ENGINEERING MATHEMATICS - II

UE20MA151

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Unit 4 : Inverse Laplace Transform

Session : 5

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- **Multiplication by *s***

If $L^{-1}\{F(s)\} = f(t)$ and $f(0) = 0$ then $L^{-1}\{sF(s)\} = \frac{df(t)}{dt}$

Recall !!!

$$L\{f'(t)\} = sF(s) - f(0) \quad (s > 0)$$

- **Division by powers of *s***

If $L^{-1}\{F(s)\} = f(t)$ and then $L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(t)dt$

Note : In general, $L^{-1}\left\{\frac{F(s)}{s^n}\right\} = \int_0^t \int_0^t \dots \dots \dots . \int_0^t f(t)dt^n$

Recall !!!

$$L\left\{\int_0^t f(t)dt\right\} = \frac{F(s)}{s}, s > 0$$

- **Second Shifting Theorem**

If $L^{-1}\{F(s)\} = f(t)$ then

$$L^{-1}[e^{-as}F(s)] = f(t-a)u(t-a)$$

Recall !!!

$$L[f(t-a)u(t-a)] = e^{-as}F(s)$$

1) Obtain the inverse Laplace Transform of

$$\frac{s}{9s^2 - 16}$$

Solution : Let $F(s) = \frac{1}{9s^2-16}$.

Then $L^{-1}[F(s)] = \frac{1}{9} L^{-1}\left[\frac{1}{s^2-16/9}\right] = \frac{1}{9} . \frac{3}{4} L^{-1}\left[\frac{1}{s^2-16/9}\right] = \frac{1}{12} sinh\frac{4}{3} t = f(t)$

Therefore,

$$L^{-1}[sF(s)] = L^{-1}\left[\frac{s}{9s^2-16}\right] = \frac{df(t)}{dt} = \frac{d}{dt}\left\{\frac{1}{12} sinh\frac{4}{3} t\right\} = \frac{1}{9}\cosh\frac{4}{3}\,\mathrm{t}$$

2) Find the inverse Laplace transform of $\frac{1}{s(s+4)}$

Solution : Let $F(s) = \frac{1}{s+4}$.

Then $L^{-1}[F(s)] = L^{-1}\left[\frac{1}{s+4}\right] = e^{-4t} = f(t)$.

Therefore, $L^{-1}\left[\frac{1}{s(s+4)}\right] = \int_0^t e^{-4t}dt = \left[\frac{e^{-4t}}{-4}\right]_0^t = \frac{1-e^{-4t}}{4}$

3) Find the inverse Laplace transform of $\frac{s+1}{s^2(s^2+1)}$

Solution : Let $F(s) = \frac{s+1}{(s^2+1)}$.

Then $L^{-1}[F(s)] = L^{-1}\left[\frac{s+1}{(s^2+1)}\right] = L^{-1}\left[\frac{s}{(s^2+1)}\right] + L^{-1}\left[\frac{1}{(s^2+1)}\right]$

$$= cost + sint$$

$$L^{-1}\left[\frac{s+1}{s^2(s^2+1)}\right] = \int_0^t \int_0^t cost + sint\, dt\, dt = \int_0^t [sint - cost]_0^t \, dt$$

$$= \int_0^t (sint - cost + 1)dt$$

$$= [-cost - sint + t]_0^t \quad = 1 + \mathrm{t} - \cos\mathrm{t} - \sin\mathrm{t}$$

4)Find the inverse Laplace transform of $\frac{s^2+3}{s(s^2+9)}$

Solution : $L^{-1}\left[\frac{s^2+3}{s(s^2+9)}\right] = L^{-1}\left[\frac{s^2+9-6}{s(s^2+9)}\right]$

$= L^{-1}\left[\frac{1}{s}\right] - 6L^{-1}\left[\frac{1}{s(s^2+9)}\right]$

$= 1 - 6.\frac{1}{3}\int_0^t sin3t\ dt$

$= 1 + 2\left[\frac{cos3t}{3}\right]_0^t = 1 + \frac{2}{3}cos3t - \frac{2}{3}$

$= \frac{2}{3}cos3t + \frac{1}{3}$

5)Find the inverse Laplace Transform of $\frac{e^{-3s}}{s^2+4}$

Solution :

Let F(s)= $\frac{1}{s^2+4}$

L^{-1}[F(s)] = $\frac{1}{2}\sin 2t = f(t)$

$$L^{-1}\left[\frac{e^{-3s}}{s^2+4}\right] = \mathrm{f(t-3)\ u(t-3)}$$

$$= \frac{1}{2}\sin 2(t\text{-}3)\mathrm{u(t\text{-}3)}$$

6) Find the inverse Laplace Transform of $\frac{7-4\, e^{-4s}-3e^{-8s}}{s}$

Solution :

$$L^{-1}\left[\frac{7-4\, e^{-4s}-3e^{-8s}}{s}\right] = 7L^{-1}\left[\frac{1}{s}\right] - 4L^{-1}\left[\frac{e^{-4s}}{s}\right] - 3L^{-1}\left[\frac{e^{-8s}}{s}\right]$$

$$= 7 - 4.\,u(t-4) - 3.\,u(t-8)$$

THANK YOU

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