Find Laplace transforms of:

1. $(sint - cost)^2$.
2. $cos^3 2t$.
3. Cos (at + b).
4. Sin 2t cos 3t.
5. $e^{-at} sinhbt$.
6. $\cosh at \sin at$.
7. $Sinh3t\ cos^2 t$
8. $f(t) = \begin{cases} 4, & 0 \le t \le 1 \\ 3, & t > 1 \end{cases}$
9. $f(t) = \begin{cases} sint, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$

10.$If\ \ L\{f(t)\} = \frac{1}{s(s^2+1)},\ \ find\ \ L\{e^{-t} f(2t)\}$

11.$te^{2t} sin3t$

12.$te^{-2t} sin4t$

13.$2^t + \frac{cos2t - cos3t}{t} + t\ sint$

14.Using LT Prove that $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t} dt = log \frac{b}{a}$

15.Evaluate $L\left(\int_0^t \frac{sint}{t} dt\right)$

16.Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ 1, & \pi < t < 2\pi \\ \sin t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find its LT

17.Express $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ 4t, & 2 < t < 4 \\ 8, & t > 4 \end{cases}$ in terms of unit step function and hence find its LT.

18.Evaluate $L\{(1 + 2t - 3t^2 + 4t^3)\ H(t-2)\}$

19.Find LT of the full-wave rectifier $f(t) = E \sin \omega t,\ \ 0 < t < \frac{\pi}{\omega}$, having the period $\frac{\pi}{\omega}$.

20.Find the LT of the triangular wave of period $2a$ given by $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$

Answers for the above questions:

1. $\frac{1}{s} - \frac{2}{s^2+4}$
2. $\frac{1}{4}\left[\frac{s}{s^2+9} + \frac{3s}{s^2+1}\right]$

3. $\left[\frac{s}{s^2+a^2}\,cosb\; - sinb\;\frac{a}{s^2+a^2}\right]$
4. $\frac{1}{2}\left[\frac{5}{s^2+25} - \frac{1}{s^2+1}\right]$
5. $\left[\frac{b}{(s+a)^2-b^2}\right]$
6. $\frac{1}{2}\left[\frac{a}{(s^2+a^2)^2-4a^2s^2}\right]$
7. $\frac{1}{4}\left[\frac{s-3}{(s-3)^2+1} - \frac{s+3}{(s+3)^2+1} + \frac{6}{s^2-9}\right]$
8. $\frac{4}{s} - \frac{e^{-s}}{s}$
9. $\frac{1+e^{-\pi s}}{s^2+1}$
10. $\frac{4}{(s-1)((s-1)^2+4)}$
11. $\frac{6(s-2)}{((s-2)^2+9)^2}$
12. $\frac{6(s+2)}{((s+2)^2+16)^2}$
13. $\frac{1}{s-ln2} + \frac{1}{2}ln(\frac{s^2+9}{s^2+4}) + \frac{2s}{(s^2+1)^2}$
14. $ln\left(\frac{b}{a}\right)$
15. $\frac{\frac{\pi}{2}- - tan^{-1}s}{s}$
16. $\frac{s(1+e^{-\pi s})+e^{-2\pi s}}{s^2+1} + \frac{e^{-\pi s}-e^{-2\pi s}}{s}$
17. $\frac{2}{s^3} + e^{-2s}[\frac{4}{s} - \frac{2}{s^3}] - e^{-4s}[\frac{4}{s^2} - \frac{8}{s}]$
18. $e^{-2s}\left[\frac{24}{s^4} + \frac{42}{s^3} + \frac{38}{s^2} + \frac{25}{s}\right]$
19. $\frac{E\omega}{s^2+\omega^2}\left[\frac{1+e^{-\pi s/\omega}}{1-e^{-\pi s/\omega}}\right]$
20. $\frac{1}{s^2}\left[\frac{1-e^{-as}}{1+e^{-as}}\right]$

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QUESTION BANK

1. Find $L^{-1}\left[\frac{3s-8}{4s^2+25}\right]$

   Ans : $f(t)=\frac{3}{4}cos\left(\frac{5}{2}\right)t-\frac{4}{5}sin\left(\frac{5}{2}\right)t$

2. Find $L^{-1}\left[\frac{4s-18}{9-s^2}\right]$

   Ans : $f(t)=6sinh3t-4cosh3t$

3. Find $L^{-1}\left[\frac{s^2-3s+4}{s^3}\right]$

   Ans : $f(t)=1-3t+2t^2$

4. Find $L^{-1}\left[\frac{1}{s-2}+\frac{1}{s^2-9}+\frac{1}{s^2+25}+\frac{s}{s^2+9}\right]$

   Ans : $f(t)=e^{2t}+\frac{1}{3}sinh3t+\frac{1}{5}sin5t+cos3t$

5. Find $L^{-1}\left[\frac{1}{s}e^{-\frac{1}{s}}\right]$ {Use the series expansion of e^x}

   Ans : $f(t)=\sum_{n=0}^{\infty}\frac{(-1)^n t^n}{(n!)^2}$

6. If $L^{-1}\left[\frac{s^2-1}{(s^2+1)^2}\right]=tcost$ then show that $L^{-1}\left[\frac{9s^2-1}{(9s^2+1)^2}\right]=\frac{t}{9}cos\frac{t}{3}$

7. Find $L^{-1}\left[\frac{1-3s}{s^2+8s+21}\right]$

   Ans : $f(t)=-3e^{-4t}cos(\sqrt{5}t)+\frac{13}{\sqrt{5}}e^{-4t}sin(\sqrt{5}t)$

8. Find $L^{-1}\left[log\frac{s(s+1)}{s^2+4}\right]$

   Ans : $f(t)=\frac{2cos2t-e^{-t}-1}{t}$

9. Find $L^{-1}[cot^{-1}s]$

   Ans : $f(t)=\frac{sint}{t}$

10. Find $L^{-1}\left[\frac{s}{s^2+6s+13}\right]$

    Ans : $f(t)=e^{-3t}\left[cos2t-\frac{3}{2}sin2t\right]$

11. Find $L^{-1}\left[\frac{s}{s^4+s^2+1}\right]$

    Ans : $f(t)=\frac{2}{\sqrt{3}}sin\left(\frac{\sqrt{3}}{2}t\right)sinh\left(\frac{t}{2}\right)$

12. Find $L^{-1}\left[\frac{s^2}{s^4+4a^4}\right]$

    Ans : $f(t)=\frac{sinh(at)cos(at)+cosh(at)sin(at)}{2a}$

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13. Find $L^{-1}\left[log\sqrt{\frac{s^2+1}{s^2+4}}\right]$

Ans : $f(t) = \frac{cos2t - cost}{t}$

14. Find $L^{-1}\left[\frac{1}{s^3(s^2+a^2)}\right]$

Ans : $f(t) = \frac{1}{a^4}\left[\frac{a^2t^2}{2} + cosat - 1\right]$

15. Find $L^{-1}\left[\frac{e^{-s}}{(s-1)(s-2)}\right]$

Ans: $f(t) = [e^{2(t-1)}$-$e^{(t-1)}]u(t-1)$

16. Find $L^{-1}\left[\frac{(1-e^{-s})(2-e^{-2s})}{s^2}\right]$

Ans : $f(t) = t^2 - (t-1)^2u(t-1) - \frac{(t-2)^2u(t-2)}{2} + \frac{(t-3)^2u(t-3)}{2}$

17. Using Convolution Theorem find $L^{-1}\left[\frac{s+2}{(s^2+4s+5)^2}\right]$

Ans : $f(t) = \frac{e^{-2t}tsint}{2}$

18. Using Convolution Theorem find $L^{-1}\left[\frac{s^2}{(s^2+16)(s^2+9)}\right]$

Ans : $f(t) = \frac{4sin4t - 3sin3t}{7}$

19. Solve : $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$ under the conditions $y(0) = 1$; $y'(0) = 1$.

Ans : $y = 2e^{-t} - e^{-2t}$

20. Solve : $y'' + 2y' + 5y = e^{\wedge}\{-t\}sint$ given $y(0) = 0$; $y'(0) = 0$

Ans : $y = \frac{e^{-t}(sint - sin2t)}{3}$

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