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Unit-3 Digital Electronics

Boolean Functions, Canonical and Standard Form

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Boolean Functions, Canonical and Standard Form



- ❖ Boolean function described by an algebraic expression consists of binary variables, the constants 0 and 1, and the logic operation symbols.
- * Boolean function is evaluated to logic-1 or logic-0 for a given value of the binary variables.
- * Boolean function can be represented in a **truth table**.
- ❖ A Boolean function can be implemented as **digital circuit**, Which is constructed by using logic gates.
- Example: F = X + Y'Z $F = 1 \quad \text{if } X = 1 \text{ or if } Y = 0 \text{ and } Z = 1$ $F = 0 \quad \text{Otherwise}$

Boolean Functions, Canonical and Standard Form

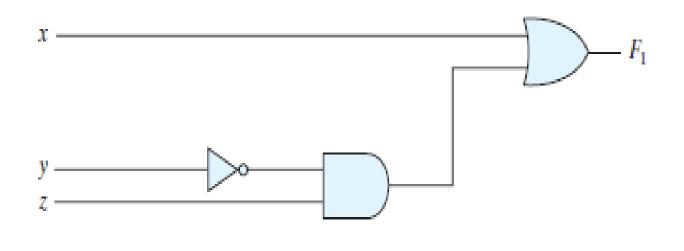


- \Leftrightarrow Example: F₁ = x + y'z
- \clubsuit F₁ contains either 0 or 1 for each of these combinations. The table shows that the function is equal to 1 when x = 1 or when yz = 01 and is equal to 0 otherwise.

Truth Table:

X	y	Z	F ₁
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Gate Level Implementation:



Reference: "Digital Design with an Introduction to Verilog HDL" M Morris Mano, Michale D Ciletti

Boolean Functions, Canonical and Standard Form



Simplify and realize the given Boolean function using basic gates

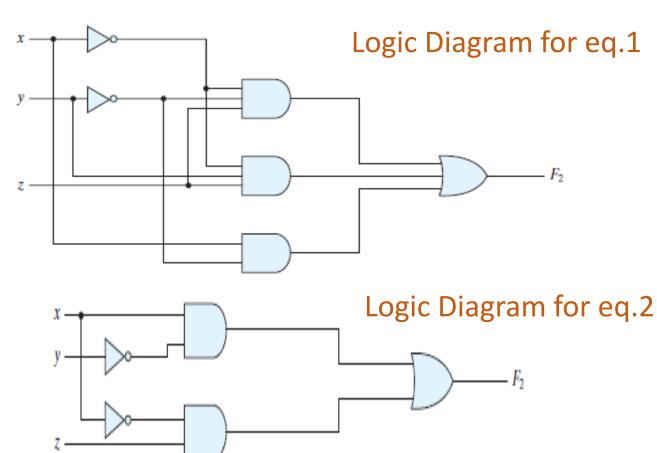
$$F_2 = x'y'z + x'yz + xy'....(1)$$

$$F_2 = x'z (y' + y) + xy'$$

$$F_2 = x'z + xy'....(2)$$

Truth Table

x	y	z	F ₂
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	. 0



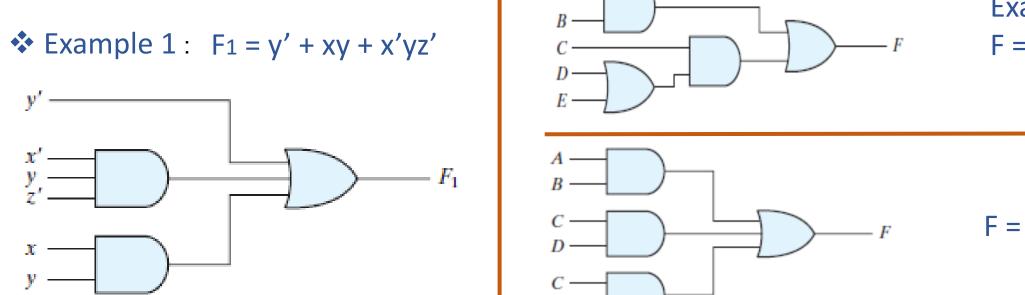
Reference: "Digital Design with an Introduction to Verilog HDL" M Morris Mano, Michale D Ciletti

Boolean Functions, Canonical and Standard Form



Sum of Products: (SOP)

- The sum of products is a Boolean expression containing AND terms, called **product terms**, with one or more literals each. The **sum** denotes the **OR**ing of these terms.
- > Two- Level Logic implementation of SOP



Example 2: F = AB + C (D+E)F = AB + CD + CE

Reference: "Digital Design with an Introduction to Verilog HDL" M Morris Mano, Michale D Ciletti

Boolean Functions, Canonical and Standard Form

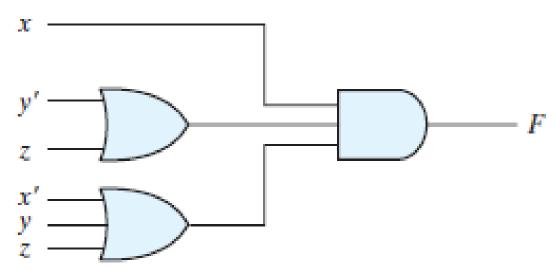


Product Of Sum: POS

➤ A product of sum (POS) is a Boolean expression containing OR terms, called sum terms. Each term may have any number of literals. The product denotes the ANDing of these terms

> Example 1:

$$F = x.(y' + z).(x' + y + z)$$



 \triangleright Example 2: Y = (A+B). (C+D)

Boolean Functions, Canonical and Standard Form



Canonical SOP Form

- ➤ Each product term contains all the literals of that function either in true or complement form
- \triangleright Example: F(x,y,z) = xyz + x'y'z + x'yz' + x'y'z'
- ➤ Each product term is called as minterm
- > For three variable function: Truth Table

			Minterms		
X	y	Z	Term	Designation	
0	0	0	x'y'z'	m_0	
0	0	1	x'y'z	m_1	
0	1	0	x'yz'	m_2	
0	1	1	x'yz	m_3	
1	0	0	xy'z'	m_4	
1	0	1	xy'z	m_5	
1	1	0	xyz'	m_6	
1	1	1	xyz	m_7	

Boolean Functions, Canonical and Standard Form



Consider the function: f1 and f2 in the truth table

X	y	Z	Function f ₁	Function f ₂
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Canonical SOP Form for f1

$$f1 = x'y'z + xy'z' + xyz = m1 + m4 + m7$$

Canonical SOP Form for f2

$$f2 = x'yz + xy'z + xyz' + xyz = m3 + m5 + m6 + m7$$

Boolean Functions, Canonical and Standard Form



Canonical POS Form

- ➤ Each sum term contains all the literals of that function either in true or complement form
- **Example:** F(x, y, z) = (x + y + z) (x' + y' + z) (x' + y + z')
- > Each sum term is called as Maxterm
- > For three variable function: Truth Table

			Maxterms		
X	y	z	Term	Designation	
0	0	0	x + y + z	M_0	
0	0	1	x + y + z'	M_1	
0	1	0	x + y' + z	M_2	
0	1	1	x + y' + z'	M_3	
1	0	0	x' + y + z	M_4	
1	0	1	x' + y + z'	M_5	
1	1	0	x' + y' + z	M_6	
1	1	1	x' + y' + z'	M_7	

Boolean Functions, Canonical and Standard Form



Consider the function: f1 and f2 in the truth table

X	y	Z	Function f ₁	Function f ₂	
0	0	0	0	0	
0	0	1	1	0	t
0	1	0	0	0	(
0	1	1	0	1	
1	0	0	1	0	
1	0	1	0	1	
1	1	0	0	1	
1	1	1	1	1	

Canonical POS Form for f1

$$f1 = (x + y + z)(x + y' + z)(x + y' + z')(x' + y + z')$$

(x' + y' + z)
= M0. M2.M3. M5. M6

Canonical POS Form for f2

$$f2 = (x + y + z)(x + y + z')(x + y' + z)(x' + y + z)$$

= M0. M1. M2. M4

Boolean Functions, Canonical and Standard Form



Find the Minterms for the given expression: (Convert into canonical SOP Form)

$$F = A + BC$$
 $F = A.1.1 + BC.1$
 $F = A. (B+B'). (C+C'). + BC.(A+A')$
 $F = A. (BC + BC' + B'C + B'C') + ABC + A'BC$
 $F = ABC + ABC' + AB'C + AB'C' + ABC + A'BC$
 $F = ABC + ABC' + AB'C + AB'C' + A'BC$
 $F = m7 + m6 + m5 + m4 + m3$
 $F = \Sigma (3,4,5,6,7)$



THANK YOU

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