ENGINEERING MECHANICS

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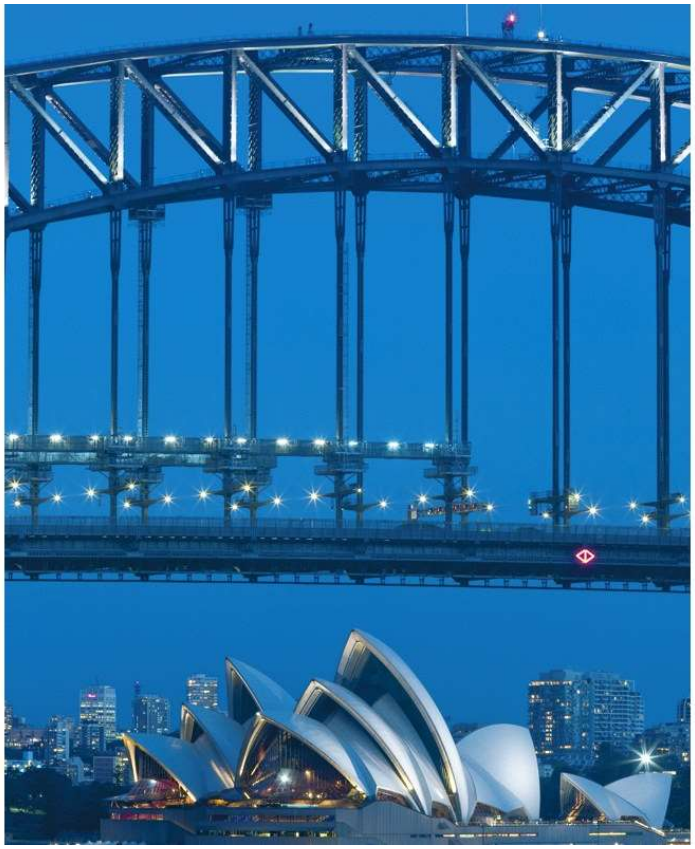
ENGINEERING MECHANICS

Structures



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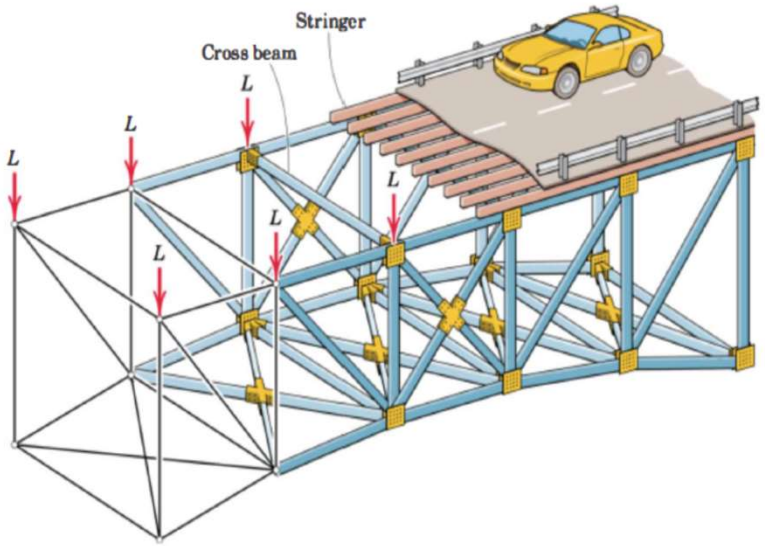
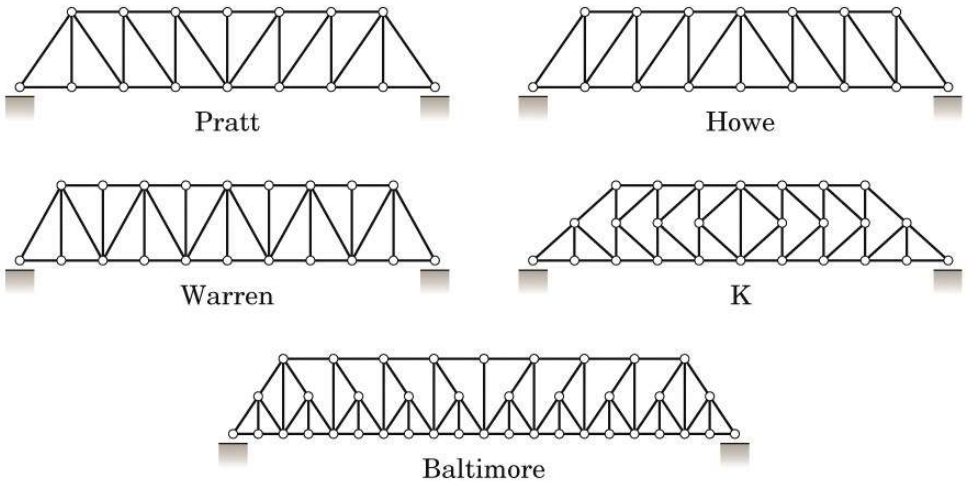
Trusses

- In the Equilibrium conditions, we have considered as single rigid body or a system of connected members treated as a single rigid body.
- In the present chapter, we focus on the determination of the forces internal to a structure-that is, forces of action and reaction between the connected members. An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.
- To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate free body diagram of individual members or combinations of members. This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.

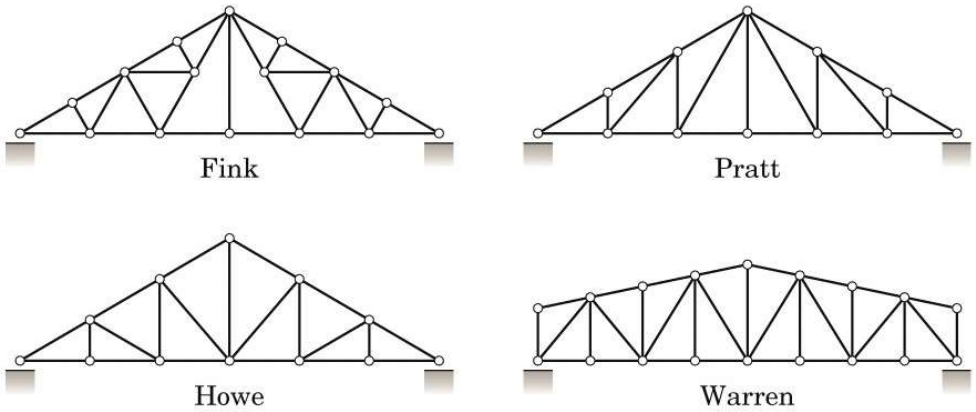


Trusses

- We analyze the internal forces acting in several types of structures-namely, trusses, frames, and machines. In this treatment we consider only statically determinate structures, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration.



Commonly Used Bridge Trusses



Commonly Used Roof Trusses

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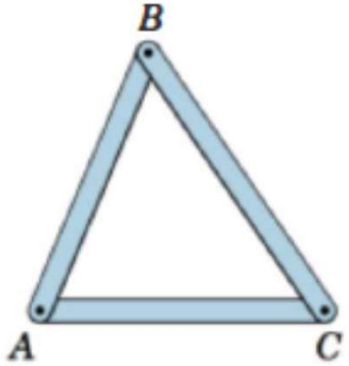
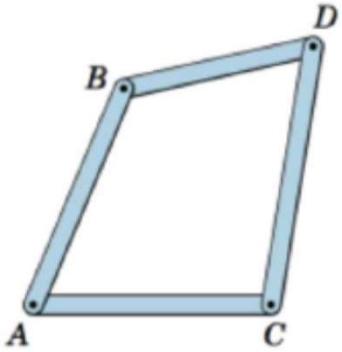
Trusses

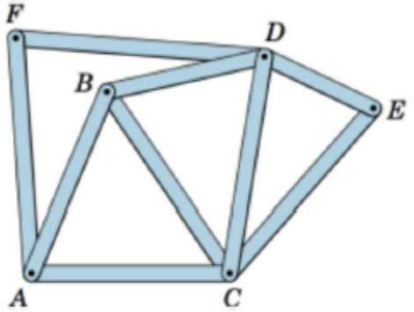
- A framework composed of members joined at their ends to form a rigid structure is called a truss. Bridges, roof supports, derricks, and other such structures are common examples of trusses.
- Structural members commonly used are :- beams, channels, angles, bars, and special shapes which are fastened together at their ends by welding, riveted connections, or large bolts or pins. When the members of the truss lie essentially in a single plane, the truss is called a plane truss.



Basic element of a plane truss is the triangle



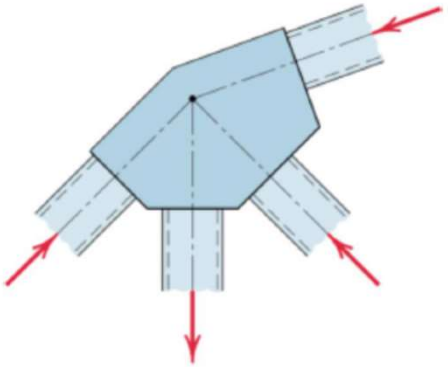
Four or more bars pin-jointed to form a polygon of as many sides constitute a nonrigid frame.

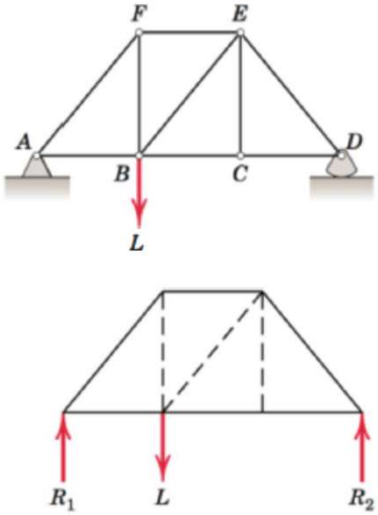


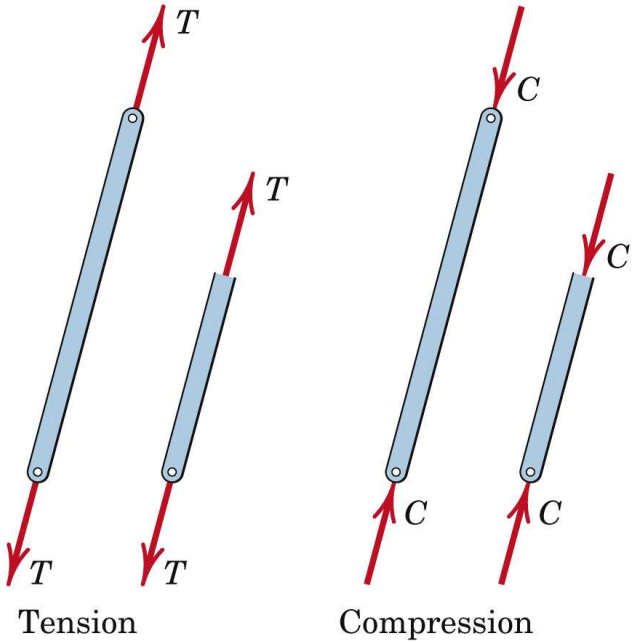
By adding a diagonal bar joining A and D or B and C and thereby forming two triangles.

Trusses

- To design a truss, we must first determine the forces in the various member and then select appropriate sizes and structural shapes to withstand the forces.
- Several assumptions are made in the force analysis of simple trusses.
- First, we assume all members to be two-force members.
- A two-force member is one in equilibrium under the action of two forces only.





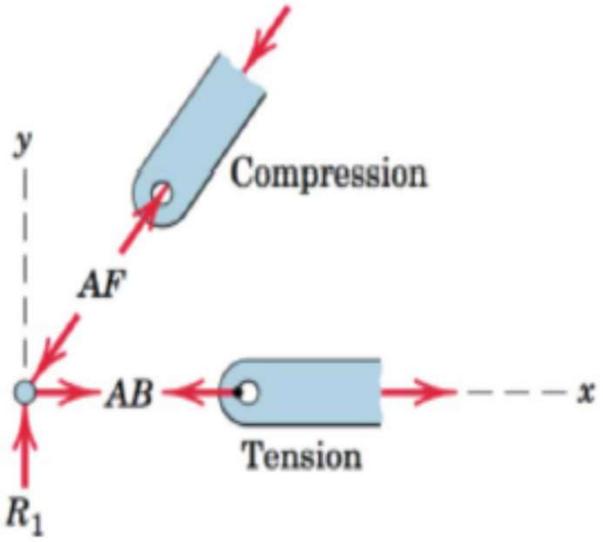


Two-Force Members

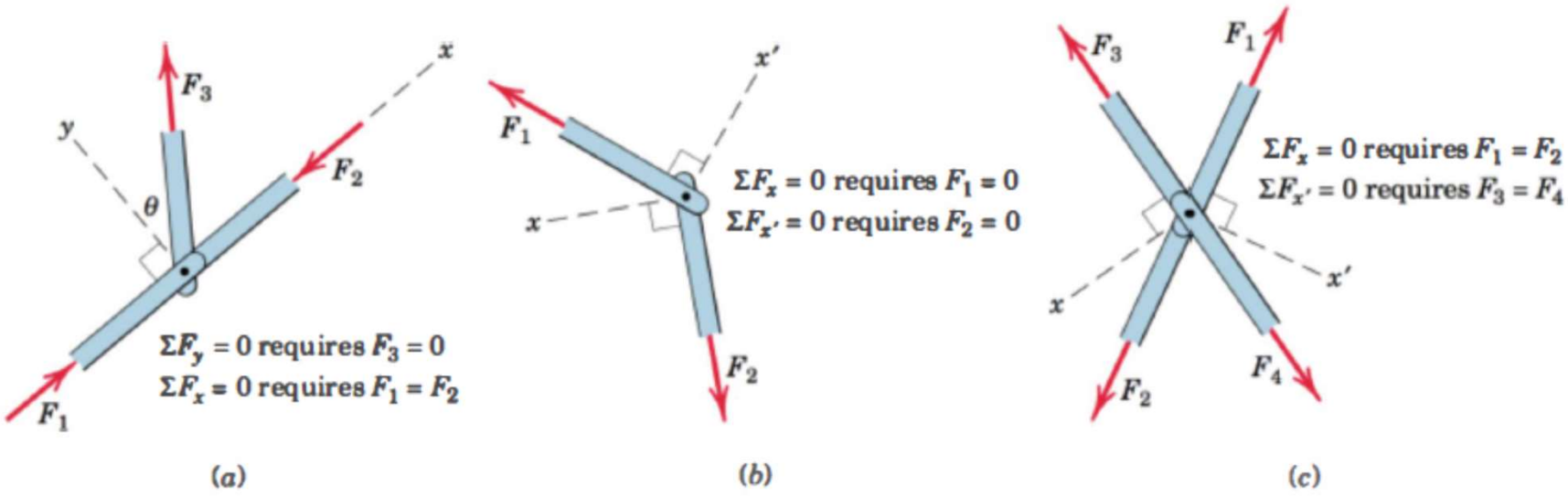
Trusses

Method of joints:

- Method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint.
- With the joints indicated by letters, we usually designate the force in each member by the two letters defining the ends of the member. The proper directions of the forces should be evident by inspection for this simple case.
- The free-body diagrams of portions of members AF and AB are also shown to clearly indicate the mechanism of the action and reaction. The member AB actually makes contact on the left side of the pin, although the force AB is drawn from the right side and is shown acting away from the pin. Thus, if we consistently draw the force arrows on the same side of the pin as the member, then tension (such as AB) will always be indicated by an arrow away from the pin, and compression (such as AF) will always be indicated by an arrow toward the pin.
- The magnitude of AF is obtained from the equation $\sum F_y = 0$ and AB is then found from $\sum F_x = 0$.



Special Conditions:
When two collinear members are under compression, as indicated in Figure, it is necessary to add a third member to maintain alignment of the two members and prevent buckling.



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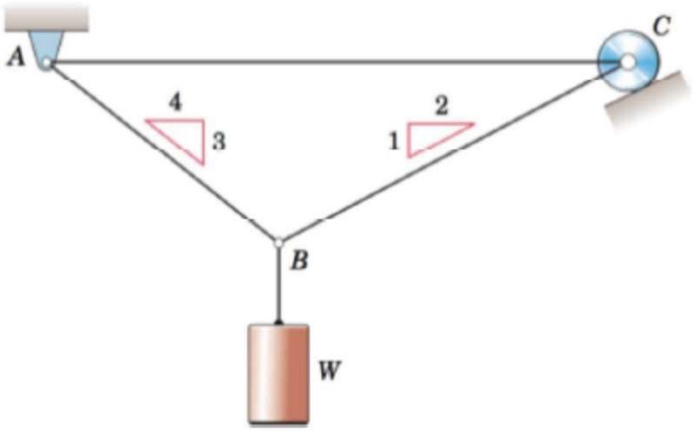
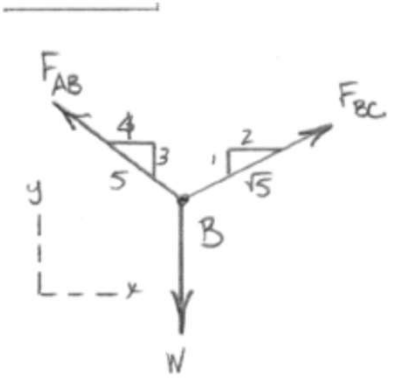
Trusses Numerical:

4/1. Determine the force in each member of the loaded truss as a result of hanging weight W.



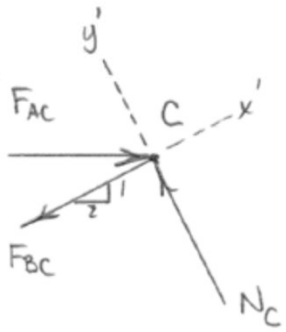


$$\Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BC} - \frac{4}{5} F_{AB} = 0$$

$$\Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BC} + \frac{3}{5} F_{AB} - W = 0$$

$F_{AB} = W$ T

$F_{BC} = \frac{2}{\sqrt{5}} W$ T



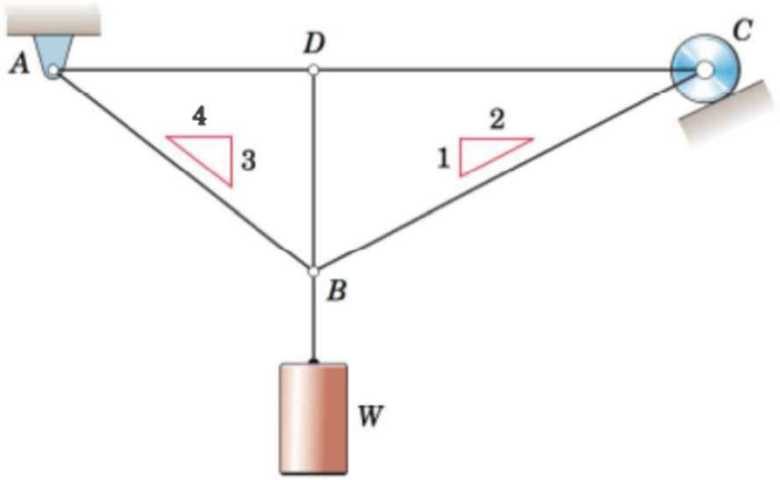
$$\Sigma F_{x'} = 0: \quad -F_{BC} + \frac{2}{\sqrt{5}} F_{AC} = 0$$

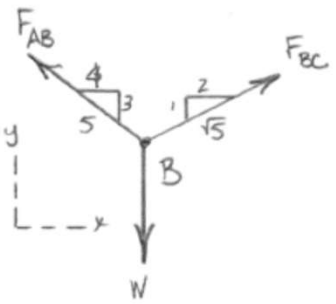
$F_{AC} = W$ C

ENGINEERING MECHANICS

Trusses Numerical:

4/2. The truss of the previous problem is modified by adding the vertical support member BD. Determine the force in each member of the modified truss as a result of hanging weight W.



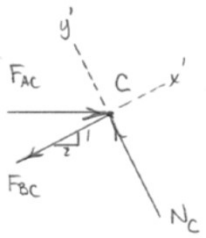


$\Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BC} - \frac{4}{5} F_{AB} = 0$

$\Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BC} + \frac{3}{5} F_{AB} - W = 0$

$F_{AB} = W \quad T$ $\qquad$ $F_{BC} = \frac{2}{\sqrt{5}} W \quad T$

SINCE $F_{BD} = 0$ (ZERO FORCE MEMBER) THE LOAD CARRIED IN THE SIDES OF THE TRUSS DO NOT CHANGE!



$\Sigma F_{x'} = 0: \quad -F_{BC} + \frac{2}{\sqrt{5}} F_{AC} = 0$

$F_{AC} = W \quad C$

FROM 4/2...

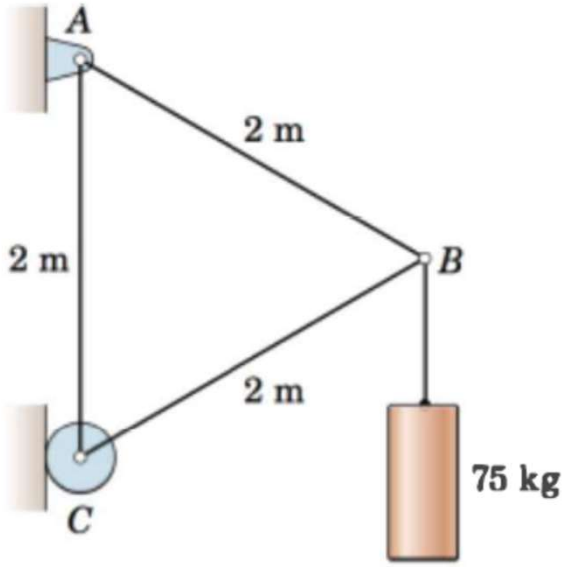
$F_{AB} = W \quad T$

$F_{BC} = \frac{2}{\sqrt{5}} W \quad T$

$F_{AD} = F_{CD} = W \quad C$

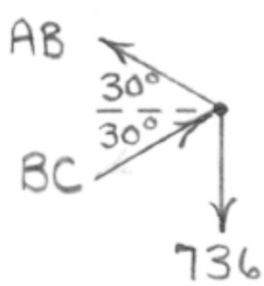
Trusses Numerical:

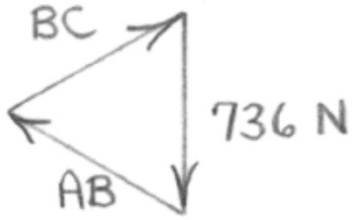
4/3. Determine the force in each member of the simple equilateral truss.



4/3 Load = 75(9.81) = 736 N

Joint B:

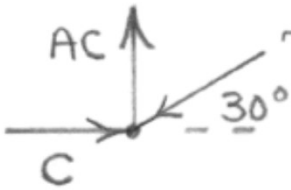


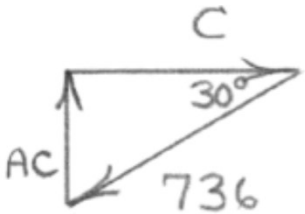


AB = 736 N T

BC = 736 N C

Joint C:



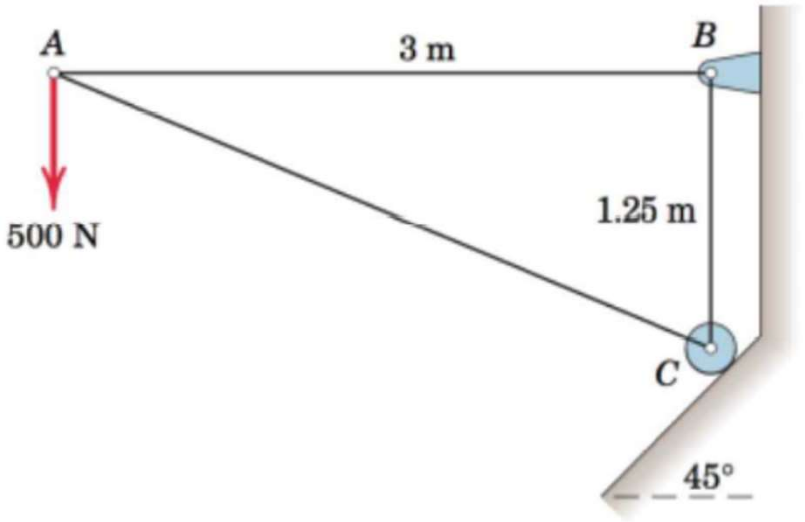


AC = 736 (1/2)

= 368 N T

Trusses Numerical:

4/4. Determine the force in each member of the loaded truss. Discuss the effects of varying the angle of the 45^0 support surface at C.



4/4

$\alpha = \tan^{-1}\frac{1.25}{3} = 22.6^\circ$

Joint A: $(\cos\alpha = \frac{12}{13},\ \sin\alpha = \frac{5}{13})$

$\Sigma F_y = 0$: $AC\sin\alpha - 500 = 0$

$AC = 1300\ N\ C$

$\Sigma F_x = 0$: $AB - 1300\cos\alpha = 0$

$AB = 1200\ N\ T$

Joint C:

$\Sigma F_{x'} = 0$: $1300\left(\frac{12}{13}\right) - C\frac{\sqrt{2}}{2} = 0$

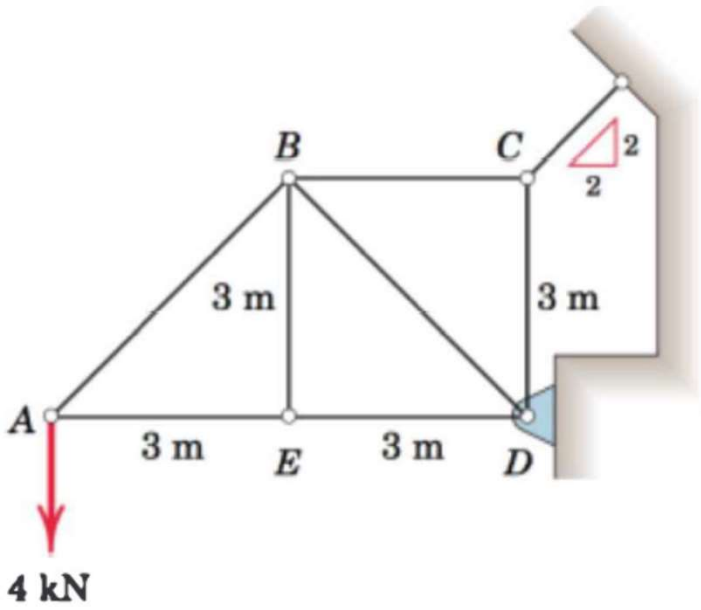
$C = 1697\ N$ (reaction)

$\Sigma F_y = 0$: $-1300\left(\frac{5}{13}\right) - BC + 1697\frac{\sqrt{2}}{2} = 0$

$BC = 700\ N\ C$

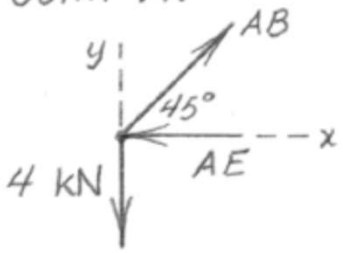
Trusses Numerical:

4/5. Calculate the force in members BE and BD of the loaded truss.



4/5

Joint A:



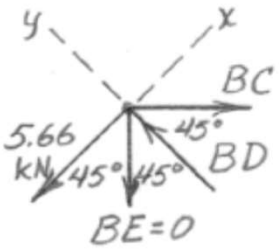
$\Sigma F_y = 0$: $AB \sin 45° - 4 = 0$

$AB = 5.66$ kN T

$\Sigma F_x = 0$: $5.66 \cos 45° - AE = 0$

$AE = 4$ kN C

Joint B:



$\Sigma F_x = 0$: $BC \cos 45° - 5.66 = 0$

$BC = 8$ kN T

$\Sigma F_y = 0$: $BD - 8 \cos 45° = 0$

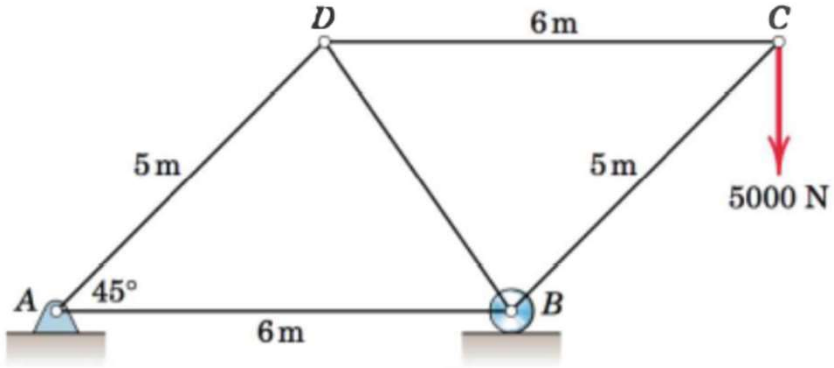
$\underline{BD = 5.66 \text{ kN C}}$

Joint E:

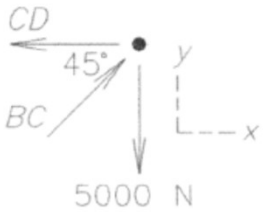
$\Sigma F_y = 0$: $\underline{BE = 0}$

Trusses Numerical:

4/6. Determine the force in each member of the loaded truss.

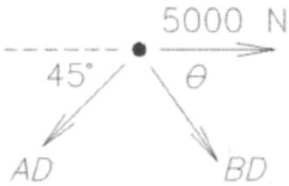


Joint *C*:



$$\begin{cases} \Sigma F_y = 0: \; BC\sin 45° - 5000 = 0, \; \underline{BC = 7070 \text{ N } C} \\ \Sigma F_x = 0: \; 7070\cos 45° - CD = 0, \; \underline{CD = 5000 \text{ N } T} \end{cases}$$

Joint *D*:



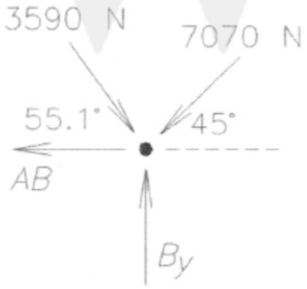
$$\overline{DB}^2 = 5^2 + 6^2 - 2(5)(6)\cos 45°, \; \overline{DB} = 4.31 \text{ m}$$

$$\frac{\sin\theta}{5} = \frac{\sin 45°}{4.31} \Rightarrow \theta = 55.1°$$

$$\begin{cases} \Sigma F_x = 0: \; 5000 + BD\cos 55.1° - AD\cos 45° = 0 \\ \Sigma F_y = 0: \; -AD\sin 45° - BD\sin 55.1° = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{BD = -3590 \text{ N or } 3590 \text{ N } C}$, $\underline{AD = 4170 \text{ N } T}$

Joint *B*:

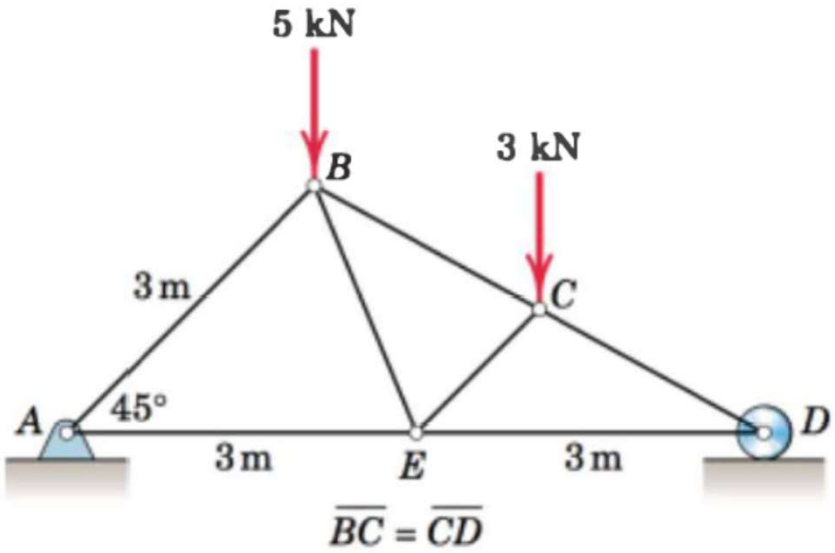


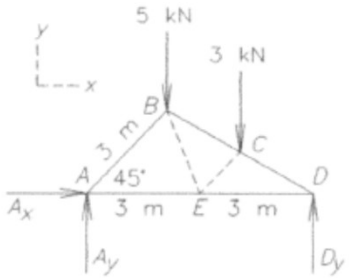
$$\Sigma F_x = 0: \; 3590\cos 55.1° - 7070\cos 45° - AB = 0$$

$$\underline{AB = -2950 \text{ N or } 2950 \text{ N } C}$$

Trusses Numerical:

4/7. Determine the force in members BE and CE of the loaded truss.





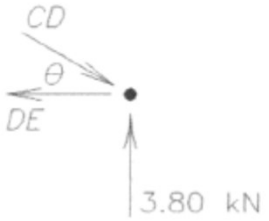
As a whole:

Note: $\overline{CE} = 1.5$ m by similar triangles

$$\Sigma M_A = 0: \; 5(3\cos 45°) + 3(3 + 1.5\cos 45°) - 6D_y = 0$$

$$D_y = 3.80 \text{ kN}$$

Joint D:



$$\theta = \tan^{-1}\frac{3\sin 45°}{6 - 3\cos 45°} = 28.7°$$

$$\begin{cases} \Sigma F_y = 0: \; 3.80 - CD\sin 28.7° = 0, \; CD = 7.92 \text{ kN } C \\ \Sigma F_x = 0: \; 7.92\cos 28.7° - DE = 0, \; DE = 6.94 \text{ kN } T \end{cases}$$

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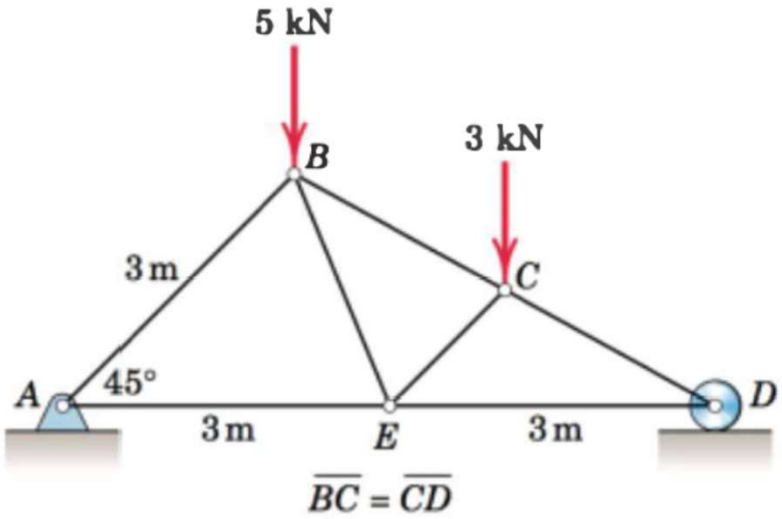
Trusses Numerical:



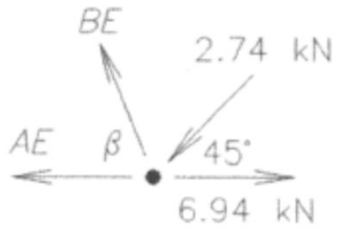
Joint C:

$$\begin{cases} \Sigma F_x = 0: & -BC\cos 28.7^\circ - CE\cos 45^\circ - 7.92\cos 28.7^\circ = 0 \\ \Sigma F_y = 0: & BC\sin 28.7^\circ - CE\sin 45^\circ + 7.92\sin 28.7^\circ - 3 = 0 \end{cases}$$

Solve simultaneously to obtain: $BC = -5.70$ kN (C)

$CE = -2.74$ kN (C)

Joint E:



$$\beta = \frac{180^\circ - 45^\circ}{2} = 67.5^\circ$$

$$\Sigma F_y = 0: \quad BE\sin 67.5^\circ - 2.74\sin 45^\circ = 0$$

$BE = 2.10$ kN T

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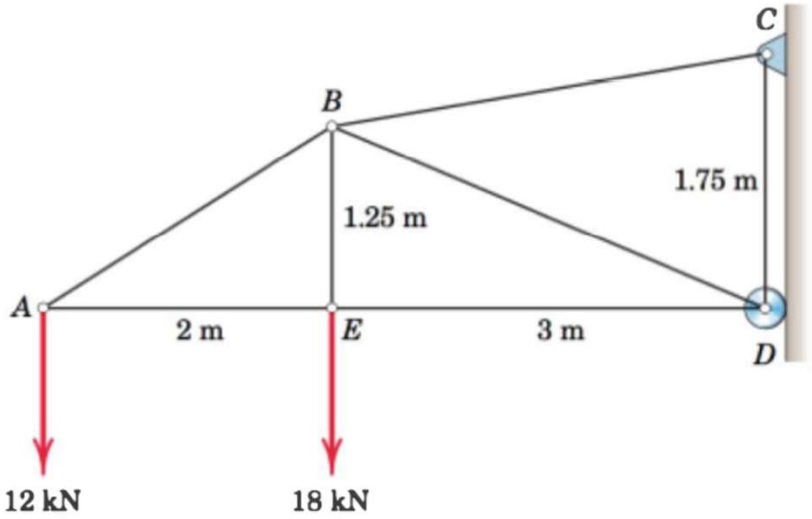
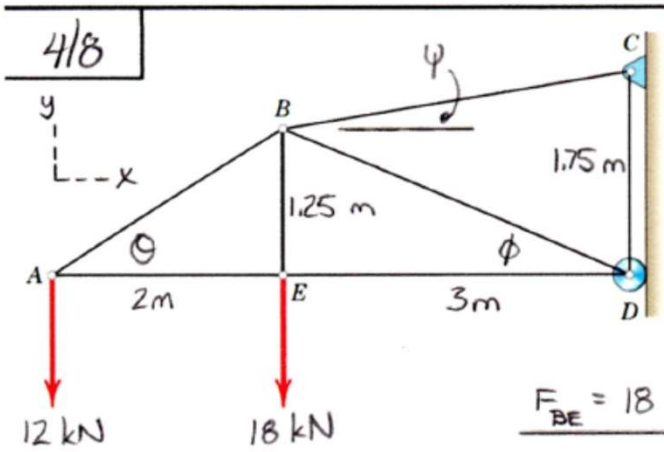
Trusses Numerical:

4/8. Determine the force in each member of the loaded truss.



4/8



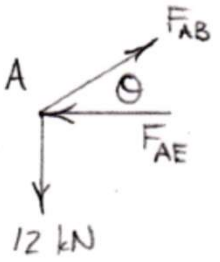
$\theta = \tan^{-1}\left(\frac{1.25}{2}\right) = 32.0^\circ$

$\phi = \tan^{-1}\left(\frac{1.25}{3}\right) = 22.6^\circ$

$\psi = \tan^{-1}\left(\frac{0.5}{3}\right) = 9.46^\circ$

$F_{BE} = 18\ \text{kN}\ T$ (INSPECTION)

JOINT A:

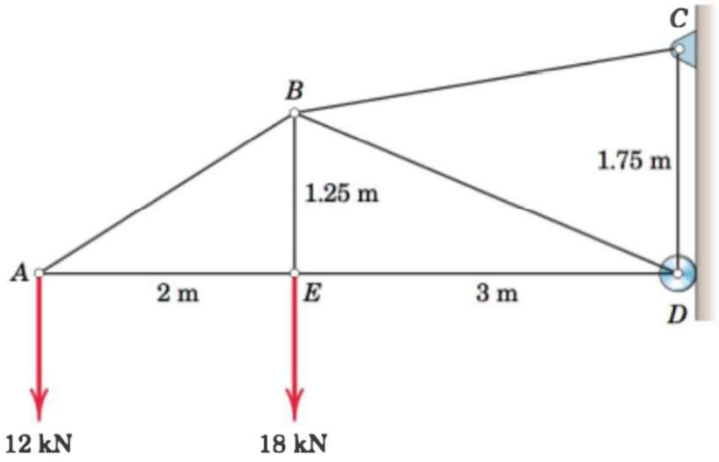


$$\begin{cases} \Sigma F_y = 0: & F_{AB}\sin\theta - 12 = 0 \\ \Sigma F_x = 0: & F_{AB}\cos\theta - F_{AE} = 0 \end{cases} \rightarrow \begin{cases} F_{AB} = 22.6\ \text{kN}\ T \\ F_{AE} = 19.20\ \text{kN}\ C \end{cases}$$

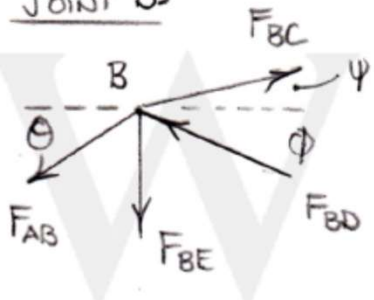
$F_{DE} = 19.20\text{kN}\ C$ (INSPECTION)

ENGINEERING MECHANICS

Trusses Numerical:



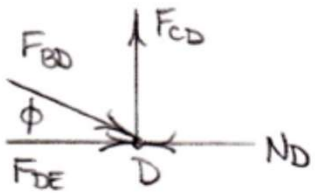
JOINT B:



$$\begin{cases} \Sigma F_x = 0: \; F_{BC}\cos\psi - F_{BD}\cos\phi - F_{AB}\cos\theta = 0 \\ \Sigma F_y = 0: \; F_{BC}\sin\psi + F_{BD}\sin\phi - F_{BE} - F_{AB}\sin\theta = 0 \end{cases}$$

$F_{BC} = 66.0$ kN T & $F_{BD} = 49.8$ kN C

JOINT D:



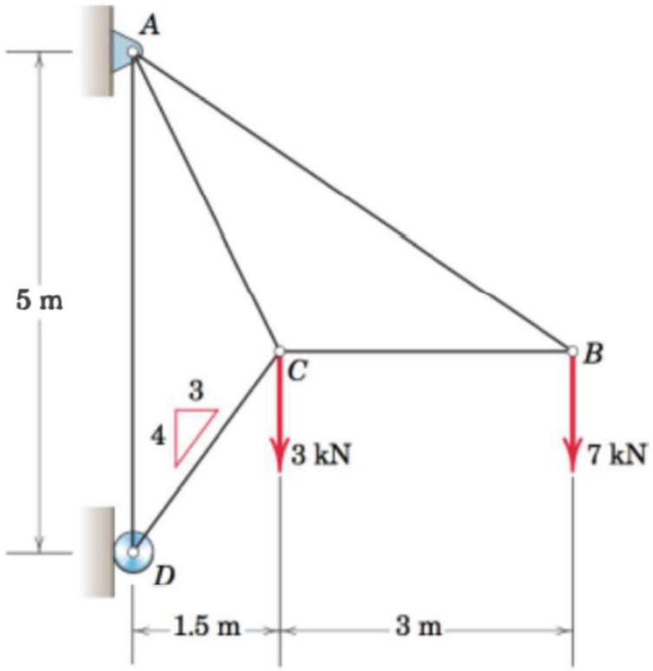
$\Sigma F_y = 0: \; F_{CD} - F_{BD}\sin\phi = 0$

$F_{CD} = 19.14$ kN T

ENGINEERING MECHANICS

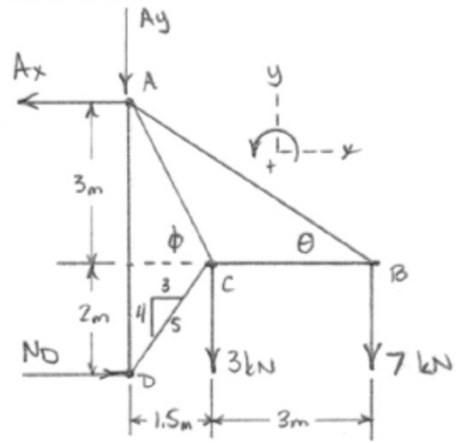
Trusses Numerical:

4/9. Determine the force in each member of the loaded truss.



4/9

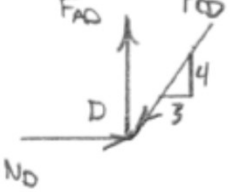
$\theta = \tan^{-1}\left(\frac{3}{4.5}\right) \rightarrow \theta = 33.69°$ $\phi = \tan^{-1}\left(\frac{3}{1.5}\right) \rightarrow \phi = 63.43°$



$\Sigma M_A = 0$: $5N_D - 1.5(3) - 4.5(7) = 0$

$N_D = 7.20$ kN

• Joint D:

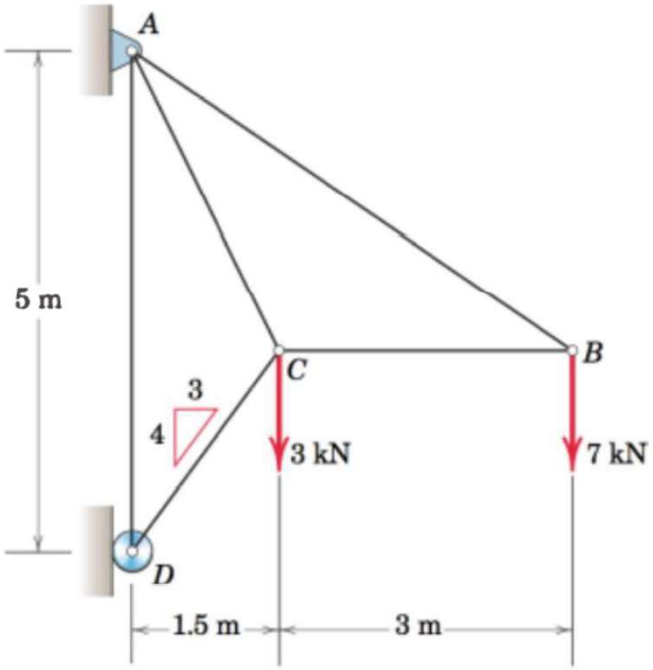


$\Sigma F_x = 0$: $N_D - \frac{3}{5} F_{CD} = 0 \rightarrow$ $F_{CD} = 12$ kN C

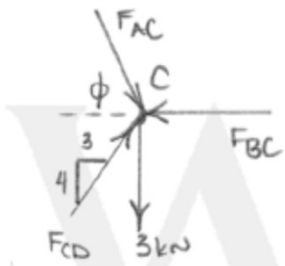
$\Sigma F_y = 0$: $F_{AD} - \frac{4}{5} F_{CD} = 0 \rightarrow$ $F_{AD} = 9.60$ kN T

ENGINEERING MECHANICS

Trusses Numerical:



• Joint C:

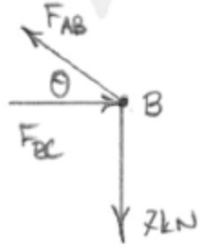


$\Sigma F_x = 0: \quad \frac{3}{5} F_{CD} + F_{AC} \cos\phi - F_{BC} = 0$

$\Sigma F_y = 0: \quad \frac{4}{5} F_{CD} - F_{AC} \sin\phi - 3 = 0$

$\longrightarrow \quad F_{BC} = 10.50 \text{ kN C}, \quad F_{AC} = 7.38 \text{ kN C}$

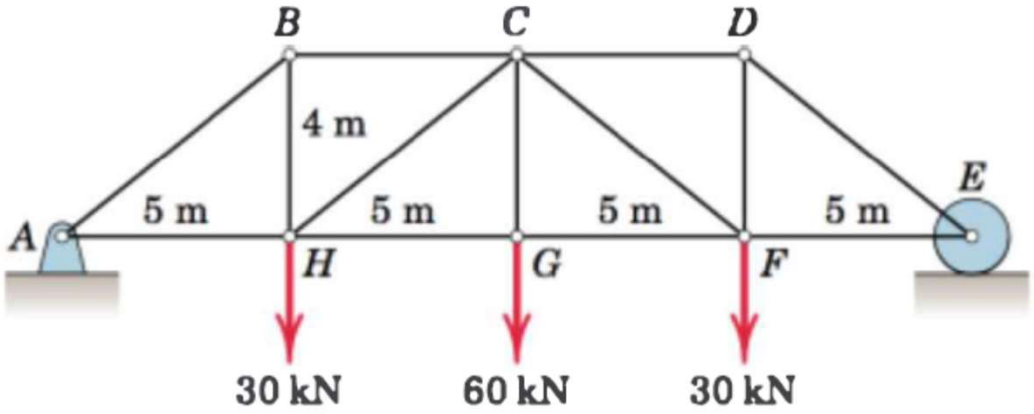
• Joint B:

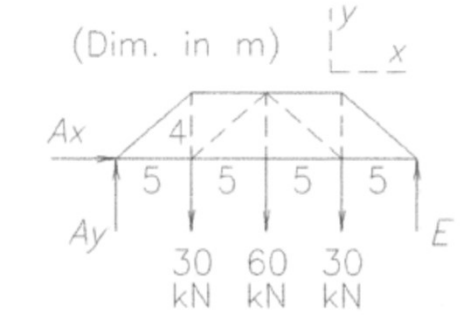


$\Sigma F_y = 0: \quad F_{AB} \sin\theta - 7 = 0 \longrightarrow F_{AB} = 12.62 \text{ kN T}$

Trusses Numerical:

4/10. Determine the force in each member of the loaded truss. Make use of the symmetry of the truss and of the loading.



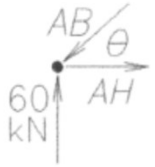


As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60$ kN by

$\Sigma F_y = 0$ and symmetry.

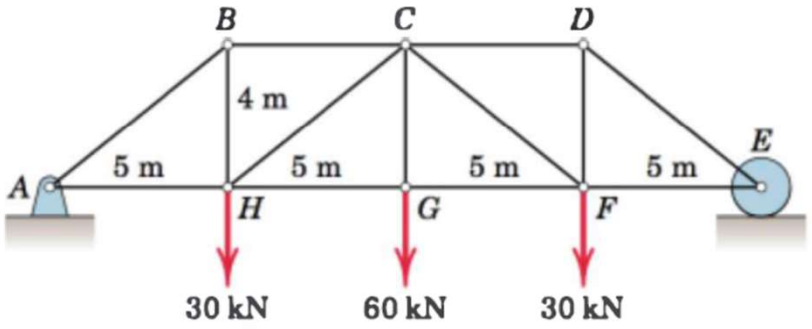
Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7^\circ\right)$



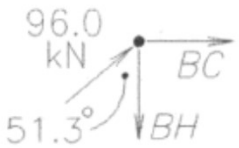
$$\begin{cases} \Sigma F_y = 0: 60 - AB\sin\theta = 0, \ \underline{AB = 96.0 \text{ kN } C} \\ \Sigma F_x = 0: AH - 96.0\cos\theta, \ \underline{AH = 75 \text{ kN } T} \end{cases}$$

ENGINEERING MECHANICS

Trusses Numerical:

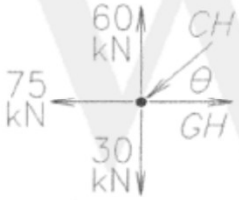


Joint *B*:



$$\begin{cases} \Sigma F_x = 0: BC + 96.0 \sin 51.3^\circ = 0, \ \underline{BC = -75 \text{ kN } (C)} \\ \Sigma F_y = 0: -BH + 96.0 \cos 51.3^\circ = 0, \ \underline{BH = 60 \text{ kN } T} \end{cases}$$

Joint *H*:



$$\begin{cases} \Sigma F_y = 0: -CH \sin\theta + 30 = 0, \ \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0 \cos\theta + GH - 75 = 0, \ \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint *G*:

$$\Sigma F_y = 0 \Rightarrow \underline{CG = 60 \text{ kN } T}$$

By symmetry:

$FG = 112.5$ kN T, $CF = 48.0$ kN C

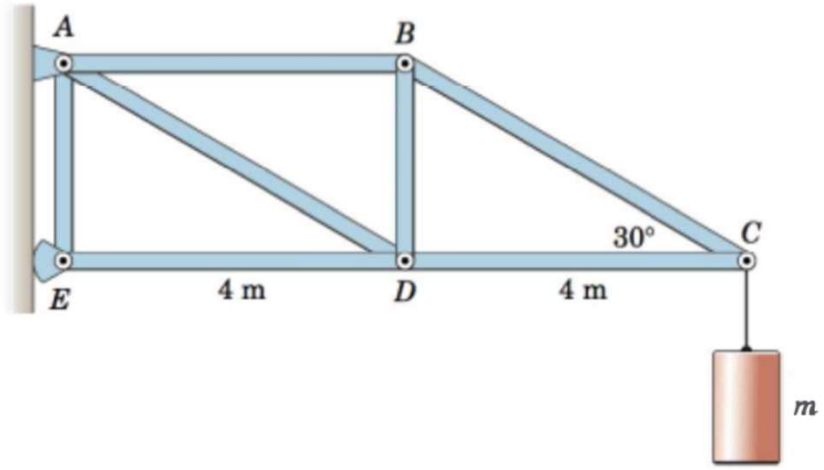
$CD = 75$ kN C, $DF = 60$ kN T

$\underline{EF = 75 \text{ kN } T, \ DE = 96.0 \text{ kN } C}$

ENGINEERING MECHANICS

Trusses Numerical:

4/11. If the maximum tensile force in any of the truss members must be limited to 24 kN, and the maximum compressive force must be limited to 35 kN, determine the largest permissible mass m which may be supported by the truss.



4/11

$F_{AE} = 0$ (INSPECTION)

• JOINT C:

$\Sigma F_x = 0: \quad -F_{BC}\cos 30^\circ + F_{CD} = 0$

$\Sigma F_y = 0: \quad F_{BC}\sin 30^\circ - mg = 0$

$\rightarrow \quad F_{BC} = 2mg \ T, \quad F_{CD} = 1.732mg \ C$

• JOINT B:

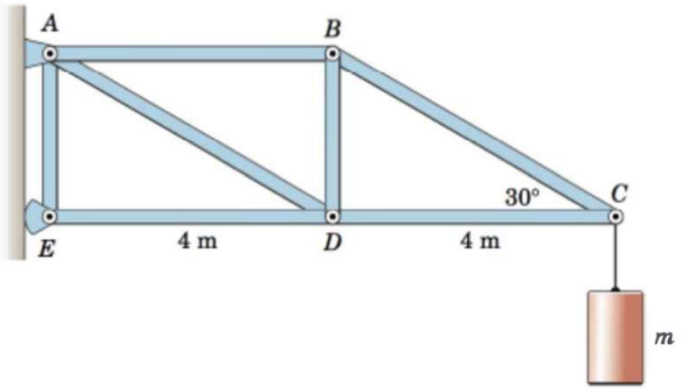
$\Sigma F_x = 0: \quad F_{BC}\cos 30^\circ - F_{AB} = 0$

$\Sigma F_y = 0: \quad F_{BD} - F_{BC}\sin 30^\circ = 0$

$\rightarrow \quad F_{AB} = 1.732mg \ T, \quad F_{BD} = mg \ C$

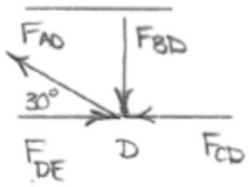
ENGINEERING MECHANICS

Trusses Numerical:



SO

• JOINT D:



$$\begin{cases} \Sigma F_x = 0: & F_{DE} - F_{CD} - F_{AD}\cos 30^\circ = 0 \\ \Sigma F_y = 0: & F_{AD}\sin 30^\circ - F_{BD} = 0 \end{cases} \longrightarrow \begin{cases} F_{AD} = 2mg \quad T \\ F_{DE} = 3.46\,mg \quad C \end{cases}$$

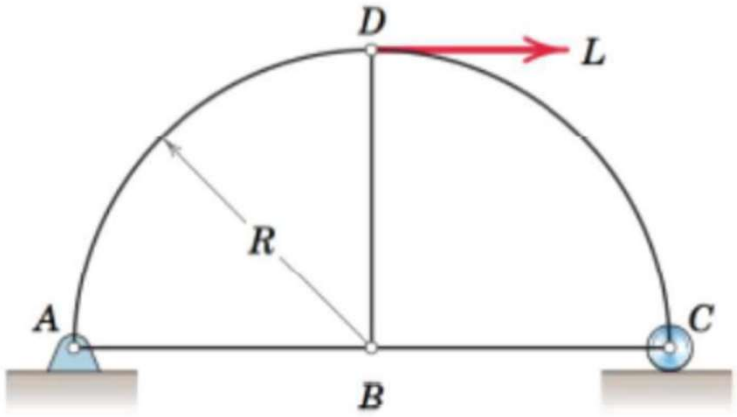
• MAX TENSION: $F_{BC} = 2mg = 24\ kN \longrightarrow m = 1223\ kg$

• MAX COMPRESSION: $F_{DE} = 3.46mg = 35\ kN \longrightarrow m = 1030\ kg$

So... $m_{max} = 1030\ kg$

Trusses Numerical:

4/12. Determine the force in members AB, BC and BD of the loaded truss.

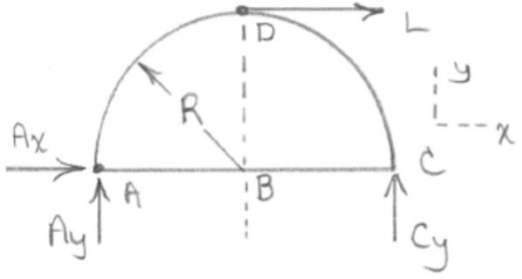


4/12 Entire truss:

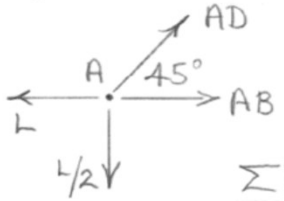
$\Sigma F_x = 0: \quad A_x = -L$

$\Sigma M_A = 0: \quad C_y = L/2$

$\Sigma F_y = 0: \quad A_y = -L/2$



Joint A:



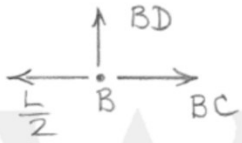
$\Sigma F_y = 0: \quad AD\frac{\sqrt{2}}{2} - \frac{L}{2} = 0$

$AD = \frac{\sqrt{2}}{2}L$

$\Sigma F_x = 0: \quad -L + \frac{\sqrt{2}}{2}L\left(\frac{\sqrt{2}}{2}\right) + AB = 0$

$AB = \frac{L}{2}\ T$

Joint B:



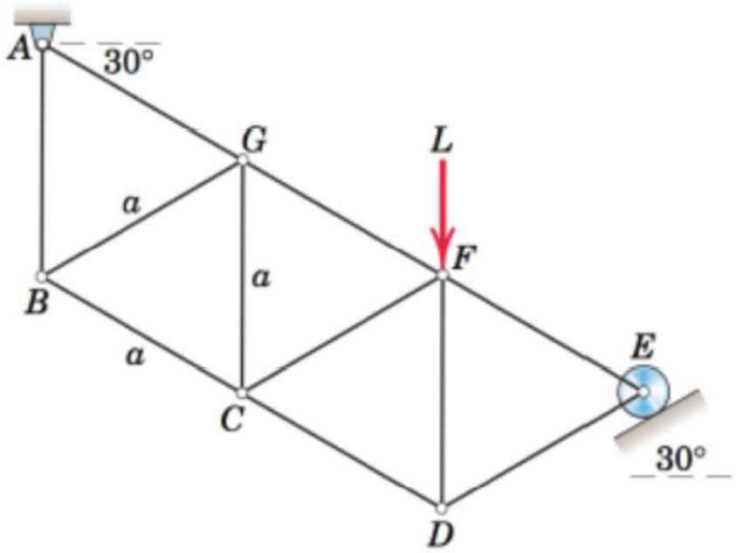
$BC = \frac{L}{2}\ T$

$BD = 0$

ENGINEERING MECHANICS

Trusses Numerical:

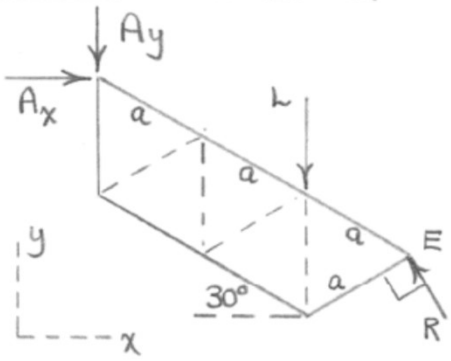
4/13. The truss is composed of equilateral triangle of sides a and is loaded and supported as shown. Determine the forces in members EF, DE and DF.



4/13

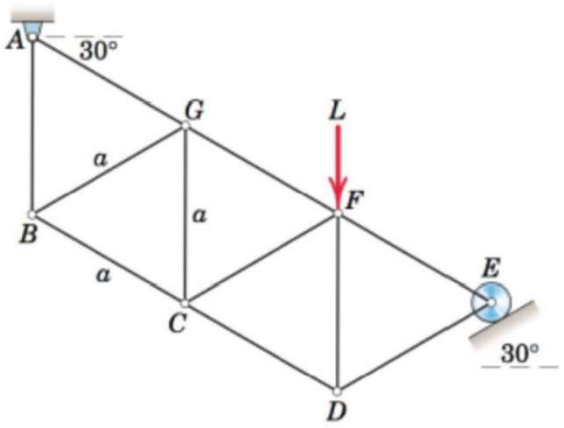
$+\circlearrowleft \Sigma M_A = 0: \quad L(2a\cos 30°) - R(3a\sin 30°) = 0$

$R = 2L/\sqrt{3}$

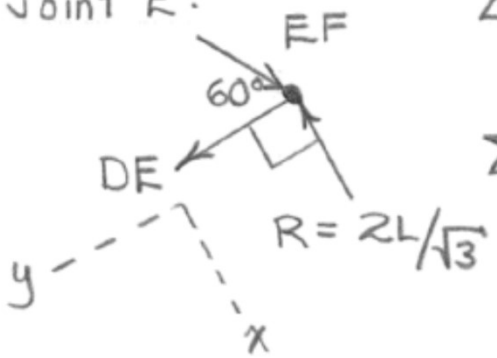


ENGINEERING MECHANICS

Trusses Numerical:



Joint E:



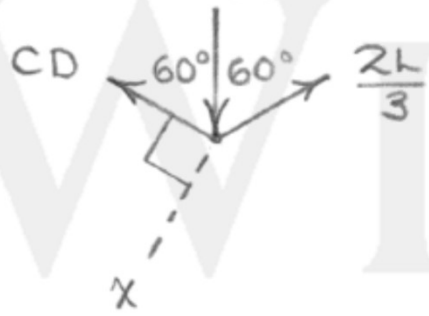
$\Sigma F_x = 0$: $EF \sin 60° - 2L/\sqrt{3} = 0$

$EF = \frac{4L}{3}$ C

$\Sigma F_y = 0$: $DE - \frac{4L}{3} \cos 60° = 0$

$DE = \frac{2L}{3}$ T

Joint D:

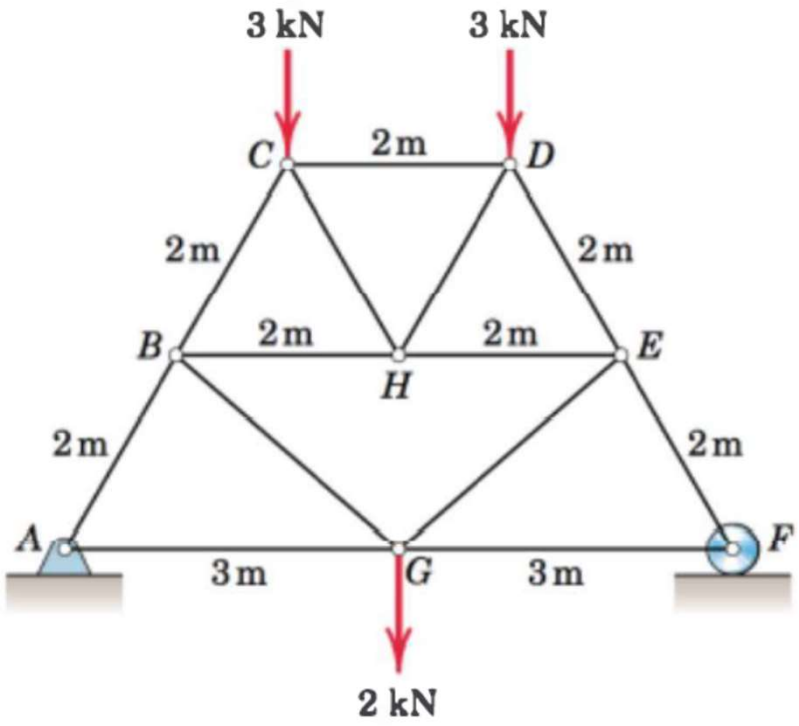


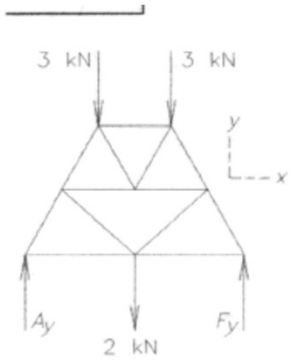
$\Sigma F_x = 0$: $DF \cos 30° - \frac{2L}{3} \cos 30° = 0$

$DF = \frac{2L}{3}$ C

Trusses Numerical:

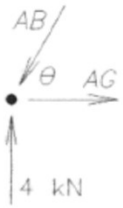
4/15. Determine the force in members BC and BG of the loaded truss.





By symmetry, $A_y = F_y = 4$ kN

Joint A:

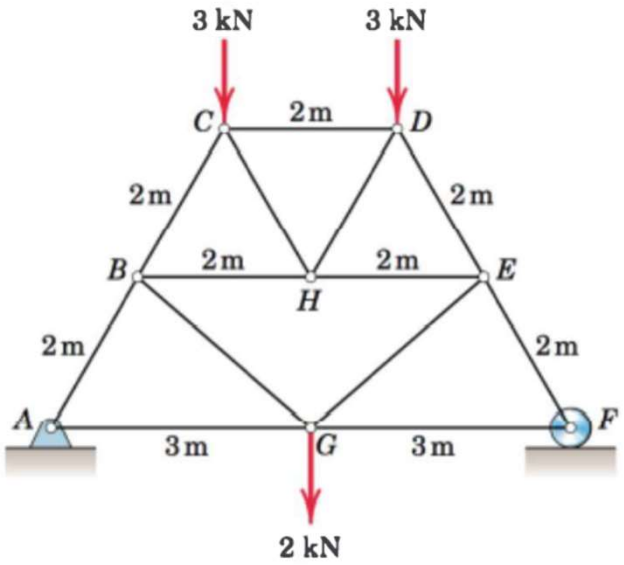


$$\theta = \cos^{-1}\frac{1}{2} = 60°$$

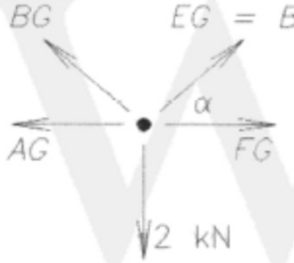
$$\begin{cases} \Sigma F_y = 0: \ 4\text{ kN} - AB\sin 60° = 0, \ AB = 4.62\text{ kN } C \\ \Sigma F_x = 0: \ AG - 4.62\cos 60° = 0, \ AG = 2.31\text{ kN } T \end{cases}$$

ENGINEERING MECHANICS

Trusses Numerical:



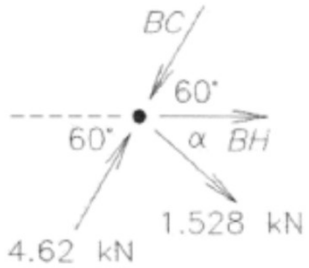
Joint G:



$$\alpha = \tan^{-1}\frac{2\sin 60°}{2} = 40.9°$$

$$\Sigma F_y = 0: \; 2BG\sin 40.9° - 2 = 0, \; \underline{BG = 1.528 \text{ kN } T}$$

Joint B:

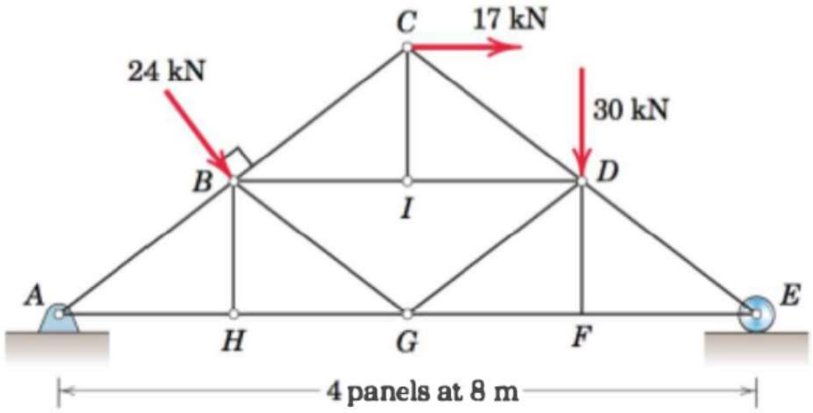


$$\Sigma F_y = 0: \; 4.62\sin 60° - BC\sin 60° - 1.528\sin 40.9° = 0$$

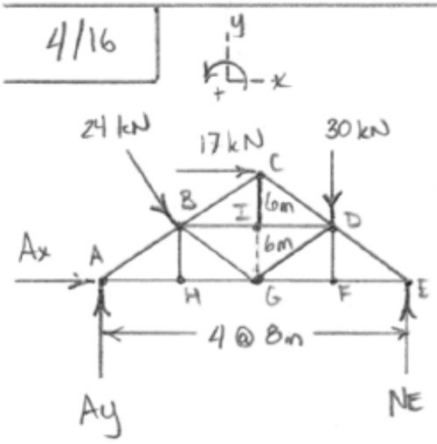
$$\underline{BC = 3.46 \text{ kN } C}$$

Trusses Numerical:

4/16. Determine the force in each member of the loaded truss. All triangles are 3-4-5.



4/16



$\Sigma F_x = 0$: $A_x + 17 + \frac{3}{5}24 = 0$

$\Sigma F_y = 0$: $A_y + N_E - \frac{4}{5}24 - 30 = 0$

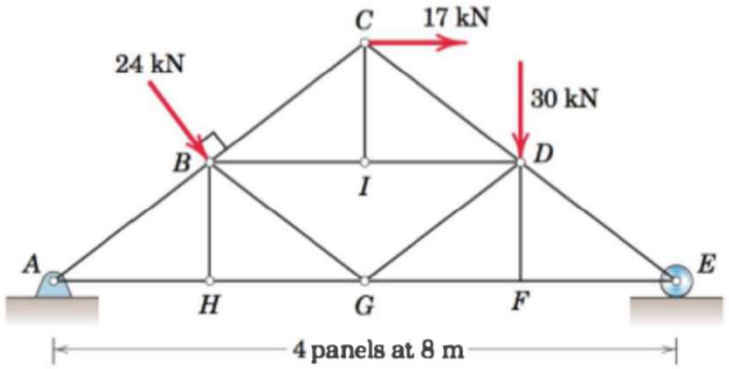
$\Sigma M_A = 0$: $32 N_E - 24(30) - 12(17) - 24(16) = 0$

$A_x = -31.4$ kN $A_y = 12.83$ kN $N_E = 36.4$ kN

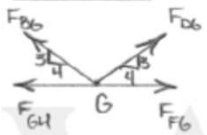
$F_{BH} = 0$ AND $F_{DF} = 0$ AND $F_{CI} = 0$

ENGINEERING MECHANICS

Trusses Numerical:



• Joint G:

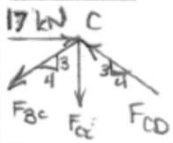


$\Sigma F_x = 0: \quad F_{FG} - F_{GH} + \frac{4}{5} F_{DG} - \frac{4}{5} F_{BG} = 0$

$\Sigma F_y = 0: \quad \frac{3}{5} F_{DG} + \frac{3}{5} F_{BG} = 0$

$F_{BG} = 0 \qquad F_{DG} = 0$

• Joint C

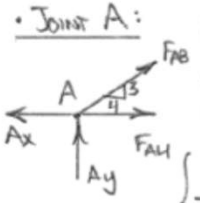


$\Sigma F_x = 0: \quad 17 - \frac{4}{5} F_{BC} - \frac{4}{5} F_{CD} = 0$

$\Sigma F_y = 0: \quad \frac{3}{5} F_{CD} - \frac{3}{5} F_{BC} = 0$

$F_{BC} = 10.63 \text{ kN T} \qquad F_{CD} = 10.63 \text{ kN C}$

• Joint A:



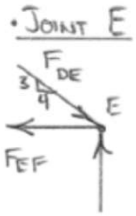
$\Sigma F_x = 0: \quad \frac{4}{5} F_{AB} + F_{AH} - A_x = 0$

$\Sigma F_y = 0: \quad A_y + \frac{3}{5} F_{AB} = 0$

$F_{AB} = -21.4 \text{ kN (C)}$

$F_{AH} = F_{GH} = 48.5 \text{ kN T}$

• Joint E



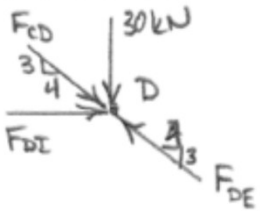
$\Sigma F_x = 0: \quad \frac{4}{5} F_{DE} - F_{EF} = 0$

$\Sigma F_y = 0: \quad N_E - \frac{3}{5} F_{DE} = 0$

$F_{DE} = 60.6 \text{ kN C}$

$F_{EF} = F_{FG} = 48.5 \text{ kN T}$

• Joint D:



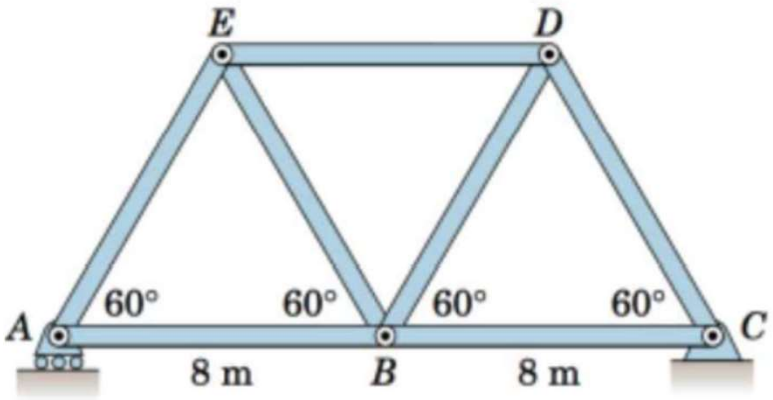
$\Sigma F_x = 0: \quad \frac{4}{5} F_{CD} - \frac{4}{5} F_{DE} + F_{DI} = 0$

$F_{DI} = F_{BI} = 40 \text{ kN C}$

ENGINEERING MECHANICS

Trusses Numerical:

4/17. Each member of the truss is a uniform 8 m bar with a mass of 400 kg. Calculate the average tension or compression in each member due to the weight of the members.



4/17 Total weight of truss $W = 7(400)(9.81)$ N
$= 27.5$ kN

By symmetry, reactions at A & C are $W/2 = 13.73$ kN

$\Sigma F_y = 0: \frac{W}{2} - \frac{W}{7} - AE \sin 60° = 0$

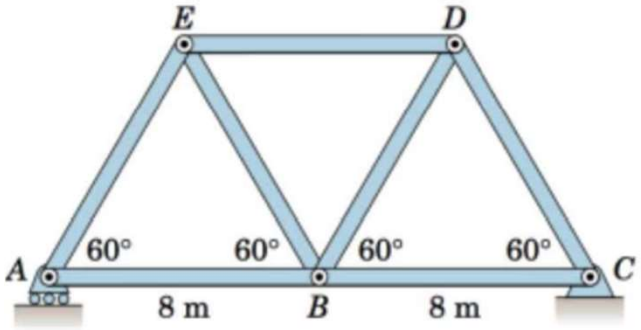
$AE = 0.412W = \underline{11.33 \text{ kN C}}$

$\Sigma F_x = 0: AB - 11.33 \cos 60° = 0$

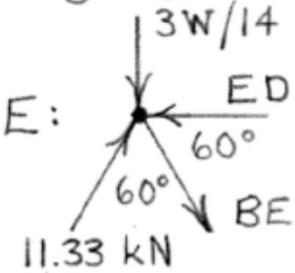
$\underline{AB = 5.66 \text{ kN T}}$

ENGINEERING MECHANICS

Trusses Numerical:



By symmetry, $BC = AB = 5.66\ kN\ T$, $DC = AE = 11.33\ kN\ C$



$\sum F_y = 0: -\frac{3W}{14} + 11.33 \cos 30° - BE \cos 30° = 0$

$BE = 4.53\ kN\ T$

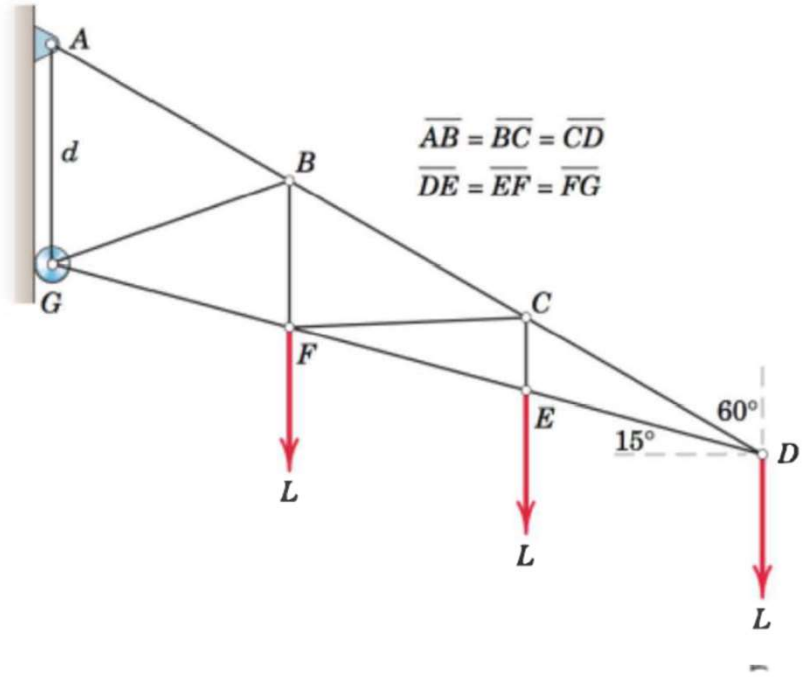
By symmetry, $BD = 4.53\ kN\ T$

$\sum F_x = 0: \quad 11.33 \sin 30° + 4.53 \sin 30° - ED = 0$

$ED = 7.93\ kN\ C$

Trusses Numerical:

4/19. Determine the force in members BG and BF of the loaded truss.



$\Sigma M_A = 0$: $N_G d - bL - 2bL - 3bL = 0$

4/19

$\frac{\overline{AD}}{\sin 105^\circ} = \frac{\overline{DG}}{\sin 60^\circ} = \frac{d}{\sin 15^\circ}$

$\overline{AD} = 3.73\,d$

$\overline{DG} = 3.35\,d$

$\theta = \tan^{-1}\left(\frac{\frac{2}{3}\overline{AD}\sin 30^\circ - \overline{DG}\sin 15^\circ}{\overline{DG}\cos 15^\circ - \frac{2}{3}\overline{AD}\cos 30^\circ}\right) \rightarrow \theta = 19.33^\circ$

$b = \frac{1}{3}\overline{DG}\cos 15^\circ \rightarrow b = 1.077\,d$

$\Sigma F_x = 0$: $N_G - A_x = 0$

$\Sigma F_y = 0$: $A_y - 3L = 0$

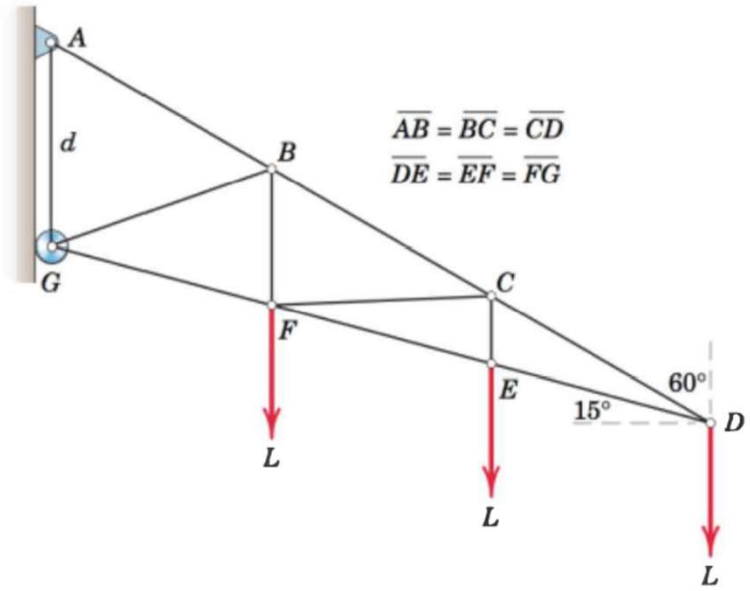
SOLVING...

$A_x = N_G = 6.46\,L$

$A_y = 3L$

ENGINEERING MECHANICS

Trusses Numerical:



• JOINT A:

Ay = 3L

$\Sigma F_x = 0$: $F_{AB}\sin 60^\circ - A_x = 0$

$\Sigma F_y = 0$: $A_y - F_{AG} - F_{AB}\cos 60^\circ = 0$

$F_{AB} = 7.46$ L T $F_{AG} = -0.732$ L (C)

• JOINT G:

$\Sigma F_x = 0$: $N_L - F_{BG}\cos\theta - F_{FG}\cos 15^\circ = 0$

$\Sigma F_y = 0$: $F_{FG}\sin 15^\circ - F_{AG} - F_{BG}\sin\theta = 0$

$F_{BG} = 1.713$ L C $F_{FG} = 5.02$ L C

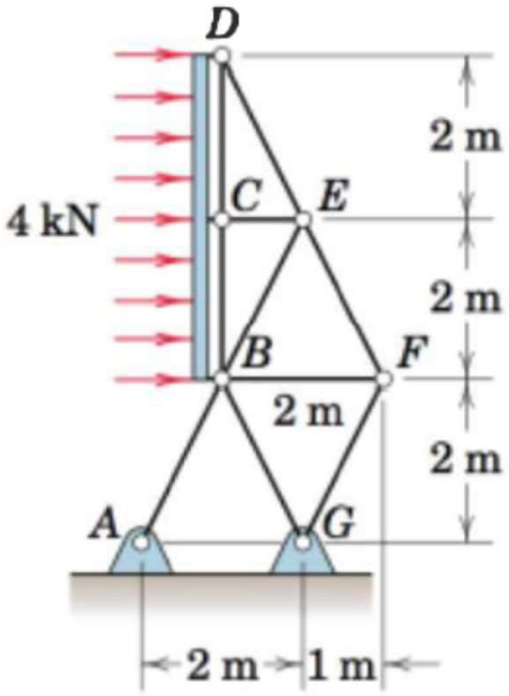
• JOINT B:

$\Sigma F_{y'} = 0$: $F_{BF}\sin 60^\circ - F_{BG}\sin(30^\circ + \theta) = 0$

$F_{BF} = 1.5$ L T

Trusses Numerical:

4/21. The signboard truss is designed to support a horizontal wind load of 4 kN. A separate analysis shows that 5/8 of this force is transmitted to the center connection at C and the rest is equally divided between D and B. Calculate the forces in members BE and BC.



4/21

Joint D

$\frac{1}{2}\left(\frac{3}{8}\right)4 = 0.75$ kN

$\theta = \tan^{-1}\frac{2}{4} = 26.6°$

$\cos\theta = \frac{2}{\sqrt{5}}$, $\sin\theta = \frac{1}{\sqrt{5}}$

$\Sigma F_x = 0$: $DE\frac{1}{\sqrt{5}} - 0.75 = 0$

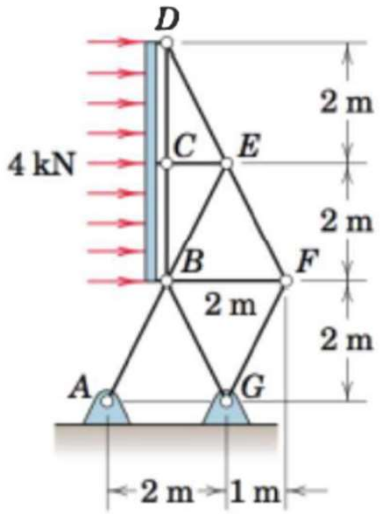
$DE = 1.677$ kN C

$\Sigma F_y = 0$: $DC - 1.677\frac{2}{\sqrt{5}} = 0$

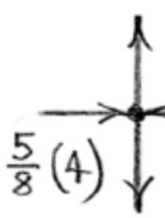
$DC = 1.5$ kN T

ENGINEERING MECHANICS

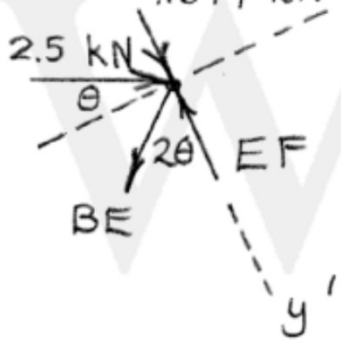
Trusses Numerical:



Joint C



Joint E



$\Sigma F_{x'} = 0: \quad 2.5 \frac{2}{\sqrt{5}} - BE \sin 2\theta = 0$

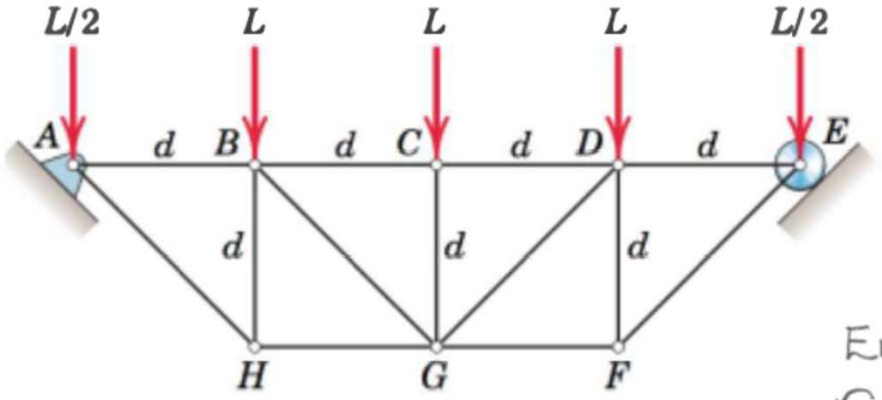
$\sin 2\theta = 0.8$

So $BE = 2.80$ kN T

ENGINEERING MECHANICS

Trusses Numerical:

4/22. Determine the forces in members AB, CG, and DE of the loaded truss.



Entire truss:

$\circlearrowleft + \Sigma M_A = 0$: $-Ld - L(2d) - L(3d) - \frac{L}{2}(4d) + E\frac{\sqrt{2}}{2}(4d) = 0$

$E = 2\sqrt{2}L$

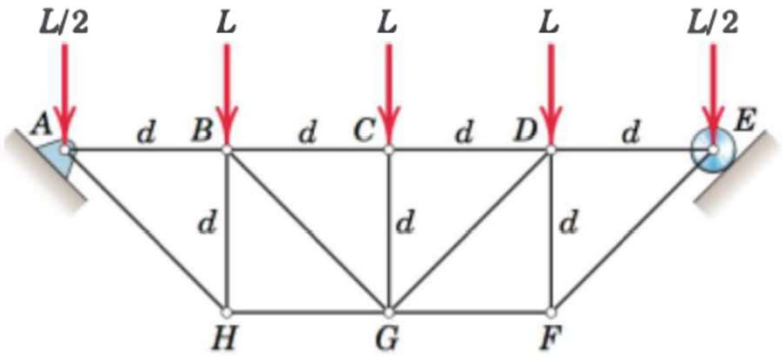
$\Sigma F_x = 0$: $A_x - 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0$, $A_x = 2L$

$\Sigma F_y = 0$: $A_y - 4L + 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0$, $A_y = 2L$

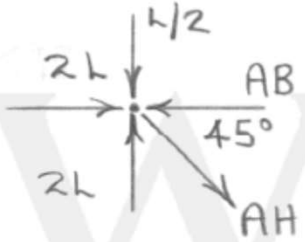
By inspection of joint C, $\underline{CG = L\ C}$

ENGINEERING MECHANICS

Trusses Numerical:



Joint A :



$\sum F_y = 0: \quad 2L - \frac{L}{2} - AH\frac{\sqrt{2}}{2} = 0$

$AH = \frac{3\sqrt{2}}{2}L \quad T$

$\sum F_x = 0: \quad 2L + \frac{3\sqrt{2}}{2}L\frac{\sqrt{2}}{2} - AB = 0$

$AB = \frac{7}{2}L \quad C$

Joint E

$\sum F_y = 0: \quad -\frac{L}{2} + 2\sqrt{2}L\frac{\sqrt{2}}{2} - FE\frac{\sqrt{2}}{2} = 0$

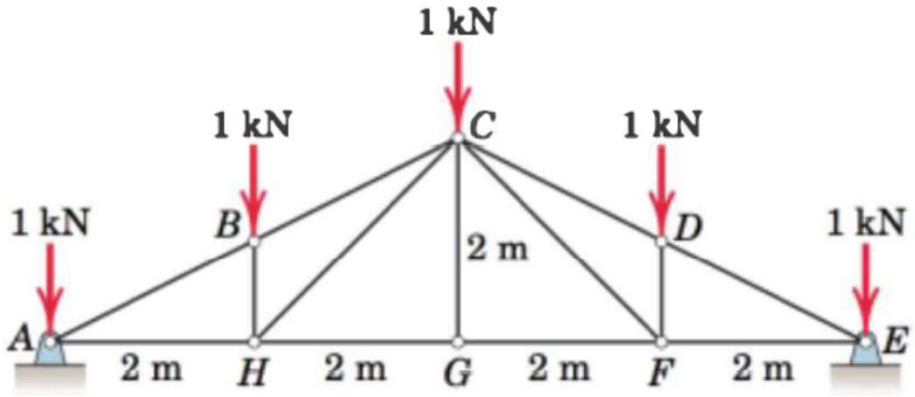
$FE = \frac{3\sqrt{2}}{2}L \quad T$

$\sum F_x = 0: \quad DE - \frac{3\sqrt{2}}{2}L\frac{\sqrt{2}}{2} - 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0$

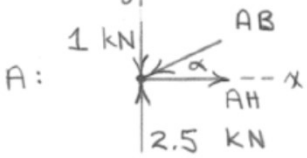
$DE = \frac{7L}{2} \quad C$

Trusses Numerical:

4/23. A snow load transfers the forces shown to the upper joints of a Pratt roof truss. Neglect any horizontal reactions at the supports and solve for the forces in all members.



4/23 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$



$\Sigma F_y = 0$: $2.5 - 1 - AB \sin\alpha = 0$

$AB = 3.35$ kN C

$\Sigma F_x = 0$: $-3.35 \cos\alpha + AH = 0$

$AH = 3$ kN T

$\Sigma F_x = 0$: $3.35 \cos\alpha - BC \cos\alpha = 0$

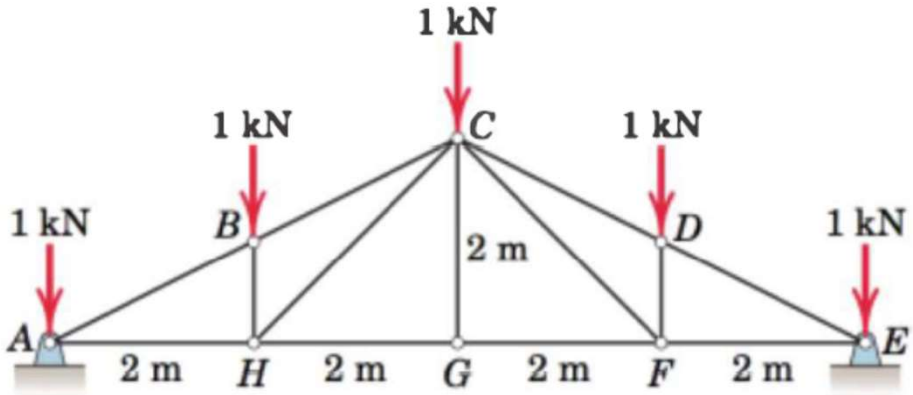
$BC = 3.35$ kN C

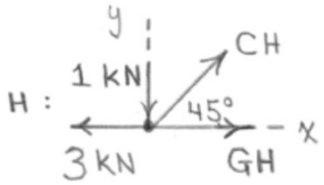
$\Sigma F_y = 0$: $-1 + (3.35 - 3.35)\sin\alpha + BH = 0$

$BH = 1$ kN C

ENGINEERING MECHANICS

Trusses Numerical:





$\Sigma F_y = 0$: $-1 + CH \sin 45° = 0$

$CH = 1.414$ kN T

$\Sigma F_x = 0$: $-3 + 1.41 \cos 45° + GH = 0$

$GH = 2$ kN T

By inspection of joint G and $\Sigma F_y = 0$, $CG = 0$.

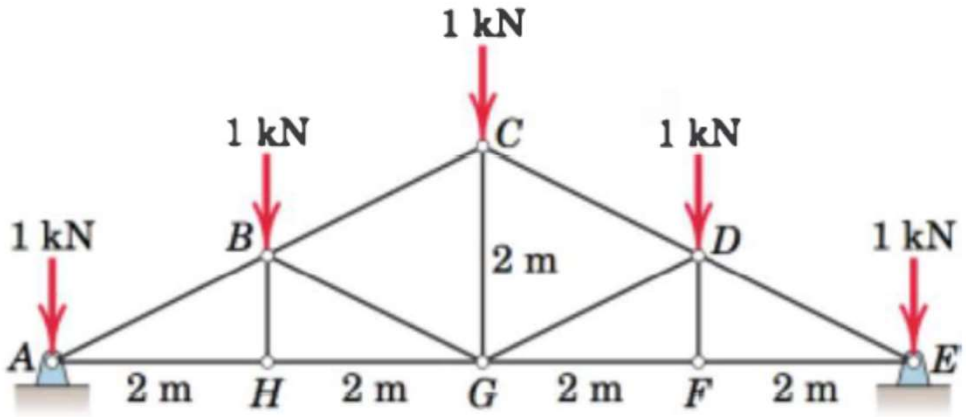
By symmetry,

- $DE = AB = 3.35$ kN C
- $CD = BC = 3.35$ kN C
- $EF = AH = 3$ kN T
- $DF = BH = 1$ kN C
- $CF = CH = 1.414$ kN T
- $FG = GH = 2$ kN T

ENGINEERING MECHANICS

Trusses Numerical:

4/24. The loading of Problem 4/23 is shown applied to a Howe roof truss. Neglect any horizontal reactions at the supports and solve for the forces in all members. Compare with the results of problem 4/23.



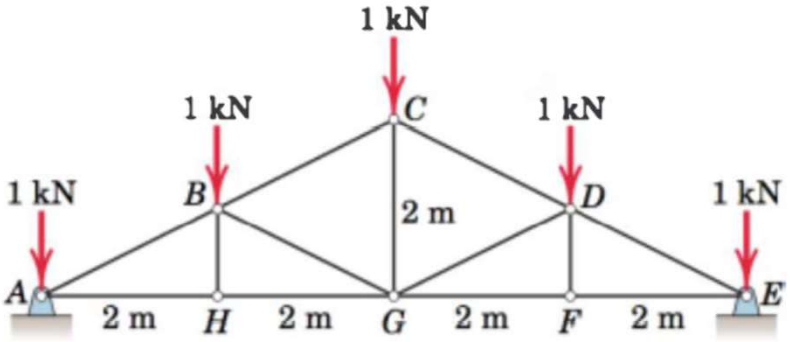
4/24 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

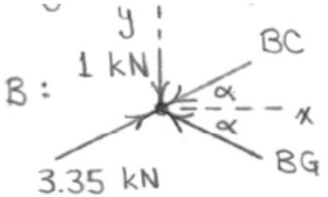
Joint A analysis same as Prob. 4/19: $\begin{cases} AB = 3.35 \text{ kN C} \\ AH = 3 \text{ kN T} \end{cases}$

By inspection, $BH = 0$ and $GH = AH$.

ENGINEERING MECHANICS

Trusses Numerical:

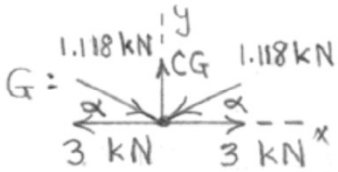




$\Sigma F_y = 0: \; -1 + 3.35 \sin\alpha + BG \sin\alpha - BC \sin\alpha = 0$

$\Sigma F_x = 0: \; 3.35 \cos\alpha - BC \cos\alpha - BG \cos\alpha = 0$

$\Rightarrow BC = 2.24 \text{ kN C}, \quad BG = 1.118 \text{ kN C}$



$\Sigma F_y = 0: \; CG - 2(1.118)\sin\alpha = 0$

$CG = 1 \text{ kN T}$

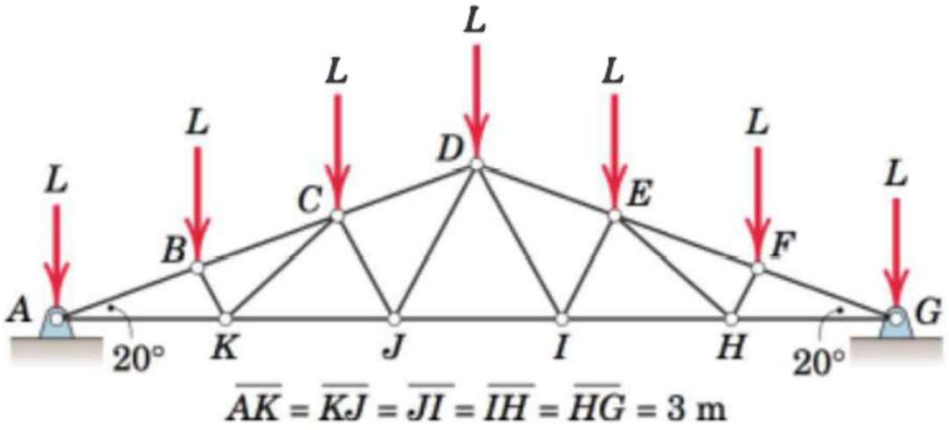
By symmetry,

- $DE = AB = 3.35 \text{ kN C}$
- $CD = BC = 2.24 \text{ kN C}$
- $EF = AH = 3 \text{ kN T}$
- $DF = BH = 0$
- $FG = GH = 3 \text{ kN T}$
- $DG = BG = 1.118 \text{ kN C}$

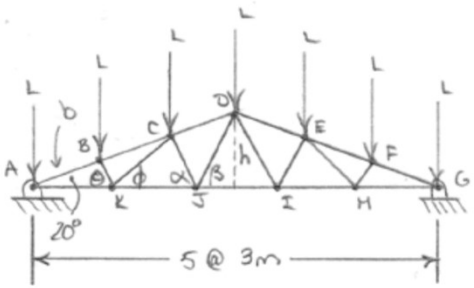
ENGINEERING MECHANICS

Trusses Numerical:

4/26. Determine the forces in members EH and EI of the double Fink truss. Neglect any horizontal reactions at the supports and note that joints E and F divide DG into thirds.



4/26



$h = \frac{15}{2}\tan 20^\circ \longrightarrow h = 2.73\ m$

$\overline{AD} = \frac{h}{\sin 20^\circ} = \frac{2.73}{\sin 20^\circ} \longrightarrow \overline{AD} = 7.98\ m$

$b = \frac{1}{3}\overline{AD} = \frac{1}{3}(7.98) \longrightarrow b = 2.66\ m$

$\theta = \tan^{-1}\left(\frac{b\sin 20^\circ}{3 - b\cos 20^\circ}\right) \longrightarrow \theta = 61.2^\circ$

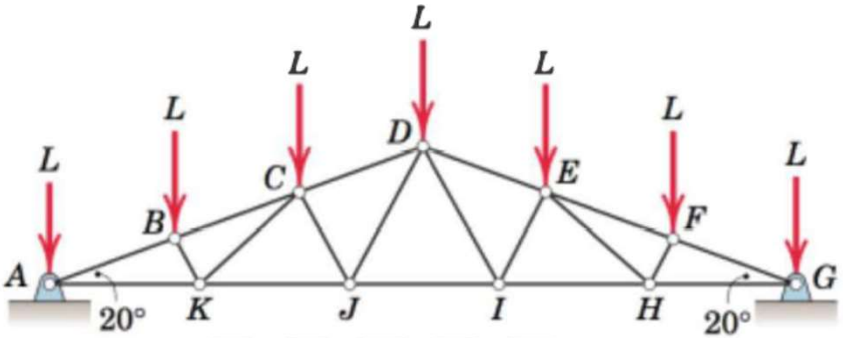
$\phi = \tan^{-1}\left(\frac{2b\sin 20^\circ}{2b\cos 20^\circ - 3}\right) \longrightarrow \phi = 42.3^\circ$

$\alpha = \tan^{-1}\left(\frac{2b\sin 20^\circ}{6 - 2b\cos 20^\circ}\right) \longrightarrow \alpha = 61.2^\circ$

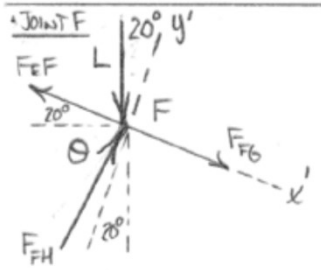
$\beta = \tan^{-1}\left(\frac{h}{1.5}\right) \longrightarrow \beta = 61.2^\circ$

ENGINEERING MECHANICS

Trusses Numerical:

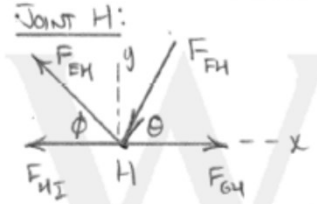


$\overline{AK} = \overline{KJ} = \overline{JI} = \overline{IH} = \overline{HG} = 3$ m



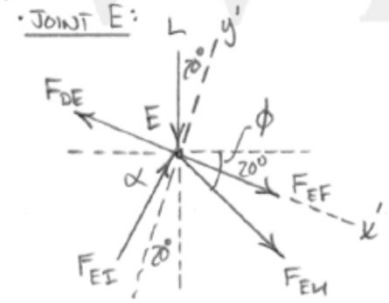
$\Sigma F_{y'} = 0: \quad F_{FH}\cos(90^\circ - 20^\circ - \theta) - L\cos 20^\circ = 0$

$F_{FH} = 0.951L \quad C$



$\Sigma F_y = 0: \quad F_{EH}\sin\phi - F_{FH}\sin\theta = 0$

$F_{EH} = 1.238L \quad T$



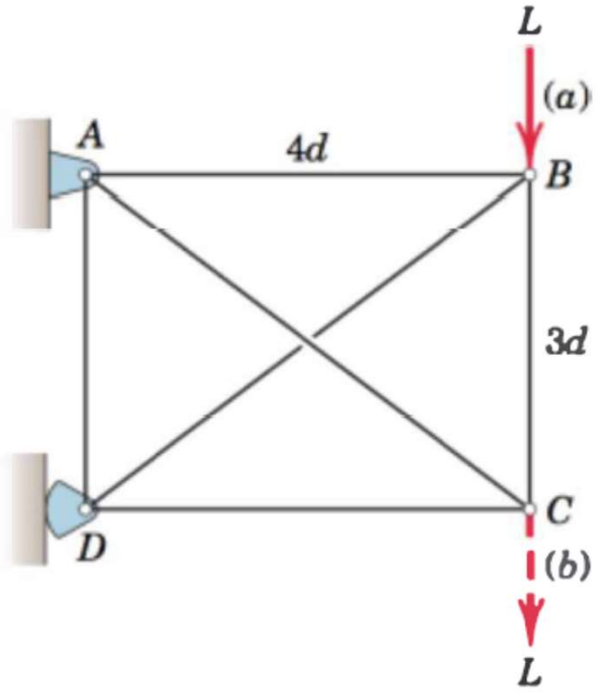
$\Sigma F_{y'} = 0: \quad F_{EI}\cos(90^\circ - \alpha - 20^\circ) - L\cos 20^\circ - F_{EH}\sin(\phi - 20^\circ) = 0$

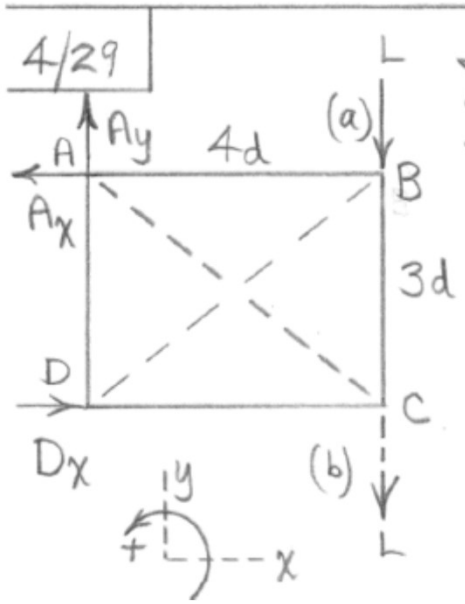
$F_{EI} = 1.426L \quad C$

ENGINEERING MECHANICS

Trusses Numerical:

4/29. The rectangular frame is composed of four perimeter two forces members and two cables AC and BD which are incapable of supporting compression. Determine the forces in all members due to the load L in position (a) and then in position (b).





$\Sigma F_y = 0: \quad A_y - L = 0, \quad A_y = L$

$\Sigma M_A = 0: \quad D_x(3d) - L(4d) = 0$

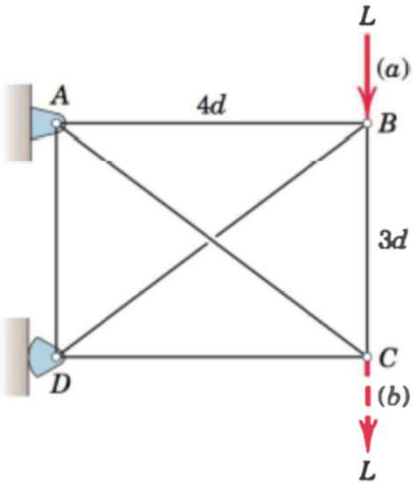
$D_x = \frac{4L}{3}$

$\Sigma F_x = 0: \quad A_x = D_x = \frac{4L}{3}$

(Reactions hold for (a) & (b).)

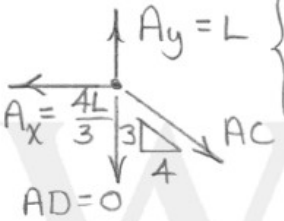
ENGINEERING MECHANICS

Trusses Numerical:



(a) Assume that BD goes slack. From an inspection of joint B, $\underline{AB = 0}$ and $\underline{BC = L\ C.}$ Similarly, from joint D, $\underline{AD = 0}$ and $\underline{CD = \frac{4L}{3}\ C.}$

Joint A:

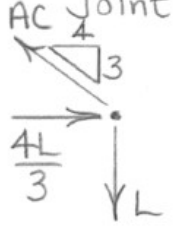


$$\begin{cases} \Sigma F_y = 0: & L - \frac{3}{5}AC = 0, & \underline{AC = \frac{5L}{3}\ T} \\ \Sigma F_x = 0: & -\frac{4L}{3} + \frac{5L}{3}\cdot\frac{4}{5} = 0 \ \checkmark \end{cases}$$

Because AC is in tension, assumption is valid.

(b) Assume that BD goes slack. From joint B, $\underline{AB = BC = 0}$. From joint D, $\underline{AD = 0}$ & $\underline{CD = \frac{4L}{3}\ C.}$

Joint C:



$$\begin{cases} \Sigma F_y = 0: & AC\left(\frac{3}{5}\right) - L = 0, & \underline{AC = \frac{5L}{3}\ T} \\ \Sigma F_x = 0: & \frac{4L}{3} - \frac{5L}{3}\left(\frac{4}{5}\right) = 0 \ \checkmark \end{cases}$$

THANK YOU

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