PARTIAL DIFFERENTIAL EQUATIONS

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LAGRANGE'S LINEAR EQUATION

WORKING METHOD

To solve the equation $Pp + Qq = R$

(i) form the auxiliary equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

(ii) solve the auxiliary equations by the method of grouping or the method of multipliers or both to get two independent solutions $u = a \,\&\, v = b,$ where a, b are arbitrary constants.

(iii) then $\phi(u, v) = 0$ or $u = f(v)$ is the general solution of the equation

$$Pp + Qq = R.$$

PROBLEMS

1. Solve the equation $yzp + zxq = xy$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

From the first two and last two terms, we get, respectively.

$$\frac{dx}{y} = \frac{dy}{x} \quad \text{or} \quad xdx - ydy = 0$$

$$\frac{dy}{z} = \frac{dz}{y} \quad \text{or} \quad xdx - ydy = 0$$

Integration $x^2 - y^2 = a, y^2 - z^2 = b$

Hence, a general solution of the given equation is $\phi\left(x^2 - y^2, y^2 - z^2\right) = 0$

PROBLEMS

2. Solve the equation $pz - qz = z^2 + (x+y)^2$

$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2}$$

From the first two terms

$$dx + dy = 0 \Rightarrow x + y = a \ldots\ldots(1)$$

First and third term

$$2dx = \frac{2z\,dz}{z^2 + a^2}$$; since $x + y = a$

$$2x + b = \log(z^2 + a^2)$$ or $$\log(z^2 + (x+y)^2) - 2x = b \ldots\ldots(2)$$

From (1) and (2), $\phi\left(x + y, \log(x^2 + y^2 + z^2 + 2xy) - 2x\right) = 0$

PROBLEMS

3. Solve the equation $y^2 p - xyq = x(z-2y)$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x\left(z-2y\right)}$$

From the first two terms

$$x^2 + y^2 = a$$

First and third term

$$\frac{dy}{-y} = \frac{dz}{z-2y} \Rightarrow \frac{dz}{dy} + \frac{z}{y} = 2 \Rightarrow yz - y^2 = b$$

$$\phi\left(x^2 + y^2, yz - y^2\right) = 0$$

PROBLEMS

4. Solve the equation $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

$$\frac{dx}{x^2(y-z)}=\frac{dy}{y^2(z-x)}=\frac{dz}{z^2(x-y)}$$

Using $\frac{1}{x^2},\frac{1}{y^2},\frac{1}{z^2}$ as a multipliers, we get

$$\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}\Rightarrow \log(xyz)=\log b$$

$$\Rightarrow xyz=b$$

$$\phi\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z},xyz\right)=0$$

PROBLEMS

5. Solve the equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

$$\frac{dx}{(x^2 - y^2 - z^2)} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

from two and three

$$\frac{dy}{y} = \frac{dz}{z} \Rightarrow \frac{y}{z} = a$$

Using x, y, z as a multipliers, we get

$$\text{each fraction} = \frac{xdx + ydy + zdz}{x(x^2 + y^2 + z^2)}$$

PROBLEMS

$$\therefore \frac{xdx + ydy + zdz}{x(x^2 + y^2 + z^2)} = \frac{dz}{2xz}$$

$$\log\left(x^2 + y^2 + z^2\right) = \log z + \log b$$

$$\log\left(\frac{x^2 + y^2 + z^2}{z}\right) = \log b$$

General solution is

$$\phi\left(\frac{y}{z}, \frac{x^2 + y^2 + z^2}{z}\right) = 0$$

THANK YOU

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