Unit-1 class-1

Sunday, 11 February 2024 7:14 AM

Unit-1 class-1

Unit-1 class-1

1. $\int_1^2\int_3^4 (xy+e^y)\,dy\,dx$ ans: $\frac{21}{4}+e^4-e^3$

A. $\int_1^2 \left(\frac{xy^2}{2}+e^y\right)_3^4 dx = \int_1^2\left(\frac{16x}{2}+e^4-\frac{9x}{2}-e^3\right)dx$

$= \int_1^2 \left(\frac{7x}{2}+e^4-e^3\right)dx = \left[\frac{7x^2}{4}+x(e^4-e^3)\right]_1^2 = 7+2(e^4-e^3)-\frac{7}{4}-(e^4-e^3)$

$= \frac{21}{4}+e^4-e^3$

2. $\iint (x+y)^2\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ans: $\frac{1}{4}\pi ab\left(a^2+b^2\right)$

A. Assume, $u=\frac{x}{a}$ & $v=\frac{y}{b}$ $\Rightarrow$ $a\,du=dx$ & $b\,dv=dy$

$I = \iint_R (au+bv)^2.\,(ab\,du\,dv)$

$= ab\iint_R (a^2u^2+b^2v^2+2ab\,uv)\,du\,dv \Rightarrow$ bounded by $u^2+v^2=1$

Also, $\iint_R u^2\,du\,dv = \iint_R v^2\,du\,dv$ & $\iint_R uv\,du\,dv=0$

$I = ab\left(a\iint_R u^2\,du\,dv + \cancel{2ab\iint_R uv\,du\,dv}^{\,0} + b\iint_R v^2\,du\,dv\right)$

$= ab(a+b)\iint_R u^2\,du\,dv$

Assume some $J = \iint_R u^2\,du\,dv = \iint_R v^2\,du\,dv$

Then, $J = \frac{1}{2}\iint_R (u^2+v^2)\,du\,dv = \frac{1}{2}\int_0^{2\pi}\int_0^1 r^2(r\,dr\,d\theta)$

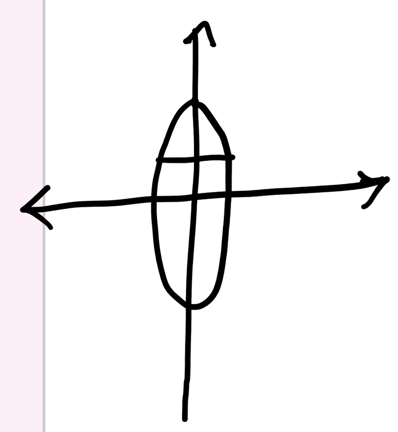
$= \frac{1}{2}\int_0^{2\pi}\left[\frac{r^4}{4}\right]_0^1 d\theta = \frac{1}{8}[\theta]_0^{2\pi} = \frac{\pi}{4}$

$I = \frac{\pi ab(a+b)}{4}$ //

(Or) $I = \iint_A (x+y)^2\,dx\,dy = \iint_A (x^2+y^2+2xy)\,dx\,dy$

$x \to -a\sqrt{1-\frac{y^2}{b^2}}$ to $a\sqrt{1-\frac{y^2}{b^2}}$

$y \to -b$ to b



$I = \int_{-b}^{b}\int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} (x^2+y^2+2xy)\,dx\,dy$

$\iint 2xy = 0$ cuz $a^2\left(1-\frac{y^2}{b^2}\right)-a^2\left(1-\frac{y^2}{b^2}\right)=0$

$= 2\times 2\int_0^b\int_0^{a\sqrt{1-\frac{y^2}{b^2}}} (x^2+y^2)\,dx\,dy$

$= 4\int_0^b \left[\frac{x^3}{3}+xy^2\right]_0^{a\sqrt{1-\frac{y^2}{b^2}}} dy$

$= 4\int_0^b \left(\frac{a^3}{3}\left(1-\frac{y^2}{b^2}\right)^{3/2} + ay^2\sqrt{1-\frac{y^2}{b^2}}\right)dy$

$\frac{y}{b}=\sin t \Rightarrow dy = b.\cos t.dt$

$y=0 \Rightarrow t=0$; $y=b \Rightarrow t=\pi/2$

$= 4\int_0^{\pi/2}\left(\frac{a^3}{3}\left(1-\sin^2 t\right)^{3/2}.\,b.\cos t\,dt + a\,b^2\sin^2 t.\cos t.\,b\cos t\,dt\right)$

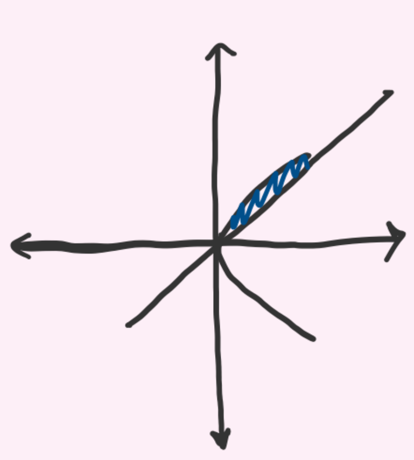
$= 4\int_0^{\pi/2}\left(\frac{a^3b}{3}\cos^4 t\,dt + \frac{ab^3}{4}\sin^2 t.\cos^2 t\,dt\right)$ [Solved using calculator]

$= 4\left(\frac{a^3b}{3}\left(\frac{\sin 4t+8\sin 2t+12t}{32}\right)_0^{\pi/2} + \frac{ab^3}{4}.\left(-\frac{\sin 4y-4y}{32}\right)_0^{\pi/2}\right)$

$= 4\left(\frac{a^3b}{\cancel{3}}.\frac{\cancel{3}\pi}{16} + \frac{ab^3}{4}.\frac{\pi}{4}\right)$

$= \frac{4ab\pi}{16}\left(a^2+b^2\right) = \frac{\pi ab\left(a^2+b^2\right)}{4}$

3. $\iint xy(x+y)\,dx\,dy$ over the area between $y=x^2$ and $y=x$ ans: $\frac{3}{56}$



$\parallel^{el}$ to y-axis: y : x^2 to x

x : 0 to 1

$\int_0^1\int_{x^2}^{x} xy(x+y)\,dy\,dx$

$= \int_0^1 \left(\frac{x^2y^2}{2}+\frac{xy^3}{3}\right)_{x^2}^{x} dx$

$= \int_0^1 \left(\frac{x^4}{2}+\frac{x^4}{3}-\frac{x^6}{2}-\frac{x^7}{3}\right)dx$

$= \int_0^1 \left(\frac{5x^4}{6}-\frac{x^6}{2}-\frac{x^7}{3}\right)dx$

$= \left[\frac{5x^5}{30}-\frac{x^7}{14}-\frac{x^8}{24}\right]_0^1$

$= \frac{5}{30}-\frac{1}{14}-\frac{1}{24}$

$= 3/56$