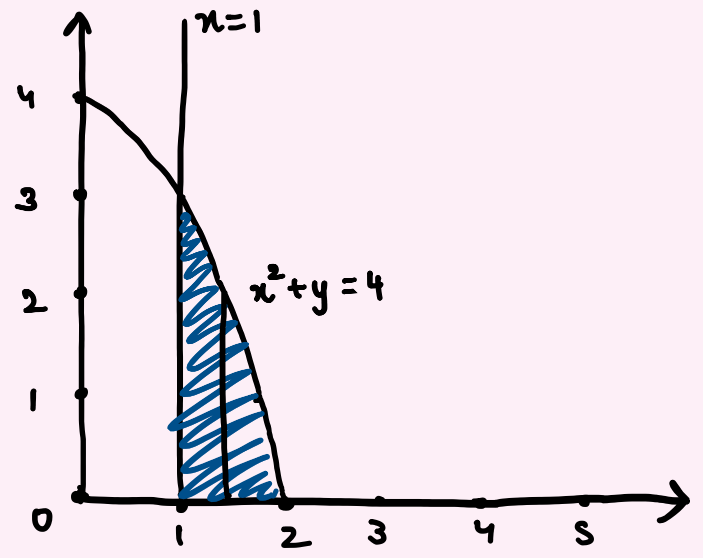
7Unit-1 class 5

1. $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy$ ans: $\frac{241}{60}$

x varies from 1 to $\sqrt{4-y}$

y varies from 0 to 3



$\parallel^{el}$ to y axis,

y varies from 0 to $4-x^2$

x varies from 1 to 2

$$\int_1^2 \int_0^{4-x^2} (x+y)\,dy\,dx = \int_1^2 \left[xy + \frac{y^2}{2}\right]_0^{4-x^2} dx = \int_1^2 \left(x(4-x^2) + \frac{(4-x^2)^2}{2}\right) dx$$

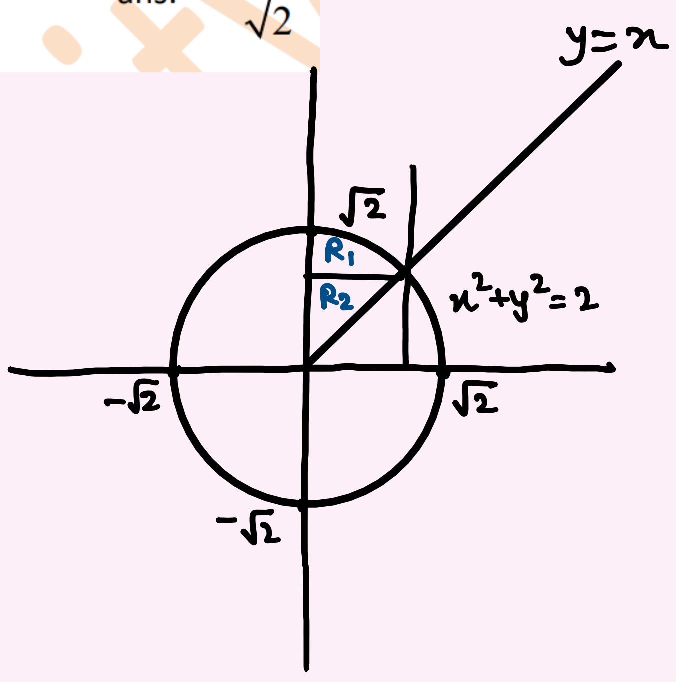
$$= \int_1^2 \left(4x - x^3 + \frac{16 + x^4 - 8x^2}{2}\right) dx = \left[2x^2 - \frac{x^4}{4} + 8x + \frac{x^5}{10} - \frac{4x^3}{3}\right]_1^2$$

$$= \left(8 - 4 + 16 + \frac{16}{5} - \frac{32}{3}\right) - \left(2 - \frac{1}{4} + 8 + \frac{1}{10} - \frac{4}{3}\right) = \frac{241}{60}$$

2. $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ ans: $1-\frac{1}{\sqrt{2}}$

y varies from x to $\sqrt{2-x^2}$

x varies from 0 to 1



$\parallel^{el}$ to x axis,

For R_1,

x varies from 0 to $\sqrt{2-y^2}$

y varies from 1 to $\sqrt{2}$

For R_2,

x varies from 0 to y

y varies from 0 to 1

$$R_1 + R_2 \Rightarrow \int_0^1 \int_0^y \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx + \int_1^{\sqrt{2}} \int_0^{\sqrt{2-y^2}} \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy$$

$$t = x^2 + y^2$$

$$\frac{dt}{2} = x\,dx$$

$x = 0 \Rightarrow t = y^2$ $\qquad$ $x = 0 \Rightarrow t = y^2$

$x = y \Rightarrow t = 2y^2$ $\qquad$ $x = \sqrt{2-y^2} \Rightarrow t = 2$

$$\int_0^1 \int_{y^2}^{2y^2} t^{-1/2} \frac{dt}{2}\,dy + \int_1^{\sqrt{2}} \int_{y^2}^{2} t^{-1/2} \frac{dt}{2}\,dy$$

$$= \int_0^1 \left[\frac{t^{1/2}}{\cancel{\frac{1}{2}}\cdot\cancel{2}}\right]_{y^2}^{2y^2} dy + \int_1^{\sqrt{2}} \left[\frac{t^{1/2}}{\cancel{\frac{1}{2}}\cdot\cancel{2}}\right]_{y^2}^{2} dy$$

$$= \int_0^1 \left(\sqrt{2}y - y\right) dy + \int_1^{\sqrt{2}} \left(\sqrt{2} - y\right) dy$$

$$= (\sqrt{2}-1)\left[\frac{y^2}{2}\right]_0^1 + \left[\sqrt{2}y - \frac{y^2}{2}\right]_1^{\sqrt{2}}$$

$$= \frac{\sqrt{2}-1}{2} + 2 - 1 - \sqrt{2} + \frac{1}{2} = 1 - \frac{1}{\sqrt{2}}$$