B. Tech – II

Department of Science and Humanities

CLASS-9

SECOND SHIFT PROPERTY

STATEMENT

Second Shifting Property

If $\mathcal{L}\{f(t)\} = F(s)$, and $g(t) = \begin{cases} f(t-a) & t > a \\ 0 & t < a \end{cases}$

then,

$$\mathcal{L}\{g(t)\} = e^{-as}F(s)$$

Proof of Second Shifting Property

$$g(t) = \begin{cases} f(t-a) & t > a \\ 0 & t < a \end{cases}$$

$$\mathcal{L}\left\{g(t)\right\} = \int_0^\infty e^{-st} g(t)\, dt$$

$$\mathcal{L}\left\{g(t)\right\} = \int_0^a e^{-st} (0)\, dt + \int_a^\infty e^{-st} f(t-a)\, dt$$

$$\mathcal{L}\left\{g(t)\right\} = \int_a^\infty e^{-st} f(t-a)\, dt$$

Let

$z = t - a$

$t = z + a$

$dt = dz$

Let

$z = t - a$

$t = z + a$

$dt = dz$

when $t = a,\ z = 0$

when $t = \infty,\ z = \infty$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-s(z+a)} f(z)\, dz$$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-sz-sa} f(z)\, dz$$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-sz} e^{-sa} f(z)\, dz$$

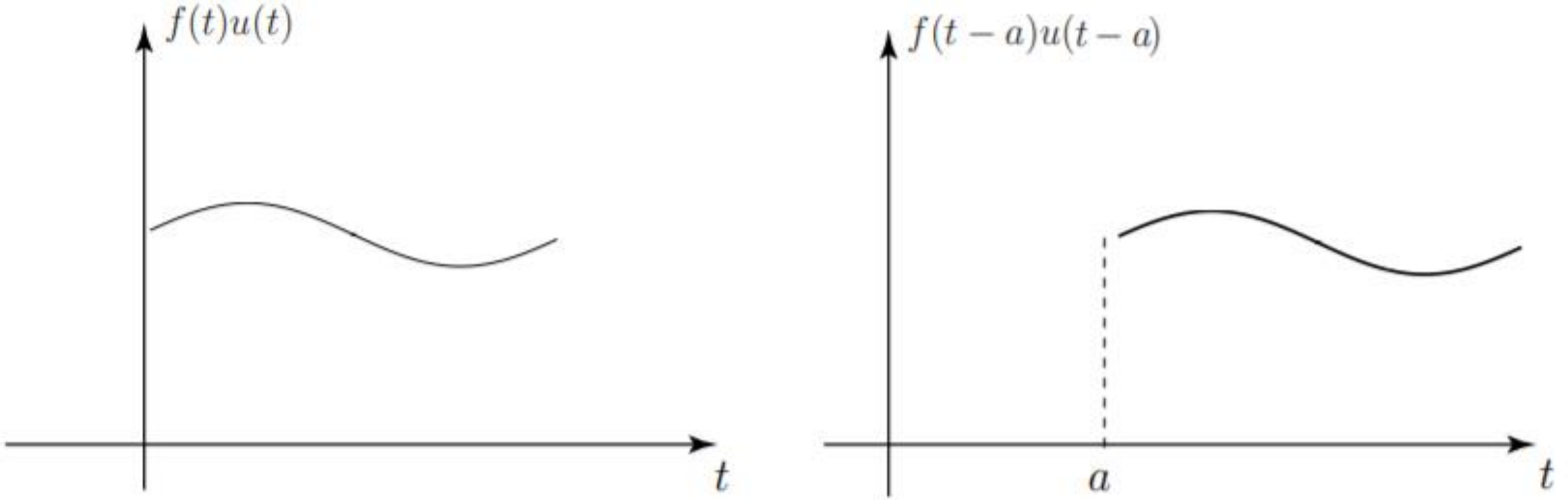
$$\mathcal{L}\{g(t)\} = e^{-sa} \int_0^\infty e^{-sz} f(z)\, dz$$

$$\mathcal{L}\{g(t)\} = e^{-as} \mathcal{L}\{f(z)\}$$

$$\mathcal{L}\{g(t)\} = e^{-as} \mathcal{L}\{f(t-a)\}$$

$$\mathcal{L}\{g(t)\} = e^{-as} F(s) \qquad okay$$

The second shift theorem is similar to the first except that, in this case, it is the time-variable that is shifted not the s-variable. Consider a causal function $f(t)u(t)$ which is shifted to the right by amount a, that is, the function $f(t-a)u(t-a)$ where $a > 0$. Figure 13 illustrates the two causal functions.



EXAMPLE

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Find the Laplace transform of $g(t) = \begin{cases} f(t-1)^2 & t > 1 \\ 0 & t < 1 \end{cases}$

$$\mathcal{L}\left\{g(t)\right\} = e^{-as}F(s)$$

$$F(s) = \mathcal{L}(t^2) \quad \text{and} \quad a = 1$$

$$F(s) = \frac{2}{s^3}$$

Thus,

$$\mathcal{L}\left\{g(t)\right\} = e^{-s}\left(\frac{2}{s^3}\right)$$

$$\mathcal{L}\left\{g(t)\right\} = \frac{2e^{-s}}{s^3} \qquad \textit{answer}$$

observe

If we use the direct method of solving by definition Laplace transform .. The previous problem would be ….

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-st} g(t)\, dt$$

$$\mathcal{L}\{g(t)\} = \int_0^1 e^{-st}(0)\, dt + \int_1^\infty e^{-st}(t-1)^2\, dt$$

$$\mathcal{L}\{g(t)\} = \int_1^\infty e^{-st}(t-1)^2\, dt$$

Let

$z = t - 1$

$t = z + 1$

$dt = dz$

when $t = 1,\ z = 0$

when $t = \infty,\ z = \infty$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-s(z+1)} z^2\, dz$$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-sz-s} z^2\, dz$$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-sz} e^{-s} z^2\, dz$$

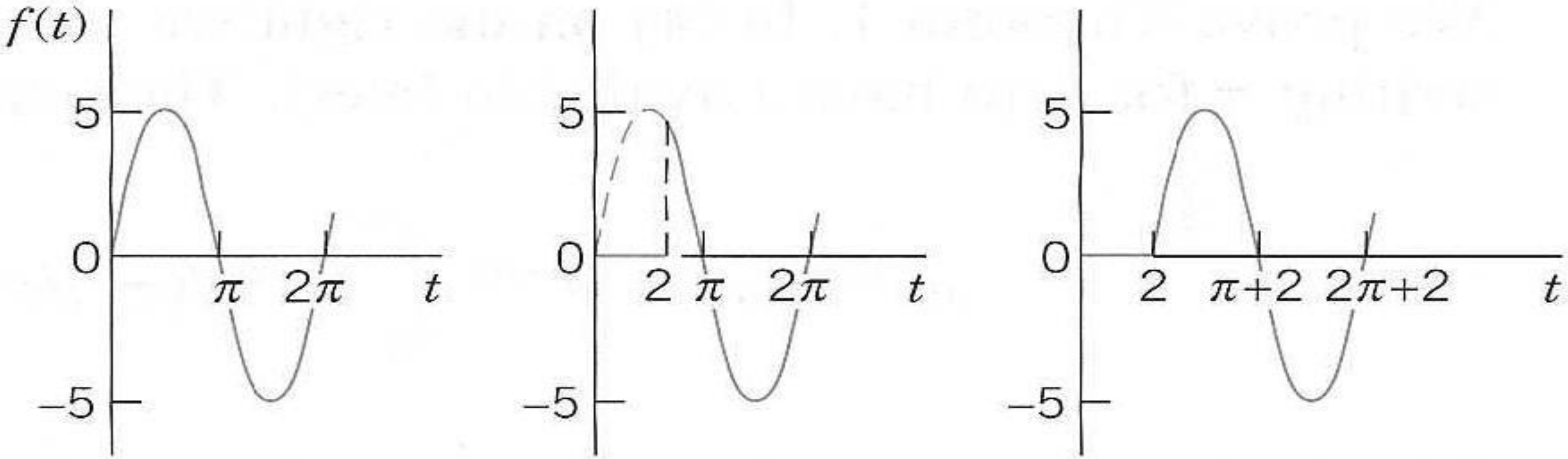
$$\mathcal{L}\{g(t)\} = e^{-s} \int_0^\infty e^{-sz} z^2\, dz$$

$$\mathcal{L}\{g(t)\} = e^{-s} \mathcal{L}(z^2)$$

$$\mathcal{L}\{g(t)\} = e^{-s} \left(\frac{2}{s^3}\right)$$

$$\mathcal{L}\{g(t)\} = \frac{2e^{-s}}{s^3} \qquad okay$$

Representation of time shifting property



(A) $f(t) = 5 \sin t$ (B) $f(t)u(t-2)$ (C) $f(t-2)u(t-2)$

Effects of unit step function

(A) Given function (B) Switching off and on (C) Shift

Express f (t) in terms of the Heavisides unit step function and find its Laplace transform:

$f(t) = \{(t^2, 0 < t < 2)\ (4t, 2 < t < 4)\ (8, t > 4)$

$$f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & 2 < t < 4 \\ 8, & t > 4 \end{cases}$$

We get

$f(t) = t^2 + (4t - t^2)\, u(t - 2) + (8 - 4t)\, u(t - 4)$

$f(t) = t^2 + [4 - (t - 2)^2]\, u(t - 2) + [-4(t - 4) - 8]\, u(t - 4)$

Taking Laplace transform on both sides, we get,

$$L\{f(t)\} = L(t^2) + L\{[4-(t-2)^2]\,u(t-2)\} + L\{[-4(t-4)-8]\,u(t-4)\}$$

$$= \frac{2}{s^3} + e^{-2s} L\left(4-t^2\right) + e^{-4s} L(-4t-8)$$

Using Heaviside shift theorem.

$$= \frac{2}{s^3} + e^{-2s}\left(\frac{4}{s} - \frac{2}{s^3}\right) + e^{-4s}\left(\frac{-4}{s^2} - \frac{8}{s}\right)$$

$$= \frac{2}{s^3} + 2e^{-2s}\left(\frac{2}{s} - \frac{1}{s^3}\right) - 4e^{-4s}\left(\frac{1}{s^2} + \frac{2}{s}\right).$$

Thanks all