ENGINEERING MECHANICS

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Friction

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Frictional Forces

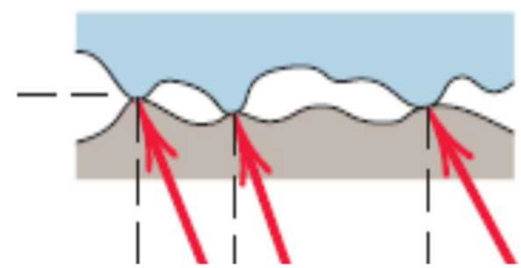
Tangential forces generated between contacting surfaces

- Occur in the interaction between all *real surfaces*
- Always act in a direction *opposite to the direction of motion*

Frictional forces are Desired in some cases:

- Brakes, clutches, belt drives and wedges
- Walking depends on friction between the shoe and the ground

Frictional forces are Not Desired in some cases:



- Bearings, power screws, gears, flow of fluids in pipes, propulsion of aircraft and missiles through the atmosphere, etc
- Friction often results in a loss of energy, which is dissipated in the form of heat
- Friction causes Wear

Types of Friction

Idea case: Friction small enough to be neglected

Real Case: Friction must be taken into account

Dry Friction (Coulomb Friction):

Occurs between unlubricated surfaces of two solids.

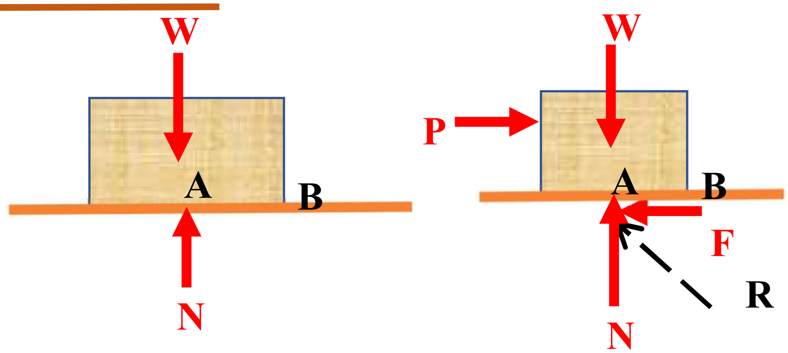
Fluid Friction:

Occurs when adjacent layers in a fluid (liquid or gas) move at a different velocities. Fluid friction also depends on viscosity of the fluid.

Internal Friction:

Occurs in all solid materials subjected to cyclic loading, especially in those materials, which have low limits of elasticity.

Mechanism of Dry Friction

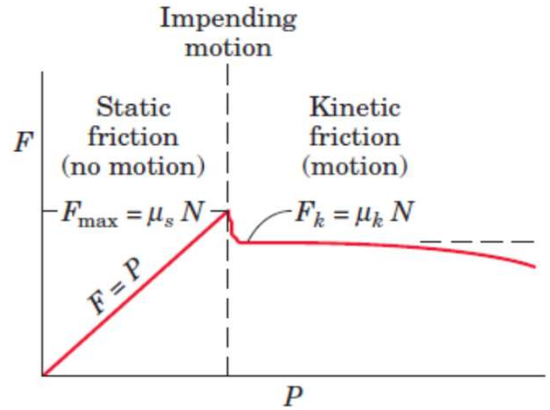


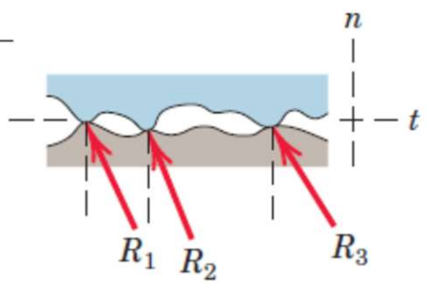
- Block of weight W placed on horizontal surface. Forces acting on block are its weight and reaction of surface N.
- Small horizontal force P applied to block. For block to remain stationary, in equilibrium, a horizontal component F of the surface reaction is required. *F is a Static-Friction force.*
- As P increases, static-friction force F increases as well until it reaches a maximum value F_m.

$$F_m = \mu_s N$$

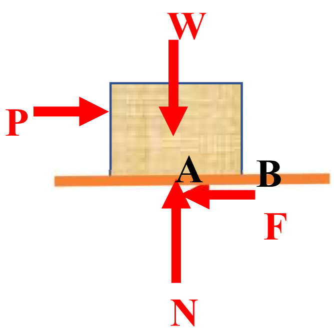
- Further increase in P causes the block to begin to move as F drops to a smaller Kinetic-Friction force F_k.

$$F_k = \mu_k N$$



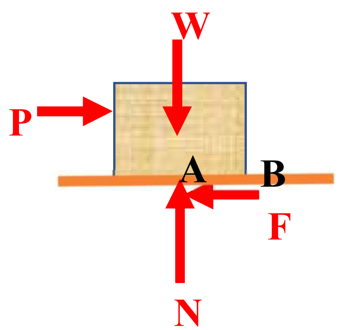


Four possible situations for a rigid body in contact with a horizontal surface:



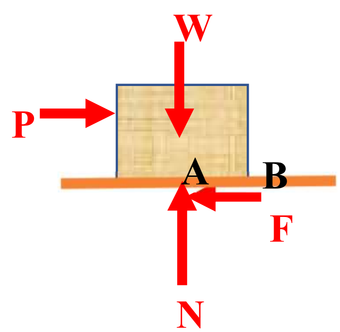
$F = 0$

(i) When P= 0
- No friction
- Equations of equilibrium valid



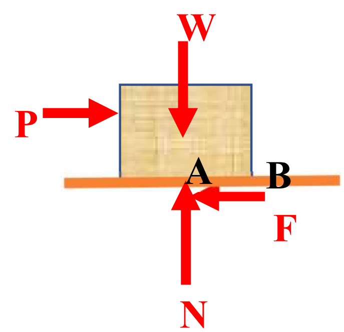
$F < \mu_s N$

(ii) When P $<F_m$
- No motion
- Equations of equilibrium valid



$F_s = \mu_s N$

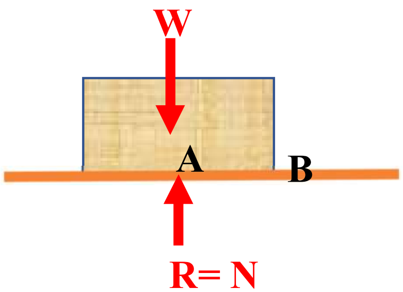
(iii) When P = F_m
- Impending motion
- Equations of equilibrium valid



$F_k = \mu_k N$

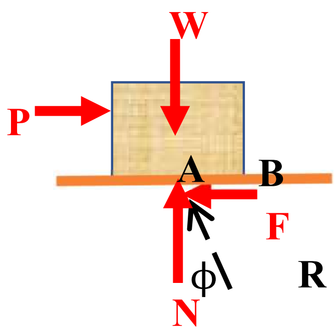
(iv) When P$>F_m$
- Motion
- Equations of equilibrium not valid

Four possible situations for a rigid body in contact with a horizontal surface:



(i) When P= 0

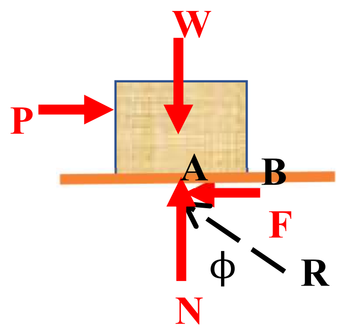
- No friction



$\boxed{\phi < \phi_s}$

(ii) When P $< F_m$

- No motion

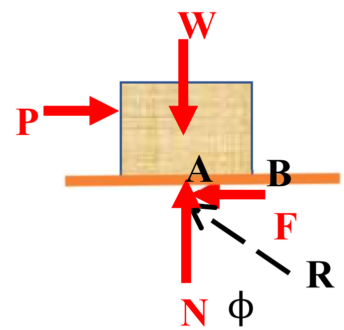


$\boxed{\phi = \phi_s}$

(iii) When P = F_m

Impending motion

$$\tan \phi_s = \frac{F_m}{N} = \mu_s$$



$\boxed{\phi = \phi_k}$

(iv) When P$>F_m$

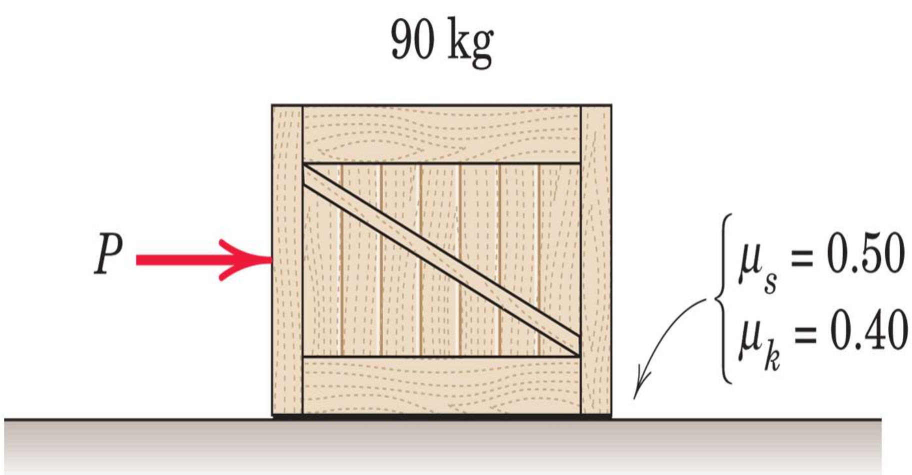
Motion

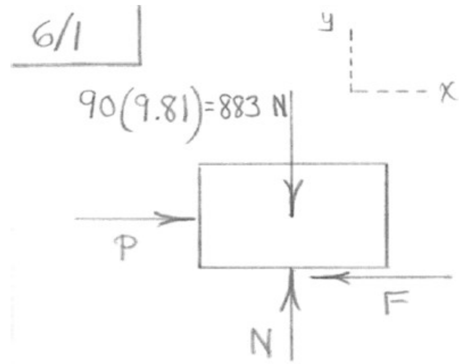
$$\tan \phi_k = \frac{F_k}{N} = \mu_k$$

ϕ_s= angle of static friction,

ϕ_k = angle of kinetic friction

6/1 The force P is applied to the 90-kg crate, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the crate if (a) P = 300 N, (b) P = 400 N, and (c) P = 500 N.





$\Sigma F_y = 0: \quad N - 883 = 0, \quad N = 883\ N$

$F_{max} = \mu_s N = 0.5(883) = 441\ N$

$\Sigma F_x = 0$ yields $F = P$ for equilibrium

(a) $P = 300\ N$, $\underline{F = 300\ N} < F_{max}$, OK

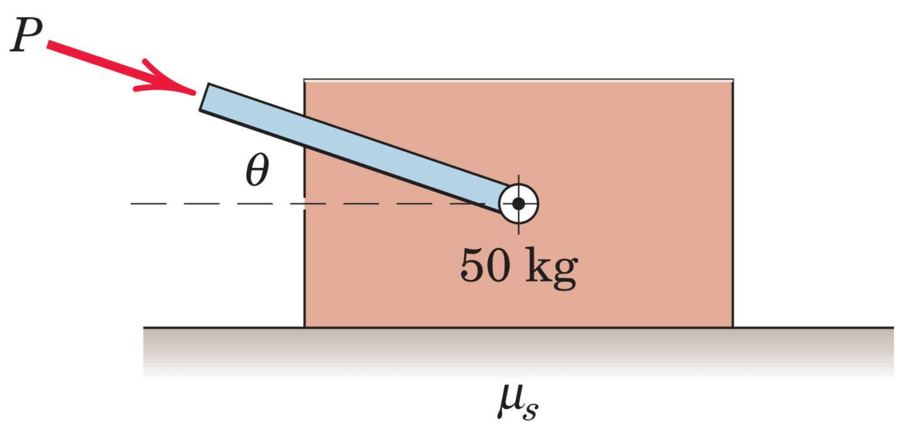
(b) $P = 400\ N$, $\underline{F = 400\ N} < F_{max}$, OK

(c) $P = 500\ N$, $F = 500\ N > F_{max}$, motion

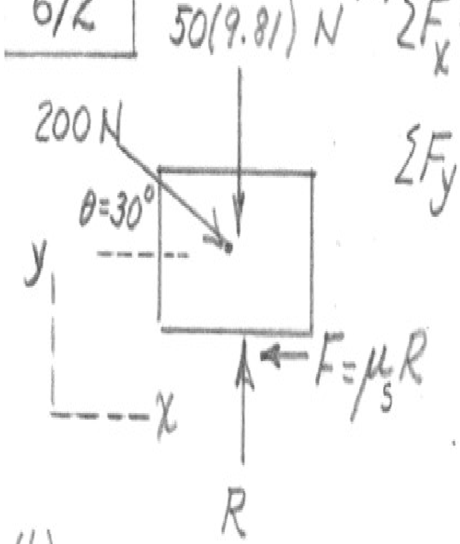
So $F = \mu_k N = 0.4(883) = \underline{353\ N}$

(all to the left)

6/2 The 50-kg block rests on the horizontal surface, and a force P = 200 N, whose direction can be varied, is applied to the block. (a) If the block begins to slip when θ is reduced to 30^0, calculate the coefficient of static friction μs. between the block and the surface. (b) If P is applied with θ = 45°, calculate the friction force F.



6/2



(a) $\Sigma F_x = 0;\ 200\cos 30^\circ - \mu_s R = 0$

$\Sigma F_y = 0;\ R - 200\sin 30^\circ - 50(9.81) = 0$

$R = 590.5$ N

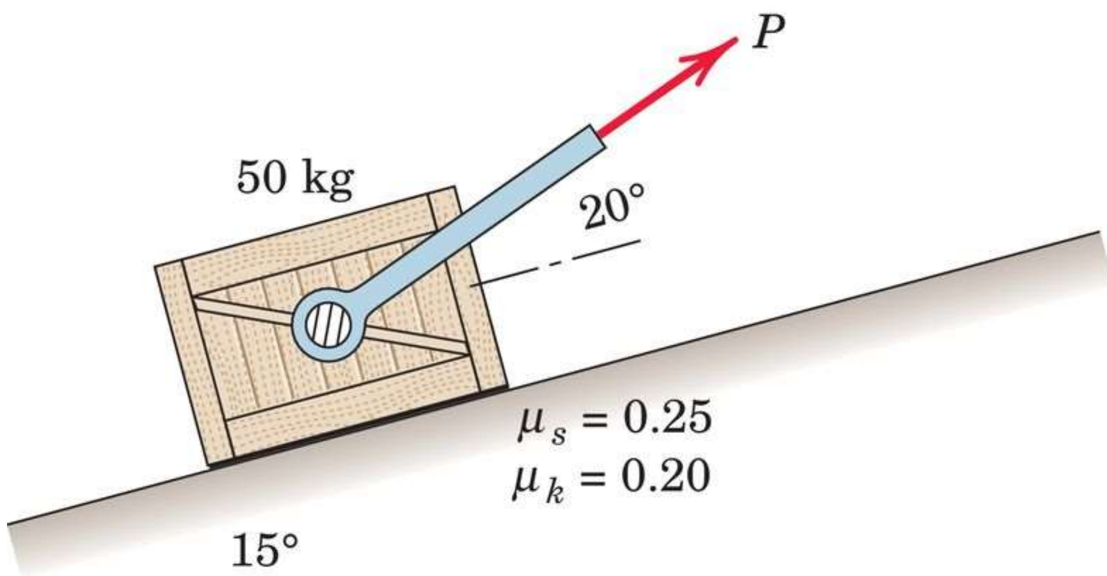
So $\mu_s = \dfrac{200\cos 30^\circ}{590.5} = \underline{0.293}$

(b)

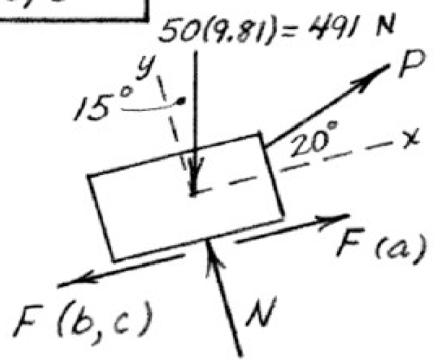
For $\theta = 45^\circ$, $\Sigma F_x = 0$ gives $F = 200\cos 45^\circ = \underline{141.4\ N}$

which is $< \mu_s R_b$

6/3 The force P is applied to the 50-kg block when it is at rest. Determine the magnitude and direction of the friction force exerted by the surface on the block if (a) P = 0, (b) P = 200 N, and (c) P = 250 N. (d) What value of P is required to initiate motion up the incline? The coefficients of static and kinetic friction between the block and the incline are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively.



6/3



(a) $P = 0$

$\Sigma F_y = 0$: $N - 491\cos 15° = 0$, $N = 474$ N

Assume equilibrium:

$\Sigma F_x = 0$: $F - 491 \sin 15° = 0$, $F = 127.0$ N

$F_{max} = \mu_s N = 0.25(474) = 118.4$ N $< F$;

assumption invalid and

$F = F_k = \mu_k N = 0.2(474) = \underline{94.8 \text{ N}}$ up the incline

(b) $P = 200$ N; assume equilibrium

$\Sigma F_y = 0$: $N - 491\cos 15° + 200 \sin 20° = 0$, $N = 405$ N

$\Sigma F_x = 0$: $200\cos 20° - 491 \sin 15° - F = 0$, $\underline{F = 61.0 \text{ N}}$

$F_{max} = \mu_s N = 0.25(405) = 101.3$ N > 61.0 N so assumption OK

(c) $P = 250$ N; assume equilibrium

$\Sigma F_y = 0$: $N - 491\cos 15° + 250\sin 20° = 0$, $N = 388$ N

$\Sigma F_x = 0$: $250\cos 20° - 491\sin 15° - F = 0$, $F = 108.0$ N

$F_{max} = \mu_s N = 0.25(388) = 97.1$ N $< F$; assumption invalid

$F = \mu_k N = 0.2(388) = \underline{77.7 \text{ N}}$ down the incline

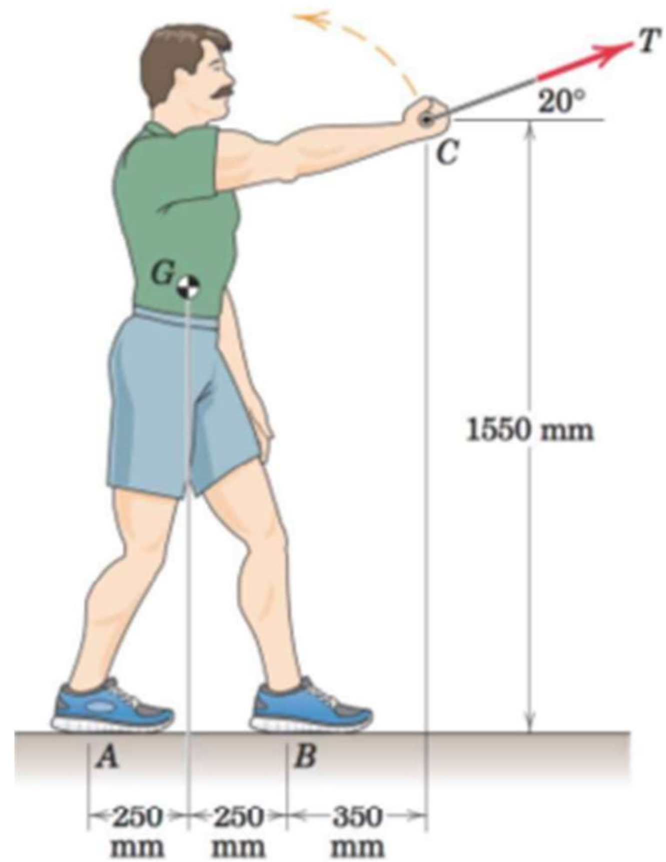
(d) To initiate motion, set $F = \mu_s N = 0.25N$ down the incline:

$\Sigma F_y = 0$: $N - 491\cos 15° + P\sin 20° = 0$

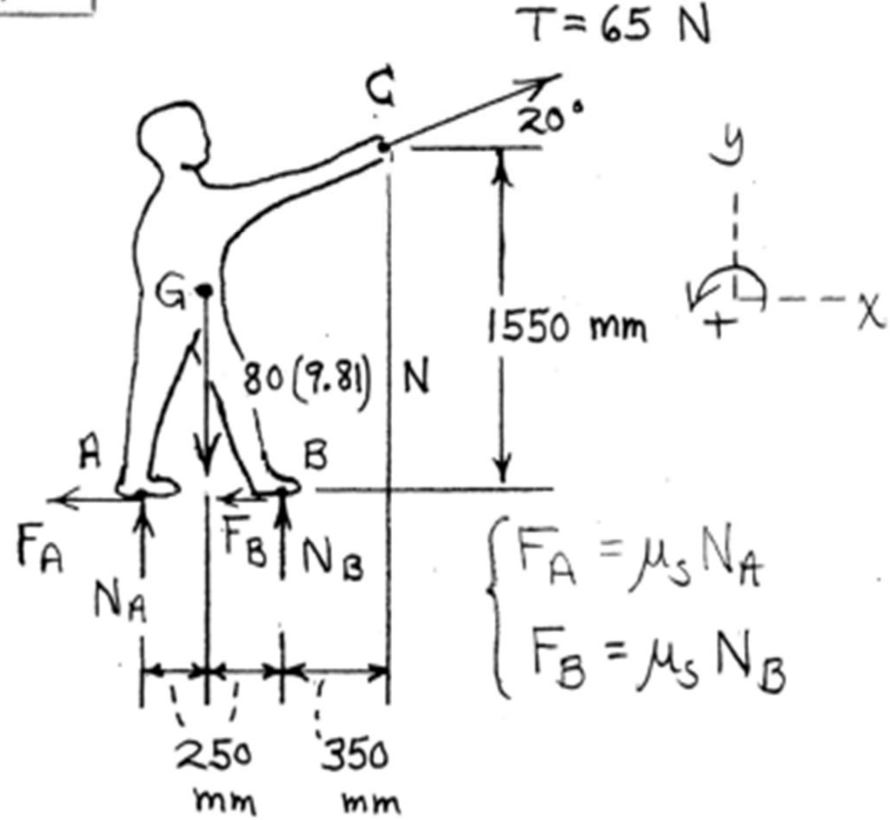
$\Sigma F_x = 0$: $P\cos 20° - 491\sin 15° - 0.25N = 0$

Solve to obtain $\begin{cases} N = 392 \text{ N} \\ \underline{P = 239 \text{ N}} \end{cases}$

6/5 The 80 kg exerciser is repeated from Prob. 3/23. The tension T = 65 N is developed against an exercise machine (not shown) as he is about to begin a biceps curl. Determine the minimum coefficient of static friction which must exit between his shoes and the floor if he is not to slip.



6/5



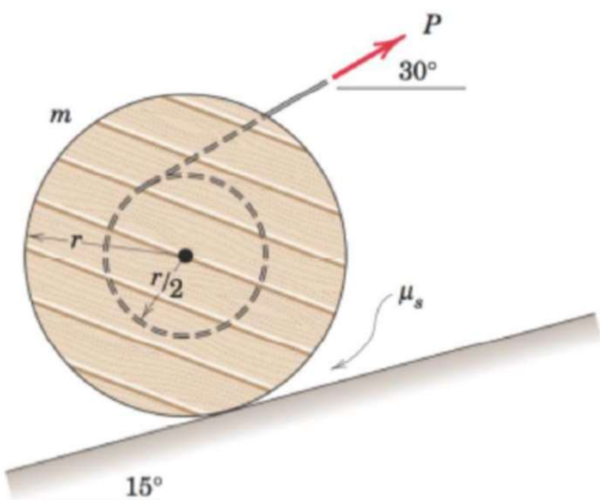
$$\begin{cases} F_A = \mu_s N_A \\ F_B = \mu_s N_B \end{cases}$$

$$\begin{cases} \Sigma F_x = 0: \ -\mu_s(N_A + N_B) + 65\cos 20° = 0 \\ \Sigma F_y = 0: \ N_A + N_B - 80(9.81) + 65\sin 20° = 0 \\ \Sigma M_B = 0: \ 80(9.81)(250) - N_A(500) - 65[1550\cos 20° \\ \qquad - 350\sin 20°] = 0 \end{cases}$$

Solve to obtain $N_A = 219$ N, $N_B = 544$ N

$\underline{\mu_s = 0.0801}$

6/6 Determine the minimum coefficient of static friction μ, which will allow the drum with fixed inner hub to be rolled up the 150 inclined at a steady speed without slipping. What are the corresponding values of the force P and the friction force F?



$$\Sigma F_x = 0: \quad P\cos 15^\circ + F - mg\sin 15^\circ = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad P\sin 15^\circ - mg\cos 15^\circ + N = 0 \qquad (2)$$

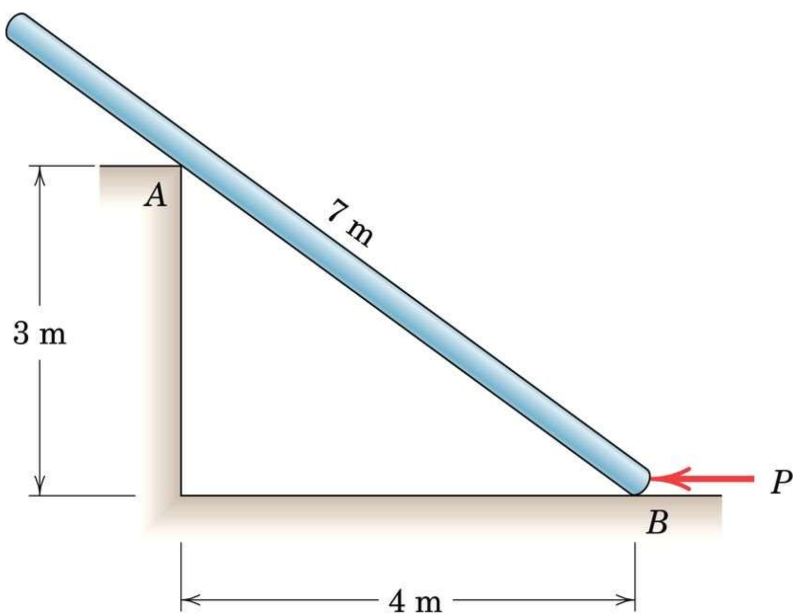
$$\Sigma M_G = 0: \quad Fr - P\left(\frac{r}{2}\right) = 0 \qquad (3)$$

Also, for impending slip: $F = \mu_s N$ (4)

Algebraically solve Eqs. (1)-(4) to obtain

$\underline{\mu_s = 0.0959}$, $N = 0.920mg$, $\underline{F = 0.0883mg}$, $\underline{P = 0.1766mg}$

6/9 The uniform 7-m pole has a mass of 100 kg and is supported as shown. Calculate the force P required to move the pole if the coefficient of static friction for each contact location is 0.40.



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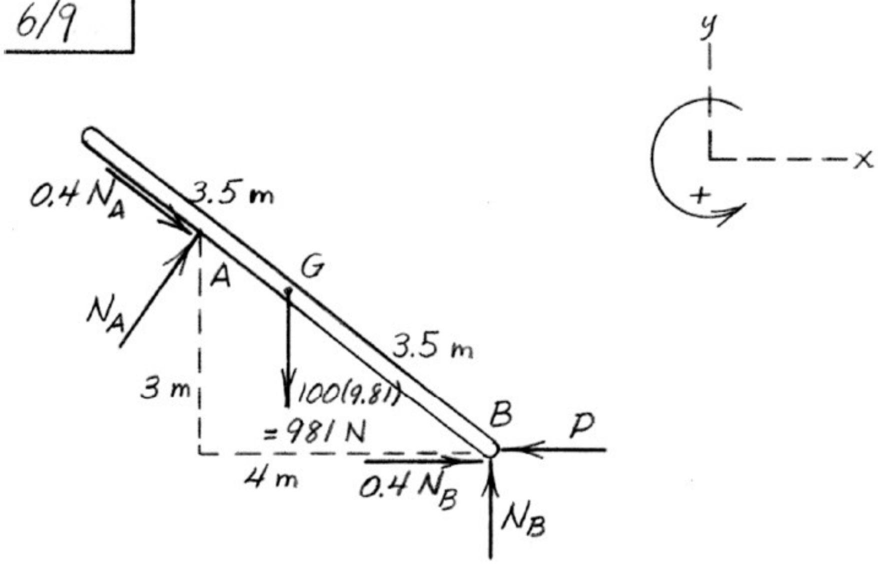


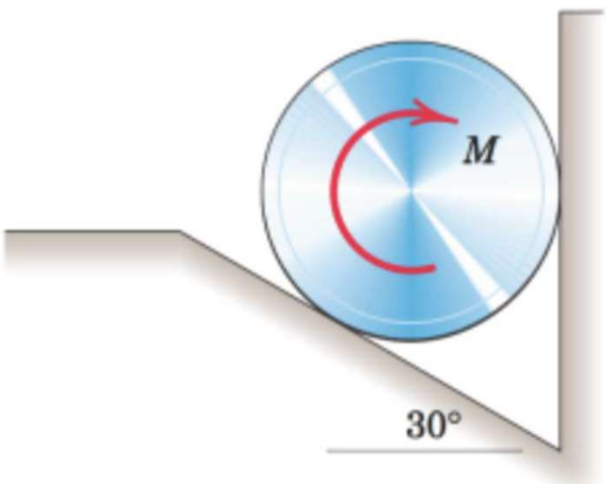
$\Sigma M_B = 0: \; 981\left(\frac{4}{5}3.5\right) - 5N_A = 0, \quad N_A = 549 \text{ N}$

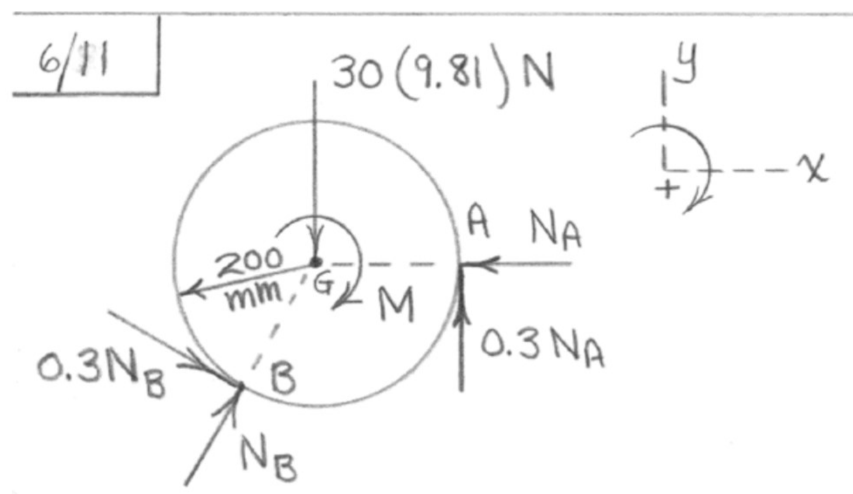
$\Sigma F_y = 0: \; N_B - 981 + \frac{4}{5}(549) - 0.4(549)\frac{3}{5} = 0, \quad N_B = 673 \text{ N}$

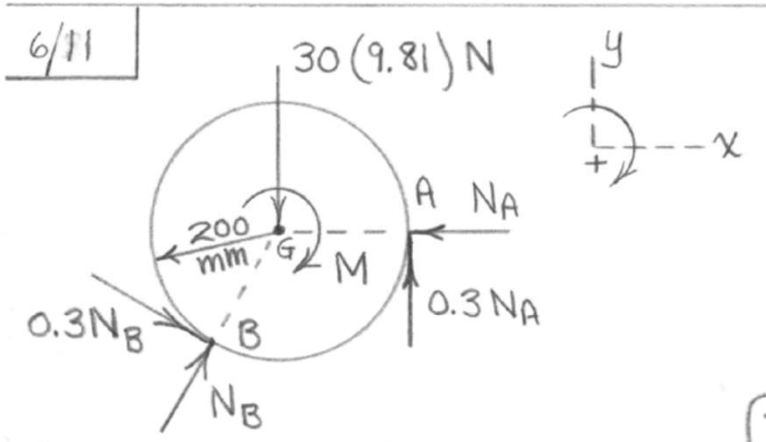
$\Sigma F_x = 0: \; -P + 0.4(673) + 549\left(\frac{3}{5}\right) + 0.4(549)\frac{4}{5} = 0$

$\underline{P = 775 \text{ N}}$

6/11 The 30-kg homogeneous cylinder of 400-mm diameter rests against the vertical and inclined surfaces as shown. If the coefficient of static friction between the cylinder and the surfaces is 0.30, calculate the applied clockwise couple M which would cause the cylinder to slip.



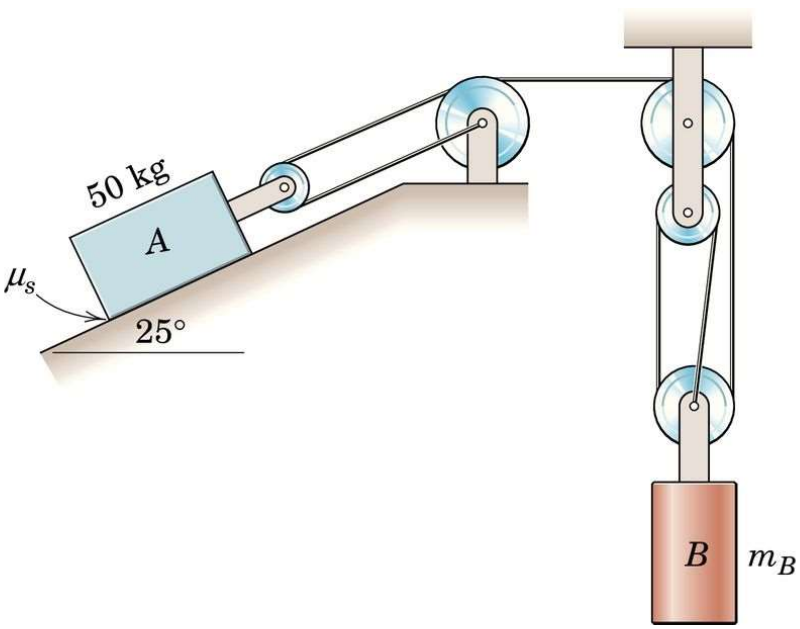




$$\begin{cases} \Sigma M_G = 0: & M - 0.3\,(N_A + N_B)\,0.2 = 0 & (1) \\ \Sigma F_x = 0: & N_B \sin 30^\circ + 0.3\,N_B \cos 30^\circ - N_A = 0 & (2) \\ \Sigma F_y = 0: & N_B \cos 30^\circ - 0.3\,N_B \sin 30^\circ - 30(9.81) \\ & \quad + 0.3\,N_A = 0 & (3) \end{cases}$$

Solution of Eqs. (1)–(3): $\begin{cases} N_B = 312\text{ N} \\ N_A = 237\text{ N} \\ M = 32.9\text{ N·m} \end{cases}$

6/12 If the coefficient of static friction between block A and the incline is $\mu_s = 0.30$, determine the range of cylinder masses m_B for which the system will remain in equilibrium. Neglect all pulley friction



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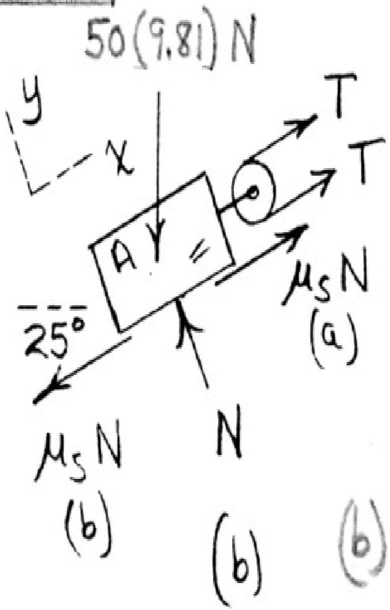


(a) Motion impends down incline

$\Sigma F_y = 0: \; N = 50(9.81)\cos 25°$

or $N = 445$ N throughout

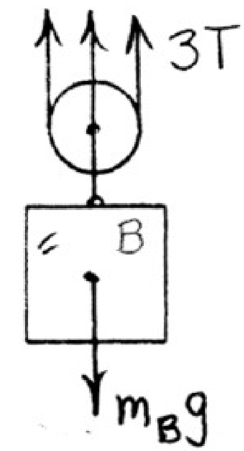
$\Sigma F_x = 0: \; 2T - 50(9.81)\sin 25°$

$+ 0.30(445) = 0, \quad T = 37.0$ N

(b) Motion impends up incline

$$\sum F_x = 0: \quad 2T - 50(9.81)\sin 25^\circ - 0.30(445) = 0$$

$$T = 170.3 \text{ N}$$



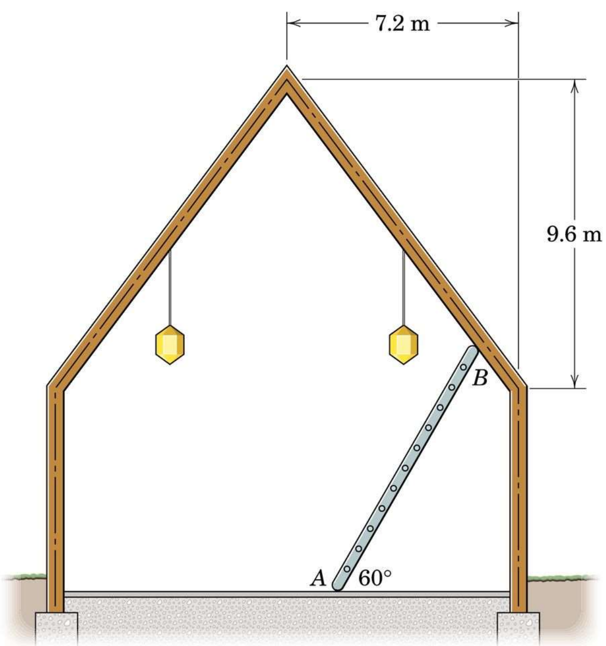
$$\uparrow \sum F = 0 \quad 3T - m_B g = 0, \quad m_B = \frac{3T}{g}$$

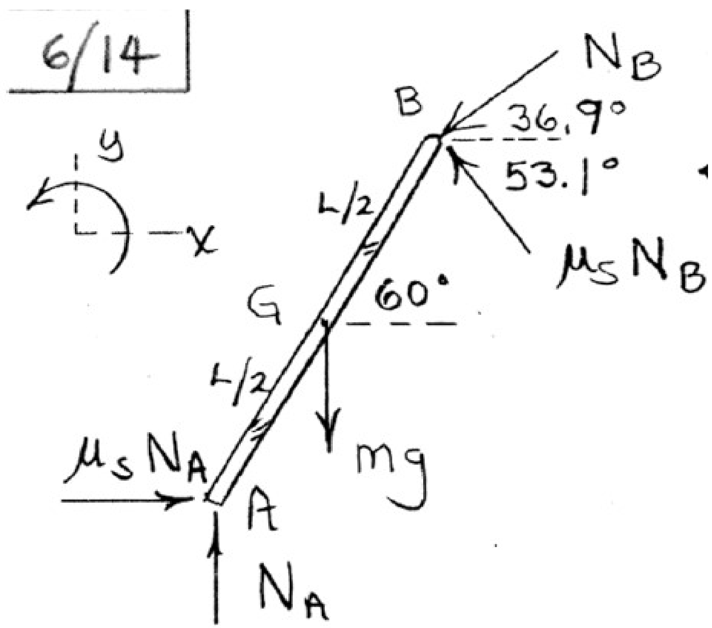
(a) $m_B = 3(37.0)/9.81 = 11.30 \text{ kg}$

(b) $m_B = 3(170.3)/9.81 = 52.1 \text{ kg}$

So $\underline{11.30 \le m_B \le 52.1 \text{ kg}}$

6/14 A uniform ladder is positioned as shown for the purpose of maintaining the light fixture suspended from the cathedral ceiling. Determine the minimum coefficient of static friction required at ends A and B to prevent slipping. Assume that the coefficient is the same at A and B.





(1) $F = \mu_s N$ at both A and B

(2) $\tan^{-1}\left(\frac{9.6}{7.2}\right) = 53.1°$

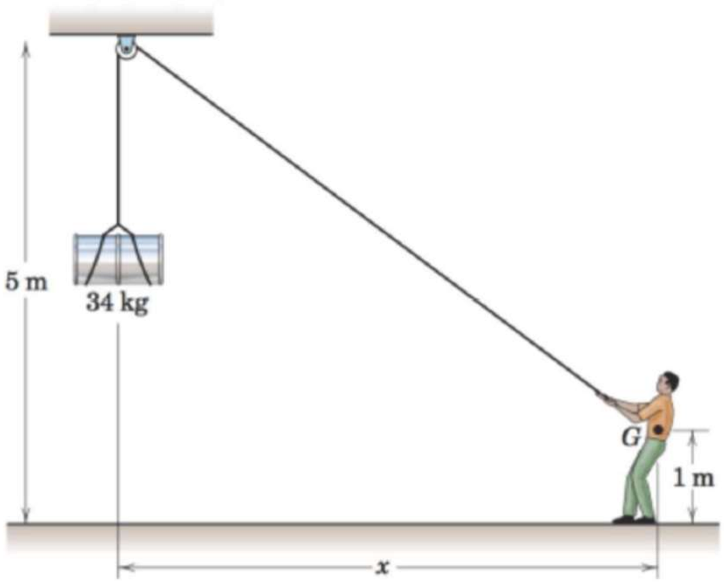
9.6

7.2

$$\begin{cases} \Sigma F_x = 0: \ \mu_s N_A - N_B \cos 36.9^\circ - \mu_s N_B \cos 53.1^\circ = 0 \\ \Sigma F_y = 0: \ N_A - N_B \sin 36.9^\circ + \mu_s N_B \sin 53.1^\circ - mg = 0 \\ \Sigma M_B = 0: \ mg \frac{L}{2} \cos 60^\circ + \mu_s N_A L \sin 60^\circ \\ \qquad\qquad - N_A L \cos 60^\circ = 0 \end{cases}$$

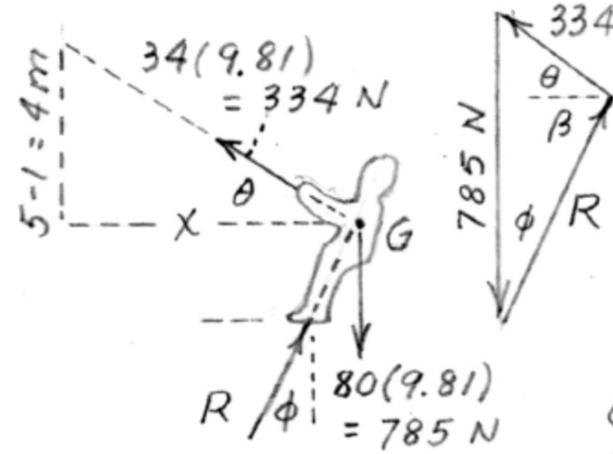
Solve to obtain
$$\begin{cases} N_A = 1.125\,mg \\ N_B = 0.364\,mg \\ \mu_s = 0.321 \end{cases}$$

6/17 The 80 kg man with center of mass G supports the 34 kg drum as shown. Find the greatest distance x at which the man can position himself without slipping if the coefficient of static friction between his shoes and the ground is 0.4.



6/17

$$\phi = \phi_{max} = \tan^{-1} 0.40 = 21.8^\circ$$



$$\beta = 90 - 21.8 = 68.2^\circ$$

Law of sines

$$\frac{785}{\sin(\theta+\beta)} = \frac{334}{\sin 21.8}$$

$$\theta + \beta = \sin^{-1} \frac{785 \sin 21.8}{334}$$

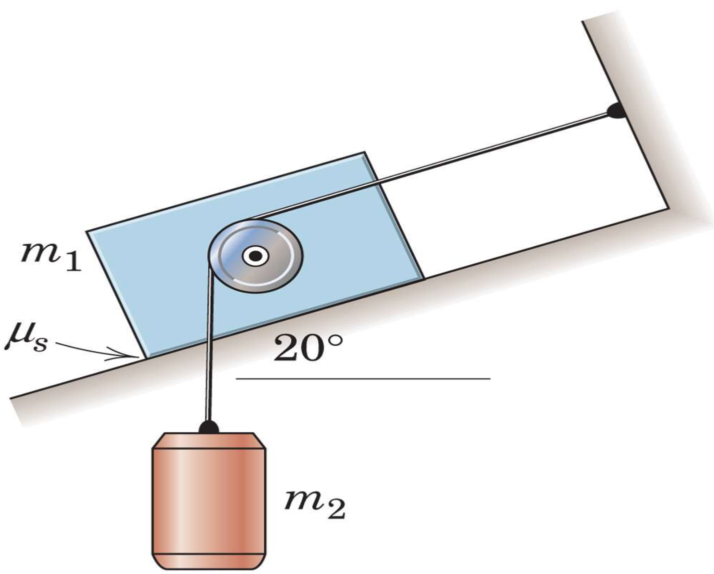
$$= 60.9^\circ \text{ or } 119.1^\circ$$

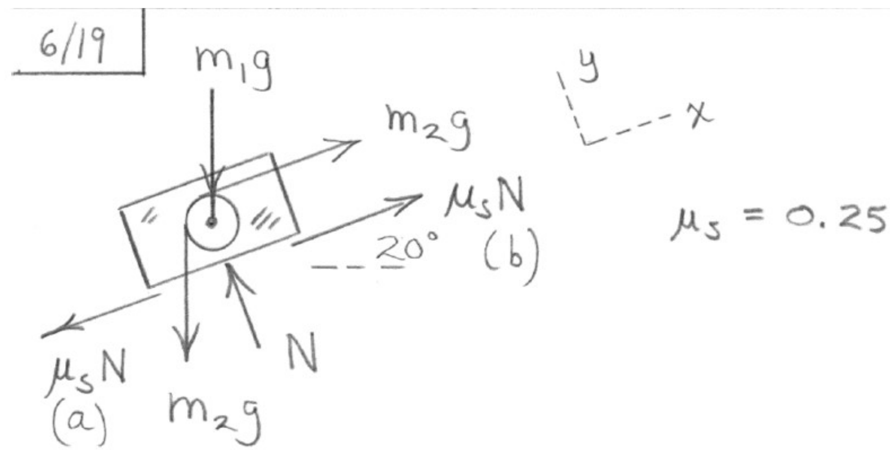
60.9° sol. not possible

So $\theta = 119.1 - 68.2 = 50.9^\circ$

So $\frac{4}{x} = \tan 50.9^\circ$, $\underline{x = 3.25 \text{ m}}$

6/19 Determine the range of mass m_2 for which the system is in equilibrium. The coefficient of static friction between the block and the incline is μ_s= 0.25.Neglect friction associated with the pulley.





(a) Motion impends up the incline

$$\Sigma F_x = 0: -\mu_s N - m_1 g \sin 20° - m_2 g \sin 20° + m_2 g = 0$$

$$\Sigma F_y = 0: \quad N - m_1 g \cos 20° - m_2 g \cos 20° = 0$$

Solving, $m_2 = 1.364 m_1$

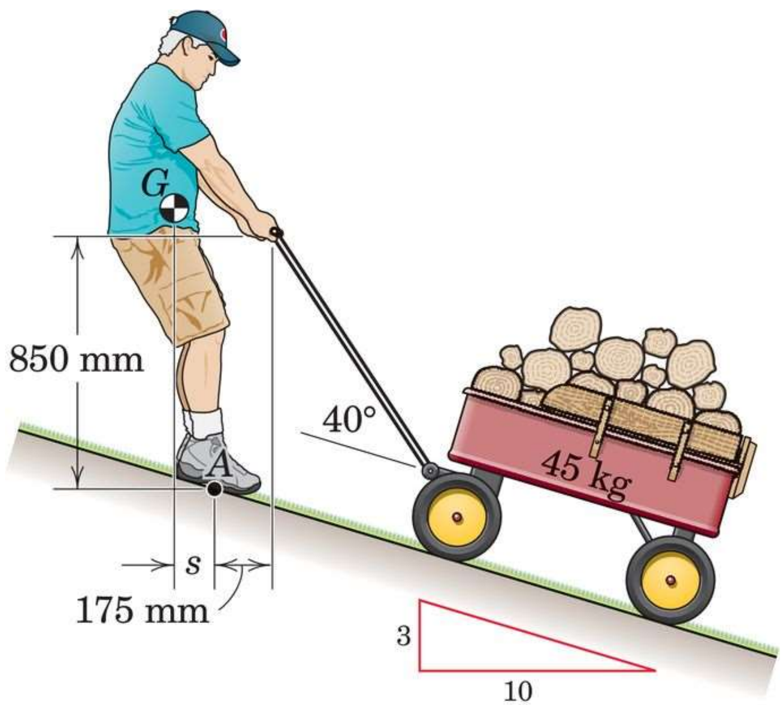
(b) Motion impends down the incline:

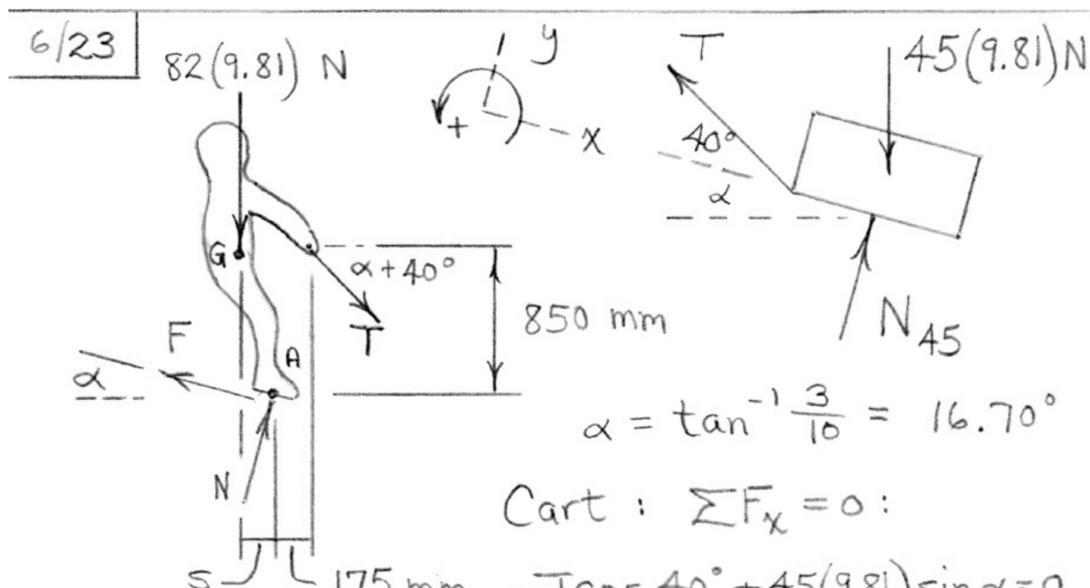
$$\begin{cases} \Sigma F_x = 0: & \mu_s N - (m_1 + m_2) g \sin 20° + m_2 g = 0 \\ \Sigma F_y = 0: & \text{(Does not change)} \end{cases}$$

Solving, $m_2 = 0.1199\, m_1$

So $\underline{0.1199\, m_1 \leq m_2 \leq 1.364\, m_1}$

6/23 A 82-kg man pulls the 45-kg cart up the incline at steady speed. Determine the minimum coefficient μ_s of static friction for which the man's shoes will not slip. Also determine the distance s required for equilibrium of the man's body.





$\alpha = \tan^{-1}\frac{3}{10} = 16.70°$

Cart : $\Sigma F_x = 0$:

$-T\cos 40° + 45(9.81)\sin\alpha = 0$

$T = 165.6\ \text{N}$

Man:

$$\sum F_x = 0: \; -F + 82(9.81)\sin\alpha + 165.6\cos 40^\circ = 0$$

$$F = 358 \text{ N}$$

$$\sum F_y = 0: \; N - 82(9.81)\cos\alpha - 165.6\sin 40^\circ = 0$$

$$N = 877 \text{ N}$$

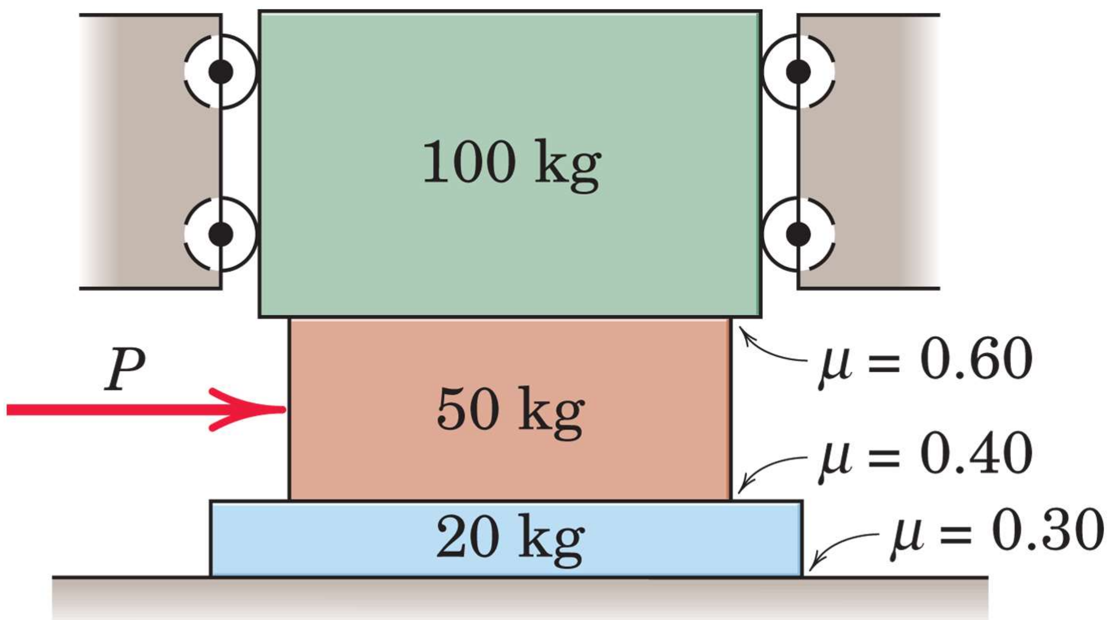
$$\mu_s = \frac{F}{N} = \frac{358}{877} = \underline{0.408}$$

$$\sum M_A = 0: \; 82(9.81)s - 165.6\cos(\alpha + 40)(850)$$

$$- 165.6\sin(\alpha + 40)(175) = 0$$

$$s = 126.2 \text{ mm}$$

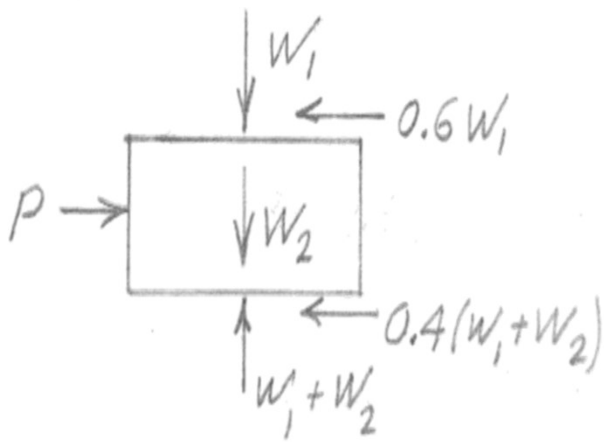
6/24 Determine the horizontal force P required to cause slippage to occur. The friction coefficients for the three pairs of mating surfaces are indicated. The top block is free to move vertically .



ENGINEERING MECHANICS
Friction

6/24 There are two possibilities

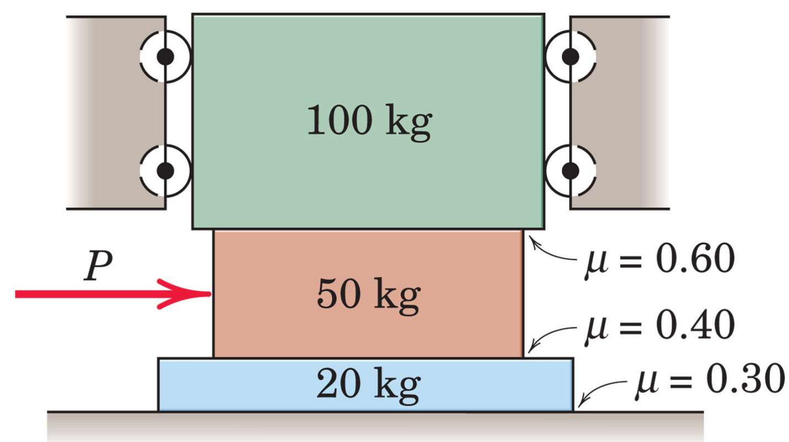
(a) Middle block moves; bottom one does not



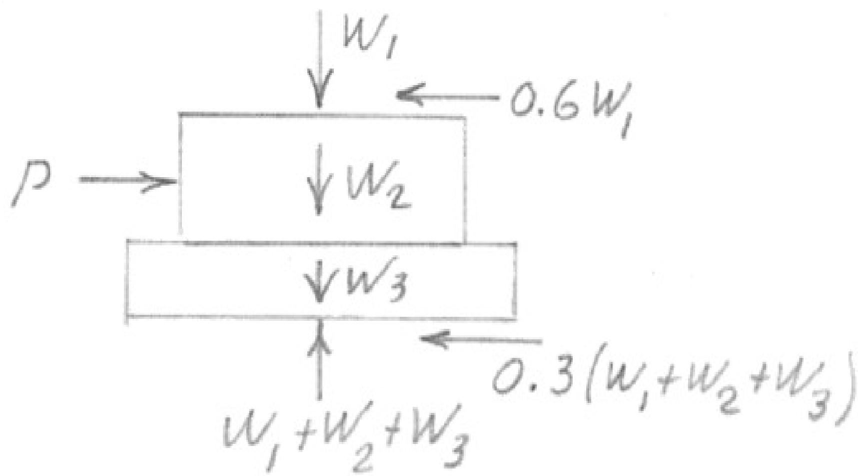
$W_1 = 100(9.81) = 981$ N

$W_2 = 50(9.81) = 490.5$ N

$\Sigma F = 0$; $P = 0.6(981) + 0.4(981 + 490.5) = 1177$ N



(b) Bottom block moves with middle block

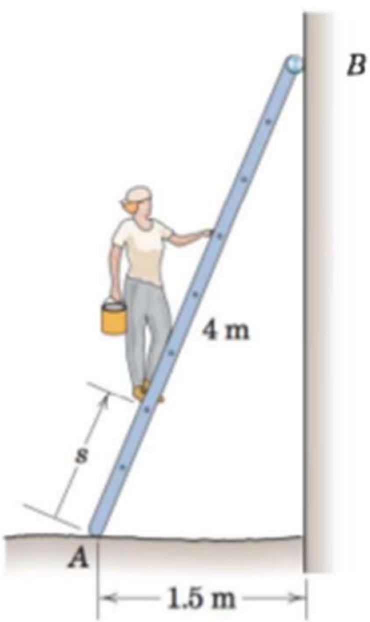


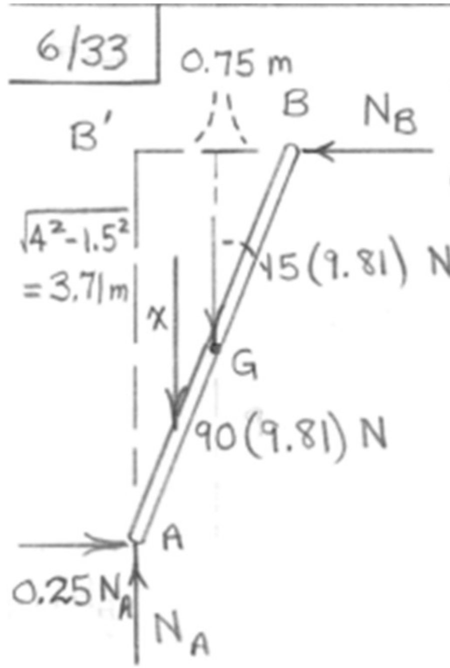
$W_3 = 20(9.81) = 196.2\ N$

$\Sigma F = 0;\ P = 0.6(981) + 0.3(981 + 490.5 + 196.2) = 1089\ N$

$1088 < 1177$ so case (b) occurs & $\underline{P = 1089\ N}$

6/33 Determine the distance s to which the 90 kg painter can climb without causing the 4 m ladder to slip at its lower end A. The top of the 15 kg ladder has a small roller, and at the ground the coefficient of static friction is 0.25. The mass center of the painter is directly above her feet.





$+\uparrow\Sigma F = 0 \Rightarrow N_A = (90+15)9.81 = 1030 \text{ N}$

$\circlearrowleft + \Sigma M_{B'} = 0 : 0.25 N_A (3.71) - 90(9.81)x - 15(9.81)(0.75) = 0, \quad x = 0.957 \text{ m}$

$\dfrac{s}{x} = \dfrac{4}{1.5}, \quad \underline{s = 2.55 \text{ m}}$

THANK YOU

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