



**PES UNIVERSITY, BANGALORE**  
(Established under Karnataka Act 16 of 2013)

**UE18MA151/  
UE19MA151**

**END SEMESTER ASSESSMENT B. Tech. II SEMESTER– May 2022**  
**UE18/19MA151 – Engineering Mathematics - II**

Time: 3 Hrs

Answer All Questions

Max Marks: 100

1.	a)	Find the directional derivative of the function $f = x^2yz + 4xz^2$ at $P(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$ .	6														
	b)	Show that $\vec{F} = 2x(y^2 + z^3)\hat{i} + 2x^2y\hat{j} + 3x^2z^2\hat{k}$ is conservative force field and find the scalar potential $\phi$ and work done in moving a particle from $(-1,2,1)$ to $(2,3,4)$ .	7														
	c)	Using Greens theorem, evaluate $\int_C x^2ydx + x^2dy$ where C is the boundary describing counter-clockwise of the triangle with vertices $(0,0)$ , $(1,0)$ $(1,1)$ .	7														
2.	a)	Prove that $\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx = \frac{\pi}{16\sqrt{2}}$	7														
	b)	Establish the Jacobi series and hence prove that $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x\sin\theta) d\theta$ where n is a positive integer.	7														
	c)	Express $J_{5/2}(x)$ interms of sine and cosine functions.	6														
3.	a)	Find the Laplace transform of $te^{-2t}\cos 3t + \frac{\sin at}{t} + 5^t + t^4\delta(t - 5)$	6														
	b)	A periodic function $f(t)$ with period $2a$ is defined by $f(t) = \left\{ \begin{matrix} a & 0 \leq t \leq a \\ -a & a < t \leq 2a \end{matrix} \right\}$ . Show that $L\{f(t)\} = \frac{a}{s} \tanh\left(\frac{as}{2}\right)$	7														
	c)	Express the following function $f(t)$ in terms of unit step function and hence find the Laplace transform $f(t) = \begin{cases} t - 1 & 1 < t < 2 \\ 3 - t & 2 < t < 3 \end{cases}$	7														
4.	a)	Evaluate $L^{-1}\left\{\frac{3s+7}{(s^2+6s+10)^2}\right\}$ , $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s^2}\right)\right\}$	6														
	b)	Solve the differential equation $y'' - 6y' + 9y = 0$ , if $y(0) = 2$ , $y'(0) = 9$ using Laplace transform.	7														
	c)	Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$	7														
5.	a)	Find the Fourier Series of $f(x) = x + x^2$ in $(-\pi, \pi)$ and hence deduce that $\sum_{n=1}^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$	7														
	b)	Find the Fourier half range Cosine series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$	6														
	c)	Express y as a Fourier series up to first harmonics. <table border="1"><tr><td>x</td><td>0</td><td><math>\pi/3</math></td><td><math>2\pi/3</math></td><td><math>\pi</math></td><td><math>4\pi/3</math></td><td><math>5\pi/3</math></td></tr><tr><td>y</td><td>7.9</td><td>7.2</td><td>3.6</td><td>0.5</td><td>0.9</td><td>6.8</td></tr></table>	x	0	$\pi/3$	$2\pi/3$	$\pi$	$4\pi/3$	$5\pi/3$	y	7.9	7.2	3.6	0.5	0.9	6.8	7
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