Class 1:

Evaluate the following double integrals

1. $\int_{1}^{2}\int_{3}^{4}\left(xy+e^{y}\right)dydx$ ans: $\frac{21}{4}+e^{4}-e^{3}$
2. $\iint(x+y)^{2}dxdy$ over the area bounded by the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ ans: $\frac{1}{4}\pi ab\left(a^{2}+b^{2}\right)$
3. $\iint xy(x+y)dxdy$ over the area between $y=x^{2}$ and $y=x$ ans: $\frac{3}{56}$

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Class 2:

1. Find by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$
   ans: 4.5
2. Find the volume bounded by the xy-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$
   ans: 3π
3. Find the average value of the function e^{x+y} over the region $R = [0,2] \times [0,2]$ ans: $\frac{(e^2-1)^2}{4}$

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Class 3:

Evaluate the following integrals by changing to polar coordinates:

1. $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$ ans: $\frac{\pi a}{4}$
2. $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2 + y^2}}\,dx\,dy$ ans: $\frac{a^3}{3}\log(\sqrt{2}+1)$

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Class 4:

Evaluate the following integrals by changing to polar coordinates:

1. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\left(1+x^2+y^2\right)^{3/2}} dxdy$ ans: 2π

2. $\iint y^2 dxdy$ over the area outside $x^2+y^2-ax=0$ and inside $x^2+y^2-2ax=0$ ans: $\frac{15\pi a^4}{64}$

3. $\iint \frac{\sqrt{1-x^2-y^2}}{1+x^2+y^2} dxdy$ the integral being extended over all positive values of x and y subject to $x^2+y^2 \le 1$. ans: $\frac{\pi^2}{8}-\frac{\pi}{4}$

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Class 5:

Evaluate the following integrals by changing the order of integration:

1. $\int_{0}^{3}\int_{1}^{\sqrt{4-y}}(x+y)\,dx\,dy$ ans: $\frac{241}{60}$

2. $\int_{0}^{1}\int_{x}^{\sqrt{2-x^2}}\frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ ans: $1-\frac{1}{\sqrt{2}}$

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Class 6:

Evaluate the following integrals:

1. $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}\left(x^2+y^2+z^2\right)dxdydz$ ans: $\frac{8abc}{3}\left(a^2+b^2+c^2\right)$
2. $\int_{0}^{a}\int_{0}^{x}\int_{0}^{x+y} e^{x+y+z}\,dzdydx$ ans: $\frac{1}{8}e^{4a}-\frac{3}{4}e^{2a}+e^{a}-\frac{3}{8}$
3. $\int_{0}^{\frac{\pi}{2}}\int_{0}^{a\sin\theta}\int_{0}^{\frac{a^2-r^2}{a}} r\,dzdrd\theta$ ans: $\frac{5\pi a^3}{64}$

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Class 7:

1. Find the volume of the tetrahedron bounded by co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

   ans: $\frac{abc}{6}$

2. Find the volume of the region bounded by $z=x^2+y^2, z=0, x=-a, x=a, y=-a, y=a$

   ans: $\frac{8a^4}{3}$

3. Find the volume cut off from the cylinder $x^2+y^2=ax$ by the planes z=0 and z=x ans: $\frac{\pi a^3}{8}$

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Class 8:

1. Find the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$ ans: $\dfrac{16a^3}{3}$
2. Find the volume cut from the sphere $x^2+y^2+z^2=a^2$ by the cone $x^2+y^2=z^2$ above XY plane.

ans: $\dfrac{\pi a^3}{3}\left(2-\sqrt{2}\right)$

3. Find the average value of $f(x,y,z)=x+y+z$, using triple integrals over the region D= $\{(x,y,z)\ \vert 0\le x\le 1, 0\le y\le 3, 0\le z\le 5\}$. ans: $\dfrac{9}{2}$

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1. If R is the region bounded by $x = 0, y = 0, z = 0, z = 1$ and the cylinder $x^2 + y^2 = 1$ evaluate $\iiint xyzdxdydz$ by changing to cylindrical coordinates. Ans: 1/16
2. Evaluate $\iiint (x^2 + y^2) dxdydz$ over the region bounded by the paraboloid $x^2 + y^2 = 3z$ and the plane $z = 3$ ans: $\frac{81\pi}{2}$
3. Evaluate $\iiint \frac{dxdydz}{\sqrt{x^2 + y^2 + z^2}}$ over the region bounded by the sphere $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$ $a > b > 0$ ans: $2\pi(a^2 - b^2)$

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Class 11:

1. A lamina is bounded by the curves $y = x^2 - 3x$ *and* $y = 2x$. If the density at any point is given by λxy, find by double integration, mass of the lamina. Ans: $182\frac{7}{24}\lambda$
2. Find the mass of the lamina in the form of the cardioid $r = a(1+\cos\theta)$ whose density at any point varies as the square of its distance from the initial line. ans: $\frac{21\pi a\mu^4}{32}$
3. Find the mass of a solid in the form of the positive octant of the sphere $x^2 + y^2 + z^2 = 9$ if the density at any point is 2xyz ans:30.375
4. Find the centroid of the area enclosed by the parabola $y^2 = 4ax$, the axis of x and its latus rectum. Ans:$\left(\frac{3a}{20}, \frac{3a}{16}\right)$

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Class 12:

1. Using double integrals find moment of inertia about the x-axis of the area enclosed by the lines $x=0, y=0, \frac{x}{a}+\frac{y}{b}=1$ Ans: $\frac{ab^3}{12}$

2. Find the moment of inertia of an octant of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ about x-axis.

Ans: $\frac{abc\left(b^2+c^2\right)\pi}{30}$

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