ENGINEERING MATHEMATICS - II

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Dr. SIVASANKARI. V
Department of Science & Humanities

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Unit 4 : Inverse Laplace Transform

Session : 4

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Department of Science & Humanities

Contents

- **Inverse Laplace Transform of Derivatives**

If $L^{-1}\{F(s)\} = f(t)$ then, for $n = 1,2,3 \ldots \ldots \ldots,$

$$L^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

Recall !!!

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

for n= 1,2,3,....

- **Inverse Laplace Transform of Integrals**

If $L^{-1}\{F(s)\} = f(t)$ then $L^{-1}\{\int_s^\infty F(s)ds\} = \frac{f(t)}{t} = \frac{L^{-1}\{F(s)\}}{t}$

Recall !!!

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)ds$$

1) Find the inverse Laplace Transforms of $\frac{s}{(s^2+a^2)^2}$

Solution : We know that, $\frac{d}{ds}\left\{\frac{1}{s^2+a^2}\right\} = \frac{-2s}{(s^2+a^2)^2}$

Or, $\frac{s}{(s^2+a^2)^2} = -\frac{1}{2}\frac{d}{ds}\left\{\frac{1}{s^2+a^2}\right\}$

Therefore, $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = -\frac{1}{2}L^{-1}\left[\frac{d}{ds}\left\{\frac{1}{s^2+a^2}\right\}\right]$

$$= (-1)\frac{1}{2}\left\{-tL^{-1}\left[\frac{1}{s^2+a^2}\right]\right\} = \frac{tsinat}{\mathbf{2}a}$$

2) Obtain the inverse Laplace Transforms of

$$L^{-1}\left[\int_{s}^{\infty}\left(\frac{1}{s}-\frac{1}{s+1}\right)ds\right]$$

Solution : Let $F(s)=\frac{1}{s}-\frac{1}{s+1}$

Then $L^{-1}\left[\int_s^{\infty}\left(\frac{1}{s}-\frac{1}{s+1}\right)ds\right]=\frac{L^{-1}\left[\frac{1}{s}-\frac{1}{s+1}\right]}{t}=\frac{1-e^{-t}}{t}$

3) Find the inverse Laplace Transforms of $\frac{s^2-a^2}{(s^2+a^2)^2}$

Solution : We know that, $\frac{d}{ds}\left\{\frac{s}{s^2+a^2}\right\} = \frac{a^2-s^2}{(s^2+a^2)^2}$

Or, $\frac{s^2-a^2}{(s^2+a^2)^2} = -\frac{d}{ds}\left\{\frac{s}{s^2+a^2}\right\}$

Therefore, $L^{-1}\left[\frac{s^2-a^2}{(s^2+a^2)^2}\right] = -L^{-1}\left[\frac{d}{ds}\left\{\frac{s}{s^2+a^2}\right\}\right]$

$= (-1)\left\{-tL^{-1}\left[\frac{s}{s^2+a^2}\right]\right\}$= tcosat

4) Obtain the inverse Laplace Transforms of

$$L^{-1}\left[\int_{s}^{\infty} log\left(\frac{u+2}{u+1}\right) du\right]$$

Solution:

Let $F(u) = log\left(\frac{u+2}{u+1}\right) = = log(u+2) - log(u+1)$

$$F'(u) = \frac{1}{u+2} - \frac{1}{u+1}$$

$$-F'(u) = \frac{1}{u+2} - \frac{1}{u+1}$$

$$L^{-1}\{-F'(u)\} = L^{-1}\left(\frac{1}{u+1}\right) - L^{-1}\left(\frac{1}{u+2}\right)$$

$$tf(t) = e^{-t} - e^{-2t}$$

Thus, $\boldsymbol{f(t)} = \frac{\boldsymbol{e^{-t}-e^{-2t}}}{\boldsymbol{t}}$

Then $L^{-1}\left[\int_{s}^{\infty} log\left(\frac{u+2}{u+1}\right) du\right] = \frac{L^{-1}\left[log\left(\frac{u+2}{u+1}\right)\right]}{t} = \frac{\boldsymbol{e^{-t}-e^{-2t}}}{\boldsymbol{t^{2}}}$

THANK YOU

Dr. SIVASANKARI. V
Department of Science & Humanities
sivasankariv@pes.edu