



B-Tech-II

Department of Science and Humanities



PROBLEMS ON LAPLACE TRANSFORMS

Problems on Linearity property



1. Find
$$L[2 + 5t^3 + 4e^{-3t} + 10e^t + \sin 2t]$$

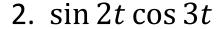
Let
$$f(t) = 2 + 5t^3 + 4e^{-3t} + 10e^t + \sin 2t$$

$$L[f(t)] = L[2 + 5t^3 + 4e^{-3t} + 10e^t + \sin 2t]$$

$$L[f(t)] = L[2] + 5L[t^3] + 4L[e^{-3t}] + 10L[e^t] + L[\sin 2t]$$

$$L[f(t)] = \frac{2}{s} + 5\frac{6}{s^4} + 4\frac{1}{s+3} + 10\frac{1}{s-1} + \frac{2}{s^2+4}$$

Problems on Linearity property...



Let
$$f(t) = \sin 2t \cos 3t$$

$$L[f(t)] = L[\sin 2t \cos 3t]$$

$$= \frac{1}{2}L[\sin 5t - \sin t] = \frac{1}{2}\left[\frac{5}{s^2 + 25} - \frac{1}{s^2 + 1}\right]$$

3.
$$L[\sin t \sin 3t \sin 5t] = \frac{1}{4} \left[\frac{3}{s^2+9} - \frac{1}{s^2+1} - \frac{9}{s^2+81} + \frac{7}{s^2+49} \right]$$

4.
$$L[\cos^2 4t] = \frac{1}{2}[1 + \cos 8t]$$

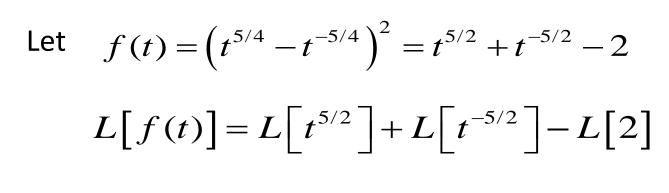
= $\frac{1}{2}[\frac{1}{s} + \frac{s}{s^2 + 64}]$



Problems on Linearity property...



$$\left(t^{5/4}-t^{-5/4}\right)^2$$



$$L[f(t)] = \frac{\Gamma(\frac{5}{2}+1)}{s^{7/2}} + \frac{\Gamma(-\frac{5}{2}+1)}{s^{-3/2}} - \frac{2}{s}$$



Problems on Linearity property...



Consider

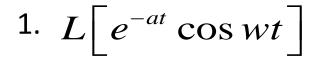
$$\Gamma\left(\frac{5}{2}+1\right) = \frac{5}{2}\Gamma\left(\frac{5}{2}\right) = \frac{5}{2}\Gamma\left(\frac{3}{2}+1\right) = \frac{5}{2}\frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{15}{4}\Gamma\left(\frac{1}{2}+1\right) = \frac{15}{4}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{15}{8}\sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}+1\right) = \Gamma\left(-\frac{3}{2}\right) = \frac{\Gamma\left(-\frac{3}{2}+1\right)}{(-3/2)} = \frac{-2}{3}\Gamma\left(-\frac{1}{2}\right) = \frac{-2}{3}\frac{\Gamma\left(-\frac{1}{2}+1\right)}{(-1/2)} = \frac{4}{3}\sqrt{\pi}$$

$$L[f(t)] = \frac{15}{8s^{7/2}} \sqrt{\pi} + \frac{4}{3s^{-3/2}} \sqrt{\pi} - \frac{2}{s}$$

Therefore

Problems on Frequency shift property





In this case
$$f(t) = \cos wt$$

$$F(s) = \frac{s}{s^2 + w^2}$$
and
$$F(s+a) = \frac{(s+a)}{(s+a)^2 + w^2}$$

$$L[e^{-at}\cos(wt)] = \frac{(s+a)}{(s+a)^2 + (w)^2}$$



Illustration of Frequency shift property

1.
$$L\left[e^{at}t^n\right] = \frac{n!}{(s-a)^{n+1}}$$

2.
$$L[e^{at}\cosh bt] = \frac{s-a}{(s-a)^2 - b^2}$$

3.
$$L\left[e^{at}\sinh bt\right] = \frac{b}{(s-a)^2 - b^2}$$

4.
$$L[e^{at}\cos bt] = \frac{s-a}{(s-a)^2 + b^2}$$

$$L\left[e^{at}\sin bt\right] = \frac{b}{\left(s-a\right)^2 + b^2}$$



Laplace transform of derivatives...

1. Find the Laplace transform of y'' - 10y' + 9y = 5t

$$L[y''] - 10L[y'] + 9L[y] = L[5t]$$

$$s^{2}Y(s) - sy(0) - y'(0) - 10(sY(s) - y(0)) + 9Y(s) = \frac{5}{s^{2}}$$

2. Find the Laplace transform of $x'' - 3x' + 2x = e^{-4t}$

$$L[x''] - 3L[x'] + 2L[x] = L[e^{-4t}]$$

$$s^{2}X(s) - sx(0) - x'(0) - 3(sX(s) - x(0)) + 2X(s) = \frac{1}{s+4}$$



Problem on Transform of integral of a function

Find
$$L\left[e^{-4t}\int_{0}^{t}t\sin 3tdt\right]$$

$$L[t\sin 3t] = \frac{6s}{(s^2+9)^2}$$

$$L\left[\int_{0}^{t} t \sin 3t dt\right] = \frac{6}{\left(s^{2} + 9\right)^{2}}$$

$$L\left[e^{-4t}\int_{0}^{t}t\sin 3tdt\right] = \frac{6}{\left[(s+4)^{2}+9\right]^{2}}$$



$$\begin{array}{rcl}
 & L\left[2+5t^{3}+4e^{-3t}+10e^{t}+8in\,at\right] \\
 & = L\left[2\right]+5L\left[t^{3}\right]+4L\left[e^{-3t}\right]+10L\left[e^{t}\right]+L\left[Min\,at\right] \\
 & = \frac{2}{8}+5.\frac{3!}{8^{3+1}}+4\frac{1}{8+2}+10.\frac{1}{8^{-1}}+\frac{2}{8^{2}+2^{2}} \\
 & = \frac{2}{8}+\frac{30}{8^{4}}+\frac{4}{8+3}+\frac{10}{8-1}+\frac{2}{8^{2}+4}
\end{array}$$







$$L[Jt] = L[t^2] = I[2+1] = I[3]$$

$$\frac{3}{2}$$

$$f(t) = \cos^2 4t - 1 + \cos 8t$$

$$L[f(t)] = LL[H\cos 8t]$$

$$= \frac{1}{2} \left[L(1) + L[\cos 8t] \right]$$

$$=\frac{1}{2}\left[\frac{1}{8}+\frac{8}{8^{2}+8^{2}}\right]$$



L(4(t))

given flf)- mint. sin 3t nin 5 t

MinA. Min B= 1 (68 (A-B)-108 (A+B)

sint. (mist.mist) = 1 [ws(2)f - ws8t]. Sint

= 1 [usat. smit - ws8t. mint] = 1 [\f [\lin 3t - \lin t] - \frac{1}{2} \left[\lin 9t - \sin 74] \]



$$L[f(t)] = L[L(min3t) - L(mint) - L(min9t) + L(min9t)$$

$$= \int \frac{3}{8^{2}+9} - \frac{1}{8^{2}+1} - \frac{9}{8^{2}+8} + \frac{7}{8^{2}+49}$$

9) find
$$L(f(t))$$
 given $f(t) = \int 0$ for $0 < t < 2$

$$= \int_{0}^{\infty} e^{-St} f(t) dt$$

$$= \int_{0}^{\infty} e^{-St} f(t) dt$$

$$= 0 + 4 \int_{-S}^{\infty} e^{-St} dt = -4 \left[0 - \overline{e}^{28} \right]$$

$$= \frac{4 \cdot e^{-28}}{8}$$



$$\cos\left[\pi\right] = 1 - \frac{\chi^{2}}{2!} + \frac{\pi^{4}}{4!} - \frac{\chi^{6}}{6!} + \dots$$

$$los(t) = 1 - \frac{t}{2!} + \frac{t^2}{4!} - \frac{t^3}{6!} + \cdots$$

$$\mathcal{L}(\omega s) = \mathcal{L} \left[-\frac{t}{\alpha!} + \frac{t^2}{4!} - \frac{t^3}{6!} + \cdots \right]$$

$$= \frac{1}{8} - \frac{1!}{8!} + \frac{1}{4!} \frac{2!}{8^3} - \frac{1}{6!} \frac{3!}{5!} + \cdots$$

$$-\frac{1}{5}$$
 $L[COSVE] = \frac{1}{5} - \frac{1}{25^2} + \frac{1}{128^3} - \frac{1}{1208^4} - \dots$







THANK YOU

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