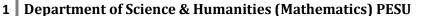**Session-01: Review of probability; Random variables; Functions of random variables**

1. A target on a test firing range consists of a bull's-eye with two concentric rings around it. A projectile is fired at the target. The probability that it hits the bull's-eye is 0.10, the probability that it hits the inner ring is 0.25, and the probability that it hits the outer ring is 0.45. What is the probability that the projectile hits the target? What is the probability that it misses the target?

   Answer: P(hits target)=0.80; P(misses target)=0.20

2. In a process that manufactures aluminum, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. What is the probability that a randomly chosen can have a flaw? What is the probability that it has no flaw?

   Answer: P(flaw on side or flaw on top)=0.04; P(a can has no flaw)=0.96

Session-02: Discrete probability distribution

1. The probability density function of a random variable X is

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

Find $P(X < 4)$; $P(X \geq 5)$; $P(3 < X \leq 6)$. What will be the minimum value of k so that $P(X \leq 2) > 0.3$.

Answer: $k = \frac{1}{49}$; $P(X < 4) = \frac{16}{49}$; $P(X \geq 5) = \frac{24}{49}$; $P(3 < X \leq 6) = \frac{33}{49}$;

For $P(X \leq 2)$, $k = \frac{1}{30}$

2. A coin is tossed three times. Let X denote the number of heads showing up. Find the distribution of X. Also, find its mean and variance.

Answer:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean = $\frac{3}{2}$; Variance=$\frac{3}{4}$

Session-03: Continuous random variables

1. The proportion of people who respond to a mail order solicitation is a continuous random variable X that has a density function given by:

$$f(x) = \begin{cases} \dfrac{2(x+2)}{5} & if\ 0 < X < 1 \\ 0 & otherwise \end{cases}$$

a) Show that $P(0 < X < 1) = 1$.

(Hint: Show that the given function integrates to 1 between the limits 0 and 1.)

b) Find the probability that more than $\frac{1}{4}$ but fewer than $\frac{1}{2}$ of the people contacted will respond to this type of solicitation.

(Hint: Integrate the given function between the limits 1/4 and 1/2.)

2. The pdf of the samples of speech waveforms is found to decay exponentially at a rate α, so that the following pdf is poposed:

$$f(x) = c\ e^{\alpha|x|} \qquad -\infty < X < \infty$$

Find the constant C and then find the probability $P(|X| < v)$.

3. The CDF of checkout time duration X is $F(x) = \begin{cases} 0 & if\ x < 0 \\ \dfrac{x^2}{4} & if\ 0 \le x < 2, \\ 1 & if\ x \ge 2 \end{cases}$

Use this information to compute a) $P(X \le 1)$ b) $P(0.5 \le X \le 1)$ c) Find the density function of X.

(Hint: Derivative of the CDF is the density function. The area under the density curve in an interval gives the probabilities.)

Session-04: Bernoulli distribution

1. Assume that out of every 50 people in a city, one is a business owner. So, if one one citizen is selected randomly, what is the distribution of business owners?

   Solution: Given $p = \frac{1}{50}$ hence $q = \frac{49}{50}$

   Let $X = 1$ represent that the person selected at random is a business owner.

   and let $X = 0$ represent that the person selected is not a business owner.

   Then, $P(X = 1) = p = \frac{1}{50} = 0.02$ and $P(X = 0) = 1 - p = 1 - \frac{1}{50} = \frac{49}{50} = 0.98$.

2. If one out of 15 stocks perform extraordinarily, then what is the performance of a stock randomly selected from the portfolio?

   Solution: Let X be the Bernoulli random variable that denotes the performance of a stock.

   Given $p = \frac{1}{15}$. Thus, $P(X = 1) = p = \frac{1}{15}$.

3. In a medical examination the chances of error are 15%. Find the Bernoulli distribution if one patient is randomly selected out of 60 patients.

   Solution: Let X be the Bernoulli random that represents the number of correct reports.

   It is given that 15% of 60 patients have error reports. This means that there are

   $\frac{15\%}{60} = $ 9 patients out of 60 who have received error reports. Thus, the number of patients receiving

   correct reports are 60-9 = 51.

   Thus, $P(X = 1) = \frac{51}{60} = 0.85$ and $P(X = 0) = \frac{9}{60} = 0.15$

Session-05: Binomial Distribution

1. Find the probability of getting a total of 7 at least once in three tosses of a fair dice.

   Ans: 91/216

2. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random (i) 1,   (ii) 0,   (iii) less than 2 bolts will be defective.

   Ans: (i) 0.4096
   (ii)0.4096
   (iii) 0.8192

3. A communication system consists of 'n' components, each of which will, independently function with probability 'p'. the total system will be able to operate effectively if at least one-half of its components function. For what values of 'p' is a 5-component system more likely to operate effectively than a 3-component system?

   Ans: $p > 1/2$ '

Session-06: Poisson Distribution

1. The random variable X has a Poisson distribution. If $P(X = 1) = 0.01487, P(X = 2) = 0.04461$, then find $P(X = 3)$.
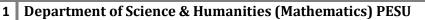
   Ans: 0.08922

2. Wireless sets are manufactured with 25 soldered joints each. On the average, 1 joint in 500 is defective. How many sets can be expected to be free from defective joints in a consignment of 10000 sets?

   Ans: 9512

3. Suppose the number of telephone calls on an operator received from 9.00 to 9.25 follow a Poisson distribution with mean 3. Find the probability that
   i. The operator will receive no calls in that time interval tomorrow.
   ii. In the next three days, the operator will receive a total of 1 call in that time interval.

   Ans: (i) o.0497 (ii) 0.0011

Session-07 & 08: Normal Distribution

1. The mean height of 500 male students in a certain college is 151 cm and the standard deviation is 15 cm. Assuming the heights are normally distributed, find how many students have heights between 120 and 155cm.
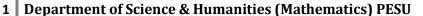
   Answer: 300

2. An aptitude test for selecting officers in a bank is conducted on 1000 candidates. The average score is 42 and the standard deviation of the score is 24. Assuming a normal distribution for the scores, find

   i)The number of candidates whose scores exceed 60

   ii) The number of candidates whose scores lie between 30 and 60.

   Answer: 252 and 533

3. In a normal distribution, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?

   Answer: 50.3 and 10.33

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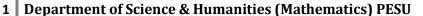
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Central Limit Theorem

1. An unknown distribution has a mean of 90 and a standard deviation of 1.5. Samples of size n=25 are drawn randomly from the population
   a) Find the probability that the sample mean is between 85 and 92.
   b) Find the value that is two standard deviation above the expected value 90 of the sample mean.

2. The mean of a certain normal population is equal to the standard error of the mean of the samples of 100 from that distribution. Find the probability that the mean of the sample of 25 from the distribution will be negative?

3. An experiment consists of rolling a die 81 times and computing the average of the numbers on the top face of the die. Estimate the probability that the sample mean will be less than 3.