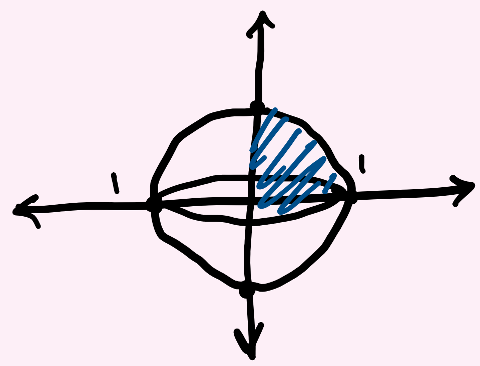
Unit-1 class-9

1. evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical coordinates. Ans: $\frac{\pi^2}{8}$

$$\left.\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\cos\theta\sin\phi \\ z &= r\cos\theta \end{aligned}\right\} J = r^2\sin\theta$$

$$x^2+y^2+z^2 = r^2$$



$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} \frac{r^2\sin\theta\, dr\, d\theta\, d\phi}{\sqrt{1-r^2}}$$

$r = \sin t$

$dr = \cos t dt$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{\pi/2} \frac{\sin^2 t\sin\theta}{\cancel{\cos t}}\cancel{\cos t}\, dt\, d\theta\, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{\pi/2}\left(\frac{1-\cos 2t}{2}\right)\sin\theta\, dt\, d\theta\, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\left(\frac{t}{2}-\frac{\sin 2t}{4}\right)_0^{\pi/2}\sin\theta\, d\theta\, d\phi = \frac{\pi}{4}\times(-\cos\theta)_0^{\pi/2}\times\frac{\pi}{2} = \frac{\pi^2}{8}$$

2. Evaluate $\iiint_V \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$ where V is the volume of the sphere $x^2+y^2+z^2=a^2$ ans: $2\pi a^2$

$r \Rightarrow 0$ to a

$\theta \Rightarrow 0$ to 2π

$\phi \Rightarrow 0$ to 2π

$$\int_0^{2\pi}\int_0^{2\pi}\int_0 \frac{r^2\sin\theta\, dr\, d\theta\, d\phi}{\sqrt{r^2}} = \int_0^{\pi}\int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^a\sin\theta\, d\theta\, d\phi$$

$$= \frac{a^2}{2}\int_0^{2\pi}\left[-\cos\theta\right]_0^{2\pi} d\phi = \frac{a^2}{2}[2]\times 2\pi = 2\pi a^2$$