Class 7 - Cauchy's Linear Differential Equations

 Solve the following Differential Equations :

1. $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + y = 2\cos^2(\log x)$

Answer : $y = a\cos(\log x) + b\sin(\log x) + 1 - \dfrac{1}{3}\cos(2\log x)$

2. $x^2 \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} + 4y = \log x + \cos(\log x)$

Answer : $y = x(a\cos\sqrt{3}\log x + b\sin\sqrt{3}\log x) + \dfrac{1}{13}(3\cos(logx) - 2\sin(\log x)) + \dfrac{1}{4}(\log x + 1/2)$

3. $(x^2 D^2 + xD + 1)y = \log x \sin(\log x)$

Answer : $y = a\cos(logx) + b\sin(\log x) - \dfrac{1}{4}(\log x)^2\cos(\log x) + \dfrac{1}{4}\log x \sin(\log x)$

4. $x^2 \dfrac{d^2y}{dx^2} - 3x \dfrac{dy}{dx} + 5y = x^2\sin(\log x)$

Answer : $y = x^2\{a\cos(\log x) + b\sin(\log x)\} - \dfrac{x^2}{2}\log x \cos(\log x)$

5. $x^2 \dfrac{d^2y}{dx^2} - 3x \dfrac{dy}{dx} + 4y = 2x^2 + \dfrac{1}{x}$

Answer : $y = (a + b\log x)x^2 + 2x^2\log x + \dfrac{1}{9x}$

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