Unit-2 class-4

1. If $L^{-1}\left[\frac{e^{-1/s}}{\sqrt{s}}\right] = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ show that $L^{-1}\left[\frac{e^{-a/s}}{\sqrt{s}}\right] = \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$, where $a > 0$

We know that $L[f(t)] = F(s)$

So, $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$

$$f(at) = L^{-1}\left[\frac{1}{a} F\left(\frac{s}{a}\right)\right]$$

$$f(t) = L^{-1}[F(s)]$$

$$F(s) = \frac{e^{-1/s}}{\sqrt{s}}$$

$$F\left(\frac{s}{a}\right) = \frac{e^{-a/s}}{\sqrt{\frac{s}{a}}} \Rightarrow \frac{1}{a} \times F\left(\frac{s}{a}\right) = \frac{e^{-a/s}}{\sqrt{as}}$$

$$f(t) = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}} \Rightarrow f(at) = \frac{\cos 2\sqrt{at}}{\sqrt{\pi a t}}$$

then, $L^{-1}\left[\frac{e^{-a/s}}{\sqrt{as}}\right] = \frac{\cos 2\sqrt{at}}{\sqrt{\pi a t}} \Rightarrow L^{-1}\left[\frac{e^{-as}}{\sqrt{s}}\right] = \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$

2. Find $L^{-1}\left[\frac{s^2}{(s-2)^3}\right]$

Ans: $e^{2t} + 4te^{2t} + 2e^{2t}t^2$

$$L^{-1}\left[\frac{s^2}{(s-2)^3}\right]$$

$$\Rightarrow F(s) = \frac{s^2}{(s-2)^3} = \frac{A}{s-2} + \frac{B}{(s-2)^2} + \frac{C}{(s-2)^3}$$

$$= \frac{A(s-2)^2 + B(s-2) + C}{(s-2)^3}$$

$$\Rightarrow A(s^2-4s+4) + B(s-2) + C = s^2$$

$A = 1$

$-4A + B = 0 \Rightarrow B = 4$

$4A - 2B + C = 0 \Rightarrow C = 4$

$$\Rightarrow L^{-1}\left[\frac{1}{s-2} + \frac{4}{(s-2)^2} + \frac{4}{(s-2)^3}\right] = e^{2t} + 4.te^{2t} + 4.\frac{t^2}{2}.e^{2t}$$

$$= e^{2t} + 4te^{2t} + 2t^2e^{2t}$$

3. Find $L^{-1}\left[\frac{14s+10}{49s^2+28s+13}\right]$

Ans: $\frac{2}{7}e^{-(2/7)t}\left\{\cos\frac{3}{7}t + \sin\frac{3}{7}t\right\}$

$$L^{-1}\left[\frac{14s+10}{49s^2+28s+13}\right]$$

$$= L^{-1}\left[\frac{14s+10}{49\left(s^2+\frac{4s}{7}+\frac{13}{49}\right)}\right]$$

$$= L^{-1}\left[\frac{\frac{2s}{7}+\frac{10}{49}}{s^2+\frac{4s}{7}+\frac{13}{49}}\right] = L^{-1}\left[\frac{\frac{2s}{7}+\frac{10}{49}}{\left(s+\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2}\right]$$

$$= \frac{2}{7}L^{-1}\left[\frac{s+5/7}{\left(s+\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2}\right] = \frac{2}{7}\left\{L^{-1}\left[\frac{s+\frac{2}{7}}{\left(s+\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2}\right] + \left[\frac{3/7}{\left(s+\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2}\right]\right\}$$

$$= \frac{2}{7}e^{-2t/7}\left[L^{-1}\left(\frac{s}{s^2+\left(\frac{3}{7}\right)^2}\right) + L^{-1}\left(\frac{3/7}{s^2+\left(\frac{3}{7}\right)^2}\right)\right]$$

$$= \frac{2}{7}e^{\frac{-2t}{7}}\left[\cos\frac{3t}{7} + \sin\frac{3t}{7}\right]$$

4. Find $L^{-1}\left[\log\left(\frac{s^2-1}{s^2}\right)\right]$

Ans: $\frac{2}{t}[1-\cosh t]$

$$L^{-1}\left[\log\left(\frac{s^2-1}{s^2}\right)\right]$$

$$= L^{-1}\left[\log(s^2-1) - \log s^2\right]$$

$$\Rightarrow F(s) = \log(s^2-1) - \log s^2$$

$$\frac{dF(s)}{ds} = \frac{1}{s^2-1} \times 2s - \frac{1}{s^2} \times 2s$$

$$\frac{dF(s)}{ds} = \frac{2s}{s^2-1} - \frac{2}{s}$$

$$-\frac{dF(s)}{ds} = \frac{2}{s} - \frac{2s}{s^2-1}$$

$$L^{-1}\left[-\frac{dF(s)}{ds}\right] = L^{-1}\left[\frac{2}{s} - \frac{2s}{s^2-1}\right]$$

$$t f(t) = 2 - 2 \times \cosh at$$

$$f(t) = \frac{2 - 2\cosh at}{t}$$

5. Find $L^{-1}\left[\tan^{-1}\left(\frac{2}{s}\right)\right]$

Ans: $\frac{\sin 2t}{t}$

$$L^{-1}\left[\tan^{-1}\left(\frac{2}{s}\right)\right]$$

$$\Rightarrow F(s) = \tan^{-1}\left(\frac{2}{s}\right)$$

$$\frac{dF(s)}{ds} = \frac{1}{1+\left(\frac{2}{s}\right)^2} \times \left(\frac{-2}{s^2}\right)$$

$$= \frac{-2s^2}{\frac{s^2+4}{s^2}} = \frac{-2}{s^2+4}$$

$$-\frac{dF(s)}{ds} = \frac{2}{s^2+4}$$

$$L^{-1}\left[-\frac{dF(s)}{ds}\right] = L^{-1}\left[\frac{2}{s^2+4}\right]$$

$$t f(t) = \sin 2t$$

$$f(t) = \frac{\sin 2t}{t}$$