

Problems on method of separation of Variables

1. $\frac{\partial z}{\partial x} y^3 + \frac{\partial z}{\partial y} x^2 = 0$

Ans: $z = c_1 e^{\frac{kx^3}{3}} \cdot c_2 e^{\frac{-ky^4}{4}}$

2. $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$

Ans: $u = c_1 e^{kx} c_2 e^{\left(\frac{k-1}{2}\right)t}$

3. $xp = yq$

Ans: $z = c_1 x^k c_2 y^k$

4. $\frac{\partial^2 z}{\partial x \partial y} = z$

Ans: $z = c_1 e^{kx} \cdot c_2 e^{\frac{y}{k}}$

5. $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

Ans: $z = \left(c_1 e^{(1+\sqrt{1+k})x} + c_2 e^{(1-\sqrt{1+k})x} \right) c_3 e^{-ky}$