ENGINEERING MECHANICS - STATICS

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ENGINEERING MECHANICS

Couple

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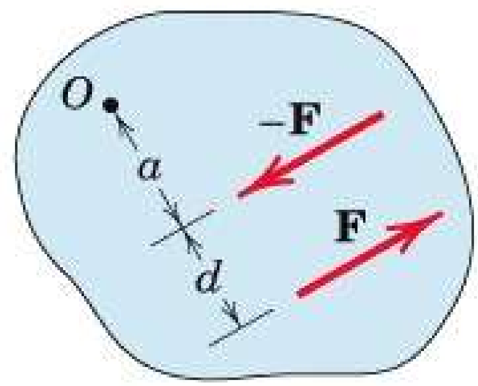
Couple

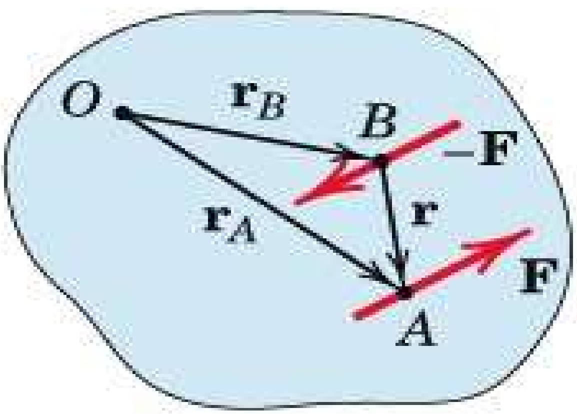
- The moment produced by two equal, opposite, and noncollinear forces is called a ***couple.***
- Consider the action of two equal and opposite forces **F** and **-F** a distance *d* apart, as shown in *Fig.*
- The combined moment of the two forces about an axis normal to their plane and passing through any point such as *O* in their plane is the couple ***M.***
- This couple has a magnitude $M = Fd$



(a)

Vector Algebra Method

- We may also express the moment of a couple by using vector algebra. With the cross-product notation , the combined moment about point *O of the forces forming the couple of Fig is*



(b)

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

Characteristics of the couple

- The algebraic sum of the forces, having the couple, is zero.
- The algebraic sum of moment of the forces, constituting couple, about any point is the same, and equal to the moment of couple itself.
- A couple can't be balanced by a single force, but can be balanced only by a couple, however of opposite sense.
- Any number of coplanar couples can be reduced to single couple, whose magnitude will be equal to algebraic sum of moments of all the couples.

Equivalent Couples

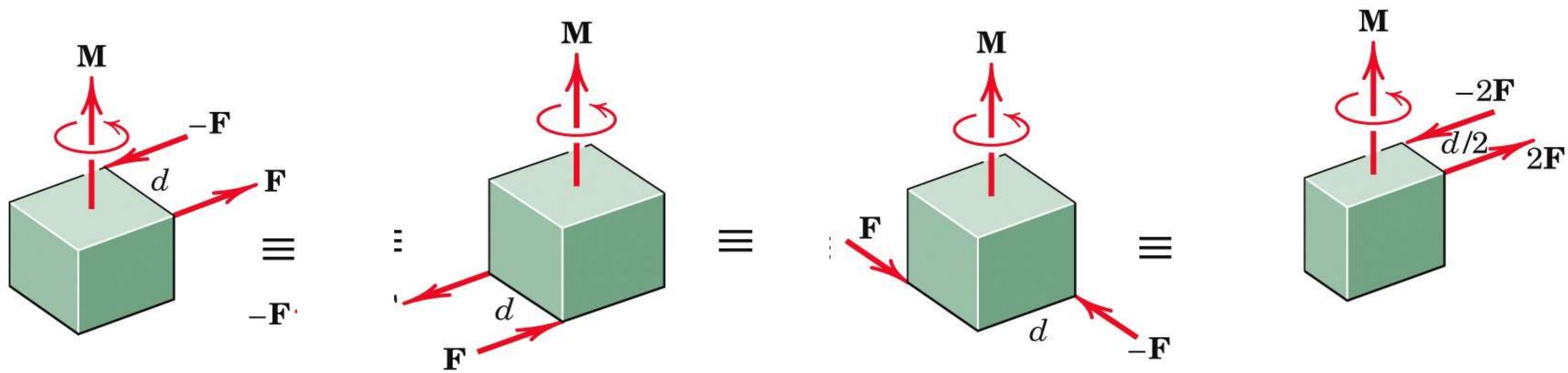


Figure 2-11

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. Figure 2/11 shows four different configurations of the same couple M. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

Force–Couple Systems

- The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force.
- We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

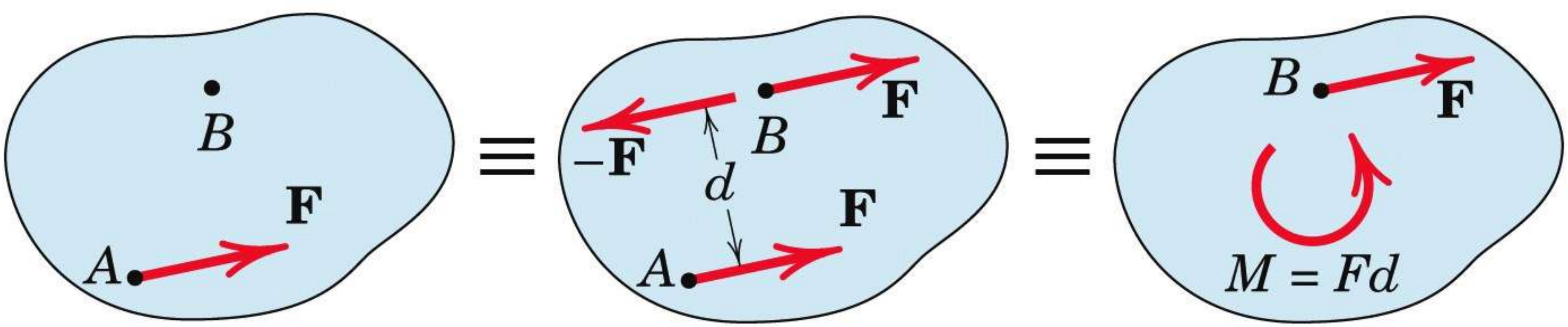
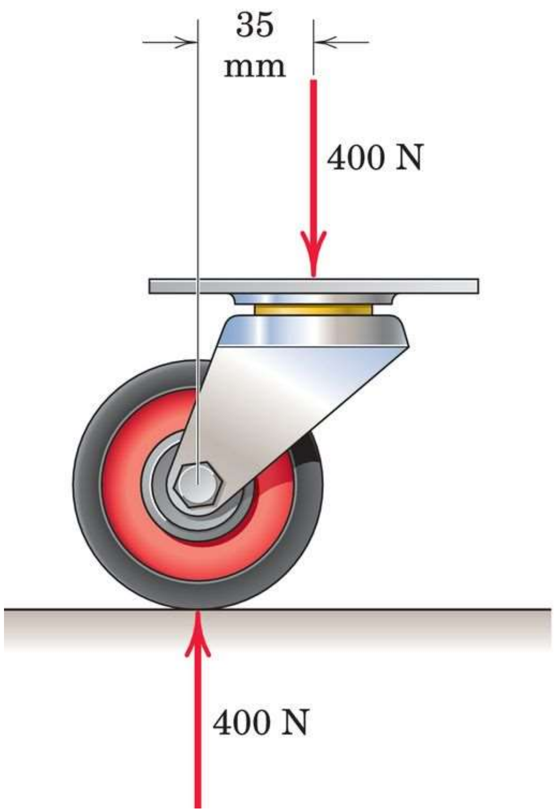


Figure 2-12

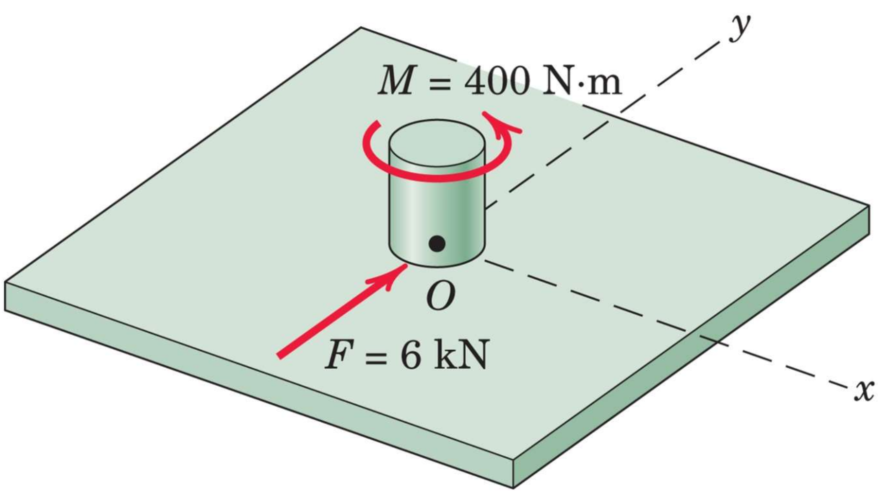
2/59) The caster unit is subjected to the pair of 400-N forces shown. Determine the moment associated with these forces.



$$\overset{+}{\curvearrowright} \quad M = Fd = 400(0.035)$$

$$= \underline{14 \text{ N·m} \quad CW}$$

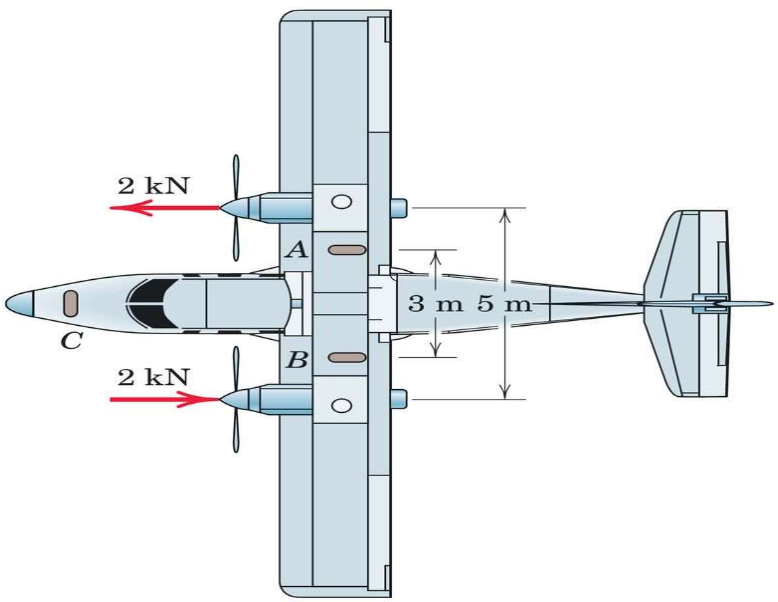
2/61) The indicated force-couple system is applied to a small shaft at the center of the plate. Replace this system by a single force and specify the coordinate of the point on the x-axis through which the line of action of this resultant force passes.

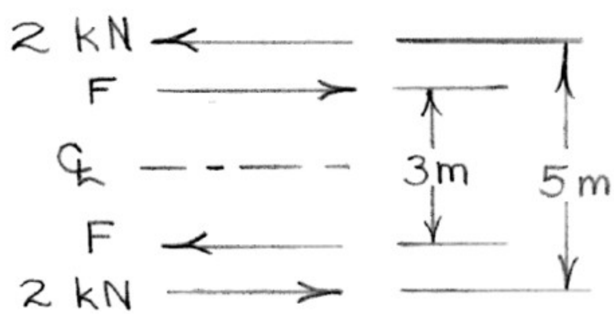


$\underline{R} = 6\underline{j}$ kN @ $x = \frac{400}{6000} = 0.0667$ m

or $x = 66.7$ mm

2/63) As part of a test, the two aircraft engines are revved up and the propeller pitches are adjusted so as to result in the fore and aft thrusts shown. What force F must be exerted by the ground on each of the main braked wheels at A and B to counteract the turning effect of the two propeller thrusts? Neglect any effects of the nose wheel C, which is turned 90° and unbraked.

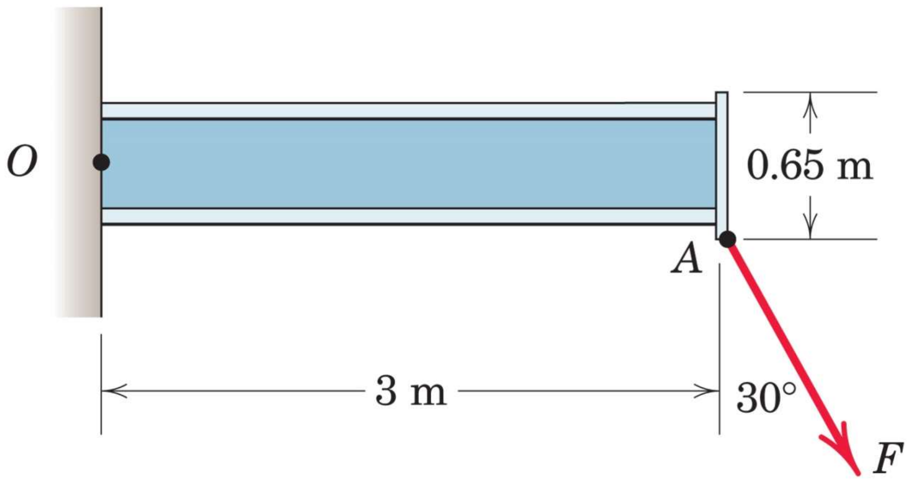




$$\circlearrowleft + \quad \Sigma M = 2(5) - F(3) = 0$$

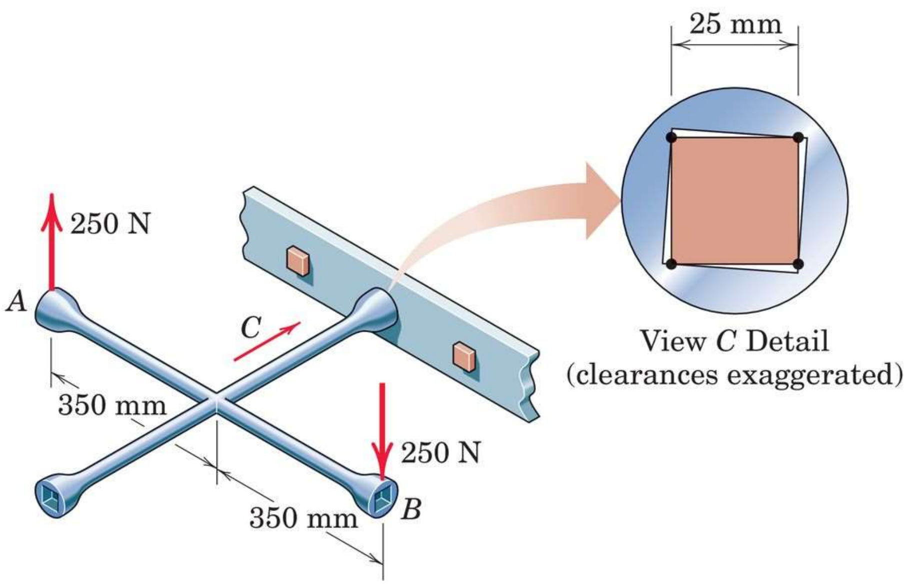
$$\underline{F = 3.33 \text{ kN}}$$

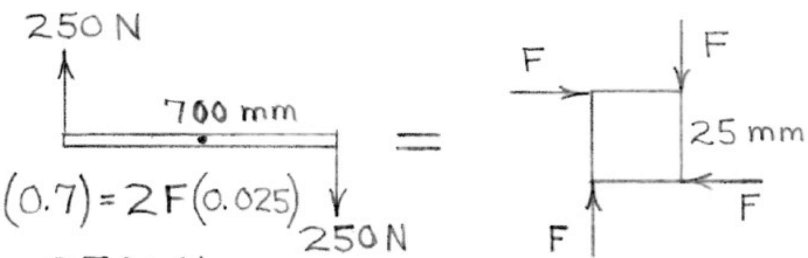
2/64) The cantilevered W530 X 150 beam shown is subjected to an 8-kN force F applied by means of a welded plate at A. Determine the equivalent force couple system at the centroid of the beam cross section at the cantilever 0.



$$M_o = 8\cos 30^\circ (3) - 8\sin 30^\circ \left(\frac{0.65}{2}\right) \longrightarrow \underline{M_o = 19.48 \text{ kN}\cdot\text{m CW}}$$

2/67) A lug wrench is used to tighten a square-head bolt. If 250-N forces are applied to the wrench as shown, determine the magnitude F of the equal forces exerted on the four contact points on the 25-mm bolt head so that their external effect on the bolt is equivalent to that of the two 250-N forces. Assume that the forces are perpendicular to the flats of the bolt head.

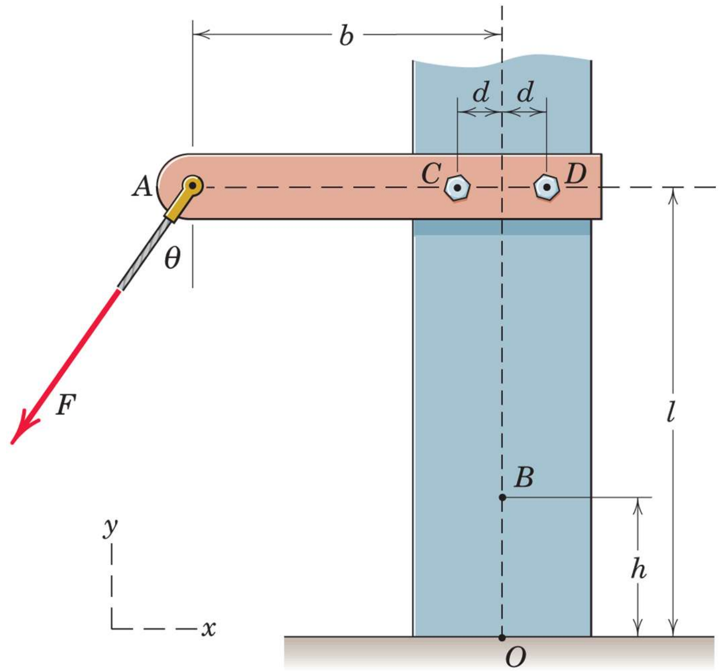




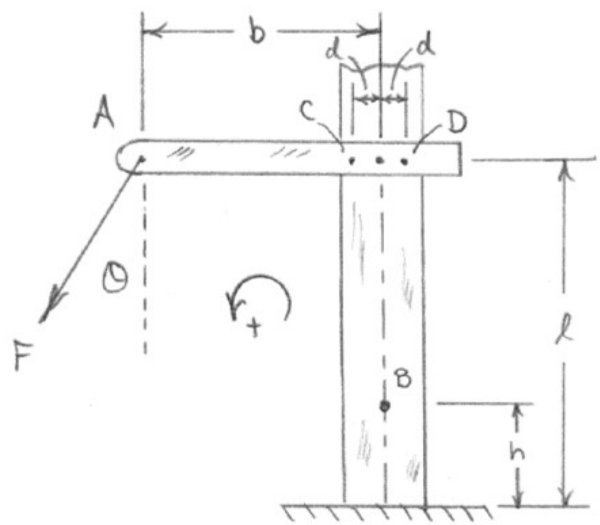
$$M = 250(0.7) = 2F(0.025)$$

$$\underline{F = 3500\ N}$$

2/68) The force F is applied at the end of arm ACD, which is mounted to a vertical post. Replace this single force F by an equivalent force-couple system at B. Next, redistribute this force and couple by replacing it with two forces acting in the same direction as F, one at C and the other at D, and determine the forces supported by the two hex-bolts. Use values of F = 425 N, θ = 30^0, b = 1 .9 m, d = 0.2 m, h = 0.8 m, and l = 2.75 m.



ENGINEERING MECHANICS
Couple - Numerical



$F = 425\ N$
$\theta = 30^\circ$
$b = 1.9\ m$
$d = 0.2\ m$
$h = 0.8\ m$
$\ell = 2.75\ m$

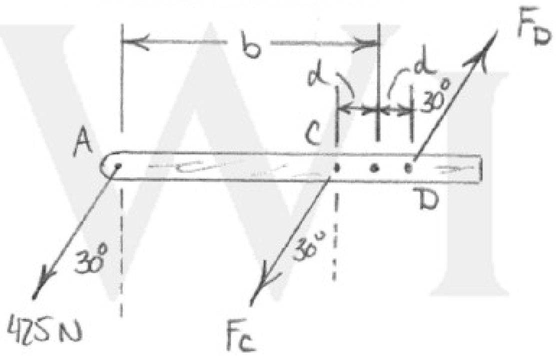
a) Force - Couple At B:

F = 425 N @ 120° CW Below Horizontal

$$M_B = F\cos\theta\, b + F\sin\theta\,(\ell - h) = 425\cos 30^\circ (1.9) + 425 \sin 30^\circ (2.75 - 0.8)$$

$$\therefore M_B = 1114\ N\cdot m\ CCW$$

b) Forces at C and D:



$\Sigma F_{\nearrow}$ (+x): $F_C - F_D = 425$

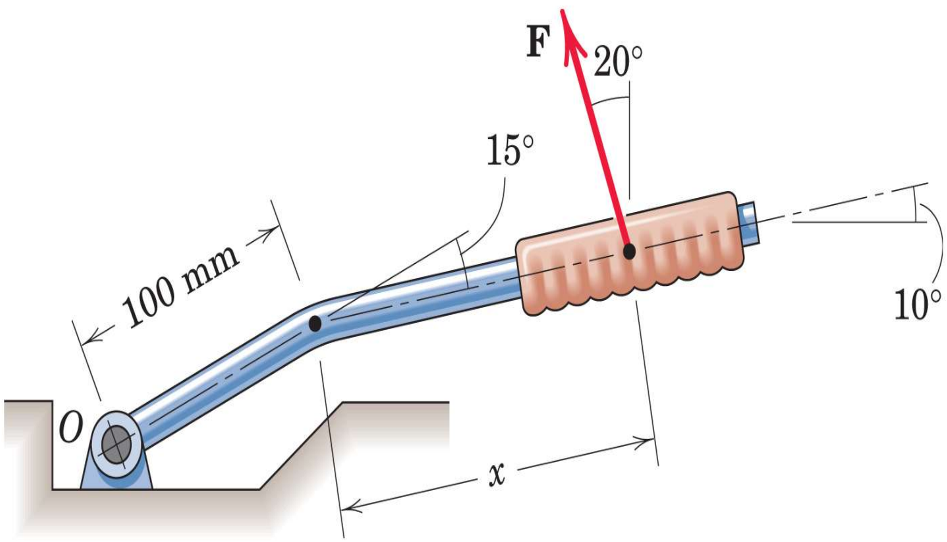
$\Sigma M_A = 0$: $F_d \cos 30°(b+d) - F_C \cos 30°(b-d) = 0$

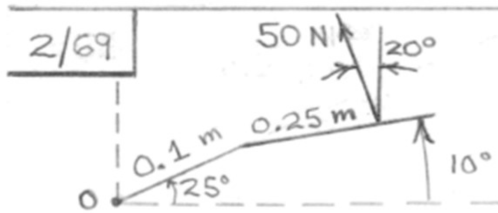
Solving...

$F_C = 2230$ N @ 120° CW below horizontal

$F_D = 1806$ N @ 60° CCW above horizontal

2/69) A force F of magnitude 50 N is exerted on the automobile parking-brake lever at the position x = 250 mm. Replace the force by an equivalent force-couple system at the pivot point 0.





Use principle of moments.

$$\curvearrowleft + \Sigma M_O = 50 \cos 20° [0.1 \cos 25° + 0.25 \cos 10°]$$

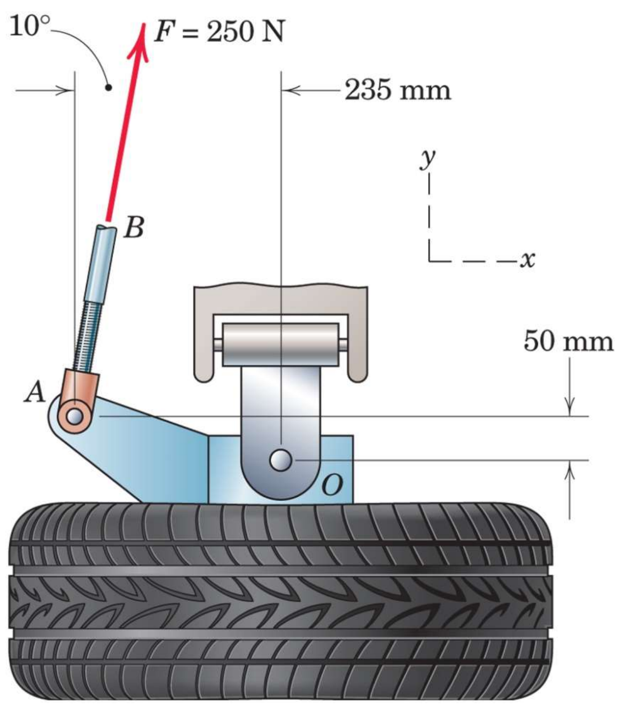
$$+ 50 \sin 20° [0.1 \sin 25° + 0.25 \sin 10°]$$

$$= 17.29 \text{ N·m}$$

Force - Couple System at O:

$$\begin{cases} R = 50 \text{ N} \quad \angle 110° \\ M_O = 17.29 \text{ N·m} \quad \curvearrowleft \end{cases}$$

2/71) The tie-rod AB exerts the 250-N force on the steering knuckle AO as shown. Replace this force by an equivalent force-couple system at 0.



2/71

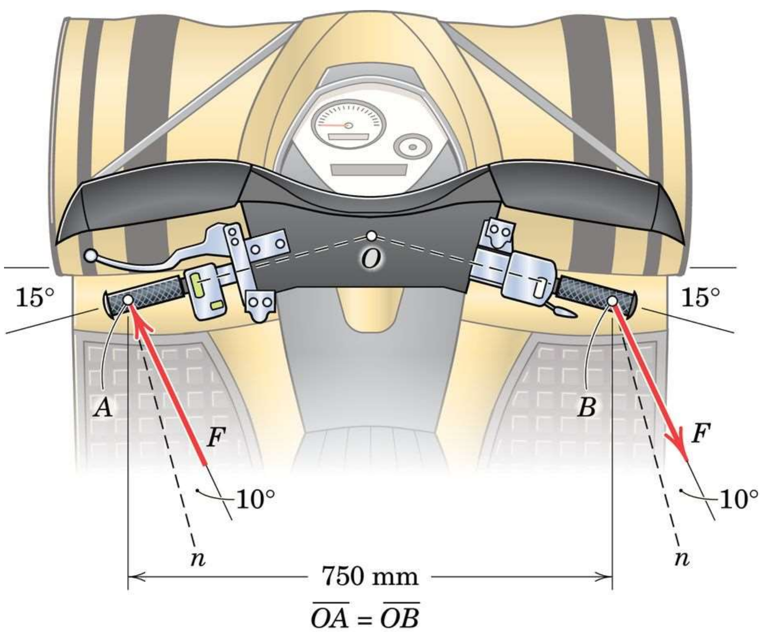
$$\underline{F} = 250(\sin 10°\,\underline{i} + \cos 10°\,\underline{j})$$

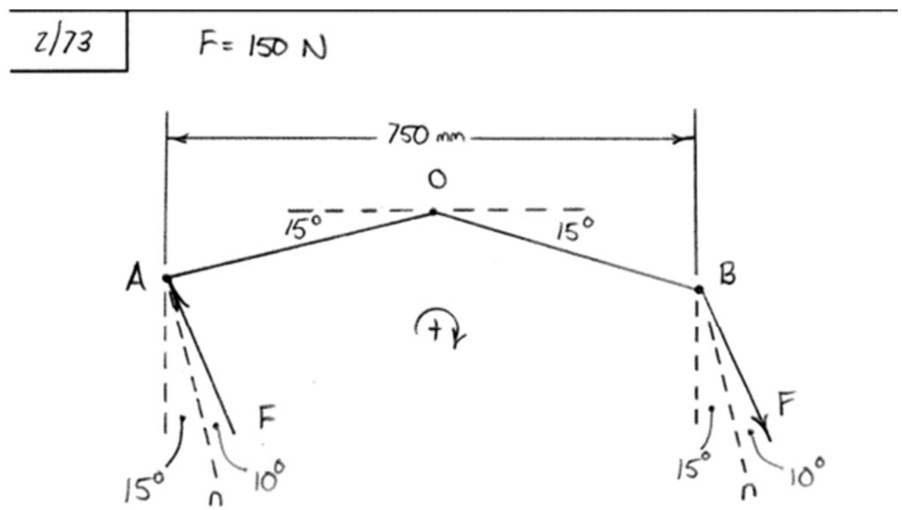
$$= \underline{43.4\,\underline{i} + 246\,\underline{j}\ \text{N}}$$

$$M_O = 250\,[\cos 10°(0.235) + \sin 10°(0.050)]$$

$$= \underline{60.0\ \text{N}\cdot\text{m}\ \ \text{CW}}$$

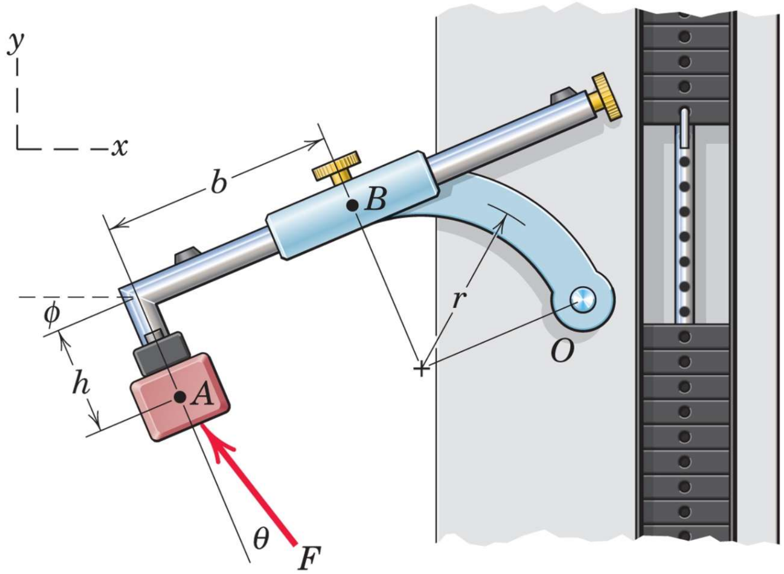
2/73) An overhead view of the handlebars on an all terrain vehicle is shown. If the indicated forces have a magnitude of F = 150 N, determine the moment created by the two forces about the vertical steering axis through point 0. Both n-axes are perpendicular to the left handlebar. Treat the problem as two dimensional.

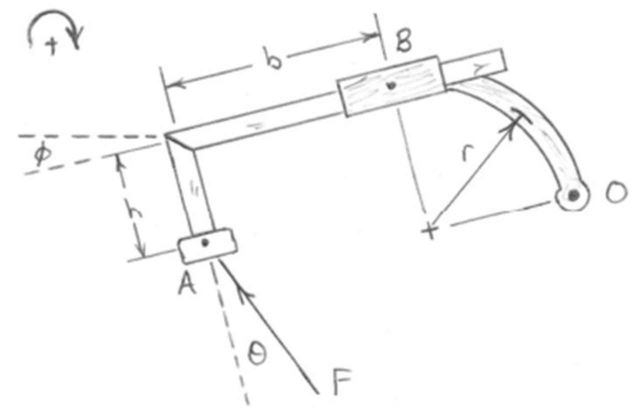




$M_O = 0.750\,(150 \cos 25^\circ) \longrightarrow \underline{M_O = 102.0 \text{ N·m CW}}$

2/74) The force F is applied to the leg-extension exercise machine as shown. Determine the equivalent force couple system at point 0. Use values of F = 520 N, b = 450 mm, h = 2 15 mm, r = 325 mm, $\theta = 15^\circ$, and $\phi = 10^0$





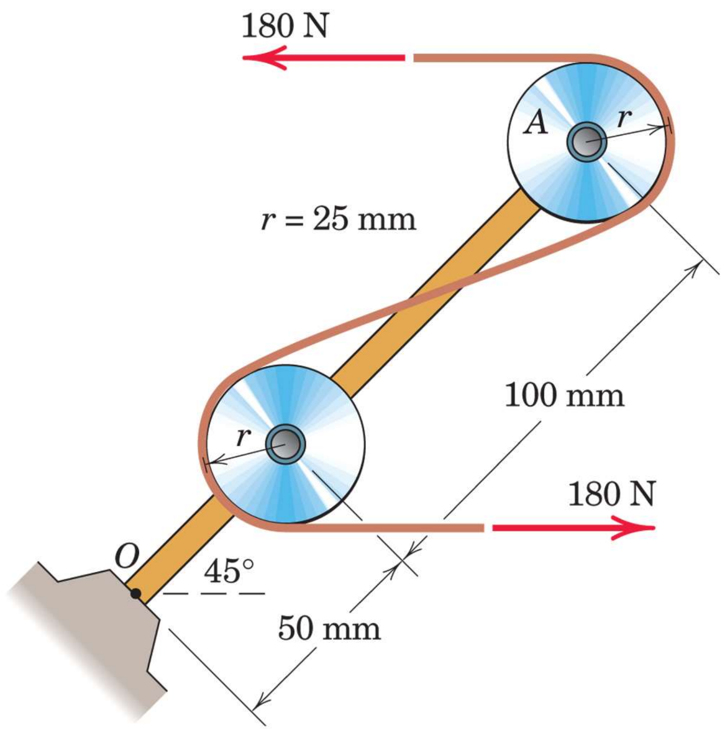
F = 520 N @ 115° CCW ABOVE HORIZONTAL

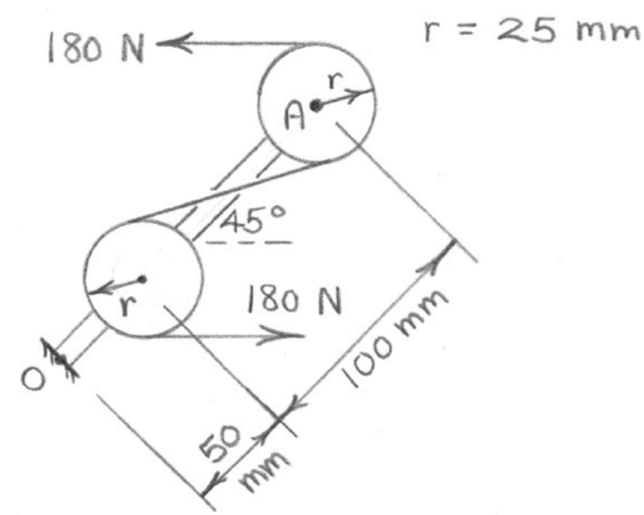
$$M_0 = F\cos\theta(b+r) - F\sin\theta(r-h)$$

$$= 520\cos 15^\circ\left(\frac{450+325}{1000}\right) - 520\sin 15^\circ\left(\frac{325-215}{1000}\right)$$

$\therefore$ $M_0 = 374$ N·m CW

2/75) The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force-couple system at point 0.



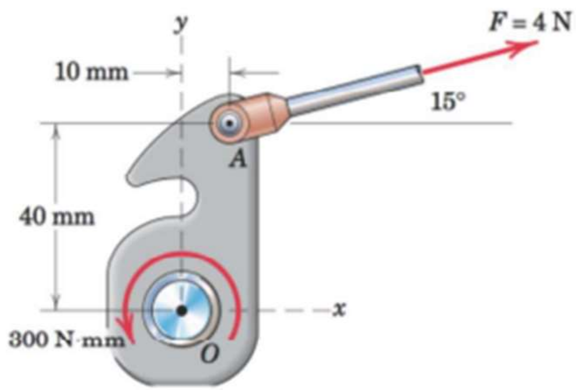


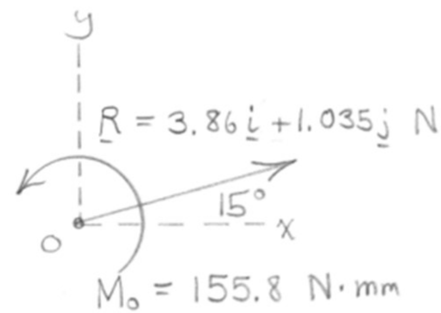
The system at O is a couple.

$\curvearrowleft + M = Fd = 180(100 \sin 45° + 25 + 25)$

$= 21\,700$ N·mm or 21.7 N·m CCW

2/76) The device shown is a part of an automobile seatback- release mechanism. The part is subjected to the 4-N force exerted at A and a 300-N.mm restoring moment exerted by a hidden torsional spring. Determine the y-intercept of the line of action of the single equivalent force.





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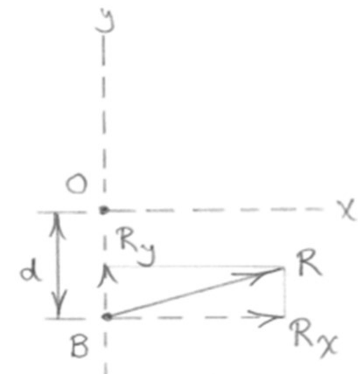


At O:

$\underline{R} = 4(\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j}) = 3.86\underline{i} + 1.035\underline{j}$ N

$\curvearrowleft + \; M_O = 300 - 4\cos 15^\circ(40) + 4\sin 15^\circ(10)$

$= 155.8$ N·mm CCW

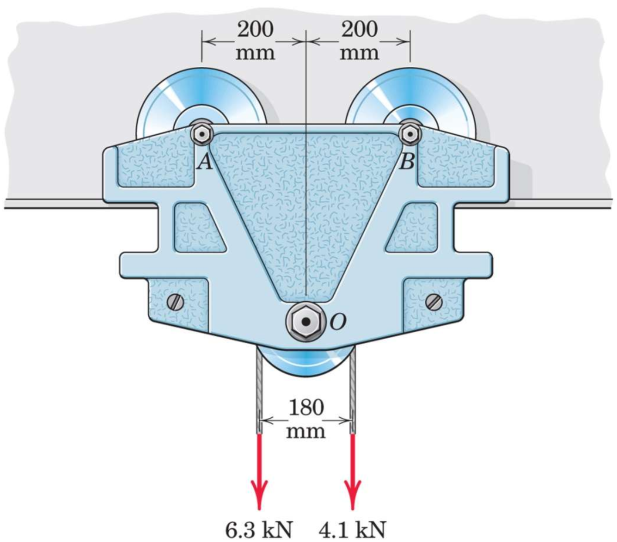
Condition: $R_x d = M_O$

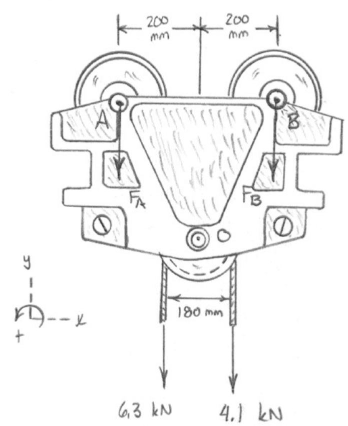
$3.86\, d = 155.8$, $d = 40.3$ mm

So $\underline{y = -40.3 \text{ mm}}$

ENGINEERING MECHANICS
Couple - Numerical

2/77) Replace the two cable tensions which act on the pulley at 0 of the beam trolley by two parallel forces which act at the track-wheel connections A and B.





F_A & F_B ARE DOWN AT A & B.

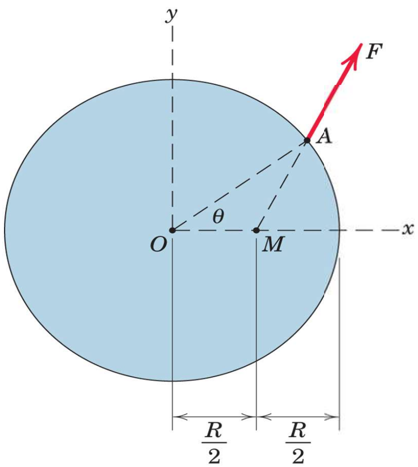
$$\begin{cases} \Sigma F_y: & F_A + F_B = 6.3 + 4.1 \\ \Sigma M_O: & 200\,F_A - 200\,F_B = 90(6.3) - 90(4.1) \end{cases}$$

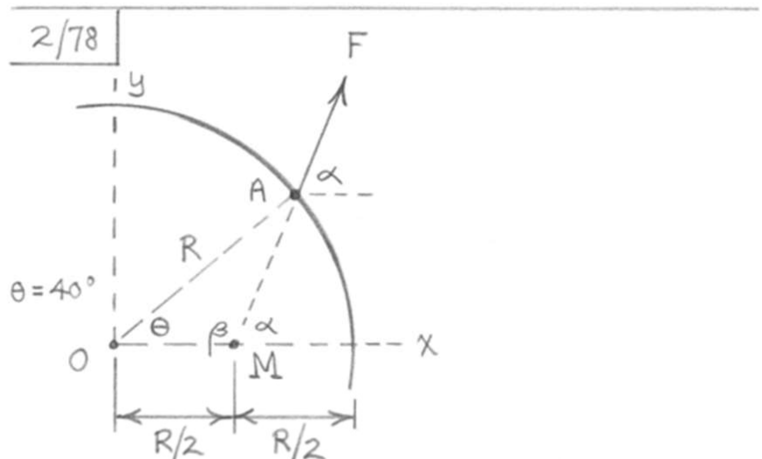
SOLVING...

$$\begin{cases} \underline{F_A = 5.70 \text{ kN}} \\ \underline{F_B = 4.70 \text{ kN}} \end{cases}$$

(BOTH DOWN AS SHOWN)

2/78) The force F acts along line MA, where M is the midpoint of the radius along the x-axis. Determine the equivalent force-couple system at 0 if θ = 40°.





$\overline{AM}^2 = R^2 + (R/2)^2 - 2(R)(\frac{R}{2})\cos 40°$

$= 0.484R^2, \quad \overline{AM} = 0.696R$

$\frac{\sin\beta}{R} = \frac{\sin 40°}{0.696R}, \quad \beta = 112.5°$

$\alpha = 180° - \beta = 180° - 112.5° = 67.5°$

$M_O = (F\sin\alpha)\frac{R}{2} = (F\sin 67.5°)\frac{R}{2}$

$= 0.462FR$

So the force-couple system for θ = 40° is

$$\begin{cases} F \quad \angle 67.5° \\ M_o = 0.462FR \quad CCW \end{cases}$$

THANK YOU

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