CL1_Q1. Magnetic monopoles do not exist. Justify.

Answer

From Maxwell's equation 2 we have

$$\nabla . B=0$$

Which implies that the magnetic field does not diverge ruling out the possibility of a magnetic monopole.

CL1_Q2. What is the physical meaning of a gradient?

Answer

The gradient is the operation of the directional derivative on a scalar field. Given a scalar function $\phi\,(x, y, z)$ the gradient, $\nabla\phi(x, y, z) = \hat{\imath}\,\frac{\partial\phi(x)}{\partial x} + \hat{\jmath}\,\frac{\partial\phi(y)}{\partial y} + \hat{k}\,\frac{\partial\phi(z)}{\partial z}$. This yields the direction and magnitude of the maximum rate of change of the field.

CL1_Q3. What is the physical significance of divergence and curl?

Answer

The divergence and curl of a vector field are two vector operators. The physical significance of divergence is the indication of the spreading of the vector from a particular point. The positive divergence signifies that the point or region is a source, and the negative divergence signifies that the point or region is a sink. Thus, divergence represents something spreading out of or converging into a volume element. When the divergence is zero, then the amount that flows in must be equal to the amount that flows out.

The curl is a measure of the rotation of a vector field. The curl of a vector field measures the tendency for the vector field to swirl around.

CL1_Q4. Explain the significance of Faraday's law of electromagnetic induction

Answer

The law of electromagnetic induction put forward by Michael Faraday states that if flux through a loop of wire changes, then a electromagnetic force is induced in the wire which has the capability of driving a current through it. Mathematically, this law is stated as $\int E.dl = - d\phi_B/dt$, where ϕ_B is the magnetic flux through the circuit.

CL2_Q1: What is the fundamental difference between the integral and the differential form of Maxwell's equations?

Answer

Integral form of Maxwell's equations apply to a large region of space and many a times it is very difficult to calculate terms like ∫E.dS, ∫E.dl, ∫B.dS and ∫B.dl. Only if E and B are constants or make constant angles with lines and surfaces, then these integrals can be calculated. It is desirable to convert Maxwell's equations into the so called differential forms which apply to every point in space in contrast to the integral forms that apply to large regions of space.

CL2_Q2: Which are the Maxwell's equations that contain 'sources'?

Answer

- $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_o}$
- $\vec{\nabla}x\vec{B} = \mu_o\vec{J} + \mu_o\varepsilon_o \frac{\partial\vec{E}}{\partial t}$

CL2_Q3: How do Maxwell's equations describe electromagnetic waves?

Answer

Using Maxwell's equations we can construct wave equations for both electric and magnetic fields as

$$\nabla^2\vec{E} = \left(\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}\right) \text{ and } \nabla^2\vec{B} = \left(\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}\right)$$

Maxwell concluded that they should be electric and magnetic vector in free space travelling at the speed of light $c = \sqrt{\frac{1}{\mu_o\varepsilon_o}}$

The electric and magnetic waves must therefore be representing light and hence Maxwell proposed that light could be treated as electromagnetic waves, where the electric and magnetic vectors are mutually perpendicular and perpendicular to the direction of propagation of the radiation.

CL2_Q4: Discuss the phase correlation and direction of the E and B fields of an EM Wave.

Answer

The electric field and the magnetic field are described by

$$E = E_0 \sin(\omega t - kx) \; \; and \; B = B_0 \sin(\omega t - kx).$$

There is no phase difference between them. However, they are perpendicular to each other.

CL3_Q1. Explain how Poynting vector explains the energy flow.

Answer

EM waves carry energy in the direction perpendicular to the E and B field variations and are described by the Poynting vector as, $\boldsymbol{s} \equiv \boldsymbol{c^2 \varepsilon_0 E} \times \boldsymbol{B} = \frac{\boldsymbol{1}}{\boldsymbol{\mu_o}} \boldsymbol{E} \times \boldsymbol{B}$

In terms of time varying electric field component and hence to determine the average energy of the wave transmitted per unit time through unit area can be found out as

$$\text{Average Energy} < S > = \frac{c\varepsilon_0}{T} \int_0^T E_x{}^2 \, dt = \frac{c\varepsilon_0}{T} \int_0^T \boldsymbol{E_{ox}}^2 \cos^2(\boldsymbol{\omega t} + \boldsymbol{kz}) \, dt$$
$$= \frac{1}{2} \varepsilon_0 c E_{ox}{}^2$$

CL3_Q2. Differentiate between circularly and elliptically polarized light?

Answer

If light is composed of two plane waves of equal amplitude but differing in phase by 90°, then the light is said to be circularly polarized. If two plane waves of differing amplitude are related in phase by 90°, or if the relative phase is other than 90° then the light is said to be elliptically polarized.

CL3_Q3. Find the energy density of electromagnetic wave, if the electric field of amplitude 6.2 V/m oscillates with a frequency of 2.4 X 10^{10} Hz.

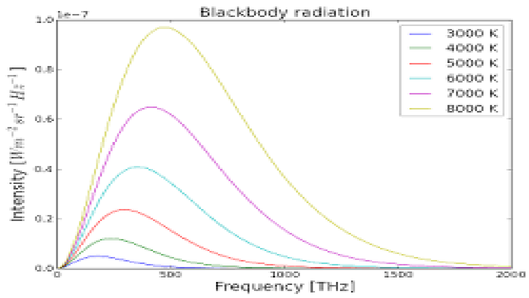
Answer

From Maxwell's equation, we know, Energy density = $\varepsilon_0 E^2$
Here, E_{max} = 6.2 V/m and ε_0 = 8.85 X 10^{-12} C^2/Nm^2
Therefore,

Energy Density = 8.85 X 10^{-12} X 6.2 X 6.2 = 3.4 X 10^{-10} J/m^3.

CL4_Q1. Mention the characteristics of a black body spectrum.

Answer



From the spectrum of black body radiation the following inferences can be made.

- At all temperatures, energy radiated by the body first increases, reaches a maximum at a particular frequency and then decreases.
- As the temperature increases, the peak shifts to higher frequencies
- An increase in temperature causes an increase in energy emission for all frequencies

The total area bounded by the curves with the X – axis gives the rate of radiation through unit area of the body and is found to be the fourth power of the temperature of the body.

CL4_Q2. Mention Planck's formula for black body radiation.

Answer

Based on the assumptions of quantum theory of radiation, Planck derived a formula according to which, the energy density of radiation can be evaluated as

$$\rho(\nu)d\nu = \langle E\rangle dN = \frac{8\pi}{c^3}\nu^2 d\nu \frac{h\nu}{e^{h\nu/kT} - 1} = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT} - 1}\, d\nu$$

CL4_Q3. Calculate the average energy of Planck's oscillator of frequency 5.6×10^{12}Hz at 330 K.

Answer

Given: v=5.6×10^{12}Hz ; T=330K

The average energy of Planck's oscillator = $\frac{h\nu}{e^{h\nu/kT}-1} = 2.945 \times 10^{-21} Joules$

CL6_Q1. Why classical physics cannot explain the results of Compton's experiment?

Answer

According to the wave theory, X-rays force electrons in the atoms of the target material to execute forced oscillations. The oscillating electrons emit radiation with the same frequency as that of the incident radiation. This radiation is called Rayleigh-scattered radiation. Further, the electrons radiate waves uniformly in all directions. Thus, as per wave theory

i) The scattered radiation should have the same wavelength as that of the incident radiation

ii) The wavelength of the scattered radiation should not show dependence on the scattering angle, θ.

The above conclusions are contrary to the experimental observations. It means that the wave theory fails to explain the Compton Effect.

CL6_Q2. What are the angles at which the Compton shift is minimum and maximum? What are the conclusions drawn from these angles?

Answer

Compton shift $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$ varies from zero for $\theta = 0^0$, corresponding to grazing collision with incident photon being scarcely deflected.

$\frac{2h}{m_e c}$ for $\theta = 180^0$ corresponding to a head on collision, the incident photon being reversed in direction.

Conclusions:

- when $\theta = 0^0$, the change in wavelength $\Delta\lambda = 0$. Therefore there is no loss of energy for the photon if the scattering angle is zero.
- when $\theta = 180^0$, the change in wavelength will be twice the Compton wavelength.

CL7_Q1. What are matter waves? State De-Broglie hypothesis.

Answer

Waves associated with material particles in motion are called matter waves or de-Broglie waves.
De-Broglie in his study, assuming that what is true with energy (X- ray/light) is also true with matter and put forward a hypothesis stating that like light, matter also has a wave-particle dual nature. He proposed that all particles in motion behave as waves. This is known as de-Broglie hypothesis.

The wavelength of a moving particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where *h* is the Planck's constant and *p* is the momentum of moving particles.

CL7_Q2. Why is the wave nature of matter not apparent for macroscopic particles?

Answer

According to de-Broglie hypothesis a moving body is associated with matter waves and the wavelength of the waves is given by

$$\boldsymbol{\lambda = \frac{h}{mv}}$$

Where ‘v’ is the velocity with which the body moves.
As the mass ‘m’ of the body increases, the wavelength tends to be insignificant. Therefore, the wavelength associated with macroscopic bodies become insignificant in comparison to the size of the bodies themselves even at very low velocities. Because of the smaller magnitude of Planck’s constant h, the wavelength λ will be significant only in the case of micro-particles.

CL7_Q3. The mass of the oxygen molecule is 5.4×10^{-26} kg. If this molecule moves with a speed of 500 m/s, calculate the de-Broglie wavelength associated with the molecule?

Answer

Given: Mass of the oxygen molecule m= 5.4×10^{-26} kg

V= 500 m/s

We know that the de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{5.4 \times 10^{-26} \times 500}$$

$$\lambda = 2.45 \times 10^{-11} m$$

CL7_Q1. Distinguish between phase and group velocities.

Answer

Phase velocity: The phase velocity of the waves is defined as the velocity of an arbitrary point marked on the wave (the high frequency component) as the wave propagates and is given by

$$v_p = \frac{\omega}{k}$$

Phase velocity refers to the velocity of a monochromatic wave.

Group velocity: The velocity of the wave packet (wave group) is defined as the group velocity and given by $v_g = \frac{d\omega}{dk}$

The group velocity is the velocity with which the entire group of waves would travel.

CL7_Q2. Show that the phase velocity of the de-Broglie waves for a particle is a function of the wavelength?

Answer

If we consider a harmonic wave, the wave has a single wavelength and a single frequency. The velocity of propagation of the wave is given by

$$v_p = \nu\lambda$$

Using, $\nu = \frac{\omega}{2\pi}$ and $\lambda = \frac{2\pi}{k}$ into the above equation, we get

$$v_p = \frac{\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k}$$

v_p is called the phase velocity. The velocity with which the plane of equal phase travels through the medium is known as the phase velocity. It thus represents the velocity of propagation of the wave front.

As $E = h\nu$ and $p = \frac{h}{\lambda}$, we get

$$v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p}$$

CL7_Q3. Distinguish between phase velocity and group velocity. Give the relation between them and under what circumstances is the group velocity equal to phase velocity?

Answer

The phase velocity is the velocity of a travelling wave whereas group velocity is the velocity of the wave packet formed due to the superposition of two or more individual waves.

$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk}$$

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

In a non-dispersive medium, all the individual waves travel with the same velocity, therefore $\frac{dv_{ph}}{d\lambda} = 0$ or the phase velocity is a constant with respect to wavelength. Then $v_g = v_p$

CL9_Q1. Distinguish between phase and group velocities.

Answer

Phase velocity: The phase velocity of the waves is defined as the velocity of an arbitrary point marked on the wave (the high frequency component) as the wave propagates and is given by $v_p = \frac{\omega}{k}$

Phase velocity refers to the velocity of a monochromatic wave.

Group velocity: The velocity of the wave packet (wave group) is defined as the group velocity and given by $v_g = \frac{d\omega}{dk}$

The group velocity is the velocity with which the entire group of waves would travel.

CL9_Q2. Show that the phase velocity of the de-Broglie waves for a particle is a function of the wavelength?

Answer

If we consider a harmonic wave, the wave has a single wavelength and a single frequency. The velocity of propagation of the wave is given by

$$v_p = \nu\lambda$$

Using, $\nu = \frac{\omega}{2\pi}$ and $\lambda = \frac{2\pi}{k}$ into the above equation, we get

$$v_p = \frac{\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k}$$

v_p is called the phase velocity. The velocity with which the plane of equal phase travels through the medium is known as the phase velocity. It thus represents the velocity of propagation of the wave front.

As $E = h\nu$ and $p = \frac{h}{\lambda}$, we get $v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p}$

CL8_Q1. The Uncertainty principle is not significant in the case of macro-bodies. Justify.

Answer

The Heisenberg's Principle is of no practical importance for heavy bodies where the de Broglie wavelength is negligibly small.

For example, let us take the case of a cricket ball of mass 0.5kg in flight. The indeterminacy in the position of the ball is, say, 1 mm. We can determine the indeterminacy of the velocity of the ball from the uncertainty principle.

$$\Delta p.\Delta \mathrm{x} \approx \frac{h}{4\pi}$$

$$\mathrm{m}\Delta v.\Delta \mathrm{x} \approx \frac{h}{4\pi}$$

$$\Delta v \approx \frac{h}{4\pi m \Delta \mathrm{x}} = \frac{6.62 \times 10^{-34}}{4 \times 3.142 \times 0.5 \times 10^{-3}} = 1.05 \times 10^{-31} m/s$$

The above inaccuracy is negligible and not detectable. It implies that the uncertainties are of no importance in the case of macro bodies; and the position and velocity of a macro body can be simultaneously determined with a high degree of accuracy. As a result, the macroscopic body follows a well-defined trajectory.

CL8_Q2. Monochromatic light passes through a shutter that opens for a time $\Delta t = 10^{-10}$ s. What is the spread in the frequency caused by the shutter?

Answer

We know that the statement of Heisenberg's Uncertainty Principle $\Delta E.\Delta t \geq \frac{h}{4\pi}$

$$\mathrm{h}\Delta\mathrm{v}.\Delta t \geq \frac{h}{4\pi}$$

$$\Delta\mathrm{v} = \frac{1}{\Delta t \; 4\pi}$$

$$\Delta\mathrm{v} = \frac{1}{10^{-10} \; 4\pi}$$

$$\Delta\mathrm{v} = 0.079 \times 10^{10}\mathrm{Hz}$$

CL8_Q3. An electron and a 150 gm baseball are travelling at a velocity of 220 m/s, measured to an accuracy of 0.065%. Calculate and compare the uncertainty in position of each.

Answer

The uncertainty in the velocity is $\Delta v = v \times 0.065\% = 220 \times \frac{0.065}{100} = 0.143\ m/s$

The uncertainty in the position of an electron is,

$$\Delta x_e \approx \frac{h}{4\pi \mathrm{m} \Delta v} = \frac{6.62 \times 10^{-34}}{4 \times 3.142 \times 9.1 \times 10^{-31} \times 0.143} = 0.4mm$$

The uncertainty in the position of baseball is,

$$\Delta x_b \approx \frac{h}{4\pi \mathrm{m} \Delta v} = \frac{6.62 \times 10^{-34}}{4 \times 3.142 \times 0.15 \times 0.143} = 2.5 \times 10^{-33} m$$

CL8_Q4. What are the implications of Uncertainty Principle?

Answer

The uncertainty Principle points out that in the microscopic world,

1. The dynamical variables of a particle are combined in sets of simultaneously determined quantities which are known as complete sets of quantities
2. The coordinates and momentum components of a particle etc. are pairs of concepts which are interrelated and fall in different complete sets of quantities. They cannot be defined simultaneously in a precise way.

Thus, the uncertainty principle implies that we can never define the path of an atomic particle with the absolute precision indicated in classical mechanics. Therefore, concepts such as velocity, position, and acceleration are of limited used in the quantum world. To describe the quantum particle the concept of energy becomes important since it is related to the state of the system rather than to its path.

CL9_Q1. Give physical interpretation of the wave function.

Answer

The wave function $\psi(x, y, z, t)$is a probability amplitude and the intensity of the wave (the point at which the energy of the wave is likely to be concentrated) is the square of the probability amplitude. Since the wave function can be a real or an imaginary function, it is evident that the square of the wave function $|\psi^2| = \psi^*.\psi$. ψ^* is the complex conjugate of the wave function. Thus the product is representative of the intensity of the wave or the probability of finding the particle at any point in the wave packet and is called the probability density.

CL9_Q2. Prove that ψ*(x,t) ψ(x,t) is necessarily real and either positive or zero.

Answer

As we know that wave functions are usually complex in nature, it consists of real and imaginary parts. But the probability must be a real and positive quantity.

Let $\psi(x,t)$ = A+iB, then its complex conjugate is $\psi^*(x,t)$ = A-iB

$\psi^*(x,t)\ \psi(x,t) = \psi^2$

$\psi 2 = \psi^*\psi = (A-iB)\ (A+iB) = A^2 - i^2B^2$

$\psi 2 = \psi^*\psi = A^2 + B^2$

Hence the proof.

CL9_Q3. Mention important properties of wave function.

Answer

1. ψ must be finite, continuous and single valued in the regions of interest
2. The derivatives of the wave function must be finite, continuous and single valued
   in the regions of interest.

3. The wave function ψ must be normalisable. i.e.. $\int_{-\infty}^{+\infty} \psi * \psi \, dV = 1$

CL9_Q4. What is the difference between probability density and probability?

Answer

Particles exhibit wave like properties, such as interference and diffraction. Though we do not see a physical wave, it is evident that a certain wavelength has to be introduced with each particle to explain experiments like interference and diffraction with regard to particles. The wave associated with a particle is not observable, however, mathematically quantified in terms of de Broglie wavelength. As with every wave a certain quantity is expressed as a function of position and time, with constant parameters like frequency ω and propagation constant *k*.

$$\psi = A \sin(\omega t - k x) \qquad \text{or generally}$$

$$\psi = A e^{i(\omega t - k x)}$$

There is no physical quantity which ψ can be associated with. However, it is seen that $|\psi|^2$ will be related to probability of locating particles. This can be thought of as the square of the resultant amplitude in Young's double slit experiment giving us intensity; intensity is directly related to probability.

$|\psi|^2 = \psi^* \psi$ where ψ^* is the complex conjugate of ψ.

$To\ obtain\ a\ complex\ conjugate\ replace\ all\ (i)\ in\ \psi\ with\ (-i)$

$|\psi|^2$ is called Probability Density

$|\psi|^2\, \Delta x$, gives the probability of finding a particle in a region Δx

$|\psi|^2\, \Delta A$, gives the probability of finding a particle in a region ΔA

$|\psi|^2\, \Delta V$ gives the probability of finding a particle in a region ΔV

CL12_Q4. What is the difference between probability density and probability?

Answer

Particles exhibit wave like properties, such as interference and diffraction. Though we do not see a physical wave, it is evident that a certain wavelength has to be introduced with each particle to explain experiments like interference and diffraction with regard to particles. The wave associated with a particle is not observable, however, mathematically quantified in terms of de Broglie wavelength. As with every wave a certain quantity is expressed as a function of position and time, with constant parameters like frequency ω and propagation constant *k*.

$$\psi = A\, sin\,(\omega t - k\, x) \qquad \text{or generally}$$

$$\psi = A\, e^{\,i(\omega t - k\, x)}$$

There is no physical quantity which ψ can be associated with. However, it is seen that $|\psi|^2$ will be related to probability of locating particles. This can be thought of as the square of the resultant amplitude in Young's double slit experiment giving us intensity; intensity is directly related to probability.

$|\psi|^2 = \psi^*\psi$ where ψ^* is the complex conjugate of ψ.

$To\ obtain\ a\ complex\ conjugate\ replace\ all\ \ (i)\ \ in\, \psi\ with\ \ \ (-i)$

$|\psi|^2$ is called Probability Density

$|\psi|^2\ \Delta x$, gives the probability of finding a particle in a region Δx

$|\psi|^2\ \Delta A$, gives the probability of finding a particle in a region ΔA

$|\psi|^2\ \Delta V$ gives the probability of finding a particle in a region ΔV

CL12_Q1. What is the physical significance of normalization of wave function?

Answer

The wave function is generally probability amplitude and the intensity of the wave is propotional to the square of the amplitude. This gives the probability density of finding the particle at any position. When the propobability density is integrated over the entire region it gives us the total probability of finding the particle in the region. Thus the normalization of the wave function is given by $\int_{-\infty}^{+\infty} \psi * \psi \, dV = 1$ gives the total probability of finding the particle.

CL12_Q2. Normalize the wave function given by $\psi = A\,sin\left(\frac{\pi}{2}x\right)$ for $0 < x < 1$ and $\psi = 0$ for all ther x.

Answer

The given wave function $\psi = A\,sin\left(\frac{\pi}{2}x\right)$ when intergrated over the region 0 to 1 should give us the total probability.

$$\int_0^1 \psi * \psi \, dx = \int_0^1 A^2\, sin^2\left(\frac{\pi}{2}x\right) dx = A^2 \int_0^1 \frac{1 - COS(\pi x)}{2} dx = \frac{A^2}{2}[1 - 0] = 1$$

Thus, $A = \sqrt{2}$

The normalized wave function is $\psi = \sqrt{2}\, sin\left(\frac{\pi}{2}x\right)$ for $0 < x < 1$

CL14_Q1. Explain the concept of "expectation value of a physical measurable quantity" in quantum mechanics?

Answer

Expectation value is the average value that you would expect to get for some observable quantity like x or p if you measured it many times.
Quantum mechanics deals with probabilities and hence predicts only the most probable values of the observables of a physical system which are called the expectation values. These expectation values could be the average of repeated measurements on the system.

In the general case for an operator $\hat{A}$ when and hence the expectation value of the parameter A can be written as $\langle A \rangle = \frac{\int \psi^* \hat{A} \psi dV}{\int \psi^* \psi dV}$.

CL14_Q2. Explain operators and observables.

Answer

Observables: The physical parameters associated with the particle such as energy, momentum, kinetic energy, spin, etc. are observables of the state of a system.

Operators: The values of the observables can be obtained using a mathematical operator operating on the wave function.

CL14_Q3. Find the expectation value <X> of the position of a particle trapped in a box 'L' wide.

Answer

$$< X >= \int_{-\infty}^{\infty} x[\psi]^2 \, dx$$
$$= \frac{2}{L}\int_0^L x sin^2(\frac{n\pi}{L}x) \, dx = \frac{2}{L}\left[\frac{L^2}{4}\right] = \frac{L}{2}$$

CL14_Q4. Describe an Eigen value equation explaining each term.

Answer

Eigen Value Equation $\hat{G}\,\psi_n = \lambda_n \,\psi_n$

Where $\hat{G}$ is an operator, λ_n are called Eigen values (Real) and ψ_n are called Eigen wave functions. The above equation holds only for a discrete set of λ_n and ψ_n. The Eigen values are the possible outcomes of the measurement on the observable represented by the operator $\hat{G}$. Any measurement cannot result in a value which is not an Eigen value of the equation. Eigen functions give us the probability of finding the particle for that given Eigen value.

CL14_Q5. Write any five operators associated with dynamical variables.

Answer

Observables and Operators

A dynamical physical quantity which can be measured in an experiment is called an observable.
For example, position, momentum, kinetic energy etc.
All observables are represented by operators. Generally, such operators are either matrices or they involve some type of differential operators.

Observable	Operator
Position. x	$\hat{x} = x$
Momentum p	$\hat{p} = -i\hbar \frac{\partial}{\partial x}$
Kinetic energy K	$\hat{K} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
Potential energy V	$\hat{V}$= V(x)
Total energy E	$\hat{E} = i\hbar \frac{\partial}{\partial t}$
Total energy Hamiltonian H	$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

CL12_Q1. Why is Schrodinger's equation referred to as a linear equation?

Ans:

The Schrödinger's wave equation is $\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$.

Schrodinger equation is a linear, partial differential equation. An important property of the equation is that it is linear in the wave function ψ, i.e. all the terms in the equation contain ψ and there is no term independent of ψ. As a result, a linear combination of solutions of Schrodinger's equation for a given system is also itself a solution. Thus the wave equation is linear in ψ and obeys linear superposition implying if ψ_1 and ψ_2 are solutions of Schrodinger equation then $a_1\psi_1 + a_2\psi_2$ is also a solution for arbitrary a_1 and a_2.

CL12_Q2. Schrodinger's equation is an operator equation. Explain

Ans:

The energy expression can be written as $E = KE + V$

Multiplying throughout with the wave function ψ we get

$E\Psi(x,t) = KE\Psi(x,t) + V\Psi(x,t)$ ----- **(1)**

This equation can be written in terms of the corresponding operators as $\hat{E}\Psi(x,t) = \widehat{KE}\Psi(x,t) + V\Psi(x,t)$

The total energy operator is $\left\{i\hbar\frac{d}{dt}\right\}$, the kinetic energy operator is $\{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\}$.

Replacing the total energy and the kinetic energy terms with the respective operators, we can rewrite the expression (1) to obtain the time dependent form of Schrodinger's equation as $i\hbar\frac{d\Psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi$

Therefore, Schrodinger's equation is an operator equation.

CL13_Q1: A free particle is a classical entity. Justify.

Answer

A particle is said to be a free particle when it experiences no external forces. Thus, the force given by $F = -\frac{dV}{dx} = 0$. This implies that either V is zero or V is a constant.

The simplest case, then could be when the particle is moving in a region of zero potential i.e., V=0.

The Schrodinger's time independent one dimensional wave equation for the system simplifies to

$$\frac{\hbar^2}{2m}\frac{d^2\psi(x).}{dx^2} + E\psi(x) = 0 \quad \text{or} \quad \frac{d^2\psi(x).}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

$\boldsymbol{\frac{d^2\psi(x).}{dx^2} + k^2\psi = 0}$ where $\boldsymbol{k = \sqrt{\frac{2mE}{\hbar^2}}}$ is the propagation constant.

The solution of this differential equation is $\boldsymbol{\psi = Ae^{ikx} + Be^{-ikx}}$ where A and B are constants.

The energy of the particle is given by $\boldsymbol{E = \frac{\hbar^2 k^2}{2m}}$.

Since the free particle is moving in a zero potential region, there is no restriction on wave number or the energy of the particle. The free particle can have any energy and there is no discreteness in the allowed energy values. In other words, there is no quantization of energy in the case of a free particle and the problem is dealt in classical mechanics. Thus a free particle is a "classical entity".

CL13_Q2. What is step potential in quantum mechanics?

Answer

In quantum mechanics, the 1D step potential is an idealized system used to model incident, reflected and transmitted matter waves. The problem consists of solving the time-independent Schrodinger equation for a particle with a step-like potential in one dimension.