Engineering Mathematics - II (UE23MA141B)

Unit - 1: Integral Calculas

Problems on double integrals

1. Evaluate: $\int_0^1 \int_0^3 x^3 y^3 dxdy$. Answer: $\frac{81}{16}$
2. Evaluate: $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. Answer: $\frac{\pi^2}{4}$
3. Evaluate: $\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 ydxdy$. Answer: $\frac{a^5}{15}$
4. **Home work problem**: Evaluate: $\int_1^4 \int_0^{\sqrt{4-x}} xydxdy$. Answer: $\frac{9}{2}$
5. Evaluate $\int\int_R dxdy$ where R is the region bounded by the lines $y = x$, $x + y = 4$, $y = 1$ and $y = 0$.
   Answer: 3
6. Evaluate $\int\int_R dxdy$ where R is the region bounded by x-axis, the ordinate $x = 2a$ and the parabola $x^2 = 4ay$. Answer: $\frac{a^4}{3}$
7. **Home work problem**: If R is the region bounded by the parabolas $y^2 = x$ and $x^2 = y$, then show that
   $\int\int_R xy(x + y)dxdy = \frac{3}{28}$.
8. Find the volume of the solid which is bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y + z = 1$ and $z = 0$ using double integration.
   Answer: π
9. **Home work problem**: Find the volume of the solid which is below the plane $z = 2x + 3$ and above the $x - y$ plane and bounded by $y^2 = x; x = 0; x = 2$ using double integration.
   Answer: $\frac{72\sqrt{2}}{\pi}$
10. Explain "Jacobian". If $x = rcos(\theta); y = rsin(\theta); z = z$, then find the "Jacobian transformation" from cartesian coordinates to cylindrical coordinates. Answer: $J = r$
11. Show that $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dxdy = \frac{1}{2}(e-1)$, by using transformation $x+y = u$, $y = uv$.
12. Find the area enclosed by pair of curves $y = 2 - x$ and $y^2 = 2(2 - x)$.
    Answer: $\pi + 8$

Problems on change of variables in double integrals

13. Transform the integral $\int_0^1 dx \cdot \int_0^1 f(x, y)dxdy$ in polar coordinates:
    Answer: $\int_0^{\frac{\pi}{4}} \left[\int_0^{sec(\theta)} f(rcos(\theta), rsin(\theta))rdr\right] d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\int_0^{cosec(\theta)} f(rcos(\theta), rsin(\theta))rdr\right] d\theta$
14. Change into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$. Answer: $\frac{\pi}{4}$

15. Change into polar coordinates and evaluate $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2+y^2)dxdy$. Answer: $\frac{3\pi}{8} - 1$

16. Find the area inside the circle $r = 2acos(\theta)$ and outside the circle $r = a$. Answer: $2a^2 \left[\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right]$ square units.

17. **Home work problem**: Evaluate $\int\int_R xydxdy$ over the region in polar coordinates. Given, $R : r = sin(2\theta), 0 \leq \theta \leq \frac{\pi}{2}$. Answer: $\frac{1}{15}$

18. **Home work problem**: Change into polar coordinates and evaluate $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}dxdy$. Answer: $\frac{a^3}{3}log(\sqrt{2}+1)$

Problems on change of order of integration

19. Evaluate $\int_0^1 \int_{3y}^3 e^{x^2}dxdy$ by changing the order of integration. Answer: $\frac{e^9-1}{6}$

20. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}dxdy$ by changing the order of integration. Answer: 1

21. Evaluate $\int_0^1 \int_{x^2}^{2-x} xydxdy$ by changing the order of integration. Answer: $\frac{3}{8}$

22. Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}dydx$ by changing the order of integration. Answer: $\frac{\sqrt{2}-1}{\sqrt{2}}$

23. **Home work problem**: Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} dxdy$ by changing the order of integration. Answer: $\frac{7}{24}$

Problems on triple integrals

24. Evaluate: $\int_2^3 \int_1^2 \int_2^5 xy^2dzdydx$. Answer: $\frac{35}{2}$

25. Evaluate: $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)dxdydz$. Answer: 0

26. Find the volume of the solid bounded by the surfaces $z = 0; z = 1 - x^2 - y^2; y = 0; y = 1 - x; x = 0$ and $x = 1$. Answer: $\frac{1}{3}$

27. The temperature at a point (x, y, z) of a solid E bounded by the planes $x = 0; y = 0; z = 0$ and the plane $x+y+z = 1$ is $\frac{1}{(1+x+y+z)^3}$ degree Celsius. Find the average temperature over the solid. Answer: $6\left(\frac{log2}{2} - \frac{5}{16}\right)$

Problems on change of variables in triple integrals: cylindrical and spherical coordinates

28. Use cylindrical coordinates to evaluate $\int\int\int_V (x^2+y^2)dxdydz$ taken over the region V bounded by the paraboloid $z = 9 - x^2 - y^2$ and the plane $z = 0$. Answer: $\frac{243\pi}{2}$

29. Calculate the volume of the solid bounded by the paraboloid $z = 2 - x^2 - y^2$ and the cone $z = \sqrt{x^2 + y^2}$. Answer: $\frac{5\pi}{6}$

30. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical co-ordinates.
Answer: $\frac{\pi^2}{8}$

31. Find volume bounded by cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. Answer: 16π

32. Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ay$. Answer: $\frac{2a^3}{9}(3\pi - 4)$

Problems on the center of mass and moment of inertia

33. Find the total mass of the region in the cube $0 \le x \le 1; 0 \le y \le 1; 0 \le z \le 1$ with density at any point given by xyz. Answer: $\frac{1}{8}$

34. Find the mass of a sphere of radius b if the density varies inversely as the square of the distance from the center. Answer: $4k\pi b$

35. Compute the moment of inertia of a right circular cylinder of altitude $2h$ and radius b, relative to the diameter of its median section with density equals k (a constant).
Answer: $k\left(\frac{2\pi h^3 b^2}{3} + \frac{hb^4}{2}\right)$