ENGINEERING MECHANICS

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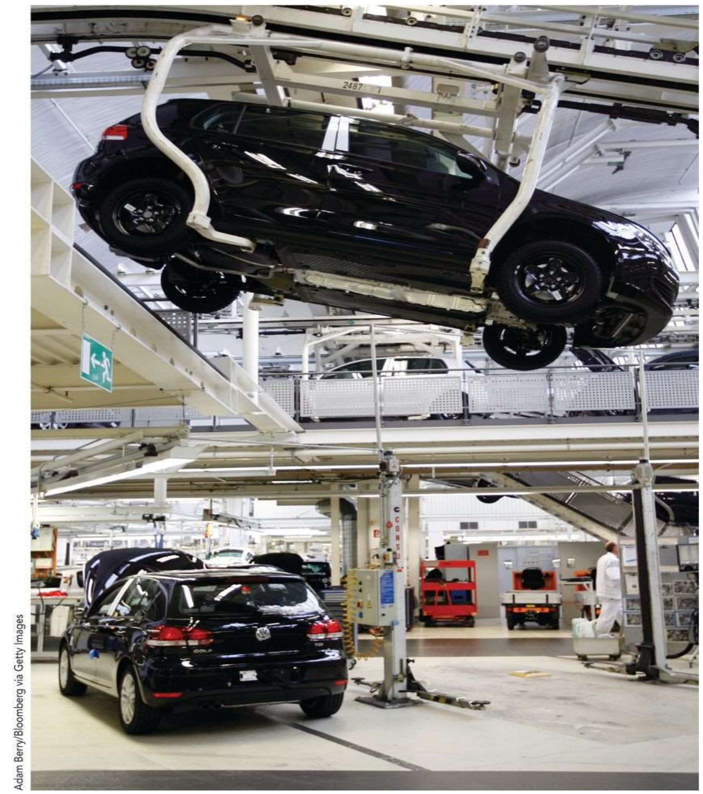
ENGINEERING MECHANICS

Unit: 3 Beams



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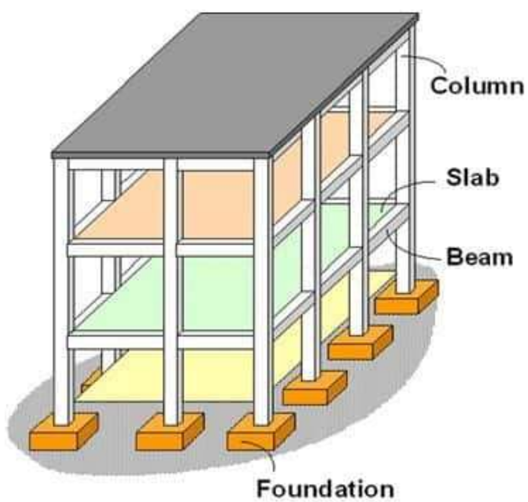
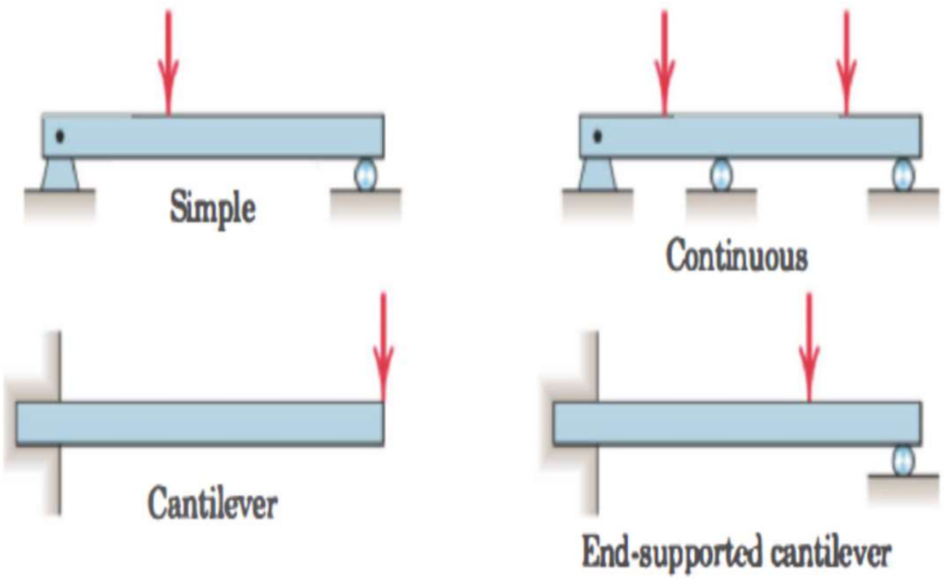
A beam is a horizontal structural member used to support loads. Beams are used to support the roof and floors in buildings.
Beams are structural members which offer resistance to bending due to applied loads.
Most beams are long prismatic bars, and the loads are usually applied normal to the axes of the bars.



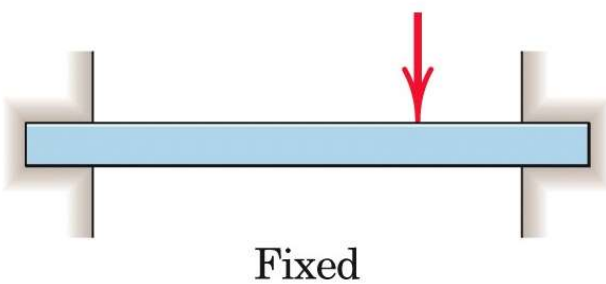
Typical RC Frame Building

Beams supported so that their external support reactions can be calculated by the methods of statics alone are called statically determinate beams.



A beam which has more supports than needed to provide equilibrium is statically indeterminate.

To determine the support reactions for such a beam we must consider its load-deformation properties in addition to the equations of static equilibrium.



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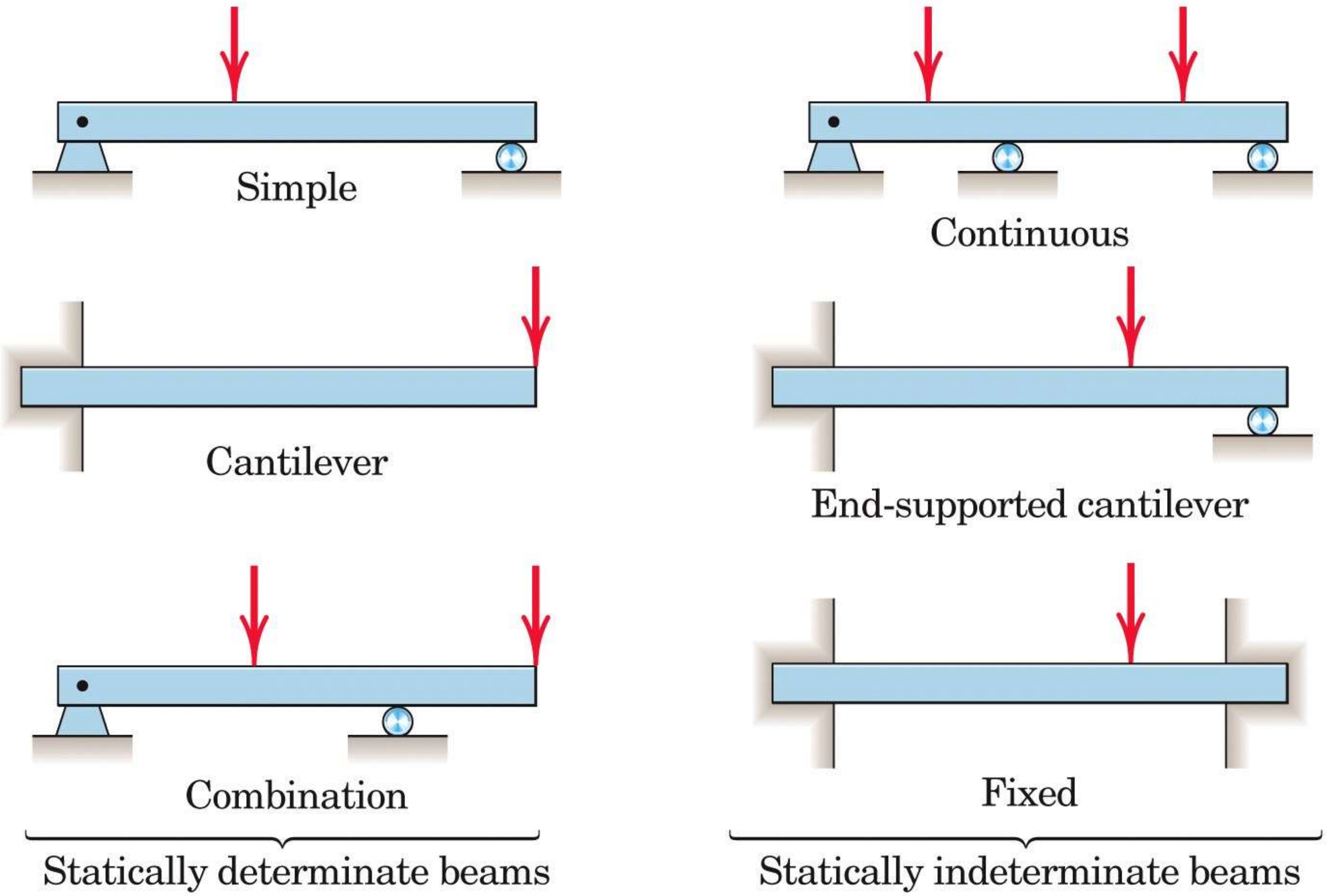
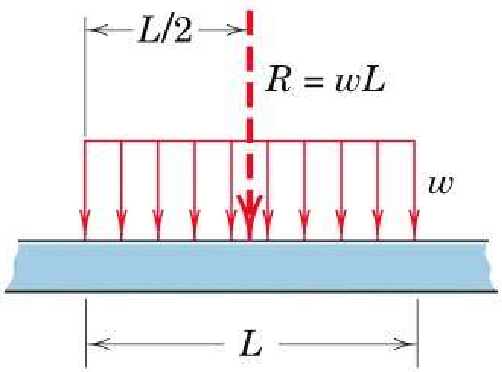
Types of Beams



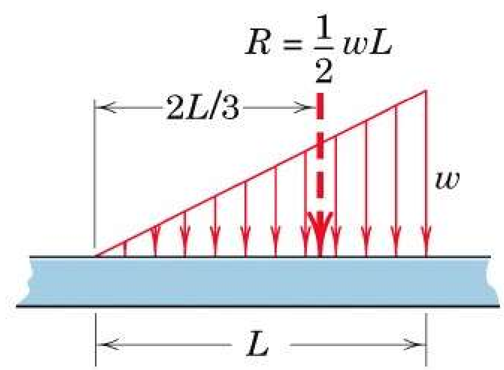
Distributed Loads:

Loading intensities which are constant or which vary linearly are easily handled. Figure illustrates most common cases and the resultants of the distributed loads in each case.

In cases a and b, we see that the resultant load R is represented by the area formed by the intensity w (force per unit length of beam) and the length L over which the force is distributed. The resultant passes through the centroid of this area.

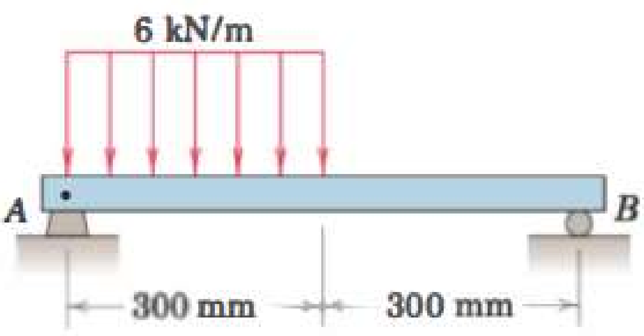


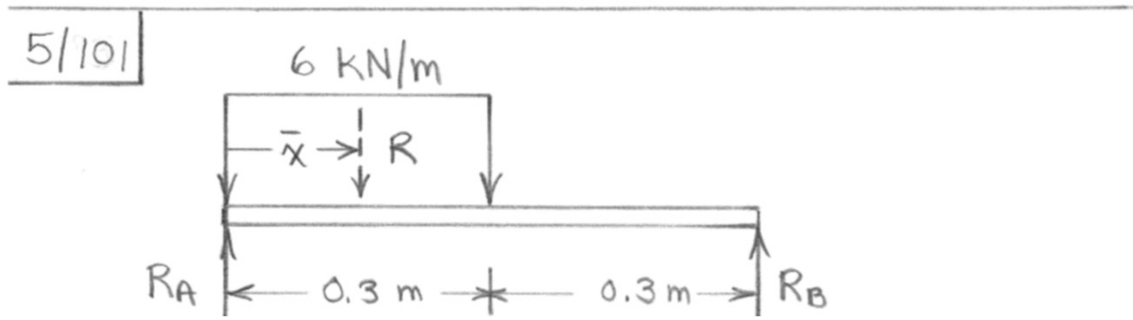
(a)



(b)

5/101 Determine the reactions at A and B for the beam subjected to the uniform load distribution.



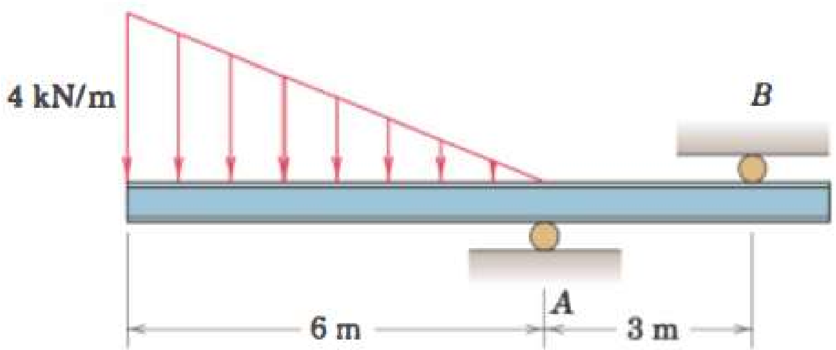


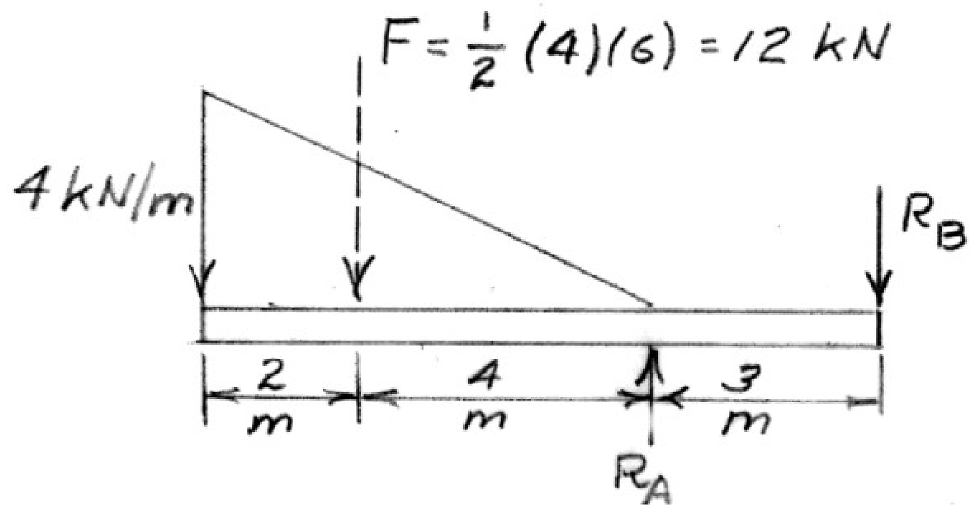
$$R = 6(0.3) = 1.8 \text{ kN} \ @ \ \bar{x} = \tfrac{1}{2}(0.3) = 0.15\text{m}$$

$$\circlearrowleft + \Sigma M_A = 0: \quad R_B(0.6) - 1.8(0.15) = 0, \quad \underline{R_B = 0.45 \text{ kN}}$$

$$+\uparrow \Sigma F = 0: \quad 0.45 - 1.8 + R_A = 0, \quad \underline{R_A = 1.35 \text{ kN}}$$

5/102 Calculate the reactions at A and B for the beam loaded as shown.





$\overset{+}{\circlearrowleft} \Sigma M_A = 0: \quad 12(4) - 3R_B = 0, \qquad \underline{R_B = 16 \text{ kN}}$

$\overset{+}{\uparrow} \Sigma F = 0: \quad R_A - 16 - 12 = 0, \qquad \underline{R_A = 28 \text{ kN}}$

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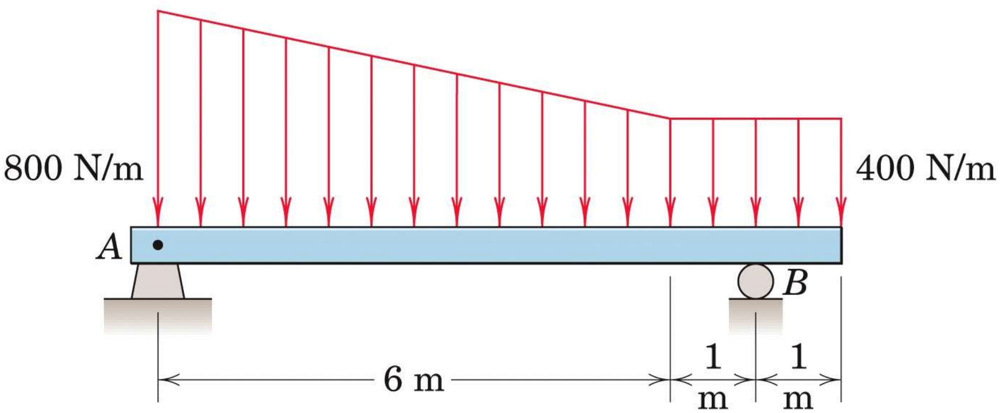
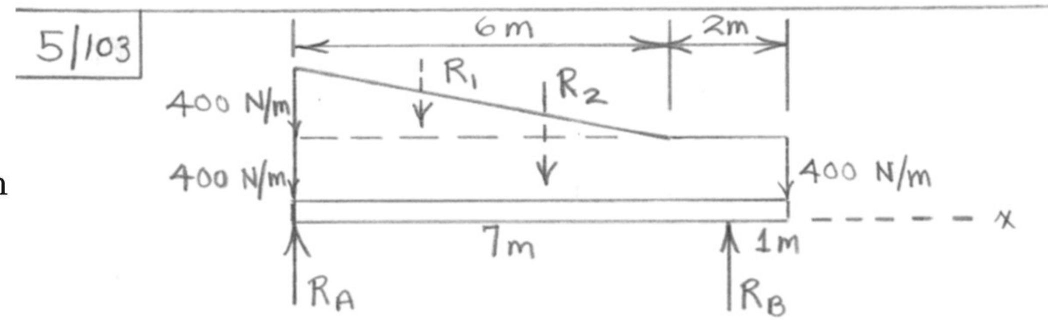
5/103 Calculate the reactions at A and B for the beam loaded as shown.





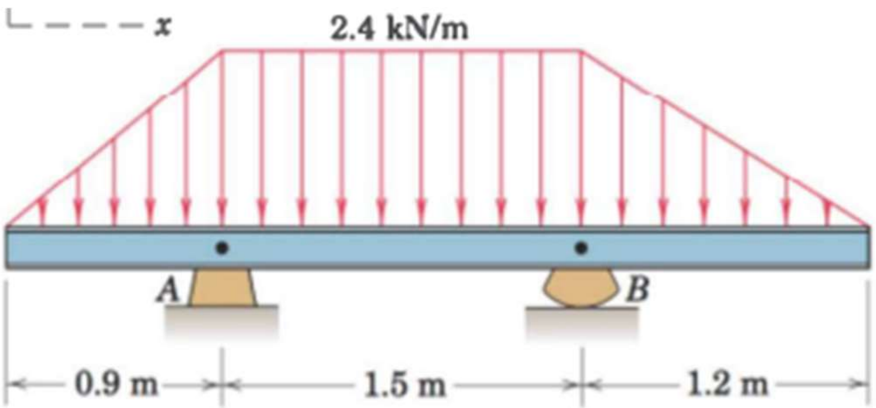
$$R_1 = \tfrac{1}{2}(400)(6) = 1200 \text{ N} \quad @ \quad \bar{x}_1 = \tfrac{1}{3}(6) = 2\text{m}$$

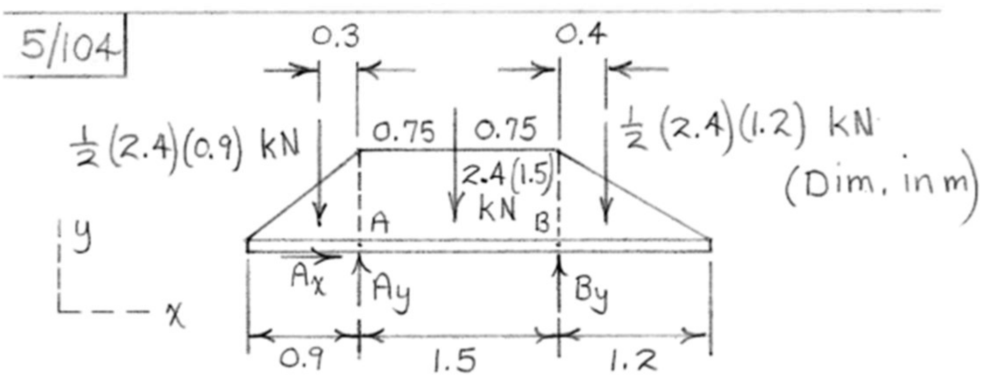
$$R_2 = 400(8) = 3200 \text{ N} \quad @ \quad \bar{x}_2 = \tfrac{1}{2}(8) = 4\text{m}$$

$$\circlearrowleft + \Sigma M_A = 0: \quad R_B(7) - 1200(2) - 3200(4) = 0, \quad \underline{R_B = 2170 \text{ N}}$$

$$+\uparrow \Sigma F = 0: \quad R_A - 1200 - 3200 + 2170 = 0, \quad \underline{R_A = 2230 \text{ N}}$$

5/104 Determine the reaction at A and B for the loaded beam.





$\circlearrowleft + \Sigma M_A = 0 : \quad 1.08(0.3) - 3.6(0.75) - 1.44(1.9) + B_y(1.5) = 0$

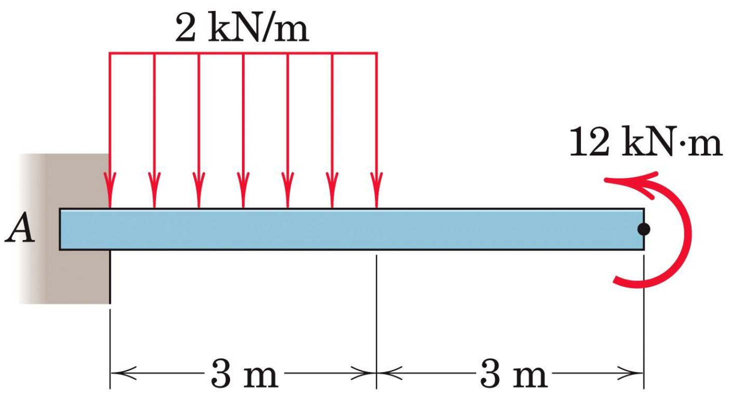
$\underline{B_y = 3.41 \text{ kN}}$

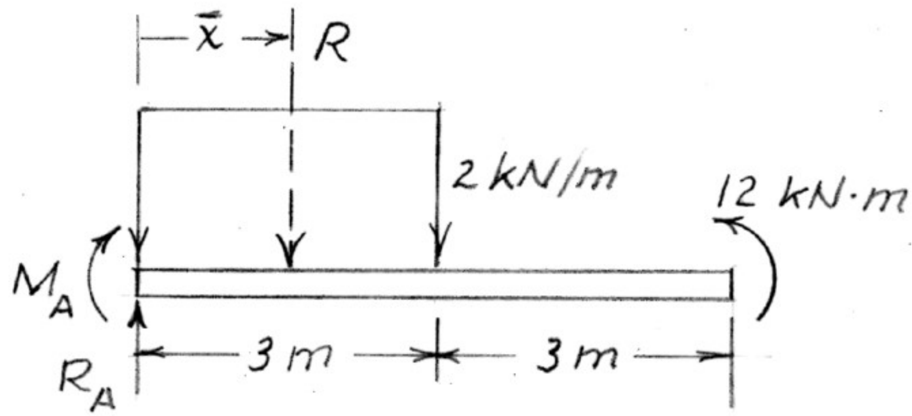
$\Sigma F_y = 0 : \quad A_y + 3.41 - 1.08 - 3.6 - 1.44 = 0$

$\underline{A_y = 2.71 \text{ kN}}$

$\Sigma F_x = 0 : \quad \underline{A_x = 0}$

5/105 Find the reaction at A due to the uniform loading and the applied couple.



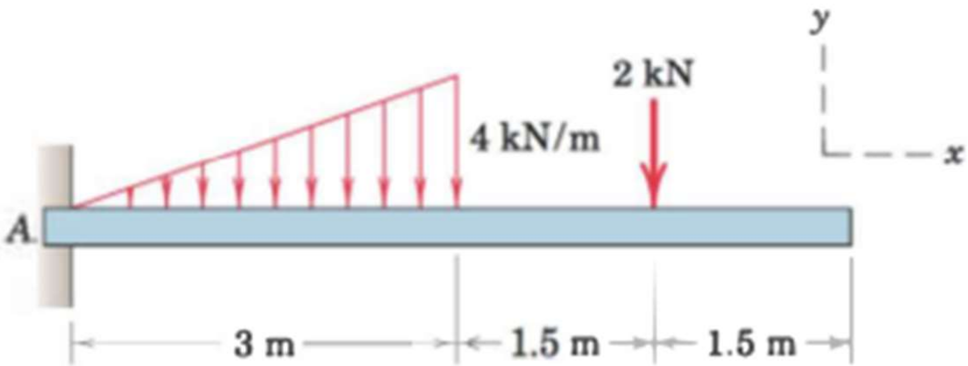


$R = 2(3) = 6 \text{ kN} \quad @ \quad \bar{x} = 1.5 \text{ m}$

$\circlearrowleft + \Sigma M_A = 0: \ -M_A - 6(3/2) + 12 = 0, \quad \underline{M_A = 3 \text{ kN·m}}$

$\uparrow + \Sigma F = 0: \ R_A - 6 = 0, \quad \underline{R_A = 6 \text{ kN}}$

5/106 Determine the reaction at A for the cantilever beam subjected to the distribution and concentrated loads.

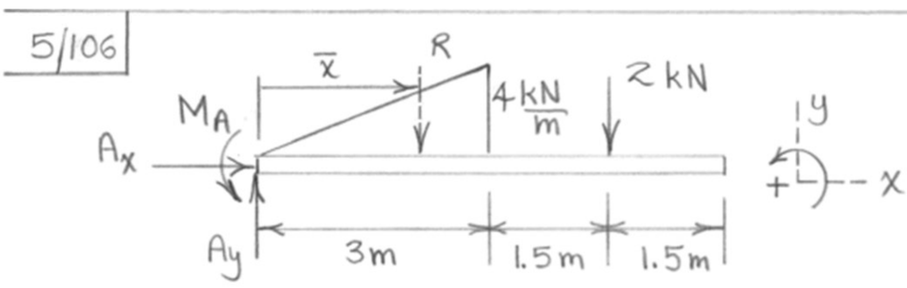


$R = \frac{1}{2}(3)(4) = 6 \text{ kN}$ @ $\bar{x} = \frac{2}{3}(3) = 2 \text{m}$

$\Sigma M_A = 0 : \quad M_A - 6(2) - 2(4.5) = 0, \quad \underline{M_A = 21 \text{ kN·m}}$

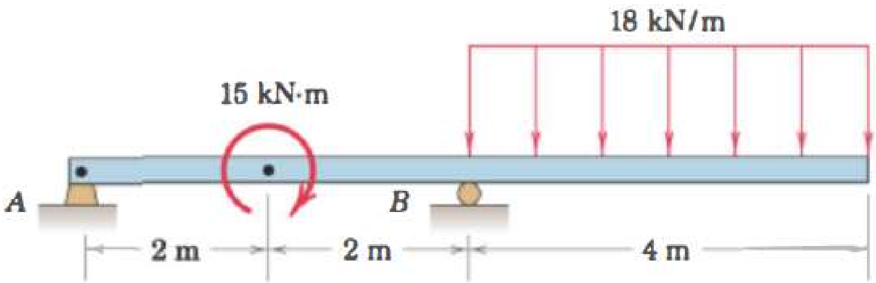
$\Sigma F_y = 0 : \quad A_y - 6 - 2 = 0, \quad \underline{A_y = 8 \text{ kN}}$

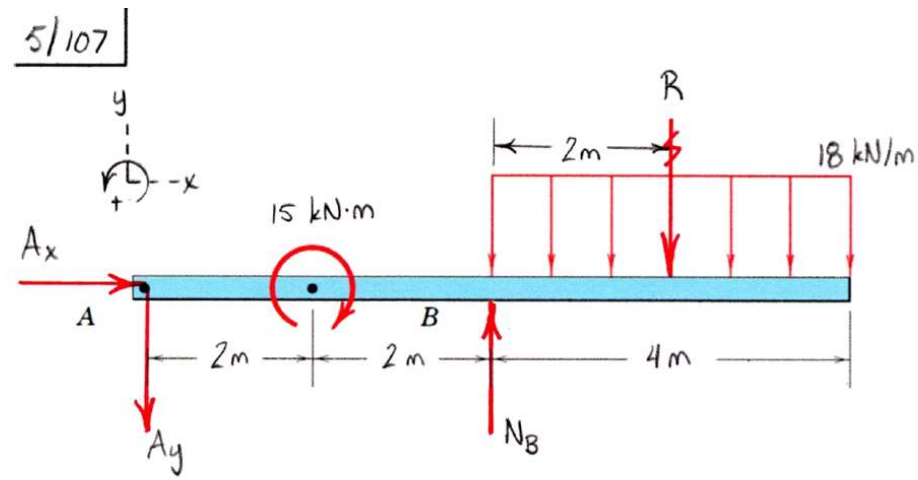
$\Sigma F_x = 0 : \quad \underline{A_x = 0}$



Numericals

5/107 Determine the reactions at A and B for the beam loaded as shown.





$R = 18(4) = 72$ kN

$$\begin{cases} \Sigma F_x = 0: & \underline{A_x = 0} \\ \Sigma F_y = 0: & -A_y + N_B - R = 0 \\ \Sigma M_A = 0: & 4N_B - 6R - 15 = 0 \end{cases} \longrightarrow \begin{cases} \underline{A_y = 39.8 \text{ kN} \downarrow} \\ \underline{N_B = 111.8 \text{ kN} \uparrow} \end{cases}$$

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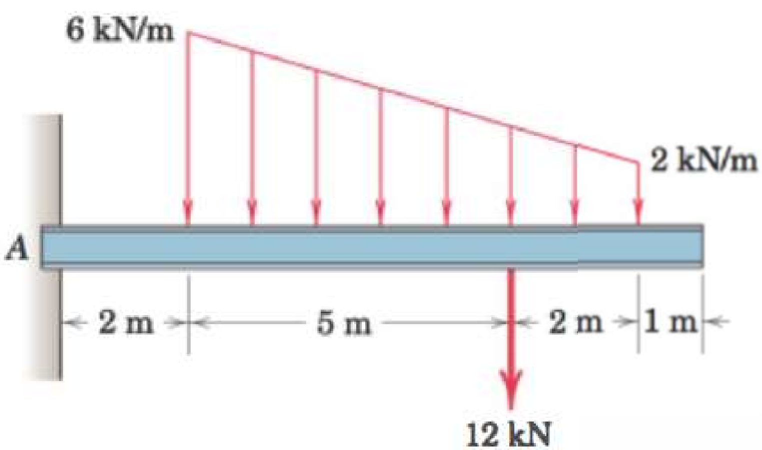
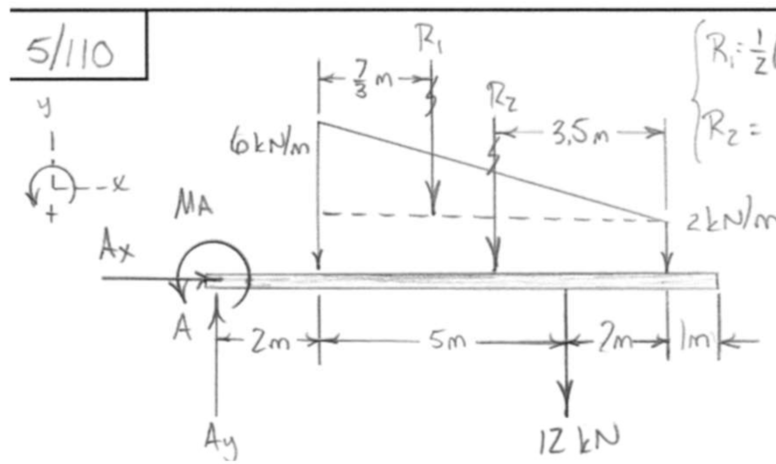
5/110 Determine the force and moment reactions at A for the cantilever beam subjected to the loading shown.





$R_1 = \frac{1}{2}(6-2)(7) = 14$ kN

$R_2 = 2(7) = 14$ kN

$\Sigma F_x = 0: \quad \underline{A_x = 0}$

$\Sigma F_y = 0: \quad A_y - R_1 - R_2 - 12 = 0$

$\Sigma M_A = 0: \quad M_A - (2 + \frac{7}{3})R_1 - (2+3.5)R_2 - 7(12) = 0$

$\underline{A_y = 40 \text{ kN} \uparrow}$ $\qquad$ $\underline{M_A = 222 \text{ kN·m CCW}}$

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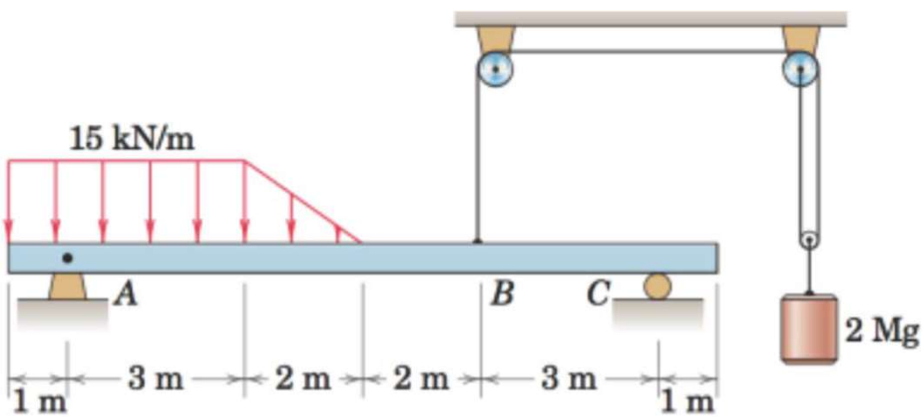
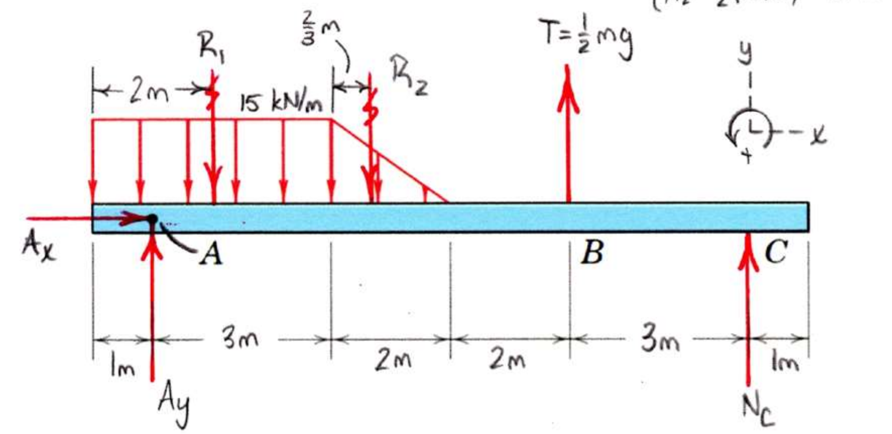
5/111 Determine the reactions at A and C for the beam subjected to the combination of point and distributed loads.



5/111 $m = 2$ Mg

$R_1 = 15(4) = 60$ kN

$R_2 = \frac{1}{2}(15)(2) = 15$ kN



$$\Sigma F_x = 0: \quad \underline{A_x = 0}$$

$$\Sigma F_y = 0: \quad A_y - R_1 - R_2 + T + N_C = 0$$

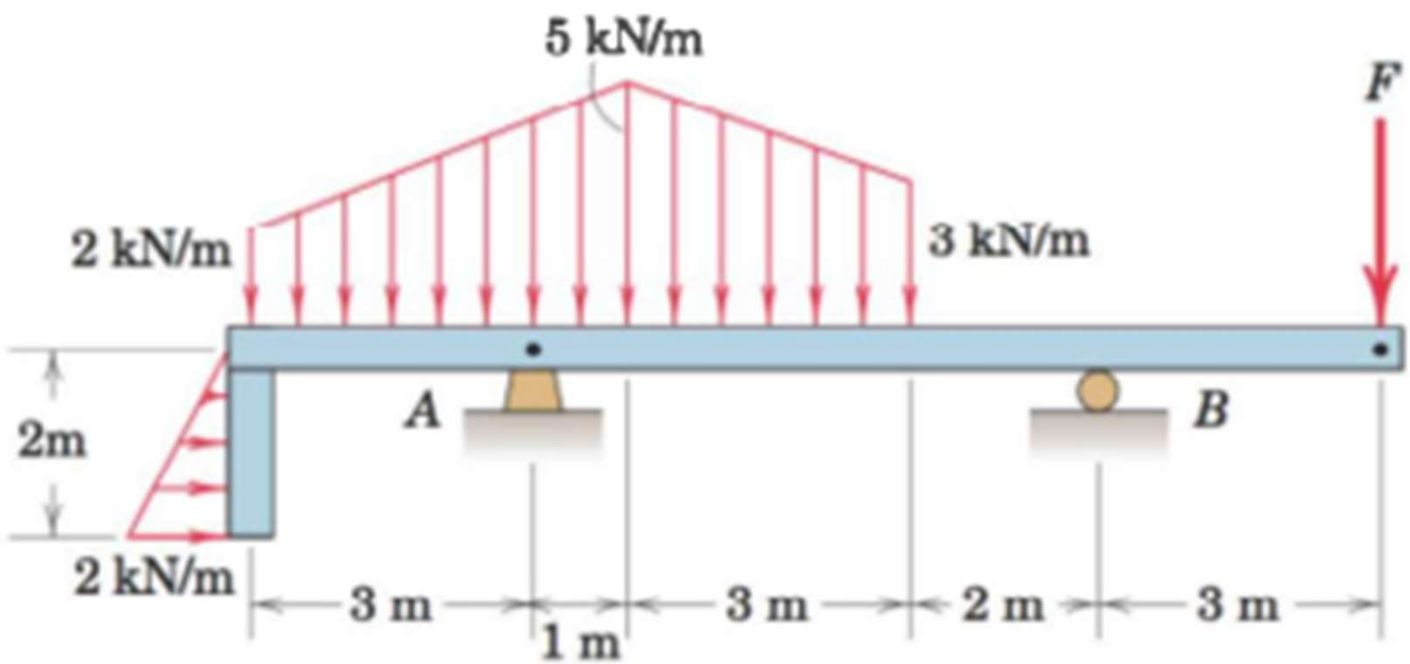
$$\Sigma M_A = 0: \quad -1R_1 - (3 + \tfrac{2}{3})R_2 + 7T + 10N_C = 0$$

$T = \frac{1}{2}(2000)(9.81) = 9810$ N

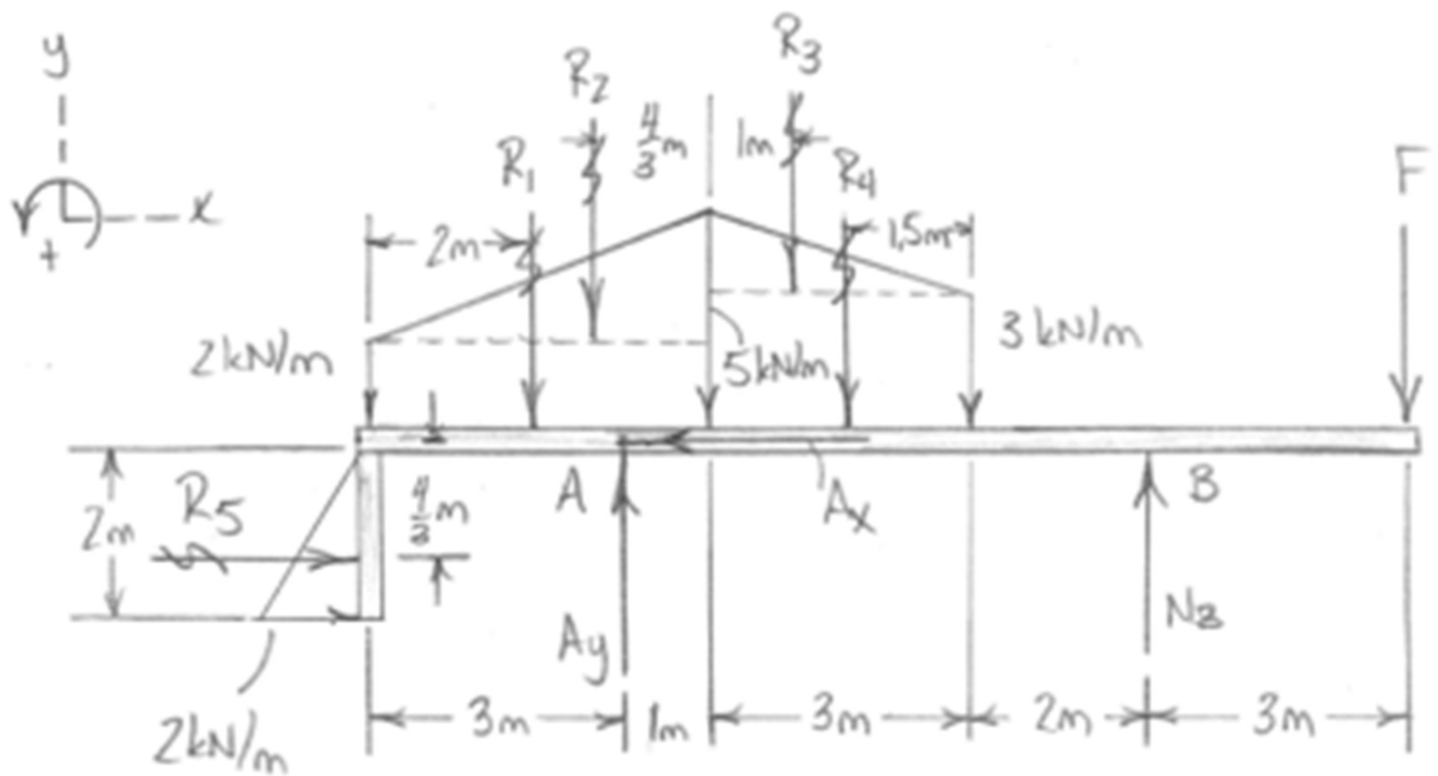
$\underline{N_C = 4.63 \text{ kN} \uparrow}$

$\underline{A_y = 60.6 \text{ kN} \uparrow}$

5/116 For the beam and loading shown, determine the magnitude of the force F for which the vertical reactions at A and B are equal. With this value of F, compute the magnitude of the pin reaction at A.



5/116 FIND F FOR $A_y = N_B$



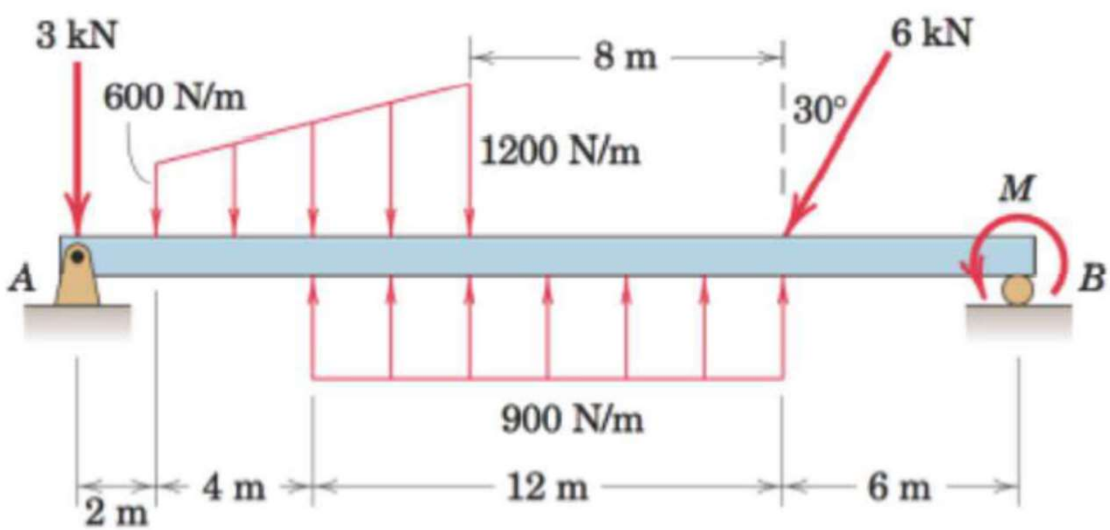
$$\begin{cases} R_1 = 2(4) = 8\ kN \\ R_2 = \frac{1}{2}(5-2)(4) = 6\ kN \\ R_3 = \frac{1}{2}(5-3)(3) = 3\ kN \end{cases} \qquad \begin{cases} R_4 = 3(3) = 9\ kN \\ R_5 = \frac{1}{2}(2)(2) = 2\ kN \end{cases}$$

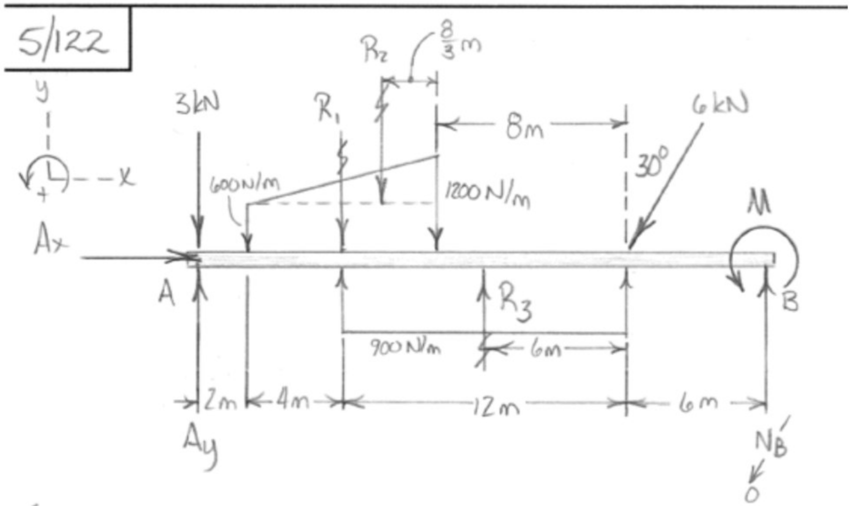
$$\begin{cases} \Sigma F_x = 0: \ -A_x + R_5 = 0 \longrightarrow A_x = 2\ kN \\ \Sigma F_y = 0: \ A_y + N_B - F - R_1 - R_2 - R_3 - R_4 = 0 \\ \Sigma M_B = 0: \ -3F + 3.5R_4 + 4R_3 + (5+\frac{4}{3})R_2 + 7R_1 + \frac{4}{3}R_5 - 6A_y = 0 \end{cases}$$

$$A_y = N_B = 18.18\ kN \qquad \underline{F = 10.36\ kN}$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{18.18^2 + 2^2} \longrightarrow \underline{R_A = 18.29\ kN}$$

5/122 Determine the magnitude of the moment M which will cause the beam to just begin to lift off the roller at B. For this value of M, determine the magnitude of the pin reaction at A.





$$R_1 = 600(8) = 4800 \text{ N}$$

$$R_2 = \frac{1}{2}(1200-600)(8) = 2400 \text{ N}$$

$$R_3 = 900(12) = 10\,800 \text{ N}$$

$$\Sigma F_x = 0: \; A_x - 6(1000)\sin 30° \longrightarrow A_x = 3000 \text{ N}$$

$$\Sigma F_y = 0: \; A_y - 3(1000) - R_1 - R_2 + R_3 + \cancel{N_B}^{\,0} - 6(1000)\cos 30° = 0$$

$$\Sigma M_A = 0: \; -6R_1 - \left(2 + \tfrac{2}{3}(8)\right)R_2 + 12R_3 - 18(6)(1000)\cos 30° + 24\cancel{N_B}^{\,0} + M = 0$$

$$A_y = 4600 \text{ N} \uparrow \qquad \underline{M = 10\,330 \text{ N·m CCW}}$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{3000^2 + 4600^2} \longrightarrow \underline{R_A = 5490 \text{ N}}$$

THANK YOU

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