Unit-2 class-7

1. Find the Laplace transform of $\frac{1-e^{at}}{t}$

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds$$

$$= \int_s^{\infty} L\left[1-e^{at}\right] ds = \int_s^{\infty}\left(\frac{1}{s} - \frac{1}{s-a}\right) ds$$

$$= \left[\log s - \log(s-a)\right]_s^{\infty}$$

$$= \left[\log\left(\frac{s}{s-a}\right)\right]_s^{\infty} = \left[\log\left(\frac{\cancel{s}}{\cancel{s}\left(1-\frac{a}{s}\right)}\right)\right]_s^{\infty}$$

$$= \log\left(\frac{1}{1-\frac{a}{s}}\right)_s^{\infty} = \log\left(\frac{1}{1-0}\right) - \log\left(\frac{1}{1-\frac{a}{s}}\right)$$

$$= 0 - \log\left(\frac{s}{s-a}\right) = \log\left(\frac{s-a}{s}\right)$$

2. Find the Laplace transform of $\frac{e^{at}-\cos bt}{t}$

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds$$

$$= \int_s^{\infty} L\left[e^{at}-\cos bt\right] ds = \int_s^{\infty}\left(\frac{1}{s-a} - \frac{s}{s^2+b^2}\right) ds$$

$$= \left[\log(s-a)\right]_s^{\infty} - \left[\frac{1}{2}\log(s^2+b^2)\right]_s^{\infty}$$

$$= \left[\log\left(\frac{s-a}{\sqrt{s^2+b^2}}\right)\right]_s^{\infty} = \left[\log \frac{\cancel{s}\left(1-\frac{a}{s}\right)}{\cancel{s}\left(\sqrt{1+\left(\frac{b}{s}\right)^2}\right)}\right]_s^{\infty}$$

$$= 0 - \log\left(\frac{s-a}{\sqrt{s^2+b^2}}\right) = \log\left(\frac{\sqrt{s^2+b^2}}{s-a}\right)$$

3. Find the Laplace transform of $\frac{e^{-at}-e^{-bt}}{t}$

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} L[f(t)]\,ds$$

$$= \int_s^{\infty} L\left[e^{-at} - e^{-bt}\right] ds$$

$$= \int_s^{\infty}\left(\frac{1}{s+a} - \frac{1}{s+b}\right) ds$$

$$= \left[\log\left(\frac{s+a}{s+b}\right)\right]_s^{\infty}$$

$$= 0 - \log\left(\frac{s+a}{s+b}\right)$$

$$= \log\left(\frac{s+b}{s+a}\right)$$