ENGINEERING MECHANICS

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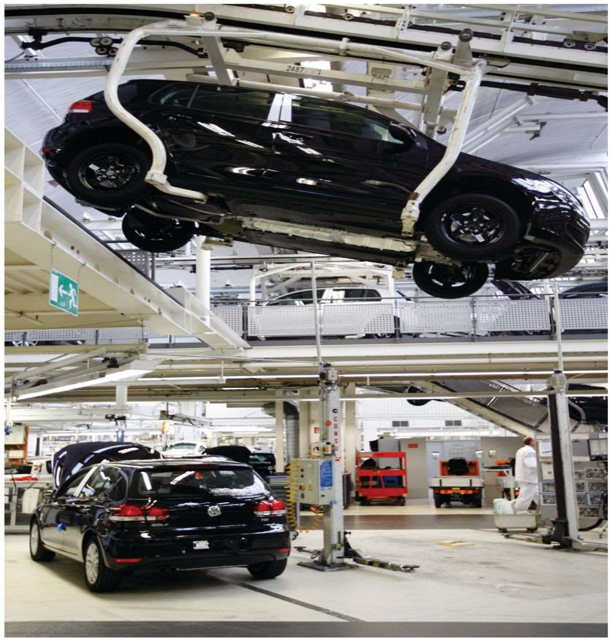
ENGINEERING MECHANICS

Unit: 2
Equilibrium and Beams


Adam Berry/Bloomberg via Getty Images

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- When a body is in equilibrium, the resultant of all forces acting on it is zero. Thus, the resultant force **R** and the resultant couple **M** are both zero, and we have the equilibrium equations
- $\mathbf{R} = \Sigma \mathbf{F} = 0 \quad \mathbf{M} = \Sigma \mathbf{M} = 0$

System Isolation and the Free-Body Diagram

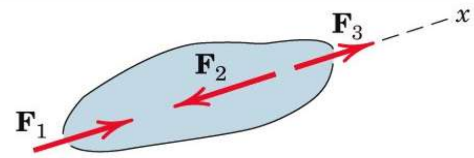
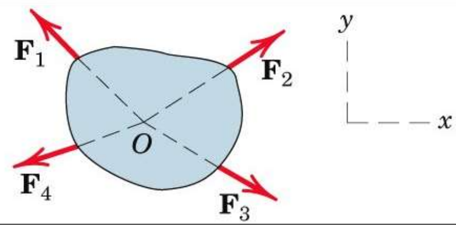
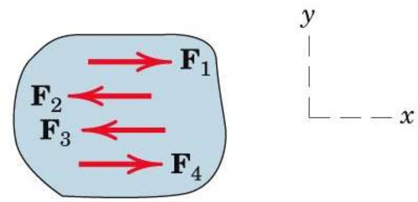
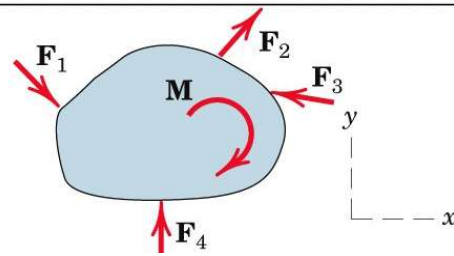
- A *mechanical system i*s defined as a body or group of bodies which can be conceptually isolated from all other bodies.
- The free-body diagram is a diagrammatic representation of the isolated system treated as a single body. *The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed. (body forces such as gravitational or magnetic attraction)*

The free-body diagram is the most important single step in the solution of problems in mechanics.

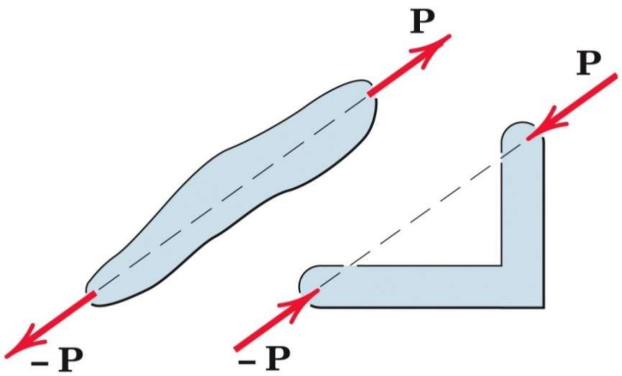
Construction of Free-Body Diagrams

- **Step 1.** *Decide which system to isolate*. The system chosen should usually involve one or more of the desired unknown quantities
- **Step 2.** Next isolate the chosen system by drawing a diagram which represents its *complete external boundary*. This boundary defines the isolation of the system from all other attracting or contacting bodies, which are considered removed. This step is often the most crucial of all. Make certain that you have completely isolated the system before proceeding with the next step.
- **Step 3.** *Identify all forces which act on the isolated system* as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system.
- **Step 4.** Show the *choice of coordinate axes directly on the diagram.*

Categories of Equilibrium

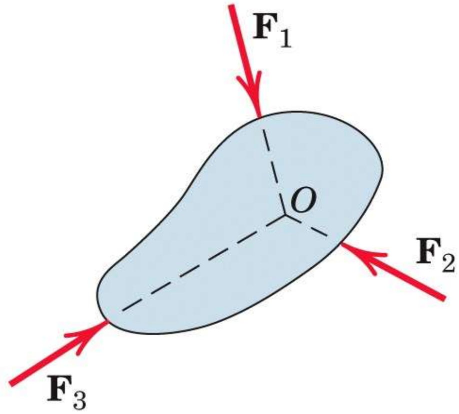
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Two- and Three-Force Members

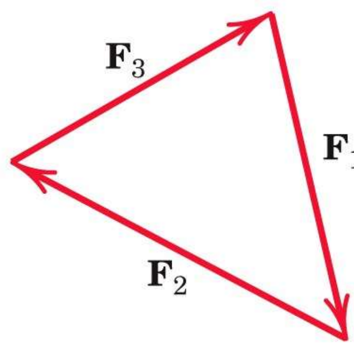


Two-force members

Figure 3-4



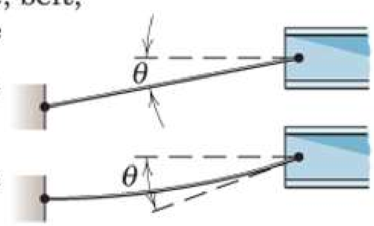
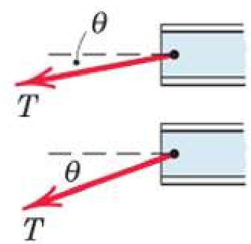
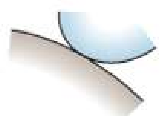
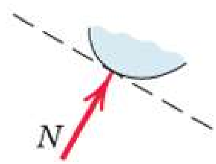
(*a*) Three-force member

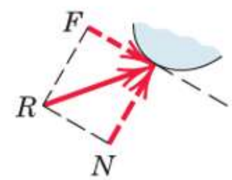
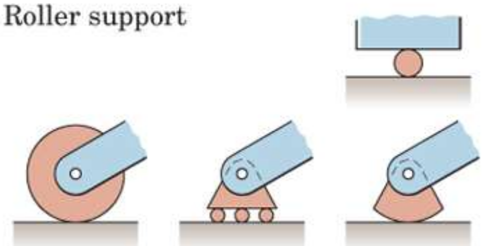
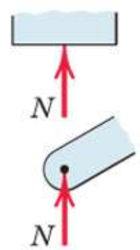
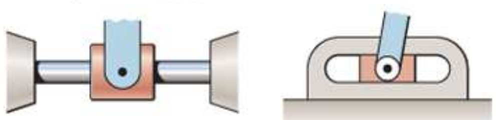
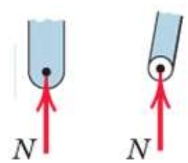


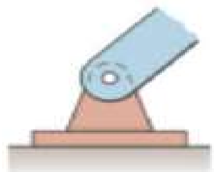
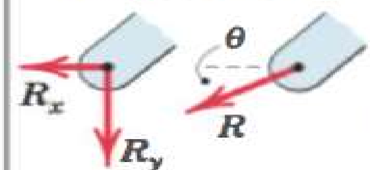
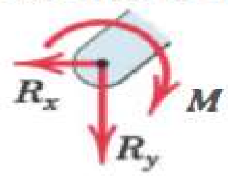
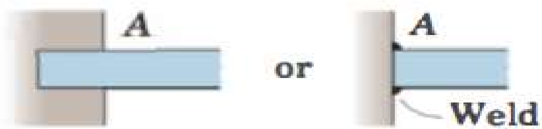
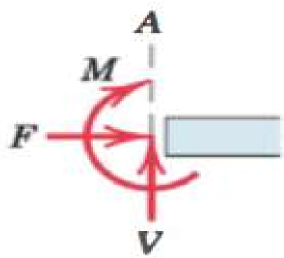
(*b*) Closed polygon satisfies $\Sigma\mathbf{F} = \mathbf{0}$

Figure 3-5

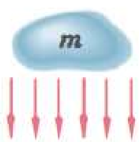
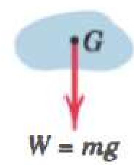
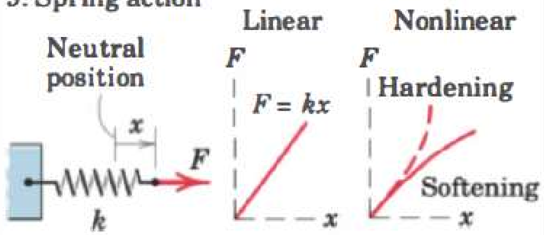
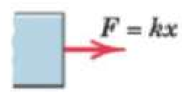
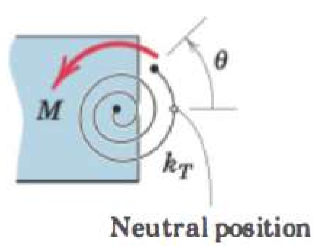
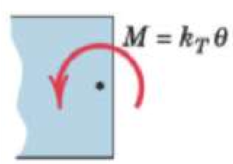
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

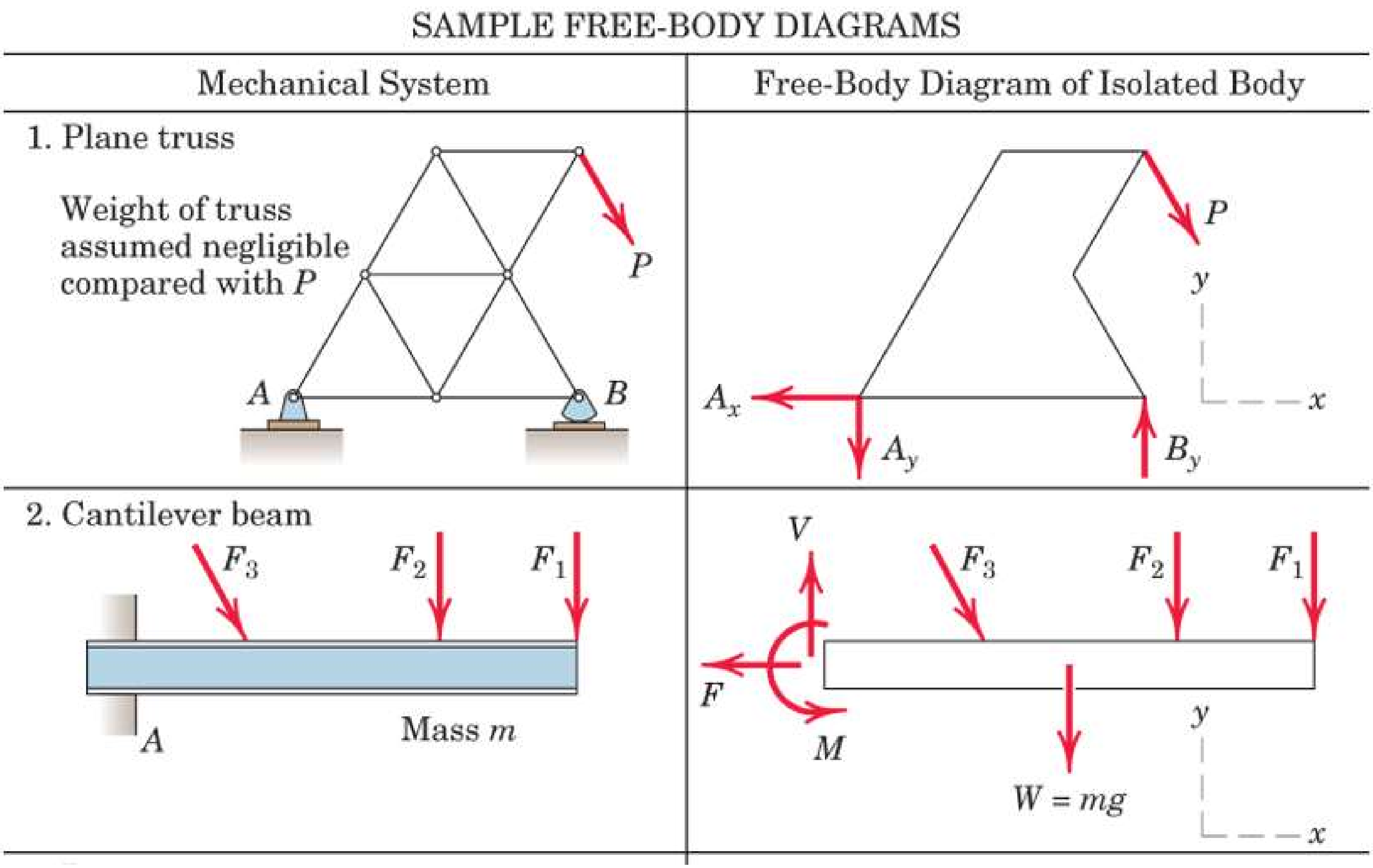
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.

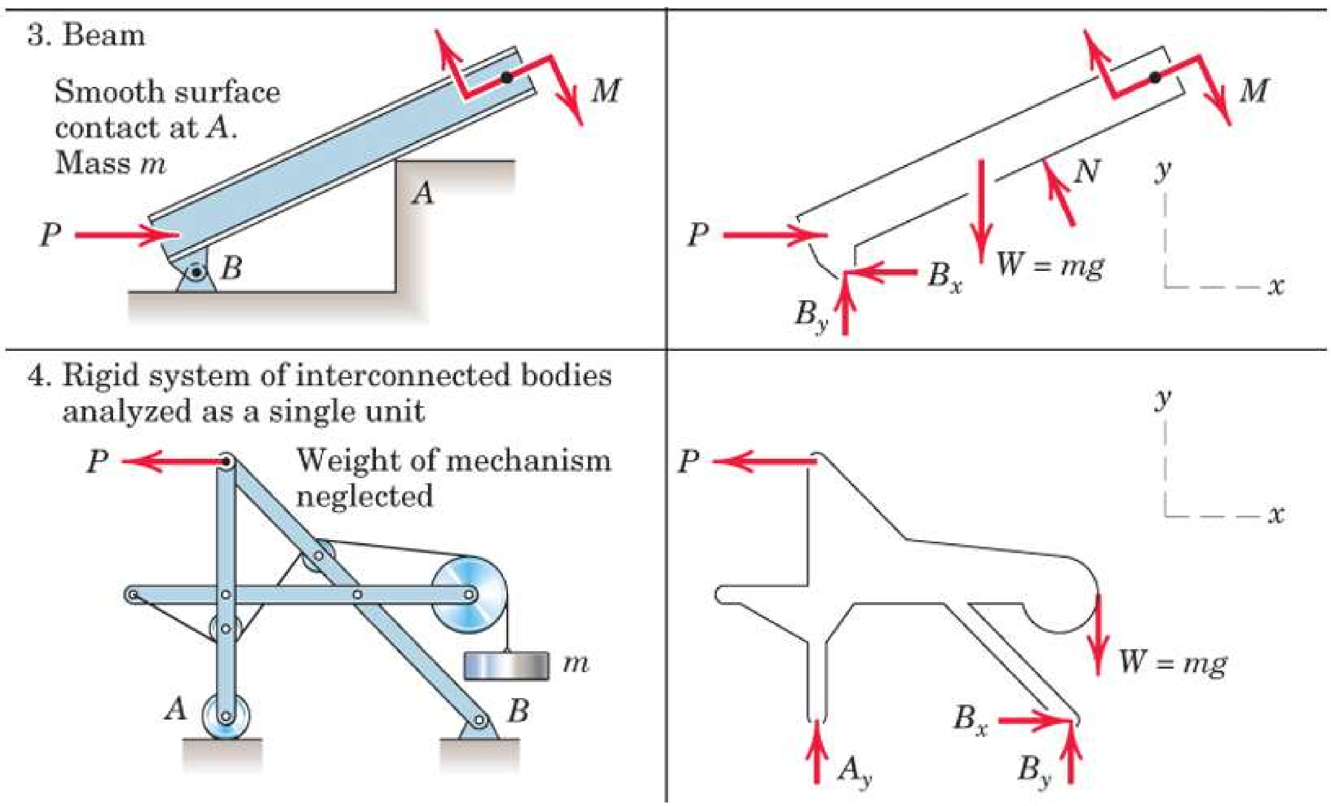
Type of contact		Force	Description
3. Rough surfaces			Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support			Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide			Collar or slider free to move along smooth guides; can support force normal to guide only.

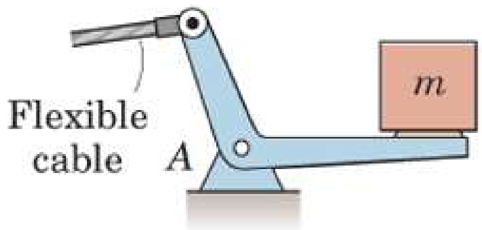
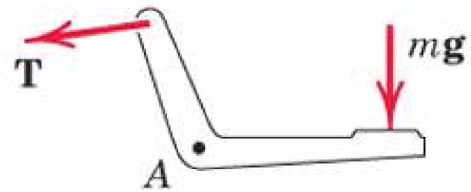
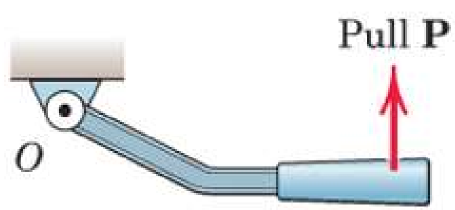
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

ENGINEERING MECHANICS
Equilibrium

8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.
9. Spring action 	 Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.
10. Torsional spring action 	 For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.


SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
A
B
P
Ax
Ay
By
y
x
2. Cantilever beam
F3
F2
F1
A
Mass m
V
F
M
W = mg
y
x


3. Beam
Smooth surface contact at A. Mass m
M
A
P
B
N
y
W = mg
B_x
B_y
x
4. Rigid system of interconnected bodies analyzed as a single unit
P
Weight of mechanism neglected
m
A
B
W = mg
B_x
A_y
B_y

<table>
<tr><th></th><th>Body</th><th>Incomplete FBD</th></tr>
<tr>
<td>1. Bell crank supporting mass m with pin support at A.</td>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>2. Control lever applying torque to shaft at O.</td>
<td>
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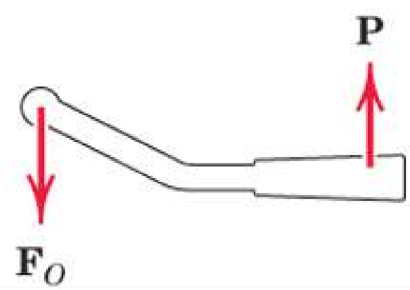
<td>
</td>
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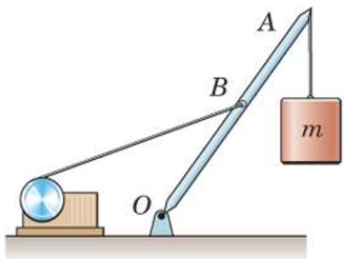
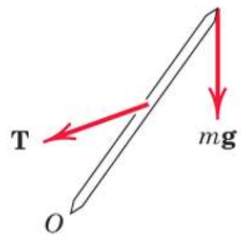
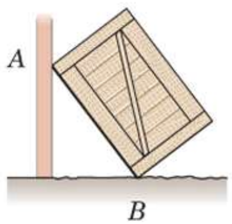
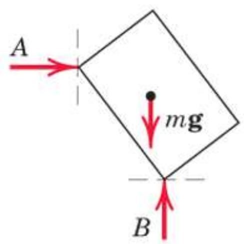
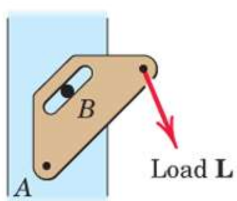
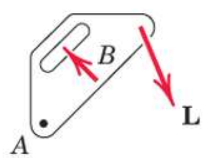
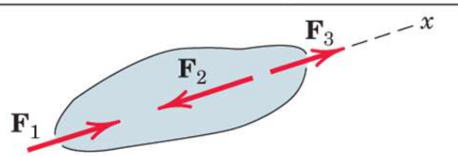
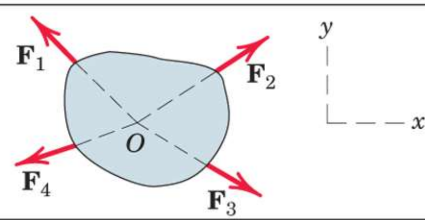
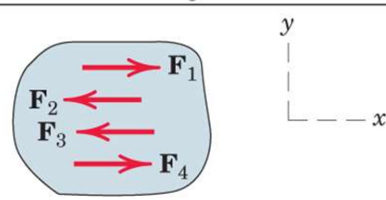
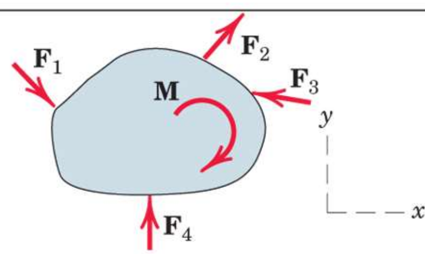
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

ENGINEERING MECHANICS

Equilibrium

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

THANK YOU

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