

Class 8 – Legendre’s Linear Differential Equations

Solve the following Legendre’s Homogeneous Linear Equations :

$$1. (x + 1)^2 y'' - 2(x + 1)y' - 10y = 3x$$

$$\text{Answer : } y = a(x + 1)^5 + \frac{b}{(x+1)^2} - 3 \left[\frac{1+x}{12} - \frac{1}{10} \right]$$

$$2. (5 + 2x)^2 y'' - 6(5 + 2x)y' + 8y = \log(5 + 2x)$$

$$\text{Answer : } y = e^{2t} (c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t}) + \frac{t}{2} + 1 \text{ where } t = \log(5 + 2x)$$

$$3. (1 + x)^2 y'' + (1 + x)y' + y = 4 \cos(\log(1 + x))$$

$$\text{Answer : } y = a \cos t + b \sin t + 2t \sin t \text{ where } t = \log(1 + x)$$

$$4. (1 + 2x)^2 y'' - 2(1 + 2x)y' - 12y = 6x$$

$$\text{Answer : } y = a(2x + 1)^3 + \frac{b}{2x+1} - \frac{3x}{8} + 1/16$$

$$5. (x + 1)^2 y'' + (x + 1)y' = (2x + 3)(2x + 4)$$

$$\text{Answer : } y = a + bt + e^{2t} + 6e^t + t^2 \text{ where } t = \log(x + 1).$$
