B. Tech – II

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Unit-3

Laplace Transform

Topics in the Module

- Laplace transforms, Advantages and sufficient conditions for Existence of Laplace transform
- Laplace transform of standard functions
- General properties of Laplace transforms and problems based on it.
- Laplace transform of periodic function: Statement and problems.
- Laplace transform of Unit step function
- Second shifting property
- Laplace transform of unit impulse function

CLASS-1

INTRODUCTION TO LAPLACE TRANSFORM

LAPLACE TRANSFORM

- TRANSFORM implies a major change in form, nature, or function.
- **Transform** in mathematics means a mathematical function from one domain to other or on to itself
- The Laplace transform is named after mathematician and astronomer Pierre-Simon Laplace.
- Laplace transform is an integral transform perhaps second only to the Fourier transform in its utility in solving physical problems. The Laplace transform is particularly useful in solving linear ordinary differential equations such as those arising in the analysis of electronic circuits.

Definition of Integral Transform

A Laplace transform is a type of integral transform of a function $f(t)$ from time domain to the complex frequency domain $F(s)$.

An **integral transform** of a function f is a relation of the form

$$F(s) = \int_{\alpha}^{\beta} K(s,t) f(t) dt, \quad \infty \leq \alpha < \beta \leq \infty$$

Given a known function $K(s,t)$, called kernel function

Plug one function in $f(t)$

Get another function out $F(s)$

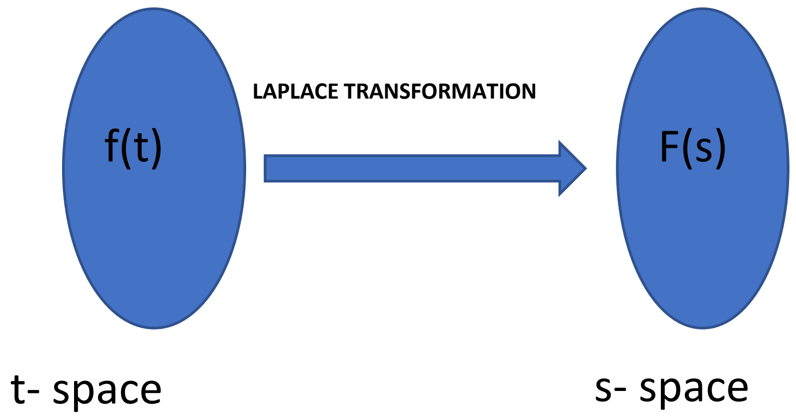
The new function is in a different domain

Definition of Laplace

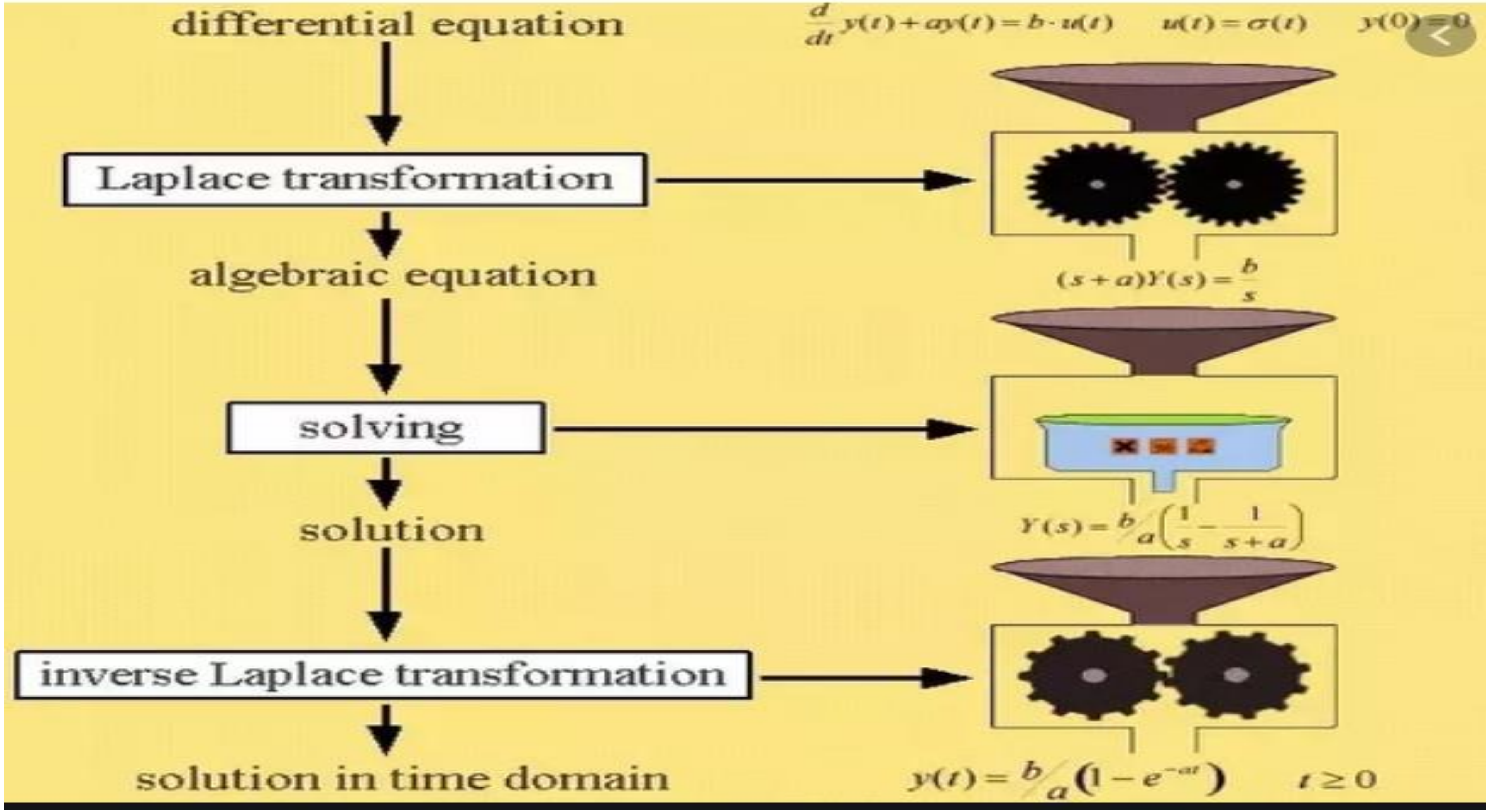
- Let $f(t)$ be defined for $t \geq 0$ and let s be a real /complexnumber. Then the *Laplace transform* of $f(t)$ is the function $F(s)$ defined by

$$\mathrm{F(s)} = \int_0^{\infty} e^{-st} f(t)\, dt = L\{f(t)\}$$

- for those values of s for which the improper integral converges.



What does Laplace transformation do?



Laplace Transform

- Laplace transforms

Time Domain

Frequency Domain

$$y^{(2)}(t)+y^{(1)}(t)+y(t)=x(t)$$

$$x(t)=1$$

Laplace transform $\longrightarrow$

$$H(s)=\frac{1}{s^2+3s+2}$$

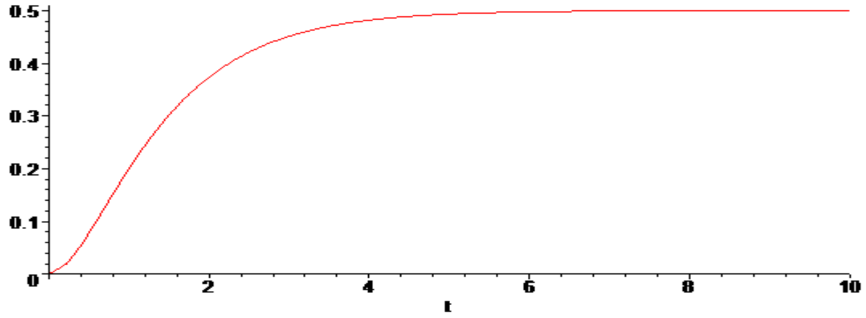
$$X(s)=\frac{1}{s}$$

$\downarrow$ Solve algebraic equation

$$\frac{1}{s}\frac{1}{s^2+3s+2}$$

$\longleftarrow$ Inverse Laplace transform

$$y(t)=\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}$$



Existence of Laplace Transforms

- Do every function has a Laplace transform?
- $\int_0^{\infty} e^{-st}\, e^{t^2}\ dt = \infty$ for very real number s. Hence, for the function $f(t) = e^{t^2}$ does not have a Laplace transform.
- Our next objective is to establish conditions that ensure the existence of the Laplace transform of a function

Piecewise Continuous Functions

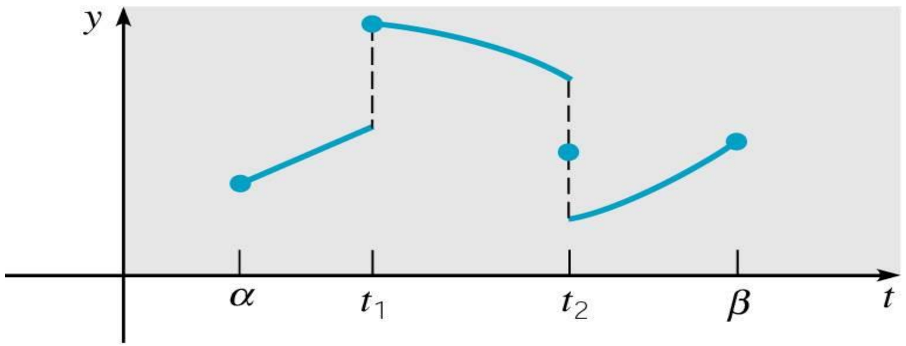
- A function f is **piecewise continuous** on an interval $[a, b]$ if this interval can be partitioned by a finite number of points

  $a = t_0 < t_1 < \ldots < t_n = b$ such that

  (1) f is continuous on each (t_k, t_{k+1})

  $$(2)\ \left|\lim_{t \to t_k^+} f(t)\right| < \infty, \quad k = 0, \ldots, n-1$$

  $$(3)\ \left|\lim_{t \to t_{k+1}^-} f(t)\right| < \infty, \quad k = 1, \ldots, n$$



- In other words, f is piecewise continuous on $[a, b]$ if it is continuous there except for a finite number of jump discontinuities.

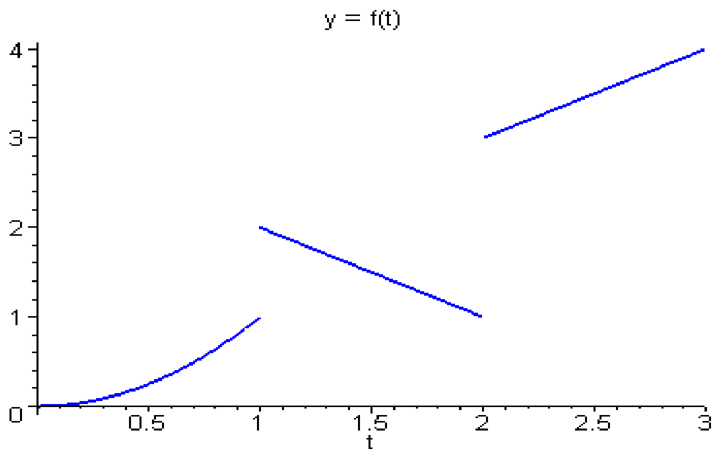
  Note that a piecewise continuous function is a function that has a finite number of breaks in it and doesnt blow up to infinity anywhere

Example

- Consider the following piecewise-defined function *f*.

$$f(t)=\begin{cases} t^2, & 0\le t\le 1 \\ 3-t, & 1<t\le 2 \\ t+1 & 2<t\le 3 \end{cases}$$

- From this definition of *f*, and from the graph of *f* below, we see that *f* is piecewise continuous on [0, 3].



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$$\int_0^{\infty} e^{-st}\, e^{t^2}\ dt = \lim_{T\to\infty} \int_0^{T} e^{-st}\, e^{t^2}\ dt$$

- do piecewise continuity alone does not guarantee that the improper integral converges

- I $\int_0^{\infty} e^{-st}\, e^{t^2}\ dt = \infty$. this occurs because e^{t^2} increases too rapidly as $t \to \infty$. The next definition provides a constraint on the growth of a function e^{t^2} that guarantees convergence of its Laplace transform for s in some interval (s_0, ∞).

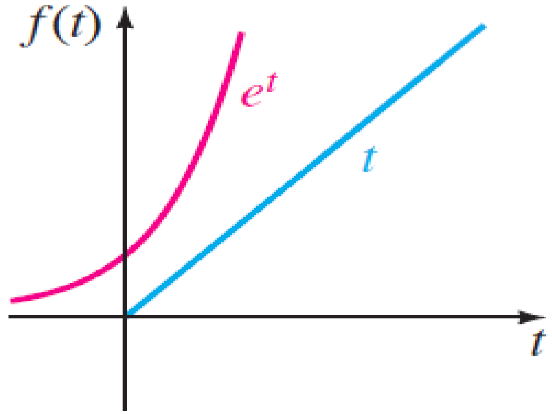
Exponential order

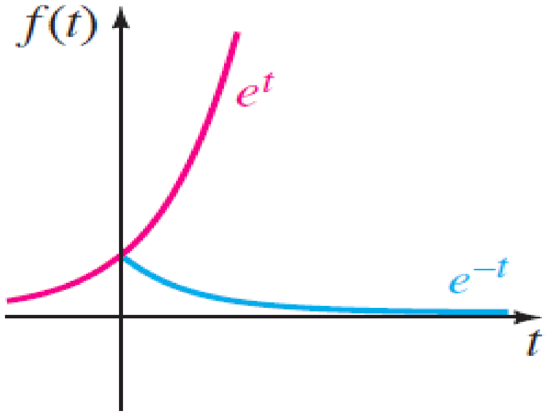
- A function f is said to be *of exponential order* s_0 if there are constants M and t_0 such that
- $|f(t)| \leq Me^{s_0 t} \quad , t > t_0$
- In situations where the specific value of s_0 is irrelevant we say simply that f is *of exponential order.*

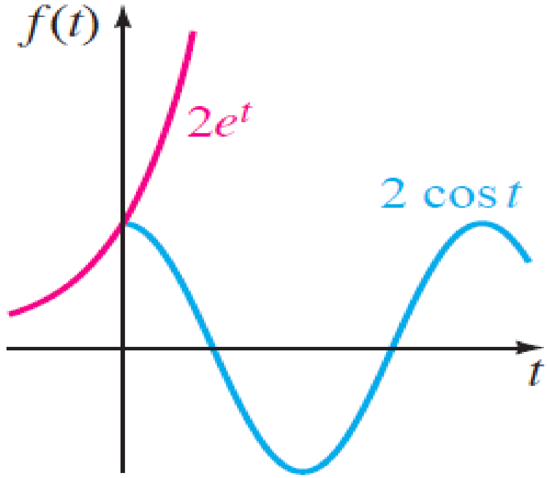
Condition for Laplace transform to exists

- If f is piecewise continuous on $[0,\infty)$and of exponential order s_0 , then $L(f)$ is defined for $s > s_0$
- The above theorem gives a sufficient condition for the existence of Laplace transforms.

Exponential order(Bounded)...

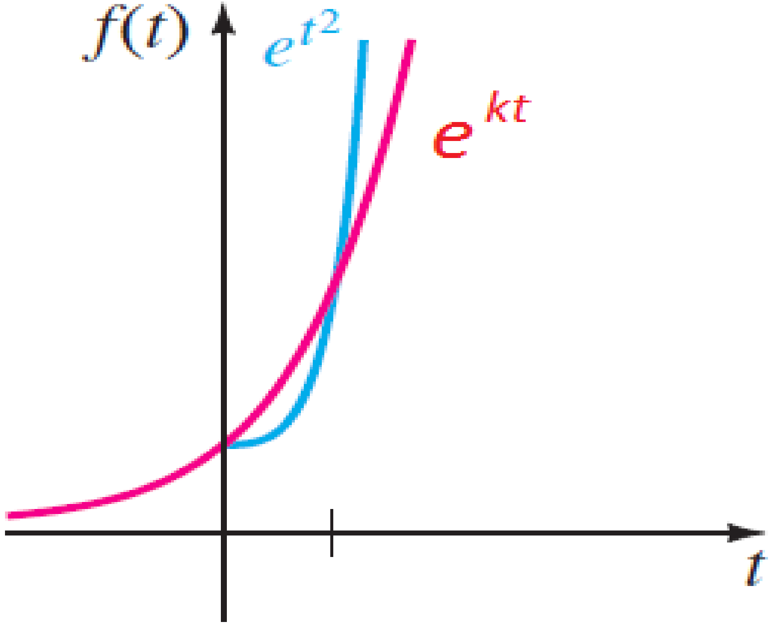






Functions with blue graphs are of exponential order

Exponential order(Bounded)...



$f(t) = e^{t^2}$ is not of exponential order since its graph grows faster

Thanks all