Course Code : UE23CV131A

ENGINEERING MECHANICS STATICS

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ENGINEERING MECHANICS

Beams – External Effects

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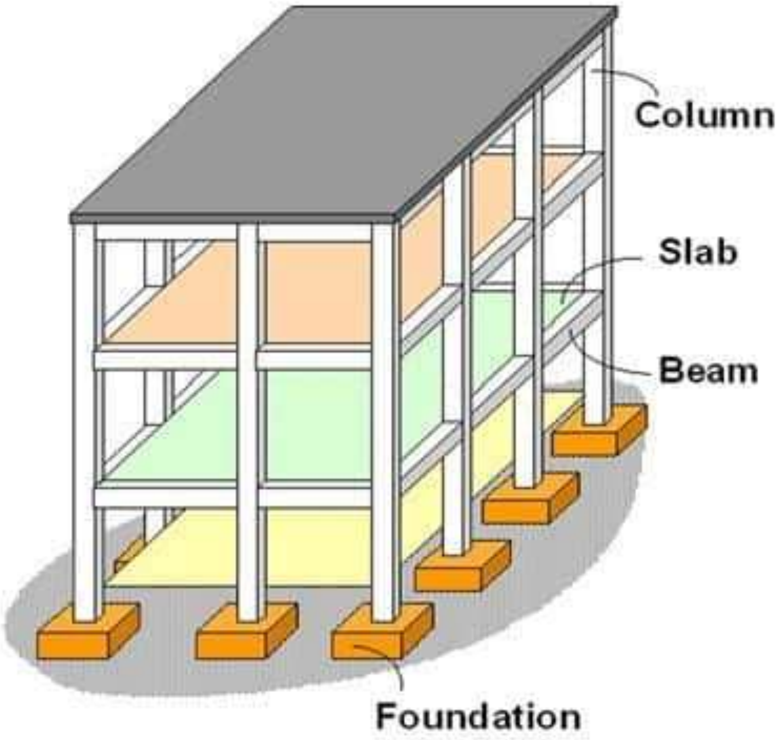
A beam is a horizontal structural member used to support loads. Beams are used to support the roof and floors in buildings.

Beams are structural members which offer resistance to bending due to applied loads. Most beams are long prismatic bars, and the loads are usually applied normal to the axes of the bars.

https://www.youtube.com/watch?v=1zd-qluq-Io

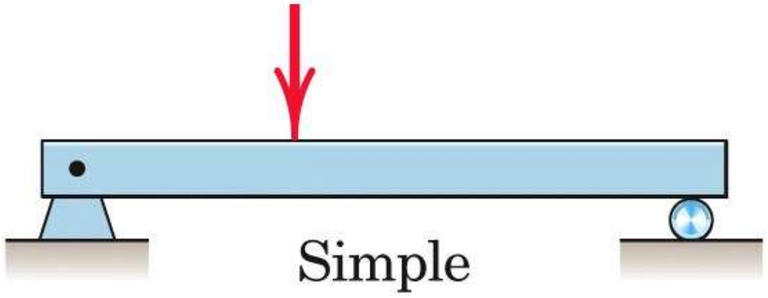
https://www.youtube.com/watch?v=lm7EqrFE4mg

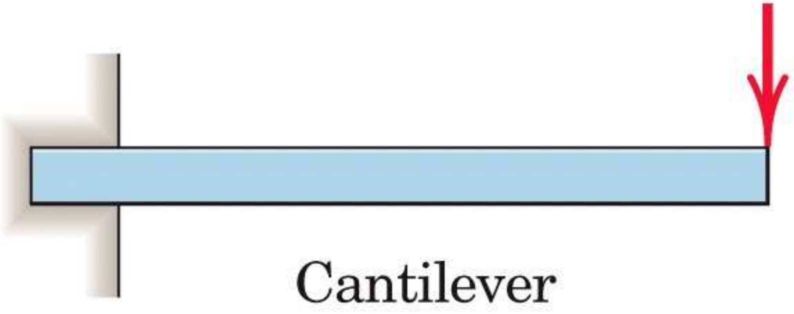
Beams



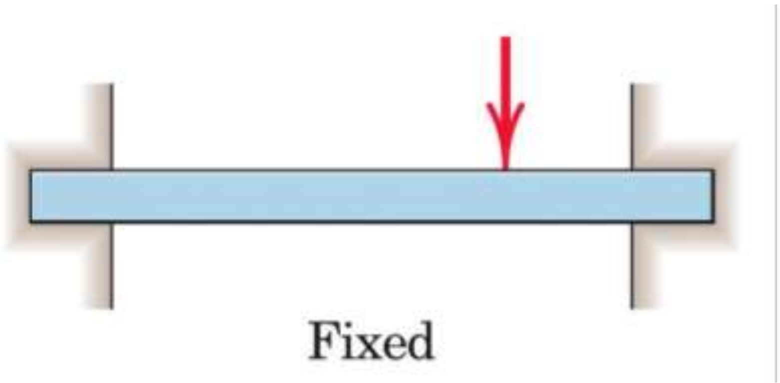
Typical RC Frame Building

Beams supported so that their external support reactions can be calculated by the methods of statics alone are called statically determinate beams.

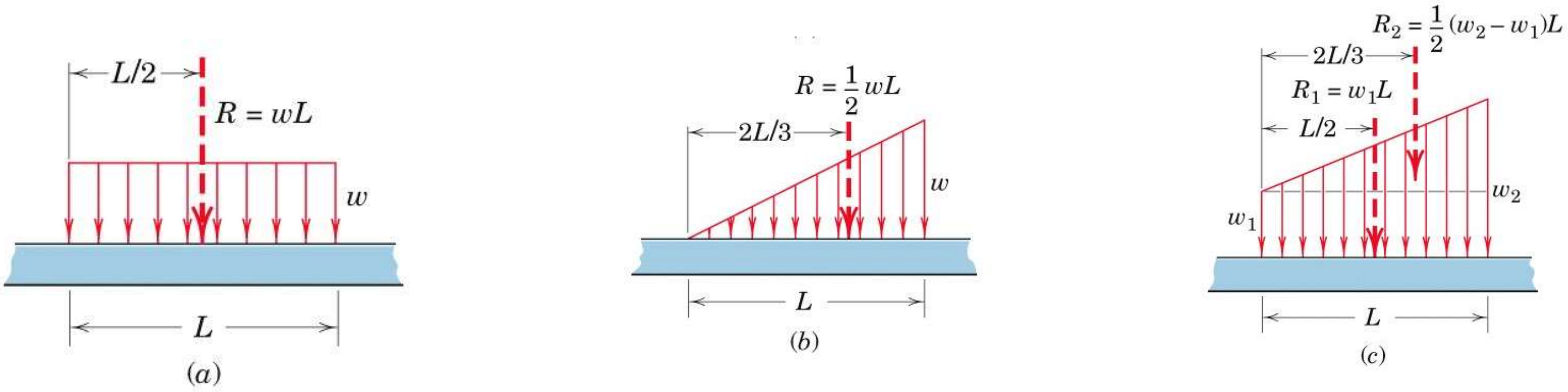




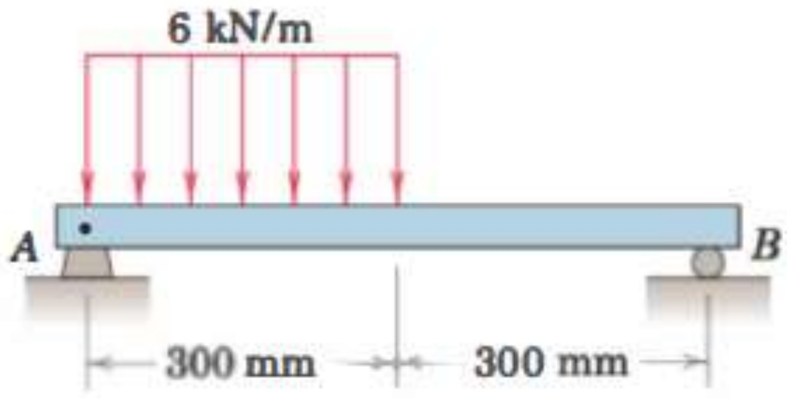
A beam which has more supports than needed to provide equilibrium is statically indeterminate.



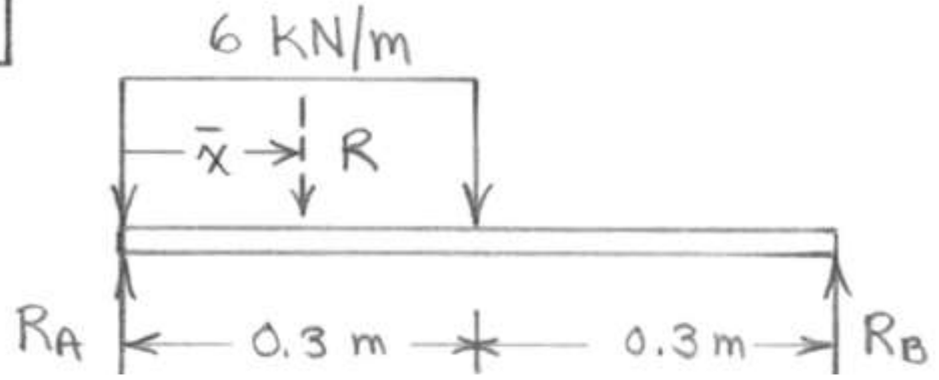
Loads on Beams



5/101) Determine the reactions at A and B for the beam subjected to the uniform load distribution.



5/101

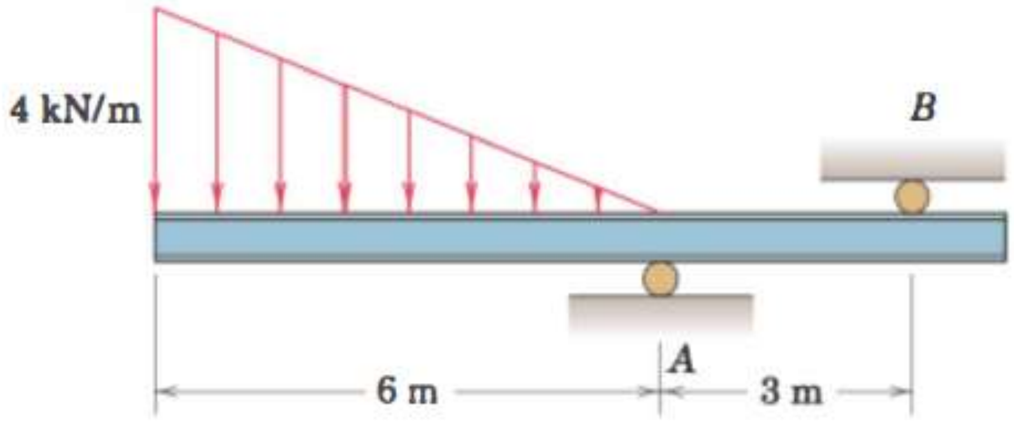


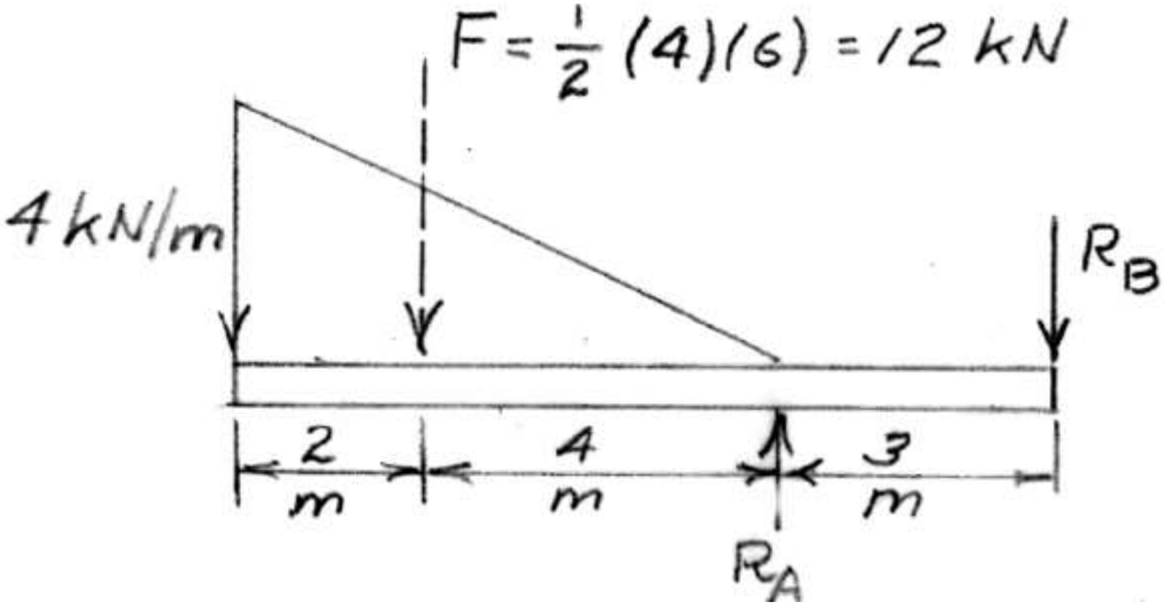
$$R = 6(0.3) = 1.8 \text{ kN} \quad @ \quad \bar{x} = \tfrac{1}{2}(0.3) = 0.15 \text{ m}$$

$$\circlearrowleft + \Sigma M_A = 0: \quad R_B(0.6) - 1.8(0.15) = 0, \quad \underline{R_B = 0.45 \text{ kN}}$$

$$+\uparrow \Sigma F = 0: \quad 0.45 - 1.8 + R_A = 0, \quad \underline{R_A = 1.35 \text{ kN}}$$

5/102) Calculate the reactions at A and B for the beam loaded as shown.

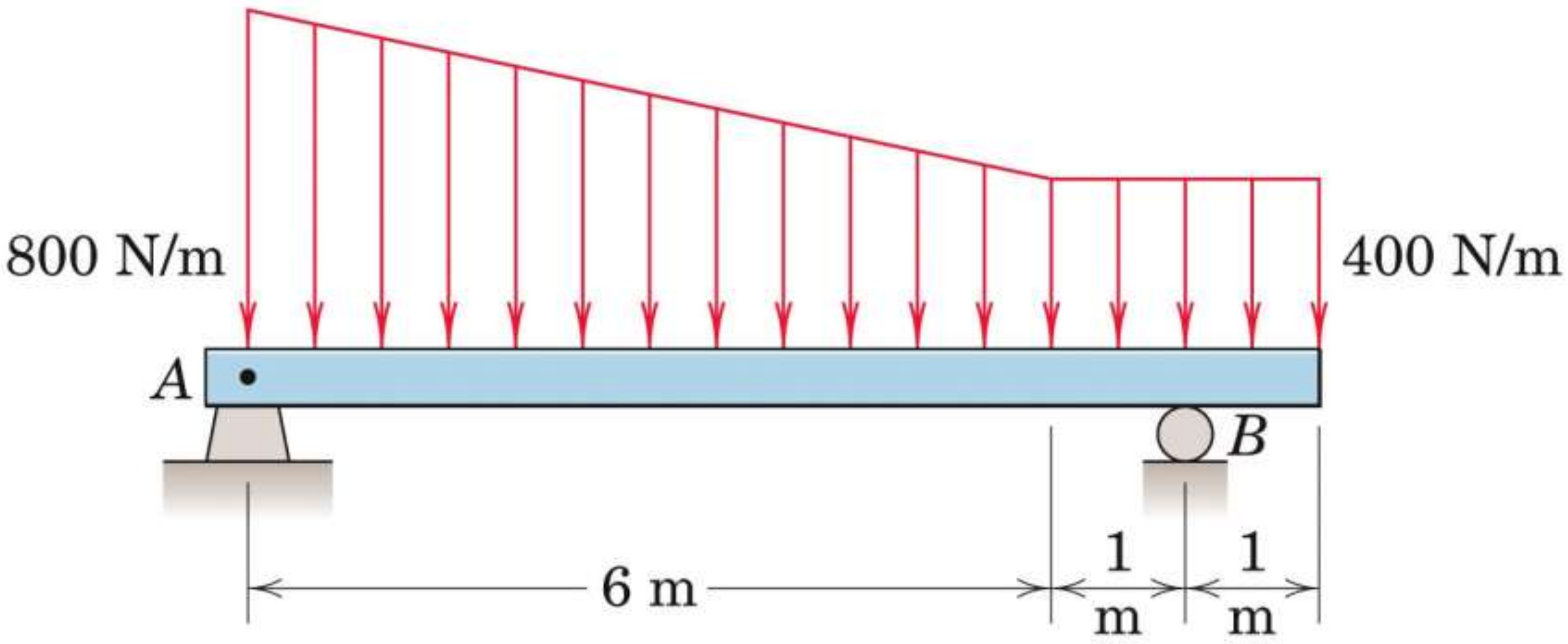




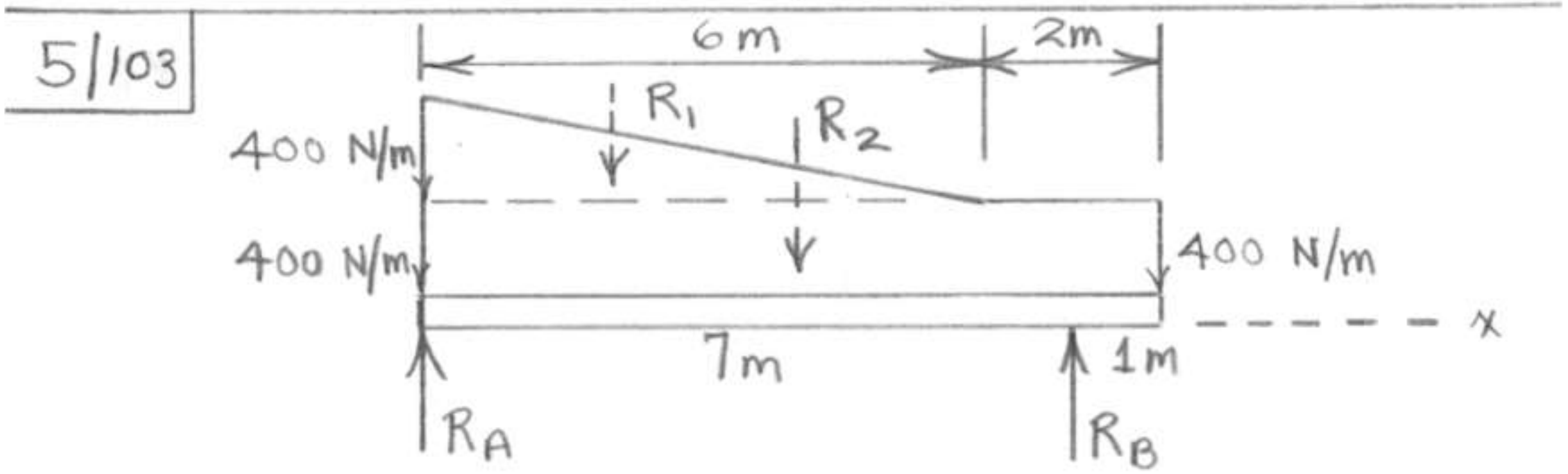
$\overset{+}{\circlearrowleft} \Sigma M_A = 0: \quad 12(4) - 3R_B = 0, \qquad \underline{R_B = 16\ kN}$

$\overset{+}{\uparrow} \Sigma F = 0: \quad R_A - 16 - 12 = 0, \qquad \underline{R_A = 28\ kN}$

5/103) Calculate the reactions at A and B for the beam loaded as shown.



Numericals



$$R_1 = \frac{1}{2}(400)(6) = 1200\text{ N} \quad @ \quad \bar{x}_1 = \frac{1}{3}(6) = 2\text{ m}$$

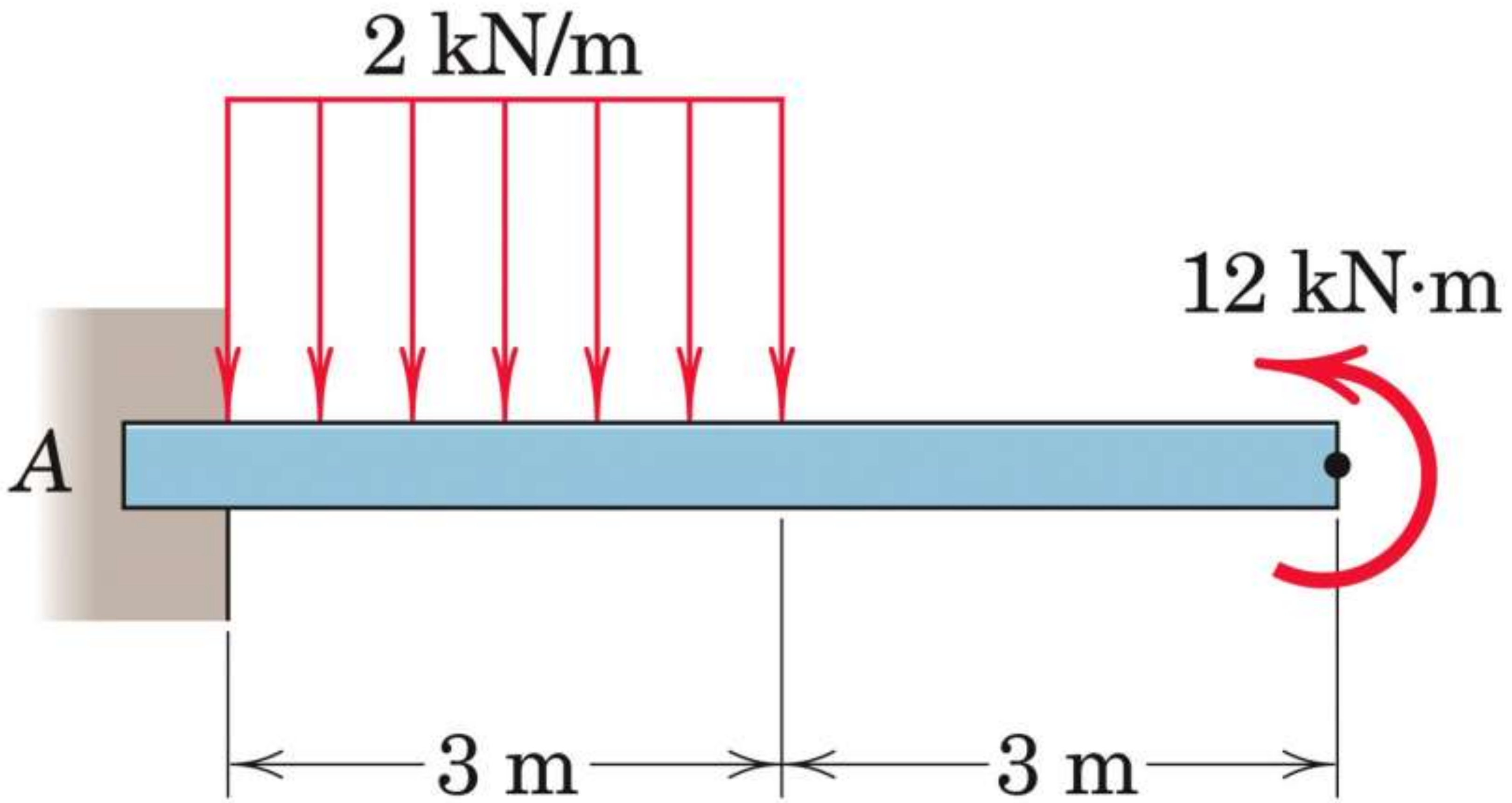
$$R_2 = 400(8) = 3200\text{ N} \quad @ \quad \bar{x}_2 = \frac{1}{2}(8) = 4\text{ m}$$

$$\circlearrowleft + \Sigma M_A = 0: \quad R_B(7) - 1200(2) - 3200(4) = 0, \quad \underline{R_B = 2170\text{ N}}$$

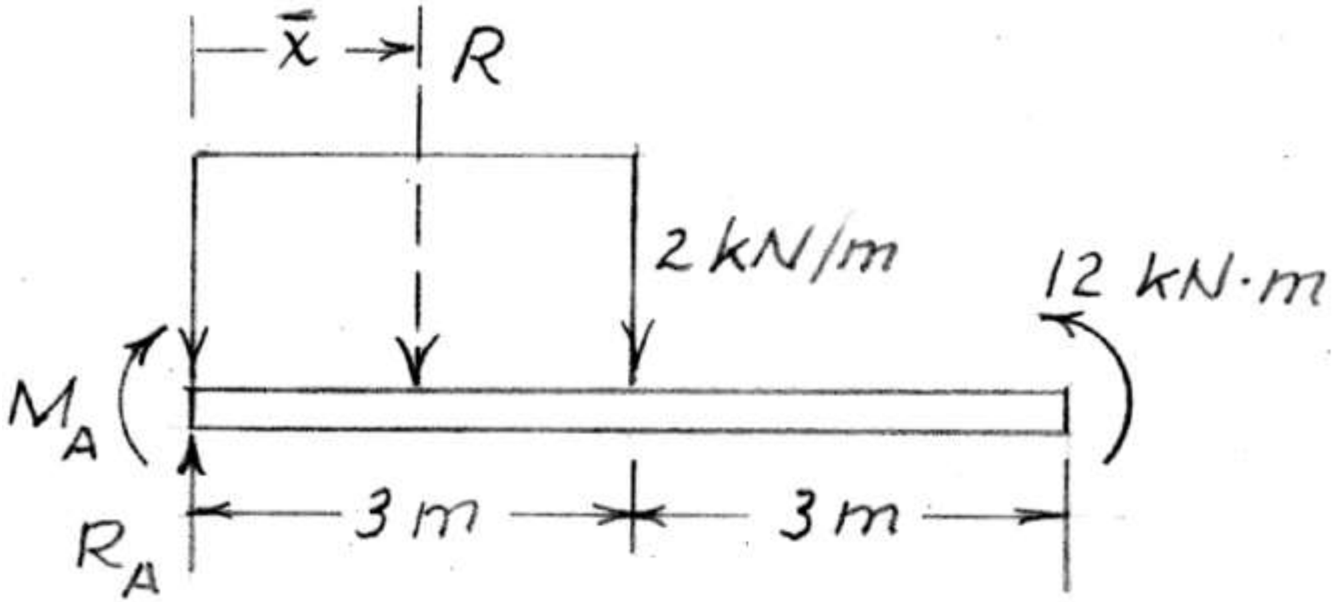
$$+\uparrow \Sigma F = 0: \quad R_A - 1200 - 3200 + 2170 = 0, \quad \underline{R_A = 2230\text{ N}}$$

Numericals

5/105) Find the reaction at A due to the uniform loading and the applied couple.



Numericals

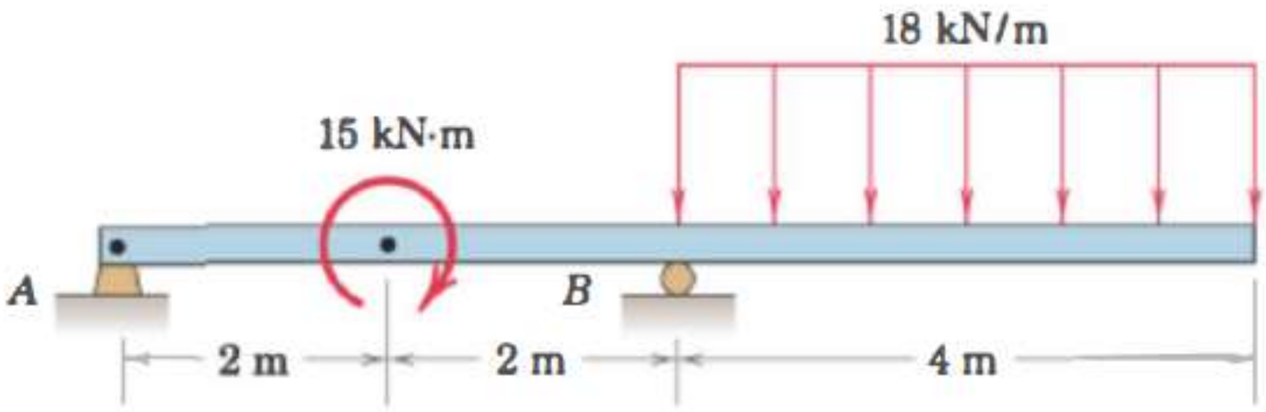


$R = 2(3) = 6 \text{ kN} \quad @ \quad \bar{x} = 1.5 \text{ m}$

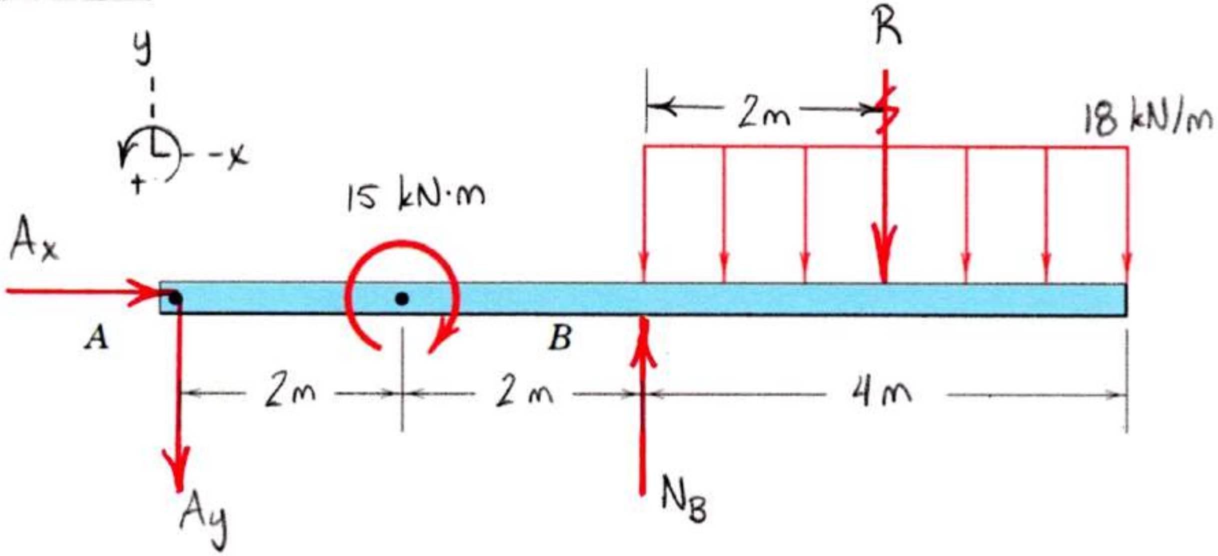
$\circlearrowleft + \Sigma M_A = 0: \ -M_A - 6(3/2) + 12 = 0, \quad \underline{M_A = 3 \text{ kN·m}}$

$\uparrow + \Sigma F = 0: \ R_A - 6 = 0, \quad \underline{R_A = 6 \text{ kN}}$

5/107) Determine the reactions at A and B for the beam loaded as shown



5/107



$R = 18(4) = 72$ kN

$\Sigma F_x = 0$: $\underline{A_x = 0}$

$\Sigma F_y = 0$: $-A_y + N_B - R = 0$

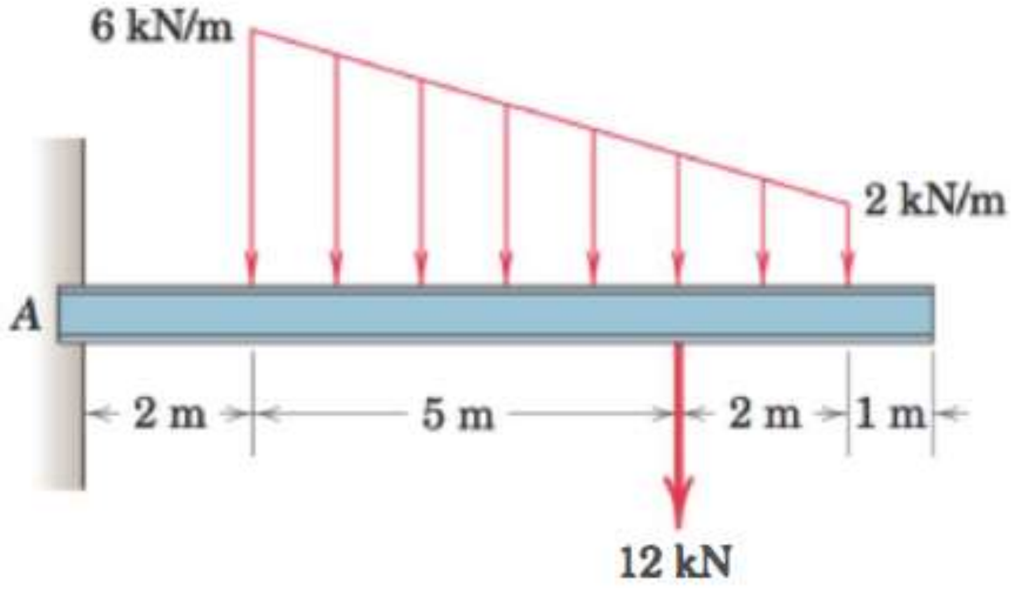
$\Sigma M_A = 0$: $4N_B - 6R - 15 = 0$

$\longrightarrow$

$\underline{A_y = 39.8 \text{ kN} \downarrow}$

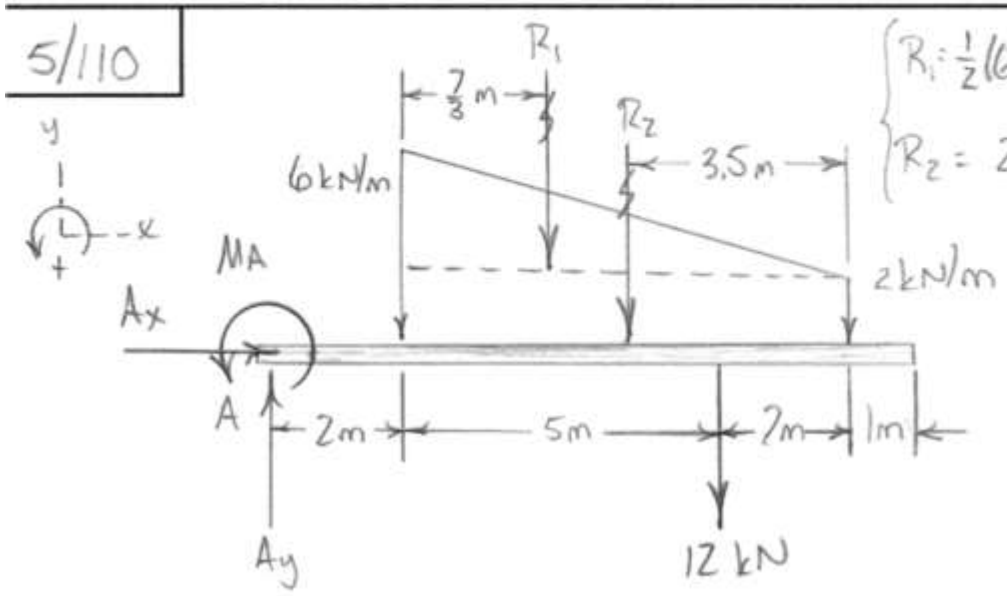
$\underline{N_B = 111.8 \text{ kN} \uparrow}$

5/110) Determine the force and moment reactions at A for the cantilever beam subjected to the loading shown.



Numericals

5/110



$$R_1 = \tfrac{1}{2}(6-2)(7) = 14 \text{ kN}$$

$$R_2 = 2(7) = 14 \text{ kN}$$

$$\Sigma F_x = 0: \quad \underline{A_x = 0}$$

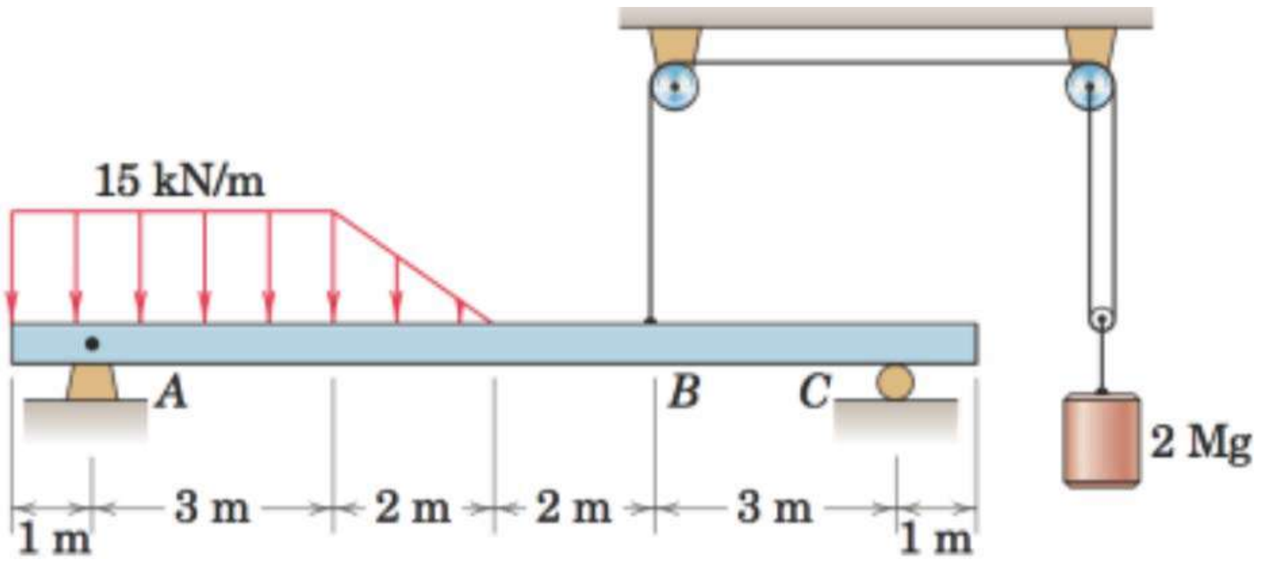
$$\Sigma F_y = 0: \quad A_y - R_1 - R_2 - 12 = 0$$

$$\Sigma M_A = 0: \quad M_A - (2 + 7/3)R_1 - (2 + 3.5)R_2 - 7(12) = 0$$

$$\underline{A_y = 40 \text{ kN} \uparrow} \qquad \underline{M_A = 222 \text{ kN}\cdot\text{m CCW}}$$

Numericals

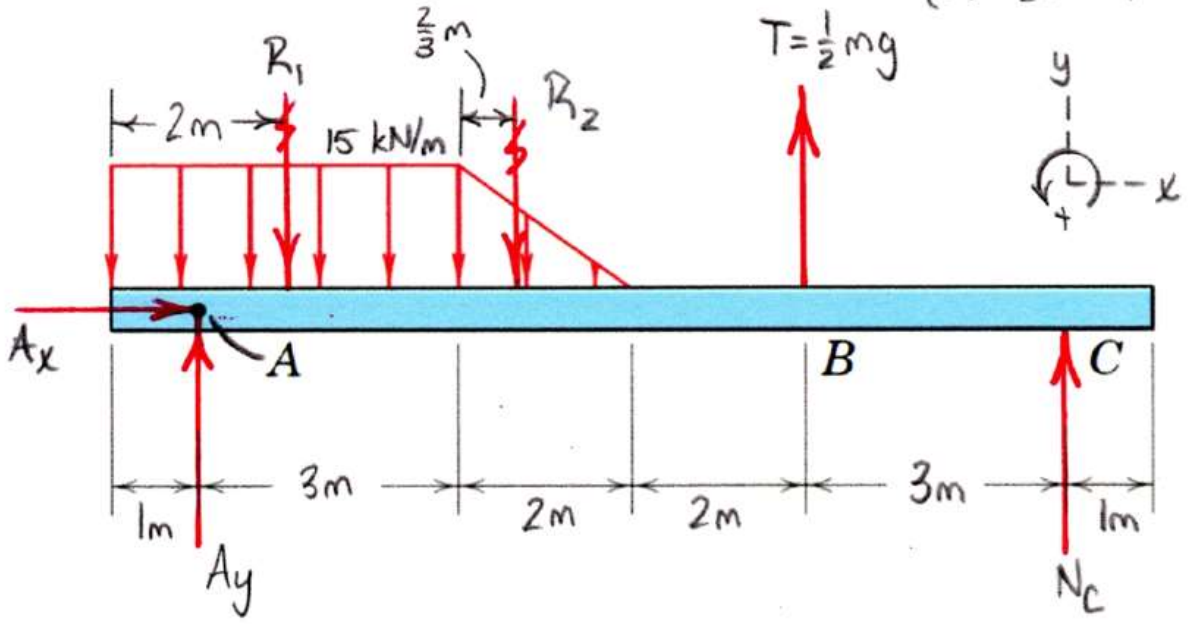
5/111) Determine the reactions at A and C for the beam subjected to the combination of point and distributed loads.



Numericals

5/111 $\quad m = 2\ \text{Mg}$

$$\begin{cases} R_1 = 15(4) = 60\ \text{kN} \\ R_2 = \frac{1}{2}(15)(2) = 15\ \text{kN} \end{cases}$$

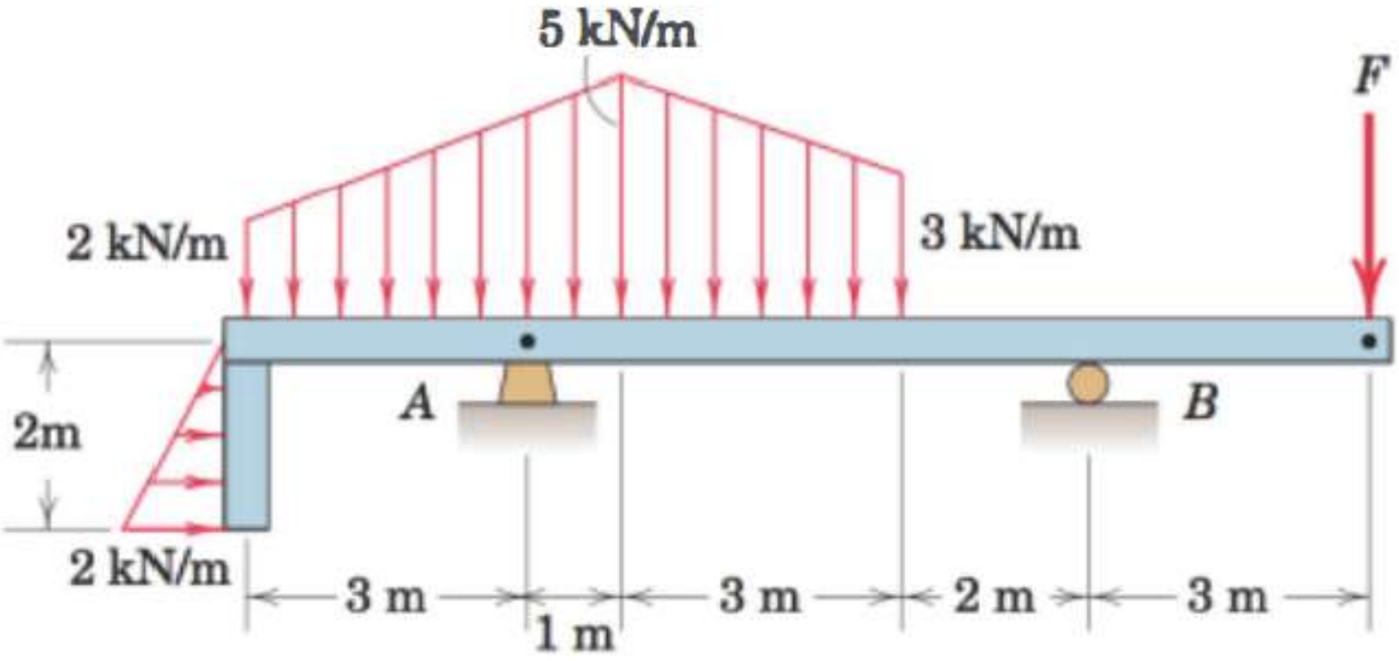


$T = \frac{1}{2}(2000)(9.81) = 9810\ \text{N}$

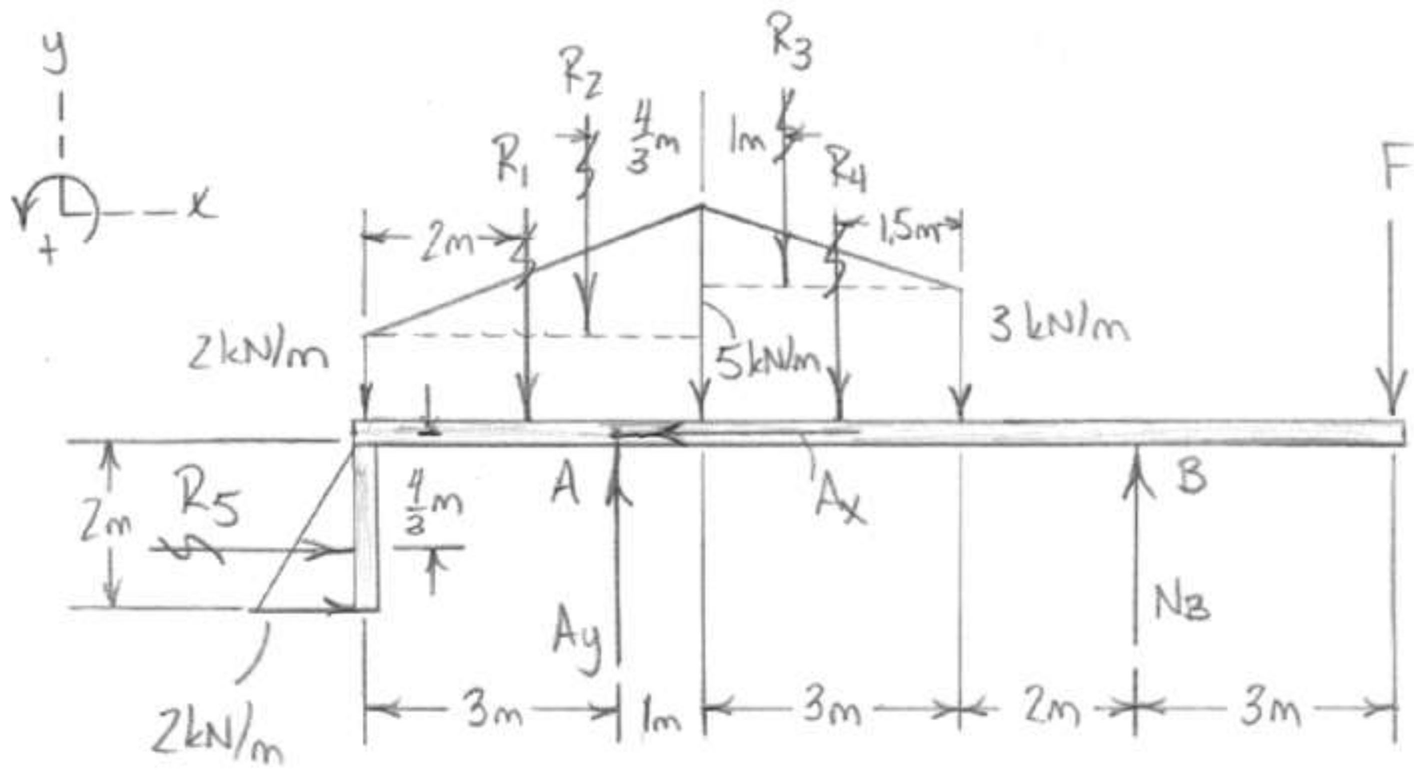
$$\begin{cases} \Sigma F_x = 0: \quad \underline{A_x = 0} \\ \Sigma F_y = 0: \quad A_y - R_1 - R_2 + T + N_C = 0 \\ \Sigma M_A = 0: \quad -1R_1 - (3 + \frac{2}{3})R_2 + 7T + 10N_C = 0 \end{cases}$$

$$\begin{cases} \underline{N_C = 4.63\ \text{kN} \uparrow} \\ \underline{A_y = 60.6\ \text{kN} \uparrow} \end{cases}$$

5/116 For the beam and loading shown, determine the magnitude of the force F for which the vertical reactions at A and B are equal. With this value of F, compute the magnitude of the pin reaction at A.



5/116 FIND F FOR $A_y = N_B$



$$\begin{cases} R_1 = 2(4) = 8\ kN \\ R_2 = \frac{1}{2}(5-2)(4) = 6\ kN \\ R_3 = \frac{1}{2}(5-3)(3) = 3\ kN \end{cases} \qquad \begin{cases} R_4 = 3(3) = 9\ kN \\ R_5 = \frac{1}{2}(2)(2) = 2\ kN \end{cases}$$

$$\begin{cases} \Sigma F_x = 0: \ -A_x + R_5 = 0 \longrightarrow A_x = 2\ kN \\ \Sigma F_y = 0: \ A_y + N_B - F - R_1 - R_2 - R_3 - R_4 = 0 \\ \Sigma M_B = 0: \ -3F + 3.5R_4 + 4R_3 + (5+\frac{4}{3})R_2 + 7R_1 + \frac{4}{3}R_5 - 6A_y = 0 \end{cases}$$

$$A_y = N_B = 18.18\ kN \qquad \underline{F = 10.36\ kN}$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{18.18^2 + 2^2} \longrightarrow \underline{R_A = 18.29\ kN}$$

THANK YOU

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ENGINEERING MECHANICS

Centroid

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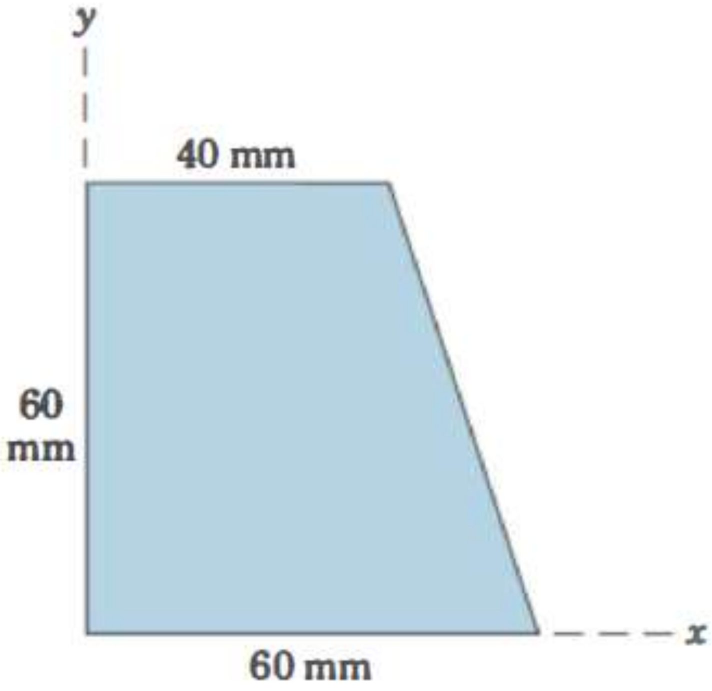
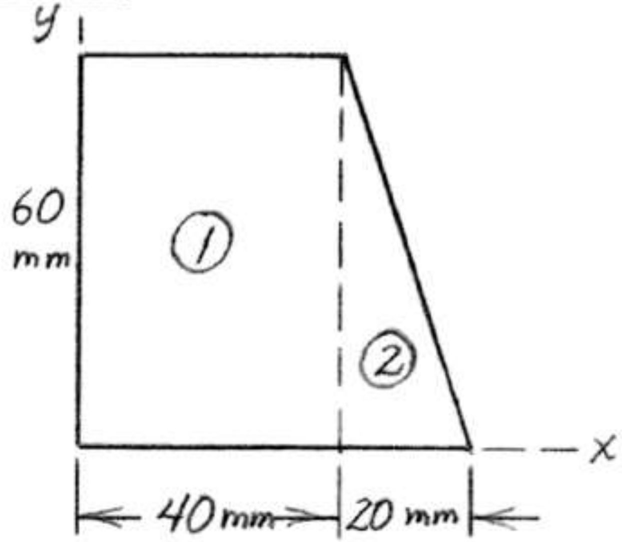
5/47 Determine the coordinates of the centroid of the trapezoidal area shown.



5/47



① $A_1 = 40(60) = 2400 \text{ mm}^2$

$\bar{x}_1 = 20 \text{ mm}, \quad \bar{y}_1 = 30 \text{ mm}$

② $A_2 = \frac{1}{2}(20)(60) = 600 \text{ mm}^2$

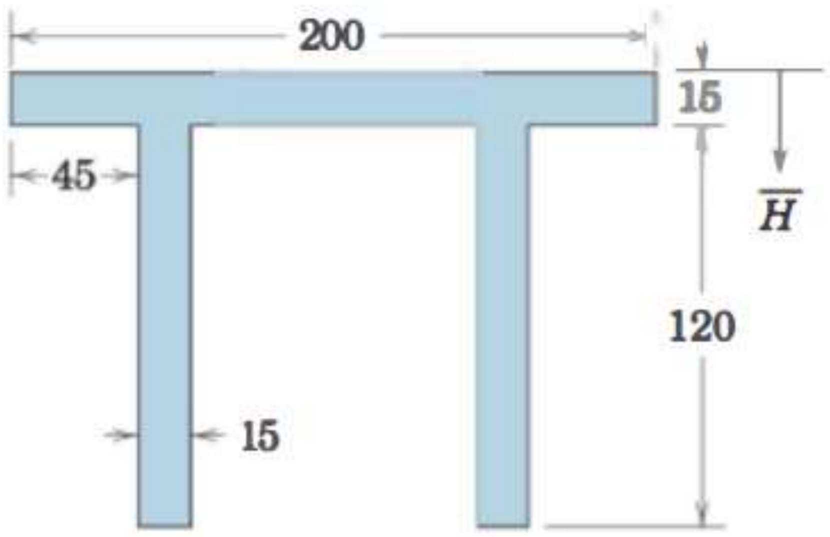
$\bar{x}_2 = 40 + \frac{20}{3} = 46.7 \text{ mm}$

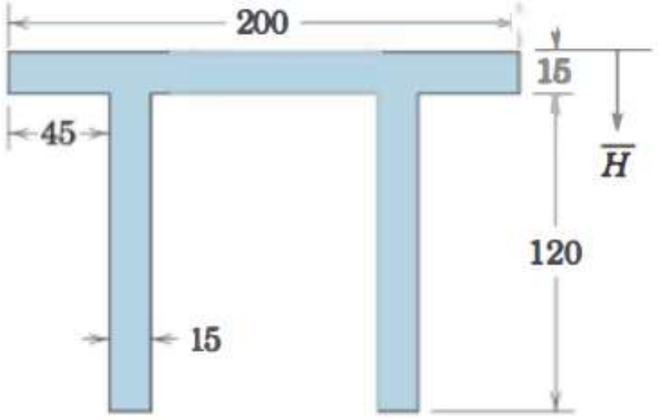
$\bar{y}_2 = \frac{60}{3} = 20 \text{ mm}$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{2400(20) + 600(46.7)}{2400 + 600} = \underline{25.3 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2400(30) + 600(20)}{2400 + 600} = \underline{28.0 \text{ mm}}$$

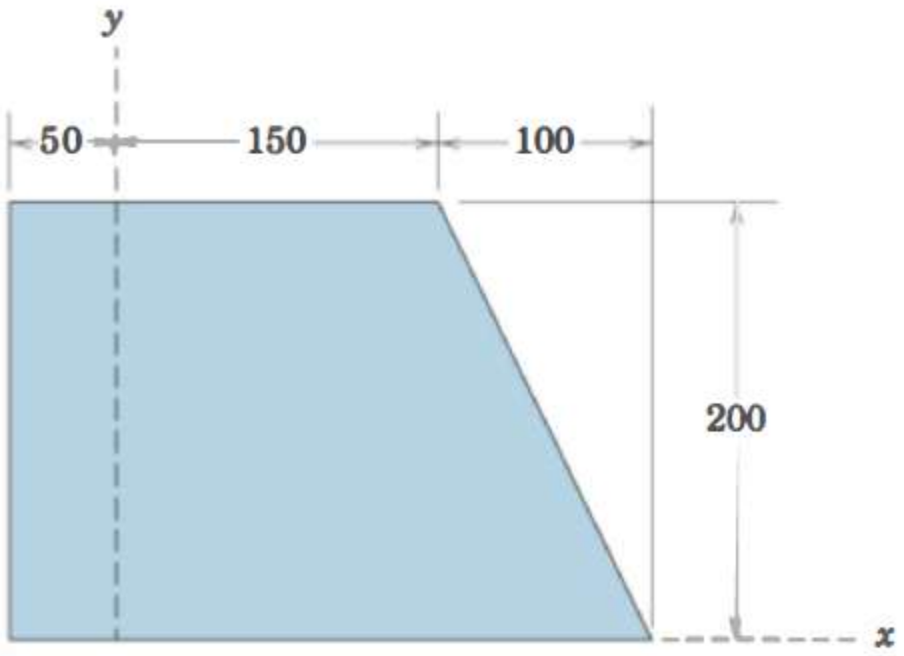
5/48 Determine the distance H from the upper surface of the symmetric double-T beam cross section to the location of the centroid.





$$\bar{H} = \frac{\Sigma A \bar{h}}{\Sigma A} = \frac{200(15)\left(\frac{15}{2}\right) + 2(15)(120)\left(15 + \frac{120}{2}\right)}{200(15) + 2(15)(120)} \longrightarrow \underline{\bar{H} = 44.3 \text{ mm}}$$

5/49 Determine the x- and y-coordinates of the centroid of the shaded area.



Dimensions in millimeters

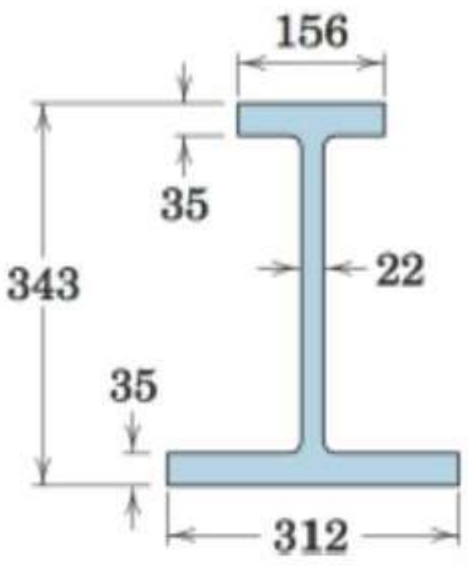
$$\bar{X} = \frac{\Sigma A \bar{x}}{\Sigma A} = \frac{50(200)(-25) + 150(200)(75) + \frac{1}{2}(100)(200)\left(150 + \frac{100}{3}\right)}{50(200) + 150(200) + \frac{1}{2}(100)(200)}$$

$$\underline{\bar{X} = 76.7 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A \bar{y}}{\Sigma A} = \frac{50(200)(100) + 150(200)(100) + \frac{1}{2}(100)(200)\left(\frac{200}{3}\right)}{50(200) + 150(200) + \frac{1}{2}(100)(200)}$$

$$\bar{Y} = 93.3 \text{ mm}$$

5/50 Determine the height above the base of the centroid of the cross-sectional area of the beam. Neglect the fillets.



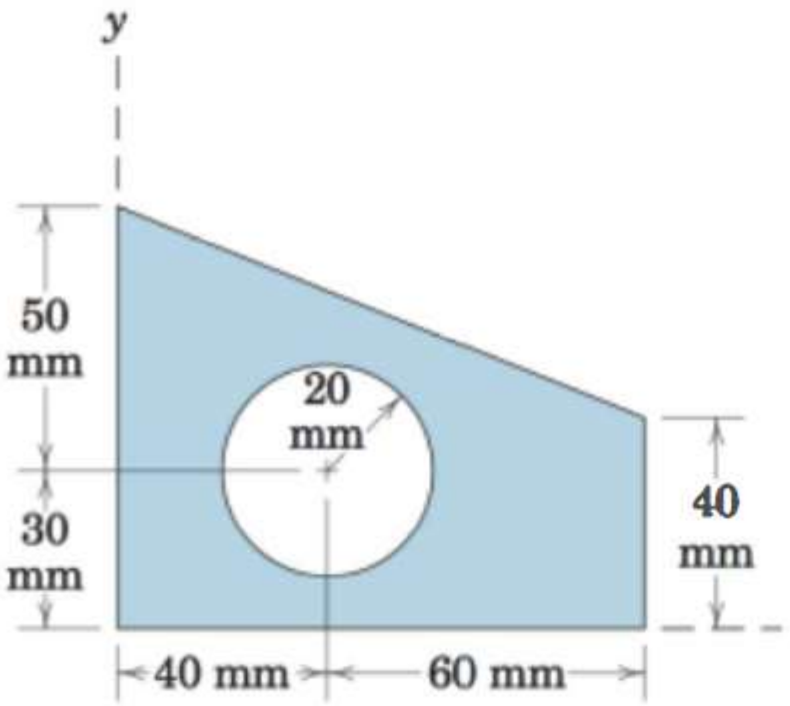
Dimensions in millimeters

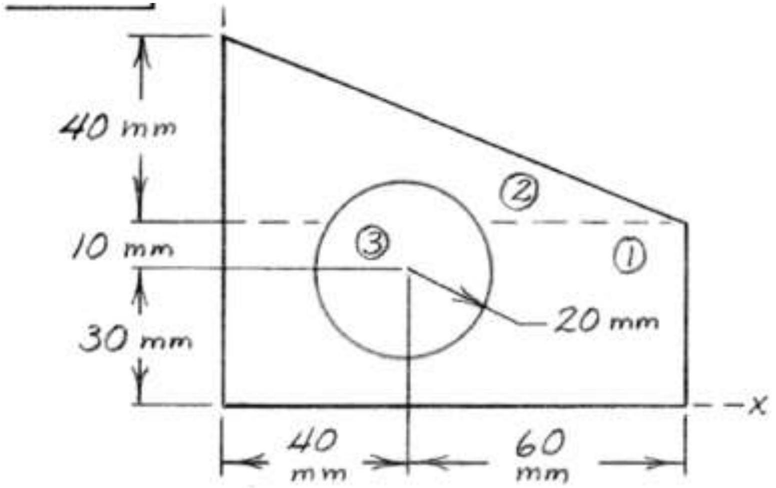
Comp.	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)
1	312(35)	$\frac{35}{2}$	191 100
2	273 (22)	$35 + \frac{273}{2}$	1 030 000
3	156 (35)	$35 + 273 + \frac{35}{2}$	1 777 000

$\Sigma A = 22\,400$ $\Sigma A\bar{y} = 3\,000\,000$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{3\,000\,000}{22400} = \underline{133.9 \text{ mm}}$$

5/51 Determine the x- and y-coordinates of the centroid of the shaded area.



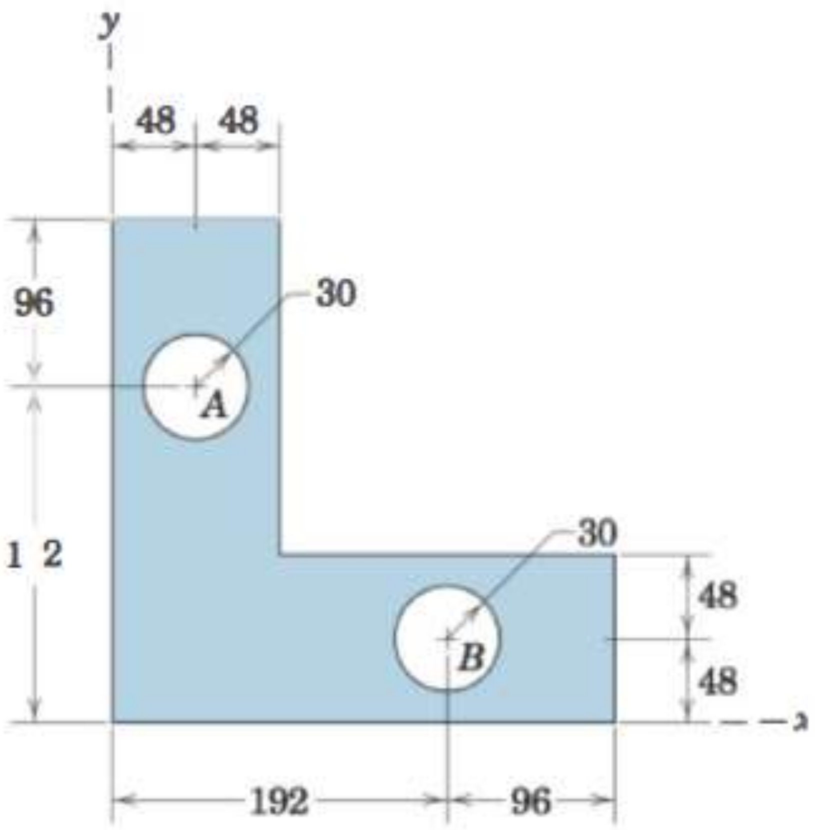


Part	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$A\bar{x}$ (mm^3)	$A\bar{y}$ (mm^3)
1	4000	50	20	$200(10^3)$	$80(10^3)$
2	2000	100/3	$40 + \frac{40}{3}$	$66.7(10^3)$	$106.7(10^3)$
3	$-\pi(20^2)$	40	30	$-50.3(10^3)$	$-37.7(10^3)$
Totals	4740			$216(10^3)$	$149.0(10^3)$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{216(10^3)}{4740} = \underline{45.6 \text{ mm}}$$

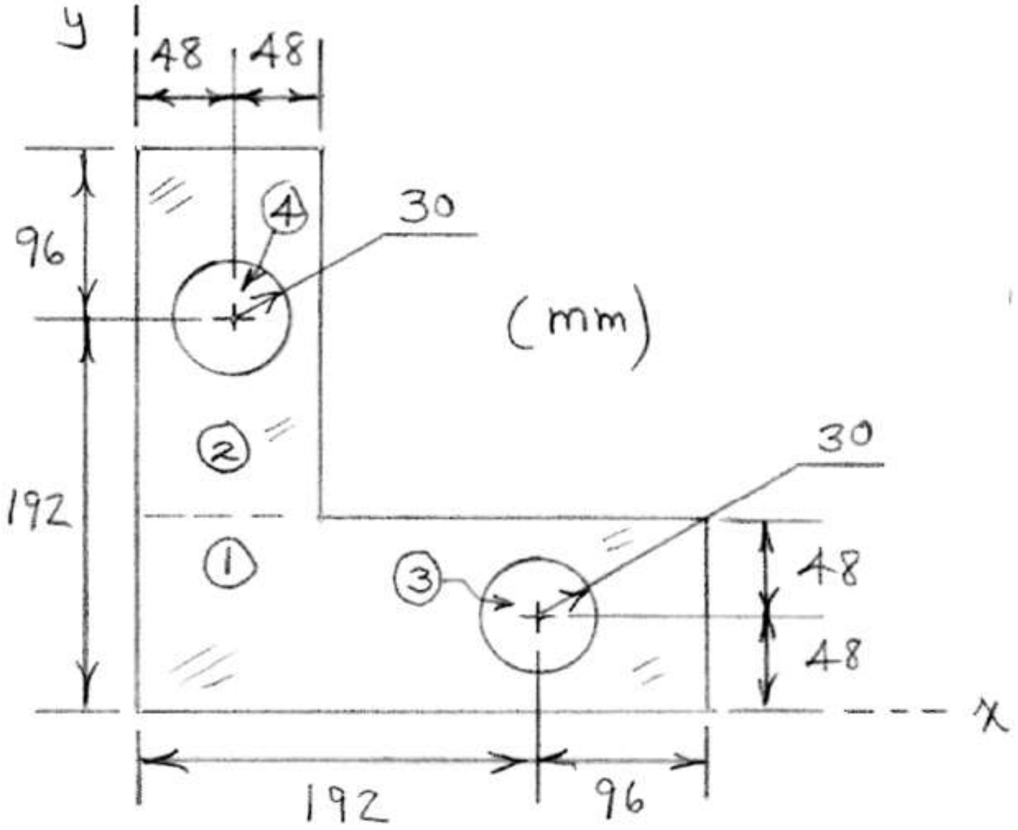
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{149.0(10^3)}{4740} = \underline{31.4 \text{ mm}}$$

5/52 Determine the x- and y-coordinates of the centroid of the shaded area.



Dimensions in millimeters

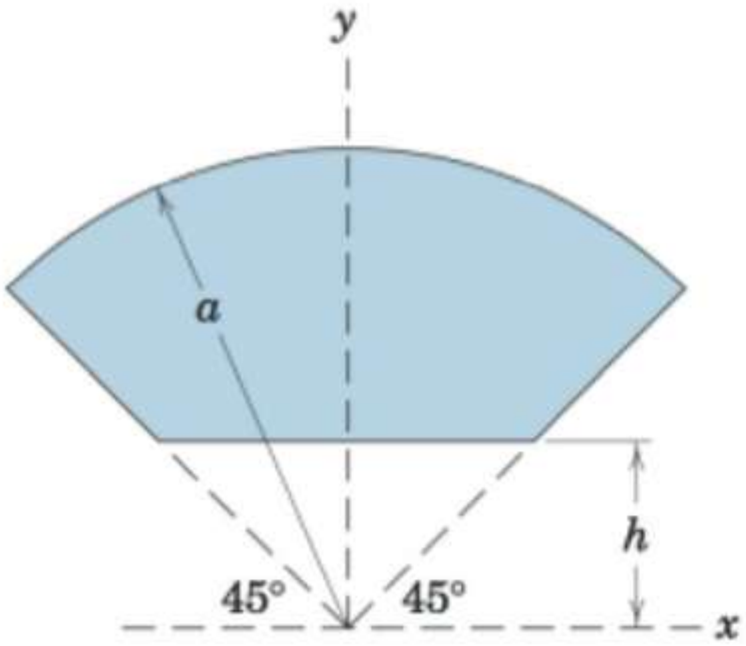
Centroid



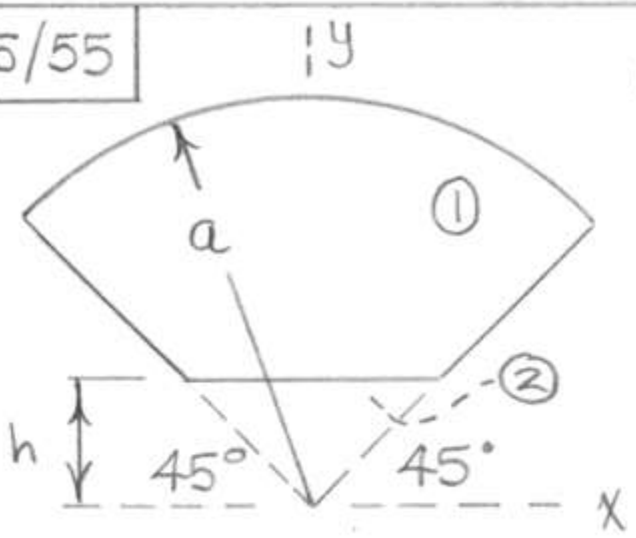
Comp.	$A\ (mm^2)$	$\bar{x}\ (mm)$	$\bar{x}A\ (mm^3)$
1	27648	144	$3.98(10^6)$
2	18432	48	$0.885(10^6)$
3	-2830	192	$-0.543(10^6)$
4	-2830	48	$-0.1357(10^6)$
	$\Sigma A = 40430$		$\Sigma \bar{x}A = 4.19(10^6)$

$$\bar{X} = \bar{Y} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{4.19(10^6)}{40430} = \underline{103.6\ mm}$$

5/55 Determine the y-coordinate of the centroid of the shaded area.



5/55



Circular sector (full) ①:

$$A_1 = \frac{\pi}{4} a^2$$

$$\bar{y}_1 = \frac{2}{3} a \frac{\sin 45°}{\pi/4}$$

$$= \frac{4\sqrt{2}}{3\pi} a$$

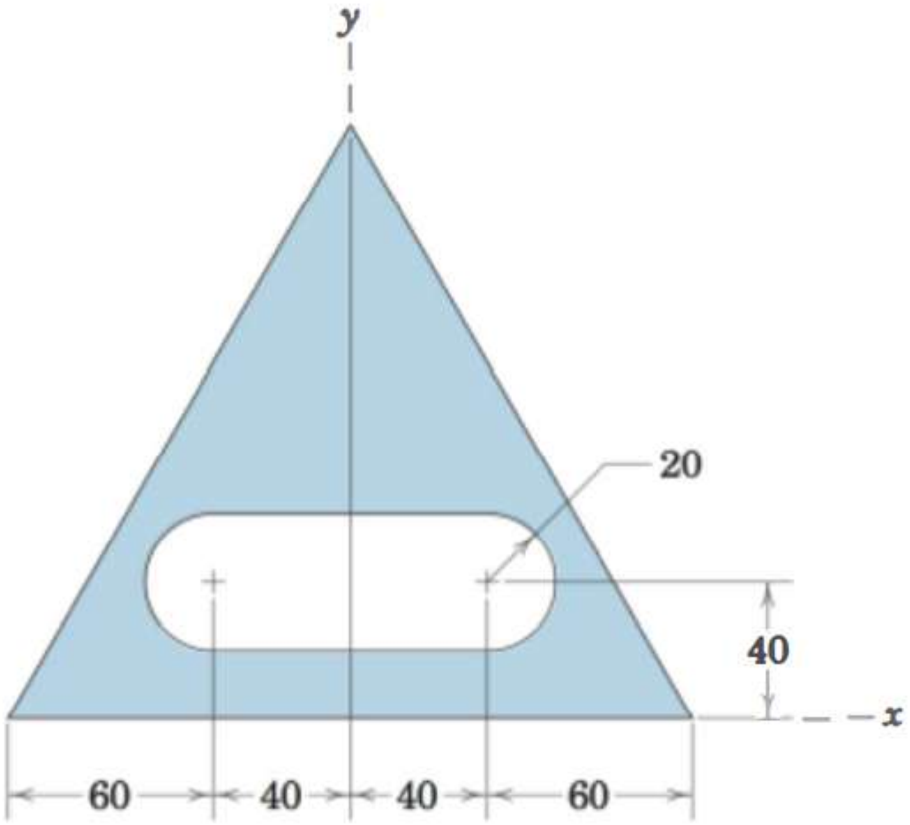
Triangular "hole" ②:

$$A_2 = \frac{1}{2} h (2h) = h^2, \quad \bar{y}_2 = \frac{2}{3} h$$

$$\bar{Y} = \frac{\Sigma A \bar{y}}{\Sigma A} = \frac{\frac{\pi}{4} a^2 \left(\frac{4\sqrt{2}}{3\pi} a \right) - h^2 \left(\frac{2}{3} h \right)}{\frac{\pi}{4} a^2 - h^2}$$

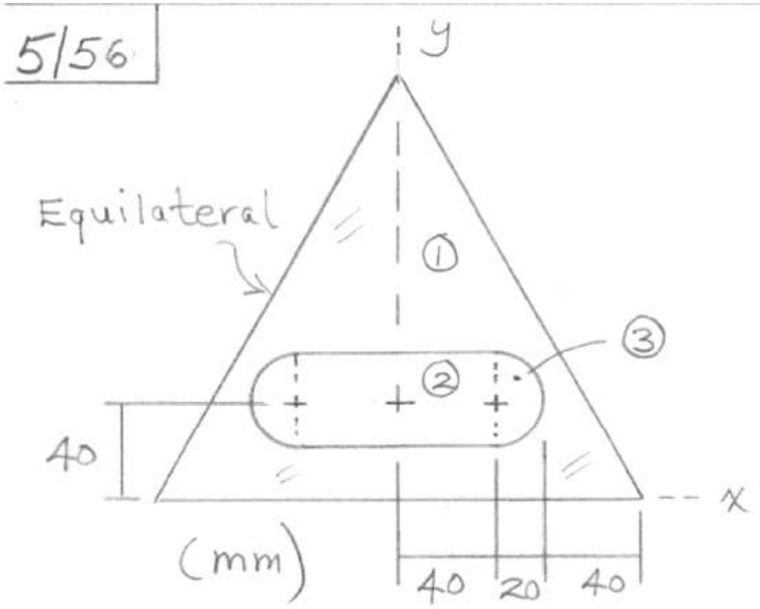
$$= \frac{4(\sqrt{2} a^3 - 2h^3)}{3(\pi a^2 - 4h^2)}$$

5/56 Determine the y-coordinate of the centroid of the shaded area. The triangle is equilateral.



Dimensions in millimeters

ENGINEERING MECHANICS
Centroid

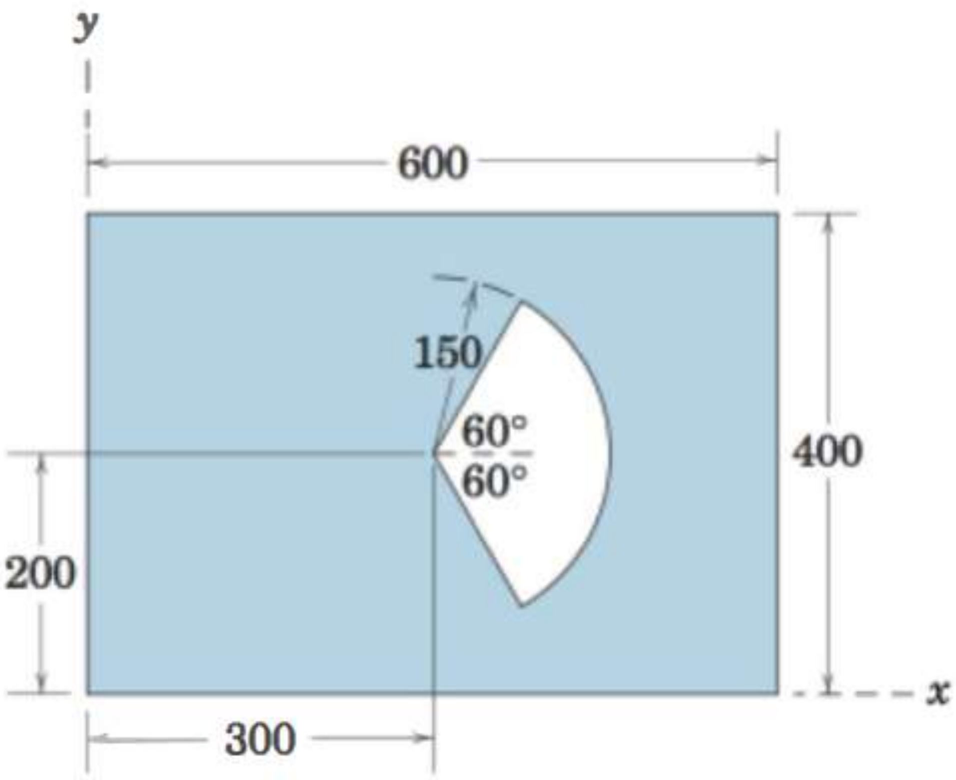


Component	A (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
Triangle 1	17 320	57.7	10^6
Rectangle 2	−3200	40	−128 000
2 semicircles 3	−1257	40	−50,300

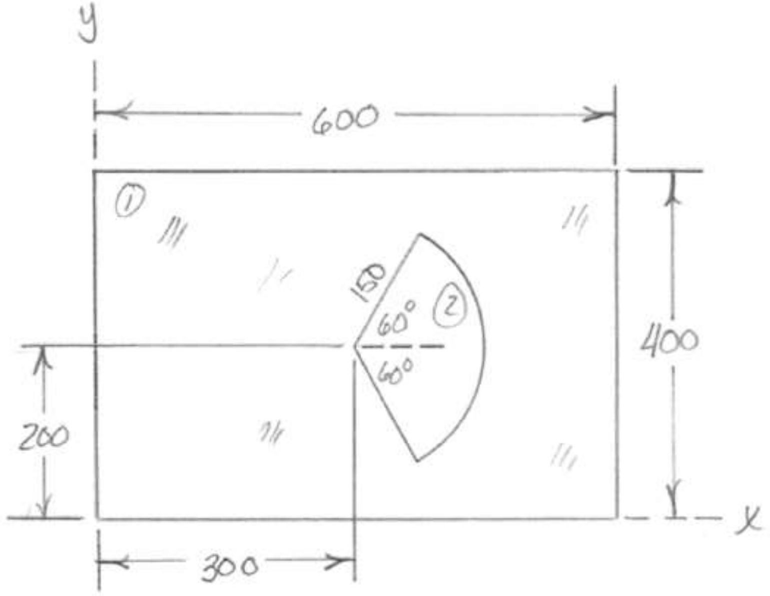
$\Sigma A = 12\,860 \qquad \Sigma \bar{y}A = 822\,000$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} \qquad \frac{822\,000}{12\,860} = 63.9 \text{ mm}$$

5/57 Determine the x- and y-coordinates of the centroid of the shaded area.



Dimensions in millimeters

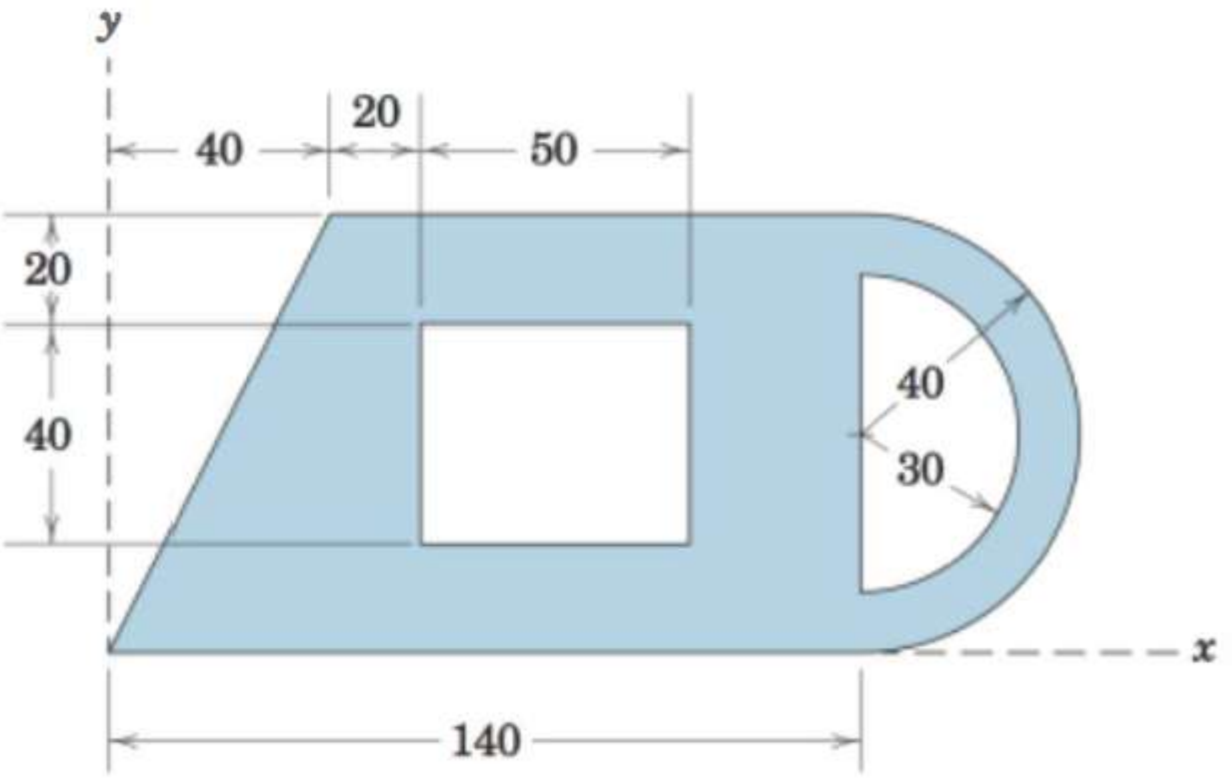


$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{400(600)(300) - \frac{1}{3}\pi(150)^2\left(300 + \frac{2}{3}(150)\frac{\sin 60}{\pi/3}\right)}{400(600) - \frac{1}{3}\pi(150)^2}$$

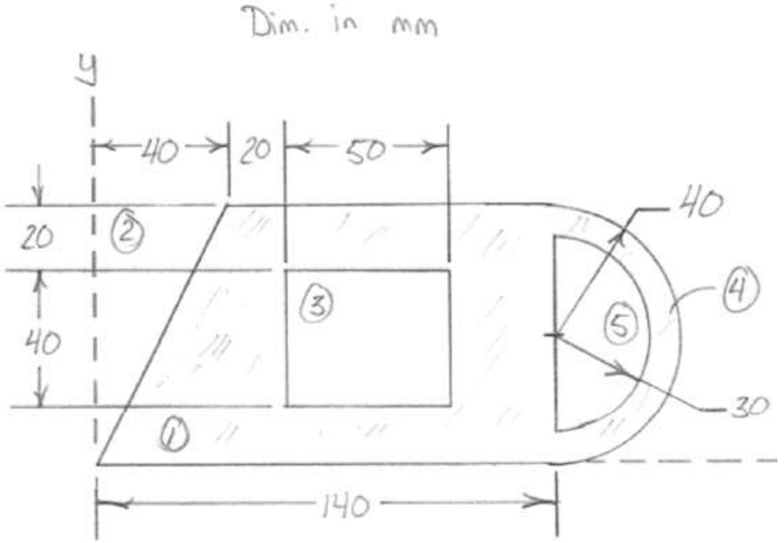
$\underline{\bar{X} = 291 \text{ mm}}$

$\underline{\bar{Y} = 200 \text{ mm}}$ (INSPECTION)

5/58 Determine the coordinates of the centroid of the shaded area.



Dimensions in millimeters

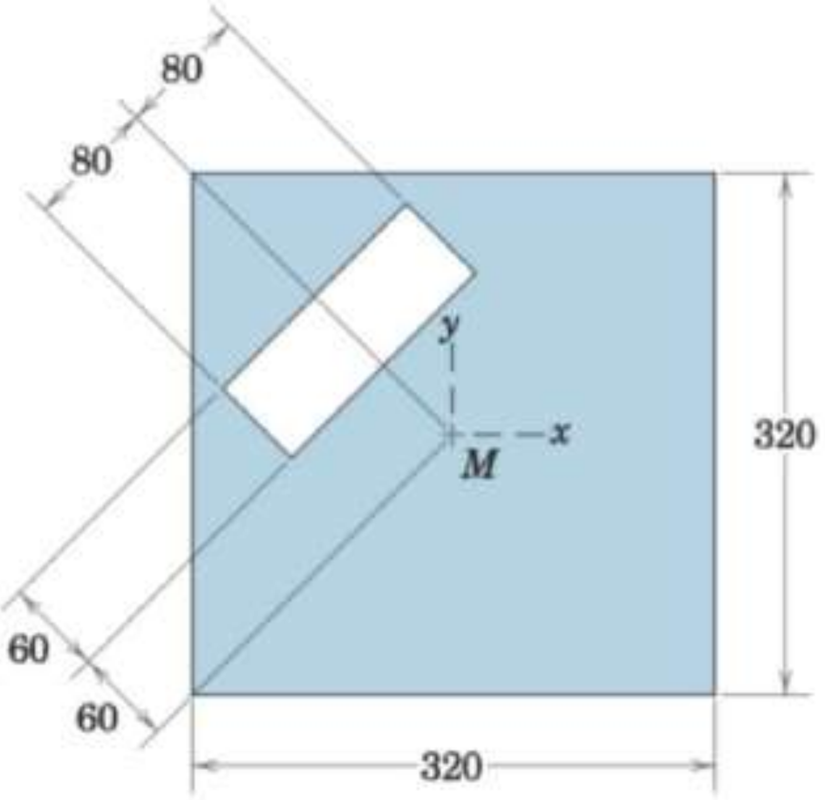


	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}$, mm^3	$A\bar{y}$, mm^3
1	$140(80) = 11200$	70	40	784×10^3	448×10^3
2	$-\frac{1}{2}(40)(80) = -1600$	$\frac{40}{3} = 13.33$	$\frac{2}{3}(80) = 53.3$	-21.3×10^3	-85.3×10^3
3	$-40(50) = -2000$	85	40	-170×10^3	-80×10^3
4	$\frac{\pi(40)^2}{2} = 800\pi$	$140 + \frac{4(40)}{3\pi} = 157.0$	40	395×10^3	32000π
5	$-\frac{\pi(30)^2}{2} = -450\pi$	$140 + \frac{4(30)}{3\pi} = 152.7$	40	-216×10^3	-18000π
Σ	8700			771×10^3	327×10^3

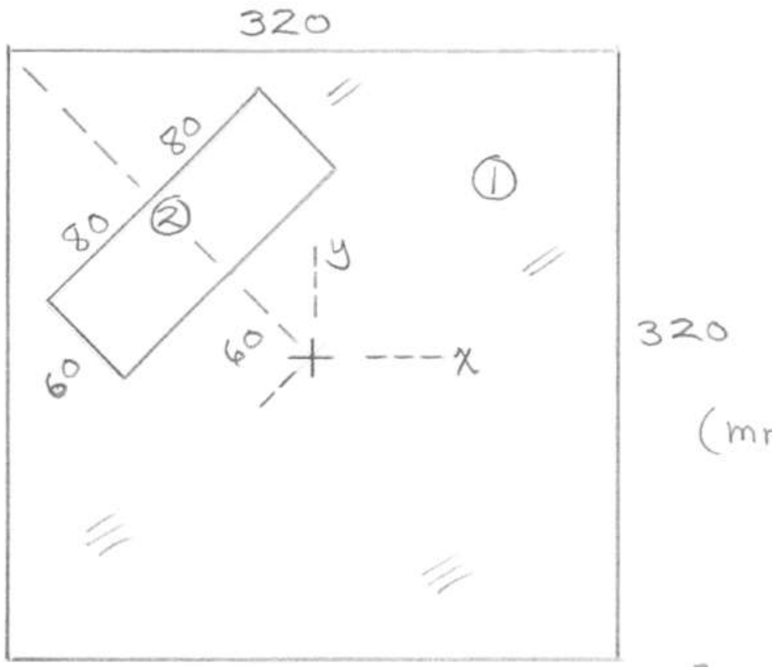
$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{771 \times 10^3}{8700} \rightarrow \underline{\bar{X} = 88.7 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{327 \times 10^3}{8700} \rightarrow \underline{\bar{Y} = 37.5 \text{ mm}}$$

5/61 By inspection, state the quadrant in which the centroid of the shaded area is located. Then determine the coordinates of the centroid. The plate center is M.



Dimensions in millimeters



Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}A$ (mm^3)	$\bar{y}A$ (mm^3)
1	$(320)^2$	0	0	0	0
2	$-160(60)$	$-90\frac{\sqrt{2}}{2}$	$90\frac{\sqrt{2}}{2}$	611 000	$-611\ 000$

$$\Sigma A = 92\ 800$$

$$\Sigma \bar{x}A = 611\ 000$$

$$\Sigma \bar{y}A = -611\ 000$$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{611\ 000}{92\ 800} = \underline{6.58\ mm}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{-611\ 000}{92\ 800} = \underline{-6.58\ mm}$$

THANK YOU

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ENGINEERING MECHANICS

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ENGINEERING MECHANICS

Moment of Inertia

Problem No. A/1 to A/19 and A/35 to A/55

Excluding A/5, A/8, A/10, A/11, A/13, A/15, A/47, A/50, A/52.

P. Ramchandra

Department of Civil Engineering

Rectangular Area moment of inertia and polar moment of inertia

Rectangular and Polar Moments of Inertia

Consider the area A in the x-y plane, Fig. A/2. The moments of inertia of the element dA about the x- and y-axes are, by definition, $dI_x = y^2\,dA$ and $dI_y = x^2\,dA$, respectively. The moments of inertia of A about the same axes are therefore

$$I_x = \int y^2\,dA$$
$$I_y = \int x^2\,dA \qquad \text{(A/1)}$$

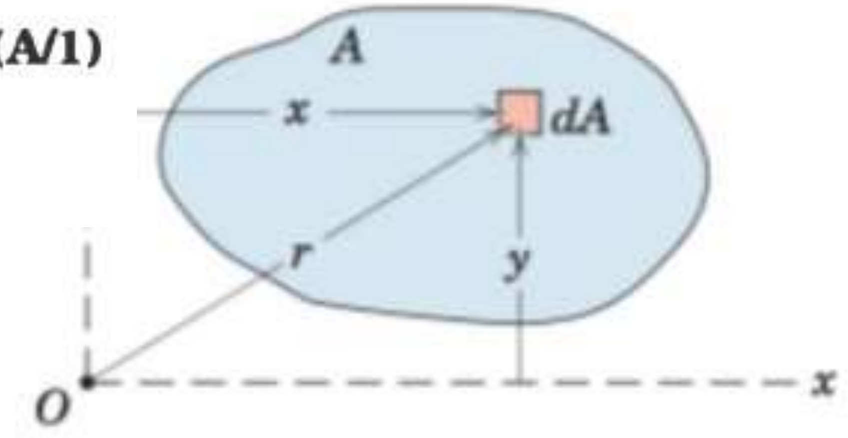


Figure A/2

Rectangular Area moment of inertia and polar moment of inertia

The moment of inertia of dA about the pole O (z-axis) is, by similar definition, $dI_z = r^2\,dA$. The moment of inertia of the entire area about O is

$$\boxed{I_z = \int r^2\,dA} \qquad \text{(A/2)}$$

The expressions defined by Eqs. A/1 are called *rectangular* moments of inertia, whereas the expression of Eq. A/2 is called the *polar* moment of inertia.* Because $x^2 + y^2 = r^2$, it is clear that

$$\boxed{I_z = I_x + I_y} \qquad \text{(A/3)}$$

Radius of gyration

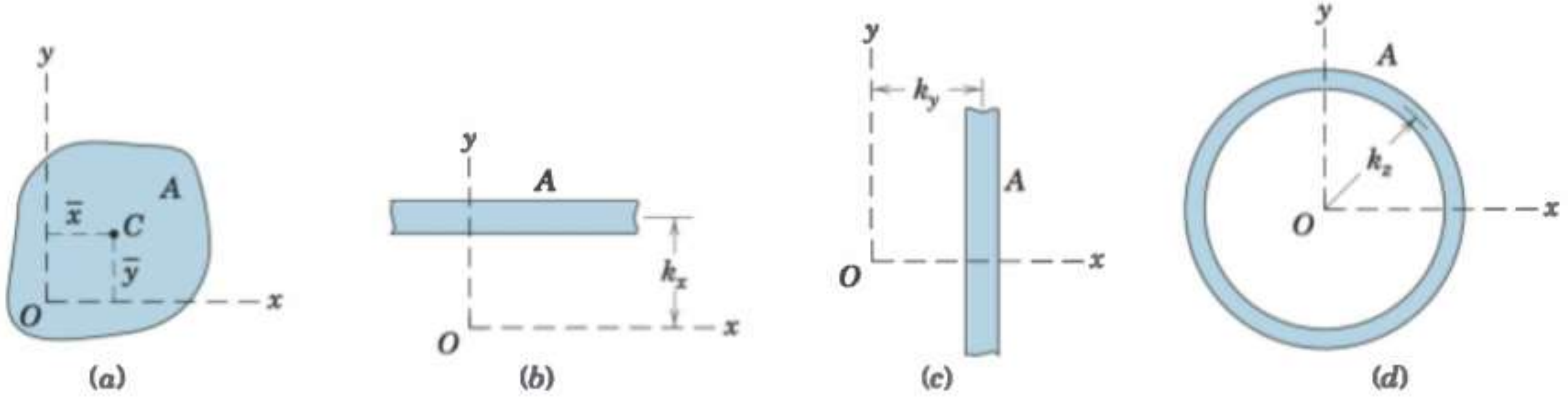


Figure A/3

$$I_x = k_x^2 A$$
$$I_y = k_y^2 A$$
$$I_z = k_z^2 A$$

or

$$k_x = \sqrt{I_x/A}$$
$$k_y = \sqrt{I_y/A}$$
$$k_z = \sqrt{I_z/A}$$

$$k_z^2 = k_x^2 + k_y^2$$

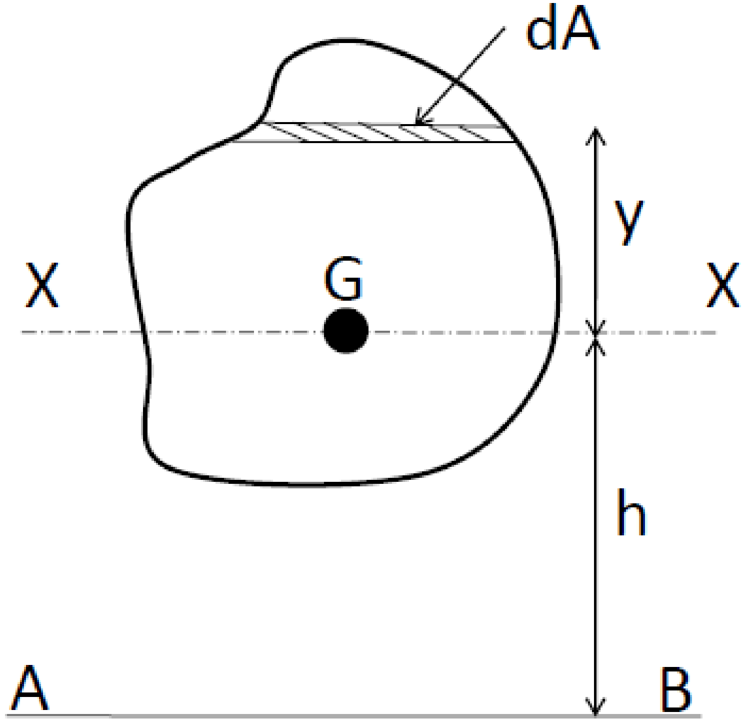
- **Conditions for parallel axis theorem**

1. **Two axis should be there and two axis must be parallel to each other**
2. **Between two axis, one axis has to pass through the centroidal axis**

Theorem of the Parallel axis

It states that if the MI of a plane area about an axis in the plane of area through the CG of the plane area is I_{GG}, then the MI of the given plane area about a parallel axis AB in the plane of area at a distance h from the CG of the area is given by

$$I_{AB} = I_G + Ah^2$$



Theorem of the Parallel axis

Consider a strip parallel to XX at a distance y.

$$(I_{XX})_{dA} = dA.y^2$$

$$I_{XX} = I_G = \sum dA.y^2$$

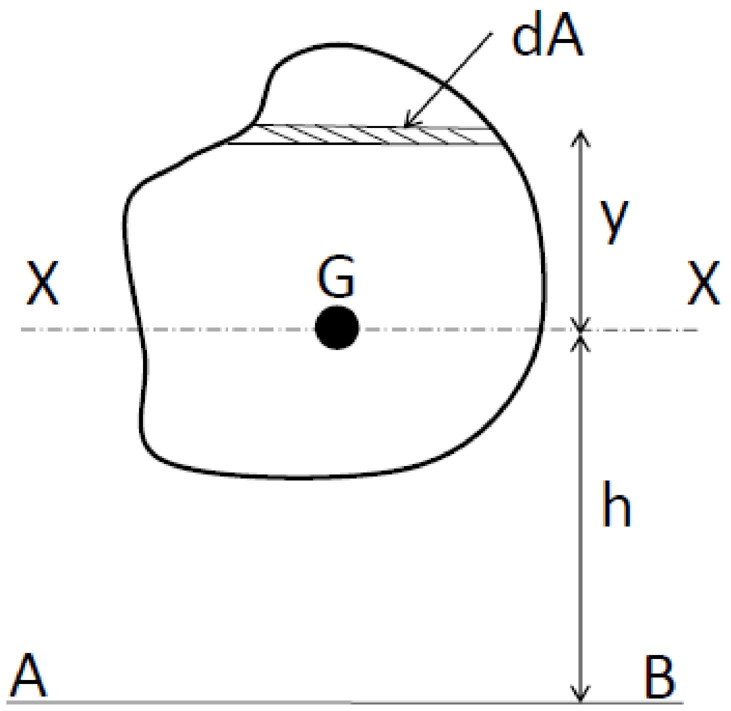
$$(I_{AB})_{dA} = dA.(y+h)^2 = dA(y^2 + h^2 + 2yh)$$

$$I_{AB} = \sum dAy^2 + \sum dAh^2 + \sum 2yhdA$$

$$I_{AB} = h^2 \sum dA + \sum dAy^2 + 2h \sum ydA$$

$$I_{AB} = h^2 A + I_G + 0$$

$$I_{AB} = I_G + Ah^2$$



Transfer axis theorem

$\left[I_x = \int y^2\,dA\right]$ $\qquad \bar{I}_x = \int_{-h/2}^{h/2} y^2 b\,dy = \frac{1}{12}bh^3$ *Ans.*

By interchange of symbols, the moment of inertia about the centroidal y_0-axis is

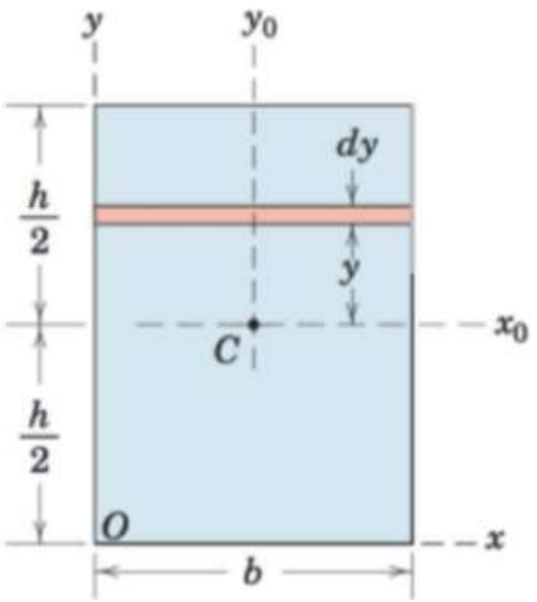
$$\bar{I}_y = \frac{1}{12}hb^3$$ *Ans.*

The centroidal polar moment of inertia is

$\left[\bar{I}_z = \bar{I}_x + \bar{I}_y\right]$ $\qquad \bar{I}_z = \frac{1}{12}(bh^3 + hb^3) = \frac{1}{12}A(b^2 + h^2)$ *Ans.*

By the parallel-axis theorem, the moment of inertia about the x-axis is

$\left[I_x = \bar{I}_x + Ad_x^{\,2}\right]$ $\qquad I_x = \frac{1}{12}bh^3 + bh\left(\frac{h}{2}\right)^2 = \frac{1}{3}bh^3 = \frac{1}{3}Ah^2$ *Ans.*



Transfer axis theorem

Solution. A strip of area parallel to the base is selected as shown in the figure, and it has the area $dA = x\,dy = [(h - y)b/h]\,dy$. By definition

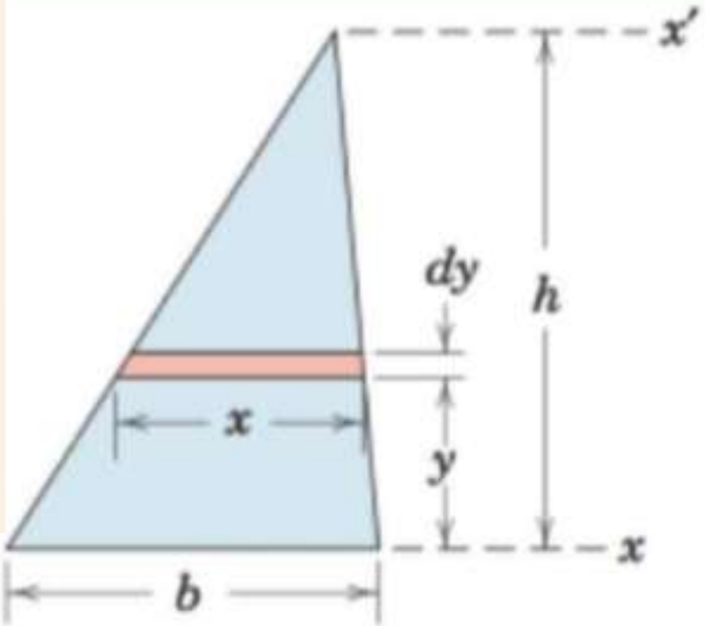
$$\left[I_x = \int y^2\,dA\right] \qquad I_x = \int_0^h y^2 \frac{h-y}{h} b\,dy = b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h = \frac{bh^3}{12} \qquad \textit{Ans.}$$

By the parallel-axis theorem, the moment of inertia $\bar{I}$ about an axis through the centroid, a distance $h/3$ above the x-axis, is

$$[\bar{I} = I - Ad^2] \qquad \bar{I} = \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36} \qquad \textit{Ans.}$$

A transfer from the centroidal axis to the x'-axis through the vertex gives

$$[I = \bar{I} + Ad^2] \qquad I_{x'} = \frac{bh^3}{36} + \left(\frac{bh}{2}\right)\left(\frac{2h}{3}\right)^2 = \frac{bh^3}{4} \qquad \textit{Ans.}$$



Transfer axis theorem

Solution. A differential element of area in the form of a circular ring may be used for the calculation of the moment of inertia about the polar z-axis through O since all elements of the ring are equidistant from O. The elemental area is $dA = 2\pi r_0\, dr_0$, and thus,

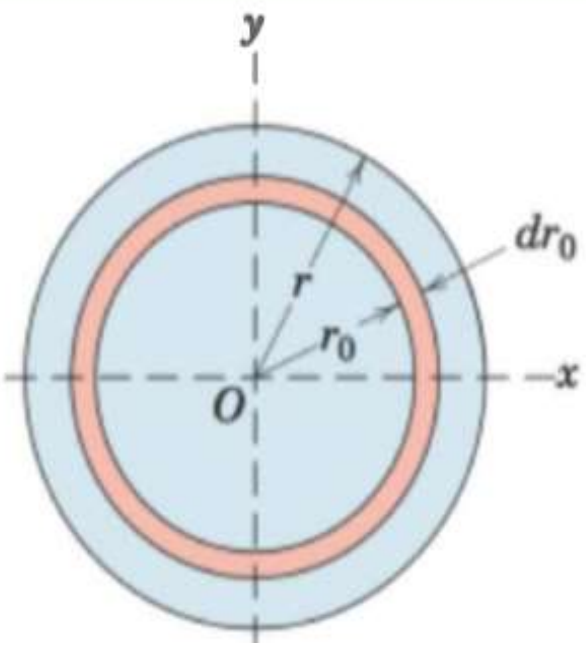
$$\left[I_z = \int r^2\, dA\right] \qquad I_z = \int_0^r r_0^{\,2}(2\pi r_0\, dr_0) = \frac{\pi r^4}{2} = \frac{1}{2}Ar^2 \qquad \textit{Ans.}$$

The polar radius of gyration is

$$\left[k = \sqrt{\frac{I}{A}}\right] \qquad k_z = \frac{r}{\sqrt{2}} \qquad \textit{Ans.}$$

By symmetry $I_x = I_y$, so that from Eq. A/3

$$[I_z = I_x + I_y] \qquad I_x = \frac{1}{2}I_z = \frac{\pi r^4}{4} = \frac{1}{4}Ar^2 \qquad \textit{Ans.}$$



ENGINEERING MECHANICS
Moment of Inertia

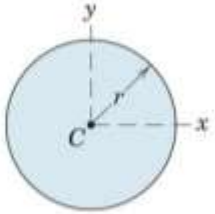
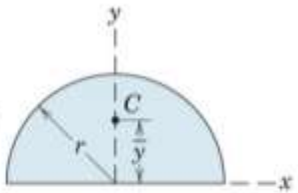
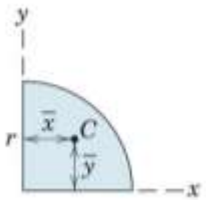
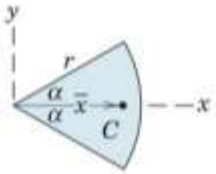
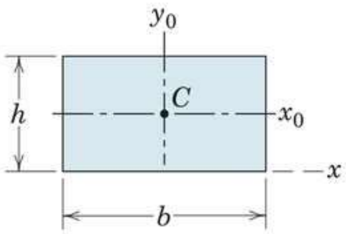
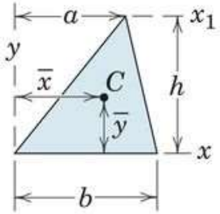
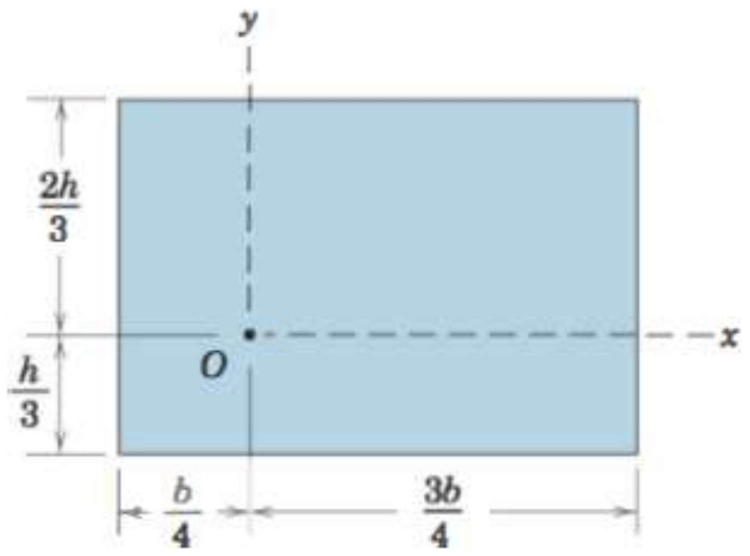
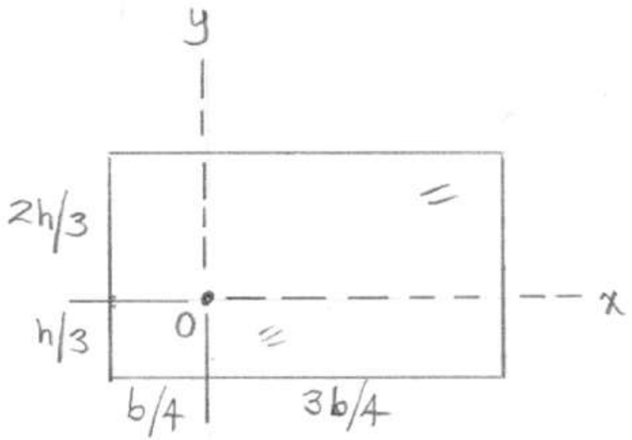
FIGURE		AREA MOMENTS OF INERTIA
Circular Area		$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
Semicircular Area		$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{4}$
Quarter-Circular Area		$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{8}$
Area of Circular Sector		$I_x = \frac{r^4}{4}\left(\alpha - \frac{1}{2}\sin 2\alpha\right)$ $I_y = \frac{r^4}{4}\left(\alpha + \frac{1}{2}\sin 2\alpha\right)$ $I_z = \frac{1}{2} r^4 \alpha$

FIGURE		AREA MOMENTS OF INERTIA
Rectangular Area		$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area		$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$

A/1 Determine the moments of inertia of the rectangular area about the x- and y-axes and find the polar moment of inertia about point 0.





$$I_x = \bar{I}_x + A d_x^2 = \frac{1}{12} bh^3 + bh\left(\frac{h}{6}\right)^2$$

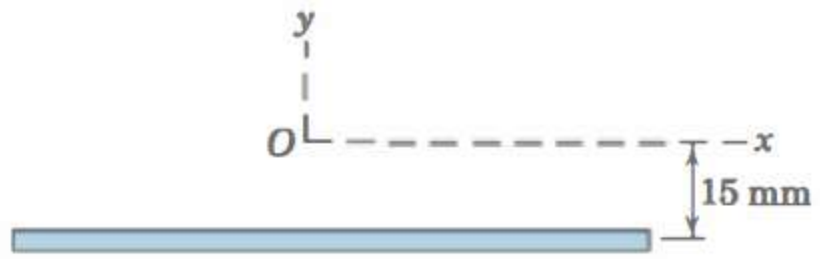
$$= \underline{\frac{1}{9} bh^3}$$

$$I_y = \bar{I}_y + A d_y^2 = \frac{1}{12} hb^3 + bh\left(\frac{b}{4}\right)^2$$

$$= \underline{\frac{7}{48} hb^3}$$

$$I_z = I_x + I_y = \underline{bh\left(\frac{h^2}{9} + \frac{7b^2}{48}\right)}$$

A/3 The narrow rectangular strip has an area of 300 mm², and its moment of inertia about the y-axis is 35(10³) mm⁴. Obtain a close approximation to the polar radius of gyration about point 0.



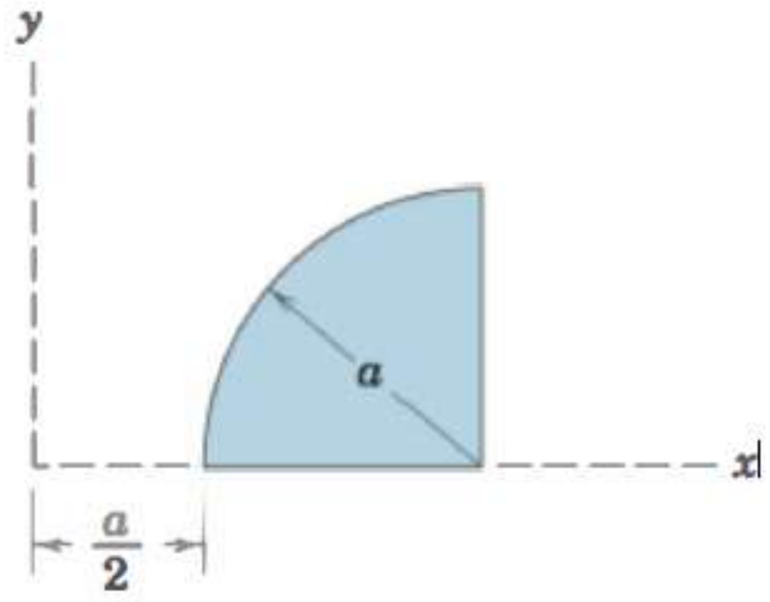
$$I_x \approx Ad^2 = 300(15)^2 = 67.5(10^3)\ mm^4$$

$$J_o = I_x + I_y = 67.5(10^3) + 35(10^3) = 102.5(10^3)\ mm^4$$

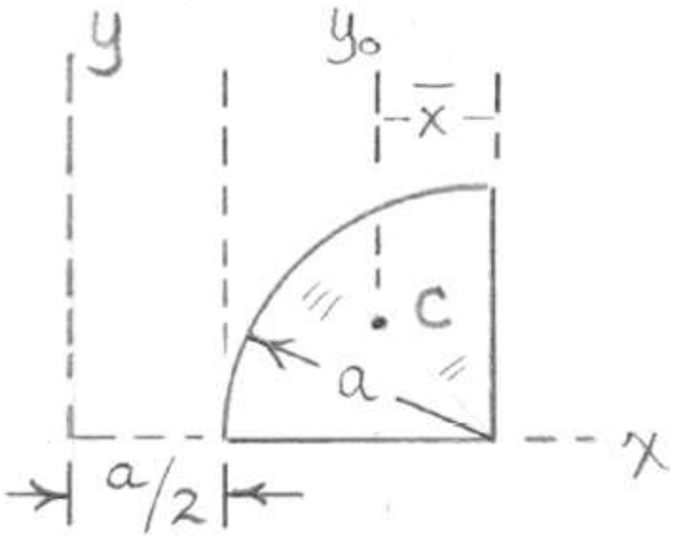
$$k_o = \sqrt{J_o/A} = \sqrt{\frac{102.5(10^3)}{300}} = \underline{18.48\ mm}$$

Moment of Inertia

A/7 Determine the moment of inertia of the quarter circular area about the y-axis.



A/7



From Table D/3:

$$\bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4$$

$$\bar{x} = \frac{4a}{3\pi}$$

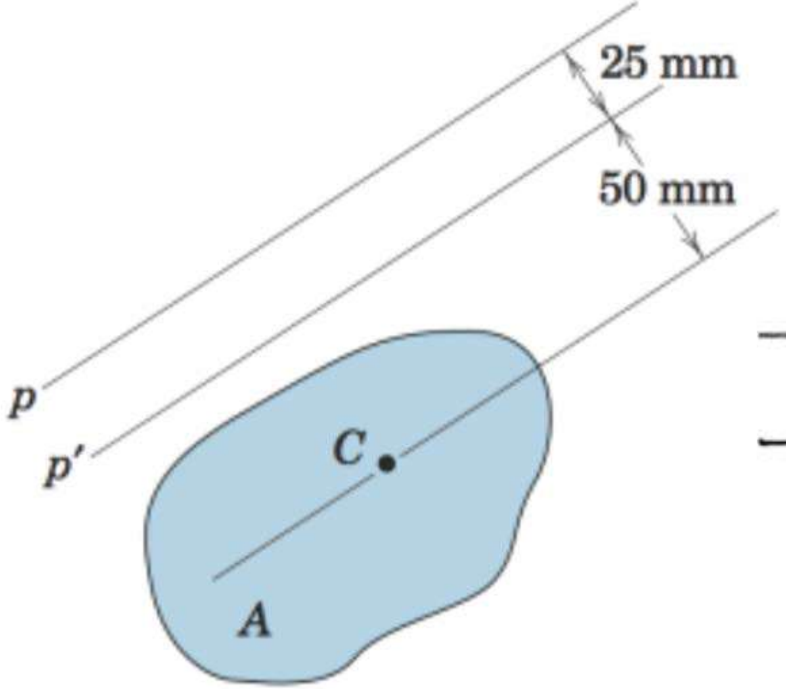
$$I_y = \bar{I}_y + Ad_y^2$$

$$= \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4 + \frac{\pi a^2}{4}\left[\frac{a}{2} + \left(a - \frac{4a}{3\pi}\right)\right]^2$$

$$= \left[\frac{5\pi}{8} - 1\right]a^4$$

Moment of Inertia

A/9 The moments of inertia of the area A about the parallel p- and p'-axes differ by 15(10^6) mm^4. Compute the area A, which has its centroid at C.



A/9

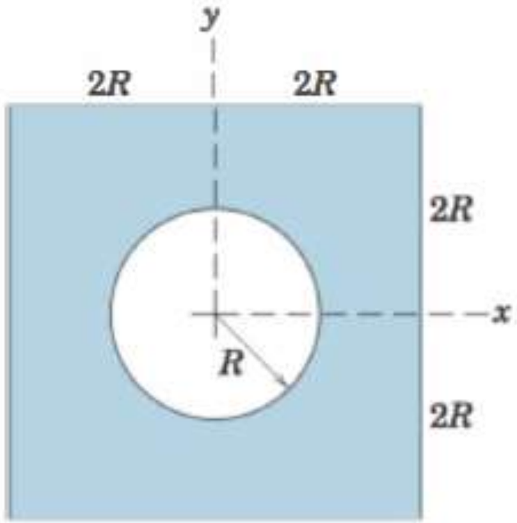
$$I_p = I_c + A(75)^2, \quad I_{p'} = I_c + A(50)^2$$

$$I_p - I_{p'} = 15(10^6) = A[(75)^2 - (50)^2]$$

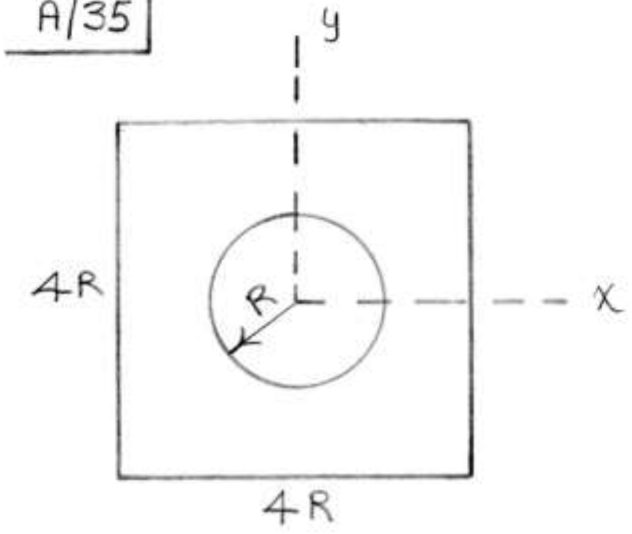
$$\underline{A = 4800 \text{ mm}^2}$$

Moment of Inertia

A/35 Determine the moment of inertia about the x-axis of the square area without and with the central circular hole.



A/35



Without hole, $I_x = \frac{1}{12}(4R)(4R)^3 = \frac{64}{3}$

$(21.3R^4)$

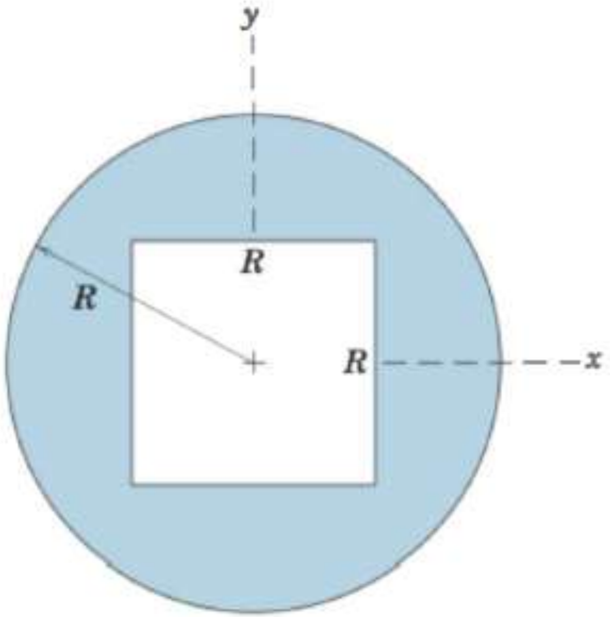
With hole, $I_x = \frac{64}{3}R^4 - \frac{1}{4}(\pi R^2)R^2$

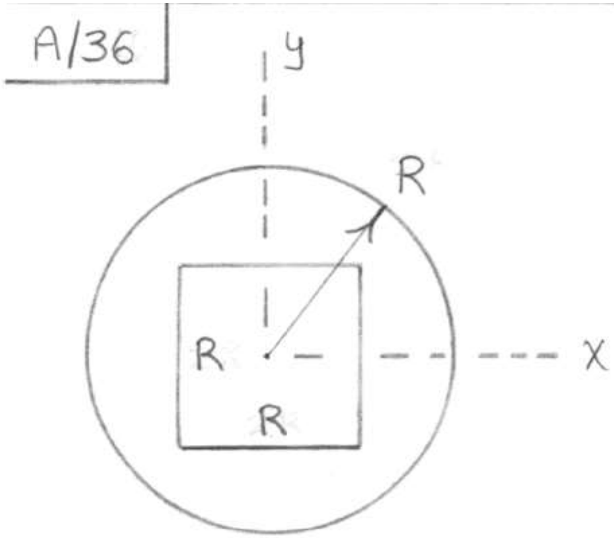
$= \underline{20.5R^4}$

(a 3.68% reduction)

Moment of Inertia

A/36 Determine the polar moment of inertia of the circular area without and with the central square hole.





Without square hole:

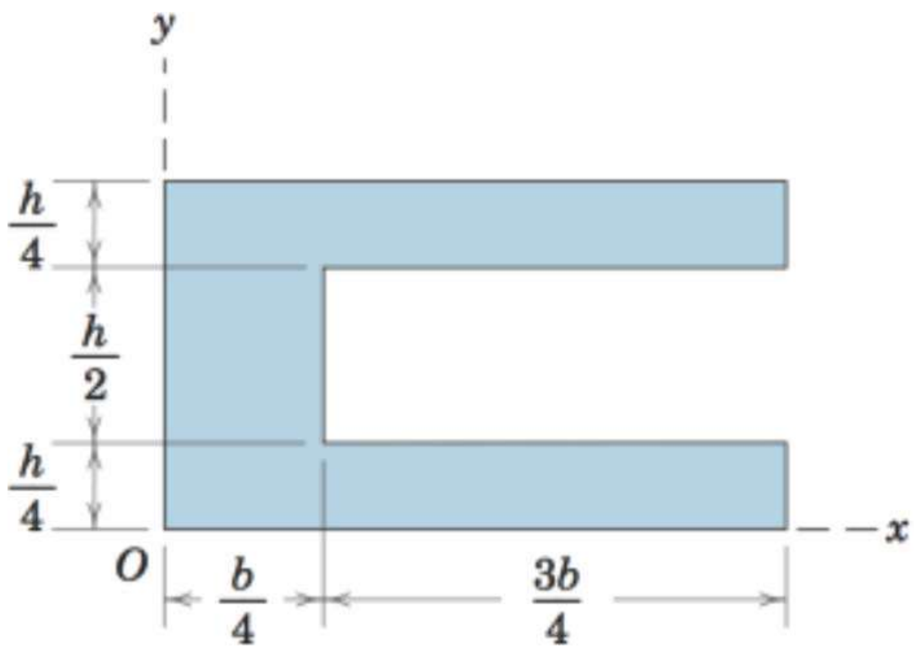
$$I_z = 2I_x = 2\left(\frac{1}{4}\pi R^2 \cdot R^2\right) = \underline{1.571 R^4}$$

With hole:

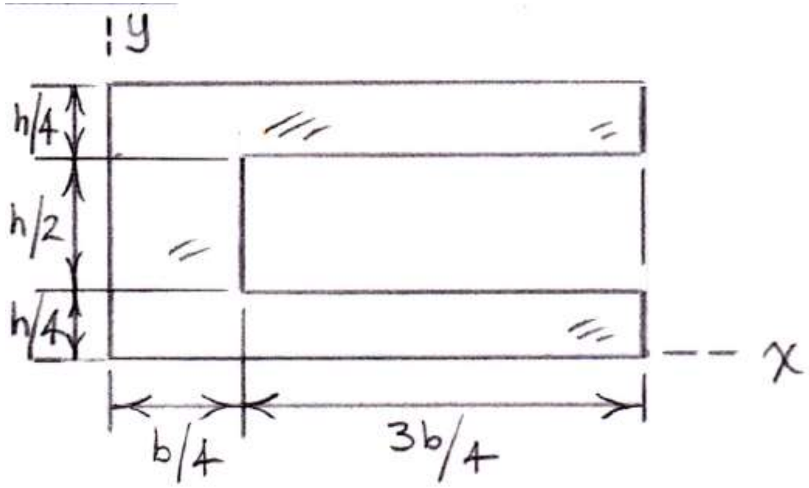
$$I_z = 1.571 R^4 - 2\left(\frac{1}{12} R \cdot R^3\right) = \underline{1.404 R^4}$$

(a reduction of 10.61%)

A/40 Determine the percent reductions in both area and area moment of inertia about the y-axis caused by removal of the rectangular cutout from the rectangular plate of baseband height h.



Moment of Inertia



Percent reductions :

$$n_A = \frac{bh - \frac{5}{8}bh}{bh}(100\%) = \underline{37.5\%}$$

$$n_{I_y} = \frac{\frac{1}{3}hb^3 - \frac{65}{384}hb^3}{\frac{1}{3}hb^3} = \underline{49.2\%}$$

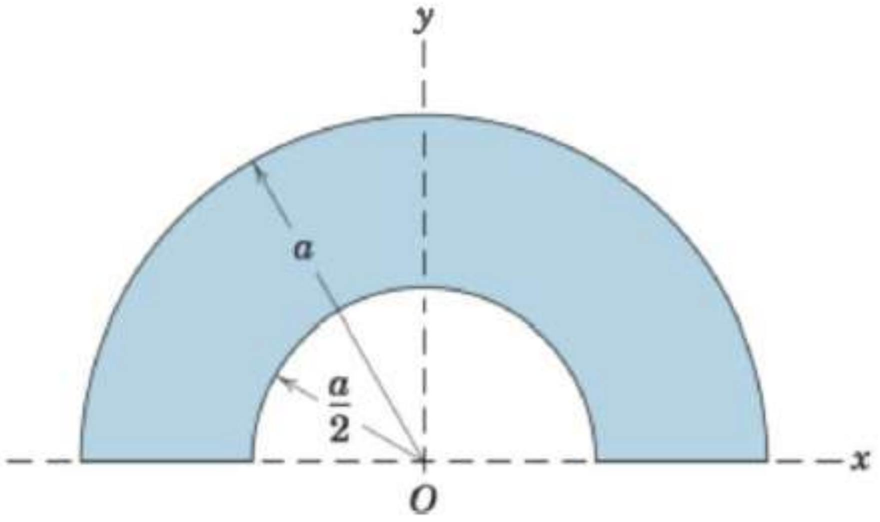
Full rectangle : $A = bh$, $I_y = \frac{1}{3}hb^3$

With cutout : $A = bh - \frac{3b}{4}\left(\frac{h}{2}\right) = \frac{5}{8}bh$

$$I_y = \frac{1}{3}hb^3 - \left[\frac{1}{12}\frac{h}{2}\left(\frac{3b}{4}\right)^3 + \frac{3}{8}bh\left(\frac{b}{4} + \frac{3b}{8}\right)^2\right]$$

$$= \frac{65}{384}hb^3$$

A/38 Calculate the polar radius of gyration of the area of the angle section about point A Note that the width of the legs is small compared with the length of each leg.



A/38

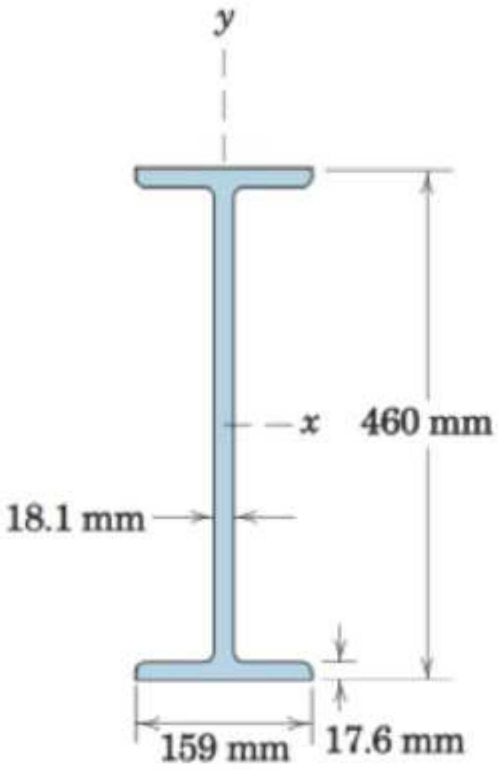
$$I_z = \frac{1}{2}\left[\frac{\pi a^4}{2} - \frac{\pi\left(\frac{a}{2}\right)^4}{2}\right] = \frac{15}{64}\pi a^4$$

$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{\frac{15}{64}\pi a^4}{\frac{3}{8}\pi a^2}} = \underline{\frac{\sqrt{10}}{4}a}$$

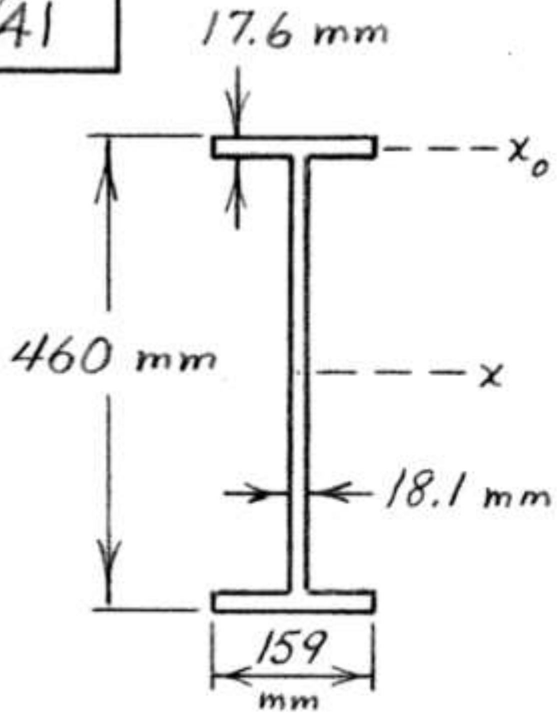
From $k_x^2 + k_y^2 = k_z^2$ and the fact that $k_x = k_y$ for the present case,

$$2k_x^2 = \left(\frac{\sqrt{10}}{4}a\right)^2, \quad k_x = k_y = \underline{\frac{\sqrt{5}}{4}a}$$

A/41 The cross-sectional area of an I-beam has the dimensions shown. Obtain a close approximation to the handbook value of Ix = 385(10^6) mm^4 by treating the section as being composed of three rectangles.



A/41



Flanges: $\bar{I}_x = I_{x_0} + Ad^2$

$$= 2\left\{\frac{1}{12}(159)(17.6^3) + 159(17.6)\left(230 - \frac{17.6}{2}\right)^2\right\}$$

$$= 2\{7.22(10^4) + 1.369(10^8)\} \text{ mm}^4$$

$$= 2.74(10^8) \text{ mm}^4$$

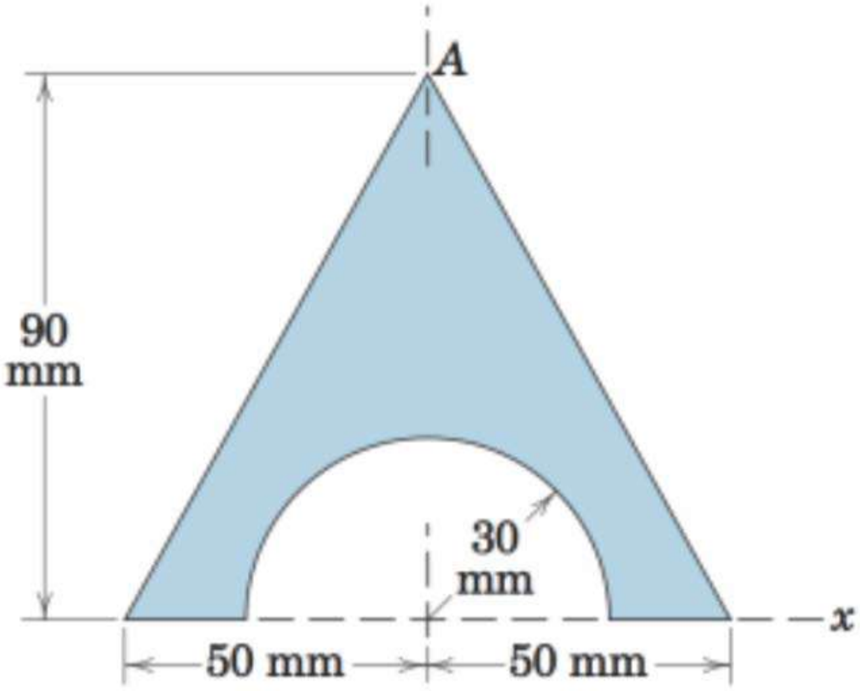
Web: $\bar{I}_x = \frac{1}{12}(18.1)(460 - 2[17.6])^3$

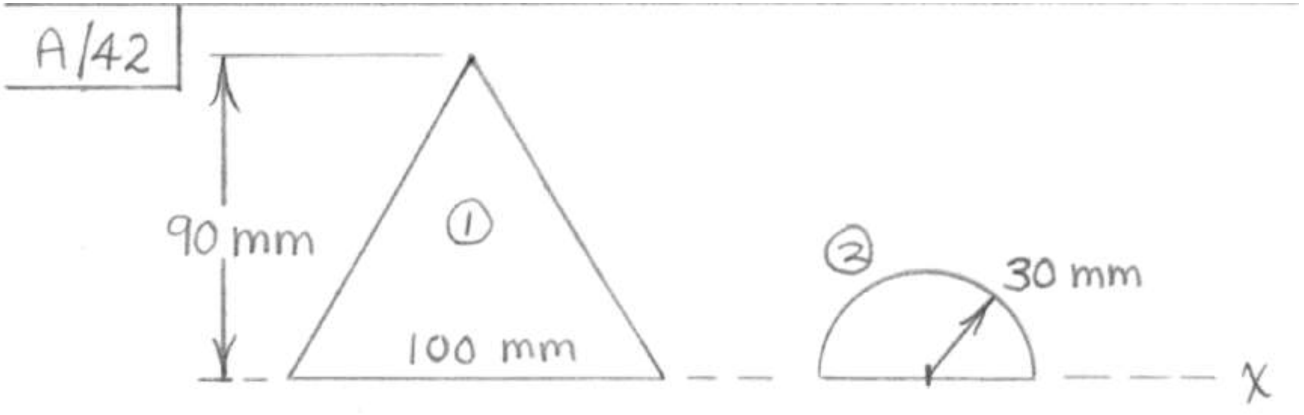
$$= 1.156(10^8) \text{ mm}^4$$

Total $\bar{I}_x = 3.90(10^8)$ mm^4

Moment of Inertia

A/42 Calculate the moment of inertia of the shaded area about the x-axis.



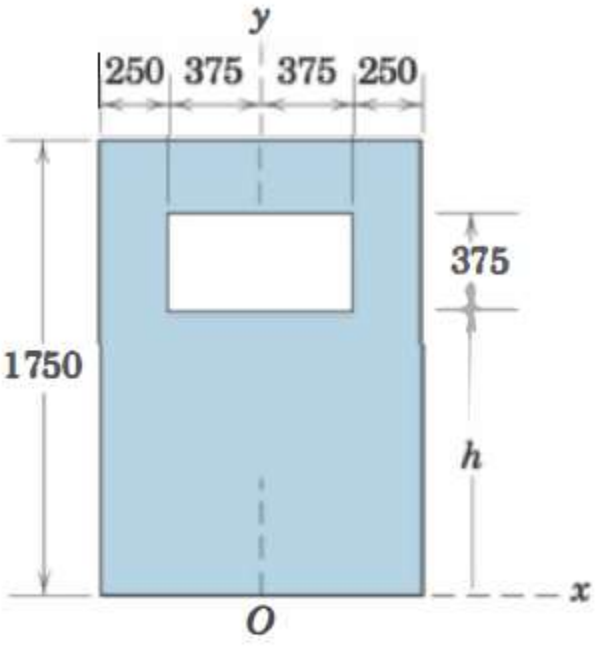


$$I_{x_1} = \frac{1}{12}(100)(90^3) = 6.08(10^6)\ \text{mm}^4$$

$$I_{x_2} = -\frac{\pi(30^4)}{8} = -0.318(10^6)\ \text{mm}^4$$

$$\text{So}\ I_x = (6.08-0.318)10^6 = \underline{5.76(10^6)\ \text{mm}^4}$$

A/43 The variable h designates the arbitrary vertical location of the bottom of the rectangular cutout within the rectangular area. Determine the area moment of inertia about the x-axis for (a) h = 1000 mm and (b) h = 1500 mm.



Dimensions in millimeters

(a) $h = 1000$ mm (hole complete)

$$I_x = \frac{1}{3}(1250)(1750^3) - \left[\frac{1}{12}(750)(375)^3 + 750(375)\left(1000 + \frac{375}{2}\right)^2\right]$$

$$= \underline{1.833(10^{12})\ \text{mm}^4} \text{ or } \underline{1.833\ \text{m}^4}$$

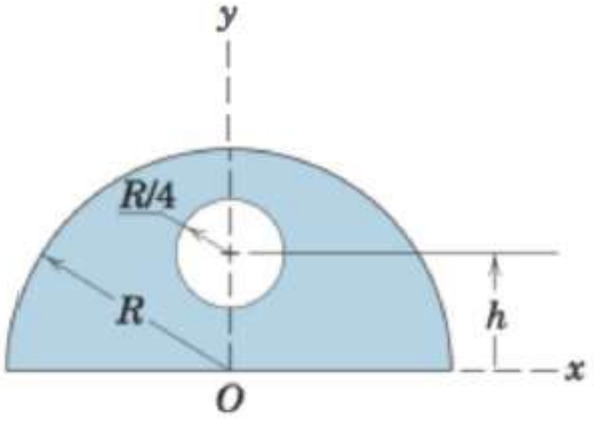
(b) $h = 1500$ mm (250 mm of hole in play)

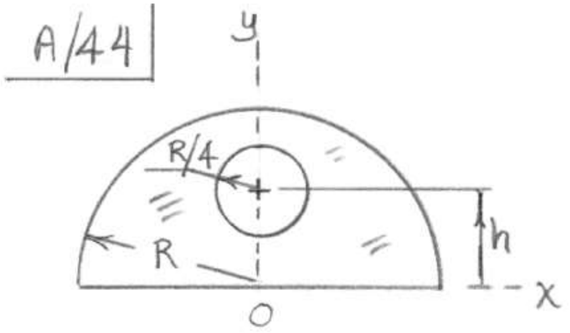
$$I_x = \frac{1}{3}(1250)(1750^3) - \left[\frac{1}{12}(750)(250)^3 + 750(250)\left(1500 + \frac{250}{2}\right)^2\right]$$

$$= \underline{1.737(10^{12})\ \text{mm}^4 \text{ or } 1.737\ \text{m}^4}$$

Moment of Inertia

A/44 The variable h designates the arbitrary vertical location of the center of the circular cutout within the semicircular area. Determine the area moment of inertia about the x-axis for (a) h = 0 and (b) h = R/2.





(a) $h = 0$ (One-half of hole considered)

$$I_x = \frac{\pi R^4}{8} - \frac{\pi (R/4)^4}{8} = \frac{255}{2048}\pi R^4$$

$$(0.391 R^4)$$

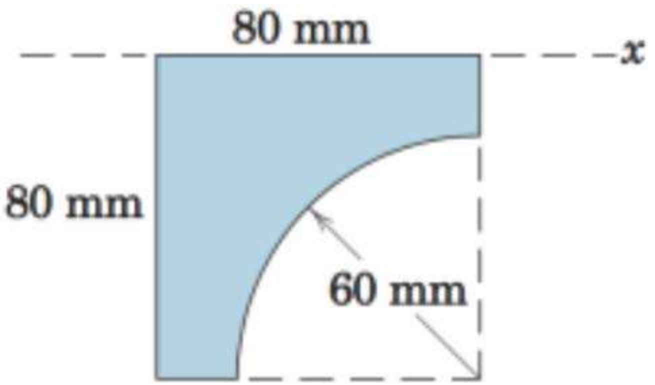
(b) $h = \frac{R}{2}$ (Entire hole now in play)

$$I_x = \frac{\pi R^4}{8} - \left[\frac{\pi (R/4)^4}{4} + \pi\left(\frac{R}{4}\right)^2\left(\frac{R}{2}\right)^2\right]$$

$$= \frac{111}{1024}\pi R^4 \quad (0.341 R^4)$$

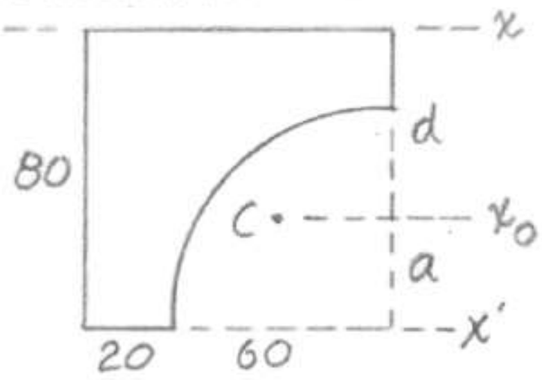
Moment of Inertia

A/45 Calculate the moment of inertia of the shaded area about the x-axis.



A/45

Dimen. in mm



Square: $I_x = \frac{1}{3}b^4 = \frac{1}{3}(80)^4 = 13.65(10^6)\text{ mm}^4$

Quarter-circle: $a = \frac{4r}{3\pi} = \frac{4(60)}{3\pi}$

$= 25.46\text{ mm}$

$d = 80 - 25.46 = 54.54\text{ mm}$

$I_x = I_{x_0} + Ad^2 = I_{x'} - Aa^2 + Ad^2$

$= -\frac{1}{4}\frac{\pi r^4}{4} - \frac{\pi r^2}{4}(d^2 - a^2) = -\frac{\pi r^2}{4}\left(\frac{r^2}{4} + d^2 - a^2\right)$

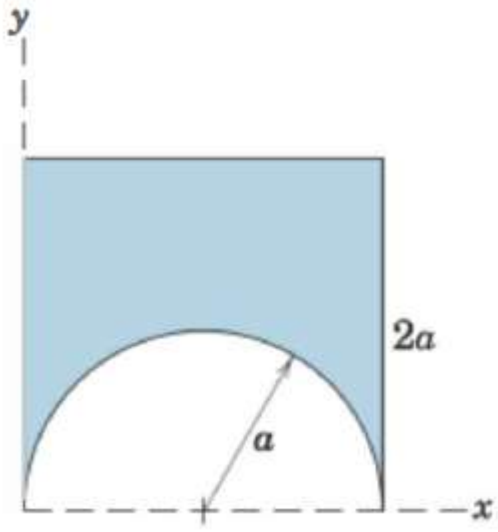
$= -\frac{\pi(60)^2}{4}\left[\frac{60^2}{4} + (54.54)^2 - (25.46)^2\right]$

$= -9.120(10^6)\text{ mm}^4$

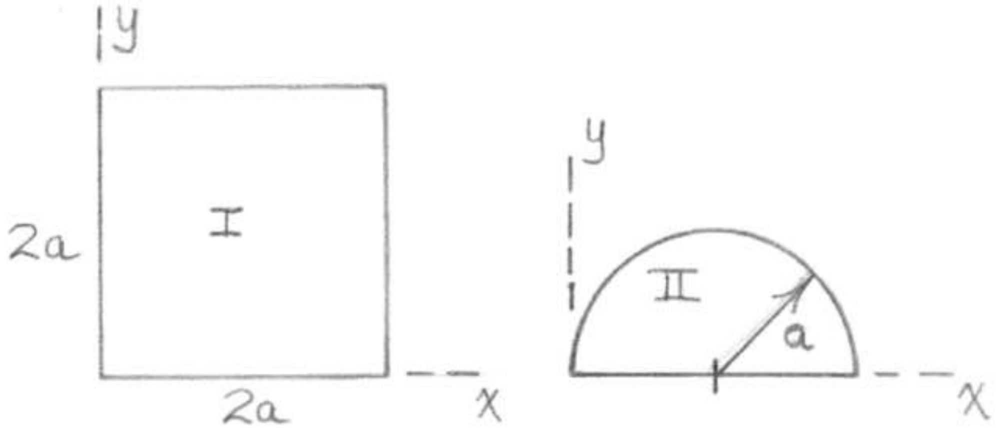
Total $I_x = (13.65 - 9.120)(10^6) = \underline{4.53(10^6)\text{ mm}^4}$

Moment of Inertia

A/46 Determine the moments of inertia of the shaded area about the x- and y-axes.



A/46



I. Square $I_x = I_y = \frac{1}{3}(4a^2)(2a)^2 = \frac{16}{3}a^4$

II. Semicircle $I_x = \frac{1}{8}\pi a^4$

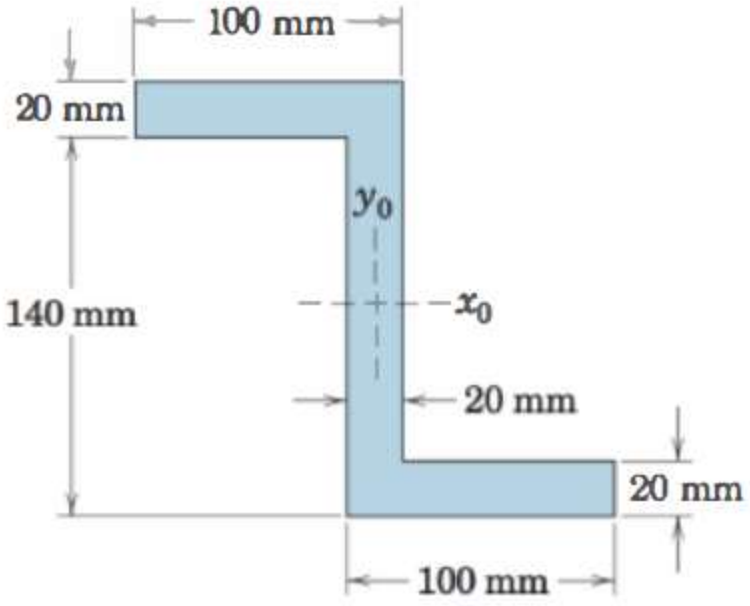
$$I_y = \frac{1}{8}\pi a^4 + \frac{1}{2}\pi a^2(a^2) = \frac{5}{8}\pi a^4$$

Combined: $I_x = \frac{16}{3}a^4 - \frac{\pi}{8}a^4 = \underline{4.94a^4}$

$$I_y = \frac{16}{3}a^4 - \frac{5}{8}\pi a^4 = \underline{3.37a^4}$$

Moment of Inertia

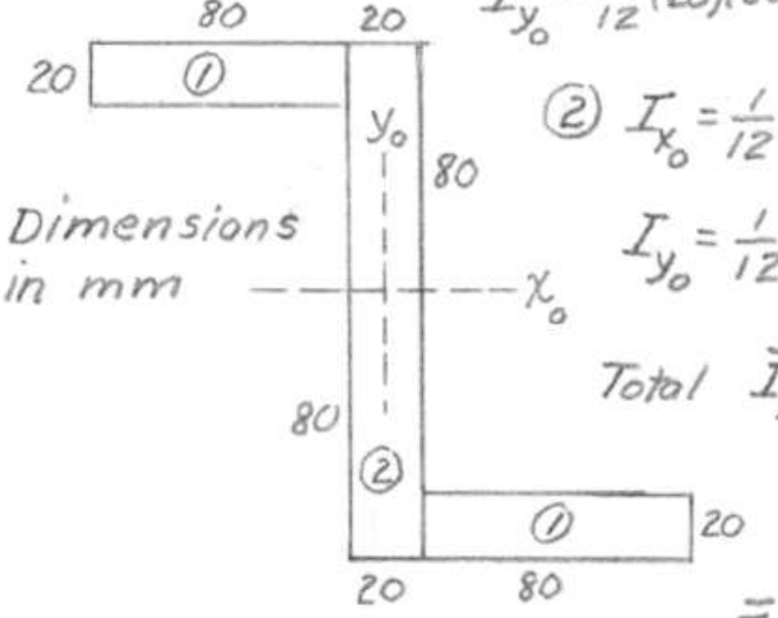
A/48 Determine the moments of inertia of the Z-section about its centroidal xo- and yo-axes.



A/48

① $I_{x_o} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6)\ \text{mm}^4$

$I_{y_o} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6)\ \text{mm}^4$



② $I_{x_o} = \frac{1}{12}(20)(160)^3 = 6.83(10^6)\ \text{mm}^4$

$I_{y_o} = \frac{1}{12}(160)(20)^3 = 0.1067(10^6)\ \text{mm}^4$

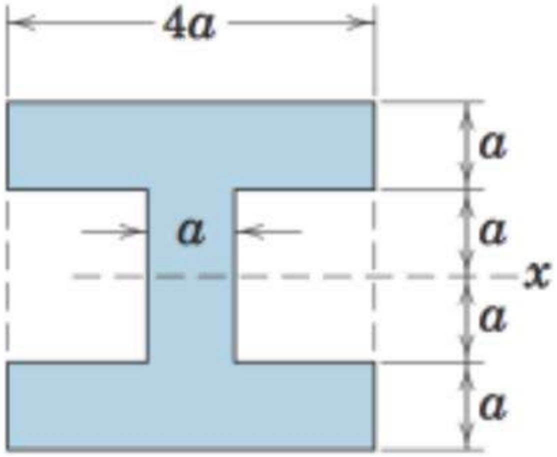
Total $\bar{I}_x = [2(7.89) + 6.83](10^6)$

$= \underline{22.6(10^6)\ \text{mm}^4}$

$\bar{I}_y = [2(4.85) + 0.1067](10^6)$

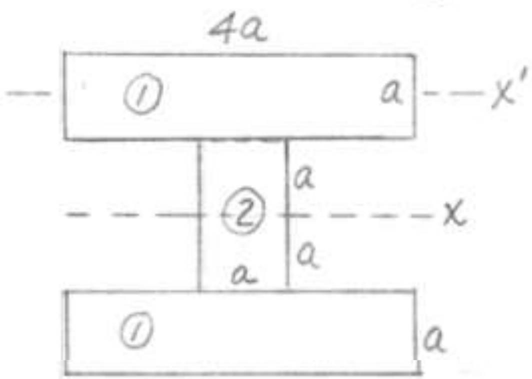
$= 9.81(10^6)\ \text{mm}^4$

A/49 Determine the moment of inertia of the shaded area about the x-axis in two different ways.



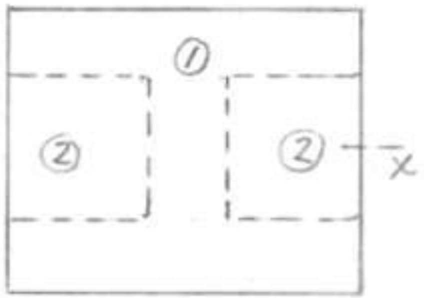
A/49

Sol. I $\quad I_{1} = 2\left[\frac{1}{12}4a(a^3) + 4a^2\left(\frac{3a}{2}\right)^2\right] = \frac{56}{3}a^4$



$I_{2} = \frac{1}{12}a(2a)^3 = \frac{2}{3}a^4$

Total $\quad I_x = \frac{58}{3}a^4$



Sol. II

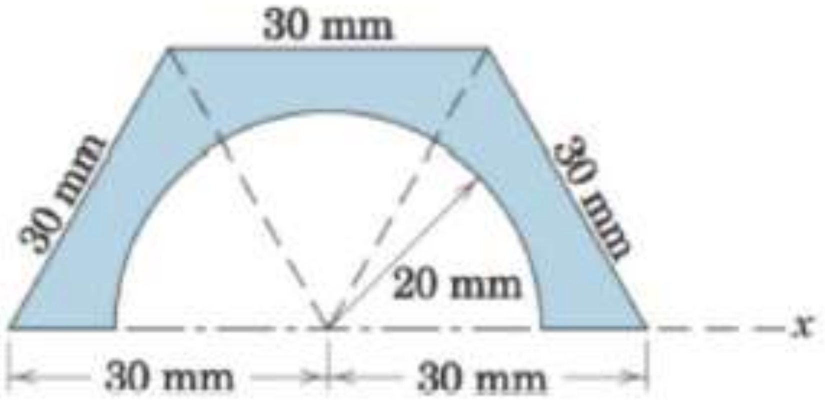
$I_{1} = \frac{1}{12}(4a)(4a)^3 = \frac{64}{3}a^4$

$I_{2} = -\frac{1}{12}(3a)(2a)^3 = -2a^4$

Total $= \left(\frac{64}{3} - \frac{6}{3}\right)a^4 = \frac{58}{3}a^4$

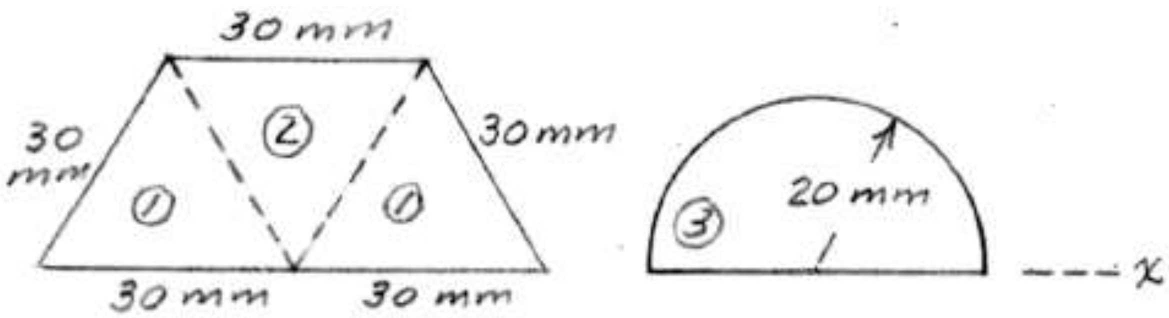
Moment of Inertia

A/51 Calculate the moment of inertia of the shaded area about the x-axis.



Problem A/51

Moment of Inertia



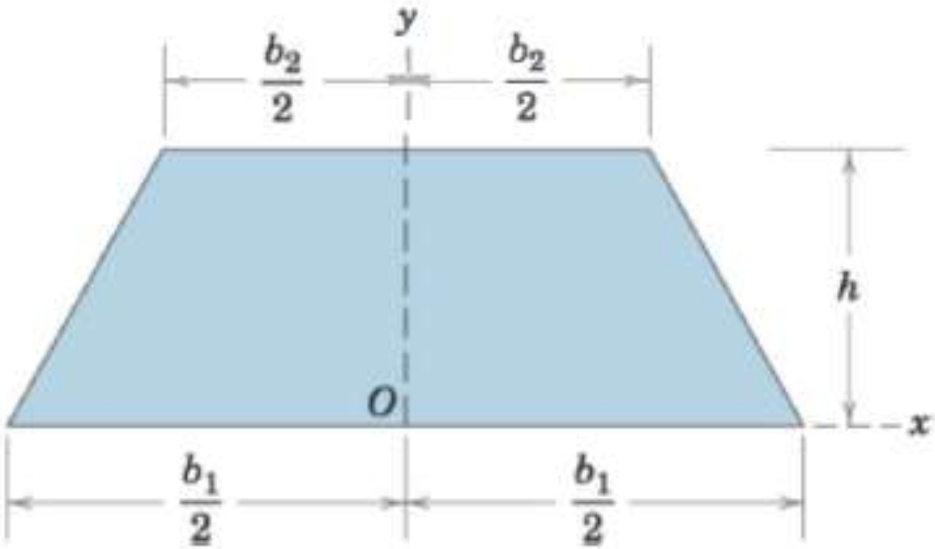
① $I_x = 2\left(\frac{1}{12}\right)(30)\left(30\frac{\sqrt{3}}{2}\right)^3 = \frac{81}{16}\sqrt{3}(10^4)\ mm^4$

② $I_x = \frac{1}{4}(30)\left(30\frac{\sqrt{3}}{2}\right)^3 = \frac{243}{32}\sqrt{3}(10^4)\ mm^4$

③ $I_x = -\frac{1}{2}\left(\frac{1}{4}\pi[20]^4\right) = -2\pi(10^4)\ mm^4$

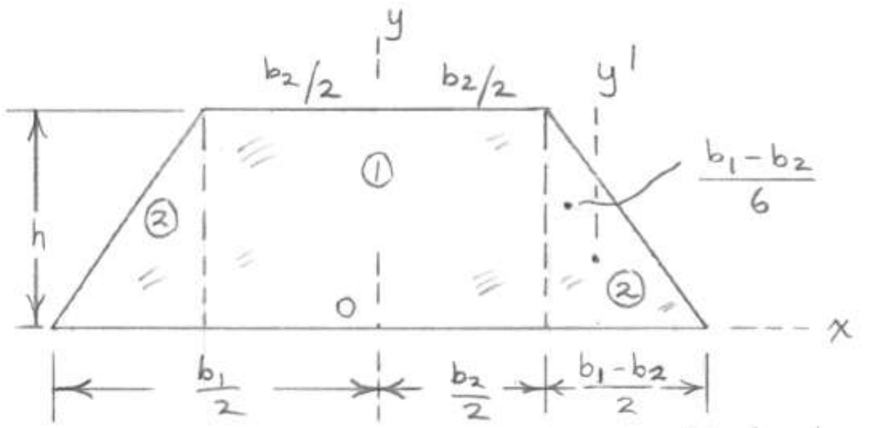
Total $I_x = \underline{15.64(10^4)\ mm^4}$

A/54 By the method of this article, determine the moments of inertia about the x- and y-axes of the trapezoidal area.



Problem A/54

Moment of Inertia



$$I_x = I_{x_1} + 2I_{x_2} = \frac{1}{3} b_2 h^3 + 2\left[\frac{1}{12}\,\frac{b_1 - b_2}{2}\, h^3\right]$$

$$= \underline{h^3\left(\frac{b_1}{12} + \frac{b_2}{4}\right)}$$

$$I_y = I_{y_1} + 2I_{y_2}$$

$$= \frac{1}{12} h b_2^3 + 2\left[\frac{1}{36} h \left(\frac{b_1 - b_2}{2}\right)^3 + \frac{1}{2} h \left(\frac{b_1 - b_2}{2}\right)\left(\frac{b_2}{2} + \frac{b_1 - b_2}{6}\right)^2\right]$$

$$\vdots$$

$$= \underline{\frac{h}{48}\left(b_1^3 + b_1^2 b_2 + b_1 b_2^2 + b_2^3\right)}$$

THANK YOU

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