ENGINEERING MATHEMATICS - II

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Unit 4 : Inverse Laplace Transform

Session : 3

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Inverse Laplace Transforms – Method of Partial Fractions

Sometimes the given $F(s) = \frac{P(s)}{Q(s)}$ where $P(s)$ and Q(s) are polynomials in s can be expressed as partial fractions for obtaining the Inverse Laplace Transform.

INVERSE LAPLACE TRANSFORM - Pre – Requisite:

Sl No.	Factors in the denominator	Corresponding Partial Fractions
1.	Non- repeated linear factors $F(s) = \frac{s}{(s+2)(3s+5)}$	$F(s) = \frac{A}{s+2} + \frac{B}{3s+5}, A \neq 0 \,\&\, B \neq 0$
2.	Repeated linear factor $F(s) = \frac{2s+5}{(s+1)^3}$	$F(s) = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3},$ $A \neq 0 , B \neq 0 \,\&\, C \neq 0$
3.	Non – repeated quadratic factor $F(s) = \frac{2s+1}{(s^2+2s+2)(s^2+2s+5)}$	$F(s) = \frac{As+B}{(s^2+2s+2)} + \frac{Cs+D}{(s^2+2s+5)}$
4.	Repeated quadratic factor $F(s) = \frac{2s+1}{(s^2+2s+5)^2}$	$F(s) = \frac{As+B}{(s^2+2s+5)} + \frac{Cs+D}{(s^2+2s+5)^2}$

1) Obtain the Inverse Laplace Transforms of $\frac{s-2}{s^2+5s+6}$

Solution :

Consider $s^2+5s+6 = (s+2)(s+3)$

To find the inverse Laplace transform, first we shall resolve given rational function into partial fractions.

Let $\frac{s-2}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$$s-2 = A(s+3) + B(s+2)$$

$$s-2 = A(s+3) + B(s+2)$$

Put $s=-3$, $\quad -5 = -B \Rightarrow B = 5$

$s=-2$, $\quad -4 = A \Rightarrow A = -4$

$$\therefore L^{-1}\left\{\frac{s-2}{s^2+5s+6}\right\} = -4L^{-1}\left\{\frac{1}{s+2}\right\} + 5L^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= -4e^{-2t} + 5e^{-3t}$$

$$\therefore L^{-1}\left\{\frac{s-2}{s^2+5s+6}\right\} = -4e^{-2t} + 5e^{-3t}$$

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2) Obtain the Inverse Laplace Transforms of $\frac{2s+3}{(s+2)^2(s-1)}$

Solution :

To find the inverse Laplace transform, first we shall resolve given rational function into partial fractions.

Let $\frac{2s+3}{(s+2)^2(s-1)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$2s+3 = A(s+2)^2 + B(s-1)(s+2) + C(s-1)$

Put $s=1$, $5 = 9A \Rightarrow A = 5/9$

$s=-2$, $-1 = -3C \Rightarrow C = 1/3$

Put

$$s=0, \quad 3 = 4A - 2B - C$$

$$3 = 4\left(\frac{5}{9}\right) - 2B - \frac{1}{3} \Rightarrow B = -\frac{5}{9}$$

$$\therefore L^{-1}\left\{\frac{2s+3}{(s-1)(s+2)^2}\right\} = L^{-1}\left\{\frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}\right\}$$ becomes

$$= L^{-1}\left\{\frac{5/9}{s-1} + \frac{-5/9}{s+2} + \frac{1/3}{(s+2)^2}\right\}$$

$$= \frac{5}{9}L^{-1}\left\{\frac{1}{s-1}\right\} - \frac{5}{9}L^{-1}\left\{\frac{1}{s+2}\right\} + \frac{1}{3}L^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

$$= \frac{5}{9}e^{t} - \frac{5}{9}e^{-2t} + \frac{1}{3}\cdot e^{-2t}\cdot t$$

3) Obtain the Inverse Laplace Transforms of $\frac{s}{(s-3)\,(s^2+4)}$

Solution :

To find the inverse Laplace transform, first we shall resolve given rational function into partial fractions.

Let $\frac{s}{(s-3)(s^2+4)} = \frac{A}{s-3} + \frac{Bs+C}{s^2+4}$

$s = A(s^2+4) + (Bs+C)(s-3)$ —— 1

Put $s=3$,

$3 = 13A \Rightarrow A = \frac{3}{13}$

① can be written as

$$s = (A+B)s^2 + (C-3B)s + (4A-3C)$$

Equating the coefficient of s^2,

$$A+B=0 \Rightarrow \frac{3}{13}+B=0 \Rightarrow B=\frac{-3}{13}.$$

Equating the coefficient of s,

$$C-3B=1$$

$$\Rightarrow C = 4/13.$$

INVERSE LAPLACE TRANSFORM

$$L^{-1}\left\{\frac{s}{(s-3)(s^2+4)}\right\} = \frac{3}{13}L^{-1}\left\{\frac{1}{s-3}\right\} + L^{-1}\left\{\frac{-\frac{3}{13}s+\frac{4}{13}}{s^2+4}\right\}$$

$$= \frac{3}{13}\cdot e^{3t} - \frac{3}{13}L^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{4}{13}L^{-1}\left\{\frac{1}{s^2+4}\right\}$$

$$= \frac{3}{13}\cdot e^{3t} - \frac{3}{13}\cdot\cos 2t + \frac{4}{13}\cdot\frac{1}{2}\sin 2t$$

$$= \frac{3}{13}e^{3t} - \frac{3}{13}\cos 2t + \frac{2}{13}\sin 2t$$

THANK YOU

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