ENGINEERING MECHANICS - STATICS

Department of Civil Engineering
Email: @pes.edu
M:

ENGINEERING MECHANICS

Course Content:

~~Unit 1: Introduction to statics~~

~~Mechanics, Basic Concepts, Scalars and Vectors, Force Systems - Introduction, Force, Rectangular Components, Moment, Numerical.~~

~~8 Hours~~

~~Unit 2: Force System~~

~~Force Systems - Couple, Resultants, Numerical.~~

~~7 Hours~~

ENGINEERING MECHANICS

Beams – External Effects

Department of Civil Engineering

A beam is a horizontal structural member used to support loads. Beams are used to support the roof and floors in buildings.

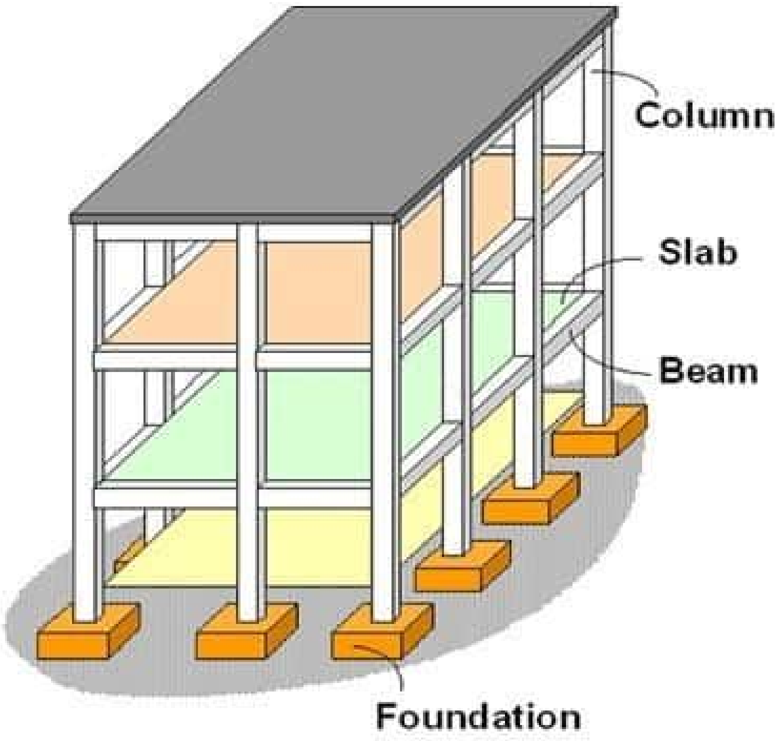
Beams are structural members which offer resistance to bending due to applied loads. Most beams are long prismatic bars, and the loads are usually applied normal to the axes of the bars.

https://www.youtube.com/watch?v=1zd-qluq-Io

https://www.youtube.com/watch?v=lm7EqrFE4mg

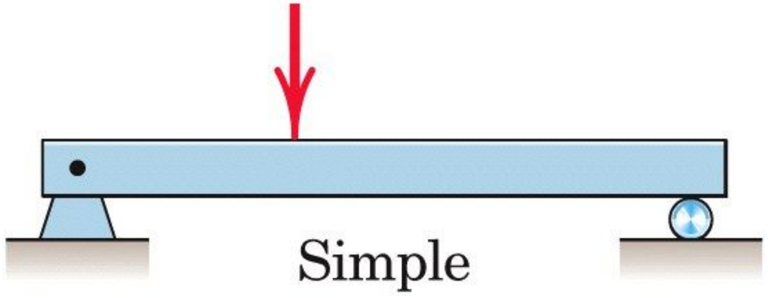
ENGINEERING MECHANICS

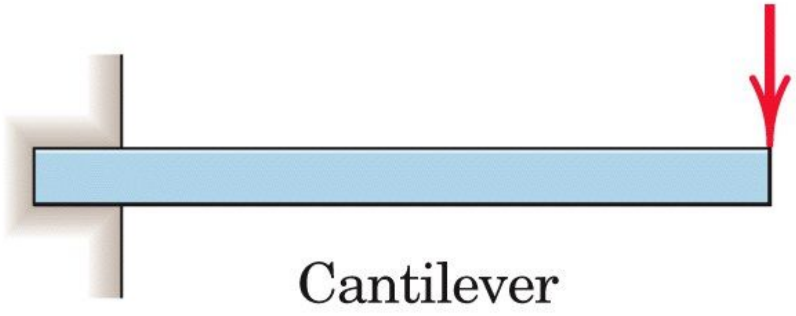
Beams



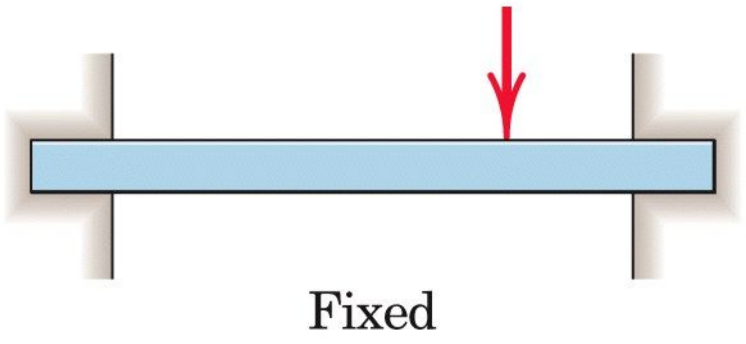
Typical RC Frame Building

Beams supported so that their external support reactions can be calculated by the methods of statics alone are called statically determinate beams.

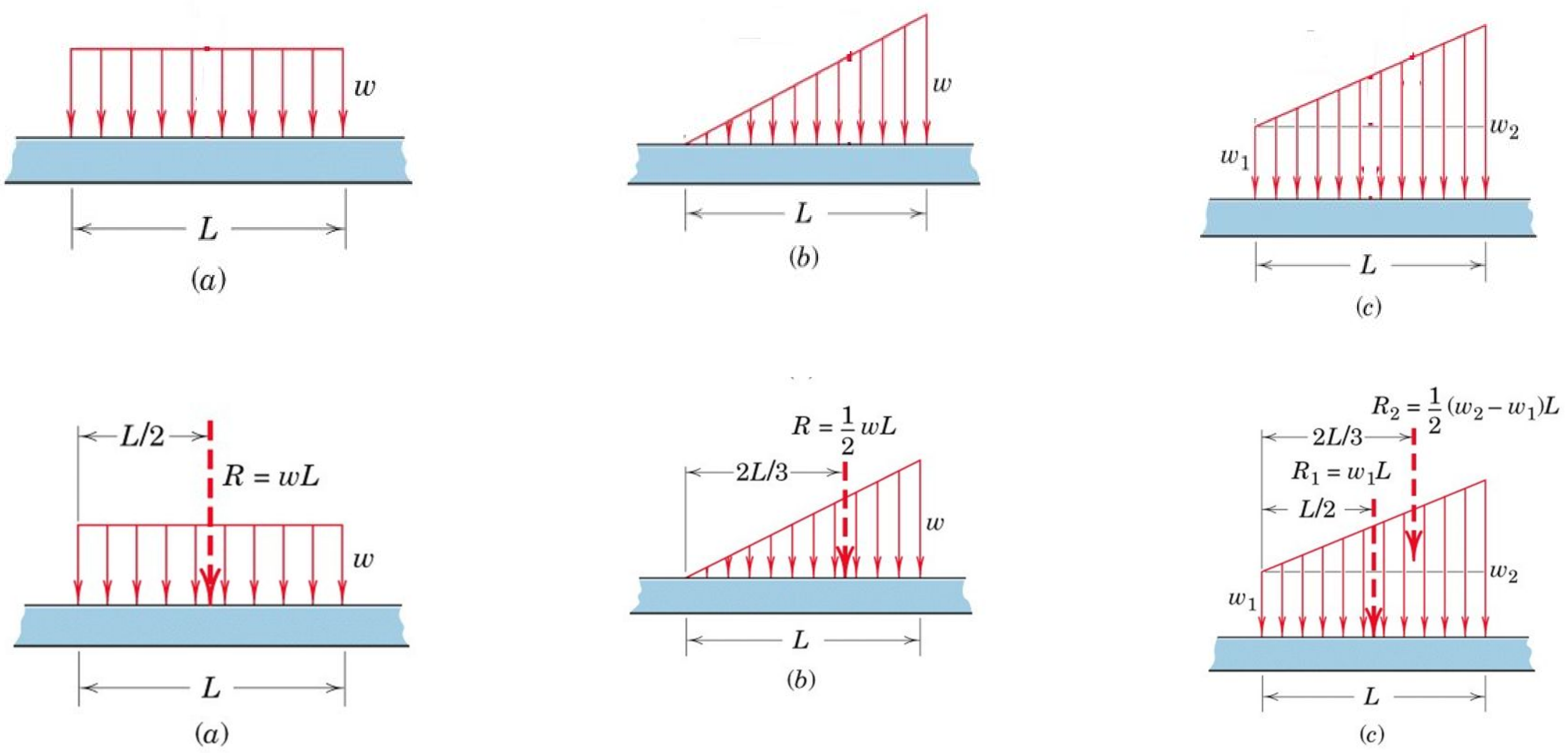




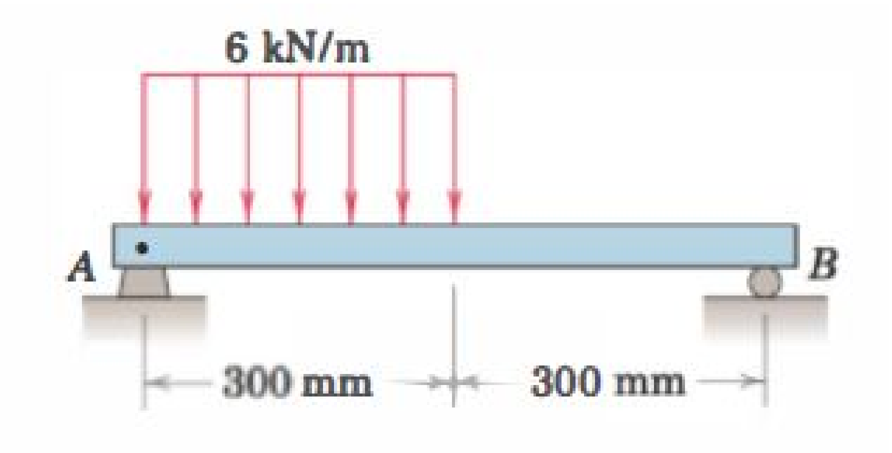
A beam which has more supports than needed to provide equilibrium is statically indeterminate.



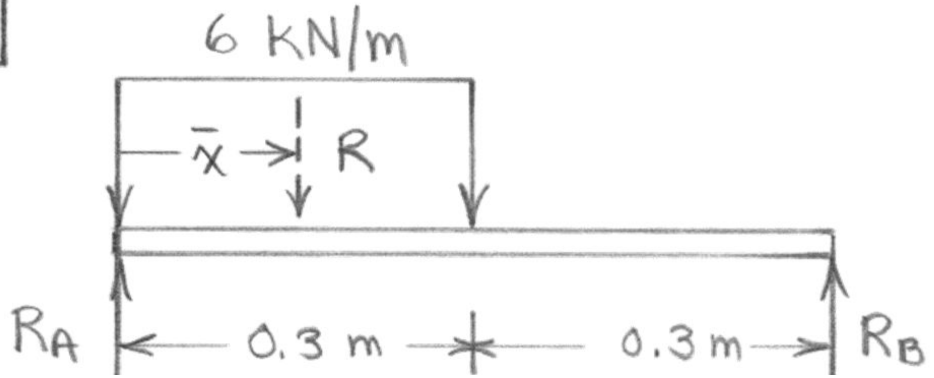
Loads on Beams



5/101) Determine the reactions at A and B for the beam subjected to the uniform load distribution.



5/101

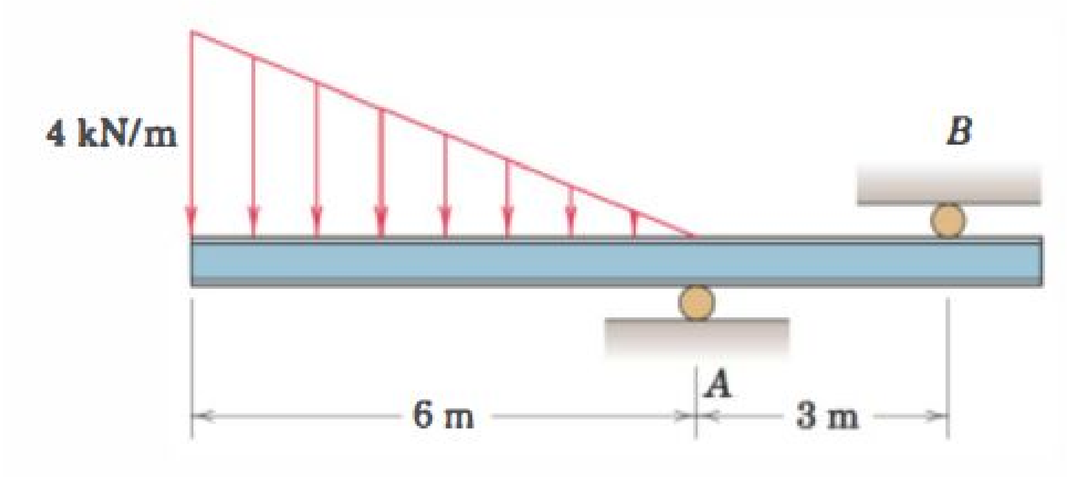


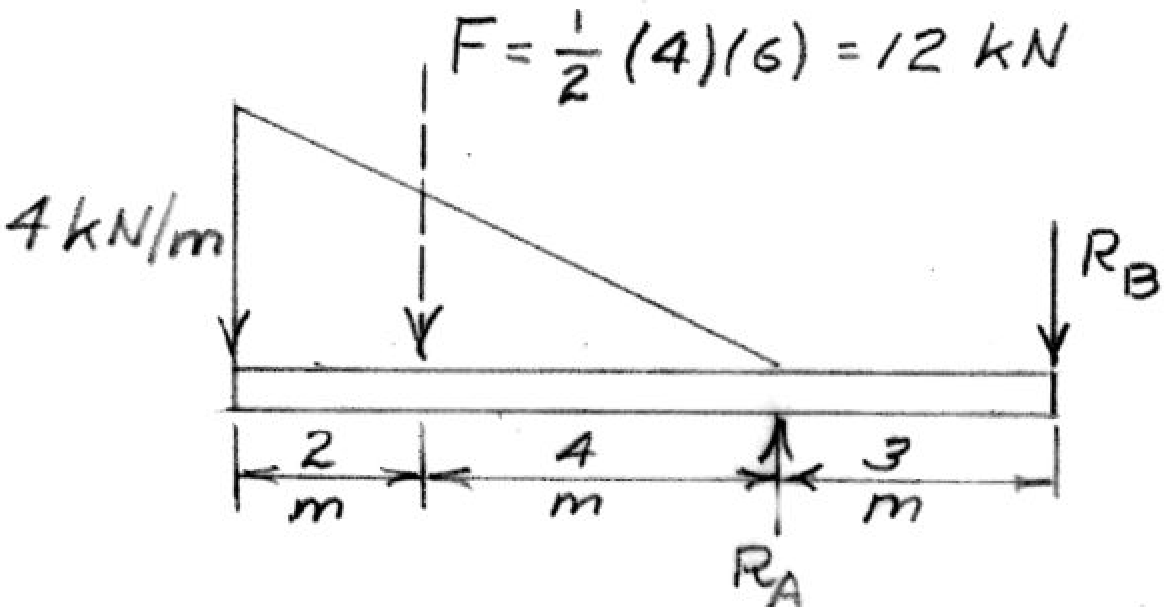
$$R = 6(0.3) = 1.8 \text{ kN} \quad @ \quad \bar{x} = \tfrac{1}{2}(0.3) = 0.15 \text{ m}$$

$$\circlearrowleft + \Sigma M_A = 0: \quad R_B(0.6) - 1.8(0.15) = 0, \quad \underline{R_B = 0.45 \text{ kN}}$$

$$+\uparrow \Sigma F = 0: \quad 0.45 - 1.8 + R_A = 0, \quad \underline{R_A = 1.35 \text{ kN}}$$

5/102) Calculate the reactions at A and B for the beam loaded as shown.

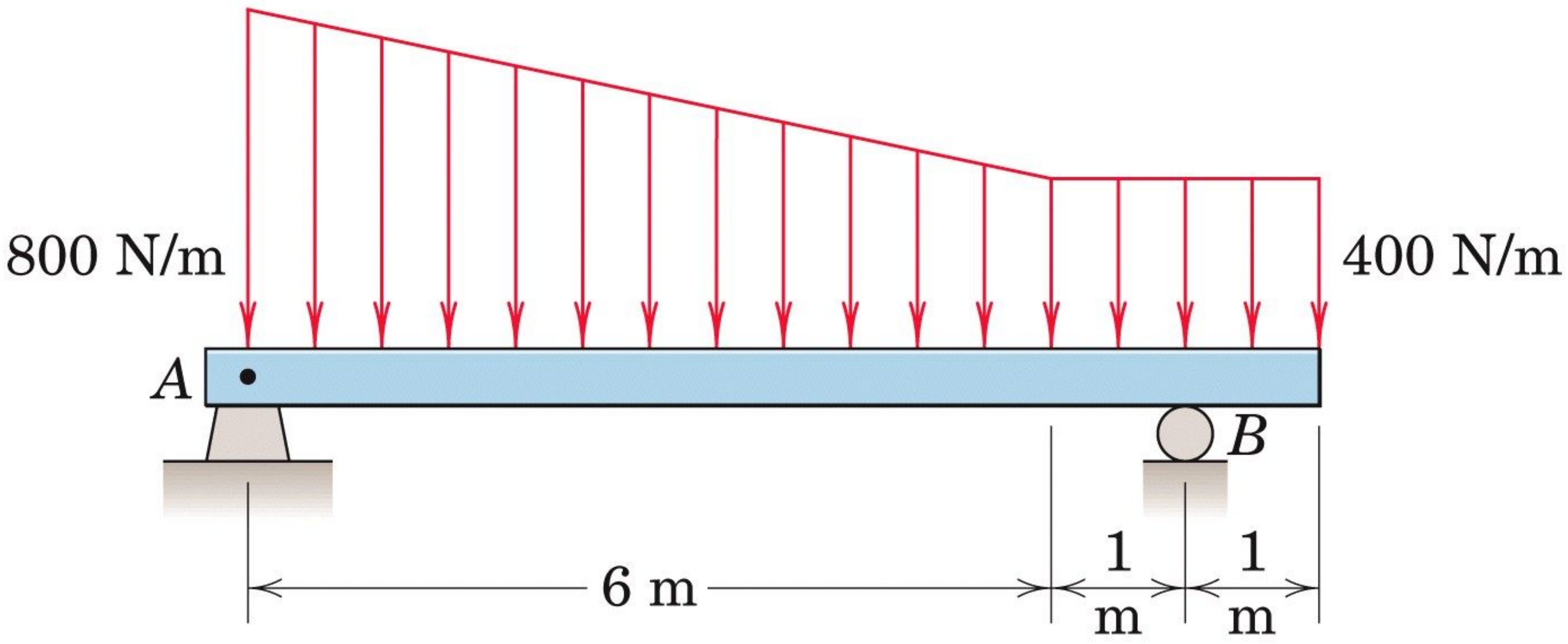




$\circlearrowleft^{+} \Sigma M_A = 0: \quad 12(4) - 3R_B = 0,$ $\quad \underline{R_B = 16\ kN}$

$+\uparrow \Sigma F = 0: \quad R_A - 16 - 12 = 0,$ $\quad \underline{R_A = 28\ kN}$

5/103) Calculate the reactions at A and B for the beam loaded as shown.



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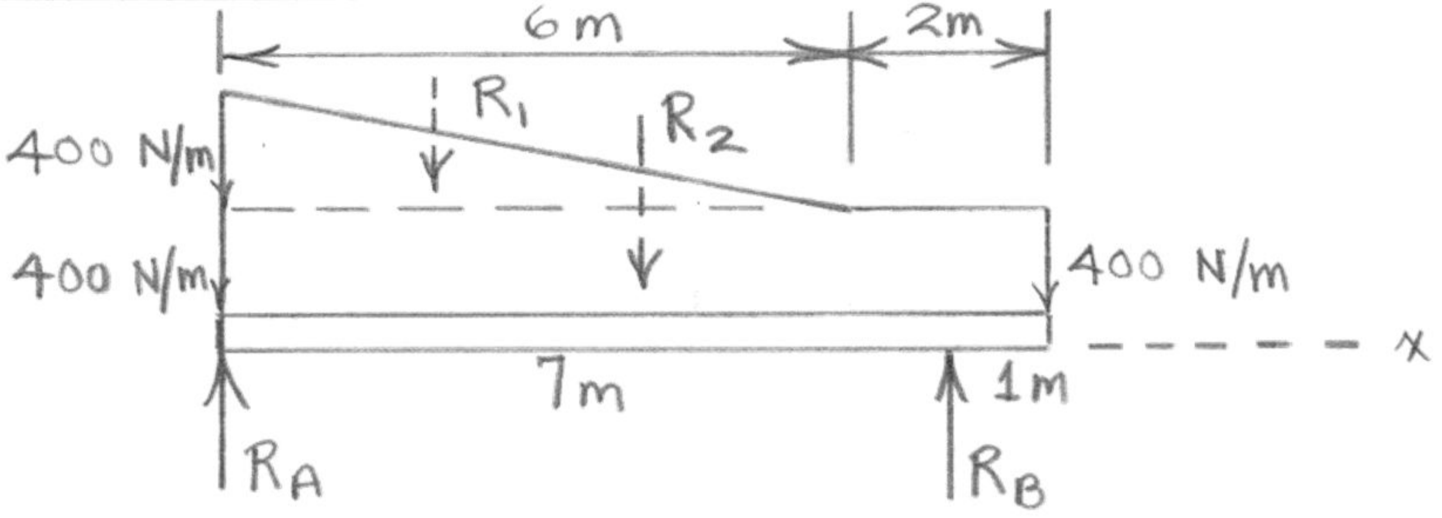


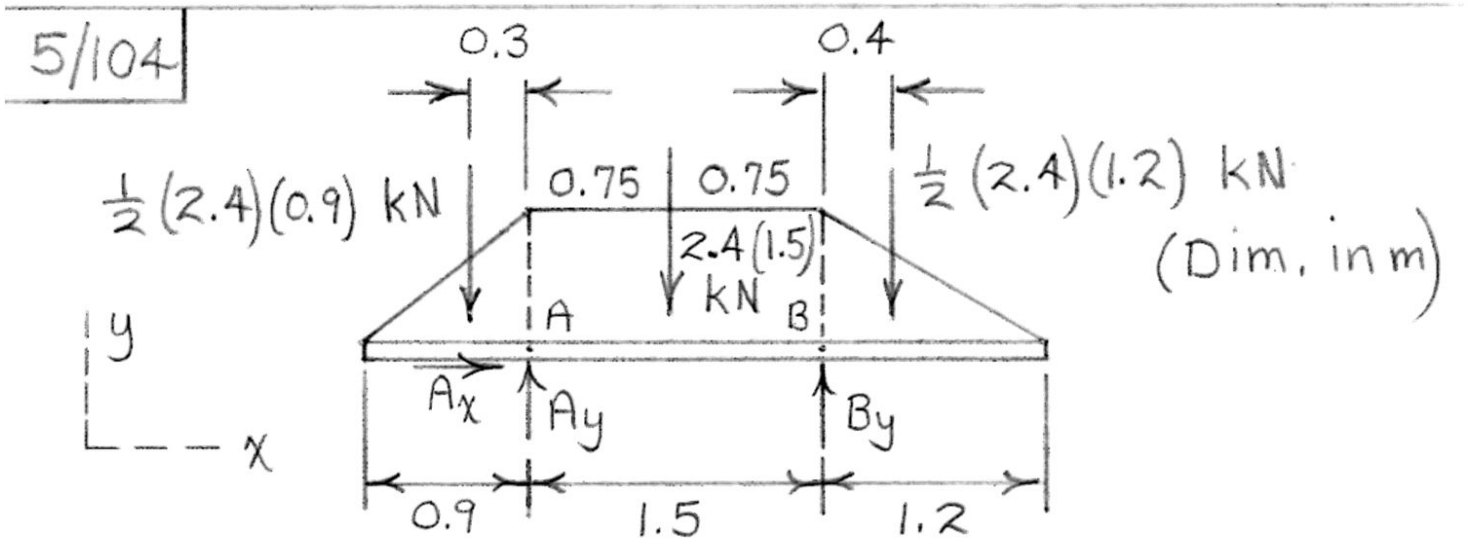
$$R_1 = \frac{1}{2}(400)(6) = 1200 \text{ N} \quad @ \quad \bar{x}_1 = \frac{1}{3}(6) = 2 \text{ m}$$

$$R_2 = 400(8) = 3200 \text{ N} \quad @ \quad \bar{x}_2 = \frac{1}{2}(8) = 4 \text{ m}$$

$$\circlearrowleft + \Sigma M_A = 0: \quad R_B(7) - 1200(2) - 3200(4) = 0, \quad \underline{R_B = 2170 \text{ N}}$$

$$+\uparrow \Sigma F = 0: \quad R_A - 1200 - 3200 + 2170 = 0, \quad \underline{R_A = 2230 \text{ N}}$$

5/104) Calculate the reactions at A and B for the beam loaded as shown.



$$\circlearrowleft + \Sigma M_A = 0 : \quad 1.08(0.3) - 3.6(0.75) - 1.44(1.9) + B_y(1.5) = 0$$

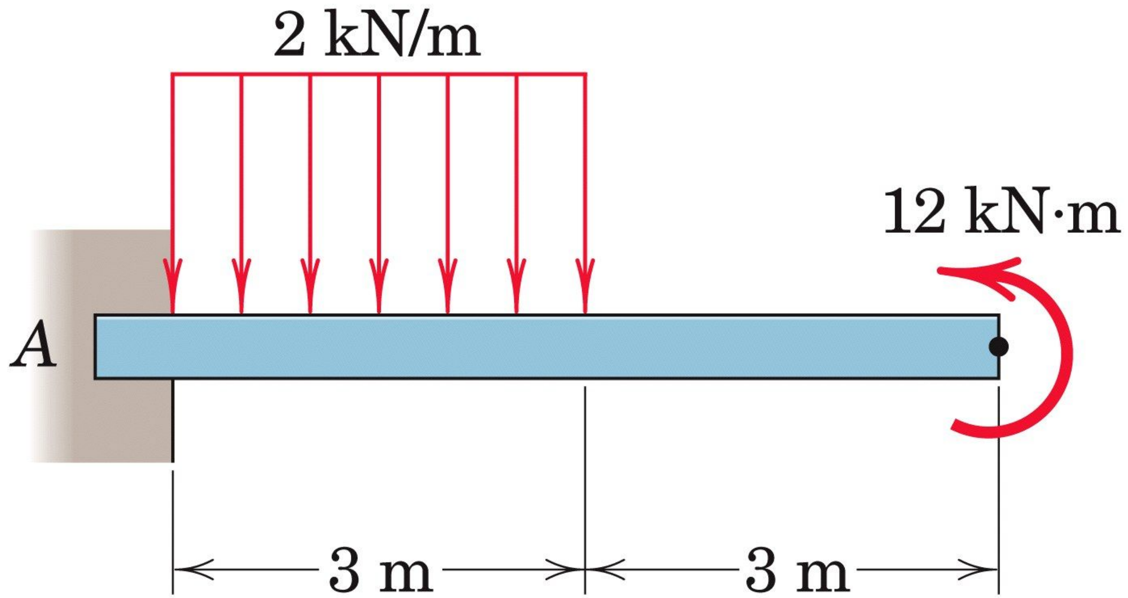
$$\underline{B_y = 3.41 \text{ kN}}$$

$$\Sigma F_y = 0 : \quad A_y + 3.41 - 1.08 - 3.6 - 1.44 = 0$$

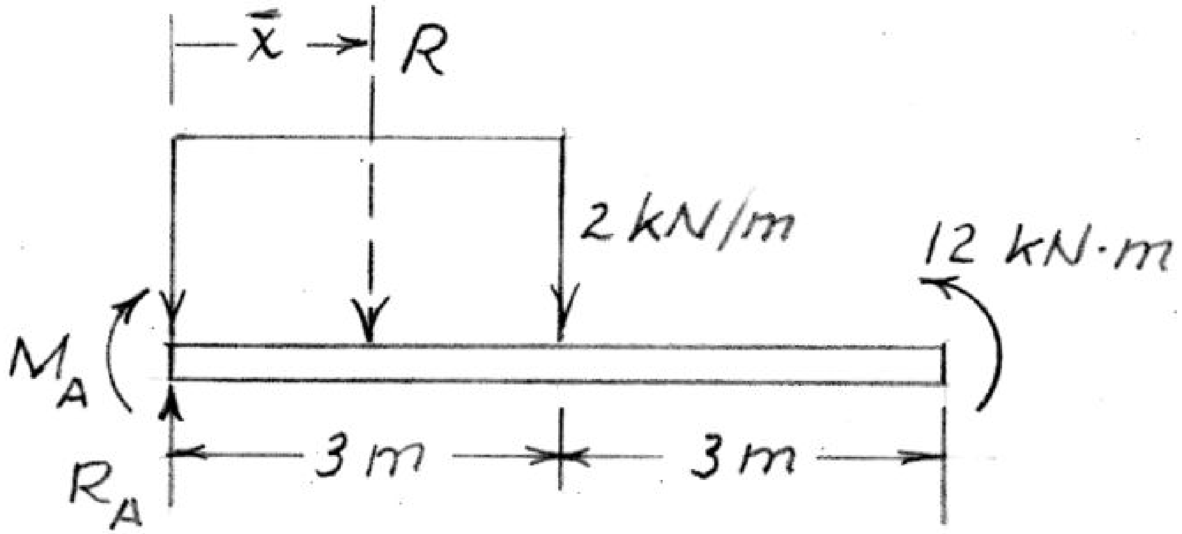
$$\underline{A_y = 2.71 \text{ kN}}$$

$$\Sigma F_x = 0 : \quad \underline{A_x = 0}$$

5/105) Find the reaction at A due to the uniform loading and the applied couple.



5/105)

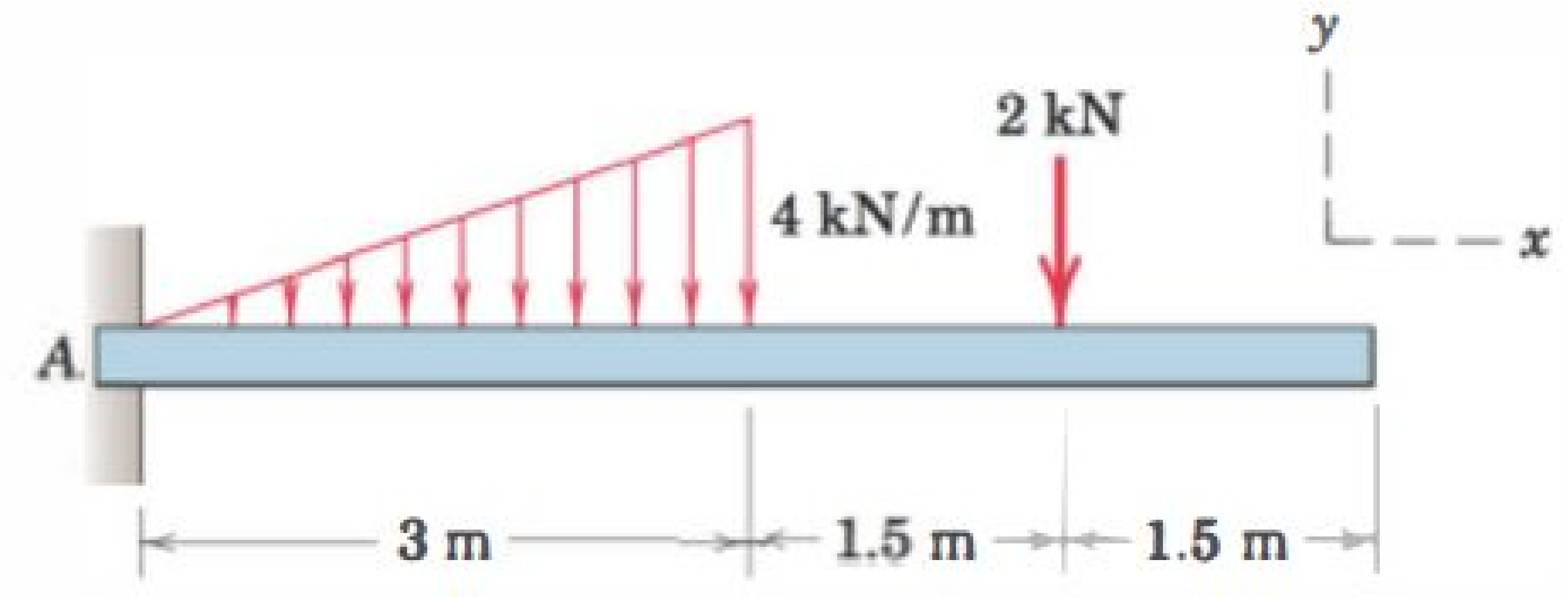


$R = 2(3) = 6\ kN \quad @ \quad \bar{x} = 1.5\ m$

$\circlearrowleft + \Sigma M_A = 0: \quad -M_A - 6(3/2) + 12 = 0, \quad \underline{M_A = 3\ kN \cdot m}$

$\uparrow + \Sigma F = 0: \quad R_A - 6 = 0, \quad \underline{R_A = 6\ kN}$

5/106) Determine the reactions at A for the cantilever beam subjected to the distributed and concentrated loads.



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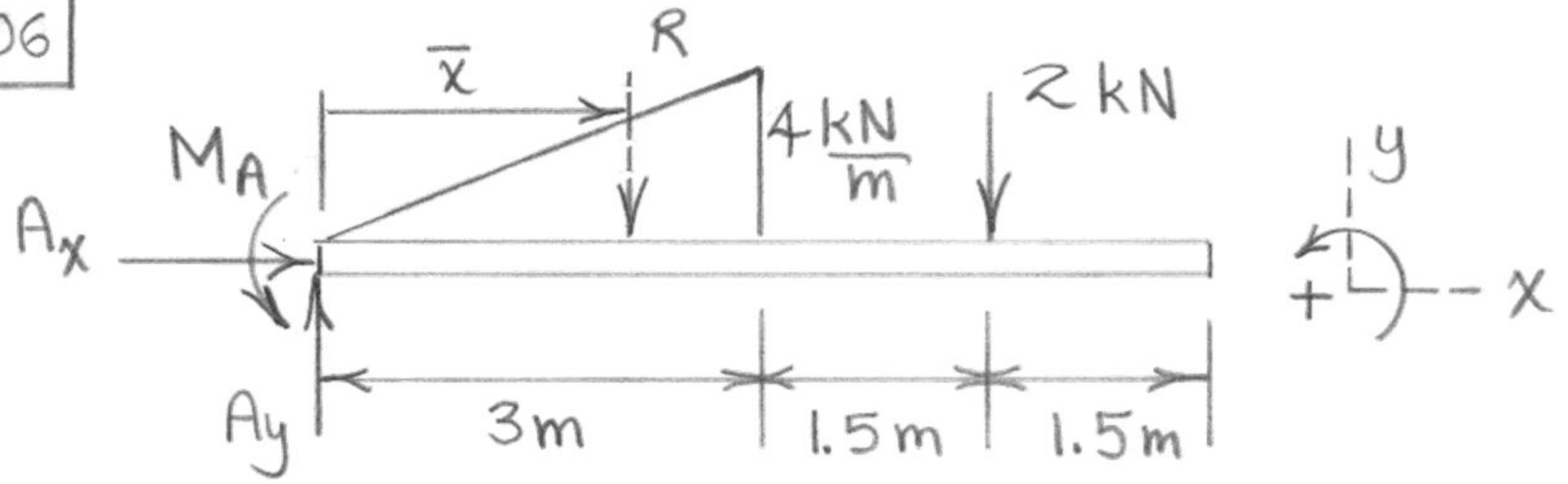


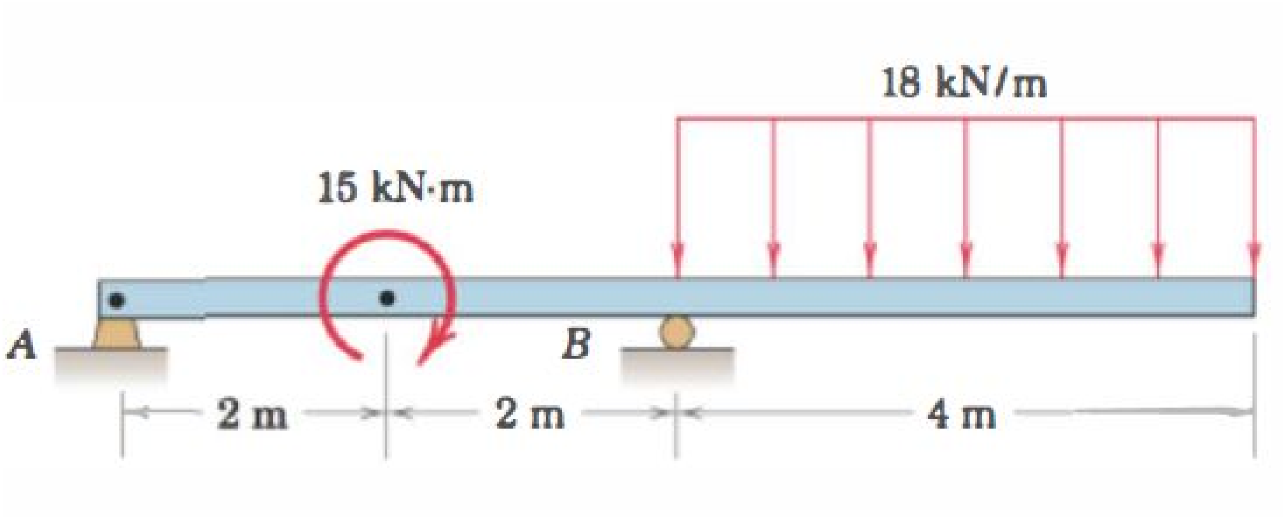
$$R = \tfrac{1}{2}(3)(4) = 6 \text{ kN} \ @ \ \bar{x} = \tfrac{2}{3}(3) = 2\text{m}$$

$$\Sigma M_A = 0 : \quad M_A - 6(2) - 2(4.5) = 0, \quad \underline{M_A = 21 \text{ kN·m}}$$

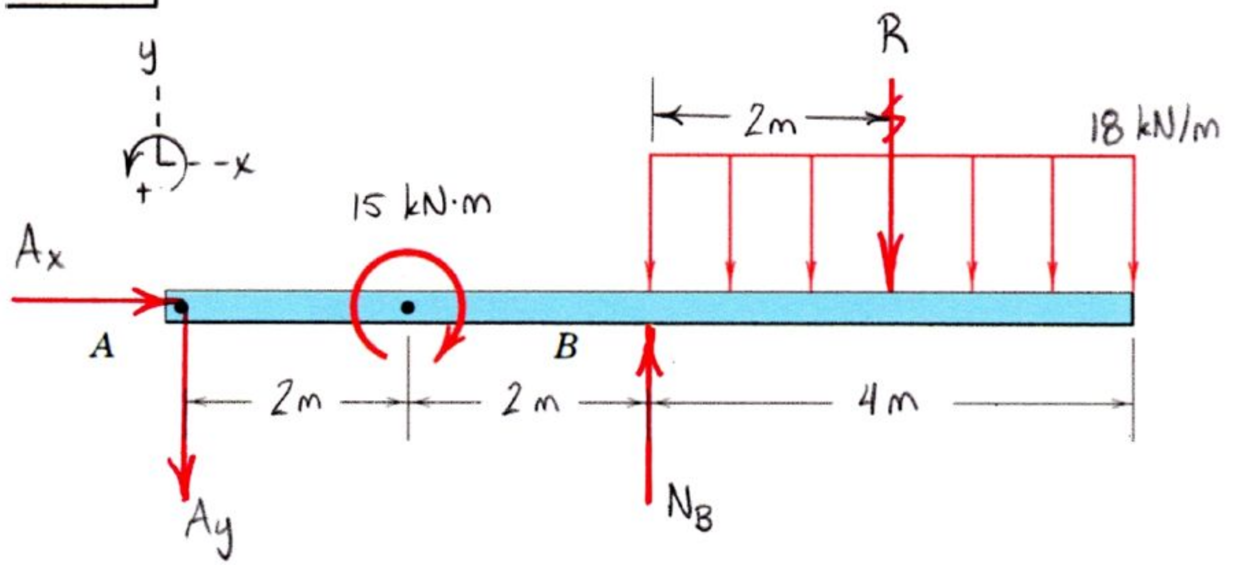
$$\Sigma F_y = 0 : \quad A_y - 6 - 2 = 0, \quad \underline{A_y = 8 \text{ kN}}$$

$$\Sigma F_x = 0 : \quad \underline{A_x = 0}$$

5/107) Determine the reactions at A and B for the beam loaded as shown



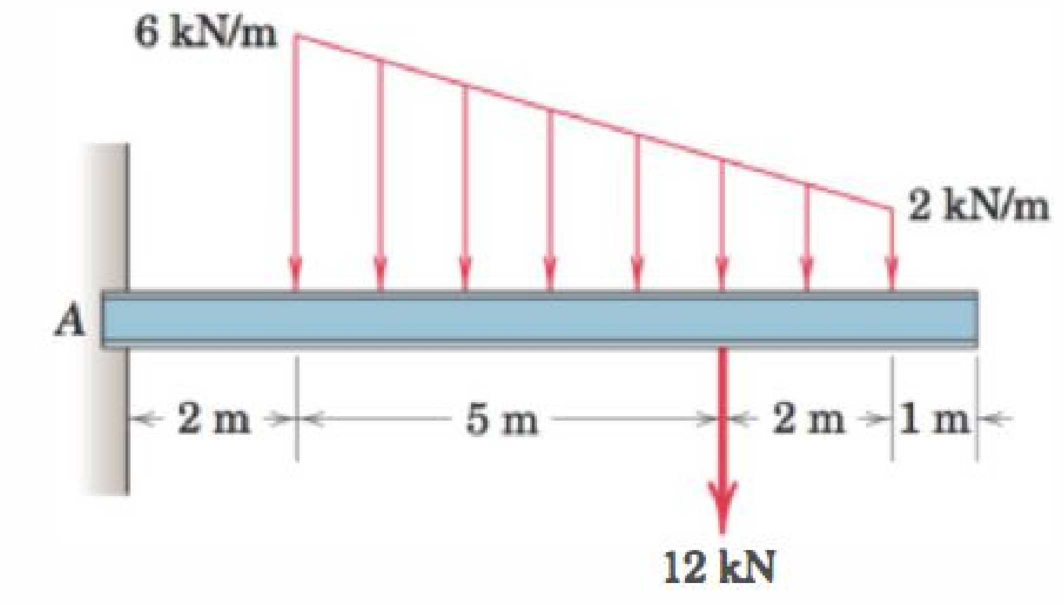
5/107



$$R = 18(4) = 72 \text{ kN}$$

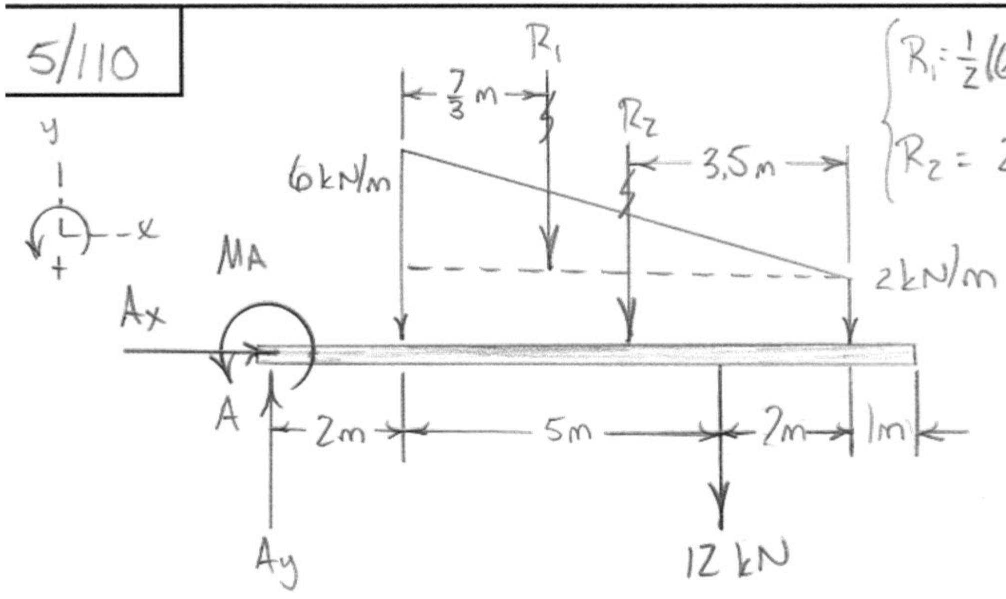
$$\begin{cases} \Sigma F_x = 0: & \underline{A_x = 0} \\ \Sigma F_y = 0: & -A_y + N_B - R = 0 \\ \Sigma M_A = 0: & 4N_B - 6R - 15 = 0 \end{cases} \longrightarrow \begin{cases} \underline{A_y = 39.8 \text{ kN} \downarrow} \\ \underline{N_B = 111.8 \text{ kN} \uparrow} \end{cases}$$

5/110) Determine the force and moment reactions at A for the cantilever beam subjected to the loading shown.



Numericals

5/110



$$R_1 = \frac{1}{2}(6-2)(7) = 14 \text{ kN}$$

$$R_2 = 2(7) = 14 \text{ kN}$$

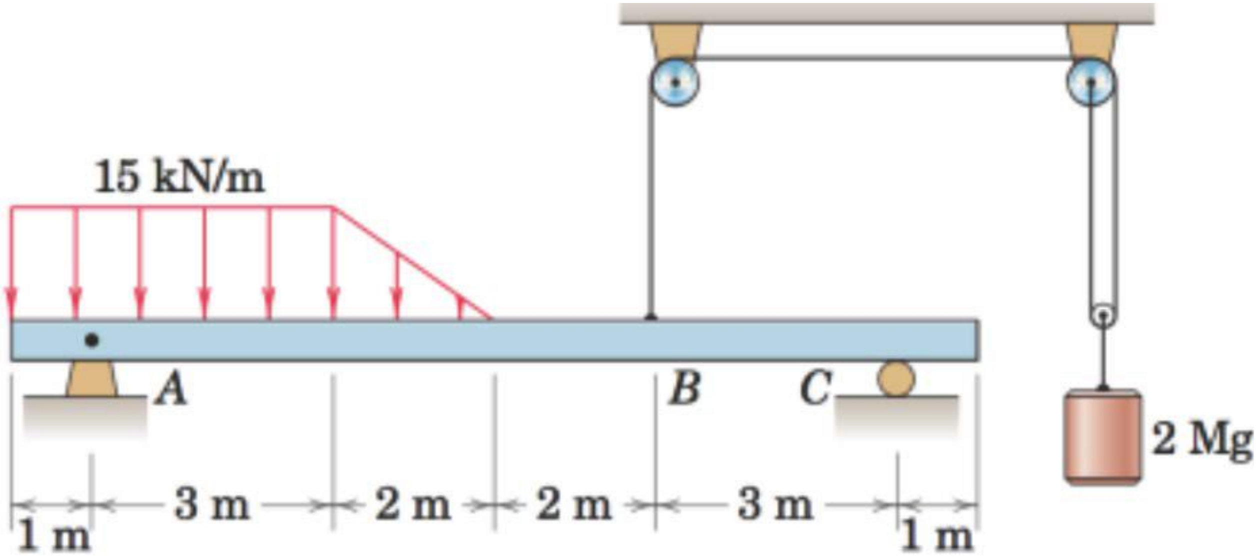
$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y - R_1 - R_2 - 12 = 0$$

$$\Sigma M_A = 0: \quad M_A - (2 + 7/3)R_1 - (2+3.5)R_2 - 7(12) = 0$$

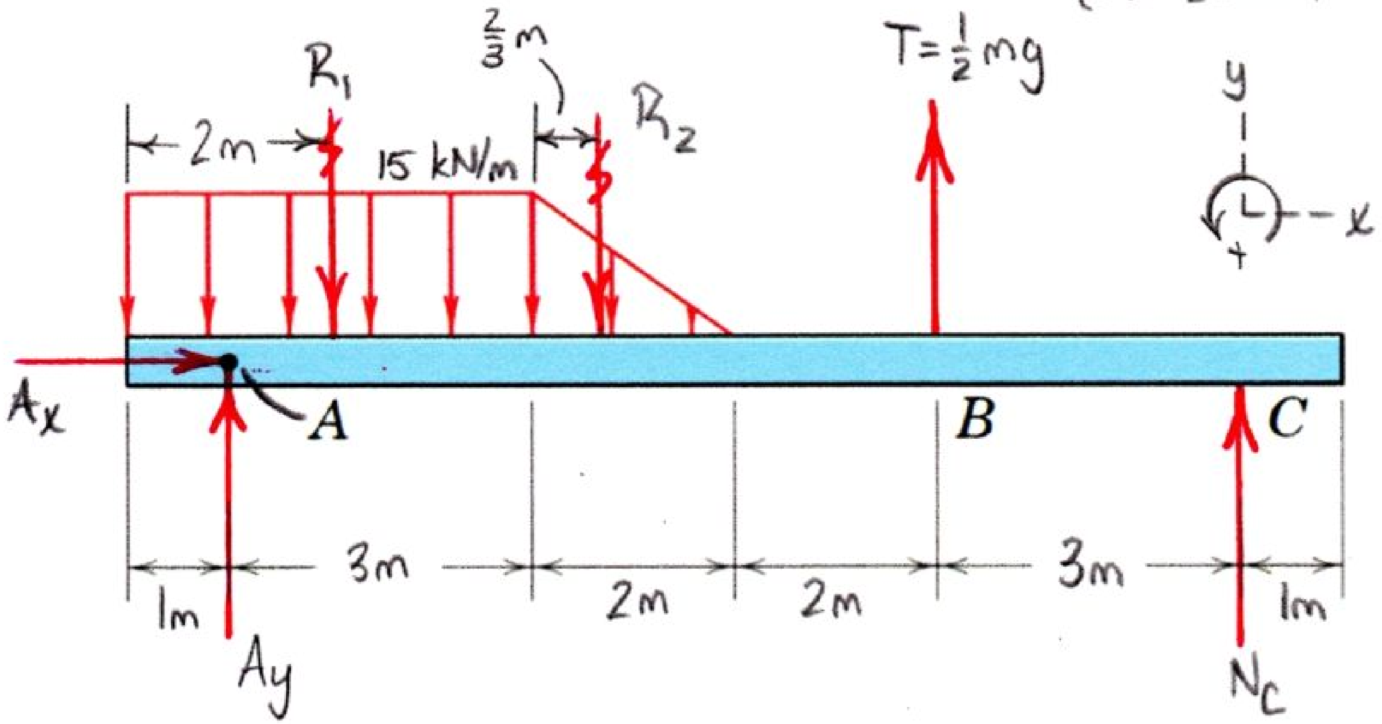
$$A_y = 40 \text{ kN} \uparrow \qquad M_A = 222 \text{ kN·m CCW}$$

5/111) Determine the reactions at A and C for the beam subjected to the combination of point and distributed loads.



5/111 $m = 2\ Mg$

$$\begin{cases} R_1 = 15(4) = 60\ kN \\ R_2 = \frac{1}{2}(15)(2) = 15\ kN \end{cases}$$



$$T = \frac{1}{2}(2000)(9.81) = 9810\ N$$

$$\Sigma F_x = 0: \quad \underline{A_x = 0}$$

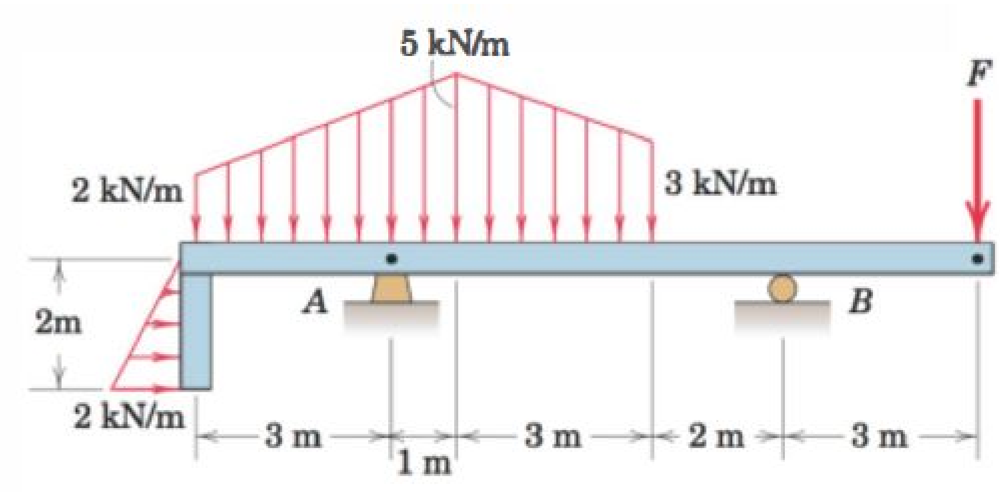
$$\Sigma F_y = 0: \quad A_y - R_1 - R_2 + T + N_c = 0$$

$$\Sigma M_A = 0: \quad -1R_1 - \left(3+\tfrac{2}{3}\right)R_2 + 7T + 10N_c = 0$$

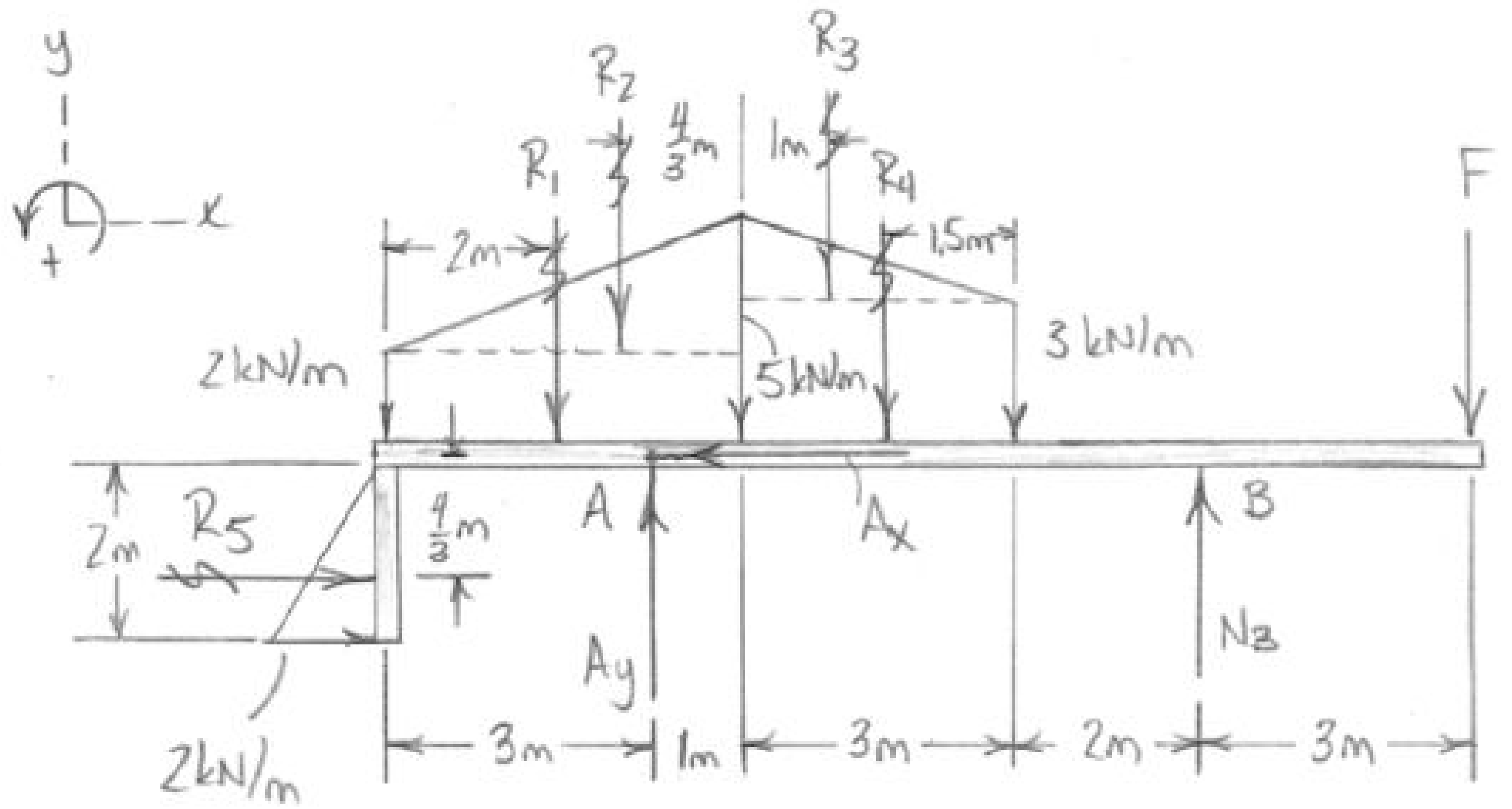
$$\underline{N_c = 4.63 \text{ kN} \uparrow}$$

$$\underline{A_y = 60.6 \text{ kN} \uparrow}$$

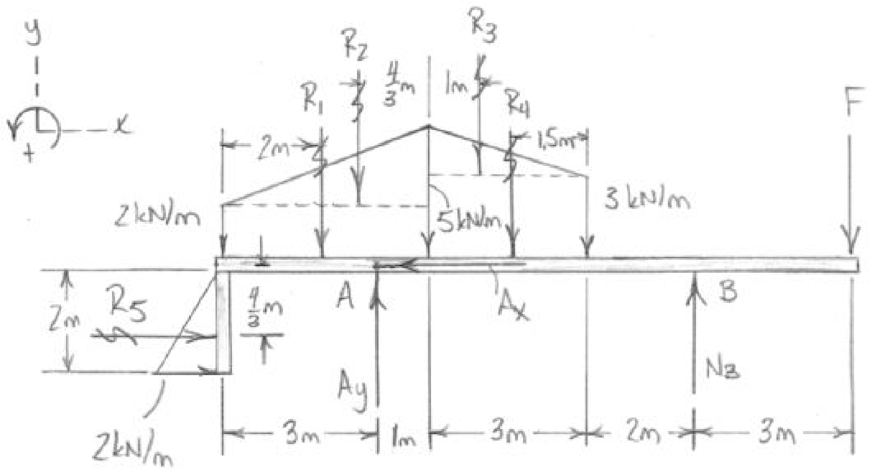
5/116) For the beam and loading shown, determine the magnitude of the force F for which the vertical reactions at A and B are equal. With this value of F, compute the magnitude of the pin reaction at A.



5/116 FIND F FOR $A_y = N_B$



5/116 Find F For $A_y = N_B$



$$R_1 = 2(4) = 8 \text{ kN}$$

$$R_2 = \tfrac{1}{2}(5-2)(4) = 6 \text{ kN}$$

$$R_3 = \tfrac{1}{2}(5-3)(3) = 3 \text{ kN}$$

$$R_4 = 3(3) = 9 \text{ kN}$$

$$R_5 = \tfrac{1}{2}(2)(2) = 2 \text{ kN}$$

$$\Sigma F_x = 0: \ -A_x + R_5 = 0 \longrightarrow A_x = 2 \text{ kN}$$

$$\Sigma F_y = 0: \ A_y + N_B - F - R_1 - R_2 - R_3 - R_4 = 0$$

$$\Sigma M_B = 0: \ -3F + 3.5R_4 + 4R_3 + \left(5+\tfrac{4}{3}\right)R_2 + 7R_1 + \tfrac{4}{3}R_5 - 6A_y = 0$$

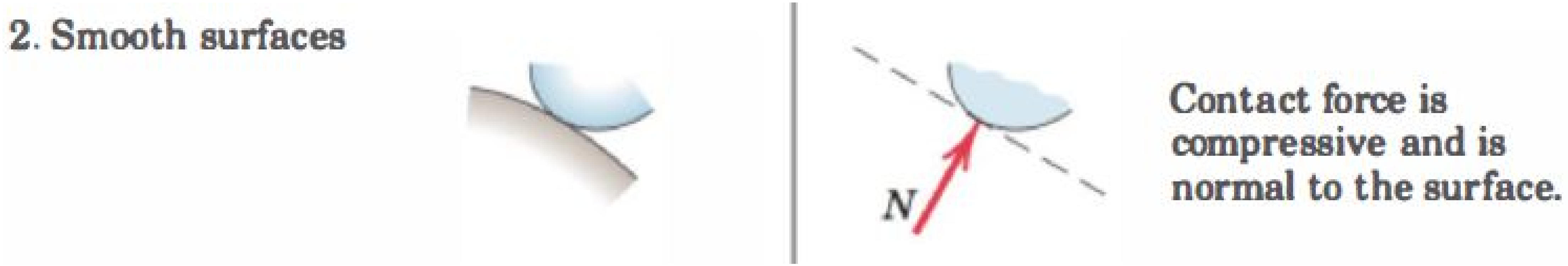
$$A_y = N_B = 18.18 \text{ kN} \qquad \underline{F = 10.36 \text{ kN}}$$

ENGINEERING MECHANICS

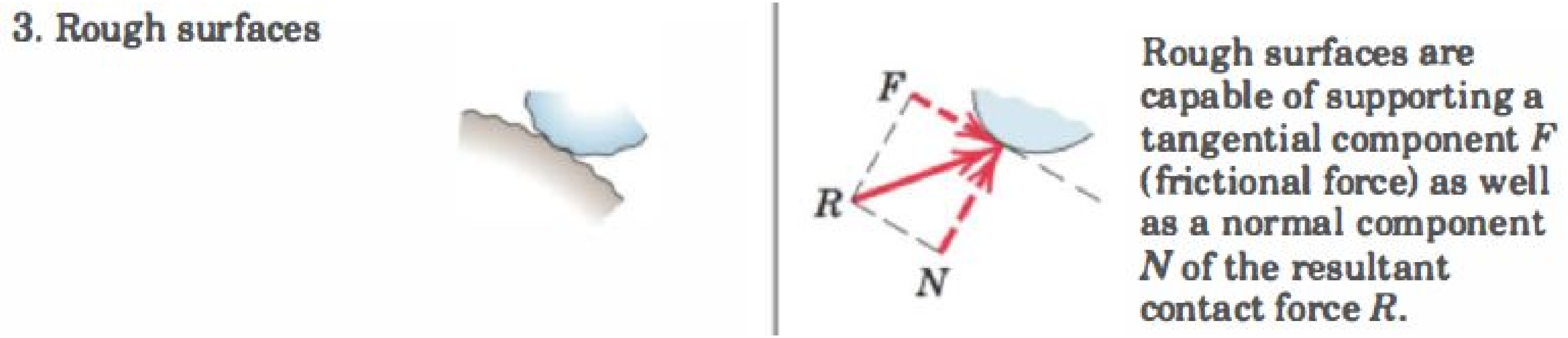
Friction

Department of Civil Engineering

- In the previous units 1 to 4, we have usually assumed that the forces of action and reaction between contacting surfaces act normal to the surfaces.
- This assumption characterizes the interaction between smooth surfaces and is as illustrated in Example below.
- This assumption yields a small error.



- But there are many cases where we must consider the ability of contacting surfaces to support tangential as well as normal forces.
- Tangential forces generated between contacting surfaces are called friction forces and occur to some degree in the interaction between all real surfaces.
- When surfaces of two solids are in contact under a condition of sliding or a tendency to slide, Friction occurs.



- Whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency.

- In some types of machines and processes we want to minimize the retarding effect of friction forces.

- *Examples are :*

- *bearings of all types, power screws, gears, the flow of fluids in pipes, and the propulsion of aircraft and missiles through the atmosphere.*

- In other situations we wish to *maximize the effects of friction, such as in brakes, clutches, belt drives, and wedges.*

- *Wheeled vehicles depend on friction for both starting and stopping.*

- *ordinary walking depends on friction between the shoe and the ground.*

- *In all cases where there is sliding motion between parts, the friction forces result in a loss of energy which is dissipated in the form of heat.*

- *Wear is another effect of friction.*

- Types of friction: *Dry Friction:*

- *Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide.*

- *A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place.*

- *The direction of this friction force always opposes the motion or impending motion.*

- *This type of friction is also called Coulomb friction.*

- Types of friction: *Fluid Friction:*

- *Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities.*

- *This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers.*

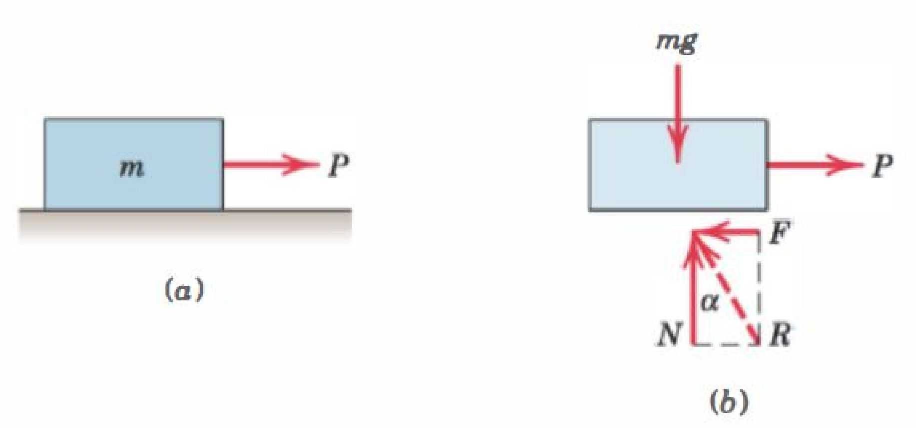
- *Internal Friction: Internal friction occurs in all solid materials which are subjected to cyclical loading.*

- *For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction.*

- Mechanism of Dry Friction:
- Consider a solid block of *mass m* resting on a horizontal surface, as shown in Fig. a below.

- We assume that the contacting surfaces have some roughness.

- The experiment involves the application of a horizontal force P which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity.

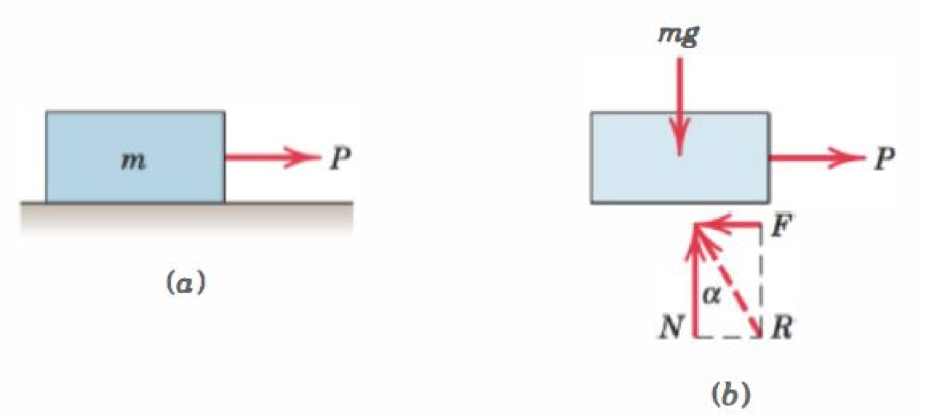


Mechanism of Dry Friction

- The free-body diagram of the block for any value of P is shown in Fig. b, where the tangential friction force exerted by the plane on the block is labeled F.

- This friction force acting on the body will always be in a direction to oppose motion or the tendency toward motion of the body.

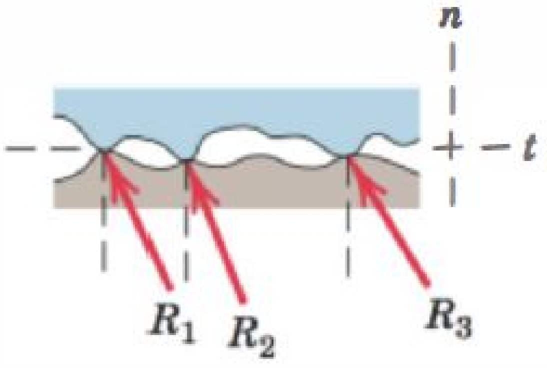
- There is also a normal force N which in this case equals mg, and the total force R exerted by the supporting surface on the block is the resultant of Nand F.



- A magnified view of the irregularities of the mating surfaces as in figure below , helps us to visualize the mechanical action of friction.

- Support is necessarily intermittent and exists at the mating humps.

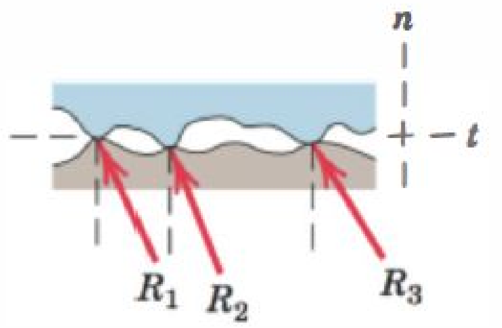
- The direction of each of the reactions on the block, R1, R2, R3, etc., depends not only on the geometric profile of the irregularities but also on the extent of local deformation at each contact point.



- The total normal force N is the sum of the n-components of the R's, and the total frictional force F is the sum of the t-components of the R’s.

- When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the R's are smaller than when the surfaces are at rest relative to one another.

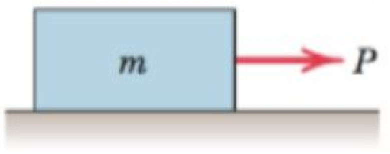
- This observation helps to explain the well known fact that the force P necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.



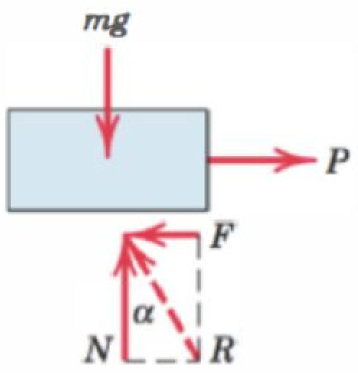
- If we perform the experiment and record the friction force F as a function of P, we obtain the relation shown in Fig. d.

- When P is zero, equilibrium requires that there be no friction force.

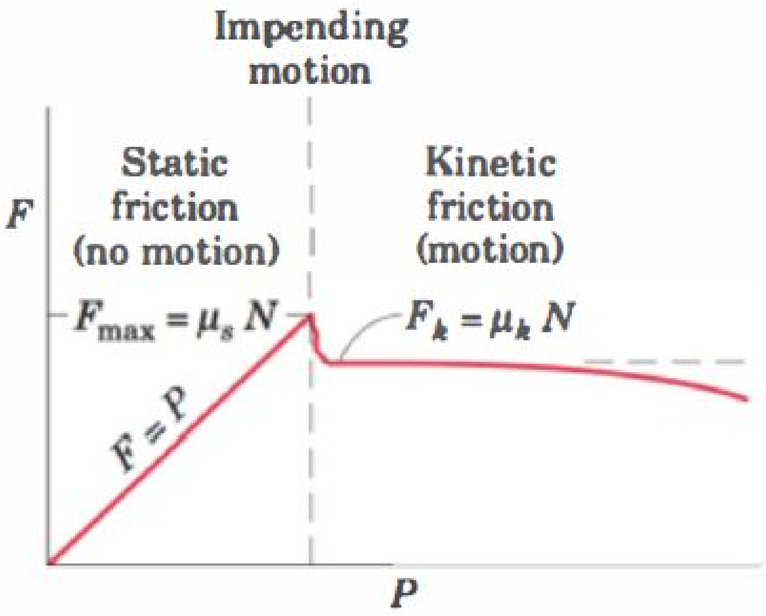
- As P is increased, the friction force must be equal and opposite to P as long as the block does not slip.



(a)



(b)



(d)

- During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations.

- Finally, we reach a value of P which causes the block to slip and to move in the direction of the applied force.

- At this same time the friction force decreases slightly and abruptly.

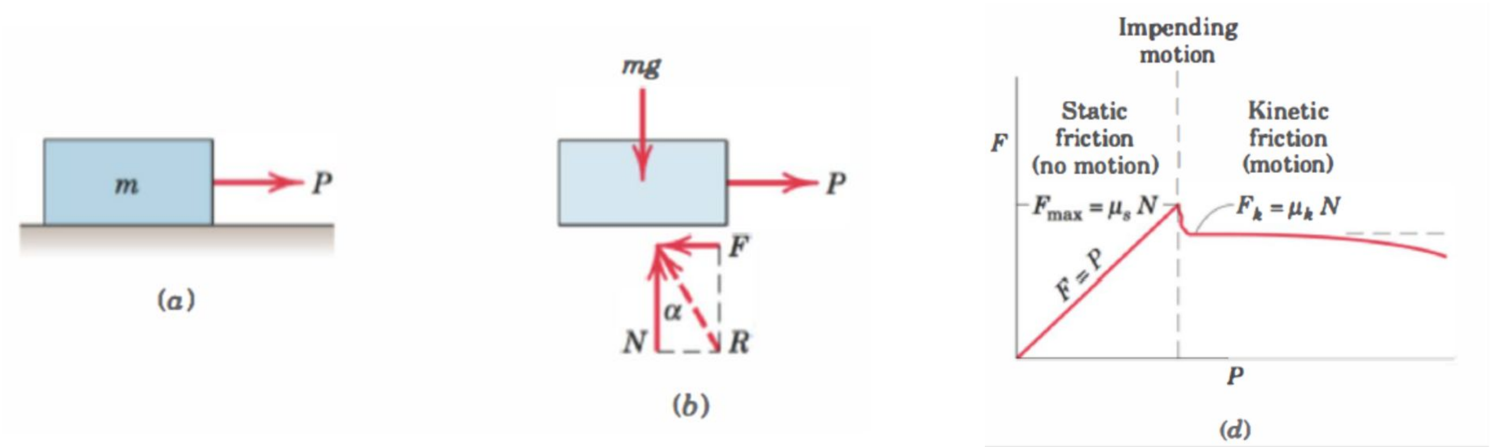
Static Friction:

- *The region in Fig. d up to the point of slippage or impending motion is called the range of static friction, and in this range the value of the friction force is determined by the equations of equilibrium.*

- *This friction force may have any value from zero up to and including the maximum value.*

- *For a given pair of mating surfaces the experiment shows that this maximum value of static friction F max is proportional to the normal force N. Thus, we may write*

$$\boxed{F_{max} = \mu_s N}$$



- Static Friction: $$\boxed{F_{max} = \mu_s N}$$

where μs is the proportionality constant, called the coefficient of static friction.

- This equation describes only the limiting or maximum value of the static friction force and not any lesser value.

- Thus, the equation applies only to cases where motion is impending with the friction force at its peak value.

- For a condition of static equilibrium when motion is not impending, the static friction force is

$$F < \mu_s N$$

- Kinetic Friction:
  - After slippage occurs, a condition of kinetic friction accompanies the ensuing motion.
  - Kinetic friction force is usually somewhat less than the maximum static friction force.
  - The kinetic friction force Fk is also proportional to the normal force. Thus,

$$\boxed{F_k = \mu_k N}$$

- Kinetic Friction:
  - Where μk is the coefficient of kinetic friction.
  - μk is generally less than μs

- *Friction Angles:*

  - The direction of the resultant R in Fig. b measured from the direction of N is specified by tan α = F / N.

  - When the friction force reaches its limiting static value F max, the angle α reaches a maximum value $\emptyset s$
  - Thus,

$$\tan \phi_s = \mu_s$$

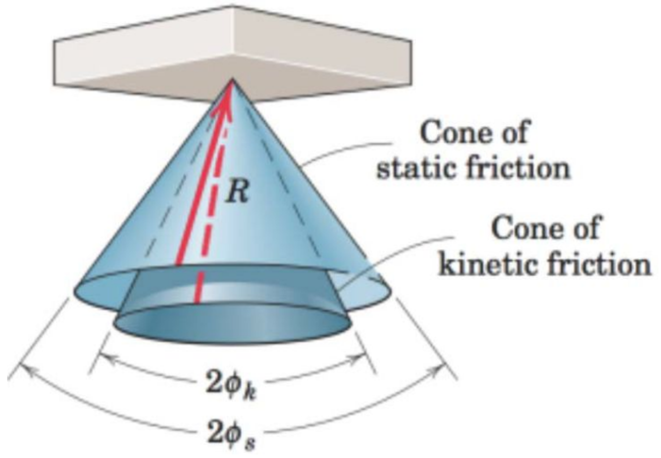
When slippage is occurring, the angle a has a value $\emptyset k$ corresponding to the kinetic friction force. In like manner,

$$\tan \phi_k = \mu_k$$

In practice we often see the expression $\tan \emptyset = \mu$, in which the coefficient of friction may refer to either the static or the kinetic case, depending on the particular problem.

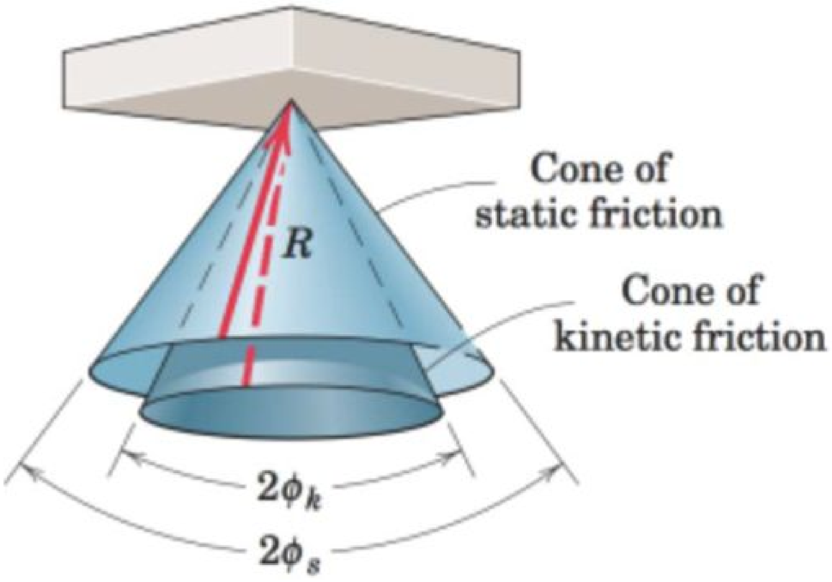
The angle $\emptyset s$ is called the *angle of static friction*, and the angle Øk is called the *angle of kinetic friction*.

The friction angle for each case clearly defines the limiting direction of the total reaction R between two contacting surfaces.



If motion is impending, R must be one element of a right-circular cone of vertex angle $2\emptyset s$, as shown in Fig. below. If motion is not impending, R is within the cone.

This cone of vertex angle $2\emptyset s$ is *called the cone of static friction* and represents the locus of possible directions for the reaction R at impending motion.



- If motion occurs, the angle of kinetic friction applies, and the reaction must lie on the surface of a slightly different cone of vertex angle $2\emptyset k$•

- *This cone is the cone of kinetic friction.*

- If motion occurs, the angle of kinetic friction applies, and the reaction must lie on the surface of a slightly different cone of vertex angle $2\emptyset k$•

- *This cone is the cone of kinetic friction.*

Types of Friction Problems

We can now recognize the following three types of problems encountered in applications involving dry friction. The first step in solving a friction problem is to identify its type.

1. In the *first* type of problem, the condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping, and the friction force equals the limiting static friction $F_{max} = \mu_s N$. The equations of equilibrium will, of course, also hold.

Types of friction problems

2. In the *second* type of problem, neither the condition of impending motion nor the condition of motion is known to exist. To determine the actual friction conditions, we first assume static equilibrium and then solve for the friction force F necessary for equilibrium. Three outcomes are possible:

(a) $F < (F_{max} = \mu_s N)$: Here the friction force necessary for equilibrium can be supported, and therefore the body is in static equilibrium as assumed. We emphasize that the *actual* friction force F is *less than* the limiting value F_{max} given by Eq. 6/1 and that F is determined *solely* by the equations of equilibrium.

Types of friction problems

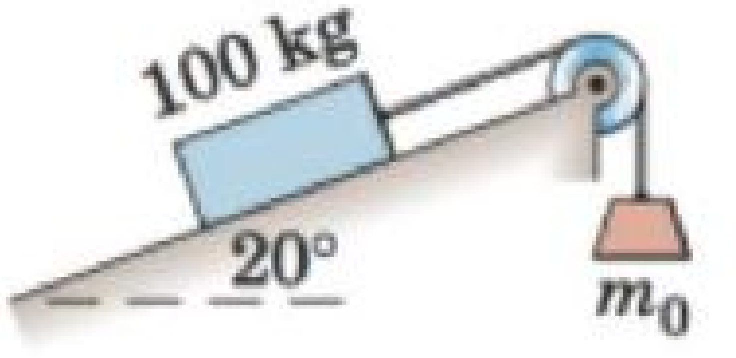
(*b*) $F = (F_{max} = \mu_s N)$: Since the friction force F is at its maximum value F_{max}, motion impends, as discussed in problem type (1). The assumption of static equilibrium is valid.

(*c*) $F > (F_{max} = \mu_s N)$: Clearly this condition is impossible, because the surfaces cannot support more force than the maximum $\mu_s N$. The assumption of equilibrium is therefore invalid, and motion occurs. The friction force F is equal to $\mu_k N$ from Eq. 6/2.

3. In the *third* type of problem, relative motion is known to exist between the contacting surfaces, and thus the kinetic coefficient of friction clearly applies. For this problem type, Eq. 6/2 always gives the kinetic friction force directly.

Sample Problem 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



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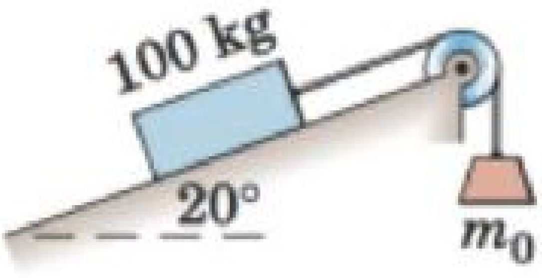
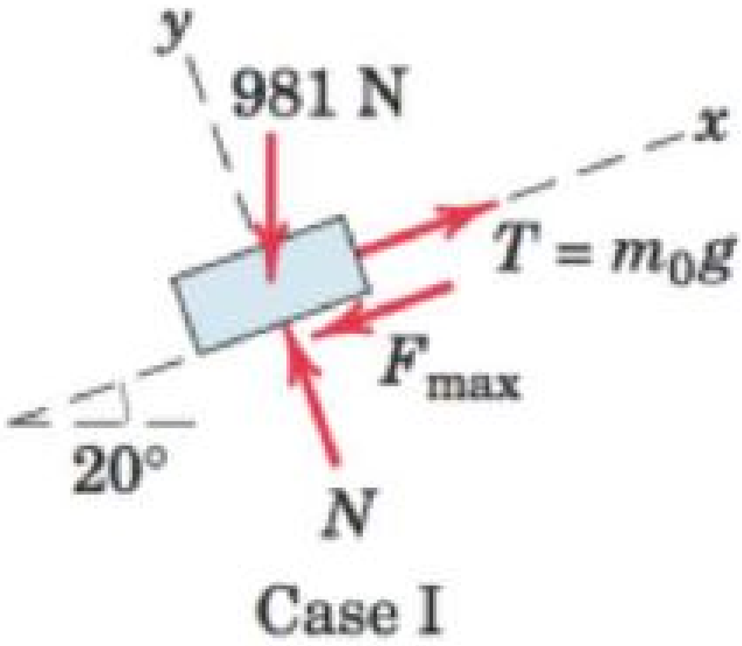
Solution. The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give





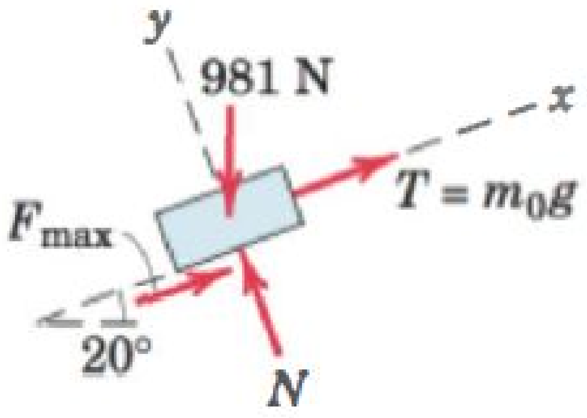
$[\Sigma F_y = 0]$ $\quad N - 981 \cos 20° = 0 \qquad N = 922\text{ N}$

$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.30(922) = 277\text{ N}$

$[\Sigma F_x = 0]$ $\quad m_0(9.81) - 277 - 981 \sin 20° = 0 \qquad m_0 = 62.4\text{ kg}$

The minimum value of m_0 is determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the x-direction requires

$$[\Sigma F_x = 0] \qquad m_0(9.81) + 277 - 981 \sin 20° = 0 \qquad m_0 = 6.01 \text{ kg}$$

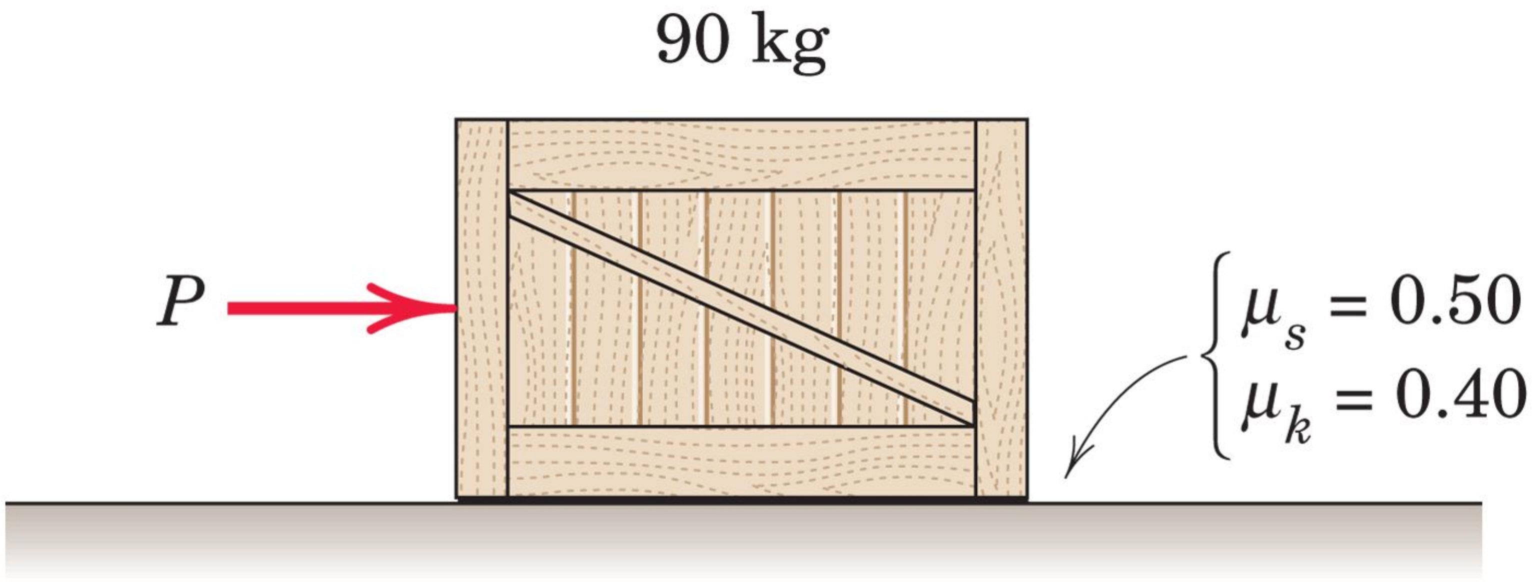


Case II

Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

In both cases equilibrium requires that the resultant of F_{max} and N be concurrent with the 981-N weight and the tension T.

6/1 The force P is applied to the 90-kg crate, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the crate if (a) P = 300 N, (b) P = 400 N, and (c) P = 500 N.



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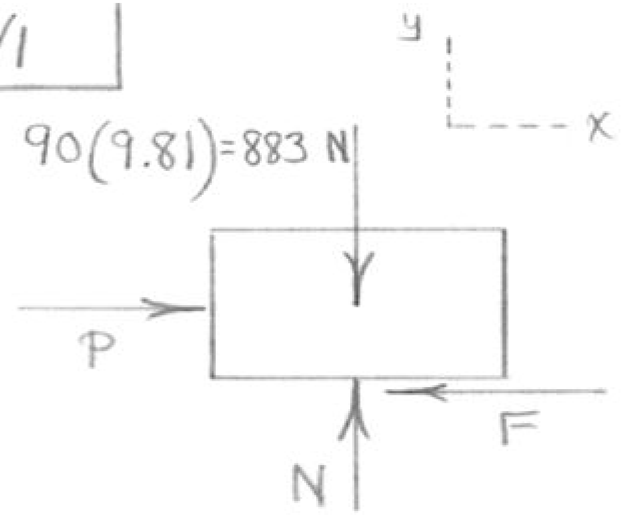


$\Sigma F_y = 0: \quad N - 883 = 0, \quad N = 883 \text{ N}$

$F_{max} = \mu_s N = 0.5(883) = 441 \text{ N}$

$\Sigma F_x = 0$ yields $F = P$ for equilibrium

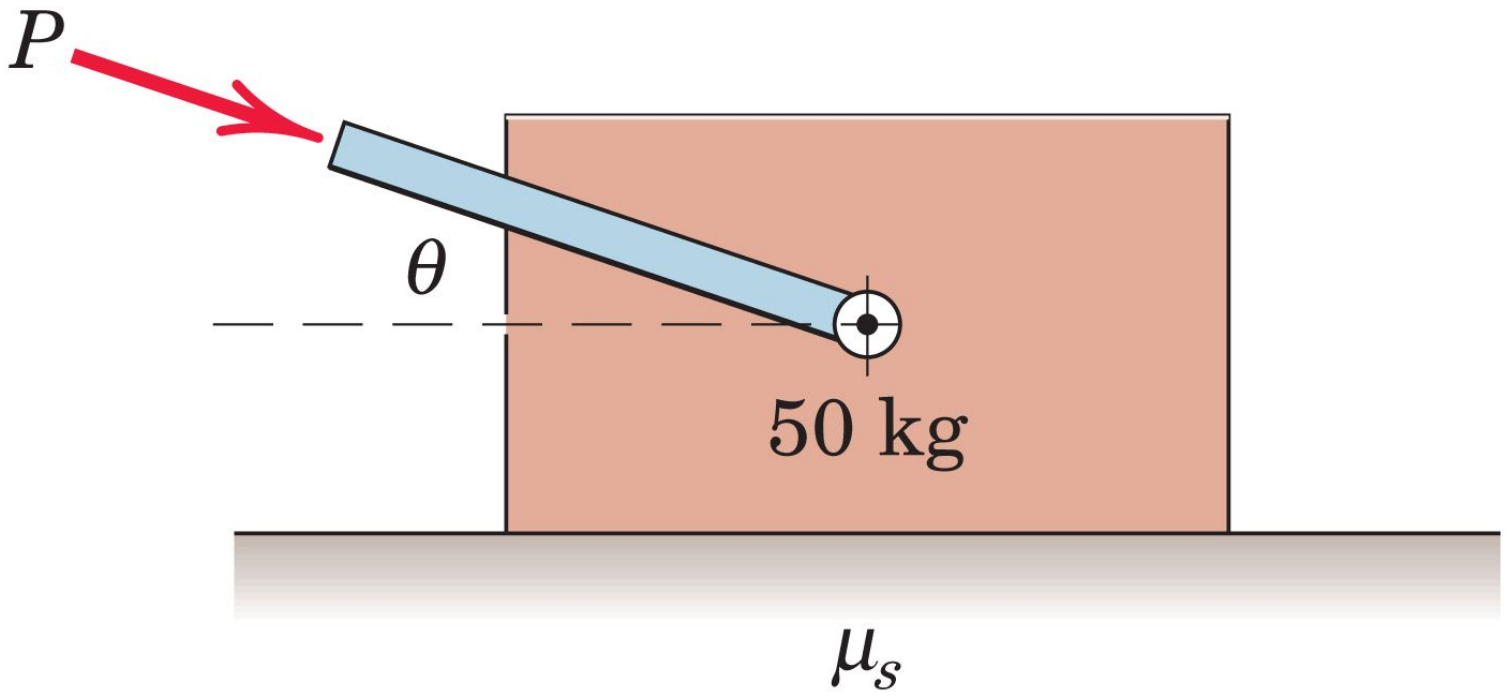
(a) $P = 300 \text{ N}$, $\underline{F = 300 \text{ N}} < F_{max}$, OK

(b) $P = 400 \text{ N}$, $\underline{F = 400 \text{ N}} < F_{max}$, OK

(c) $P = 500 \text{ N}$, $F = 500 \text{ N} > F_{max}$, motion

So $F = \mu_k N = 0.4(883) = \underline{353 \text{ N}}$

6/2 The 50-kg block rests on the horizontal surface, and a force P = 200 N, whose direction can be varied, is applied to the block. (a) If the block begins to slip when θ is reduced to 30^0, calculate the coefficient of static friction μs. between the block and the surface. (b) If P is applied with θ = 45°, calculate the friction force F .



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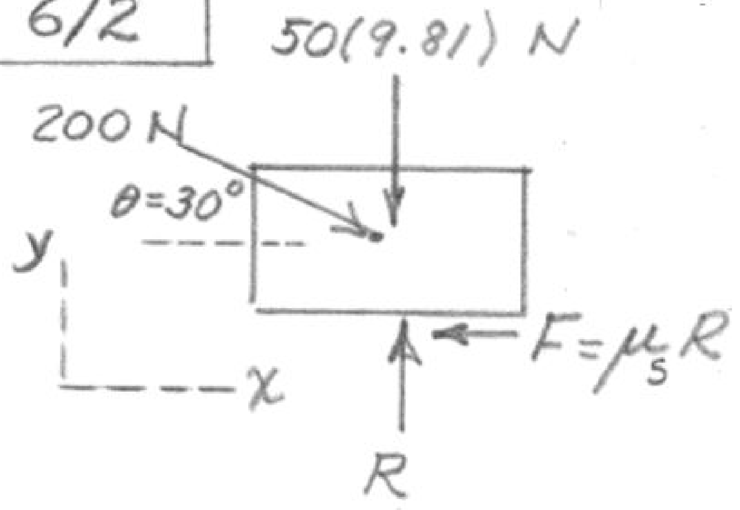


(a) $\Sigma F_x = 0;\ 200 \cos 30° - \mu_s R = 0$

$\Sigma F_y = 0;\ R - 200 \sin 30° - 50(9.81) = 0$

$R = 590.5\ N$

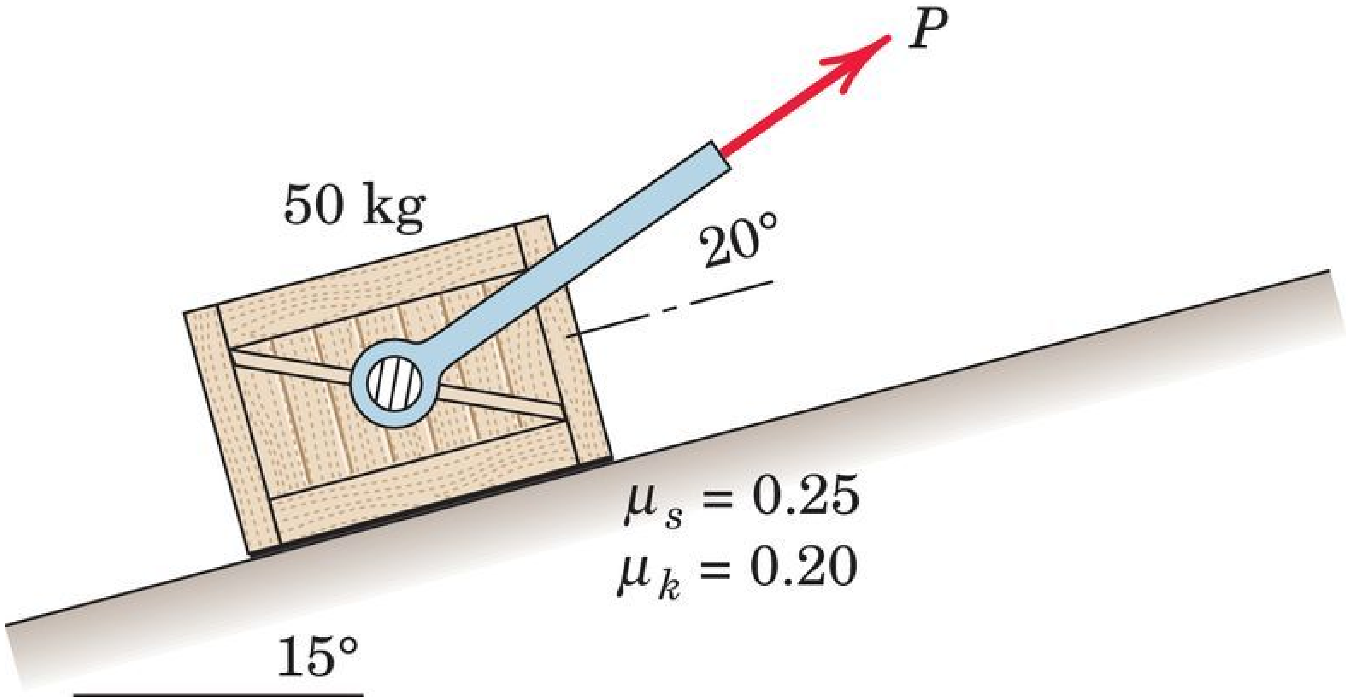
So $\mu_s = \dfrac{200 \cos 30°}{590.5} = \underline{0.293}$

(b)

For $\theta = 45°$, $\Sigma F_x = 0$ gives $F = 200 \cos 45° = \underline{141.4\ N}$

which is $< \mu_s R_b$

6/3 The force P is applied to the 50-kg block when it is at rest. Determine the magnitude and direction of the friction force exerted by the surface on the block if (a) P = 0, (b) P = 200 N, and (c) P = 250 N. (d) What value of P is required to initiate motion up the incline? The coefficients of static and kinetic friction between the block and the incline are μs = 0.25 and μk = 0.20, respectively.



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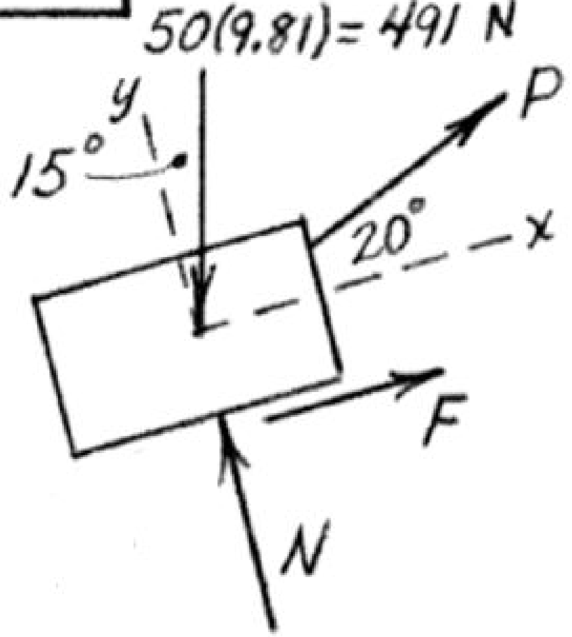


(a) $P = 0$

$\Sigma F_y = 0$: $N - 491 \cos 15° = 0$, $N = 474$ N

Assume equilibrium:

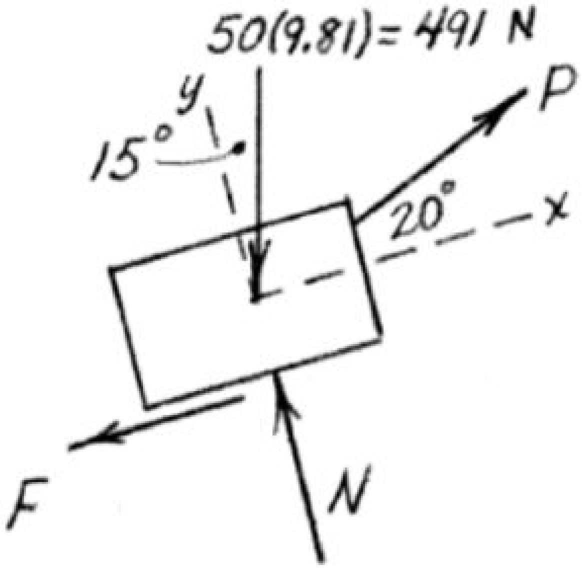
$\Sigma F_x = 0$: $F - 491 \sin 15° = 0$, $F = 127.0$ N

$F_{max} = \mu_s N = 0.25(474) = 118.4 \text{ N} < F$;

assumption invalid and

$F = F_k = \mu_k N = 0.2(474) = \underline{94.8 \text{ N}}$ up the incline

Case b   when P = 200 N

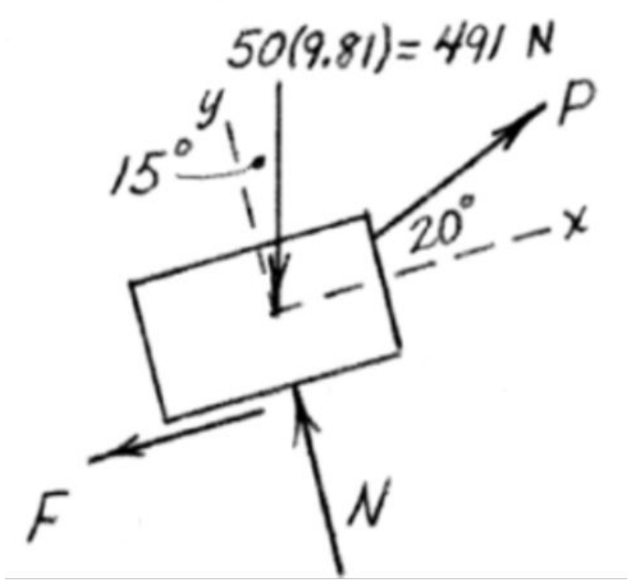


(b) $P = 200$ N; assume equilibrium

$\Sigma F_y = 0: \; N - 491\cos 15° + 200\sin 20° = 0, \quad N = 405$ N

$\Sigma F_x = 0: \; 200\cos 20° - 491\sin 15° - F = 0, \quad \underline{F = 61.0 \text{ N}}$

$F_{max} = \mu_s N = 0.25(405) = 101.3$ N > 61.0 N so assumption OK



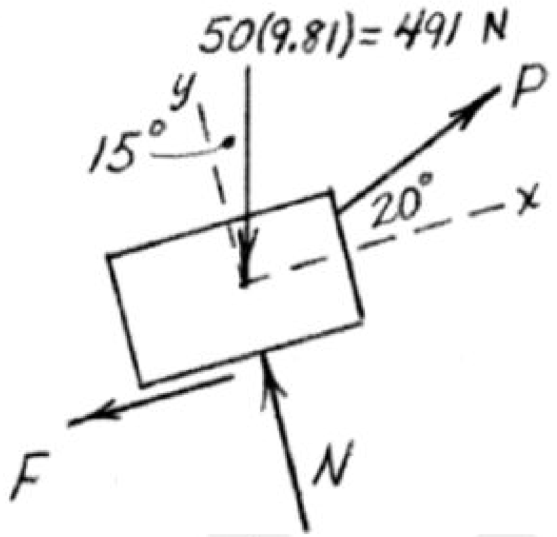
(c) $P = 250$ N; assume equilibrium

$\Sigma F_y = 0: \; N - 491\cos 15° + 250\sin 20° = 0, \; N = 388$ N

$\Sigma F_x = 0: \; 250\cos 20° - 491\sin 15° - F = 0, \; F = 108.0$ N

$F_{max} = \mu_s N = 0.25(388) = 97.1$ N $< F$; assumption invalid

$F = \mu_k N = 0.2(388) = \underline{77.7 \text{ N}}$ down the incline



(d) To initiate motion, set $F = \mu_s N = 0.25N$ down the incline:

$$\Sigma F_y = 0: \quad N - 491\cos 15° + P\sin 20° = 0$$

$$\Sigma F_x = 0: \quad P\cos 20° - 491\sin 15° - 0.25N = 0$$

Solve to obtain $\begin{cases} N = 392 \text{ N} \\ \underline{P = 239 \text{ N}} \end{cases}$

6/4 The designer of a ski resort wishes to have a portion of a beginner's slope on which a snowboarder's speed will remain fairly constant. Tests indicate the average coefficients of friction between a snowboard and snow to be μs = 0.11 and μk = 0.09. What should be the slope angle θ of the constant-speed section?

6/4

$$\begin{cases} \Sigma F_x = 0: \mu_k N - mg\sin\theta = 0 \\ \Sigma F_y = 0: N - mg\cos\theta = 0 \end{cases}$$

$$\Rightarrow \quad N = mg\cos\theta$$

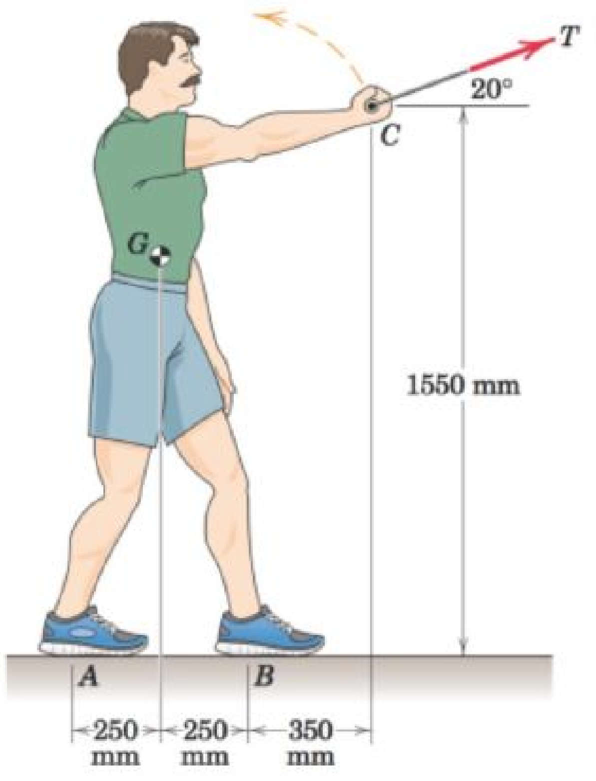
and $\mu_k mg\cos\theta = mg\sin\theta$

$$\tan\theta = \mu_k$$

$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.09)$$

$$= 5.14^\circ$$

6/5 The 80-kg exerciser is repeated from Prob. 3/23. The tension T = 65 N is developed against an exercise machine (not shown) as he is about to begin a biceps curl. Determine the minimum coefficient of static friction which must exist between his shoes and the floor if he is not to slip.



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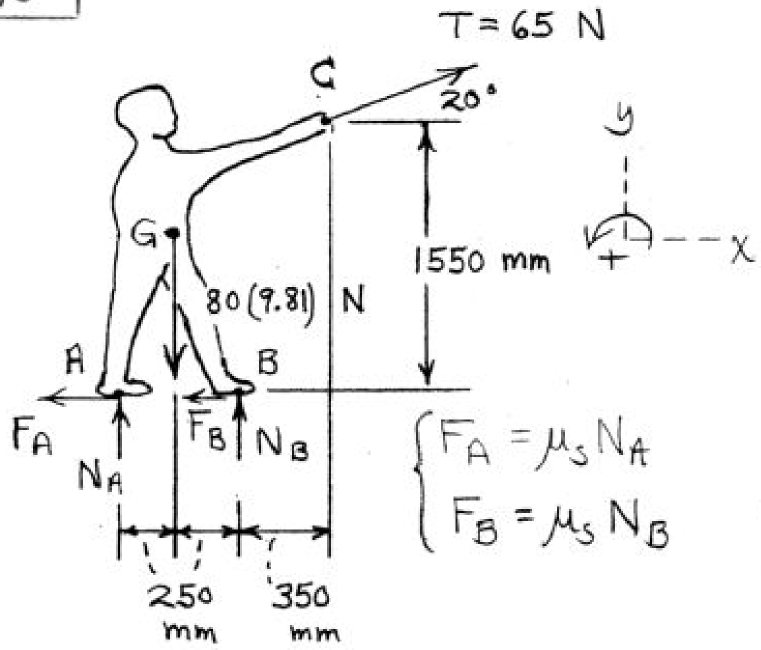


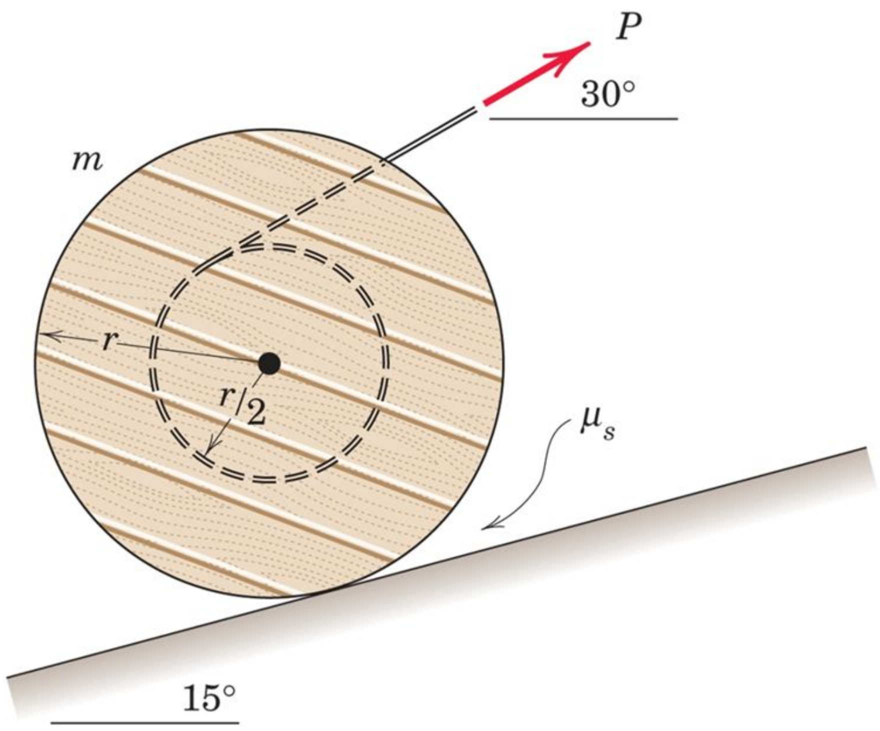
$$\begin{cases} F_A = \mu_s N_A \\ F_B = \mu_s N_B \end{cases}$$

$$\begin{cases} \Sigma F_x = 0 : \ -\mu_s (N_A + N_B) + 65 \cos 20^\circ = 0 \\ \Sigma F_y = 0 : \ N_A + N_B - 80(9.81) + 65 \sin 20^\circ = 0 \\ \Sigma M_B = 0 : \ 80(9.81)(250) - N_A(500) - 65[1550 \cos 20^\circ \\ \qquad - 350 \sin 20^\circ] = 0 \end{cases}$$

Solve to obtain $N_A = 219$ N, $N_B = 544$ N

$\underline{\mu_s = 0.0801}$

6/6 Determine the minimum coefficient of static friction μs. which will allow the drum with fixed inner hub to be rolled up the 15° incline at a steady speed without slipping. What are the corresponding values of the force P and the friction force F?



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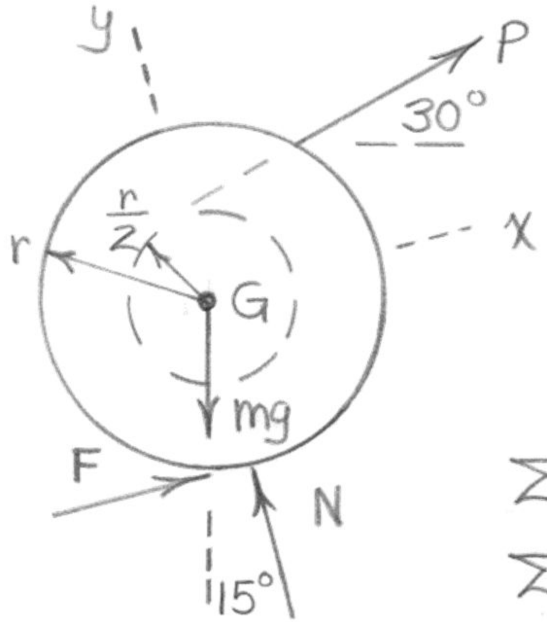
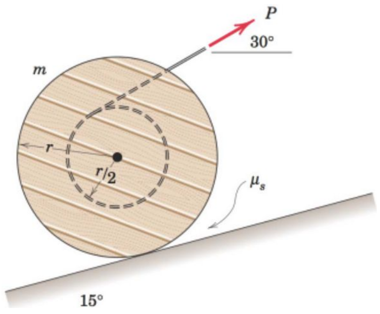




$$\Sigma F_x = 0: \quad P\cos 15^\circ + F - mg\sin 15^\circ = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad P\sin 15^\circ - mg\cos 15^\circ + N = 0 \qquad (2)$$

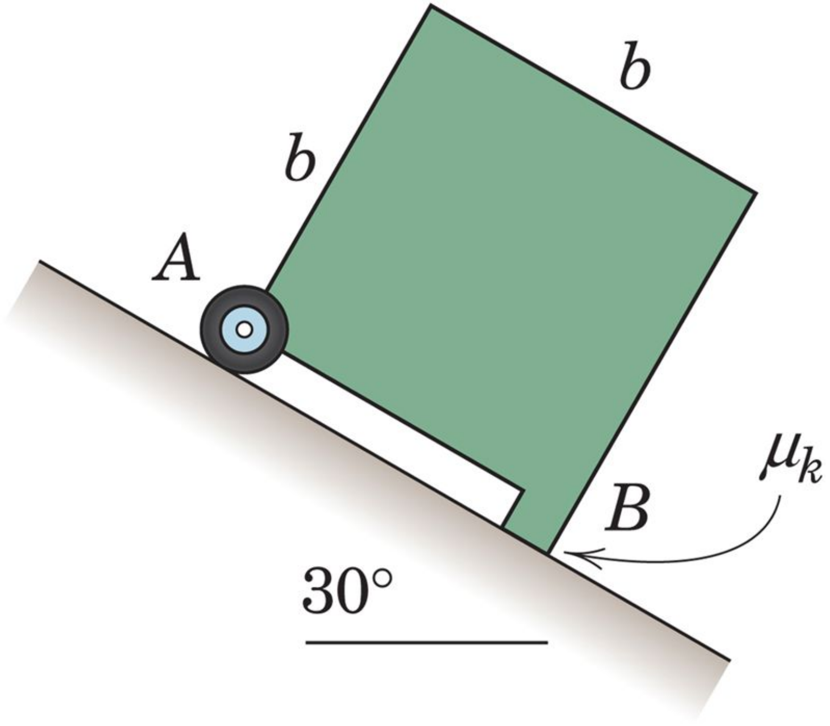
$$\circlearrowleft^{+} \Sigma M_G = 0: \quad Fr - P\left(\frac{r}{2}\right) = 0 \qquad (3)$$

Also, for impending slip: $F = \mu_s N$ (4)

Algebraically solve Eqs. (1)-(4) to obtain

$\underline{\mu_s = 0.0959}$, $N = 0.920mg$, $\underline{F = 0.0883mg}$, $\underline{P = 0.1766mg}$

6/8 Determine the coefficient μ_k of kinetic friction which allows the homogeneous body to move down the 30° incline at constant speed. Show that this constant speed motion is unlikely to occur if the ideal roller and small foot were reversed.



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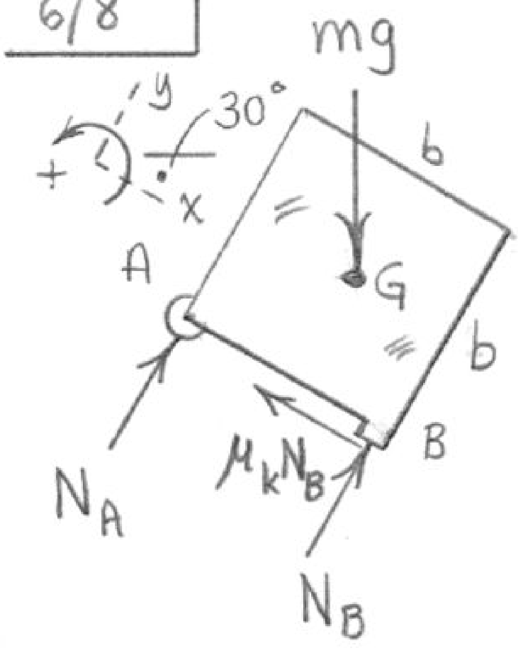


$\Sigma F_x = 0: -\mu_k N_B + mg \sin 30° = 0$

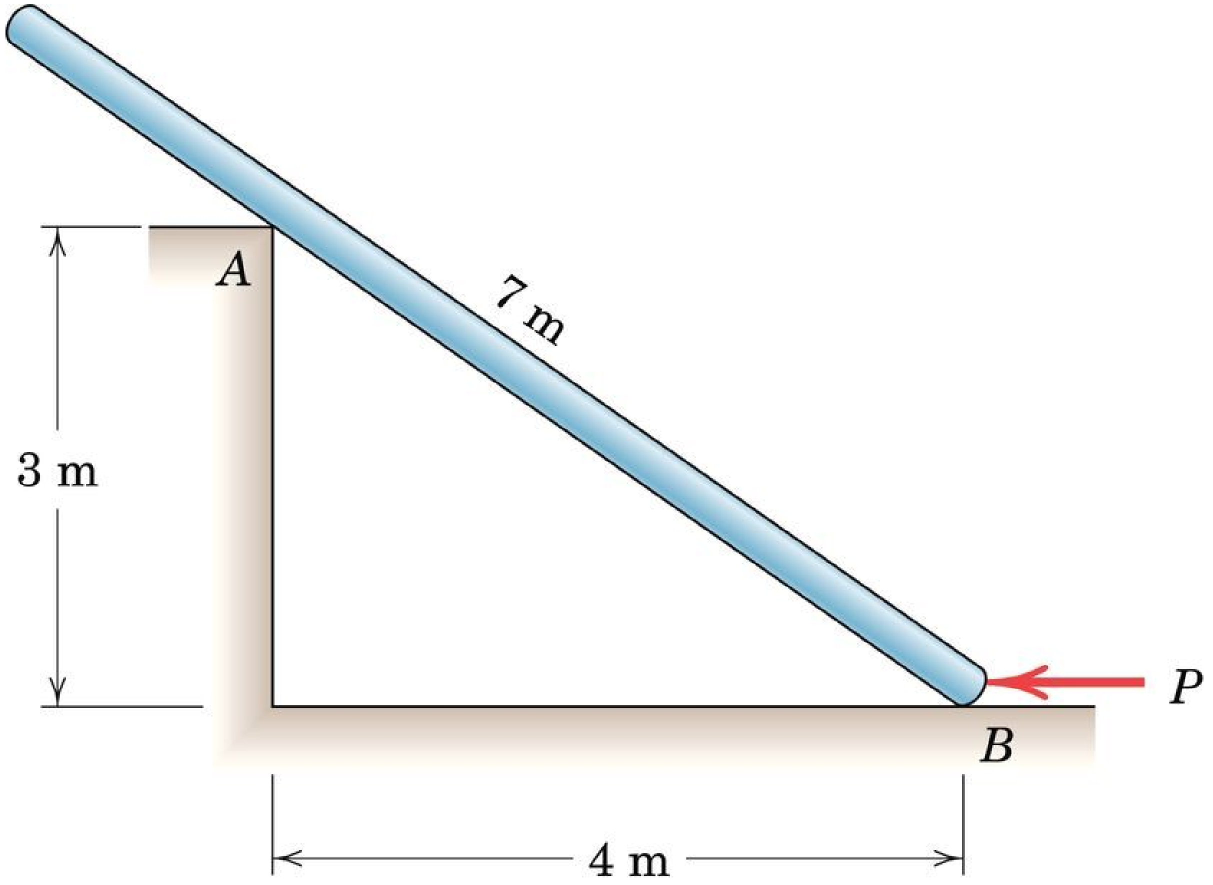
$\Sigma F_y = 0: N_A + N_B - mg \cos 30° = 0$

$\Sigma M_A = 0: N_B(b) - mg\left(\frac{b}{2}\cos 30° + \frac{b}{2}\sin 30°\right) = 0$

Solving : $\begin{cases} N_A = 0.1830\,mg \\ N_B = 0.683\,mg \\ \mu_k = 0.732 \end{cases}$

Reversing the roller and foot yields $\mu_k = 2.73$, an unlikelihood for simple contact.

6/9 The uniform 7-m pole has a mass of 100 kg and is supported as shown. Calculate the force P required to move the pole if the coefficient of static friction for each contact location is 0.40.



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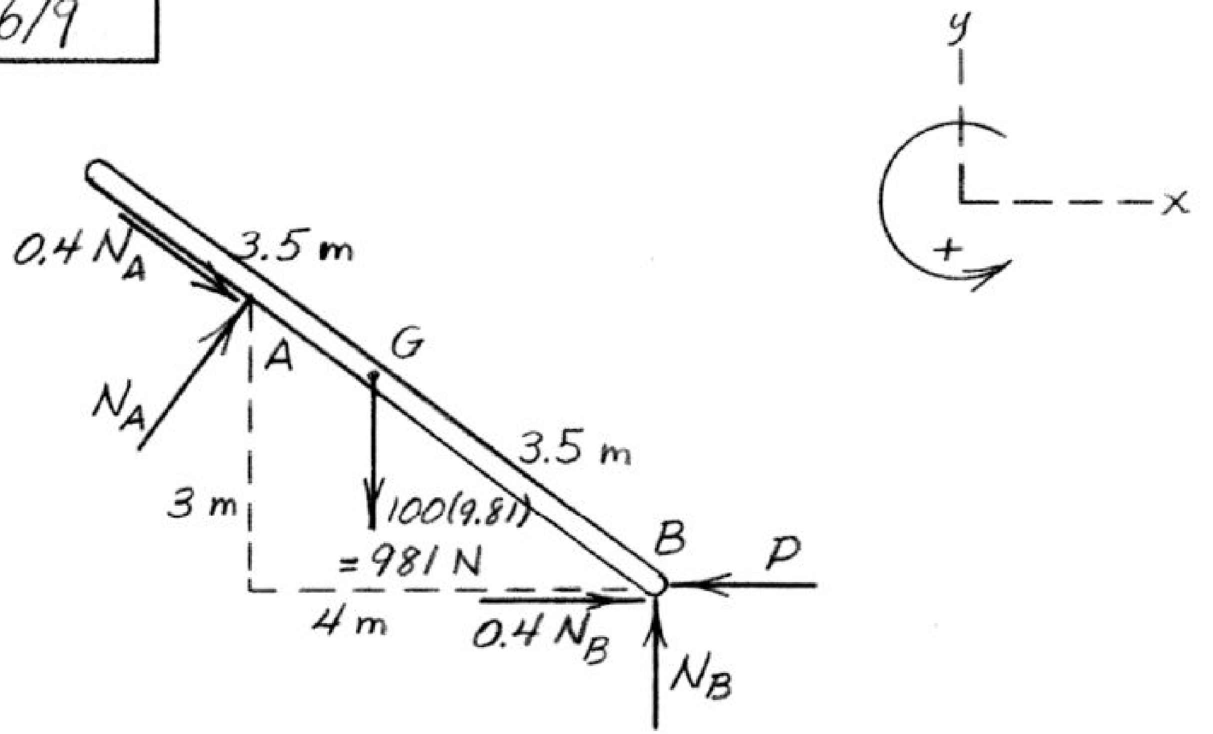
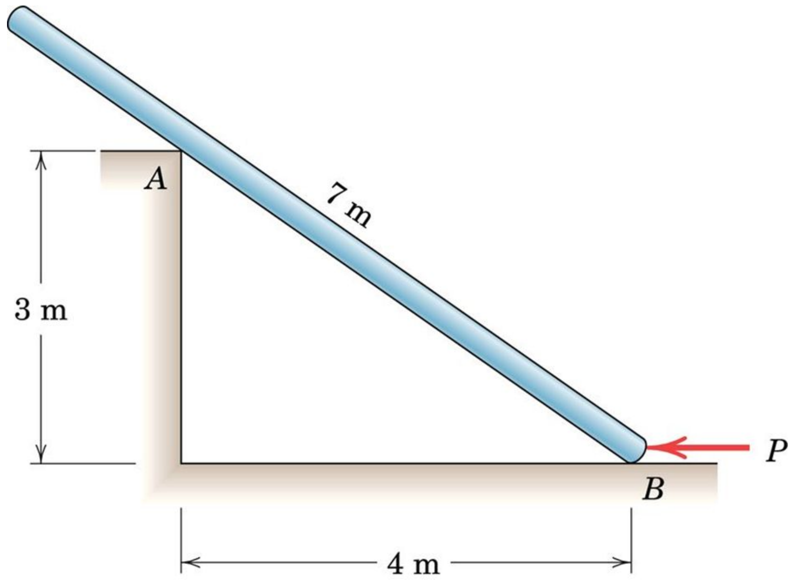




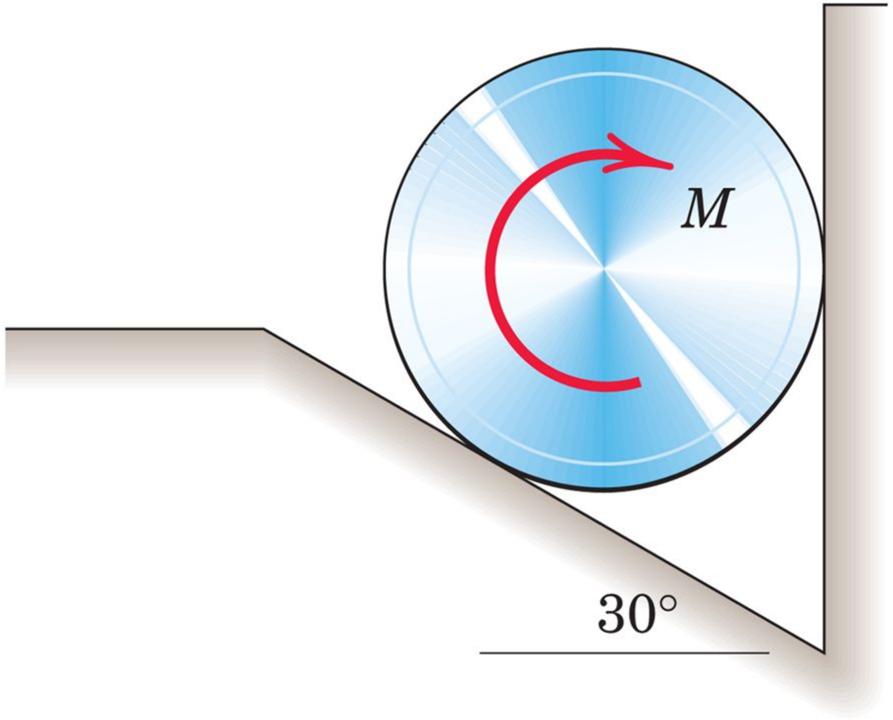
$$\Sigma M_B = 0: \quad 981\left(\tfrac{4}{5}\,3.5\right) - 5N_A = 0, \quad N_A = 549 \text{ N}$$

$$\Sigma F_y = 0: \quad N_B - 981 + \tfrac{4}{5}(549) - 0.4(549)\tfrac{3}{5} = 0, \quad N_B = 673 \text{ N}$$

$$\Sigma F_x = 0: \quad -P + 0.4(673) + 549\left(\tfrac{3}{5}\right) + 0.4(549)\tfrac{4}{5} = 0$$

$$\underline{P = 775 \text{ N}}$$

6/11 The 30-kg homogeneous cylinder of 400-mm diameter rests against the vertical and inclined surfaces as shown. If the coefficient of static friction between the cylinder and the surfaces is 0.30, calculate the applied clockwise couple M which would cause the cylinder to slip.



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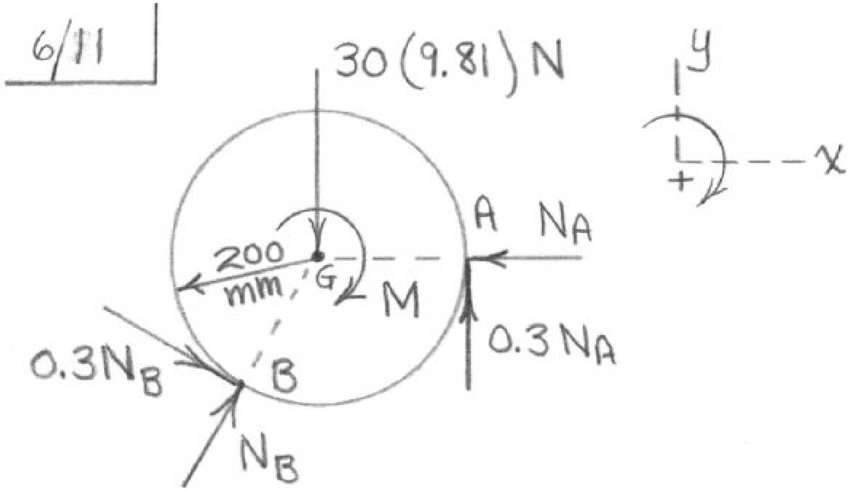


$$\Sigma M_G = 0: \quad M - 0.3(N_A + N_B)\,0.2 = 0 \qquad (1)$$

$$\Sigma F_x = 0: \quad N_B \sin 30^\circ + 0.3 N_B \cos 30^\circ - N_A = 0 \qquad (2)$$

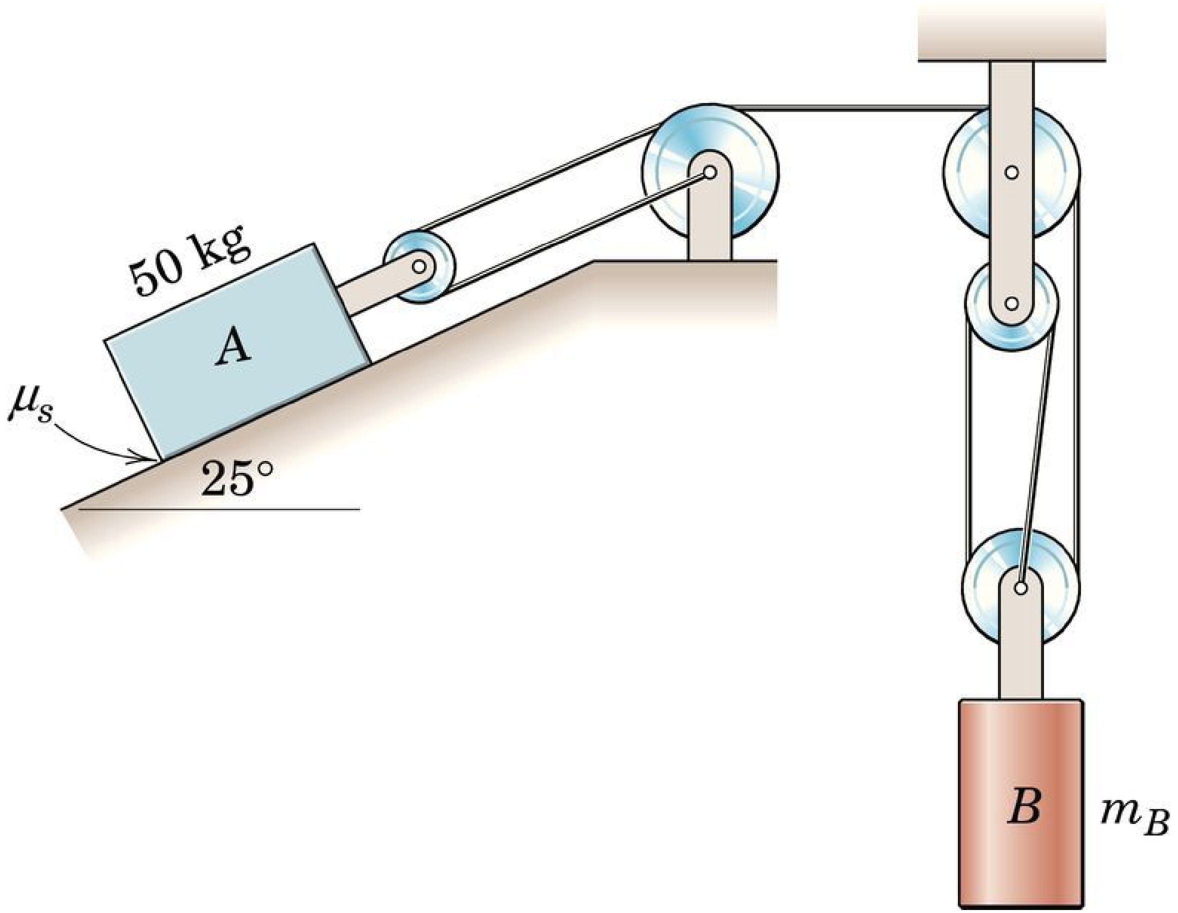
$$\Sigma F_y = 0: \quad N_B \cos 30^\circ - 0.3 N_B \sin 30^\circ - 30(9.81) + 0.3 N_A = 0 \qquad (3)$$

$N_B = 312$ N

$N_A = 237$ N

$M = 32.9$ N·m

6/12 If the coefficient of static friction between block A and the incline is µs = 0.30, determine the range of cylinder masses mB for which the system will remain in equilibrium. Neglect all pulley friction



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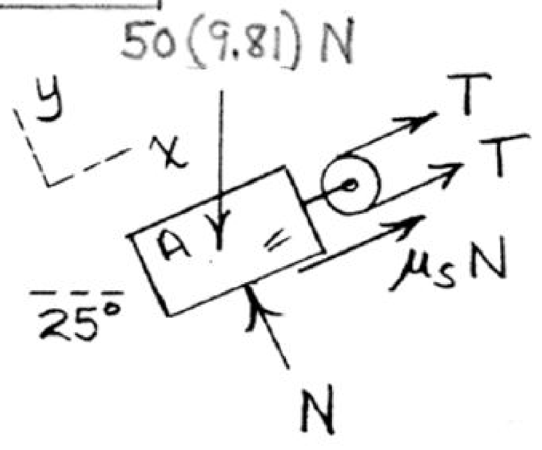


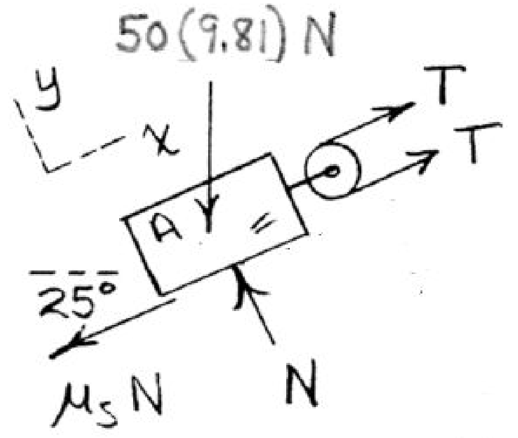
(a) Motion impends down incline

$\Sigma F_y = 0: \; N = 50(9.81)\cos 25°$

or $N = 445$ N throughout

$\Sigma F_x = 0: \; 2T - 50(9.81)\sin 25°$

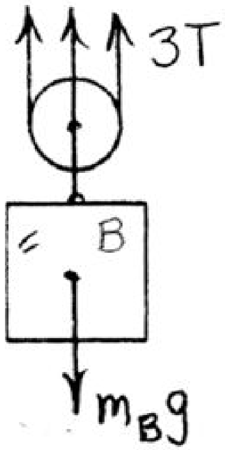
$+ 0.30(445) = 0, \quad T = 37.0$ N



(b) Motion impends up incline

$\Sigma F_x = 0: \; 2T - 50(9.81)\sin 25° - 0.30(445) = 0$

$T = 170.3$ N



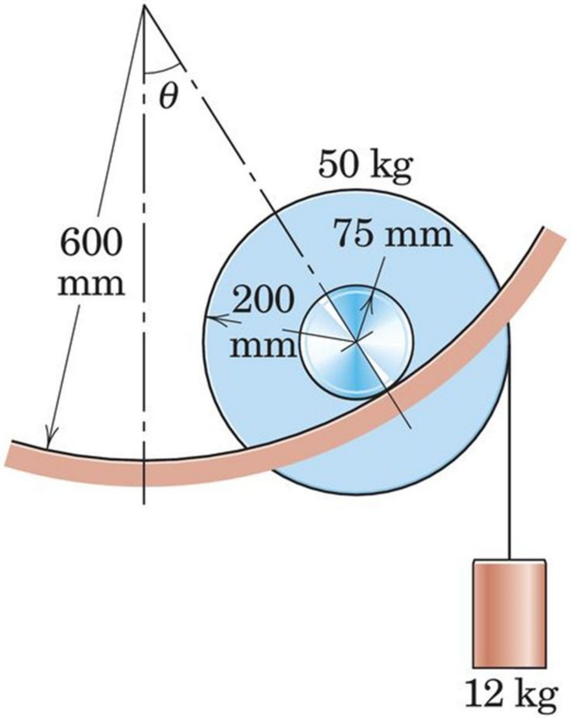
$$\uparrow \Sigma F = 0 \quad 3T - m_B g = 0, \quad m_B = \frac{3T}{g}$$

(a) $m_B = 3(37.0)/9.81 = 11.30$ kg

(b) $m_B = 3(170.3)/9.81 = 52.1$ kg

$11.30 \le m_B \le 52.1$ kg

6/13 The 50-kg wheel rolls on its hub up the circular incline under the action of the 12-kg cylinder attached to a cord around the rim. Determine the angle 0 at which the wheel comes to rest, assuming that friction is sufficient to prevent slippage. What is the minimum coefficient of static friction which will permit this position to be reached with no slipping?



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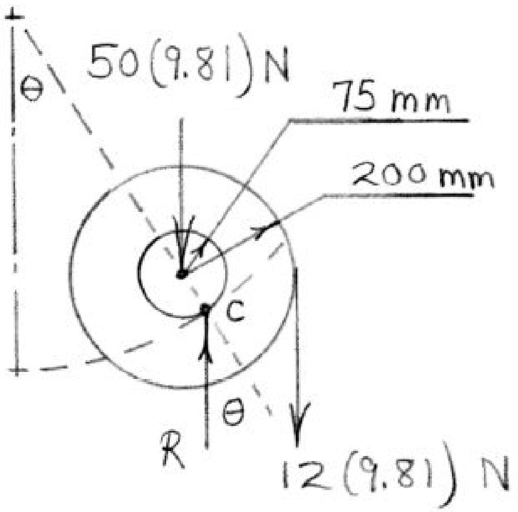
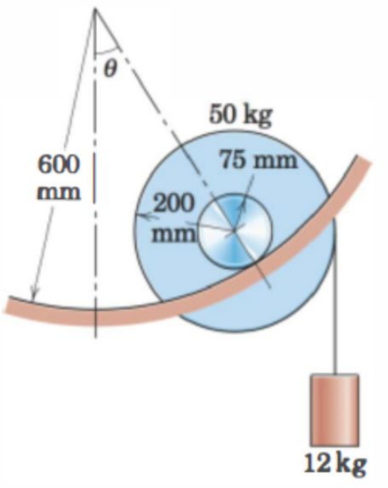


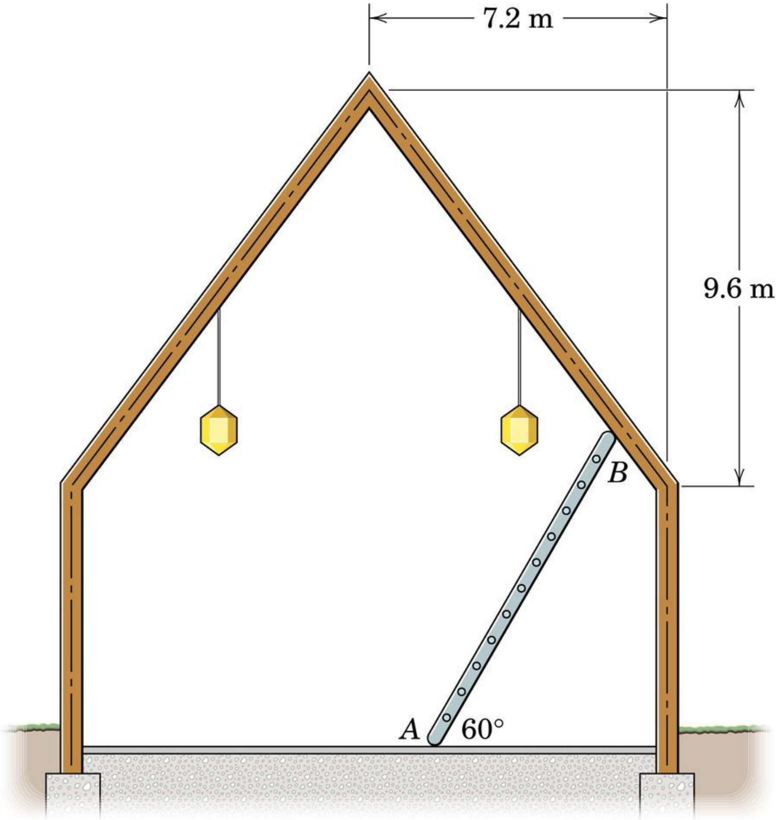


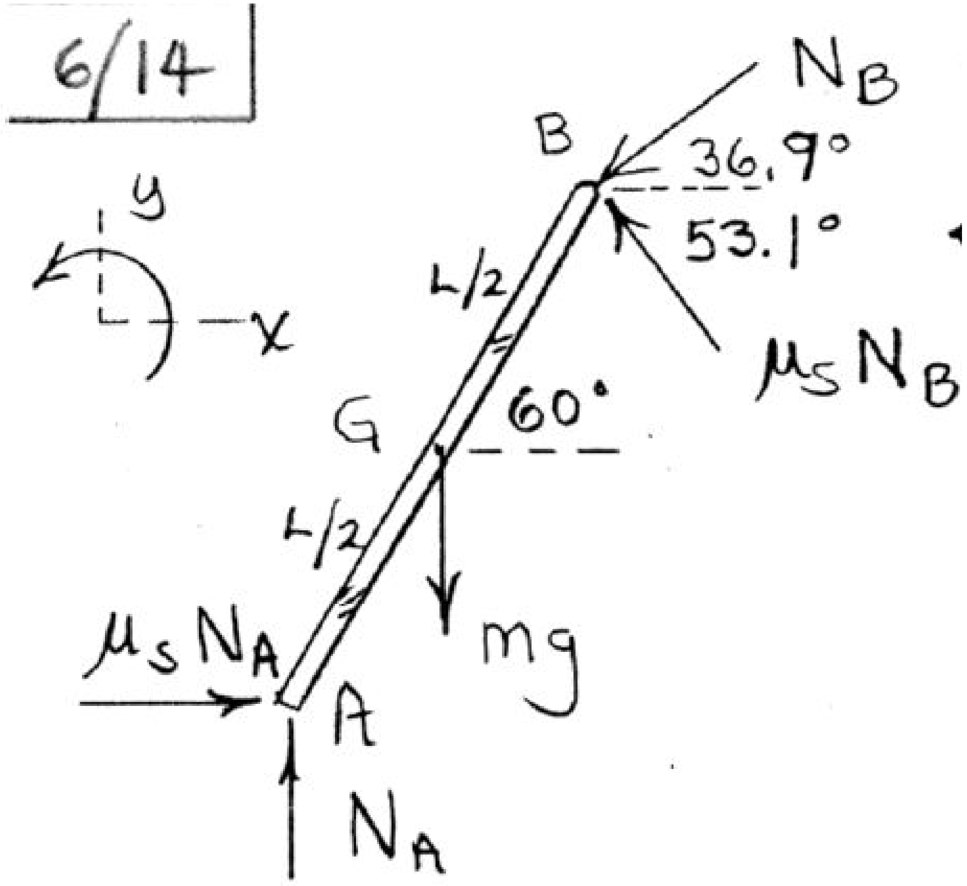
$$\circlearrowleft + \Sigma M_C = 0: \ 50(9.81)(75 \sin\theta) - 12(9.81)(200 - 75\sin\theta) = 0$$

$$\underline{\theta = 31.1^\circ}$$

$$\mu_{min} = \tan\theta = \tan 31.1^\circ = \underline{0.603}$$

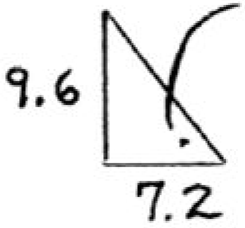
6/14 A uniform ladder is positioned as shown for the purpose of maintaining the light fixture suspended from the cathedral ceiling. Determine the minimum coefficient of static friction required at ends A and B to prevent slipping. Assume that the coefficient is the same at A and B.





(1) $F = \mu_s N$ at both A and B

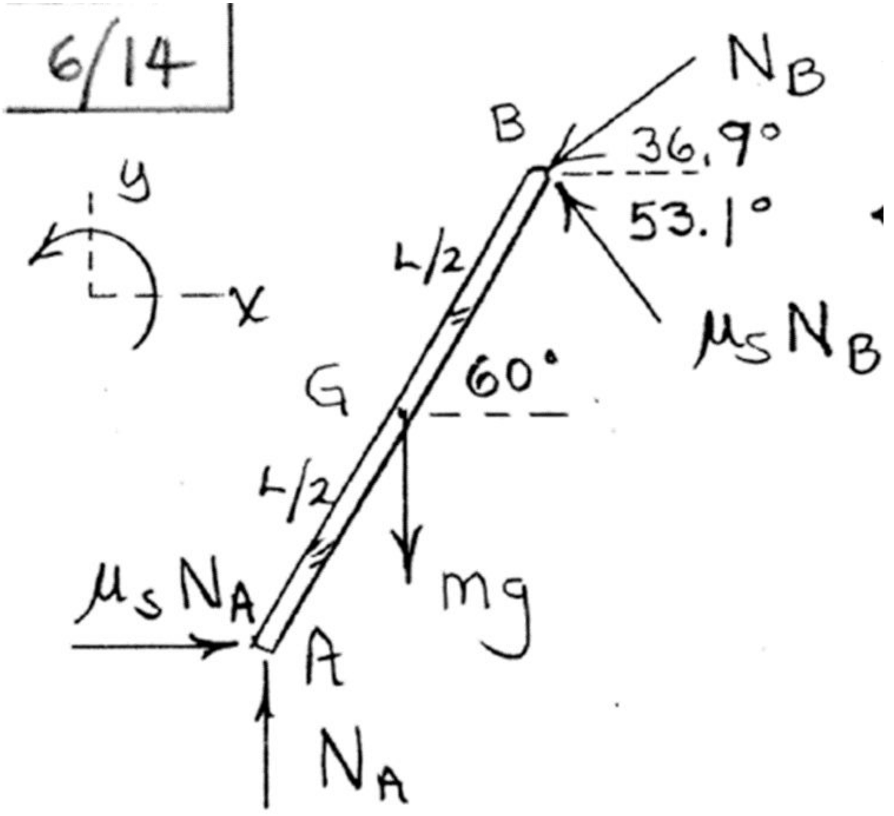
(2)



$\tan^{-1}\left(\frac{9.6}{7.2}\right) = 53.1°$

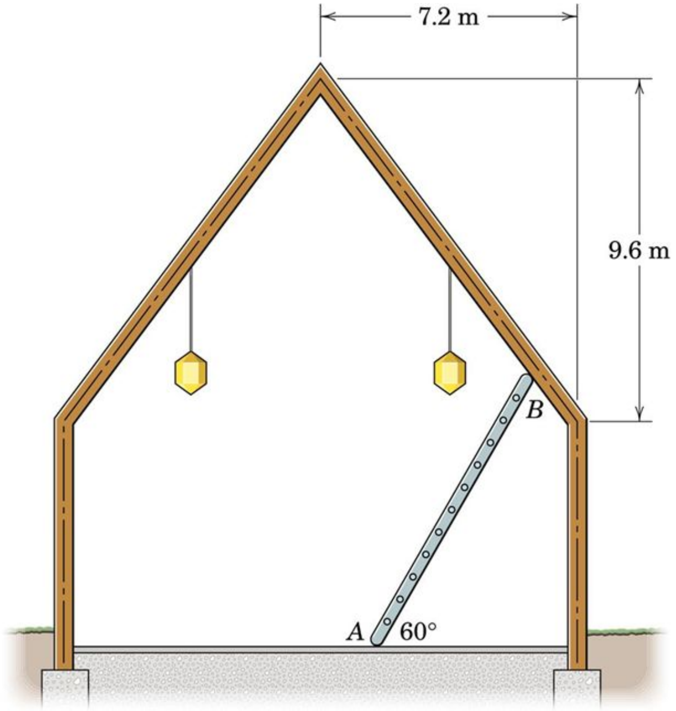
$$\begin{cases} \Sigma F_x = 0: \ \mu_s N_A - N_B \cos 36.9^\circ - \mu_s N_B \cos 53.1^\circ = 0 \\ \Sigma F_y = 0: \ N_A - N_B \sin 36.9^\circ + \mu_s N_B \sin 53.1^\circ - mg = 0 \\ \Sigma M_B = 0: \ mg \frac{L}{2} \cos 60^\circ + \mu_s N_A L \sin 60^\circ \\ \qquad - N_A L \cos 60^\circ = 0 \end{cases}$$

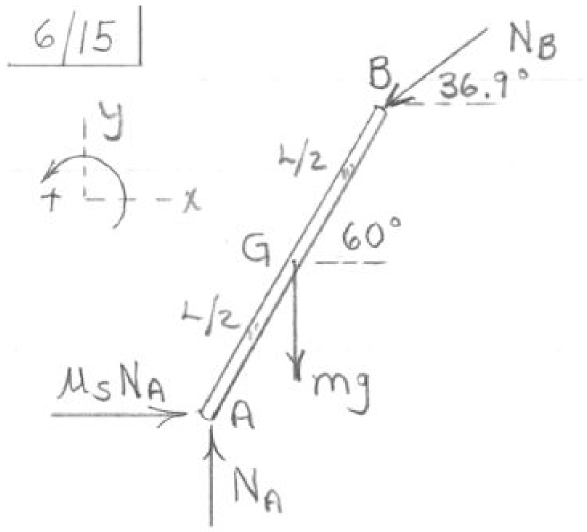
Solve to obtain
$$\begin{cases} N_A = 1.125\, mg \\ N_B = 0.364\, mg \\ \mu_s = 0.321 \end{cases}$$



μNa – 0.8 Nb – 0.6 μNb = 0
Na – 0.6 Nb + 0.8 μNb –mg =0
0.25 mg + 0.433 L μ -0.5 Na L = 0

6/15 If there is a small frictionless roller on end B of the ladder of Prob. 6/14, determine the minimum coefficient of static friction required at end A in order to provide equilibrium. Compare with the results of the previous problem.

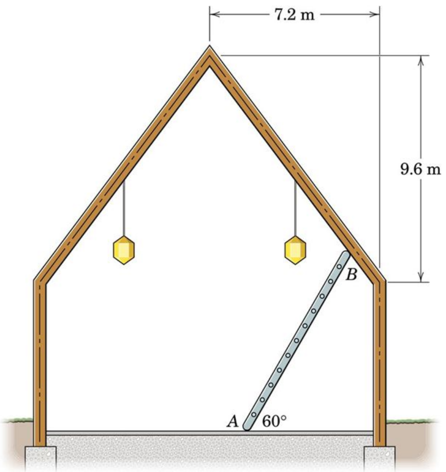




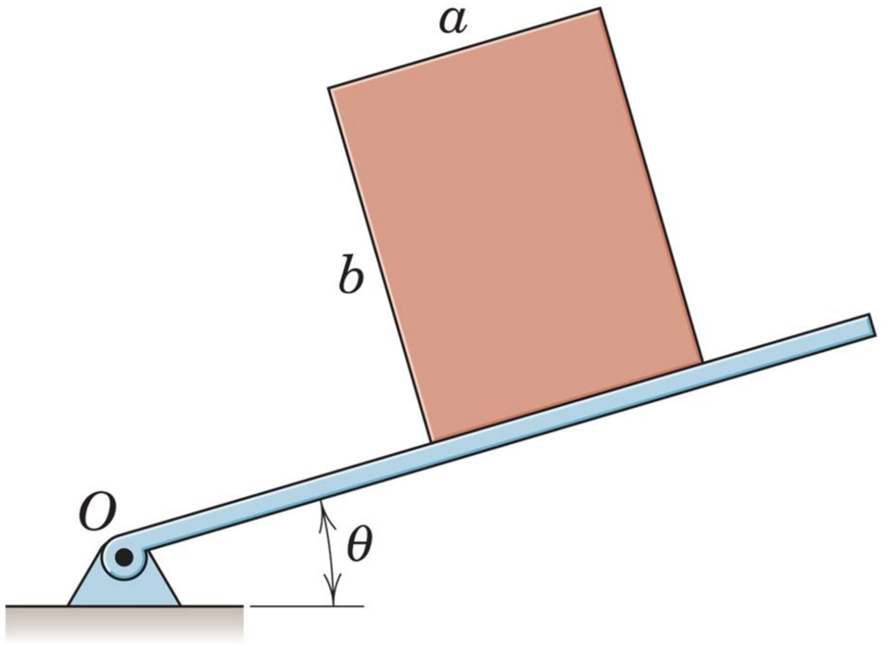
$$\begin{cases} \Sigma F_x = 0 : \ \mu_s N_A - N_B \cos 36.9^\circ = 0 \\ \Sigma F_y = 0 : \ N_A - N_B \sin 36.9^\circ - mg = 0 \\ \Sigma M_B = 0 : \ mg \frac{L}{2} \cos 60^\circ + \mu_s N_A L \sin 60^\circ \\ \qquad - N_A L \cos 60^\circ = 0 \end{cases}$$

Solve to obtain $\begin{cases} N_A = 1.382 mg \\ N_B = 0.636 mg \\ \mu_s = 0.368 \end{cases}$

(μ_s here higher than in previous problem ✓)



6/16 The homogeneous rectangular block of mass m rests on the inclined plane which is hinged about a horizontal axis through 0. If the coefficient of static friction between the block and the plane is μ, specify the conditions which determine whether the block tips before it slips or slips before it tips as the angle θ is gradually increased.

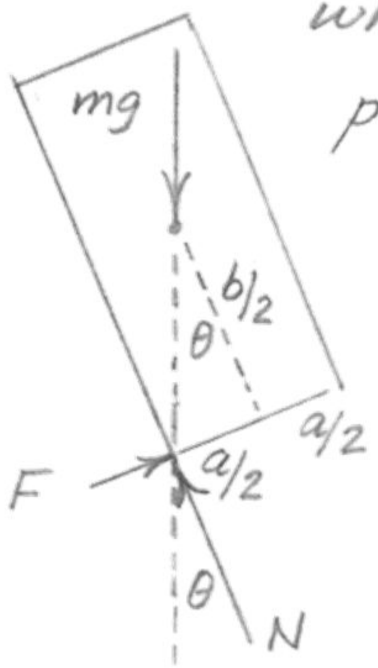


6/16

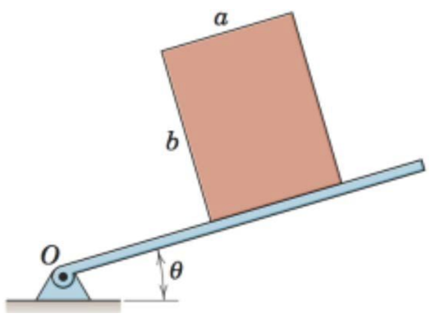
Block slips if $F = \mu N$ or $mg \sin\theta = \mu mg \cos\theta$

when angle reaches $\theta = \tan^{-1}\mu$

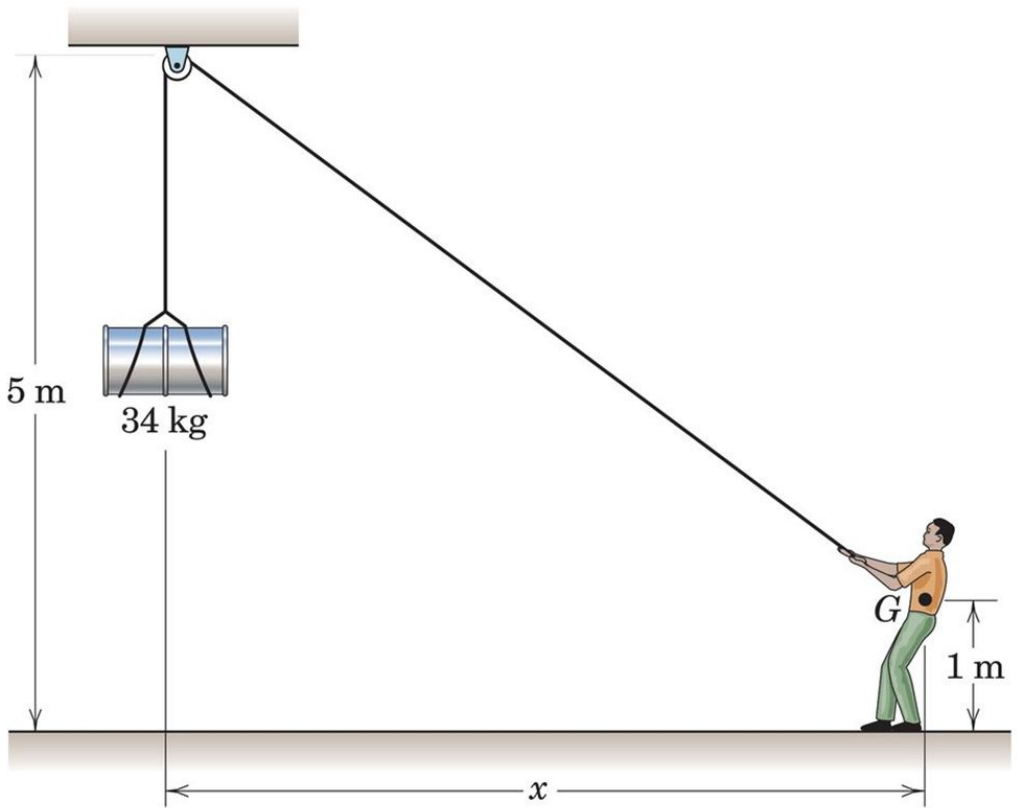
provided $\frac{a}{2} > \frac{b}{2}\tan\theta$ or $\underline{a > \mu b}$



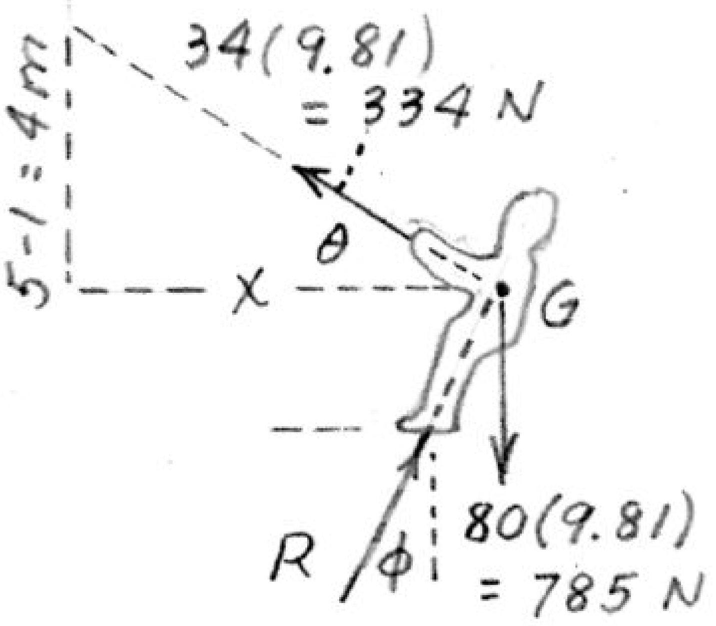
Tips first if $\underline{a < \mu b}$

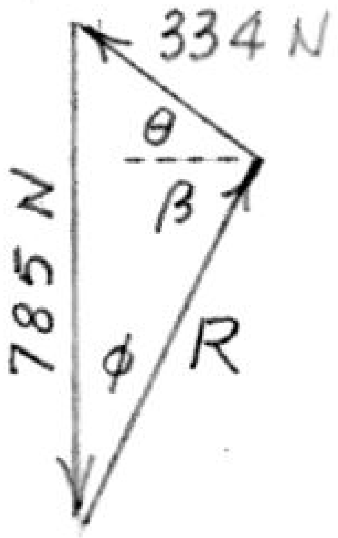


6/17 The 80-kg man with center of mass G supports the 34-kg drum as shown. Find the greatest distance x at which the man can position himself without slipping if the coefficient of static friction between his shoes and the ground is 0.40.



6/17





$\phi = \phi_{max} = \tan^{-1} 0.40 = 21.8°$

$\beta = 90 - 21.8 = 68.2°$

Law of sines

$$\frac{785}{\sin(\theta + \beta)} = \frac{334}{\sin 21.8}$$

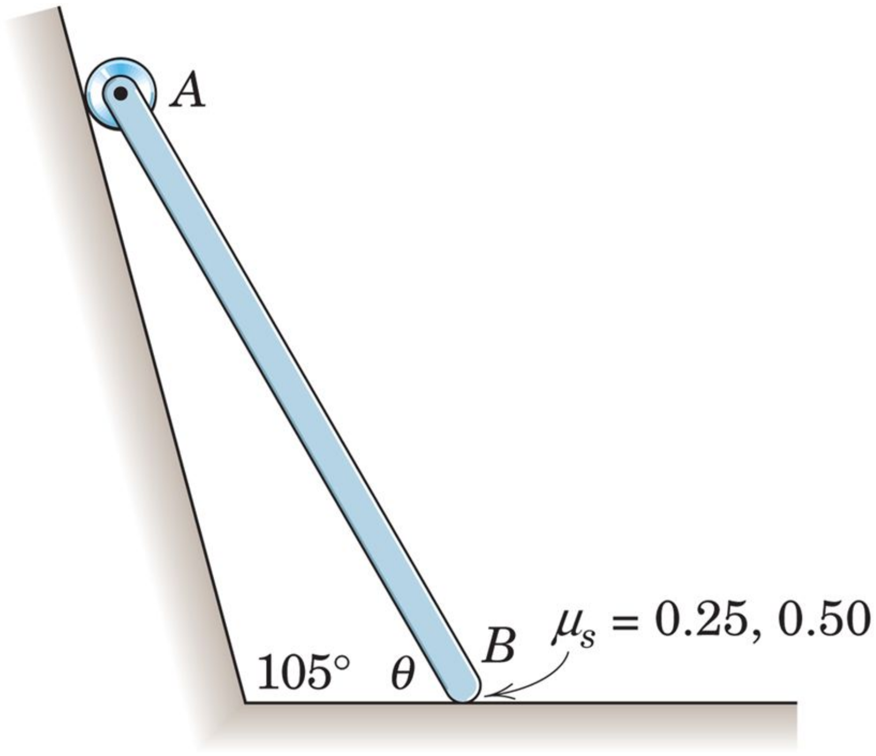
$$\theta + \beta = \sin^{-1} \frac{785 \sin 21.8}{334}$$

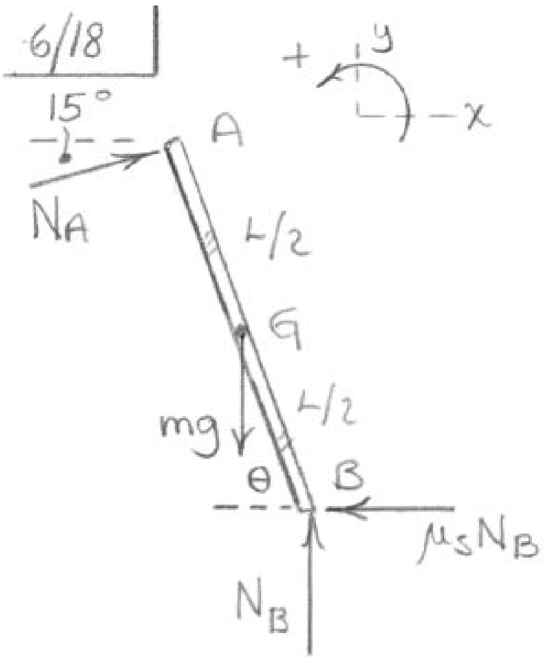
$$= 60.9° \text{ or } 119.1°$$

60.9° sol. not possible

So $\theta = 119.1 - 68.2 = 50.9°$

6/18 The uniform slender bar has an ideal roller at its upper end A. Determine the minimum value of the angle θ for which equilibrium is possible for μ= 0.25 and for μ = 0.50.





$$\Sigma F_x = 0: \quad N_A \cos 15° - \mu_s N_B = 0$$

$$\Sigma F_y = 0: \quad N_B + N_A \sin 15° - mg = 0$$

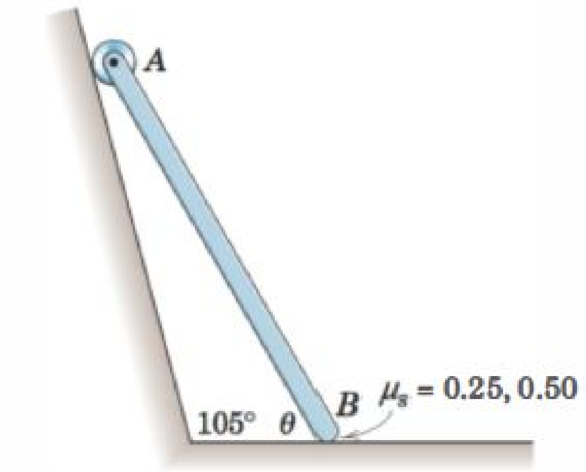
$$\Sigma M_A = 0: \quad -mg \frac{L}{2} \cos\theta + N_B L \cos\theta - \mu_s N_B L \sin\theta = 0$$

Eliminate N_A and N_B to obtain

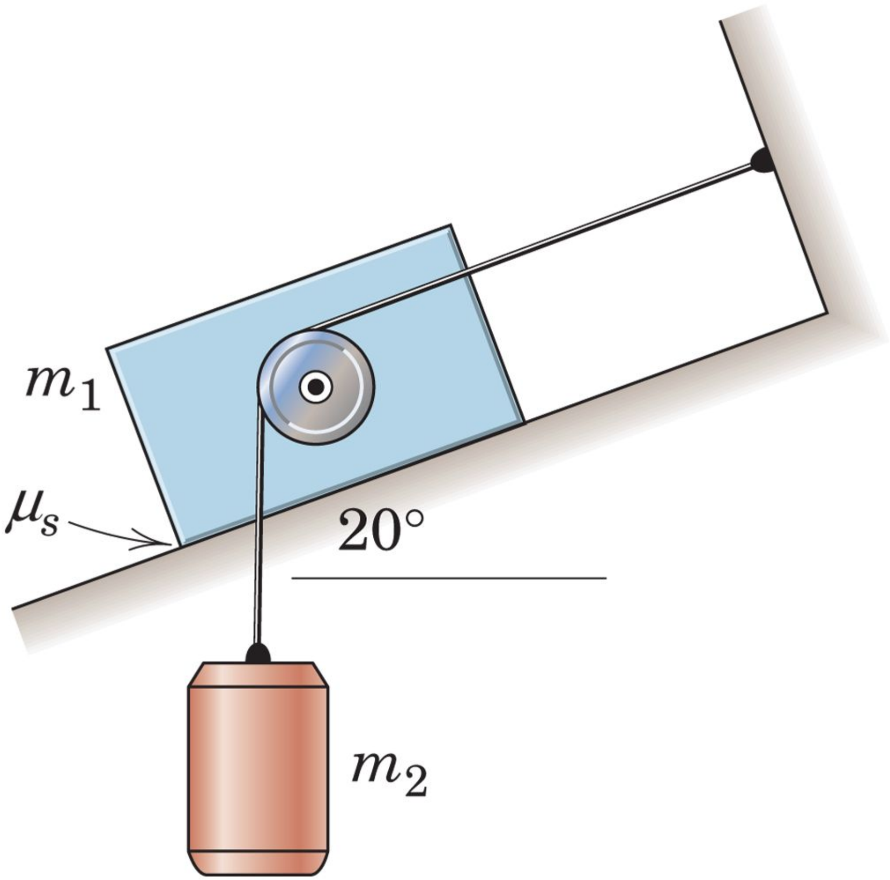
$$\tan\theta = \frac{1 - \mu_s \tan 15°}{2\mu_s}$$

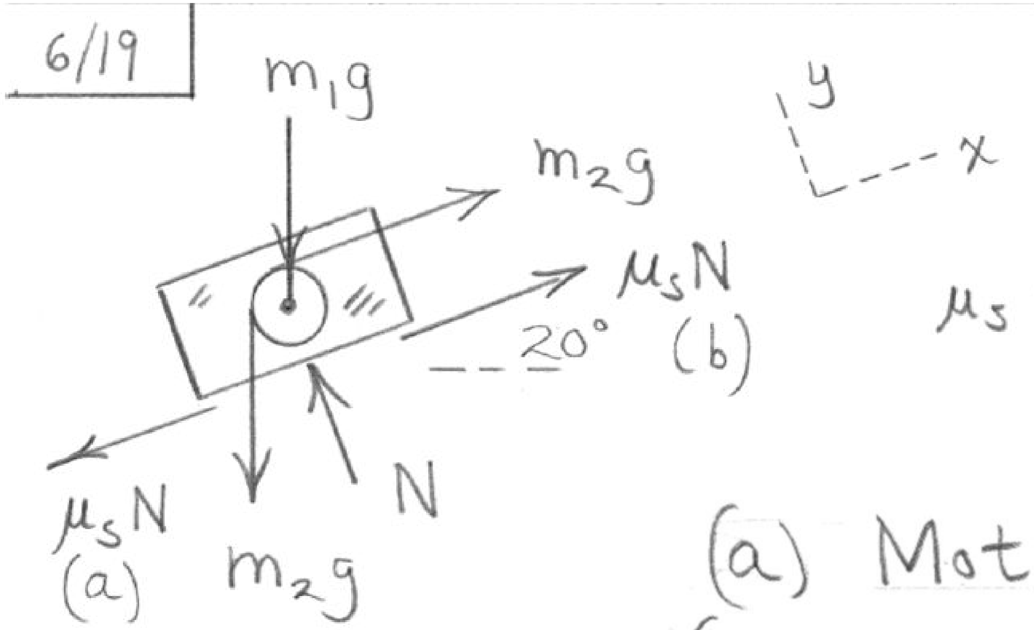
For $\mu_s = 0.25$, $\underline{\theta = 61.8°}$

For $\mu_s = 0.50$, $\underline{\theta = 40.9°}$



6/19 Determine the range of mass m2 for which the system is in equilibrium. The coefficient of static friction between the block and the incline is μs= 0.25.Neglect friction associated with the pulley.





$\mu_s = 0.25$

(a) Motion impends up the incline

$$\Sigma F_x = 0: -\mu_s N - m_1 g \sin 20° - m_2 g \sin 20° + m_2 g = 0$$

$$\Sigma F_y = 0: \quad N - m_1 g \cos 20° - m_2 g \cos 20° = 0$$

Solving, $m_2 = 1.364 m_1$

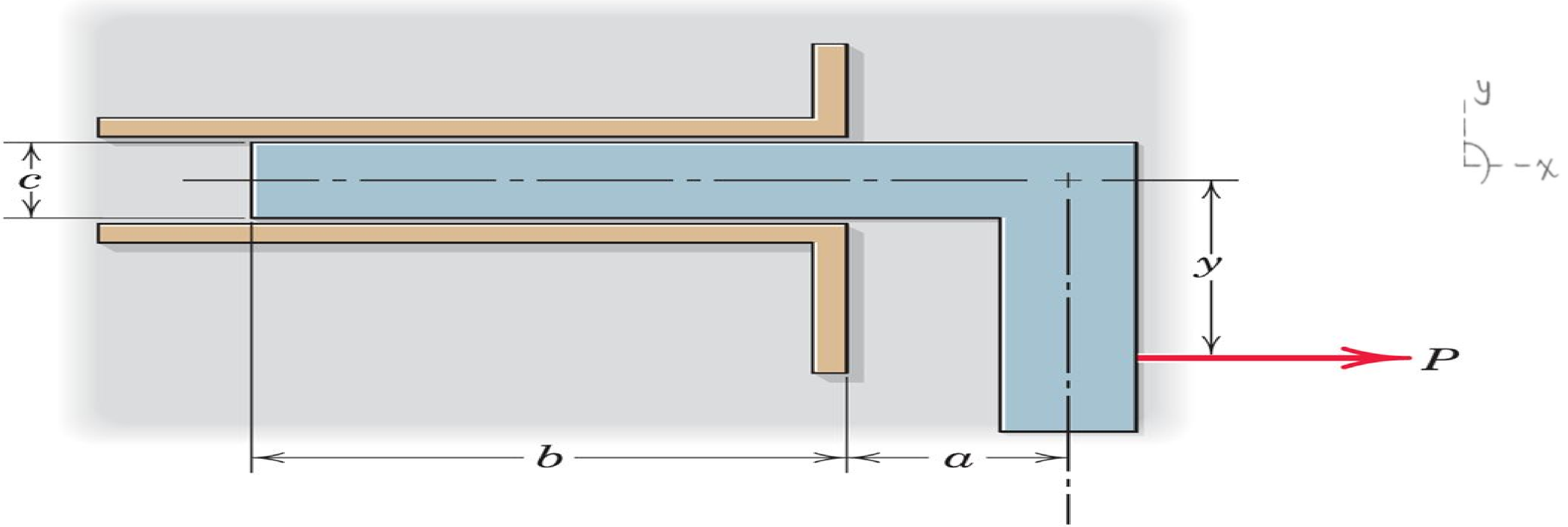
(b) Motion impends down the incline :

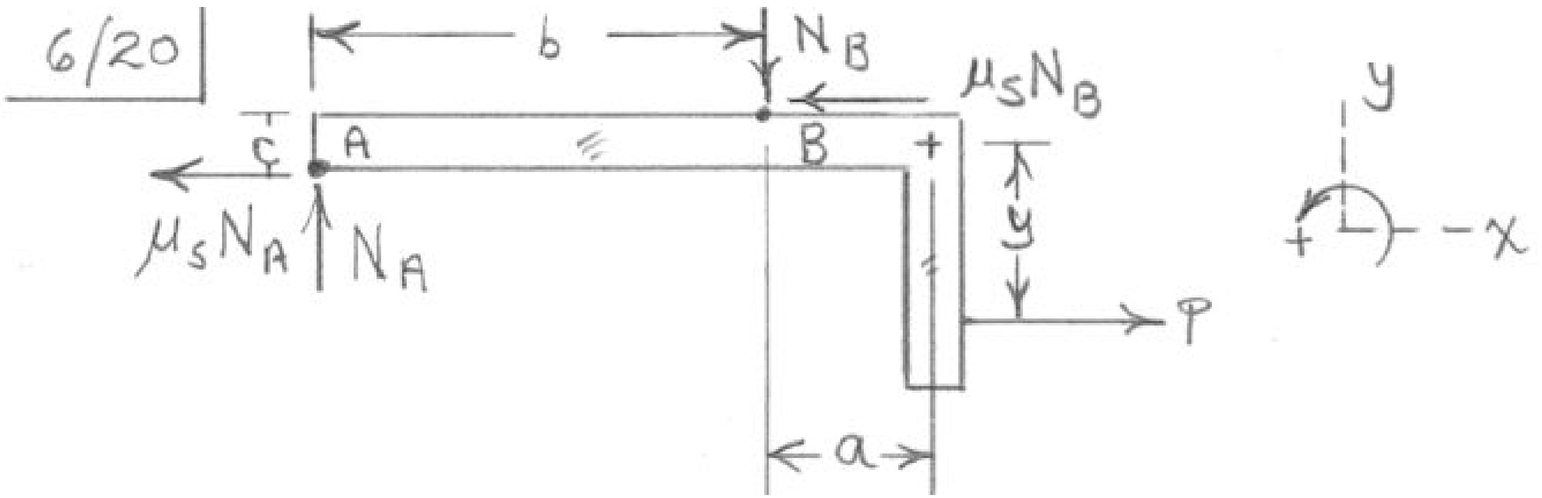
$$\begin{cases} \Sigma F_x = 0: & \mu_s N - (m_1 + m_2) g \sin 20^\circ + m_2 g = 0 \\ \Sigma F_y = 0: & \text{(Does not change)} \end{cases}$$

Solving, $m_2 = 0.1199\, m_1$

So $\underline{0.1199\, m_1 \leq m_2 \leq 1.364\, m_1}$

6/20 The right angle body is to be withdrawn from the close-fitting slot by the force P. Find the maximum distance y from the horizontal centerline at which P may be applied without binding. The body lies in a horizontal plane, and friction underneath the body is to be neglected. Take the coefficient of static friction along both sides of the slot to be μs.





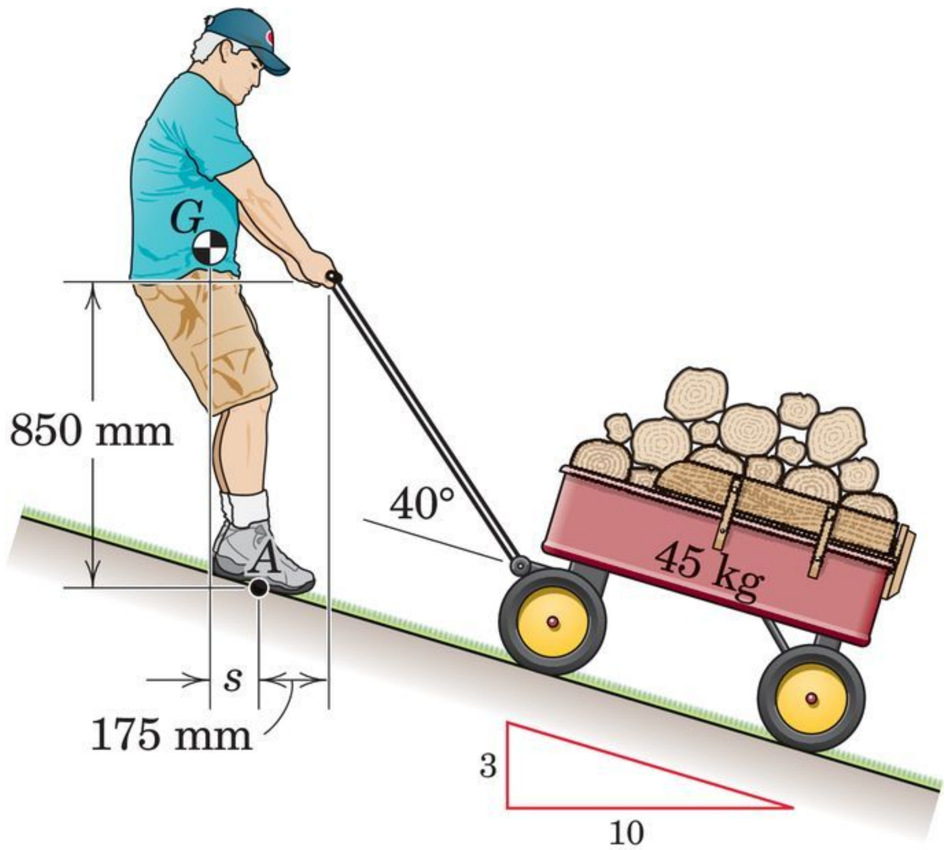
$$\Sigma F_x = 0 : -\mu_s (N_A + N_B) + P = 0$$

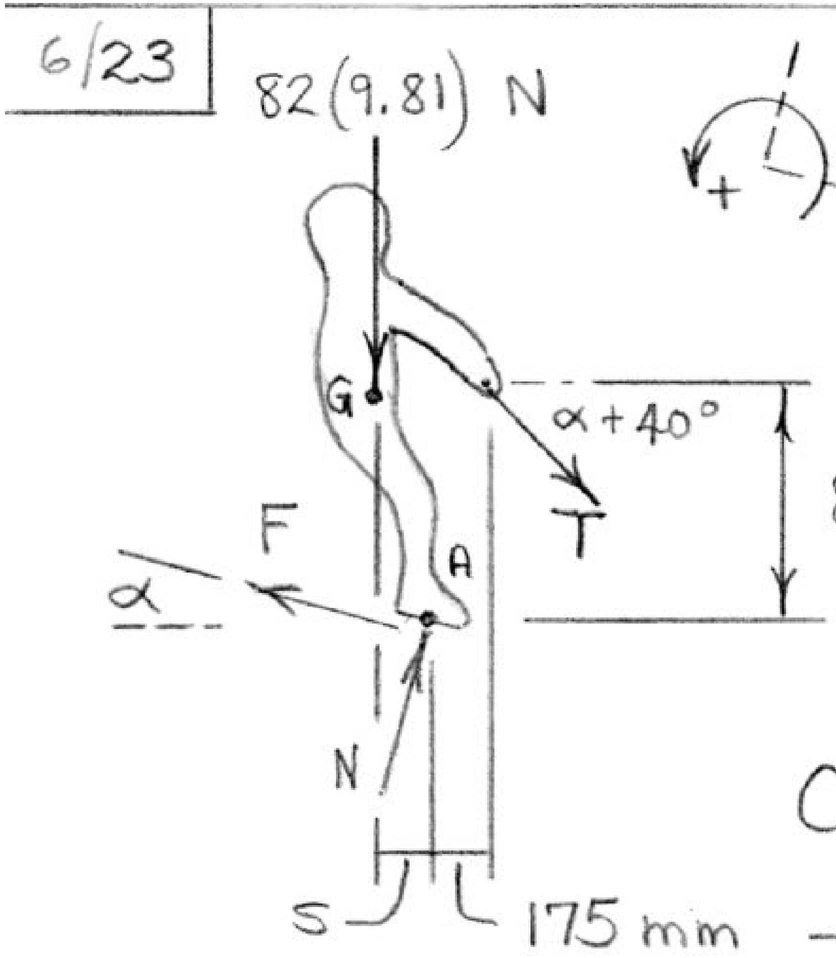
$$\Sigma F_y = 0 : \quad N_A - N_B = 0$$

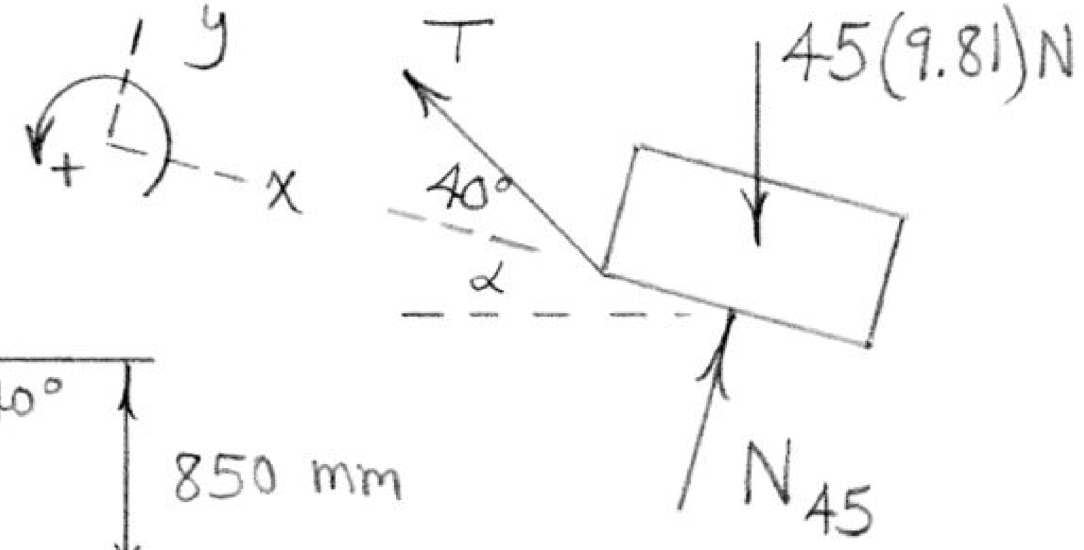
$$\Sigma M_A = 0 : -N_B(b) + \mu_s N_B (c) + P\left(y - \frac{c}{2}\right) = 0$$

Solve to obtain $\quad y = \dfrac{b}{2\mu_s}$

6/23 A 82-kg man pulls the 45-kg cart up the incline at steady speed. Determine the minimum coefficient μs of static friction for which the man's shoes will not slip. Also determine the distance s required for equilibrium of the man's body.







$$\alpha = \tan^{-1}\frac{3}{10} = 16.70^\circ$$

Cart : $\Sigma F_x = 0$:

$$-T\cos 40^\circ + 45(9.81)\sin\alpha = 0$$

$$T = 165.6 \text{ N}$$

Man:

$\Sigma F_x = 0: \; -F + 82(9.81)\sin\alpha + 165.6\cos 40° = 0$

$F = 358 \text{ N}$

$\Sigma F_y = 0: \; N - 82(9.81)\cos\alpha - 165.6\sin 40° = 0$

$N = 877 \text{ N}$

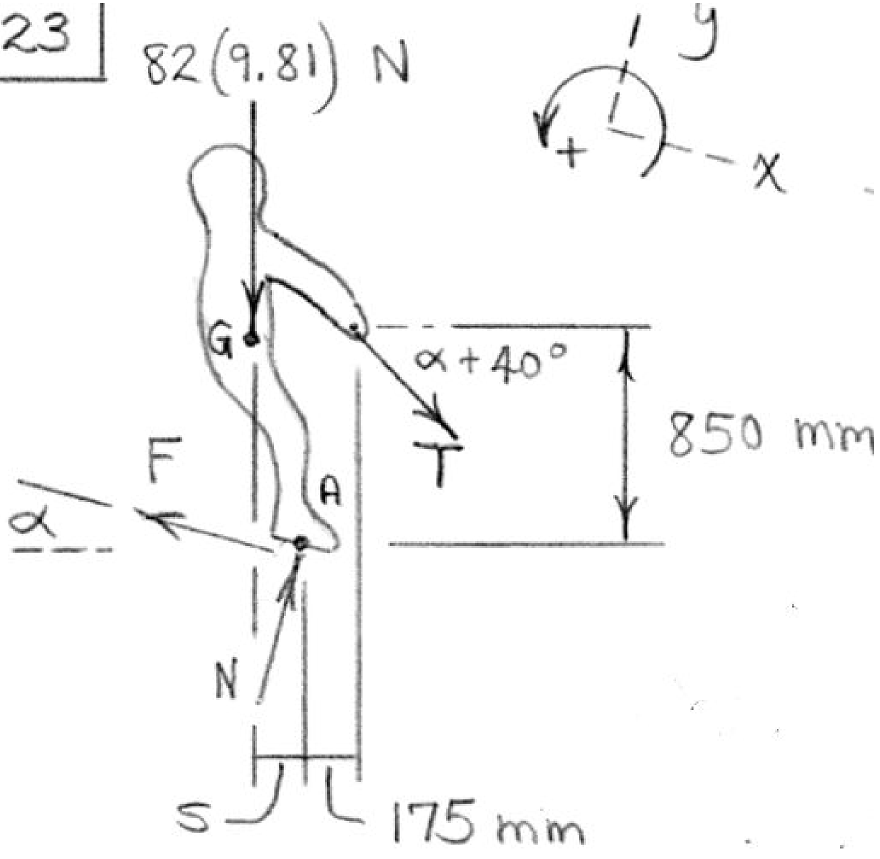
$\mu_s = \frac{F}{N} = \frac{358}{877} = \underline{0.408}$

$\Sigma M_A = 0: \; 82(9.81)s - 165.6\cos(\alpha + 40)(850)$

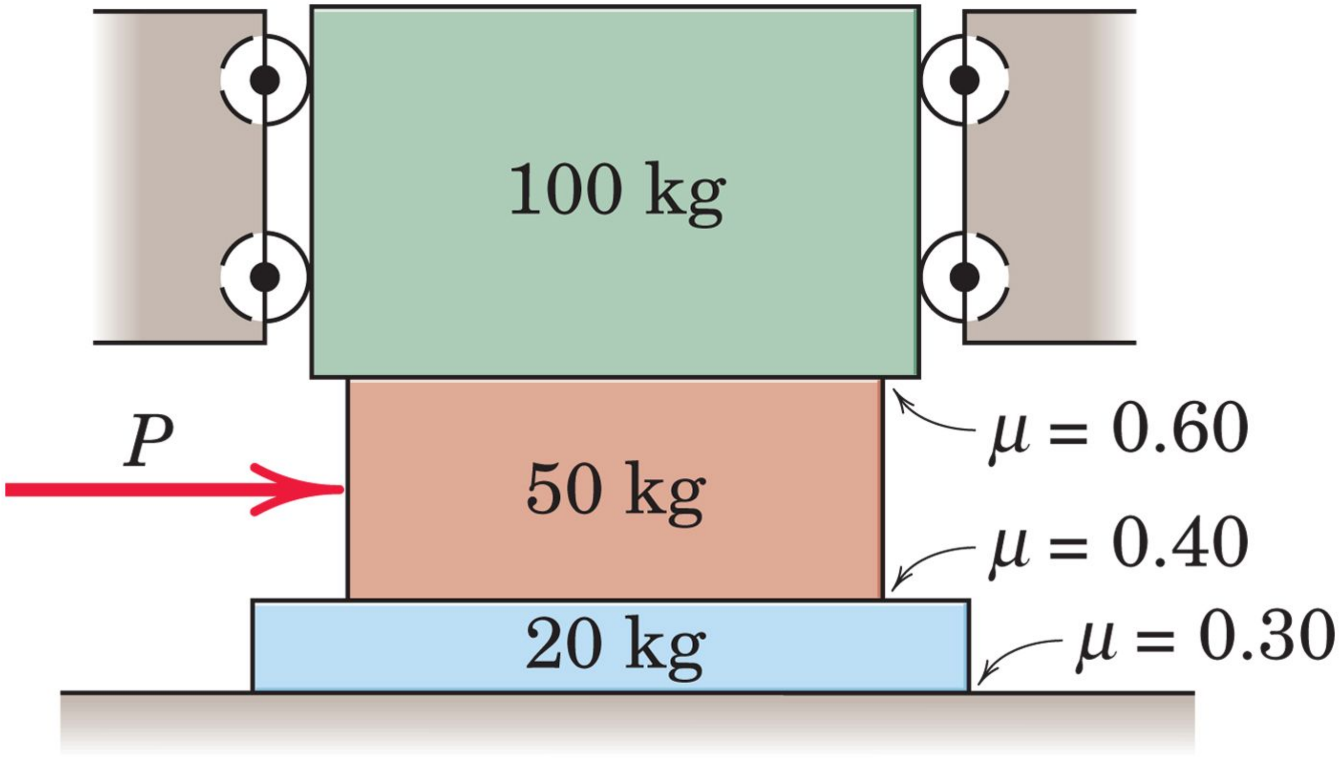
$- 165.6\sin(\alpha + 40)(175) = 0$

$s = 126.2 \text{ mm}$

6/23



6/24 Determine the horizontal force P required to cause slippage to occur. The friction coefficients for the three pairs of mating surfaces are indicated. The top block is free to move vertically .

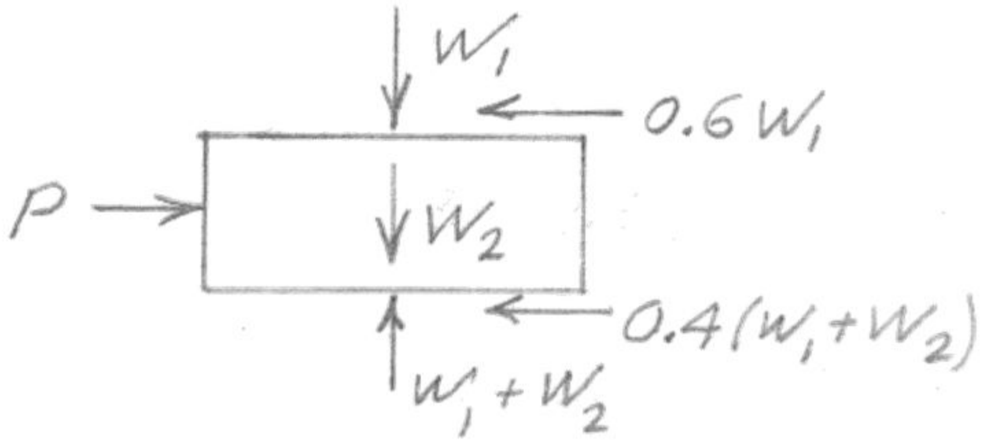


6/24 There are two possibilities

(a) Middle block moves; bottom one does not

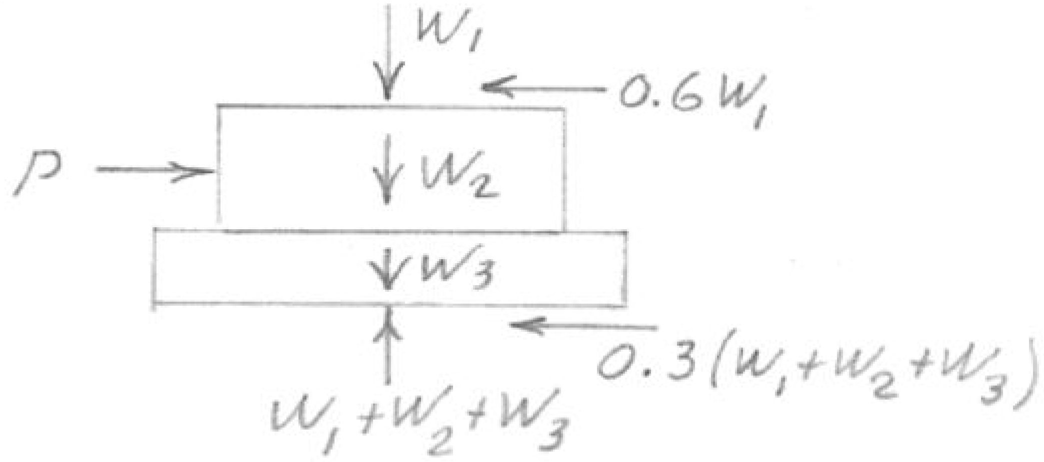
$W_1 = 100(9.81) = 981$ N

$W_2 = 50(9.81) = 490.5$ N



$\Sigma F = 0;$ $\quad P = 0.6(981) + 0.4(981 + 490.5) = 1177$ N

(b) Bottom block moves with middle block

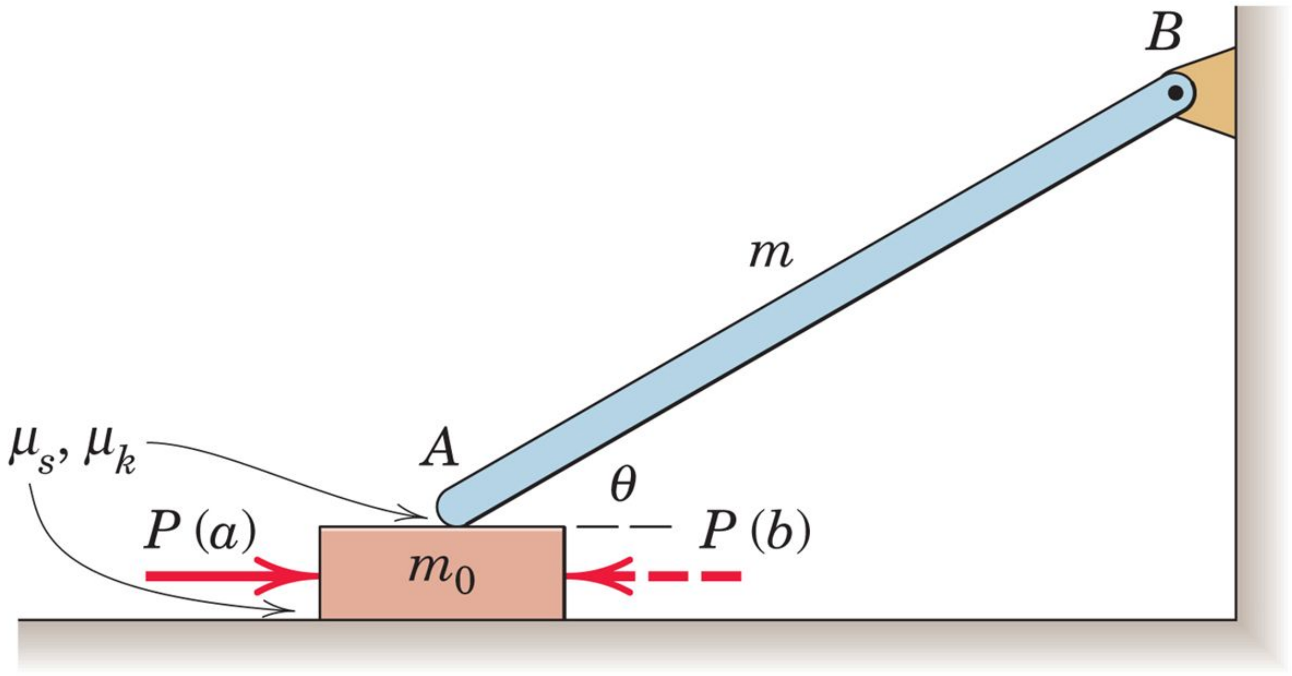


$W_3 = 20(9.81) = 196.2\ N$

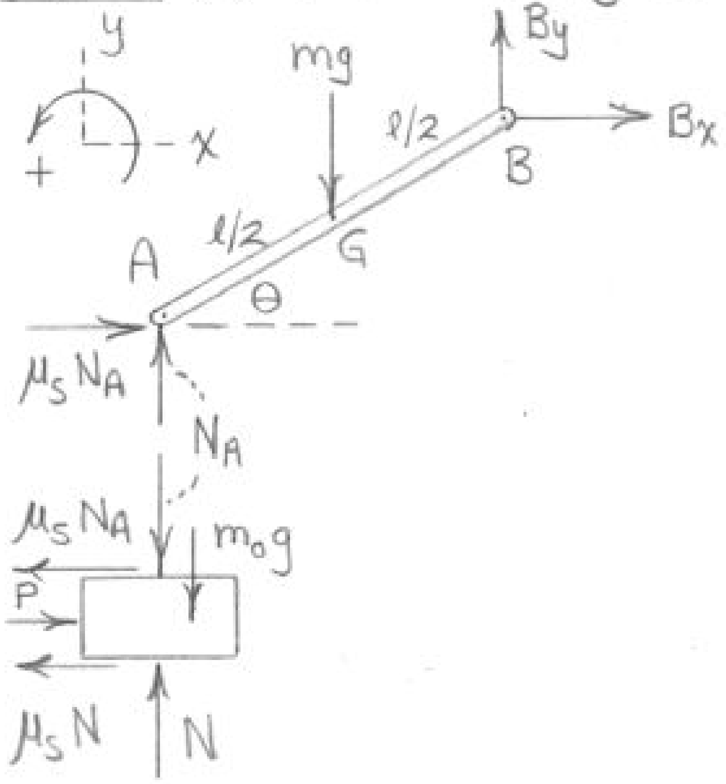
$\Sigma F = 0;\ P = 0.6(981) + 0.3(981 + 490.5 + 196.2) = 1089\ N$

$1088 < 1177$ so case (b) occurs & $\underline{P = 1089\ N}$

6/28 Determine the magnitude P of the horizontal force required to initiate motion of the block of mass m0 for the cases (a) Pis applied to the right and (b) Pis applied to the left. Complete a general solution in each case, and then evaluate your expression for the values () = 300, m = mo = 3 kg, JJ-. = 0.60, and lkk = 0.50.



6/28 (a) P to the right.



$$\Sigma M_B = 0: \; mg\frac{l}{2}\cos\theta - N_A\, l\cos\theta + \mu_s N_A\, l\sin\theta = 0 \qquad (1)$$

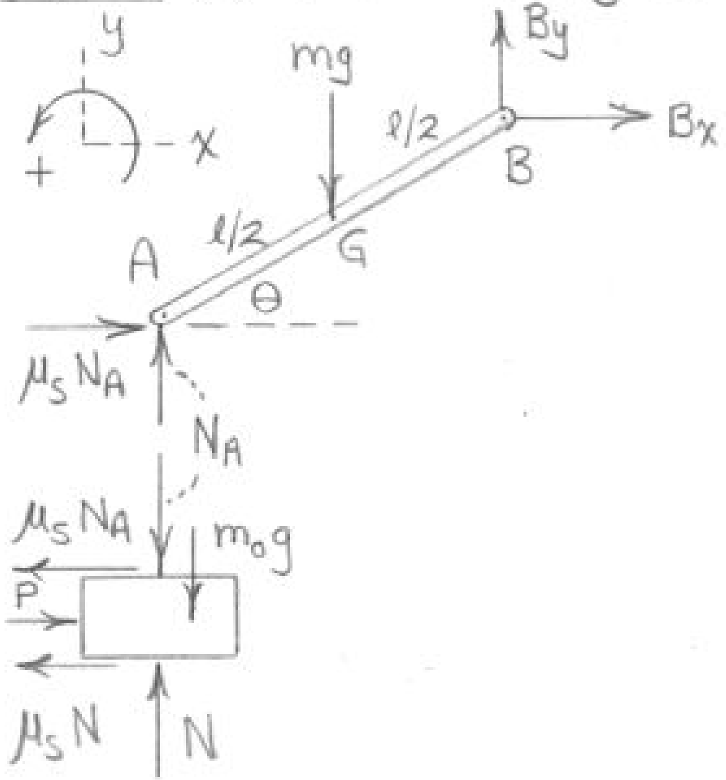
$$\text{(Box)} \quad \Sigma F_x = 0: \; P - \mu_s N_A - \mu_s N = 0 \qquad (2)$$

$$\Sigma F_y = 0: \; N - m_0 g - N_A = 0 \qquad (3)$$

Solve for P as

$$P = \mu_s g\left[\frac{m\cos\theta}{\cos\theta - \mu_s\sin\theta} + m_0\right]$$

6/28 (a) P to the right.



(b) P to the left. Reverse P and all friction forces in the above FBD's & obtain

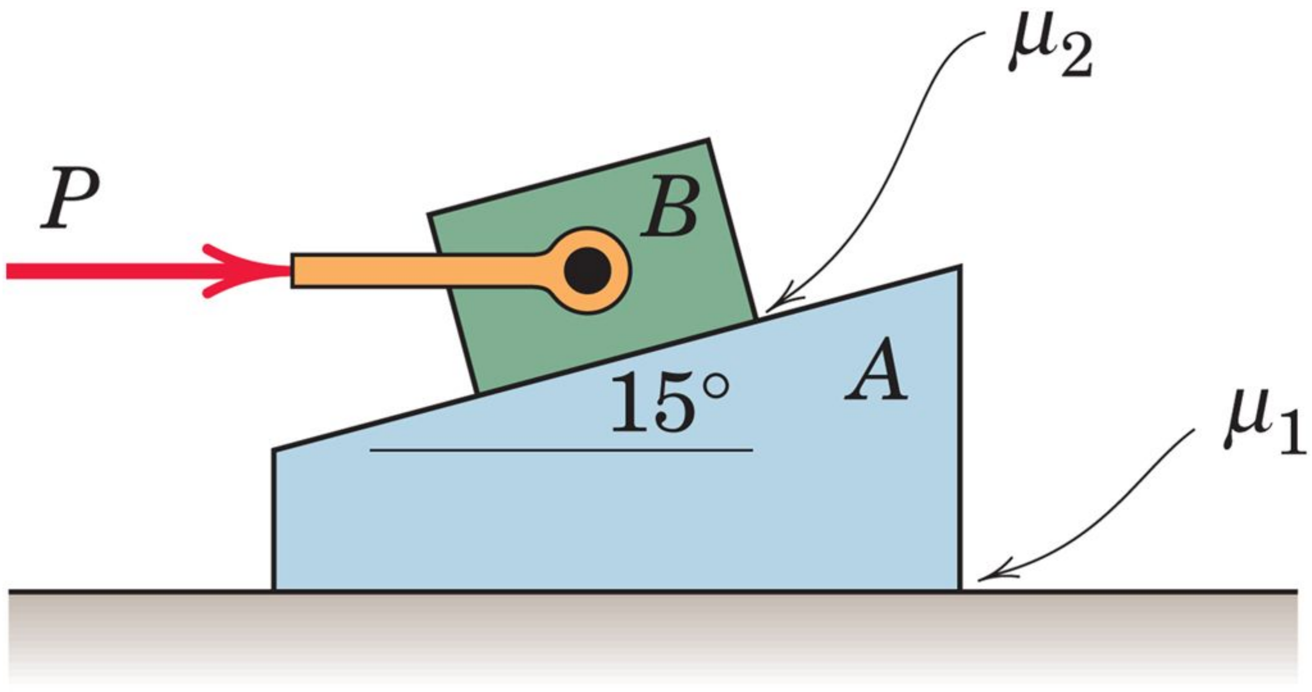
$$P = \mu_s g \left[\frac{m \cos\theta}{\cos\theta + \mu_s \sin\theta} + m_o \right]$$

With $\theta = 30°$, $m = m_o = 3\text{kg}$, and $\mu_s = 0.60$, we obtain

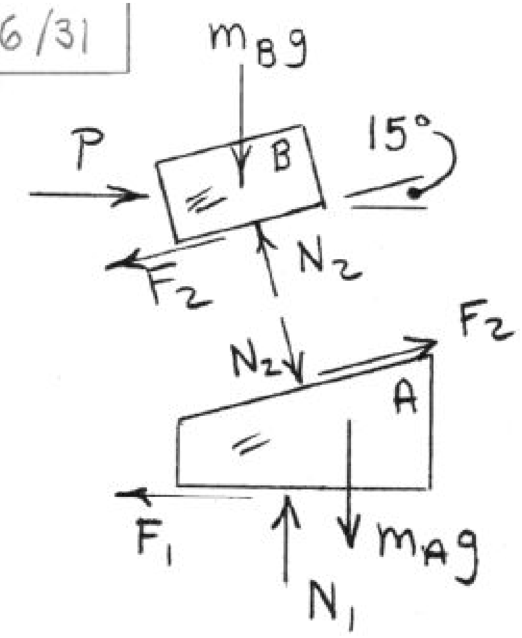
(a) $P = 44.7$ N

(b) $P = 30.8$ N

6/31 Reconsider the system of Prob. 6/30. If P = 40 N, JJ.1 = 0.30, and J.l-2 = 0.50, determine the force which block B exerts on block A. Assume that the coefficients of kinetic friction are 75 percent of the static values. The block masses remain mA = 10 kg and m8 = 5 kg.



6/31



B:
$$\Sigma F_x = 0: \quad P - F_2 \cos 15° - N_2 \sin 15° = 0$$
$$\Sigma F_y = 0: \quad N_2 \cos 15° - F_2 \sin 15° - m_B g = 0$$

A:
$$\Sigma F_x = 0: \quad N_2 \sin 15° + F_2 \cos 15° - F_1 = 0$$
$$\Sigma F_y = 0: \quad N_1 - N_2 \cos 15° + F_2 \sin 15° - m_A g = 0$$

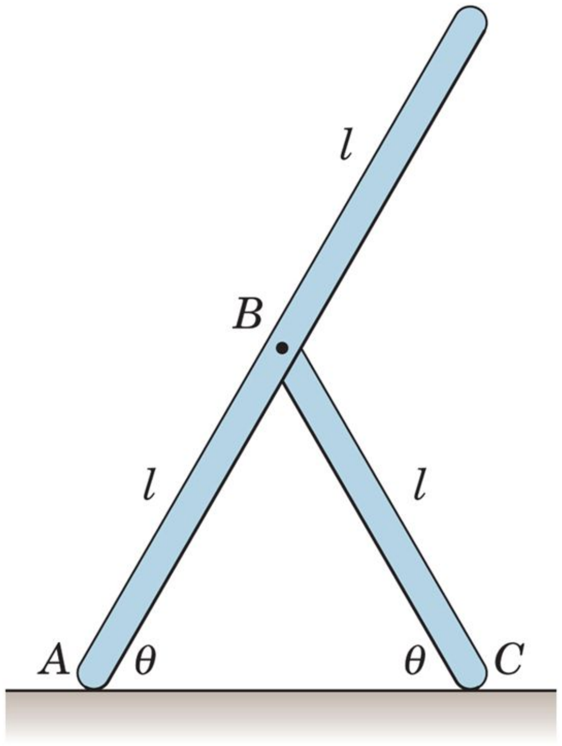
With $m_A = 10$ kg, $m_B = 5$ kg, & $P = 40$ N:

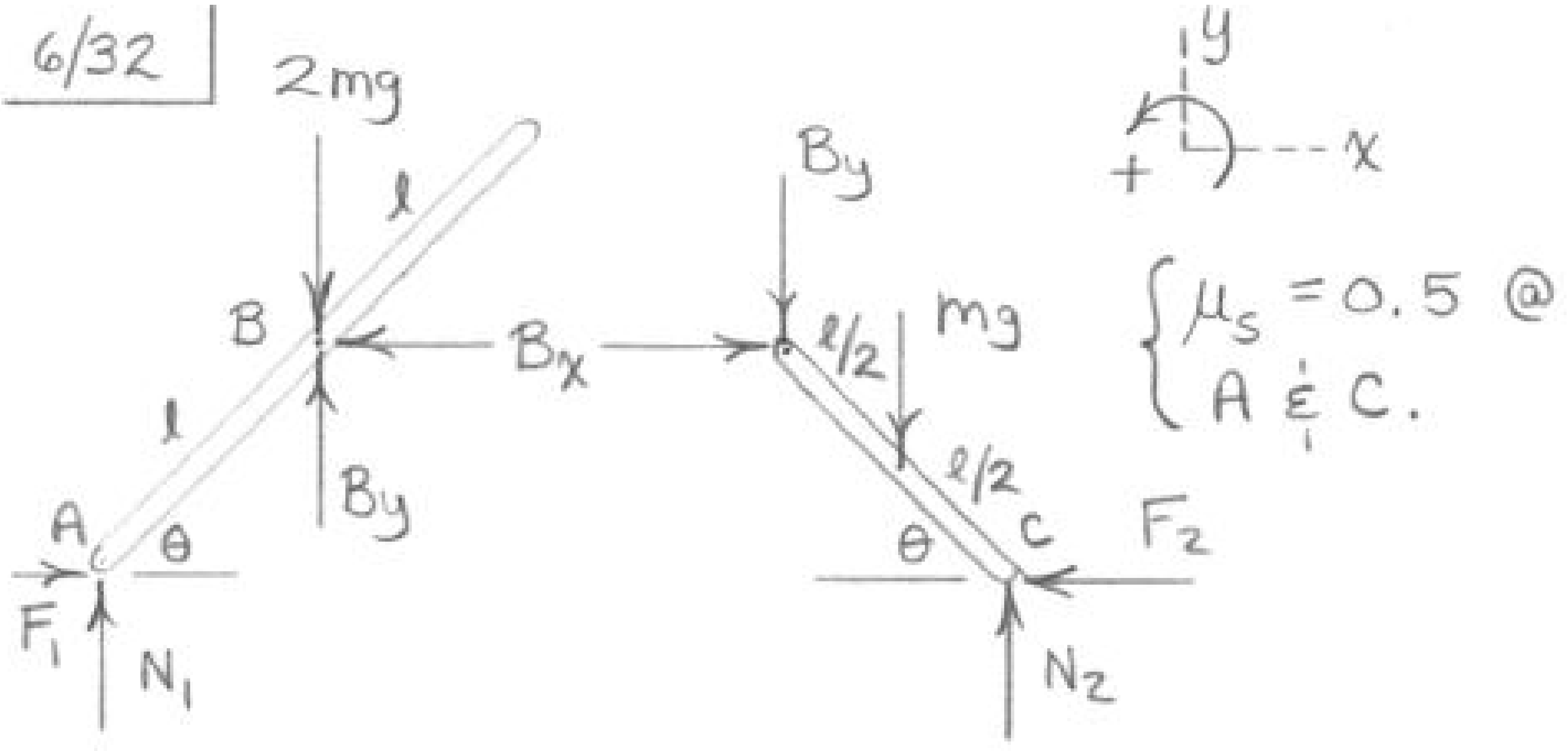
$F_1 = 40$ N, $F_2 = 25.9$ N, $N_1 = 147.2$ N, $N_2 = 57.7$ N

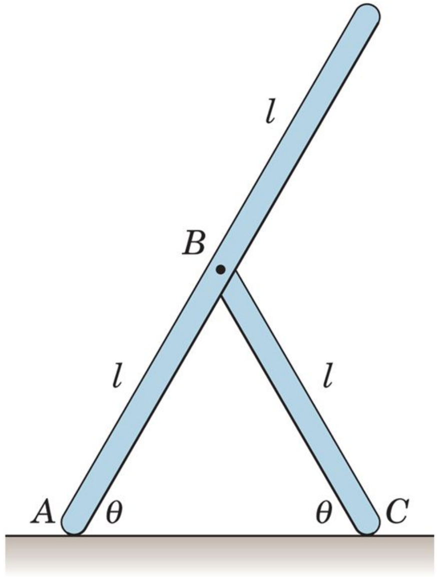
$$\left.\begin{array}{l} F_{1_{max}} = 0.30(147.2) = 44.1 \text{ N} \\ F_{2\,max} = 0.50(57.7) = 28.9 \text{ N} \end{array}\right\} \text{No slippage}$$

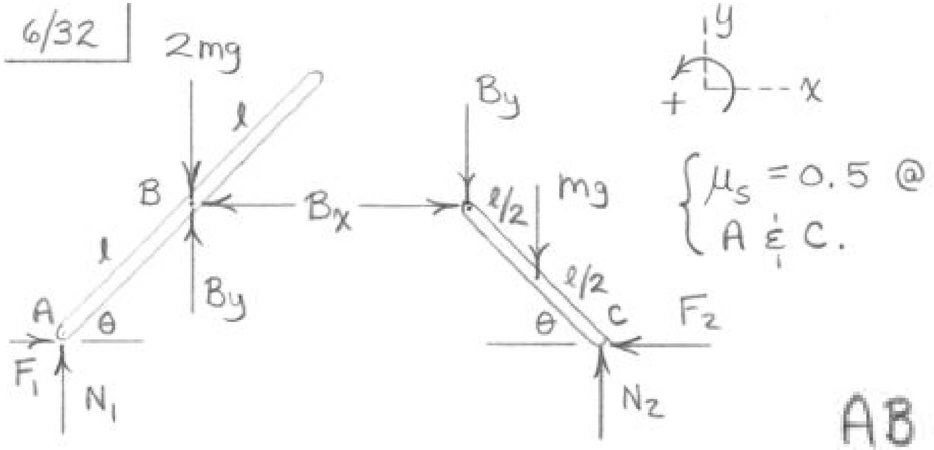
So B exerts $\underline{40\mathbf{i} - 49.0\mathbf{j}\ \text{N}}$ on A.

6/32 The two uniform slender bars constructed from the same stock material are freely pinned together at B . Determine the minimum angle 8 at which slipping does not occur at either contact point A or C. The coefficient of static friction at both A and C is ,.,.. = 0.50. Consider only motion in the vertical plane shown.



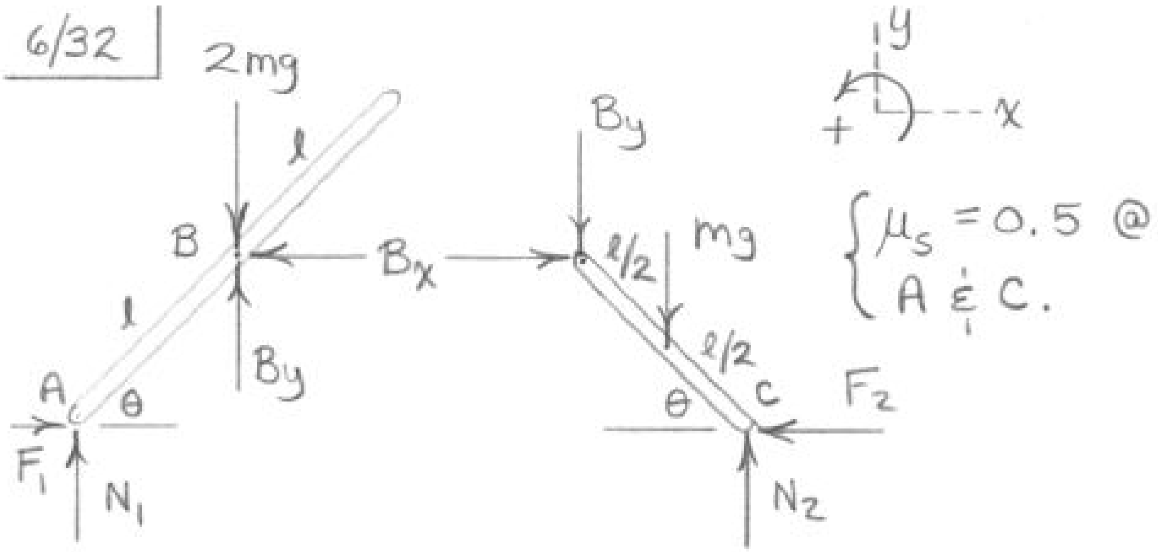

6/32
2mg
l
B
l
A
θ
F_1
N_1
B_y
B_x
B_y
l/2
mg
l/2
C
θ
F_2
N_2
y
x
+
$\mu_s = 0.5$ @
A & C.


l
B
l
l
A
θ
θ
C

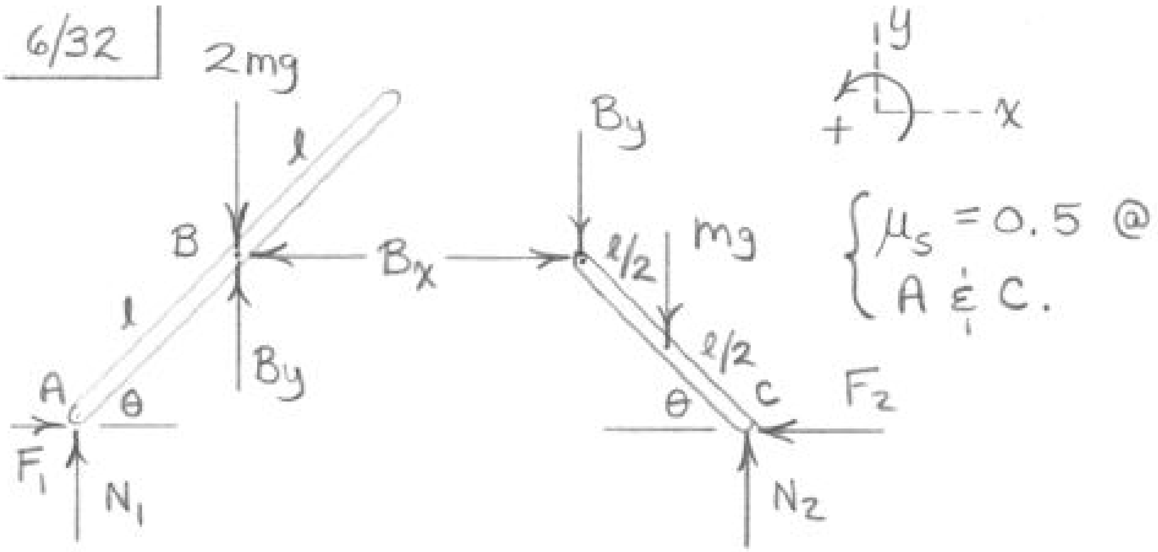


AB
$$\sum F_x = 0 : \quad F_1 - B_x = 0 \qquad (1)$$
$$\sum F_y = 0 : \quad N_1 + B_y - 2mg = 0 \qquad (2)$$
$$\sum M_A = 0 : \quad B_x(l\sin\theta) + B_y(l\cos\theta) - 2mg(l\cos\theta) = 0 \qquad (3)$$

BC
$$\sum F_x = 0 : \quad B_x - F_2 = 0 \qquad (4)$$
$$\sum F_y = 0 : \quad -B_y - mg + N_2 = 0 \qquad (5)$$
$$\sum M_C = 0 : \quad mg\left(\frac{l}{2}\cos\theta\right) + B_y(l\cos\theta) - B_x(l\sin\theta) = 0 \qquad (6)$$



Assume first slippage at A: $F_1 = 0.5 N_1$. Solve seven equations to obtain $\underline{\theta = 63.4°}$, $F_2 = 0.625\,mg$, & $N_2 = 1.75\,mg$. Note $F_2 < F_{2max} = 0.875\,mg$.

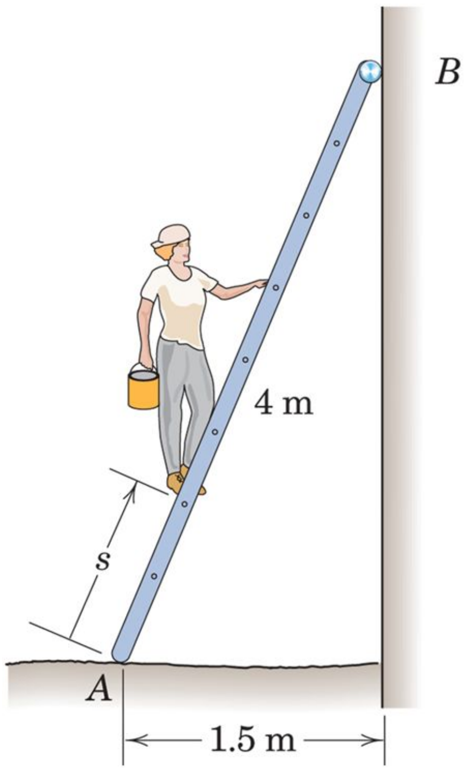


Then assume first slippage at B: $F_2 = 0.5 N_2$.

Obtain $\theta = 55.0°$, $F_1 = 0.875\,mg$ & $N_1 = 1.25\,mg$.

Note $F_1 > F_{1max} = 0.625\,mg$. So A slips first.

6/33 Determine the distances to which the 90-kg painter can climb without causing the 4-m ladder to slip at its lower end A. The top of the 15-kg ladder has a small roller, and at the ground the coefficient of static friction is 0.25. The mass center of the painter is directly above her feet.



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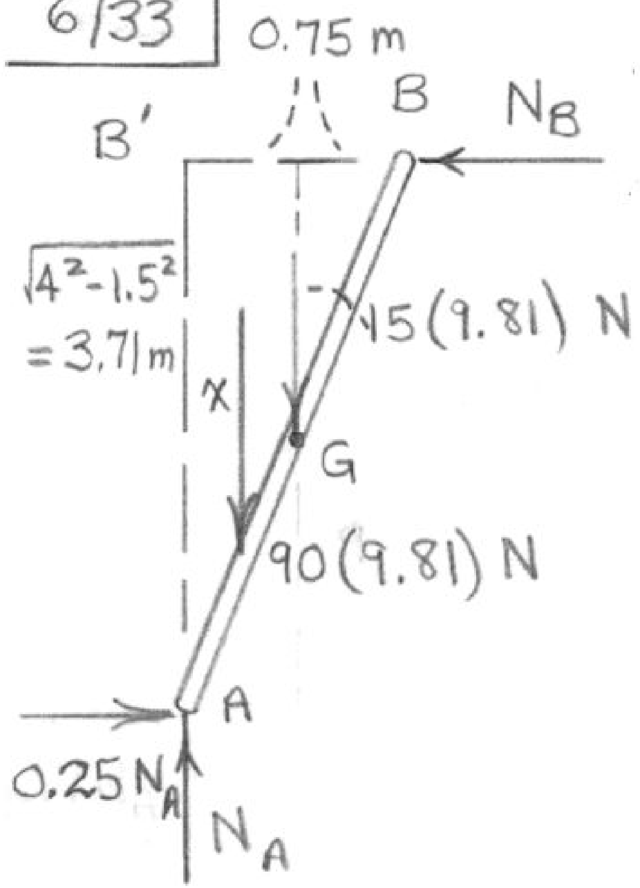

$$+\uparrow \Sigma F = 0 \Rightarrow N_A = (90+15)9.81 = 1030\ N$$

$$\circlearrowright + \Sigma M_{B'} = 0 : 0.25 N_A (3.71) - 90(9.81)x$$

$$-15(9.81)(0.75) = 0, \quad x = 0.957\ m$$

$$\frac{S}{x} = \frac{4}{1.5}, \quad \underline{S = 2.55\ m}$$

THANK YOU

Department of Civil Engineering

r@pes.edu

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