Question Bank

1. If $y = e^{2t}$ is a solution to $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + ky = 0$ then find k. **Answer:** k=6.

2. If $\frac{d^2x}{dt^2} + \frac{g}{b}(x-a) = 0$ where a, b and g are positive numbers and x =a', $\frac{dx}{dt} = 0$ *when* $t = 0$, *show that* $x = a + (a'-a)\cos\left(\sqrt{\frac{g}{b}}t\right)$.

3. Obtain the general solution of the equations:

i) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$. ii) $(D^4 + 2D^3 - 3D^2)y = 0$

Answer: i) $y = (A + Bx)e^{-4x}$ ii) $y = (c_1 + c_2x) + c_3e^x + c_4e^{-3x}$

4. Solve $y'' + 4y' + 29y = 0$, *given that when* $x = 0, y = 0$ *and* $y' = 15$.
Answer: $3e^{-2x}\sin 5x$.

5. Solve $(D^3 + 7D^2 + 16D + 10)y = 0$.
Answer: $y = C_1e^{-x} + e^{-3x}(C_2\cos x + C_3\sin x)$

6. If $\frac{d^4x}{dt^4} = m^4x$, show that $x = C_1\cos mt + C_2\sin mt + C_3\cosh mt + C_4\sinh mt$.

7. i) Solve $y'' - 2y' + 10y = 0$, $y(0) = 4$, $y'(0) = 1$.
ii) Solve $(D^3 - 2D^2 - 5D + 6)y = (e^{2x} + 3)^2 + e^{3x}\cosh x$
iii) Solve $(D^4 + D^3 - 3D^2 - 5D - 2)y = 3xe^{-x}$
iv) Solve $(D - 2)^2y = \sin 2x + \cos 3x$
v) Solve $y'' + 3y' + 2y = 3x + x^2$
Answer: i) $y = e^x(4\cos 3x - \sin 3x)$

ii) $y = c_1e^x + c_2e^{3x} + c_3e^{-2x} + \frac{e^{4x}}{12} - \frac{13}{8}e^{2x} + \frac{3}{2}$.

iii) $y = c_1e^{2x} + e^{-x}(c_2 + c_3x + c_3x^2) - e^{-x}\left(\frac{x^4}{24} + \frac{x^3}{18}\right)$.

iv) $y = e^{2x}(c_1 + c_2x) + \frac{x^2}{2}\sin 2x + \cos 3x$

v) $y = c_1e^{-x} + c_2e^{-2x} + \frac{x^2-1}{2}$

8. Find the Particular Integral of $(D^2+4D+3)y=e^{-3x}$ **Answer:** PI $=\dfrac{-xe^{-3x}}{2}$

9. Solve $(D^2+2)y=\sin(\sqrt{2}x)$. **Answer:** y = $C_1\cos(\sqrt{2}x)+C_2\sin(\sqrt{2}x)-\dfrac{x}{2\sqrt{2}}\cos(\sqrt{2}x)$

10. Solve $(D^4+4)y=\sin 3x+e^x$.

Answer: y=$e^x(C_1\cos x+C_2\sin x)+e^{-x}(C_3\cos x+C_4\sin x)+\dfrac{1}{85}\sin 3x+\dfrac{1}{5}e^x$.

11. Solve $(D^3-4D^2+D-4)y=\cos x+e^x$.

Answer: y = $C_1e^{4x}+C_2\cos x+C_3\sin x-\dfrac{x}{34}(4\sin x+\cos x)$

12. Find the general solution of the differential equation $(D^2+D+1)y=\sin 2x$

Answer: $y=e^{\frac{-x}{2}}\left(a\cos\dfrac{\sqrt{3}x}{2}+b\sin\dfrac{\sqrt{3}x}{2}\right)-\dfrac{2\cos 2x+3\sin 2x}{13}$

13. Solve $(D^4-a^4)y=x^4$. **Answer:** $y=C_1e^{ax}+C_2e^{-ax}+C_3\cos ax+C_4\sin ax-\dfrac{1}{a^4}x^4-\dfrac{24}{a^8}$

14. Solve $(D^2.+2D+1)y=\cos^2 x$ Answer: $y=(C_1+C_2x)e^{-x}+\dfrac{1}{8}\sin(2x)+\dfrac{1}{2}$

15. a) Solve the following Cauchy's Linear Equations:

i) $x^2y''-3xy'+y=\log x\dfrac{\sin(\log x)+1}{x}$

Answer:

$$y=x^2\left(c_1x^{\sqrt{3}}+\frac{c_2}{x^{\sqrt{3}}}\right)+\frac{1}{61}\left(\frac{\log x}{x}\right)[5\sin\log x+6\cos\log x]+\frac{2}{3721}\frac{1}{x}[27\sin\log x+191\cos\log x]+\frac{1}{6x}$$

ii). $x^2y''+2xy'-20y=(x+1)^2$ **Answer:** $y=c_1x^4+c_2x^{-5}-\dfrac{1}{9}\left[\dfrac{9}{14}x^2+x+\dfrac{9}{20}\right]$

iii). $x^3y'''+2x^2y''+2y=10\left(x+\dfrac{1}{x}\right)$

Answer: $y=\dfrac{c_1}{x}+x[c_2\cos\log x+c_3\sin\log x]+5x+2\left(\dfrac{\log x}{x}\right)$

iv). $x^2y'' - 3xy' + y = \dfrac{1}{(1-x)^2}$ **Answer:** $y = \dfrac{1}{x}\left(c_1 + c_2 \log x\right) + \dfrac{1}{x}\log\dfrac{x}{1-x}$

v) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = \dfrac{x^3}{1+x^2}$

Answer: y = $c_1e^{-t} + c_2e^{t}$ + = $\frac{e^t}{4}\log(1+e^{2t}) - \frac{e^{-t}}{4}\{1+e^{2t} - \log(1+e^{2t})\}$where t = log x.

vi) $x^3y''' + 3x^2y'' + xy' + 8y = 65\cos(\log x)$.

Answer: y= $c_1e^{-2t} + e^t(c_2\cos\sqrt{3}t + c_3\sin\sqrt{3}t) + 8\cos t - \sin t$ where t = log(x).

vii) $x^2y'' - 2xy' - 4y = x^4$ **Answer:** y= $c_1e^{-t} + c_2e^{4t}) + \frac{te^{4t}}{5}$ where t = log x.

16. b)The radial displacement in a rotating disc at a distance r from the axis is given by $r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u + kr^3 = 0$, where k is a constant. Solve the equation under the conditions $u = 0, when\ r = 0, u = 0\ when\ r = a$ **Answer:** $u = \frac{k}{8}a^2r - \frac{k}{8}r^3$.

17. Solve the following Legendre's Linear Equations:

i). $(1+2x)^2D^2y - 6(1+2x)Dy + 16y = 8(1+2x)^2$

Answer: $y = (2x+1)^2[c_1 + c_2\log(2x+1)] + (2x+1)^2[\log(2x+1)]^2$

ii). $(1+x)^2D^2y + (1+x)Dy + y = 4\sin\log(1+x)$

Answer: $y = c_1\cos\log(1+x) + c_2\sin\log(1+x) - 2\log(1+x)\cos\log(1+x)$

iii). $(3+x)^2D^2y - 4(3+x)Dy + 6y = x$ **Answer:** $y = c_1t^3 + c_2t^2 + \dfrac{t-1}{2}$ where t = x + 3

iv). $(1+x)^2D^2y - 3(1+x)Dy + 4y = x^2$ **Answer:** $y = t^2[c_1 + c_2\log t] + \dfrac{t^2}{2}(\log t)^2 - 2t + \dfrac{1}{4}$ where t = x+1

v). $(1+x)^2D^2y + (1+x)Dy = (2x+3)(2x+4)$ **Answer:** $y = c_1 + c_2\log(x+1) + [\log(x+1)]^2 + x^2 + 8$

vi) $(x+1)^2\dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + y = 2\sin(\log(x+1)),\quad where\ \ x > -1$

Answer: y = $C_1\cos(\log(x+1))+C_2\sin(\log(x+1))-\log(x+1)\cos(\log(x+1))$.

vii) $(x+1)^2y''+(x+1)y'+y=\sin(2\log(x+1))$

Answer: y = $c_1\cos t+c_2\sin t-\frac{\sin t}{3}$ Where t = log(x + 1)

18. Solve the following differential equations using the method of variation of parameters.

i). $(D^2+4)y=x\sin 2x$ **Answer:** y = $C_1\cos 2x+C_2\sin 2x+\frac{x}{16}\sin 2x+\frac{1}{64}\cos 2x-\frac{x^2}{8}\cos 2x$

ii) $(D^2-D-2)y=e^{(e^x+3x)}$ **Answer:** y = $C_1e^{-x}+C_2e^{2x}+\frac{e^{e^x}}{3}(3e^x-6+6e^{-x})$

iii) $(D-1)^2y=2z$ *where* $z=\log x$ **Answer:** $y=(C_1+C_2x)e^x+2\log x+4$

iv) $(D^2+4)y=\tan 2x$ **Answer:** y= $c_1\cos 2x+c_2\sin 2x\ \frac{-1}{4}\cos 2x\ \log(\sec 2x+\tan 2x)$

v) $(D^2+a^2)y=\tan ax$ **Answer:** $y=C_1\cos ax+C_2\sin ax-\frac{1}{a^2}\cos ax\log(\sec ax+\tan ax)$.

vii) $y''-y=\frac{2}{1+e^x}$. **Answer:** y = $c_1e^x+c_2e^{-x}-e^{-x}+\log\left(\frac{1+e^x}{e^x}\right)-\log(1+e^x)$.

19. Show that the frequency of forced vibrations in a closed electrical circuit with inductance L and capacitance C in series is $\frac{30}{\pi\sqrt{LC}}$ per minute.

20. The differential equation for a circuit in which self-inductance L and capacitance C neutralize each other is $L\frac{d^2i}{dt^2}+\frac{i}{c}=0$. Find the current I as a function of t given that I is the maximum current and $i=0$ when $t=0$. **Answer:** $I\sin\left(\frac{t}{\sqrt{LC}}\right)$.

21. A condenser of capacity C discharged through an inductance L and resistance R in series and the charge q at time t satisfies the equation $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=0$. Given that $L=0.25$ henries, $R=250\,ohms\ C=2+10^{-6}\,farads$ and that when $t=0$, charge $q\ is\ 0.002\ coulombs$ and the current $\frac{dq}{dt}=0$, obtain the value of q in terms of t.

Answer: $q(t)=e^{-500t}[0.002\cos(1323t)+0.0008\sin(1323t)]$.

22. An electricity circuit consists of an inductance of 0.1 Henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-Farads. Find the charge Q and the current I at any time t, with the following initial conditions

(a) $Q = 0.05 coulomb, I = \frac{dQ}{dt} = 0 \; when \; t = 0$

(b) $Q = 0.05 coulomb, I = \frac{dQ}{dt} = -0.2 \; when \; t = 0$

(c) what will be Q and I after a long time?

Answer: a) $Q(t) = e^{-100t}(0.05\cos(624.5t) + 0.0008\sin(624.5t)) \;\; and \; I(t) = -0.32e^{-100t}\sin(624.5t)$

b) $Q(t) = e^{-100t}(0.05\cos(624.5t) + 0.0077\sin(624.5t)) \;\; and \;\; I(t) = e^{-100t}(-0.2\cos(624.5t) - 32.0\sin(624.5t))$

c) $Q \; and \; I \to 0 \; as \; t \to \infty$ since both solutions are transient containing e^{-t}

23. Suppose that a spring hanging vertically is stretched to an equilibrium position of 15.36 inches by a 1.6- pound weight. If the spring is stretched 4 additional inches and released with an upward velocity of 15 inches per second, what is the equation of motion of the system? What is the velocity of the mass the first time it passes downward through the rest position?

Answer: $x(t) = \frac{5}{12}\sin\left(5t + \pi + \arctan\left(-\frac{4}{3}\right)\right) = 0.$

Which is when $5t + \pi + \arctan\left(-\frac{4}{3}\right) = k\pi.$

24. At t = 0 a current flows in an LRC circuit with resistance R = 40 ohms, inductance L= 0.2 henrys, and capacitance $C = 10^{-5}$ farads. Find the current flowing in the circuit at t > 0 if the initial change on the capacitor is 1 coulomb. Assume that E(t) = 0 for t > 0.

Answer: $I = e^{-100t}(102cos200t - \; sin200t).$

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