Class 11 : Applications of Differential Equations- Simple Harmonic Motion

1. In a simple harmonic motion of period $2\pi / \mu$, if the initial displacement be x_0 and the initial velocity be u_0 prove that

(i) amplitude = $\sqrt{x_0^{\,2} + \frac{u_0^{\,2}}{\mu^2}}$

(ii) position at time t = $\sqrt{x_0^{\,2} + \frac{u_0^{\,2}}{\mu^2}} \cos\left\{ \mu t - \tan^{-1}\left(\frac{u_0}{\mu x_0} \right) \right\}$

(iii) time to the position of rest = $\frac{1}{\mu} - \tan^{-1}\left(\frac{u_0}{\mu x_0} \right)$

2. A particle is moving in simple harmonic motion. Find a formula for the displacement x (t) of the particle (with x in meters and t in seconds) given that

(1) the period of the motion is 16 seconds

(2) the particle passes through the centre of oscillation when t = 2

(3) the particle has a velocity of 2π m/s when t = 4

Also find the amplitude of the motion.

(Hint : The general solution $x = c_1 \cos \mu t + c_2 \sin \mu t$ can be written as x(t) = C sin (μt + α) where C and α are constants. The constant α is called the phase shift of the motion and can be taken as zero when the particle starts at the origin)

Answer : $x(t) = 16\sqrt{2} \sin\left(\frac{\pi t}{8} - \frac{\pi}{4} \right)$ and amplitude is $16\sqrt{2}$.

3. A particle moves in a straight line towards the centre of force with an acceleration $\frac{\mu}{x^3}$, starting from rest at a distance " a" from the centre of force. Show that the time of reaching

a point at a distance "b" from the centre of force is $a\sqrt{\frac{a^2-b^2}{\mu}}$ and that its velocity is then $\sqrt{\frac{\mu(a^2-b^2)}{ab}}$. Also show that the time taken to reach the centre is $a^2 / \sqrt{\mu}$.

4. In a certain bay, there is a low tide of 6 meters at 1 a.m and a high tide of 10 meters at 8 a.m. Assuming that the tide motion is simple harmonic, find an expression for the height at time t after 1 a.m and find the first time after 1 a.m when the tide is 9 meters.

Hint : Assume x(t) = C sin (μt + α)

Answer : $x(t) = 8 + 2\sin(\frac{\pi t}{7} - \frac{\pi}{2})$, t = 5. 40 a.m

5. Show that if the displacement of a moving particle at any time t is given by an equation of the form $x = c_1 \cos \mu t + c_2 \sin \mu t$, the motion is simple harmonic. If $c_1 = 3$, $c_2 = 4$ and $\mu = 2$ determine the period, amplitude, maximum velocity and maximum acceleration of the motion.

Answer : π , 5 cm, 10 cm/ sec , 20 cm/ sec^2.

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