Unit-2 class-2

1. $L^{-1}\left[\frac{3s-12}{s^2+8}\right]$

Ans : $3cos2\sqrt{2}t - 3\sqrt{2}sin2\sqrt{2}t$

$$L^{-1}\left[\frac{3s-12}{s^2+8}\right] = L^{-1}\left[\frac{3s}{s^2+8}\right] - L^{-1}\left[\frac{12}{s^2+8}\right]$$

$$= 3 \times \cos 2\sqrt{2}t - \frac{\cancel{12}^{\,3\sqrt{2}}}{\cancel{2\sqrt{2}}} \times \sin 2\sqrt{2}t$$

$$= 3\cos\sqrt{2}t - 3\sqrt{2}\sin 2\sqrt{2}\,t$$

2. $L^{-1}\left[\frac{5s+10}{9s^2-16}\right]$

Ans : $\frac{5}{9}cosh\left(\frac{4}{3}\right)t + \frac{5}{6}sinh\left(\frac{4}{3}\right)t$

$$L^{-1}\left[\frac{5s+10}{9\left(s^2-\frac{16}{9}\right)}\right] = \frac{5}{9}L^{-1}\left[\frac{s}{s^2-\frac{16}{9}}\right] + \frac{10}{9}L^{-1}\left[\frac{1}{s^2-\frac{16}{9}}\right]$$

$$= \frac{5}{9} \times \cosh\frac{4t}{3} + \frac{\cancel{10}^{\,5}}{\cancel{9}_{\,3}} \times \frac{1}{\frac{\cancel{4}\,2}{\cancel{3}}} \times \sinh\frac{4t}{3}$$

$$= \frac{5}{9}\cosh\frac{4t}{3} + \frac{5}{6}\sinh\frac{4t}{3}$$

3. $L^{-1}\left[\frac{3(s^2-2)^2}{2s^5}\right]$

Ans : $\frac{3}{2} - 3t^2 + \frac{t^4}{4}$

$$3L^{-1}\left[\frac{s^4-4s^2+4}{2s^5}\right]$$

$$= \frac{3}{2} \times L^{-1}\left[\frac{1}{s}\right] - 6L^{-1}\left[\frac{1}{s^3}\right] + 6L^{-1}\left[\frac{1}{s^5}\right]$$

$$= \frac{3}{2} - 6\frac{t^2}{\lceil 3} + \frac{6t^4}{\lceil 5}$$

$$= \frac{3}{2} - 3t^2 + \frac{t^4}{4}$$

4. $L^{-1}\left[\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9}\right]$

Ans : $3e^{(3/2)t} - \frac{1}{4}sinh\left(\frac{4}{3}\right)t - \frac{4}{9}cosh\left(\frac{4}{3}\right)t + \frac{2}{3}sin\left(\frac{3}{4}\right)t - \frac{3}{8}cos\left(\frac{3}{4}\right)t$

$$L^{-1}\left[\frac{\cancel{6}\,3}{\cancel{2}(s-3/2)}\right] - L^{-1}\left[\frac{\cancel{3}}{\cancel{9}_{\,3}\left(s^2-\frac{16}{9}\right)}\right] - L^{-1}\left[\frac{4s}{9\left(s^2-\frac{16}{9}\right)}\right] + L^{-1}\left[\frac{\cancel{8}}{\cancel{16}_{\,2}\left(s^2+\frac{9}{16}\right)}\right] - L^{-1}\left[\frac{{}^{3}\cancel{6}s}{\cancel{16}_{\,8}\left(s^2+\frac{9}{16}\right)}\right]$$

$$= 3e^{3t/2} - \frac{1}{\cancel{3}} \times \frac{1}{\frac{4}{\cancel{3}}}\sinh\frac{4t}{3} - \frac{4}{9}\cosh\frac{4t}{3} + \frac{1}{\cancel{2}} \times \frac{1}{\frac{3}{\cancel{4}\,2}} \times \sinh\frac{3t}{4} - \frac{3}{8}\cosh\frac{3t}{4}$$

$$= 3e^{3t/2} - \frac{1}{4}\sinh\frac{4t}{3} - \frac{4}{9}\cosh\frac{4t}{3} + \frac{2}{3}\sinh\frac{3t}{4} - \frac{3}{8}\cosh\frac{3t}{4}$$

5. $L^{-1}\left[\frac{1}{s}sin\left(\frac{1}{s}\right)\right]$ {Use the series expansion of $sinx$}

Ans : $\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n+1}}{[(2n+1)!]^2}$

(Ans given is $\frac{(-1)^{n-1}.t^{2n-1}}{((2n-1)!)^2}$, but it isn't possible at n = 0, hence I've made it +ve)

$$L^{-1}\left[\frac{1}{s} \times \left(\frac{1}{s} - \frac{1}{s^3 \times 3!} + \frac{1}{s^5 \times 5!} - \cdots\right)\right]$$

$$= L^{-1}\left[\frac{1}{s^2} - \frac{1}{3!} \times \frac{1}{s^4} + \frac{1}{5!} \times \frac{1}{s^6} - \cdots\right]$$

$$= \frac{t}{\lceil 2} - \frac{t^3}{3! \times \lceil 4} + \frac{1}{5!} \times \frac{t^5}{\lceil 6} - \cdots$$

$$= \frac{t}{(1!)^2} - \frac{t^3}{(3!)^2} + \frac{t^5}{(5!)^2} - \cdots$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n\, t^{2n+1}}{\left(\lceil 2n+1\right)^2}$$