ENGINEERING MATHEMATICS - II

UE20MA151

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ENGINEERING MATHEMATICS - II

Unit 4 : Inverse Laplace Transform

Session : 7

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Contents

- Convolution Theorem
- Inverse Laplace transform of functions using Convolution theorem

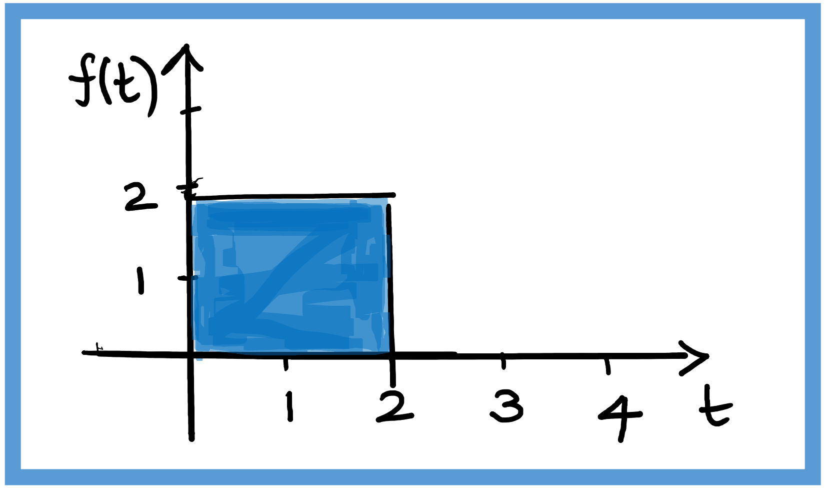
Definition :

The convolution of two functions $f(t)$ and $g(t)$ is denoted by $(f * g)(t)$ and is defined as

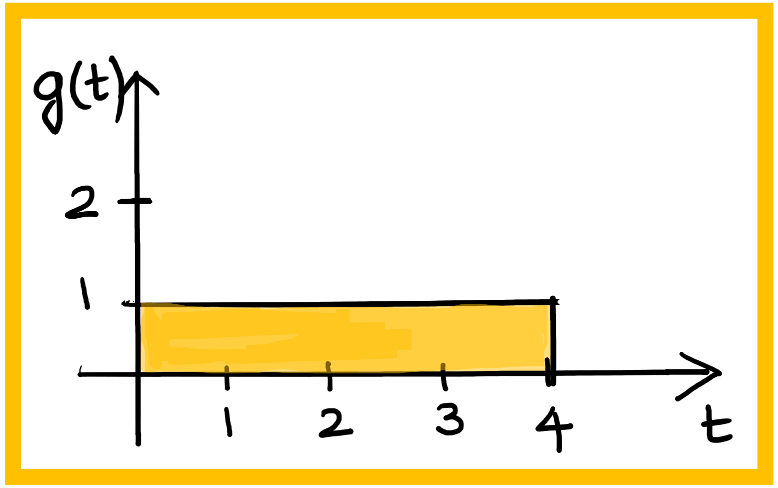
$$(f * g)(t) = \int_{0}^{t} f(u)g(t-u)du$$

$f * g$ is called as the convolution or faltung of f *and* g and is regarded as a generalized product of these functions.

CONVOLUTION- INTUITION



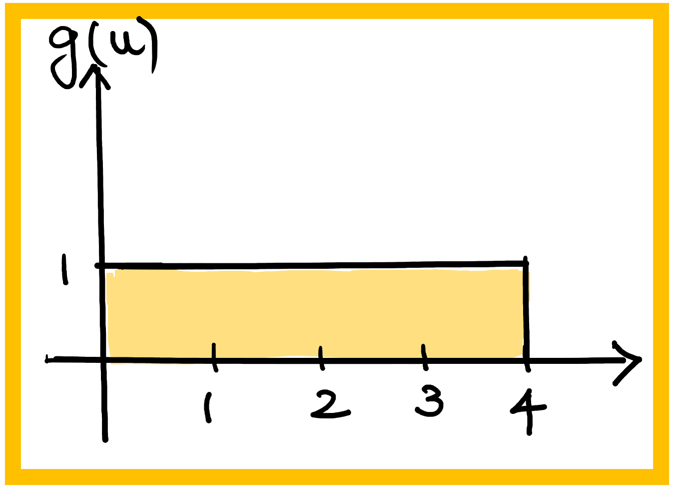
f(t)

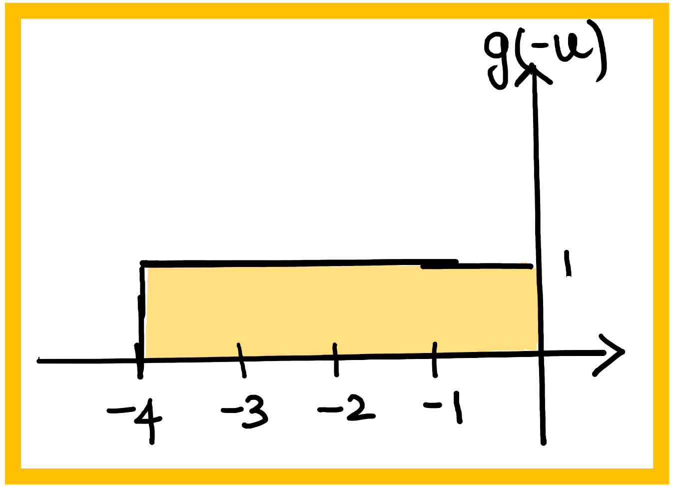


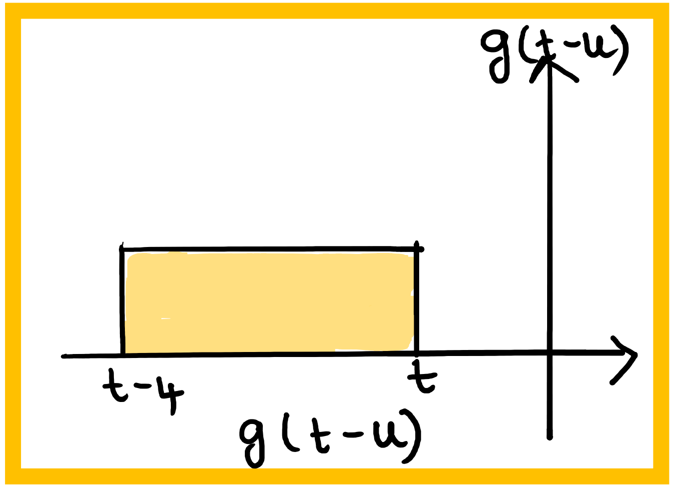
g(t)

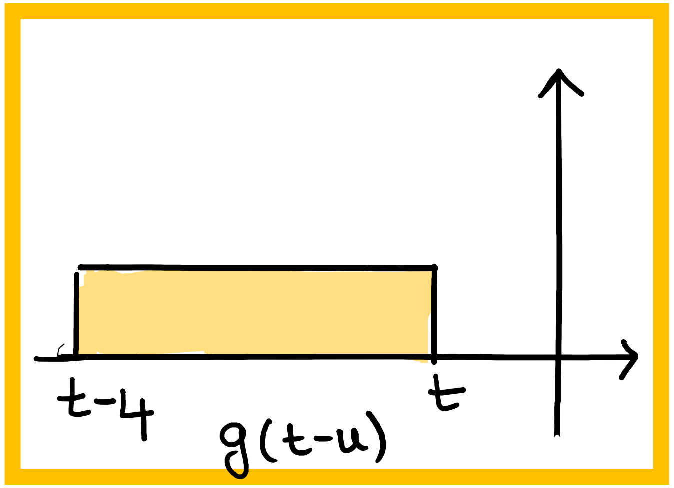
$$(f * g)(t) = \int_0^t f(u)g(t-u)\,du$$

CONVOLUTION- INTUITION









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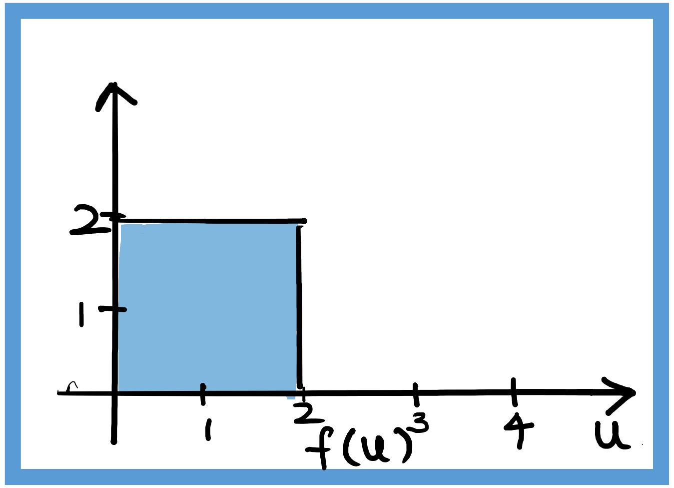
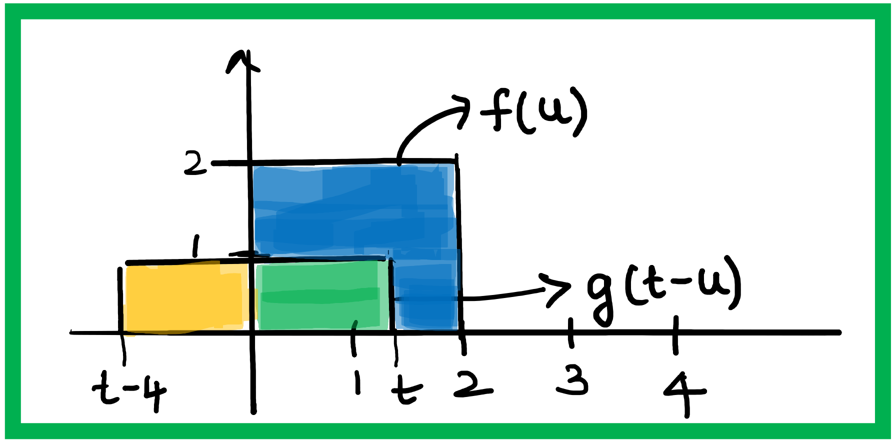
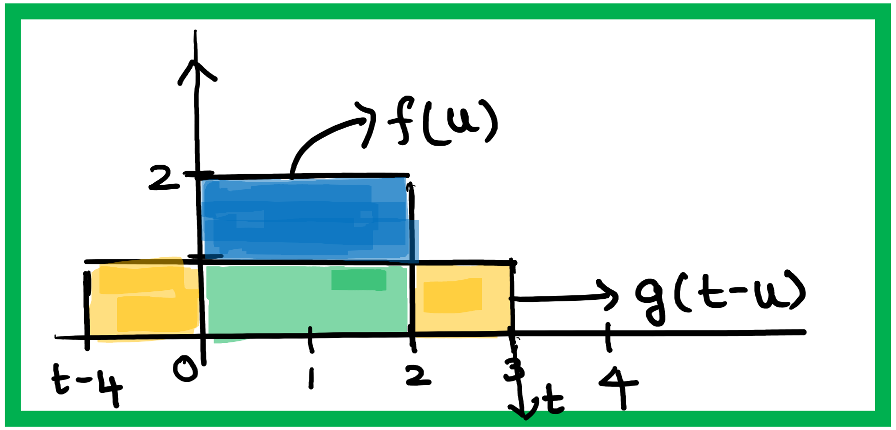
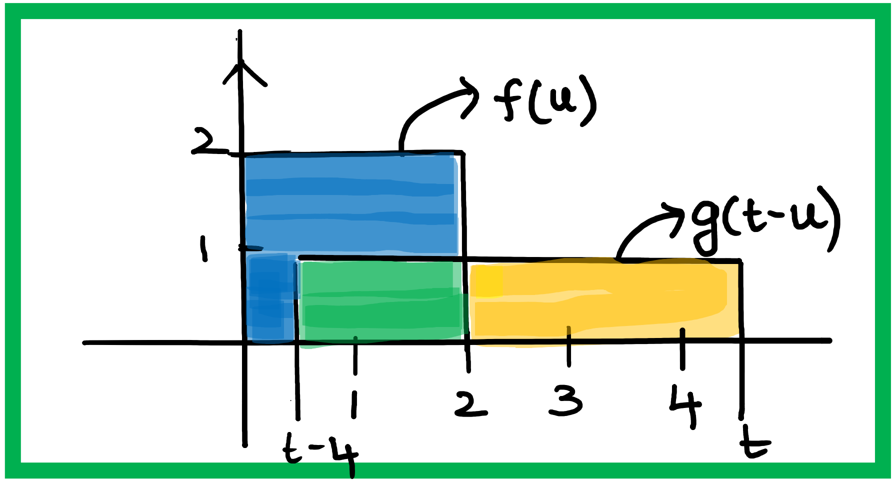
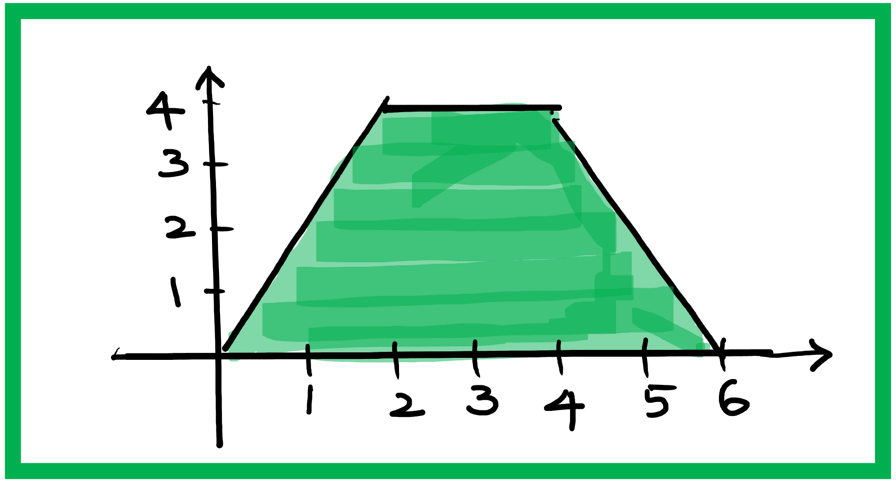


INVERSE LAPLACE TRANSFORM









$$\rightarrow \int_0^t f(u)\cdot g(t-u)\,du$$

$2t$	$0<t<2$
4	$2<t<4$
$-2t+12$	$4<t<6$

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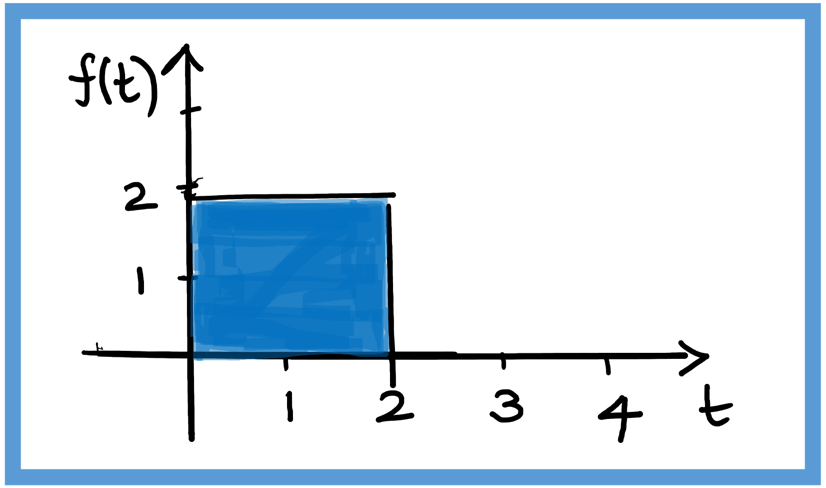
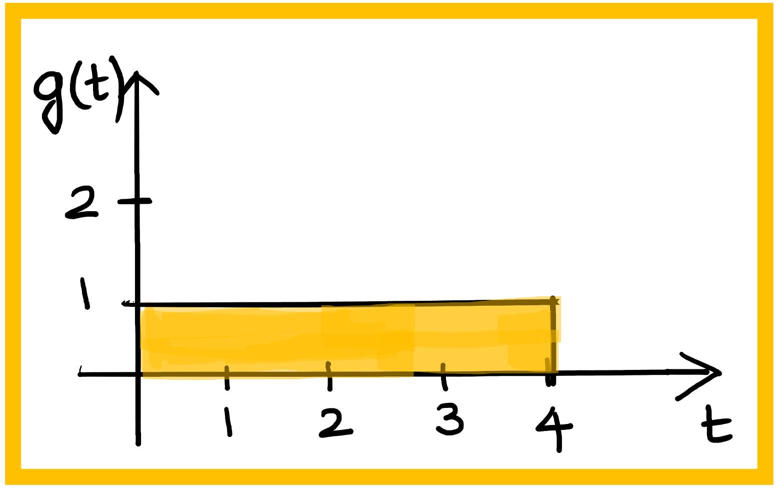
CONVOLUTION- INTUITION

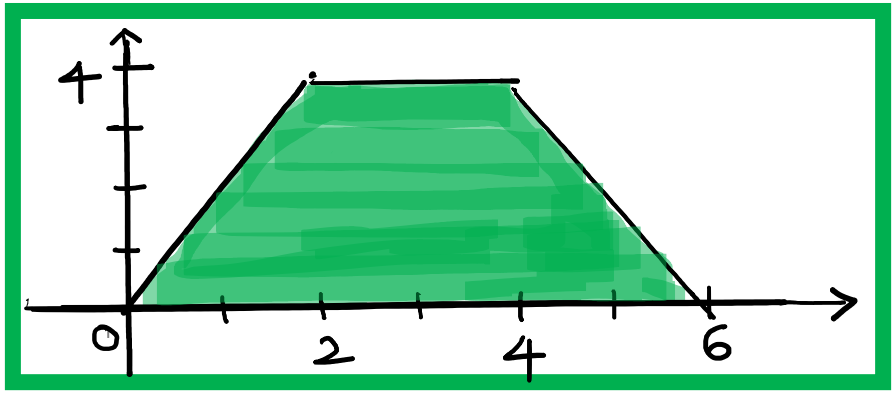


f(t)

$*$



g(t)



$$\rightarrow f(t) * g(t) = \int_0^t f(u) \cdot g(t-u)\,du$$

Properties of convolution integral

1) $(f * g)(t) = (g * f)(t)$

2) $[(f * g) * h](t) = [f * (g * h)](t)$

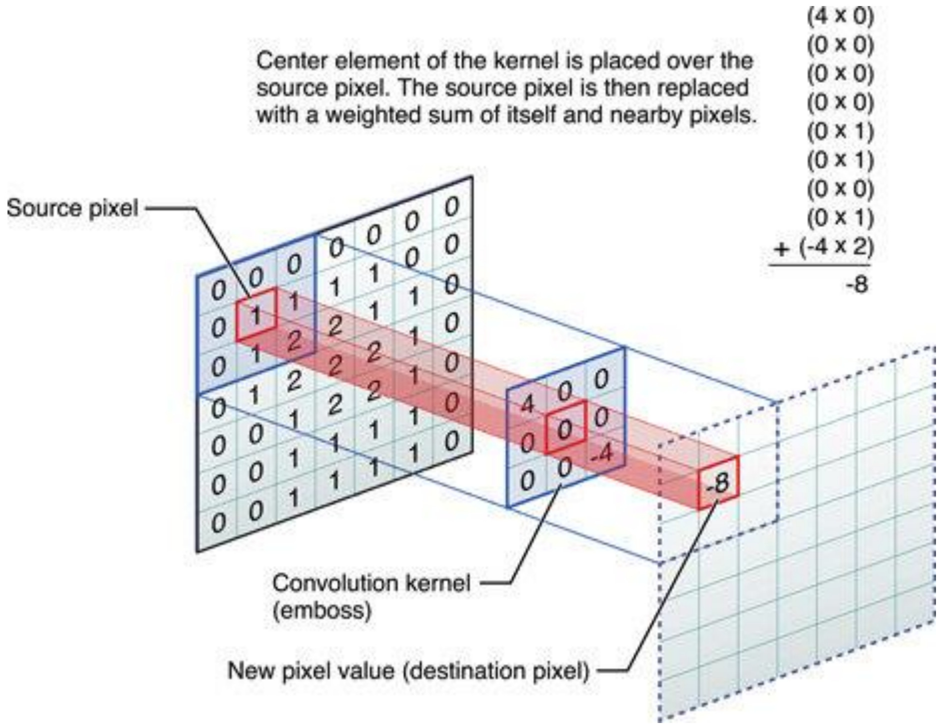
3) $[f * (g + h)](t) = (f * g)(t) + (f * h)(t)$

4) $f * 0 = 0 * f = 0$

Applications of Convolution:

CNN : Convolutional neural networks

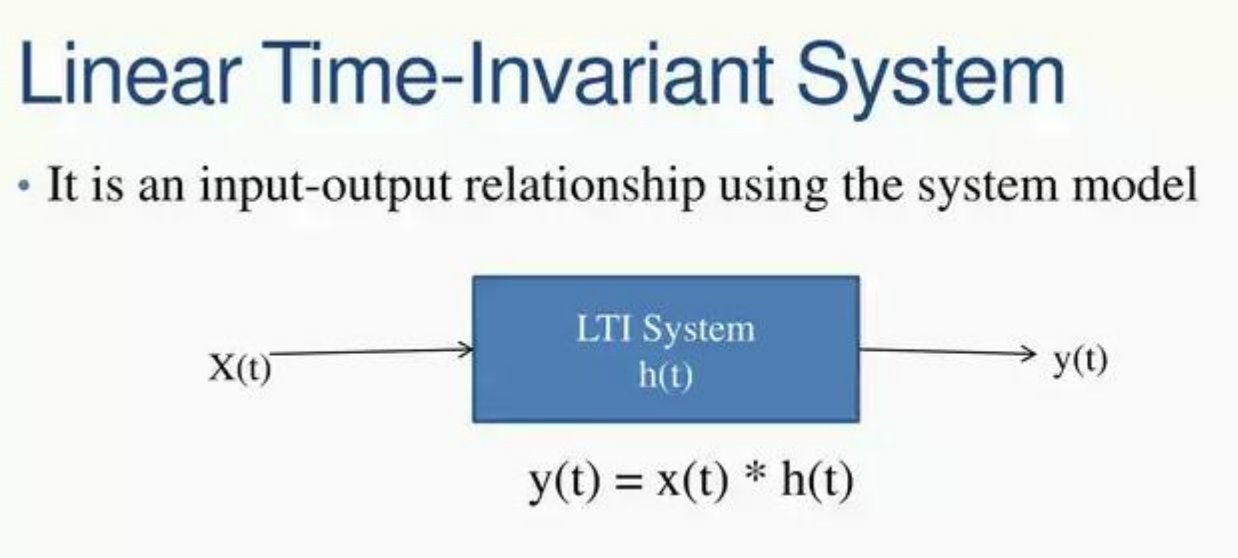
Image processing
Natural language processing
Human pose estimation
Speech recognition



- Applications of Convolution:

LTI : Linear time invariant systems

Signal processing
Circuit analysis



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1) Verify Convolution theorem for the functions f(t)= t and g(t) = cost.

Solution:

We have to verify

$$L^{-1}[F(s).G(s)] = f(t) * g(t).$$

LHS: $f(t) = t \quad L\{f(t)\} = F(s) = \frac{1}{s^2}$

$g(t) = \cos t \quad L\{g(t)\} = G(s) = \frac{s}{s^2+1}$

Therefore,

$$L^{-1}[F(s).G(s)] = L^{-1}\left[\frac{1}{s^2} \cdot \frac{s}{s^2+1}\right] = L^{-1}\left[\frac{1}{s(s^2+1)}\right] = \int_0^t \sin t \, dt$$

= 1- cos t

RHS:

$$f(t) * g(t) = \int_{0}^{t} \mathrm{f(u)g(t-u)du}$$

$$= \int_{0}^{t} \mathrm{u}\, cos(\mathrm{t-u})\mathrm{du}$$

$$=\left[u\left(\frac{\sin(t-u)}{-1}\right) - 1.\left(\frac{\cos(t-u)}{-1}\right)\right]_0^t$$

= 1-cos t

LHS = RHS.

Hence the theorem is verified.

2) Using convolution theorem find $\mathbf{L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]}$

Solution : Let $F(s) = \frac{1}{s+1}$ and $G(s) = \frac{1}{s^2+1}$.

Then $L^{-1}\left[\frac{1}{s+1}\right] = e^{-t} = f(t)$ and $L^{-1}\left[\frac{1}{s^2+1}\right] = \text{sint} = g(t)$.

Using Convolution Theorem,

$$L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right] = F(s) * G(s) = \int_0^t f(u)g(t-u)du$$

$= \int_0^t e^{-u}\sin(t-u)du$

$= \frac{e^{-u}}{2}\left[-\sin(t-u) + \cos(t-u)\right]_0^t$

$\{$Using $\int e^{ax}\text{sinbxdx} = \frac{e^{ax}}{a^2+b^2}(\text{asinbx} - \text{bcosbx})\}$

$= \frac{1}{2}\left(e^{-t} + \mathbf{sint} - \mathbf{cost}\right)$

THANK YOU

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