Unit-2 class-1

1. 1. Find $L\left\{\frac{1}{\sqrt{t}}\right\}$ ans: $\sqrt{\frac{\pi}{s}}$

$$L\left[\frac{1}{\sqrt{t}}\right] = L\left[t^{-1/2}\right] = \frac{\sqrt{\frac{1}{2}}}{s^{1/2}} = \sqrt{\frac{\pi}{s}}$$

2. Find $L\{\sin(t)\sin(5t)\}$ ans: $\frac{10s}{(s^2+16)(s^2+36)}$

$$L\left[\frac{1}{2}(\cos 4t - \cos 6t)\right] = \frac{1}{2}\times\left(\frac{s}{s^2+16} - \frac{s}{s^2+36}\right) = \frac{1}{2}\left(\frac{s^3+36s - s^3 - 16s}{(s^2+16)(s^2+36)}\right)$$

$$= \frac{10s}{(s^2+16)(s^2+36)}$$

3. 3. Find $L\{\cosh(at)\}$ ans: $\frac{s}{s^2-a^2}(s>a)$

$$L[\cosh(at)] = L\left[\frac{e^{at}+e^{-at}}{2}\right] = \frac{1}{2}\left(L[e^{at}] + L[e^{-at}]\right)$$

$$= \frac{1}{2}\left(\int_0^\infty e^{-st}\cdot e^{at}\,dt + \int_0^\infty e^{-st}\cdot e^{-at}\,dt\right)$$

$$= \frac{1}{2}\left(\left[\frac{e^{(a-s)t}}{a-s}\right]_0^\infty + \left[\frac{e^{-(a+s)t}}{-(a+s)}\right]_0^\infty\right)$$

$$= \frac{1}{2}\left(\frac{0-a}{(a-s)} - \frac{0-a}{a+s}\right)$$

$$= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right) = \frac{1}{2}\left(\frac{s+a+s-a}{s^2-a^2}\right) = \frac{s}{s^2-a^2}$$

4. 4. Find $L\{e^{2t}+3e^{-t}+5\}$ ans: $\frac{9s^2-10s-10}{s(s+1)(s-2)}$

$$L\{e^{2t}+3e^{-t}+5\} = L\{e^{2t}\} + 3L\{e^{-t}\} + L\{5\}$$

$$= \frac{1}{s-2} + \frac{3}{s+1} + \frac{5}{s} = \frac{(s+1)s + 3s(s-2) + 5(s-2)(s+1)}{(s-2)(s+1)}$$

$$= \frac{s^2+s+3s^2-6s+5s^2-5s-10}{s(s+1)(s-2)} = \frac{9s^2-10s-10}{s(s+1)(s-2)}$$

5. 5. Find $L\{\cos^2(2t-1)\}$ ans: $\frac{1}{2s}+\frac{1}{2}\left(\frac{s\cos(2)+4\sin(2)}{s^2+16}\right)$

$$L\{\cos^2(2t-1)\} = L\left\{\frac{1+\cos(4t-2)}{2}\right\}$$

$$= L\left\{\frac{1}{2}\right\} + \frac{1}{2}\cdot L\{\cos(4t-2)\}$$

$$= \frac{1}{2s} + \frac{L\{\cos 4t\cos 2 + \sin 4t\sin 2\}}{2}$$

$$= \frac{1}{2s} + \frac{1}{2}\left(\frac{s\cdot\cos 2 + 4\sin 2}{s^2+16}\right)$$