

## Unit-02: Random variables and probability distributions

1. A hole is drilled in a metal sheet component, and then a shaft is inserted through the hole. The shaft clearance is equal to the difference between the radius of the hole and the radius of the shaft. Let the random variable  $X$  denote the clearance, in millimetres. The probability density function of  $X$  is defined by :

$$f(x) = \begin{cases} 1.25(1 - x^4) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that 0.8 mm of the components are scrapped ? Find also the CDF of the random variable  $X$ .

Ans: 0.0819

2. The lifetime in months of a transistor in a certain application is random with probability density function given by

$$f(x) = \begin{cases} 0.1 e^{-0.1t} & t \geq 0 \\ 0 & t \leq 0 \end{cases}$$

a) Find the mean lifetime.      Ans:  $\frac{100}{(\ln\{e\})^2}$

b) Find the standard deviation of the lifetimes.      Ans:  $\frac{2000}{(\ln\{e\})^2} - \left(\frac{100}{(\ln\{e\})^2}\right)^2$

d) Find the probability that the lifetime will be less than 12 months.      Ans:  $1 - e^{-1.2}$

3. In a medical examination the chances of error are 15%. Find the Bernoulli distribution if one patient is randomly selected out of 60 patients.

Ans:  $P(X = 1) = \frac{51}{60} = 0.85$  and  $P(X = 0) = \frac{9}{60} = 0.15$

4. The probability of a shooter hitting a target is  $1/3$ . How many times he should shoot so that the probability of hitting the target at least once is more than  $3/4$ .
5. If the mean and standard deviation of the number of correctly answered question in a test given to 4096 students is 2.5 and  $\sqrt{1.875}$ . Find an estimate of the number of candidates answering correctly
- (i) 8 or more questions
  - (ii) 2 or less questions
  - (iii) 5 questions.
6. In a quiz contest of answering 'yes' or 'no', what is the probability of guessing at least 6 answers correctly out of ten question asked? Also find the probability of the same if there are 4 options for a correct answer.
7. A shop has 4 diesel generator sets which it hires every day. The demand for generator set on an average is a Poisson variate with value  $5/2$ . Obtain the probability that on a particular day (i) there was no demand (ii) a demand has to be refused.
- Ans : (i) 0.0820 (ii) 0.1088
8. Assume that the probability of an individual coal miner being killed in a mine accident during a year is  $1/2400$ . Calculate the probability that in a mine employing 200 miners, there will be at least one fatal accident in a year.
- Ans: 0.08
9. Suppose that a book of 600 pages contain 40 printing mistakes. Assume that these errors are randomly distributed throughout the book and the number of errors per page has a Poisson distribution. What is the probability that 10 pages selected at random will be free of errors?

Ans: 0.51

10. In an examination taken by 500 candidates, the average and the standard deviation of marks obtained (normally distributed) are 40% and 10%. Find approximately
- how many will pass, if 50% is fixed as a minimum?
  - what should be the minimum if 350 candidates are to pass?
  - how many have scored marks above 60%?
11. The mean inside diameter of a sample of 200 washers produced by a machine is 5.02mm and the standard deviation is 0.05mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.96 to 5.08mm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.
12. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for
- more than 2150 hours,
  - less than 1950 hours,
  - more than 1920 hours and but less than 2160 hours.

### Central Limit Theorem

1. A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean  $\mu = 205$  pounds and standard deviation  $\sigma = 15$  pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?
2. A manufacturing process produces cylindrical component parts for the automotive industry. It is important that the process produces parts having a mean of 5 millimeters. The engineer involved conjectures that the population means (i.e., the expected value for the whole production) is 5.0 mm. An experiment is conducted in which 100 parts produced by the process are selected randomly and the diameter is measured on each. It is known that the population standard deviation (i.e., the standard deviation characteristic for every individual part)  $\sigma$  is 0.1. The experiment gives a sample average diameter  $\bar{x} = 5.027$  mm. Does this sample information appear to support or refute the engineer's conjecture?
3. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with mean  $\mu$  (expected value) equal to 800 hours and a standard deviation of 40 hours. Find the probability that a random sample of 16 bulbs will have an average life of fewer than 775 hours.