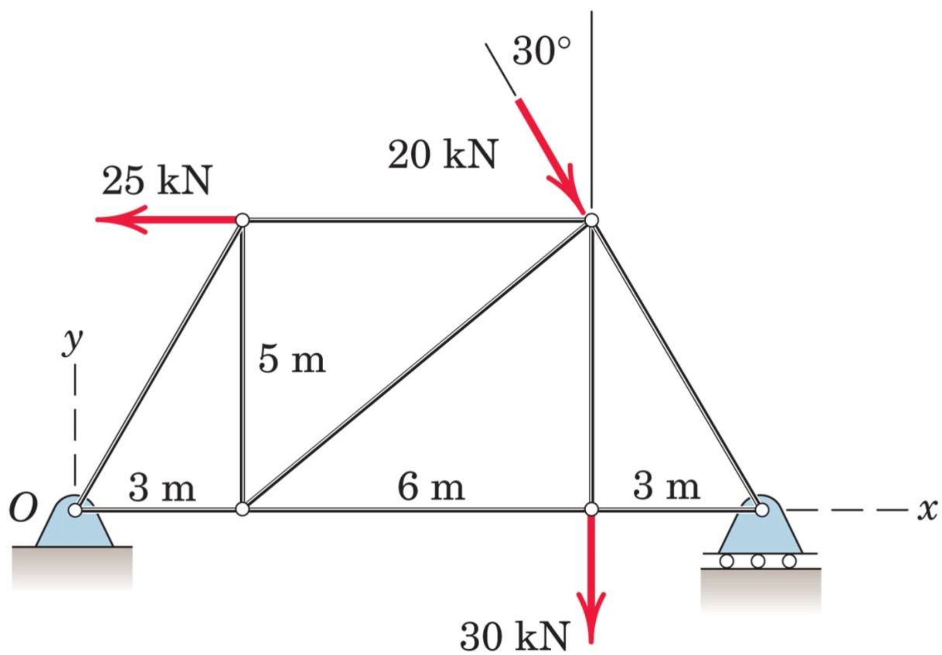
$$\Sigma M_O = (2.4(0.2) + 1.5(0.12) + 3.6(0.3)) \cos 20^\circ = 1.635 \text{ kN·m}$$

$$\underline{r} \times \underline{R} = \underline{M_O} : (x\underline{i} + y\underline{j}) \times (-1.641\underline{i} - 0.999\underline{j}) = -1.635$$

$$\Rightarrow -0.999x + 1.641y = -1.635$$

Axis intercepts : $x = 1.637$ m, $y = -0.997$ m

2/96) Determine the resultant R of the three forces acting on the simple truss. Specify the points on the x- and y-axes through which R must pass.



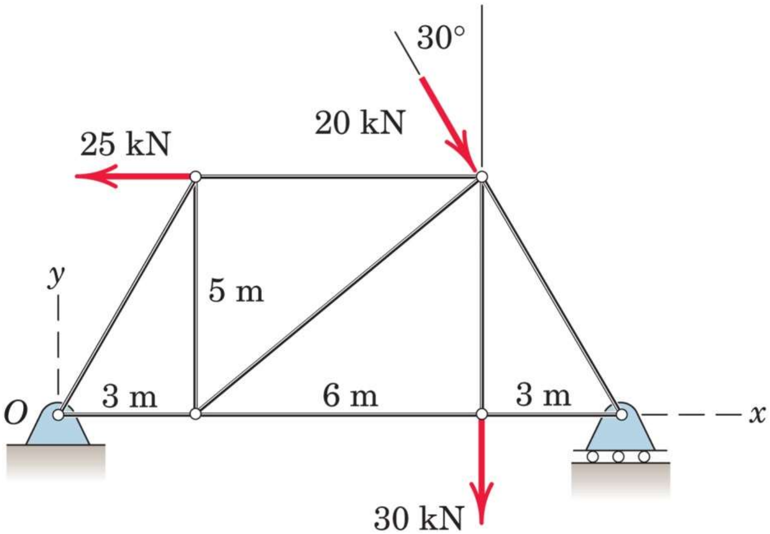
2/96 Equivalent force - couple system at point O:

$\underline{R} = \Sigma \underline{F} = (-25 + 20 \sin 30°)\,\underline{i}$

$+ (-30 - 20 \cos 30°)\,\underline{j} = \underline{-15\,\underline{i} - 47.3\,\underline{j}\ \text{kN}}$

$\curvearrowleft + M_O = 25(5) - 30(9) - (20 \cos 30°)9$

$-(20 \sin 30°)5 = -351\ \text{kN}\cdot\text{m}$



For final location of $\underline{R}$:

$\underline{r} \times \underline{R} = \underline{M}_o$, $(x\underline{i}+y\underline{j}) \times (-15\underline{i} - 47.3\,\underline{j})$

$= -351\,\underline{k}$

$-47.3x + 15y = -351$

Axis intersections : $\underline{x = 7.42\text{ m},\ y = -23.4\text{ m}}$

THANK YOU

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