1. The proportion of people who respond to a mail order solicitation is a continuous random variable X that has a density function given by:

$$f(x) = \begin{cases} \frac{2(x+2)}{5} & if\ 0 < X < 1 \\ 0 & otherwise \end{cases}$$

a) Show that $P(0 < X < 1) = 1$.

(Hint: Show that the given function integrates to 1 between the limits 0 and 1.)

b) Find the probability that more than $\frac{1}{4}$ but fewer than $\frac{1}{2}$ of the people contacted will respond to this type of solicitation.

(Hint: Integrate the given function between the limits 1/4 and 1/2.)

A. a) $f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

$$\int_0^1 f(x)\,dx = \int_0^1 \frac{2x+4}{5}\,dx = \left[\frac{x^2}{5} + \frac{4x}{5}\right]_0^1 = \left[\frac{1}{5} + \frac{4}{5} - 0\right] = 1$$

b) $$\int_{1/4}^{1/2} f(x)\,dx = \int_{1/4}^{1/2} \frac{2x+4}{5}\,dx = \left[\frac{x^2}{5} + \frac{4x}{5}\right]_{1/4}^{1/2} = \frac{1}{20} + \frac{2}{5} - \frac{1}{80} - \frac{1}{5} = \frac{3}{80} + \frac{1}{5} = \frac{19}{80}$$

2. The pdf of the samples of speech waveforms is found to decay exponentially at a rate α, so that the following pdf is poposed:

$$f(x) = c\,e^{\alpha|x|} \qquad -\infty < X < \infty$$

Find the constant C and then find the probability $P(|X| < v)$.

A. $f(x) = ce^{\alpha|x|}$, $-\infty < x < \infty$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} ce^{-\alpha|x|}\,dx = \int_{-\infty}^{0} ce^{-\alpha|x|}\,dx + \int_{0}^{\infty} ce^{-\alpha|x|}\,dx = 1$$

$$= \frac{c}{\alpha}(1-0) + \frac{c}{-\alpha}(0-1) = 1 \Rightarrow \frac{2c}{\alpha} = 1 \Rightarrow c = \frac{\alpha}{2}$$

3. The CDF of checkout time duration X is $F(x) = \begin{cases} 0 & if\ x < 0 \\ \frac{x^2}{4} & if\ 0 \le x < 2, \\ 1 & if\ x \ge 2 \end{cases}$

Use this information to compute a) $P(X \le 1)$ b) $P(0.5 \le X \le 1)$ c) Find the density function of X .

(Hint: Derivative of the CDF is the density function. The area under the density curve in an interval gives the probabilities.)

A. $\text{PDF} = \frac{d}{dx}(\text{CDF})$

$f(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \le x < 2 \\ 0, & x \ge 2 \end{cases}$

a) $$P(x \le 1) = \int_0^1 f(x)\,dx = \int_0^1 \frac{x}{2}\,dx = \left[\frac{x^2}{4}\right]_0^1 = \frac{1}{4}$$

b) $$P(0.5 \le x \le 1) = \int_{0.5}^{1} f(x)\,dx = \int_{1/2}^{1} \frac{x}{2}\,dx = \left[\frac{x^2}{4}\right]_{1/2}^{1} = \left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3}{16}$$

c) P.D.F $\Rightarrow$ $f(x) = \begin{cases} x/2, & 0 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$