ENGINEERING MECHANICS

Prof. Vinay Papanna
Department of Mechanical Engineering

ENGINEERING MECHANICS

Friction Applications in Wedges

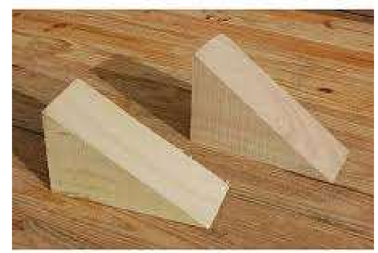
Prof. Vinay Papanna
Department of Mechanical Engineering

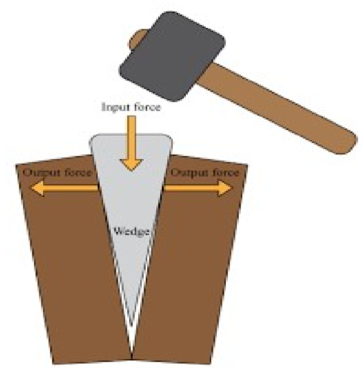
Wedges:

A wedge is one of the simplest and most useful machines. A wedge is used to produce small adjustments in the position of a body or to apply large forces.

Wedges largely depend on friction to function. When sliding of a wedge is impending, the resultant force on each sliding surface of the wedge will be inclined from the normal to the surface by an amount equal to the friction angle.

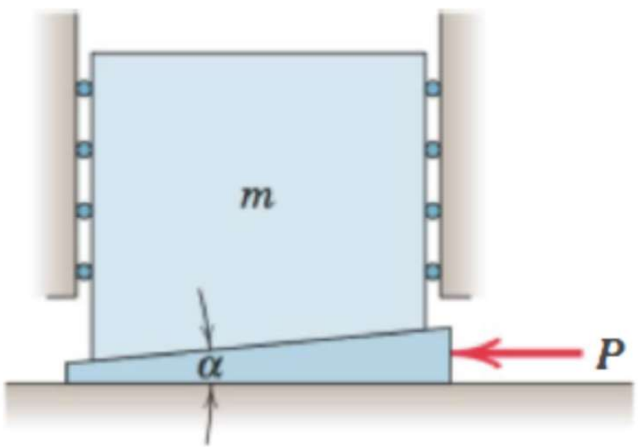
The component of the resultant along the surface is the friction force, which is always in the direction to oppose the motion of the wedge relative to the mating surfaces.

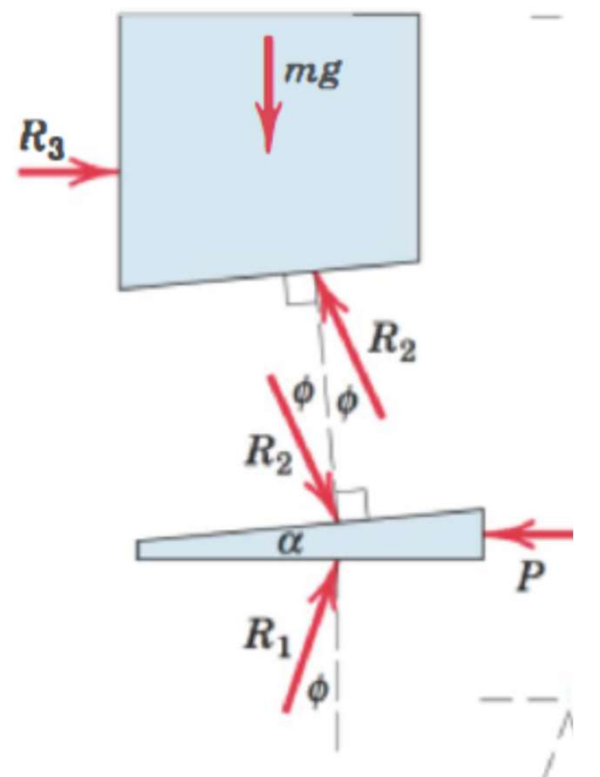




Friction Application in Wedges

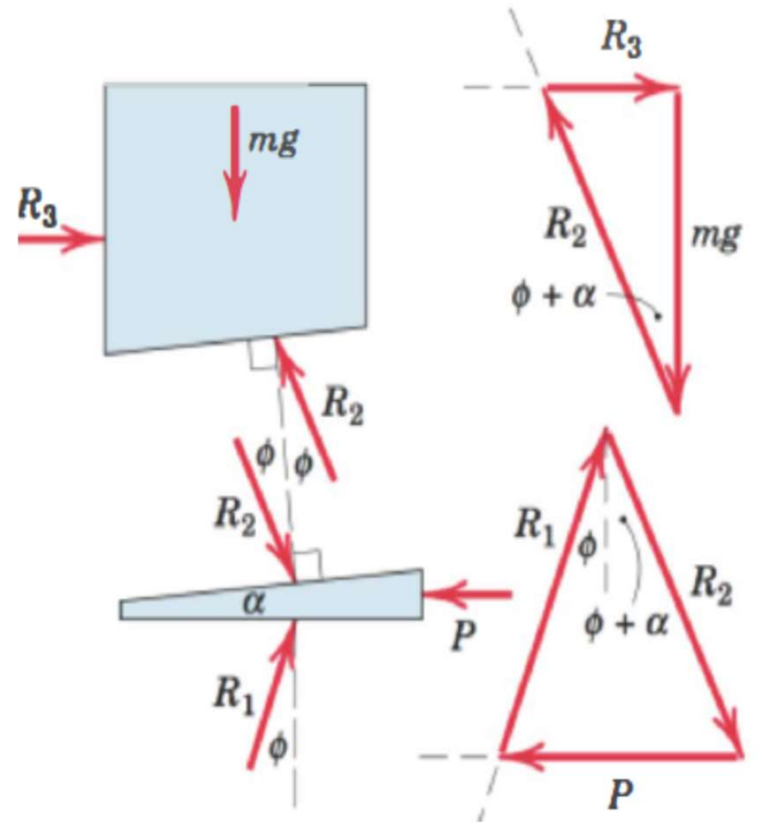
- Figure shows a wedge used to position or lift a large mass m , where the vertical loading is m_g.
- The coefficient of friction for each pair of surfaces is $\mu = \tan \Phi$. The force P required to start the wedge is found from the equilibrium triangles of the forces on the load and on the Wedge.





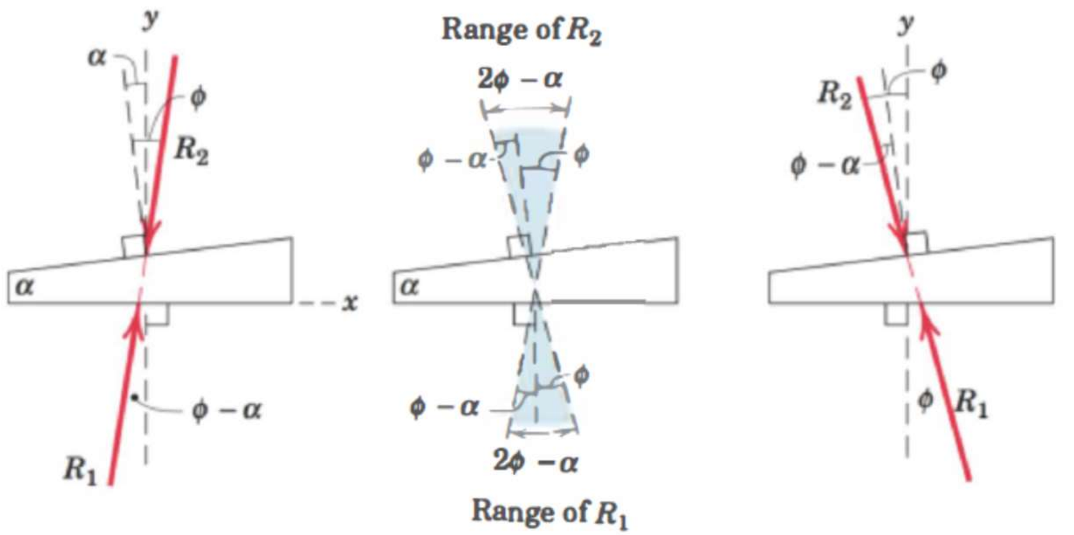
Friction Application in Wedges

- The reactions are inclined at an angle Φ from their respective normal and are in the direction to oppose the motion.
- We neglect the mass of the wedge. From the free-body diagrams we write the force equilibrium conditions by equating to zero the sum of the force vectors acting on each body.
- The solutions of these equations are shown in figure, were R_2 is found first in the upper diagram using the known value of mg. The force P is then found from the lower triangle once the value of R_2 has been established.



Friction Application in Wedges

- If P is removed and the wedge remains in place, equilibrium of the wedge requires that the equal reactions R_1 and R_2 be collinear as shown in Figure.
- Where the wedge angle α is taken to be less than Φ. Part a of the figure represents impending slippage at the upper surface, and part c of the figure represents impending slippage at the lower surface.



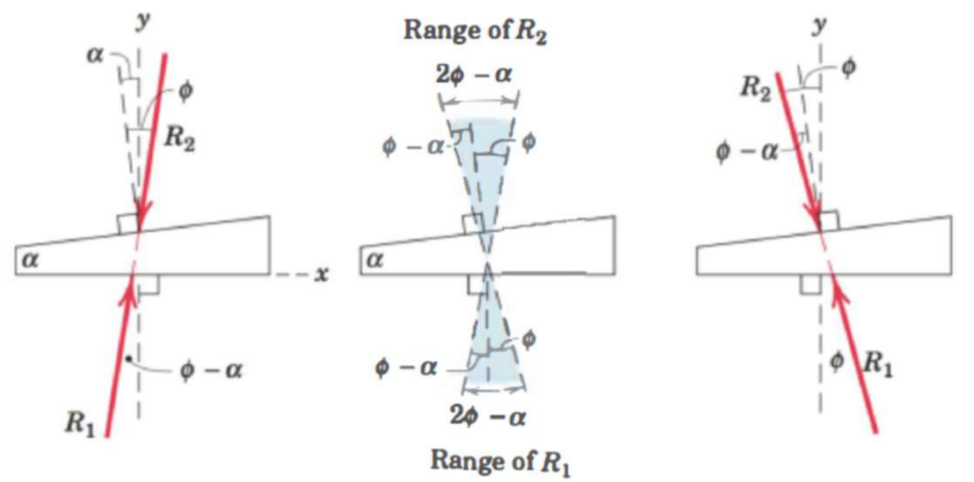
(a) Slipping impending at upper surface

(b) Range of $R_1 = R_2$ for no slip

(c) Slipping impending at lower surface

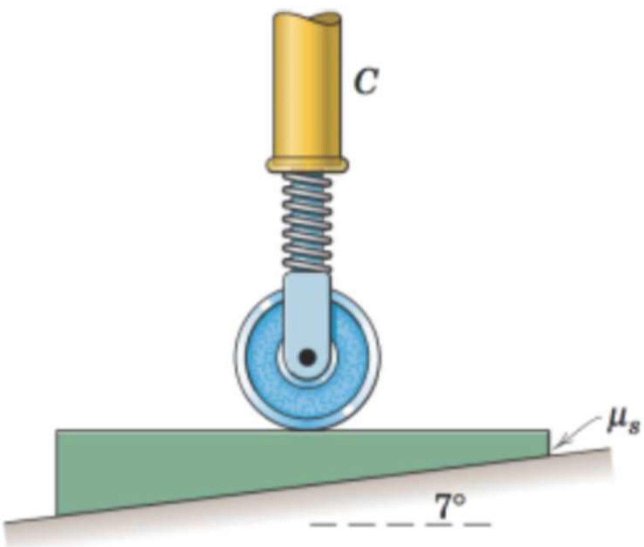
Friction Application in Wedges

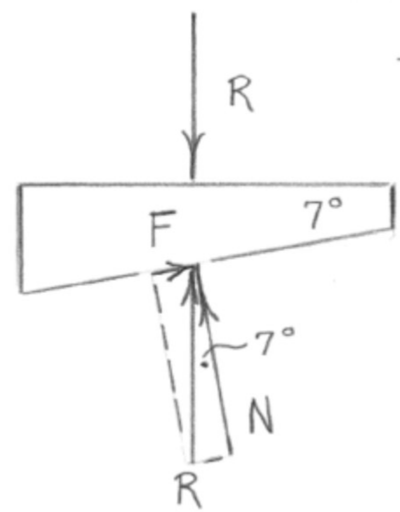
- In order for the wedge to slide out of its space, slippage must occur at both surfaces simultaneously; otherwise, the wedge is self locking, and there is a finite range of possible intermediate angular positions of R_1 and R_2 for which the wedge will remain in place.
- Figure illustrates this range and shows that simultaneous slippage is not possible, if $\alpha < 2\ \Phi$. You are encouraged to construct additional diagrams for the case were $\alpha > \Phi$ and verify that the wedge is self-locking as long as $\alpha < 2\ \Phi$.



(a) Slipping impending at upper surface (b) Range of $R_1 = R_2$ for no slip (c) Slipping impending at lower surface

6/53 The 7^0 wedge is driven under the spring-loaded wheel whose supporting strut C is fixed. Determined the minimum coefficient of static friction μ_s for which the wedge will remain in place. Neglect all friction associated with the wheel.

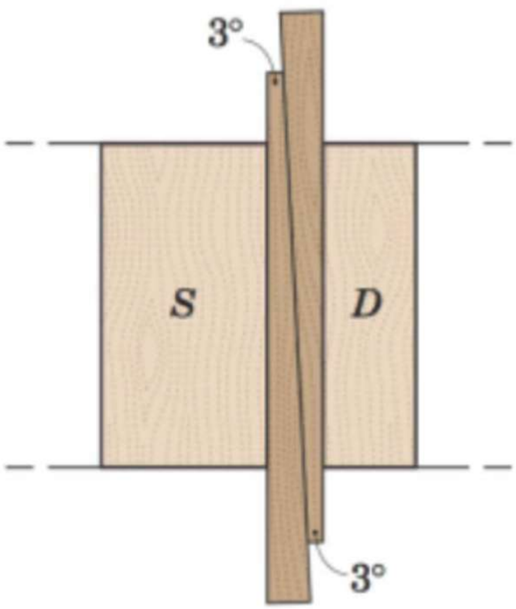




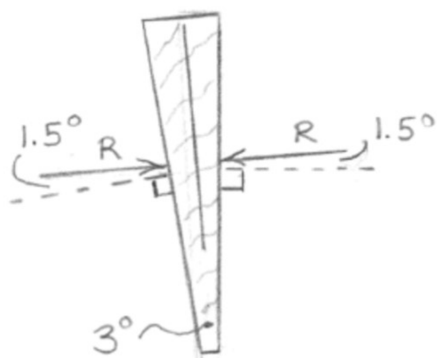
$$\frac{F}{N} = \mu_s = \tan 7^\circ$$

$$\underline{\mu_s = 0.1228}$$

6/55 In wood-frame construction, two shims are frequently used to fill the gap between the framing S and the thinner window/door jamb D. The members S and D are shown in cross section in the figure. For the 3° shims shown, determine the minimum necessary coefficient of static friction so that the shims will remain in place.



6/55 Consider the right shim (3° angle exaggerated)

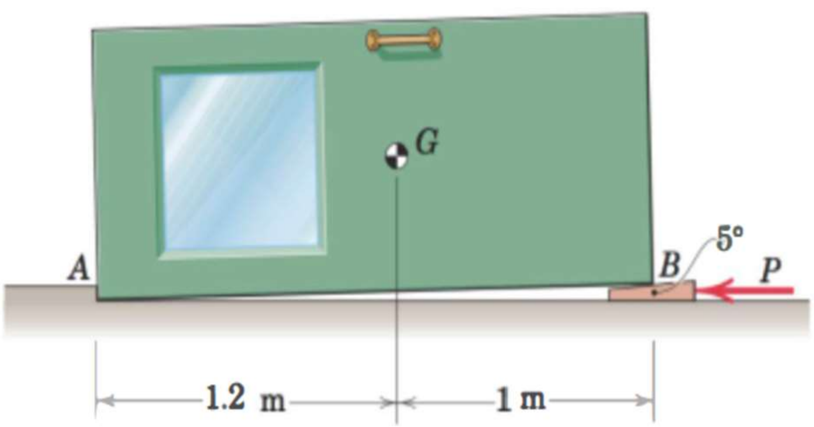


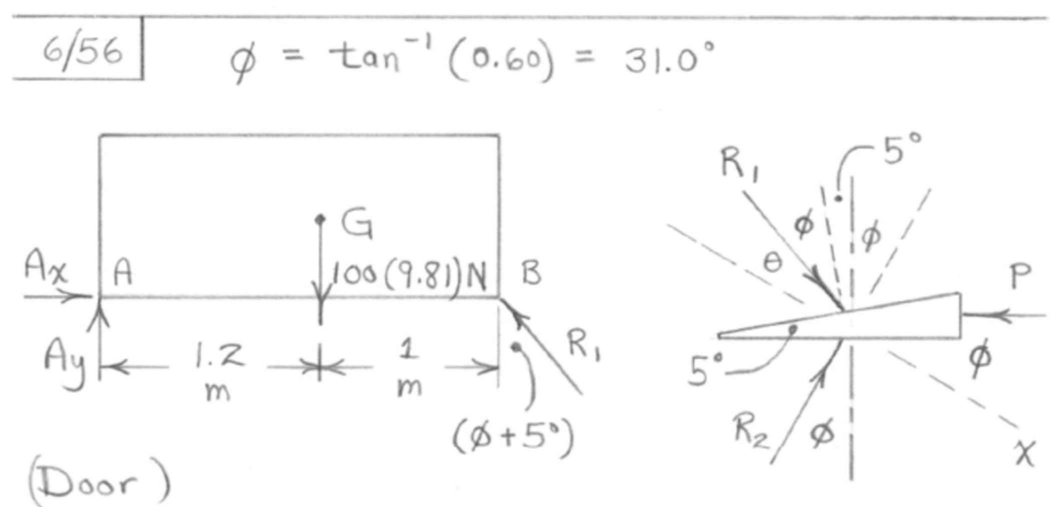
$\mu_s = \tan\phi = \tan 1.5°$

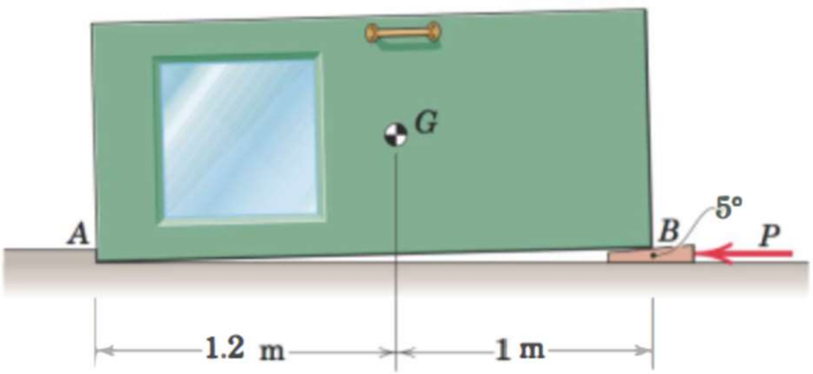
$= \underline{0.0262}$

(A very modest requirement!)

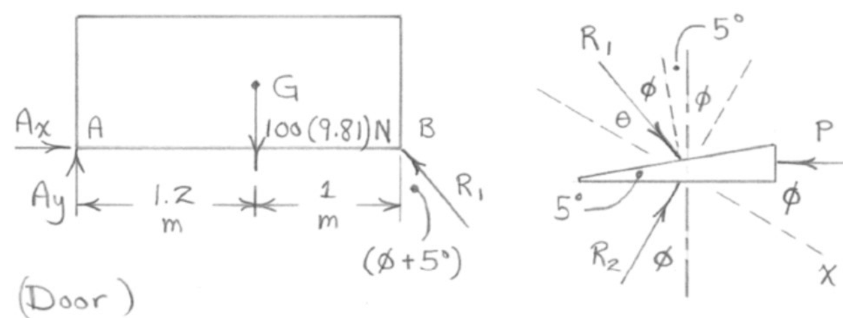
6/56 The 100-kg industrial door with mass center at G is being positioned for repair by insertion of the 5^0 wedge under corner B. Horizontal movement is prevented by the small ledge at corner A. If the coefficients of static friction at both the top and bottom wedge surfaces are 0.60, determine the force P required to lift the door at B.







6/56 $\phi = \tan^{-1}(0.60) = 31.0^\circ$



(Door)

$$\Sigma M_A = 0: \quad R_1 \cos(31.0^\circ + 5^\circ)(2.2) - 100(9.81)(1.2) = 0$$

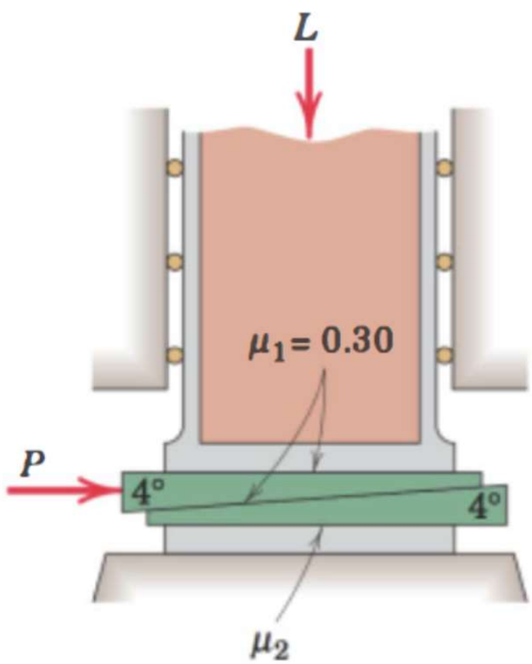
$$R_1 = 661 \text{ N}$$

(Wedge) (Note $\theta = 90^\circ - 2\phi - 5^\circ = 23.1^\circ$)

$$\Sigma F_x = 0: \quad 661 \cos 23.1^\circ - P \cos 31.0^\circ = 0$$

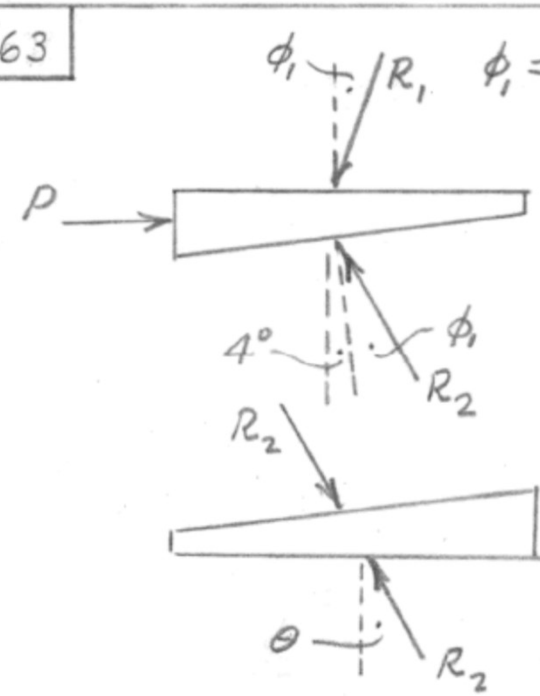
$$\underline{P = 709 \text{ N}}$$

6/63 The two 4° wedges are used to position the vertical column under a load L. What is the least value of the coefficient of friction μ_2 for the bottom pair of surfaces for which the column may be raised by applying a single horizontal force P to the upper wedge?



6/63

$\phi_1 = \tan^{-1} 0.3 = 16.70°$

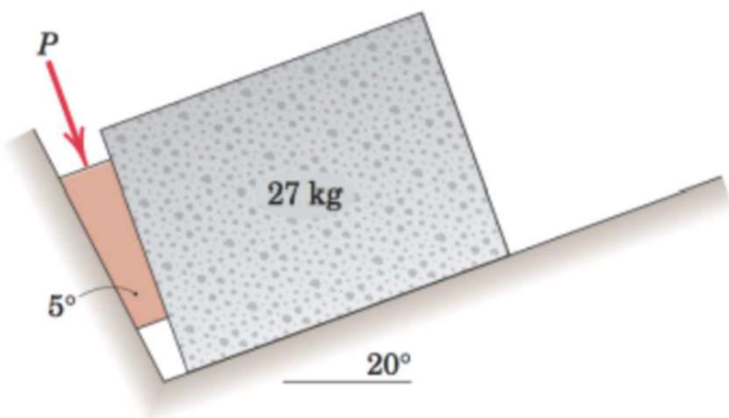


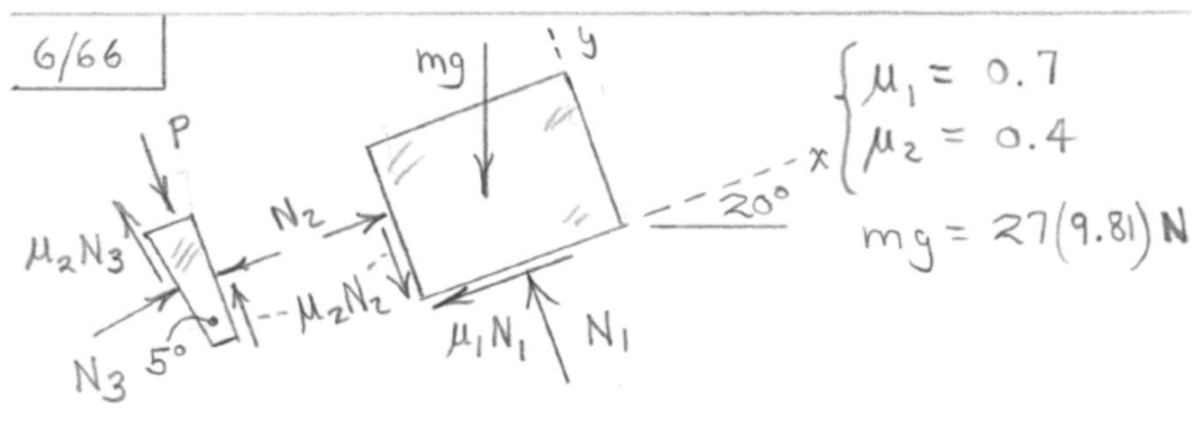
In order for column to be raised, bottom wedge must not move. Therefore μ_2 must be greater than $\tan\theta$ or

$(\mu_2)_{min} = \tan(4 + 16.70)$

$= \underline{0.378}$

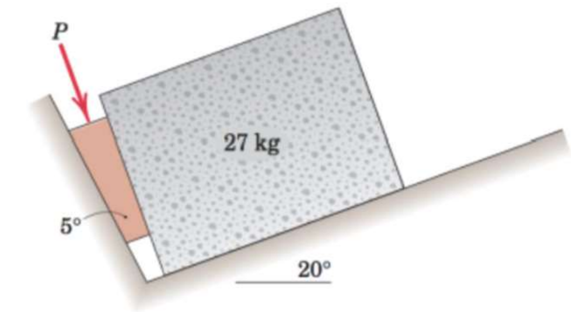
6/66 The coefficient of static friction for both wedge surfaces is 0.40 and that between the 27-kg concrete block and the 20° incline is 0.70. Determine the minimum value of the force P required to begin moving the block up the incline. Neglect the weight of the wedge.

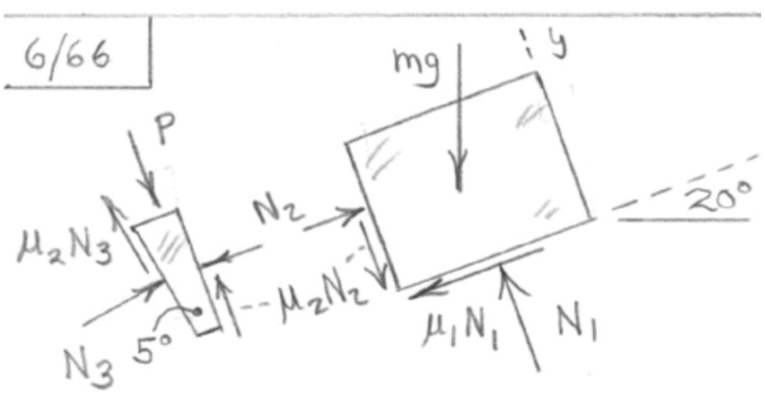




ENGINEERING MECHANICS

Application of Friction in Wedges





Block:

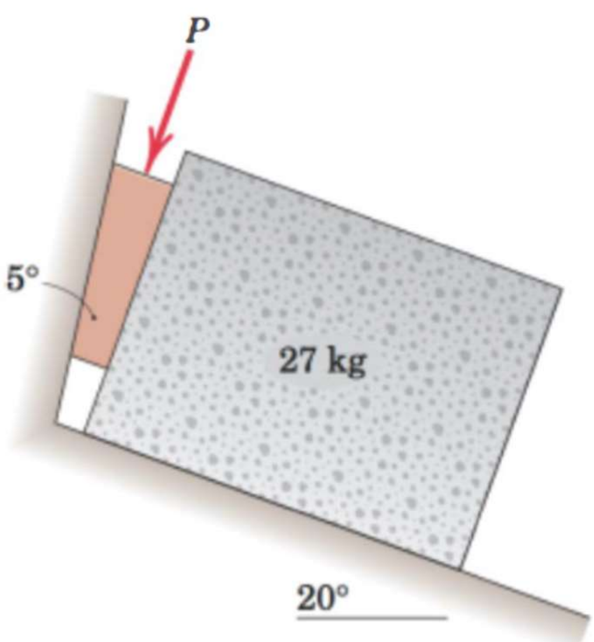
$$\begin{cases} \Sigma F_x = 0: & -mg\sin 20^\circ + N_2 - \mu_1 N_1 = 0 \quad (1) \\ \Sigma F_y = 0: & -mg\cos 20^\circ + N_1 - \mu_2 N_2 = 0 \quad (2) \end{cases}$$

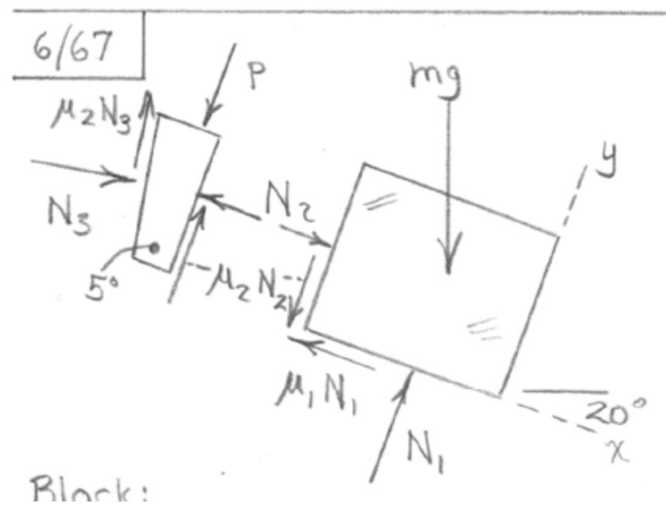
Wedge:

$$\begin{cases} \Sigma F_x = 0: & N_3\cos 5^\circ - \mu_2 N_3 \sin 5^\circ - N_2 = 0 \quad (3) \\ \Sigma F_y = 0: & N_3 \sin 5^\circ + \mu_2 N_3 \cos 5^\circ + \mu_2 N_2 - P = 0 \quad (4) \end{cases}$$

Solution: $\begin{cases} N_1 = 396\ \text{N} & N_2 = 368\ \text{N} \\ N_3 = 383\ \text{N} & \underline{P = 333\ \text{N}} \end{cases}$

6/67 Repeat Prob. 6/66, only now the 27-kg concrete block begins to move down the 20° incline as shown. All other conditions remain as in Prob. 6/66.



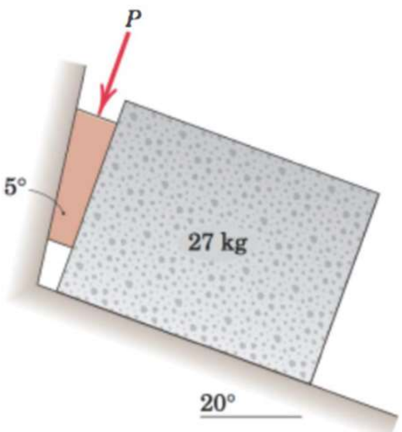


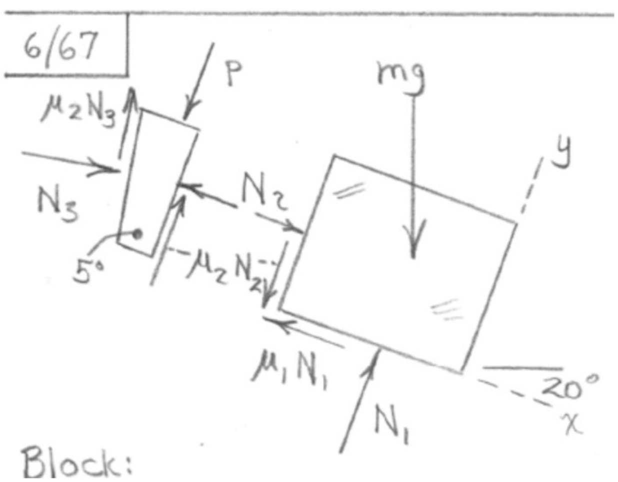
$\mu_1 = 0.7$

$\mu_2 = 0.4$

$mg = 27(9.81)$ N

ENGINEERING MECHANICS
Application of Friction in Wedges





Block:

$$\begin{cases} \sum F_x = 0 : \; mg\sin 20^\circ + N_2 - \mu_1 N_1 = 0 & (1) \\ \sum F_y = 0 : \; -mg\cos 20^\circ + N_1 - \mu_2 N_2 = 0 & (2) \end{cases}$$

Wedge:

$$\begin{cases} \sum F_x = 0 : \; N_3\cos 5^\circ - \mu_2 N_3 \sin 5^\circ - N_2 = 0 & (3) \\ \sum F_y = 0 : \; N_3 \sin 5^\circ + \mu_2 N_3 \cos 5^\circ + \mu_2 N_2 - P = 0 & (4) \end{cases}$$

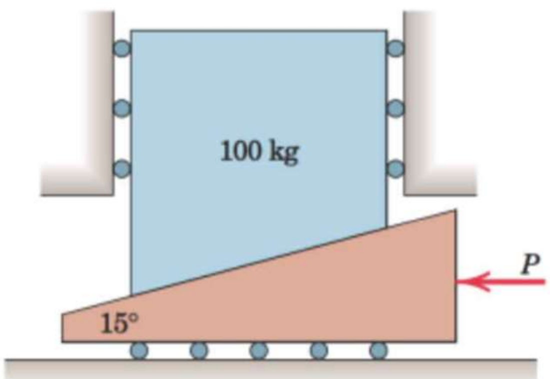
Solution: $N_1 = 295\text{ N}$ $\quad N_2 = 116.2\text{ N}$

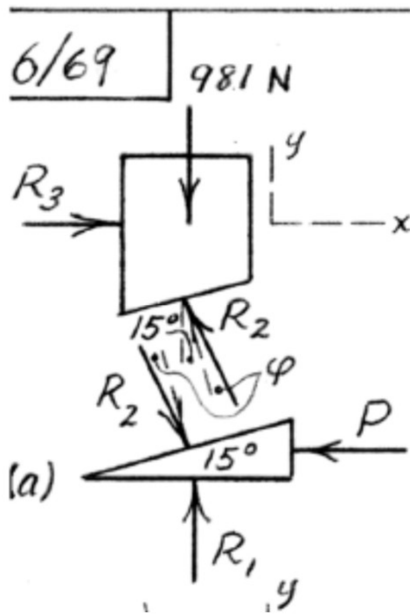
$N_3 = 120.8\text{ N}$ $\quad P = \underline{105.1\text{ N}}$

ENGINEERING MECHANICS

Application of Friction in Wedges

6/69 The coefficient of static friction μ_s between the 100-kg body and the 15° wedge is 0.20. Determine the magnitude of the force P required to begin raising the 100-kg body if (a) rollers of negligible friction are present under the wedge, as illustrated, and (b) the rollers are removed and the coefficient of static friction $\mu_s = 0.20$ applies at this surface as well.





Friction angle $\varphi = \tan^{-1}(0.20) = 11.31°$

(a) Rollers under wedge:

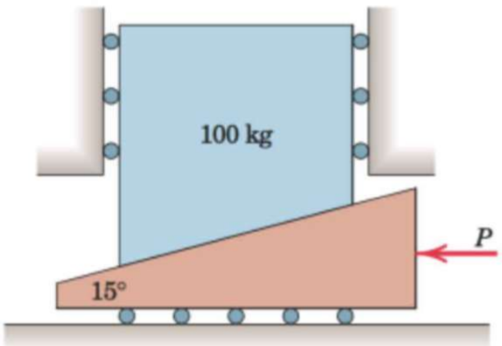
$\Sigma F_y = 0: -981 + R_2 \cos(15° + 11.31°) = 0$

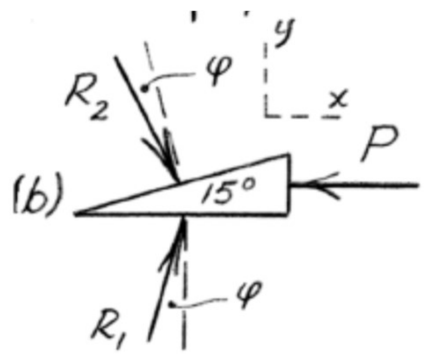
$R_2 = 1094$ N

$\Sigma F_x = 0: R_2 \sin(15° + 11.31°) - P = 0$

$\underline{P = 485 \text{ N}}$

(b) the rollers are removed and the coefficient of static friction $\mu_s = 0.20$ applies at this surface as well.





(b) Rollers removed:

Value of R_2 from 100-kg body is unchanged.

$\Sigma F_x = 0$: $R_2 \sin(15° + 11.31°) - P + R_1 \sin(11.31°) = 0$

With R_1 determined from overall equilibrium as $R_1 = 981/\cos 11.31° = 1000$ N, we solve for P as

$\underline{P = 681 \text{ N}}$

THANK YOU

Prof. Vinay Papanna
Department of Mechanical Engineering

vinayp@pes.edu

+91 9980933582