Laplace transforms of standard functions

1. Find $L\left\{\frac{1}{\sqrt{t}}\right\}$ ans: $\sqrt{\frac{\pi}{s}}$

2. Find $L\{\sin(t)\sin(5t)\}$ ans: $\frac{10s}{(s^2+16)(s^2+36)}$

3. Find $L\{\cosh(at)\}$ ans: $\frac{s}{s^2-a^2}(s>a)$

4. Find $L\{e^{2t}+3e^{-t}+5\}$ ans: $\frac{9s^2-10s-10}{s(s+1)(s-2)}$

5. Find $L\{\cos^2(2t-1)\}$ ans: $\frac{1}{2s}+\frac{1}{2}\left(\frac{s\cos(2)+4\sin(2)}{s^2+16}\right)$

Laplace transforms of standard functions

1. Prove that $L\{t^n\} = \dfrac{n!}{s^n + 1}$ where $n = 0, 1, 2, 3, \ldots$
2. Find Laplace transform of

   $(a) \sin^3 t$

   $(b) \cos(3t) \sin^2 t$

   ans: $(a) \dfrac{6}{(s^2+1)(s^2+9)}$

   $(b) \dfrac{2s(s^2-23)}{(s^2+1)(s^2+9)(s^2+25)}$

3. Find Laplace transform of $\cos^3 2t$ ans: $\dfrac{s(s^2+28)}{(s^2+36)(s^2+4)}$

ASSIGNMENT – III

III . Inverse Laplace Transforms using Partial Fractions

1. Find $L^{-1}\left[\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right]$

   Ans : $f(t) = \frac{1}{2}e^t - e^{2t} + \frac{5}{2}e^{3t}$

2. *Find* $L^{-1}\left[\frac{2s^2}{(s^2+1)(s-1)^2}\right]$

   Ans : $f(t) = -\cos t + e^t + te^t$

3. Find $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right]$

   Ans : $f(t) = e^t + e^{-t}\left\{\frac{3}{2}\sin 2t - \cos 2t\right\}$

4. Find $L^{-1}\left[\frac{s+2}{(s^2+4s+8)^2}\right]$

   Ans : $f(t) = \frac{1}{2}te^{-2t}\sin 2t$

5. Find $L^{-1}\left[\frac{s}{s^4+4a^4}\right]$

   Ans : $f(t) = \frac{\sin at \sinh at}{2a^2}$

ASSIGNMENT - IV

IV . Inverse Laplace Transforms of derivatives, integrals :

1. Find $L^{-1}\left[s\log\left(\frac{s+4}{s-4}\right)\right]$

   Ans : $f(t) = \frac{2[4t\cosh 4t - \sinh 4t]}{t^2}$

2. Find $L^{-1}\left[\cot^{-1}\left(\frac{s}{2}\right)\right]$

   Ans : $f(t) = \frac{\sin 2t}{t}$

3. Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$

   Ans : $f(t) = \frac{e^{-bt} - e^{-at}}{t}$

4. Find $L^{-1}\left[\log\left(1 - \frac{a^2}{s^2}\right)\right]$

   Ans : $f(t) = \frac{2(1-\cosh at)}{t}$

5. Find $L^{-1}\left[\log\frac{(s^2+4)}{s(s+4)(s-4)}\right]$

   Ans : $f(t) = \frac{1+2(\cosh 4t - \cos 2t)}{t}$

ASSIGNMENT - V

V. Multiplication by 's', Division by 's', Second Shifting Property :

1. Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$

   Ans : $f(t) = \frac{t \sin at}{2a}$

2. Find $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$

   Ans : $f(t) = \frac{\sin at + at \cos at}{2a}$

3. Find $L^{-1}\left[\frac{1}{s^2(s^2+a^2)}\right]$

   Ans : $f(t) = \frac{1}{a^2}\left(t - \frac{\sin at}{a}\right)$

4. Find $L^{-1}\left[\frac{1}{s^3(s^2+1)}\right]$

   Ans : $f(t) = \frac{t^2}{2} + \cos t - 1$

5. Find $L^{-1}\left[\frac{5-3e^{-3s}-2e^{-7s}}{s}\right]$

   Ans : $f(t) = \begin{cases} 5, & 0 < t < 3 \\ 2, & 3 < t < 7 \\ 0, & t > 7 \end{cases}$

6. Find $L^{-1}\left[\frac{(s+1)e^{-\pi s}}{s^2+s+1}\right]$

   Ans : $e^{-\frac{1}{2}(t-\pi)}\left\{\cos\frac{\sqrt{3}}{2}(t-\pi)u(t-\pi) + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}(t-\pi)u(t-\pi)\right\}$

Transforms of derivatives

1. Find $L\{t\cos at\}$, assuming $L\{\sin at\} = \dfrac{a}{s^2+a^2}$ ans: $\dfrac{s^2-a^2}{(s^2+a^2)^2}$

2. If $L\{\sin at\} = \dfrac{a}{s^2+a^2}$, find $L\{\sin^2 at\}$ ans: $\dfrac{2a^2}{s(s^2+4a^2)}$

Laplace transforms of integrals

1. Find Laplace transform of $\int_0^t e^{-t}\cos(3t)\cos(2t)dt$ ans: $\frac{F(s)}{s} = \frac{(s+1)(s^2+2s+14)}{s(s^2+2s+26)(s^2+2s+2)}$
2. Find Laplace transform of $\int_0^t te^{-t}\sin(2t)dt$ ans: $\frac{4(s+1)}{s(s^2+2s+5)^2}$

ASSIGNMENT - VII

VII. Convolution Theorem :

Obtain the Inverse Laplace Transform of the following using Convolution Theorem

1. $\frac{s}{(s^2+9)^2}$

   Ans : $\frac{t \sin 3t}{6}$

2. $\frac{4s+5}{(s+2)(s-1)^2}$

   Ans : $-\frac{e^{-2t}}{3}+\frac{e^t}{3}+3te^t$

3. $\frac{1}{(s^2+4s+13)^2}$

   Ans : $f(t)=\frac{e^{-2t}}{54}[\sin 3t-3t\cos 3t]$

4. $\frac{s}{(s+2)(s^2+9)}$

   Ans : $\frac{1}{13}\left[e^{-2t}(-2\cos 3t+3\sin 3t)+2\right]$

5. $\frac{s}{(s^2+a^2)^3}$

   Ans : $\frac{1}{8a^3}[\sin at-at\cos at]$

Laplace transform of unit step function

Express the following in terms of unit step function and find it's Laplace transform

1. $f(t)=\begin{cases}\cos t, 0<t<\pi \\ \cos 2t, \pi<t<2\pi \\ \cos 3t, t>2\pi\end{cases}$

2. $f(t)=\begin{cases}t^2, 0<t\le 2 \\ 0, t>2\end{cases}$ ans: $\frac{2}{s^3}(1-e^{-2s})-\frac{4e^{-2s}}{s^2}(s+1)$

3. $f(t)=\begin{cases}t, 0<t\le 2 \\ t^2, t>2\end{cases}$ ans: $\frac{1}{s^2}+\frac{e^{-2s}}{s^3}(2s^2+3s+2)$

II Shifting property

Find the Laplace transforms of the following

1. $t^2u(t-3)$ ans: $\frac{e^{-3s}}{s^3}(9s^2+6s+2)$
2. $e^{-2t}u(t-2)$ ans: $\frac{1}{e^2}\frac{e^{-s}}{(s+2)}$
3. $\sin(t)u(t-\pi)$ ans: $-\frac{e^{-\pi s}}{s^2+1}$

Laplace transform of unit impulse functions

1. Find $L[t\delta(t-1)]$ ans: e^{-s}
2. Find $L\{\delta(t-a)u(t-a)\}$ ans: e^{-as}
3. Find $L\left\{\dfrac{\delta(t-a)}{t}\right\}$ sans: $\dfrac{e^{-as}}{s}$