Formation of Partial differential equation by eliminating arbitrary constants

1) $(x-a)^2 + (y-b)^2 + z^2 = c^2$ where a, b, c are constants

solution:

Differentiating the given eq w.r.t x and y

$$2(x-a) + 2z\frac{\partial z}{\partial x} = 0 \quad \text{---} (1)$$

$$2(y-b) + 2z\frac{\partial z}{\partial y} = 0 \quad \text{---} (2)$$

Replacing $\frac{\partial z}{\partial x} = p$ and $\frac{\partial z}{\partial y} = q$ and substituting

$(x-a) = -zp$ and $(y-b) = -zq$ in given eq we obtain

$(p^2 + q^2 + 1)z^2 = c^2$ as required p.d.eq.

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2) $\log_e(az-1) = x + ay + b$.

Solution.

Diffentiating eq partially w.r.t x and y

$\frac{1}{az-1} ap = 1 \Rightarrow ap + az = -1 \Rightarrow a = \frac{1}{z-p}$

$\frac{1}{az-1} aq = a \Rightarrow q = az-1 \Rightarrow az-1 = q$

$q = az - 1 = \frac{z}{z-p} - 1 \Rightarrow \frac{p}{z-p} = q \Rightarrow p = q(z-p)$

required pd.eq

Formation of Partial differential equation by eliminating arbitrary functions

1) $z = xy + f(x^2+y^2+z^2)$

solution

Diffentiate partially w.r.t x and y

$p = y + f'(x^2+y^2+z^2)(2x+2zp)$

$q = x + f'(x^2+y^2+z^2)(2y+2zq)$

$p = \frac{\partial z}{\partial x}$

$q = \frac{\partial z}{\partial y}$

$$\frac{p-y}{x+zp} = 2f'(x^2+y^2+z^2) \text{ ——— } (1)$$

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$$\frac{q-x}{y+zq} = 2f'(x^2+y^2+z^2) \quad \text{———} \quad (2)$$

From (1) & (2) we get

$$\frac{p-y}{x+zp} = \frac{q-x}{y+zq}$$

$p(y+zq) - q(x+zp) = y^2 - x^2$ is the required p.d.eq

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2) $z = f\left(\frac{xy}{z}\right)$

solution

$$p = f'\left(\frac{xy}{z}\right)\left(\frac{z \cdot y - xyp}{z^2}\right)$$

$$\frac{pz^2}{zy - xyp} = f'\left(\frac{xy}{z}\right) \quad \text{---} (1)$$

$$q = f'\left(\frac{xy}{z}\right)\left(\frac{zx - xyq}{z^2}\right)$$

$$\frac{qz^2}{zx - xyq} = f'\left(\frac{xy}{z}\right) \quad \text{---} (2)$$

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from (1) and (2) we get

$$\frac{pz^2}{zy - xyp} = \frac{qz^2}{zx - xyq}$$

$$pzx - pxyq = qzy - qxyp$$

$px = qy$ is the required p.d.eq.

Formation of Partial differential equation by eliminating arbitrary functions

3) $z = f(x+iy) + g(x-iy)$

solution

$$p = \frac{\partial z}{\partial x} = f'(x+iy) + g'(x-iy)$$

$$q = \frac{\partial z}{\partial y} = i f'(x+iy) - i g'(x-iy)$$

$$r = \frac{\partial^2 z}{\partial x^2} = f''(x+iy) + g''(x-iy)$$

$$s = \frac{\partial^2 z}{\partial x \partial y} = i f''(x+iy) - i g''(x-iy)$$

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$$t = \frac{\partial^2 z}{\partial y^2} = if''(x+iy)\cdot i - ig''(x-iy)(-i)$$

$$t = -r \Rightarrow r + t = 0$$ is the req p.d.eq.

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$$\frac{\partial f}{\partial u}(1+q) + \frac{\partial f}{\partial v}(2y + 2zq) = 0 \quad \text{——— (2)}$$

Eliminating $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ in (1) & (2)

$$\begin{vmatrix} 1+p & 2x+2zp \\ 1+q & 2y+2zq \end{vmatrix}$$

$(y-z)p + (z-x)q = x-y$ is the required p.d.eq

Formation of Partial differential equation by eliminating arbitrary functions

4) $f(x+y+z,\ x^2+y^2+z^2) = 0$

Solution.

Let $x+y+z = u$, $x^2+y^2+z^2 = v$

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}p\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}p\right) = 0$$

$$\frac{\partial f}{\partial u}(1+p) + \frac{\partial f}{\partial v}(2x+2zp) = 0 \quad \text{--- (1)}$$

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}q\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}q\right) = 0$$