B-Tech- II

Department of Science and Humanities

Q1) $L\left[\int_0^t \frac{e^{-t}\sin t}{t}\,dt\right] = \frac{1}{s} L\left[\frac{e^{-t}\sin t}{t}\right]$

$\left\{ L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s} L[f(t)] \right.$

$$= \frac{1}{s}\left\{L\left[\frac{\sin t}{t}\right]\right\}_{s \to s+1}$$

$$= \frac{1}{s}\left[\int_s^\infty L[\sin t]\,ds\right]_{s \to s+1}$$

$$= \frac{1}{s}\left[\int_s^\infty \frac{1}{s^2+1}\,ds\right]_{s \to s+1}$$

$$= \frac{1}{s}\left[\frac{\pi}{2} - \tan^{-1}(s)\right]_{s \to s+1}$$

$$= \frac{1}{s}\left[\frac{\pi}{2} - \tan^{-1}(s+1)\right]$$

Q2) Evaluate $\int_0^\infty \frac{\cos t - \cos 5t}{t}\, dt$

$$\int_0^\infty e^{-st}\left[\frac{\cos t - \cos 5t}{t}\right] dt \qquad \text{At } s=0$$

$$= L\left[\frac{\cos t - \cos 5t}{t}\right]_{\underline{\text{At } s=0}}$$

$$L\left[\frac{\cos t - \cos 5t}{t}\right] = \int_s^\infty L[\cos t - \cos 5t]\, ds$$

$$= \int_s^\infty \left(\frac{s}{s^2+1} - \frac{s}{s^2+5^2} \right) ds$$

$$= \int_s^\infty \left[\frac{1}{2}\frac{2s}{s^2+1} - \frac{1}{2}\frac{2s}{s^2+25} \right] ds$$

$$= \frac{1}{2} \int_s^\infty \left[\frac{2s}{s^2+1} - \frac{2s}{s^2+25} \right] ds$$

$$= \frac{1}{2} \left[\log[s^2+1] - \log(s^2+25] \right|_s^\infty$$

$$= \frac{1}{2} \left[\log \left[\frac{s^2+1}{s^2+25} \right] \Big|_s^\infty \right]$$

$$= \frac{1}{2}\left[\log\left[\frac{\cancel{s^2}\left[1+1/s^2\right]}{\cancel{s^2}\left[1+25/s^2\right]}\right]\right]_s^\infty$$

$$= \frac{1}{2}\left[\log(1) - \log\left[\frac{1+1/s^2}{1+25/s^2}\right]\right]$$

$$= \frac{1}{2}\left[-\log\left[\frac{s^2+1}{s^2+25}\right]\right]$$

$$= \frac{1}{2}\log\left\{\left[\frac{s^2+1}{s^2+25}\right]^{-1}\right\}$$

$$L\left[\frac{\cos t - \cos 5t}{t}\right] = \frac{1}{2}\log\left[\frac{s^2+25}{s^2+1}\right]$$

$$L\left[\frac{\cos t - \cos 5t}{t}\right] = \frac{1}{2}\log\left[\frac{25}{1}\right] = \frac{1}{2}\log 5^2$$

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3) find the Laplace transform of $f(t)$

given $f(t) = 2^t + \frac{\cos 2t - \cos 3t}{t} + t \sin t$

<u>Solution</u> $L[f(t)] = L[2^t] + L\left[\frac{\cos 2t - \cos 3t}{t}\right] + L[t \sin t$

$$L[2^t] = L\left[e^{t\log 2}\right] = \frac{1}{s - \log 2} \quad \text{— eq}^n \text{ (1)}$$

$$L\left[\frac{\cos 2t - \cos 3t}{t}\right] = \int_s^\infty L[\cos 2t - \cos 3t]\, ds$$

$$= \int_s^\infty \left[\frac{s}{s^2+4} - \frac{s}{s^2+9}\right] ds$$

$$= \frac{1}{2}\left\{\int_s^\infty \frac{2s}{s^2+4} - \frac{2s}{s^2+9}\right] ds$$

$$= \frac{1}{2}\left[\log(s^2+4) - \log(s^2+9)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+4}{s^2+9}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left[\frac{1+4/s^2}{1+9/s^2}\right]_{s=\infty} - \log\left[\frac{s^2+4}{s^2+9}\right]\right]$$

$$= \frac{1}{2}\left[0 - \log\left[\frac{s^2+4}{s^2+9}\right]\right]$$

$$= \frac{1}{2} \log\left[\frac{s^2+9}{s^2+4}\right] \quad — \quad eq\ 2$$

$$L[t \sin t] = (-1)^1 \frac{d}{ds}\left[L[\sin t]\right]$$

$$= -\frac{d}{ds}\left[\frac{1}{s^2+1}\right] = \frac{2s}{(s^2+1)^2} \quad \ldots eq\ 3$$

Ans = eq 1 + eq 2 + eq 3

4) Obtain the Laplace transform of

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y \qquad \text{given } y(0) = 2$$
$$y'(0) = -4$$

Sol $L\left[\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y\right] = L[y'' - 3y' + 5y]$

$$= L[y''] - 3L[y'] + 5L[y]$$

$$= \left[s^2 Y(s) - s\,y(0) - y'(0)\right]$$

$$-3\left[s\,Y(s) - y(0)\right. + 5\,Y(s)$$

taking $y(0)=2$ $\quad y'(0)=4$

$$= \left(s^2 - 3s + 5\right) Y(s) - 2s + 10 \;//$$

THANK YOU

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