QUESTION BANK

1. Expand $f(x)=1-x^2$ as a Fourier series in the interval $-1<x<1$

   Ans: $a_0=\frac{4}{3}, a_n=\frac{4}{\pi^2 n^2}(-1)^{n+1}, 1-x^2=\frac{2}{3}+\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(-1)^{n+1}\cos n\pi x$

2. Find the half range Fourier cosine series for the function $f(x)=x(\pi-x)$ over the interval $(0,\pi)$. Hence deduce that $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}=\frac{\pi^2}{12}$

   Ans: $a_0=\frac{\pi^2}{3}, a_n=\frac{-2}{n^2}\left[1+(-1)^n\right], x(\pi-x)=\frac{\pi^2}{6}-2\sum_{n=1}^{\infty}\frac{1}{n^2}\left[1+(-1)^n\right]\cos nx$, put $x=\frac{\pi}{2}$,

   we get the required series .

3. Expand $f(x)=\begin{cases} x, & in\ 0<x<\pi/2 \\ \pi-x, & in\ \pi/2<x<\pi \end{cases}$ in half range Fourier cosine series.

   Ans: $a_0=\frac{\pi}{2}, a_n=\frac{2}{\pi n^2}\left[2\cos\left(\frac{n\pi}{2}\right)-1-(-1)^n\right]$

4. Find the Half range sine series for $f(x)=\begin{cases} \frac{1}{4}-x & in\ \left(0,\frac{1}{2}\right) \\ x-\frac{3}{4} & in\ \left(\frac{1}{2},1\right) \end{cases}$

   Ans: $b_n=\frac{1}{2n\pi}[1-(-1)^n]-\frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$

5. Find the Half range sine series for $f(x)=(x-1)^2$ in the interval $0\le x\le 1$

   Ans: $b_n=\frac{2}{n\pi}\left[1+\frac{2}{n^2\pi^2}\left\{(-1)^n-1\right\}\right]$

6. Find the Fourier series for the function $f(x)=x-x^2$ in $-\pi\le x\le\pi$.

   Ans: $a_0=-\frac{2\pi^2}{3}, a_n=-\frac{4(-1)^n}{n^2}, b_n=-\frac{2(-1)^n}{n}$

7. Find the Fourier series for the function $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in $0<x<2\pi$ and hence deduce that

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}=\frac{\pi^2}{12}$$

Ans: $a_0=\frac{\pi^2}{6}, a_n=\frac{1}{n^2}, b_n=0,\ put\ \ x=\pi$

8. Find the complex Fourier series for the function $f(x)=\cos 2x$ in the interval $-\pi<x<\pi$

Ans: $\frac{2}{\pi}\sin(2\pi)\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{4-n^2}e^{inx}$

9. Find the complex Fourier series for the function $f(x)=\begin{cases}0, & for\ \ 0<x<l\\ 1, & for\ \ l<x<2l\end{cases}$

Ans: $\frac{1}{2}-\frac{1}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{(2n-1)}\left[e^{\left(\frac{(2n-1)\pi}{l}\right)ix}-e^{-\left(\frac{(2n-1)\pi}{l}\right)ix}\right]$

10. Find the Fourier series up to first harmonic for $f(x)$ *given by the following table:*

x^o	0	60	120	180	240	300	360
f(x)	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Ans: $f(x)=\frac{8.9667}{2}+\left[(4.05)\cos x+(0.8949)\sin x\right]$

11. Find the Fourier series for the function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$

Ans: $a_0=\pi, a_n=\frac{2}{n^2\pi}\left[(-1)^n-1\right],\ b_n=0$

12. Develop $f(x)=e^{-x}$ in Fourier series in the interval $-3<x<3$

$$a_0=\frac{2\,sinh\,3}{3}, a_n=\frac{6\,sinh\,3\,cos\,n\,\pi}{9+n^2\pi^2}, b_n=\frac{2n\pi\,sinh\,3\,cos\,n\,\pi}{9+n^2\pi^2},$$

13. Expand $f(x)=\begin{cases}0 & for-2<x<0\\ 1 & for\ \ 0<x<2\end{cases}$ in Fourier series .

$a_0=1,\quad a_n=0,\quad b_n=\frac{1}{n\pi}\left[1-(-1)^n\right]$

14. Find the Fourier series for the function $f(x) = x - x^2$ in $-1 \le x \le 1$.

Ans: $a_0 = -\frac{2}{3}, a_n = -\frac{4(-1)^n}{n^2\pi^2}, b_n = \frac{2(-1)^{n+1}}{n\pi}$

15. Find the Fourier series for the $f(x) = e^x$ in the interval $0 < x < 2\pi$

$a_0 = \frac{1}{\pi}\left[e^{2\pi} - 1\right], \quad a_n = \frac{1}{\pi}\left[\frac{e^{2\pi} - 1}{1+n^2}\right], b_n = -\frac{n}{\pi}\left[\frac{e^{2\pi} - 1}{1+n^2}\right],$

16. Find the Fourier series for the function $f(x) = |\sin x|$ in $-\pi \le x \le \pi$.

Ans: $a_0 = \frac{4}{\pi}, a_n = -\frac{2}{\pi(n^2-1)}\left[1+(-1)^n\right], b_n = 0$

17. Find the Fourier series for the function $f(x) = x^2$ in $-1 \le x \le 1$.

Ans: $a_0 = \frac{2}{3}, a_n = \frac{4}{n^2\pi^2} \cos n\pi, b_n = 0$

18. Expand $f(x) = x \sin x$ as a Fourier series in the interval $0 < x < 2\pi$

Ans: $a_0 = -2, a_1 = -\frac{1}{2}, a_n = \frac{2}{n^2-1}(n \ne 1), \quad b_n = 0(n \ne 1), b_1 = \pi$

19. Obtain a Fourier series to represents the following periodic function

$$f(x) = \begin{cases} 0 & 0 < x < \pi \\ 1 & \pi < x < 2\pi \end{cases}$$

Ans: $a_0 = 1, a_n = 0, \quad b_n = -\frac{1}{n\pi}\left[1-(-1)^n\right]$

20. Find the Fourier series for the $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$

Ans: $a_0 = \frac{1}{\pi}\left[1 - e^{2\pi}\right], \quad a_n = \frac{1}{\pi}\left[\frac{1-e^{2\pi}}{1+n^2}\right], b_n = \frac{n}{\pi}\left[\frac{1-e^{2\pi}}{1+n^2}\right],$

21. Find the Fourier series for the $f(x) = e^{-ax}$ in the interval $-\pi < x < \pi$

Ans:$a_0 = \frac{2 \sinh a\pi}{\pi a}, a_n = \frac{2a(-1)^n \sinh a\pi}{\pi(n^2+a^2)}, b_n = \frac{2n(-1)^n \sinh a\pi}{\pi(n^2+a^2)}$

22. Express $f(x) = \cos wx$ in $-\pi < x < \pi$ as a Fourier series, where w is constant .

Ans: $a_0 = \frac{2}{\pi w} \sin w\pi, a_n = \frac{(-1)^{n+1} 2w \sin w\pi}{\pi(n^2-w^2)}, b_n = 0$

23. Obtain a Fourier series for the function

$$f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

Ans: $a_0 = \pi,\ a_n = \dfrac{2}{n^2\pi}\left[-1+(-1)^n\right],\ \ b_n = 0$

24. Find the Fourier series for the $f(x) = x$ in the interval $-\pi < x < \pi$. Draw its graph.

Ans: $a_0 = 0,\ a_n = 0,\ \ b_n = -\dfrac{2}{n}\cos n\pi$

25. Express $f(x) = x$ as a Half-range sine series in $0 < x < 2$

Ans: $a_0 = 0,\ a_n = 0,\ \ b_n = \dfrac{4}{n\pi}(-1)^n$, $f(x) = \frac{4}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} sin\frac{n\pi}{2}x$

26. Express $f(x) = x$ as a Half-range cosine series in $0 < x < 2$

Ans: $a_0 = 2,\ a_n = \dfrac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$, $f(x) = 1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos[(2n-1)\frac{\pi x}{2}$

27. Express $f(x) = x^2$ as a Half-range cosine series in $0 < x < \pi$

Ans: $a_0 = \dfrac{2}{3}\pi^2,\ \ a_n = \dfrac{4}{n^2}\left[(-1)^n\right]$

28. Find the Fourier series for the function $f(x) = x^2$ in $-l \le x \le l$.

Ans: $a_0 = \frac{2l^2}{3}, a_n = \frac{4l^2(-1)^n}{n^2\pi^2}, b_n = 0$

29. Find the value of $\sum_{n=1}^{\infty}\dfrac{1}{n^2}$ using Fourier series. [Assume $f(x) = x^2$ in the interval $(-\pi, \pi)$]

Ans: $\dfrac{\pi^2}{6}$

30. Find the value of $\sum_{n=1}^{\infty}\dfrac{1}{4n^2-1}$ using the Fourier series. Given $f(x) = \sqrt{1-\cos x}$ in the interval $0 < x < 2\pi$

Ans: $\sum_{n=1}^{\infty}\dfrac{1}{4n^2-1} = \dfrac{1}{2};\ \ a_0 = \dfrac{4\sqrt{2}}{\pi}, a_n = -\dfrac{4\sqrt{2}}{\pi(4n^2-1)},\ b_n = 0$

31. Find the Half range Fourier cosine series for the function $f(x) = x - x^2$ in $0 < x < 1$.

Ans: $a_0 = \frac{1}{3}, a_n = -\frac{2}{\pi^2 n^2}[(-1)^n + 1]$

32. Obtain a cosine series for $f(x) = \begin{cases} 1 & for\ 0 < x < 1 \\ x & for\ 1 < x < 2 \end{cases}$

Ans: $f(x)=\frac{5}{4}-\frac{4}{\pi^2}\left[\cos\left(\frac{\pi x}{2}\right)-\frac{1}{2}\cos\left(\frac{2\pi x}{2}\right)+\frac{1}{9}\cos\left(\frac{3\pi x}{2}\right)+\ldots\ldots\ldots\ldots\right]$

33. Find the Fourier series for the function $f(x)=2x-x^2$ in $0<x<2$.

Ans: $a_0=\frac{4}{3}, a_n=-\frac{4}{n^2\pi^2}, \; b_n=0$

34. Find the Fourier series up to first harmonic for $f(x)$ *given by the following table:*

x	*0*	*1*	*2*	*3*	*4*	*5*
f(x)	*9*	*18*	*24*	*28*	*26*	*20*

Ans: $f(x)=\frac{41.66}{2}+\left[(-8.33)\cos x+(-1.15)\sin x\right]$

35. Find the Fourier series up to first harmonic for $f(x)$ *given by the following table:*

x^o	*0*	*60*	*120*	*180*	*240*	*300*	*360*
f(x)	*0.8*	*0.6*	*0.4*	*0.7*	*0.9*	*1.1*	*0.8*

Ans: $f(x)=0.75+\left[(0.10)\cos x+(-0.29)\sin x\right]$

36. Find the Fourier series up to first harmonic for $f(x)$ *given by the following table:*

x^o	*0*	*60*	*120*	*180*	*240*	*300*	*360*
f(x)	*7.9*	*7.2*	*3.6*	*0.5*	*0.9*	*6.8*	*7.9*

Ans: $f(x)=\frac{8.9667}{2}+\left[(4.05)\cos x+(0.8949)\sin x\right]$

37. The turning moment T units of a crank shaft of a steam engine are given for a series of values of the crank angle θ I degrees.

θ	*0*	*30*	*60*	*90*	*120*	*150*	*180*
T	*0*	*5224*	*8097*	*7850*	*5499*	*2626*	*0*

Find the first three terms of sine series represent T

Ans: $T = b_1 \sin\theta + b_2 \sin 2\theta + b_3 \sin 3\theta = 7.850 \sin\theta + 1.500 \sin 2\theta + 0$

38. Find the Fourier series for the function $f(x)=x^2-2$ in $-2<x<2$.

Ans: $a_0=-\frac{1}{3}, a_n=\frac{16}{\pi^2 n^2}(-1)^n, \; b_n=0$

39. In the range $(-2,2)$, $f(x)$ is defined by the relation

$$f(x) = \begin{cases} 0, -2 < x < 0 \\ a, 0 < x < 2 \end{cases}$$

Expand $f(x)$ in Fourier series .

Ans: $a_0 = a,\ a_n = 0$, $b_n = \frac{a}{n\pi}[1-(-1)^n]$, $f(x) = a[\frac{1}{2} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\{1-(-1)^n\}\sin(\frac{n\pi x}{2})]$

40. Find the Fourier series for the function $f(x) = 2x - x^2$ in $0 < x < 3$.

Ans: $a_0 = 0,\ a_n = -\dfrac{9}{n^2\pi^2},\ b_n = \dfrac{3}{\pi n}$

Ans: $a_0 = -\dfrac{1}{2},\ a_n = \dfrac{8}{n^2\pi^2}\left[\cos\left(\dfrac{n\pi}{2}\right) - 1\right],\ b_n = 0$

41. Find the half range cosine series for $f(x) = \begin{cases} c & 0 < x < a \\ 0 & a < x < l \end{cases}$

Ans: $a_0 = \dfrac{ac}{l},\ a_n = \dfrac{2c}{n\pi}\sin\left(\dfrac{n\pi a}{l}\right)$

42. Find the value of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$ using Fourier series. Given

$$f(x) = \begin{cases} -1 + x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$$

Ans: $\dfrac{\pi}{4}$

43. Find the Fourier series for the function $f(x) = \begin{cases} -1, & -3 < x < 0 \\ 1, & 0 < x < 3 \end{cases}$

Ans: $\dfrac{\pi}{4}\displaystyle\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)}\sin\left\{(2n-1)\dfrac{\pi x}{3}\right\}$

44. Find the Fourier coefficients and Fourier series of the square-wave function f defined by

$$f(x) = \begin{cases} 0 \ if - \pi \le x < 0, \\ 1 \ if \ 0 \le x < \pi \end{cases} \quad and\ f(x + 2\pi) = f(x),$$

Ans:

$$\frac{1}{2} + \sum_{k=1}^{\infty}\frac{2}{(2k-1)\pi}\sin(2k-1)x$$

45. Find the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$ Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

Ans: $f(x) = \frac{\pi - x}{2} = \sum_{n=1}^{\infty} \frac{1}{n} sinnx$

46. Find the comple form of Fourier series for the function $f(t) = sint$ in $(0, \pi)$

Ans: $\frac{2}{\pi}\left[1 - \frac{e^{2it}+e^{-2it}}{1.3} - \frac{e^{4it}+e^{-4it}}{3.5} - \frac{e^{6it}+e^{-6it}}{15.7-\cdots}\right]$

47. Find the Fourier series of $f(x) = x(2\pi - x)$ in $0 < x < 2\pi$

Ans:

$$f(x) = \frac{4\pi^2}{6} + \sum_{n=1}^{\infty} \frac{-4}{n^2} cosnxx$$

48. Find the Fourier half range a) Cosine series b) Sine series of

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{cases} \boldsymbol{x}, & \mathbf{0} < \boldsymbol{x} < \mathbf{1} \\ \mathbf{2} - \boldsymbol{x}, & 1 < x < \mathbf{2} \end{cases}$$

Ans: $\frac{8}{n^2\pi^2} cos\left(\frac{n\pi}{2}\right) - \frac{4}{n^2\pi^2}[1 + (-1)^n]$, $\sum_{n=1}^{\infty} \frac{8}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \sin\frac{n\pi x}{2}$

Question Bank:

1. Find the Fourier transform of the function, $f(t) = \begin{cases} 0 & for \ -\infty \le t \le a \\ t & for \ a < t \le b \\ 0 & for \ t > b \end{cases}$

Ans: $F(\omega) = \frac{1}{i\omega}\left(ae^{-i\omega a} - be^{-i\omega b}\right) + \frac{1}{\omega^2}\left(e^{-i\omega b} - e^{-i\omega a}\right)$

2. Find Fourier transform of $f(t) = \begin{cases} \sin t & for \ 0 < t < \pi \\ 0 & otherwise \end{cases}$ Ans: $F(\omega) = \frac{\left(1+e^{-i\omega\pi}\right)}{1-\omega^2}$

3. Given $F(e^{-t^2}) = \sqrt{\pi}e^{-\frac{\omega^2}{4}}$, find the Fourier transform of i) $e^{-\frac{t^2}{3}}$ ii) $e^{-4(t-3)^2}$ Ans: $\sqrt{\pi}e^{-3\frac{\omega^2}{4}}$; $\sqrt{\pi}e^{-\omega(3i+\frac{\omega^2}{16})}$

4. Find the Fourier transform of the function,

$f(t) = \begin{cases} a^2 - t^2 & for \ |t| \le a \\ 0 & for \ |t| > a \end{cases}$ Ans: $F(\omega) = \frac{4}{\omega^3}\left(\sin a\omega - a\omega \cos a\omega\right)$

5. Find the Fourier sine and cosine transform of $f(t) = \begin{cases} k & if \ 0 < t < a \\ 0 & otherwise \end{cases}$

Ans: $F_s(\omega) = \frac{k}{\omega}\left(1 - \cos a\omega\right)$; $F_c(\omega) = \frac{k}{\omega}\sin a\omega$

6. Find the Fourier sine transform of $f(t) = \frac{1}{t(1+t^2)}, t > 0.$ Ans: $\sqrt{\frac{\pi}{2}}\left(1 - e^{-\omega}\right)$

7. Find the Fourier sine transform of $f(t) = \frac{1}{t}, t > 0.$ Ans: $\frac{\pi}{2}$

8. Find inverse Fourier sine transform of $\frac{e^{-a\omega}}{a}$ $a > 0$. Ans: $\frac{2}{\pi}\tan^{-1}\left(\frac{\omega}{a}\right)$

9. Find the Fourier cosine transform of $f(x) = e^{-at}, a > 0.$ Hence find $\int_0^\infty \frac{\cos\omega x d\omega}{\left(1+\omega^2\right)}$

Ans: $F_c(\omega) = \frac{a}{a^2+\omega^2}$; $\int_0^\infty \frac{\cos\omega x d\omega}{\left(1+\omega^2\right)} = \frac{\pi}{2a}e^{-ax}$

10. Find the Fourier cosine transform of $f(t) = e^{-at}\cos at.$ Ans: $F_c(\omega) = \frac{a\left(2a^2+\omega^2\right)}{\left(2a^2-\omega^2\right)^2}$

11. Find the Fourier sine and cosine transform of $f(t) = \begin{cases} \cos t & if \quad 0<t<a \\ 0 & otherwise \end{cases}$

Ans: $F_c(\omega) = \dfrac{\{\omega \sin \omega a \cos a - \cos \omega a \sin a\}}{\omega^2 - 1}$

12. Find $f(t)$ satisfying the integral equation, $\int_0^\infty f(t)\cos \omega t dt = e^{-\omega}$ Ans: $f(t) = \dfrac{2}{\pi(1+\omega^2)}$

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Q) $Find\ the\ Fourier\ Inverse\ of\ \frac{3i\omega}{2+i\omega}$

Ans) $3\delta(t) - 6e^{-2t}u(t)$ (Hint : 3 - $\frac{6}{2+i\omega}$)

Q) Find the real and imaginary part of Fourier Transformation of $e^{a|t|}$

Ans) Real part = $\frac{2a}{a^2+w^2}$ and Imaginary part = 0

Q) Find the Fourier transformation of $e^{-3|t|}\sin(2t)$

Hint : Use Modulation Theorem

Q) $\mathcal{F}$ ($\frac{1}{(3t)^2+1}$)

Hint : Use symmetric property

Ans) $\frac{\pi}{3}e^{-\left|\frac{w}{3}\right|}$

Q) Solve y' + 2y = $e^{-t}u(t)$

Ans) $e^{-t}u(t) + e^{-2t}u(t)$

Q) Prove that $\mathcal{F}\,[t^{n-1}e^{-at}u(t)] = \frac{(n-1)!}{(a+iw)^n}$