ASSIGNMENT - I

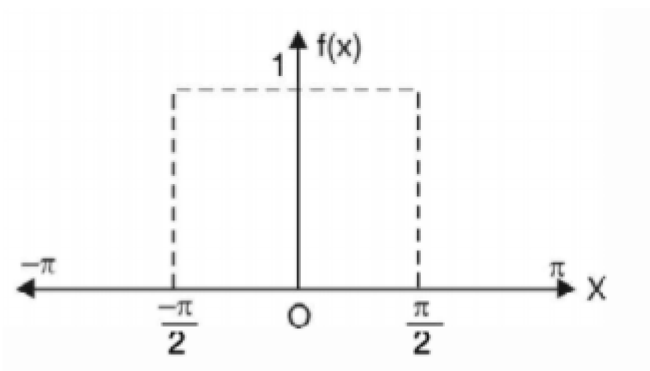
Introduction to Fourier Series, Dirchlet's conditions, Euler's formulae:

1. **Obtain the Fourier coefficient a_0 using Euler's formula for the periodic function**

$$f(x) = \begin{cases} x & -\pi < x < 0 \\ -x & 0 < x < \pi \end{cases}$$

Ans : $a_0 = -\pi$

2. **Obtain the Fourier coefficient a_0 using Euler's formula for the periodic function**



Ans : $a_0 = 1$

3. **Obtain the Fourier coefficient b_1 using Euler's formula for the periodic function**

$$I = \begin{cases} I_0 \sin\theta & for\ 0 < \theta \le \pi \\ 0 & for\ \pi < \theta \le 2\pi \end{cases}$$

Ans: $b_1 = \frac{I_0}{2}$

ASSIGNMENT - 2

Fourier Series of Even and Odd functions

1. Obtain the Fourier Series Expansion of $f(x) = |cosx|$ $(-\pi, \pi)$

   Ans: $a_0 = \frac{4}{\pi}$ $a_n = -\frac{4\cos(n\pi/2)}{\pi(n^2-1)}$, $b_n = 0$

2. Find the Fourier series of $f(x) = |x|$ in $(-1,1)$

   Ans: $a_0 = 1$, $a_n = \frac{-2}{n^2\pi^2}\{1-(-1)^n\}, f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-2}{n^2\pi^2}\{1-(-1)^n\} cos n\pi x$

3. Find the Fourier series of $f(x) = \begin{Bmatrix} 0 & -2 < x < -1 \\ 1+x & -1 < x < 0 \\ 1-x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{Bmatrix}$

   Ans: $\frac{1}{4} + \frac{4}{n^2}\sum_1^{\infty} \frac{1}{n^2}\left[1 - cos\frac{n\pi}{2}\right] cos\frac{n\pi x}{2}$

ASSIGNMENT - 3

Problems on Fourier series:

Obtain the Fourier Series for the following:

1. $f(x) = e^x$ in $(-l, l)$

   Ans: $f(x) = \frac{sinhl}{l} - \frac{2sinhl}{l}\sum_{n=1}^{\infty}\frac{(-1)^n}{1^2+n^2}(-cosnx + nsinnx)$

2. $f(x) = \begin{Bmatrix} 0 & in\ (-\pi, 0) \\ sinx & in\ (0, \pi) \end{Bmatrix}$ Deduce $\frac{\pi-2}{4} = \frac{1}{1.3} - \frac{1}{1.5} + \frac{1}{5.7} \ldots$

   Ans: $a_{0=-\frac{2}{\pi}}$ $a_n = \frac{-1}{\pi(n^2-1)}\{1-(-1)^n) where\ n \neq 1$, $b_n = 0$

3. $f(x) = xcosx\ \ in\ (-\pi, \pi)$

   Ans: $a_{0=0}$ $a_n = 0$, $b_n = \frac{2n(-1)^n}{n^2-1}(n \neq 1)$

ASSIGNMENT - 4

Problems on Fourier series:

Obtain the Fourier Series for the following:

1. $f(x) = \begin{cases} -1 & -\pi < t < -\pi/2 \\ 0 & -\pi/2 < t < \pi/2 \\ 1 & \pi/2 < t\pi \end{cases}$ in $(-l, l)$

   Ans: $f(x) = \frac{2}{\pi}\left[\!\left[sint - sin2t + \frac{1}{3}sin3t + \cdots \right]\!\right]$

2. $f(x) = |sinx| \quad -\pi < x < \pi$

   Ans: $\frac{2}{\pi} - \frac{4}{\pi}\left[\!\left[\frac{cos2x}{3} + \frac{cos4x}{15} + \quad \right]\!\right]$

ASSIGNMENT - 5

Half -range fourier series

1. Obtain the Fourier expansion of x^2 as a cosine series in $(0, \pi)$.

   Ans: $f(x) = \frac{\pi^2}{3} - 4\left[cosx - \frac{1}{2^2}cos2x + \frac{1}{3^2}cos3x + \frac{1}{4^2}cos4x + \cdots\right]$

2. Obtain the half range cosine series for the function $f(x) = x$ in the interval 0<x<2.

   Ans: $a_0 = \frac{-8}{\pi^2}, \quad a_n = \frac{4}{n^2\pi^2}[(-1)^n - 1]$

3. Obtain the half range sine series for the function $f(x) = t - t^2$ in the interval 0<t<2.

   Ans: $f(x) = \frac{8}{\pi^3}\left[\frac{sin\pi t}{1} - \frac{1}{3^3}sin3\pi t + \frac{1}{5^3}sin5\pi t + \cdots\right]$

ASSIGNMENT - 6

Parseval's Identity

1. Define $f: T \to R$ by $f(x) = x^2 for\ -\pi \leq x \leq \pi$ use Parseval's theorem to deduce that $\sum_1^\infty \frac{1}{n^4} = \frac{\pi^4}{90}$

   Ans: $a_0 = 2\frac{n^2}{3}$, $a_n = \frac{4}{n^2}(-1)^n$, $b_n = 0$

2. If $\frac{a_0}{2} + \sum a_n cos\frac{n\pi x}{l}$ is the half range cosine series of $f(x)$ of period $2l$ in$(0, l)$, then show that the mean square value of $f(x)$ in $(0, l)$ is $\frac{l}{2}\left\{\frac{{a_0}^2}{2} + \sum_{n=1}^\infty {a_n}^2\right\}$ use this result to evaluate $1^{-4} + 3^{-4} + 5^{-4} + \cdots$ from the half range cosine series of the function $f(x)$ of period 4 defined in (0,2) by $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ \pi(2-x) & 1 < x < 2 \end{cases}$

   Ans: $\frac{\pi^4}{96}$

ASSIGNMENT - 7

Practical Harmonic Analysis

1. Compute the first two harmonics of the Fourier series of f(x) given the following table

0	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Ans: $a_1 = -0.367$, $a_2 = -0.1$, $b_1 = 0.1732$, $b_2 = -0.0577$

2. The turning moment T on the crackshaft of a steam engine for the crack angle θ degree is given as follows. Expand T in a series of $sines$ up to the fourth harmonics.

θ	0	15	30	45	60	75	90	105	120	135	150	165	180
T	0	2.7	5.2	7.0	8.1	8.3	7.9	6.8	5.5	4.1	2.8	1.2	0

Ans: $T = 4.95 - 3.416Scos\theta + 1.4839sin\theta$

ASSIGNMENT - 8

Practical Harmonic Analysis

1. The following values of y gives the displacement in inches of a certain machine part for the rotation x of the flywheel. Expand y in terms of Fourier series :

X	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$
y	0	0,2	14.4	17.8	17.3	11.7

Ans: $11.733 - 7.33cos2x - 2.833cos4x + \cdots - 1.566sin2x - 0.116si4x + \cdots$

2. Determine the first two harmonics of the Fourier series for the following values:

x	30	60	90	120	150	180	210	240	270	300	330	360
y	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

Ans:$y = 2.102 + 0.558cosx + 1.531sinx + 0.354cos2x + 0.145sin2x$

ASSIGNMENT - 10

Complex form of Fourier series

1. Find the complex form of the Fourier series of $f(x) = cosh3x + sinh3x \;\; in \; (-3,3)$

   Ans: $sinh9 \sum_{-\infty}^{\infty} \frac{(-1^n)(9+n\pi)}{(81+(n\pi)^2)} e^{\frac{in\pi x}{3}}$

2. Find the complex form of the Fourier series of the periodic function $f(x) = \begin{Bmatrix} -k & in \, (-\pi, 0) \\ k & in \, (0, \pi) \end{Bmatrix}$

   Ans: $f(x) = \frac{k}{i\pi} \sum_{n=-\infty}^{\infty} \frac{(1-(-1)^n)}{n} e^{inx}$

Class 1:

1. Find the Fourier transform of the function f(t) defined as

$$f(t) = \begin{cases} t & |t| \le \alpha \\ 0 & |t| > \alpha \end{cases}$$

Ans: $\frac{2i}{\omega^2}(\omega\cos\omega\alpha - \sin\omega\alpha)$

2. Find the Fourier transform of the function f(t) defined as

$$f(t) = \begin{cases} \frac{1}{2a} & |t| \le a \\ 0 & |t| > a \end{cases}.$$

Ans: $\frac{\sin\omega a}{\omega a}$

****

Class 2:

1. Find the Fourier transform of the slit function f(t) defined as

$$f(t) = \begin{cases} \dfrac{1}{\varepsilon} & |t| \le \varepsilon \\ 0 & |t| > \varepsilon \end{cases}$$

Ans: $\dfrac{2}{iu\varepsilon}\sinh \omega\varepsilon$, $\dfrac{2}{i}$

Determine the limit of this transformation as $\varepsilon \to 0$

2. Find the Fourier transform of $e^{-|t|}$ and hence evaluate $\int_0^{\infty} \dfrac{\sin t}{1+t^2}\,dt$

Ans: $\dfrac{2i\omega}{\omega^2+1}$, $\dfrac{-\pi}{e}$

****

Class 3:

1. Find the Fourier transform of $f(t) = te^{-|t|}$ Ans: $\dfrac{4i\omega}{\left(\omega^2+1\right)^2}$

2. Find the Fourier transform of $f(t) = \begin{cases} \dfrac{\pi}{2}\cos t & for \ |t| \le \pi \\ 0 & for \ |t| > \pi \end{cases}$. Ans: $\dfrac{\pi\omega(-1)^n \sin \pi\omega}{\omega^2-1}$

3. Find the Fourier transform of $f(t) = e^{-a^2x^2}$ and hence show that $e^{-\frac{x^2}{2}}$ is self reciprocal.

Ans: $\dfrac{\sqrt{\pi}}{a} e^{-\frac{\omega^2}{4a^2}}$

****

ω Class 4:

1. Find the Fourier sine transform of the function $f(t) = \begin{cases} t, & t < \frac{\pi}{2} \\ 0, & t > \frac{\pi}{2} \end{cases}$

Ans: $\frac{1}{\omega^2} sin\left(\frac{\pi\omega}{2}\right) - \frac{\pi}{2\omega} cos\left(\frac{\pi\omega}{2}\right)$

2. Find the Fourier sine transform of $2e^{-5t} + 5e^{-2t}$. Ans: $\omega\left(\frac{2}{\omega^2+25} + \frac{5}{\omega^2+4}\right)$.

3. Find the Fourier sine transform of $f(t) = e^{-at}, a > 0$. Show that $\int_0^\infty \frac{x\sin kx}{a^2+x^2}\,dx = \frac{\pi}{2}e^{-ax}$,$(k>0)$ Ans: $\frac{\omega}{\omega^2+a^2}$

****

Class 5:

1. Find the Fourier sine transform of $f(t) = \begin{cases} cos^2 t, & 0 < t < \frac{\pi}{2} \\ 0, & t > \frac{\pi}{2} \end{cases}$

Ans: $\frac{1}{2} cos\left(\frac{\pi\omega}{2}\right)\left[\frac{\omega}{(\omega^2-4)} - \frac{1}{\omega}\right] + \frac{1}{2\omega}$

2. Find the Fourier sine transformation of $f(t) = t^{m-1}$ where $m \neq 0$ or negative integer.

Hence find $F_s\left(\frac{1}{\sqrt{t}}\right)$.

Ans: $F_s(t^{m-1}) = \frac{\Gamma(m)}{s^m}\sin\left(\frac{m\pi}{2}\right); F_s\left(\frac{1}{\sqrt{t}}\right) = \sqrt{\frac{\pi}{2s}}$

****

Class 6:

1. Find the Fourier cosine transform of $f(x) = 2t \quad for\, 0 < t < 4$.

   Ans: $\frac{2}{\omega}(1 + \cos 4\omega + 2\sin 4\omega)$

2. Find the Fourier cosine transform of $f(x) = \begin{cases} 4t & 0 < t < 1 \\ 4 - t & 1 < t < 4 \\ 0 & t > 4 \end{cases}$

   Ans: $\frac{5\cos\omega - 4 - \cos 4\omega}{\omega^2} + \frac{\sin\omega}{\omega}$

3. Find the Fourier cosine transform of $f(t) = te^{-at}, a > 0$ Ans: $\frac{a^2 - \omega^2}{\left(a^2 + \omega^2\right)^2}$

****

Assignment

1. Find $f(t)$ .given Fourier cosine transform is $\mathcal{F}_C(\omega) = \begin{cases} 1 & for\ 0 < \omega < k \\ 0 & for\ \omega > k \end{cases}$.

2. Find the function which yields $\mathcal{F}_C(\omega) = \frac{sink\omega}{\omega}$ as its Fourier cosine transform.

$$f(t) = \begin{cases} 1 & for\ \ t < |k| \\ 0 & for\ t > |k| \end{cases}$$

Class 8:

1. Find If $\mathcal{F}\{f(t)\} = \sqrt{\pi}e^{\frac{-\omega^2}{4}}$ for $f(t) = e^{-t^2}$, find Fourier transform of

   a) $e^{\frac{-x^2}{a}}$ b) $e^{-4(x-a)^2}$

2. If $\mathcal{F}[e^{-5t}u(t-3)] = \frac{e^{-b(a+i\omega)}}{3+i\omega}$ find the value of $a - b$.

****

Class-1

Assignment Questions:

1. Find the Fourier Transform of $f(t)=5e^{2t}$, where $|t|<a$.

Ans: $F[f(t)]=\dfrac{10\sinh(i\omega-2)a}{i\omega-2}$.

2. Find the Inverse Fourier Transform of the function $\dfrac{e^{-i\omega}}{2(1+i\omega)}$

Ans: $f(t)=\dfrac{1}{2}e^{-(t-1)}U(t-1)$.

3. Find the Fourier transform of $f(t)=1-t^2$, $|t|<1$.

Ans: $F[f(t)]=4\left(\dfrac{\sin\omega-\omega\cos\omega}{\omega^3}\right)$.

4. Find $F^{-1}\left[\dfrac{1}{2-3i\omega-\omega^2}\right]$.

Ans: $f(t)=e^{-t}U(t)-e^{-2t}U(t)$.

5. Find $F^{-1}\left[\dfrac{1}{(9+\omega^2)(4+\omega^2)}\right]$ [Hint: $F^{-1}\left[\dfrac{2a}{a^2+\omega^2}\right]=e^{-a|t|}$].

Ans: $f(t)=\dfrac{e^{-2|t|}}{20}-\dfrac{e^{-3|t|}}{30}$.

6. Find the Fourier Transform of $f(t)=a-|t|$, $|t|\le a$.

Ans: $F[f(t)]=\dfrac{2(1-\cos a\omega)}{\omega^2}$.

7. Find the Fourier Transform of $f(t)=e^{ikt}$, $a<t<b$.

Ans: $F[f(t)]=\dfrac{i}{k+\omega}\left[e^{i(k+\omega)b}-e^{i(k+\omega)a}\right]$

Class-2

Assignment Questions:

1. Find the finite Fourier Cosine transform of $f(t)=t$.

Ans: $C_0=\frac{\pi^2}{2}$ and $C_n=\frac{\cos n\pi-1}{n^2}$.

2. Find the finite Fourier sine transform of $f(t)=\sin(at), a>0$, if the function is defined on $[0,\pi]$. Also find the inverse finite Fourier Sine transformation.

Ans: $S_n=\frac{1}{2}\left[\frac{\sin(n-a)\pi}{n-a}-\frac{\sin(n+a)\pi}{n+a}\right]$, and

Inverse finite Fourier sine transform is $f(t)=\frac{1}{\pi}\sum_{n=1}^{\infty}\left[\frac{\sin(n-a)\pi}{n-a}-\frac{\sin(n+a)\pi}{n+a}\right]\sin(nt)$.

3. Find the Fourier Cosine Transform of $f(t)=\begin{cases} t, & 0<t<1 \\ 2-t & 1<t<2. \end{cases}$

Ans: $C_0=1$ and $C_n=\frac{1}{n^2\pi^2}[4\cos(\frac{n\pi}{2})-3\cos n\pi-1]-\frac{2}{n\pi}\sin(\frac{n\pi}{2})$.

4. Find the finite Fourier Cosine transform of $f(t)=t^2,\ 0<t<\pi$.

Ans: $C_0=\frac{\pi^3}{3}$ and $C_n=\frac{2\pi\cos n\pi}{n^2}$.

5. Find the finite Fourier sine transform of $f(t)=\begin{cases} 0, & 0\le t<\pi/2 \\ 1, & \pi/2\le t\le\pi \end{cases}$.

Ans: $S_n=\frac{-1}{n}\left[(-1)^n-\cos\left(\frac{n\pi}{2}\right)\right]$.

6. Find the finite Fourier sine transform of $f(t)=1-\frac{t}{\pi},\ 0<t<\pi$.

Ans: $S_n=\frac{1}{n}$.

7. Find the finite Fourier Cosine transform of $f(t)=\frac{\cos k(\pi-t)}{k\sin k\pi},\ 0<t<\pi$.

Ans: $C_n=\frac{1}{n^2-k^2}$.