1. $e^{2t}\cos^2 t$

$$L[e^{2t}\cos^2 t]$$

$$= L\left[e^{2t}\left(\frac{\cos 2t+1}{2}\right)\right]$$

$$= \frac{1}{2} L\left[e^{2t} + e^{2t}\cos 2t\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-2} + \frac{s-2}{(s-2)^2+4}\right]$$

$$= \frac{1}{2}\left[\frac{s^2+8-4s+s^2+4-4s}{(s-2)(s^2+8-4s)}\right]$$

$$= \frac{s^2-4s+6}{(s-2)(s^2+8-4s)}$$

2. $t^2 e^{-3t}\sin 2t$

$$\frac{d^2}{ds^2}\left[L[e^{-3t}\sin 2t]\right]$$

$$\frac{d^2}{ds^2}\left(\frac{2}{(s+3)^2+4}\right)$$

$$\frac{d^2}{ds^2}\left(\frac{2}{s^2+6s+13}\right)$$

$$\frac{d}{ds}\left(\frac{-2(2s+6)}{(s^2+6s+13)^2}\right) = \frac{d}{ds}\left(\frac{-4s-12}{(s^2+6s+13)^2}\right)$$

$$= \frac{-4(s^2+6s+13)^2 - (-4s-12)\times 2(s^2+6s+13)(2s+6)}{(s^2+6s+13)^4}$$

$$= \frac{-4(s^2+6s+13)+4(2s+6)^2}{(s^2+6s+13)^3}$$

$$= \frac{+4(-s^2-6s-13)+4(4s^2+36+24s)}{(s^2+6s+13)^3}$$

$$= \frac{4(3s^2+18s+23)}{(s^2+6s+13)^3}$$

3. $\sinh(3t)\cos^2 t$

$$L[\sinh(3t)\cos^2 t]$$

$$= L\left[\frac{e^{3t}-e^{-3t}}{2}\times\left(\frac{1+\cos 2t}{2}\right)\right]$$

$$= \frac{1}{4}\left[e^{3t}-e^{-3t}+e^{3t}\cos 2t - e^{-3t}\cos 2t\right]$$

$$= \frac{1}{4}\left[\frac{1}{s-3}-\frac{1}{s+3}+\frac{s-3}{(s-3)^2+4}-\frac{s+3}{(s+3)^2+4}\right]$$

$$= \frac{1}{4}\left[\frac{s+3-s+3}{s^2-9}+\frac{s-3}{s^2-6s+13}-\frac{s+3}{s^2+6s+13}\right]$$

$$= \frac{1}{4}\left[\frac{6}{s^2-9}+\frac{(s-3)(s^2+6s+13)-(s+3)(s^2-6s+13)}{(s^2+13)^2-36s^2}\right]$$

$$= \frac{1}{4}\left[\frac{6}{s^2-9}+\frac{6(s^2-13)}{s^4-10s^2+169}\right]$$

$$= \frac{3}{2}\left[\frac{1}{s^2-9}+\frac{s^2-13}{s^4-10s^2+169}\right]$$

4. $t^2 e^{-3t}\sin(2t)$ ⇒ repeated

5. $t\cos^3 t$

$$L[t\cos^3 t]$$

$$-\frac{d}{ds} L\left[\frac{3\cos t+\cos 3t}{4}\right]$$

$$-\frac{1}{4}\cdot\frac{d}{ds}\left[\frac{3s}{s^2+1}+\frac{s}{s^2+9}\right]$$

$$\frac{-1}{4}\left[\frac{3(s^2+1)-(3s)(2s)}{(s^2+1)^2}+\frac{(s^2+9)-(s)(2s)}{(s^2+9)^2}\right]$$

$$= \frac{-1}{4}\left[\frac{3s^2+3-6s^2}{(s^2+1)^2}+\frac{9-s^2}{(9+s^2)^2}\right]$$

$$= \frac{1}{4}\left[\frac{3(s^2-1)}{(s^2+1)^2}+\frac{s^2-9}{(s^2+9)^2}\right]$$