Class - 6

Problems on Euler's theorem

1. Verify Euler's theorem for the following function
   $u = x^2yz - 4y^2z^2 + 2xz^3$
2. If $u = e^{\frac{x}{y}} \sin\left(\frac{x}{y}\right) + e^{\frac{y}{x}} \cos\left(\frac{x}{y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.
3. If $u = sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$.
4. If $u = cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-1}{4}\sin 2u$.
5. If $u = tan^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$, find the value of

   $i)\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ **Ans:** $\mathbf{\frac{5}{4}\sin 2u}$

   $ii)\ x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ **Ans:** $\mathbf{\frac{25}{16}\sin 4u - \frac{5}{4}\sin 2u}$