5.

1. Find the probability of getting a total of 7 at least once in three tosses of a fair dice.
Ans: 91/216

A. $n = 3 \quad p = \frac{1}{6} \quad q = \frac{5}{6} \quad r = 0$

$P(X = 1) = 1 - P(X = 0)$

$P(X = 0) = {}^3C_0 \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^3$

$= \frac{125}{216}$

$P(X = 1) = 1 - \frac{125}{216} = \frac{91}{216}$

2. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random (i) 1, (ii) 0, (iii) less than 2 bolts will be defective.

A. $p = 20\% = 0.2 \Rightarrow q = 0.8$

$n = 4$

i) $P(X = r) = {}^nC_r \cdot p^r \cdot q^{n-r}$

$P(X = 1) = {}^4C_1 \cdot (0.2)^1 \cdot (0.8)^3$

$= 0.4096$

ii) $P(X = 0) = {}^4C_0 \; (0.2)^0 \cdot (0.8)^4$

$= 0.4096$

iii) $P(X < 2) = P(X = 0) + P(X = 1)$

$= 0.4096 + 0.4096$

$= 0.8192$

3. A communication system consists of 'n' components, each of which will, independently function with probability 'p'. the total system will be able to operate effectively if at least one-half of its components function. For what values of 'p' is a 5-component system more likely to operate effectively than a 3-component system?

A. For 3-Comp System:

$P_3 = {}^3C_2 \, p^2 (1-p) + {}^3C_3 \, p^3$

$= 3p^2(1-p) + p^3$

For 5-Comp System:

$P_5 = {}^5C_3 \, p^3 (1-p)^2 + {}^5C_4 \, p^4 (1-p) + {}^5C_5 \, p^5$

$= 10p^3(1-p)^2 + 5p^4(1-p) + p^5$

According to question, $P_5 > P_3$

$10p^3(1 + p^2 - 2p) + 5p^4 - 5p^5 + p^5 > 3p^2 - 3p^3 + p^3$

$10p^3 + 10p^5 - 20p^4 + 5p^4 - 5p^5 + p^5 > 3p^2 - 3p^3 + p^3$

$10p + 10p^3 - 20p^2 + 5p^2 - 5p^3 + p^3 - 3 + 3p - p > 0$

$6p^3 - 15p^2 + 12p - 3 > 0$

$2p^3 - 5p^2 + 4p - 1 > 0$

Upon solving $p > \frac{1}{2}$ & $p \neq 1$