Generation of Balanced Three Phase EMFs:

A balanced three phase system of EMFs is a set of three EMFs which are equal in amplitude (or) magnitude and displaced in phase from one another by 120°.

For instance,

$e_1(t) = E_m \sin(\omega t)$

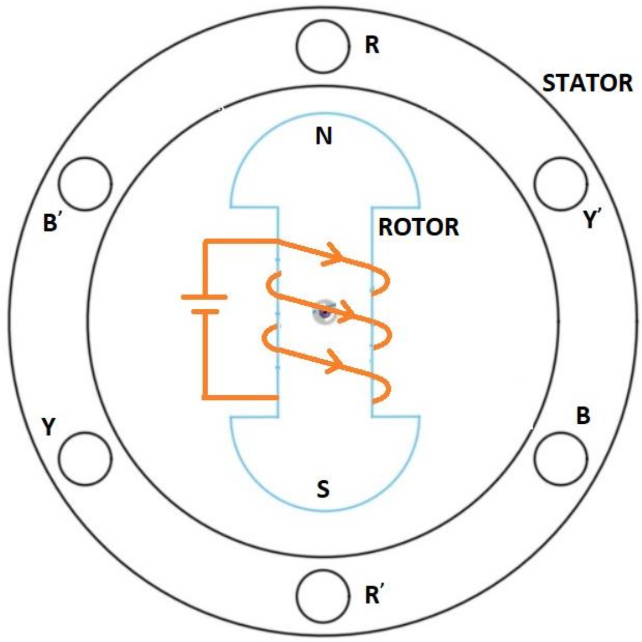
$e_2(t) = E_m \sin(\omega t - 120°)$

$e_3(t) = E_m \sin(\omega t - 240°)$

represent balanced three phase system of EMFs.

A balanced three phase system of EMFs is generated in a machine called 'Three Phase Generator', also called 'Alternator'.

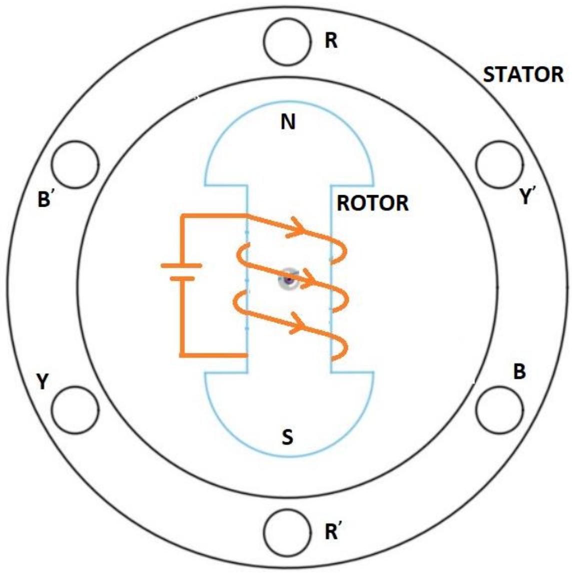
Three Phase Generator (or) Alternator – Construction:



The **'Stator'** which is the stationary member consists of three coils R, Y & B which are physically displaced from one another by 120°. The **'Rotor'** which is the rotating member consists of set of electromagnets excited by a field winding connected to a DC supply. The shaft of the rotor is coupled to a prime mover such as steam turbine (or) hydro turbine (or) Diesel engine.

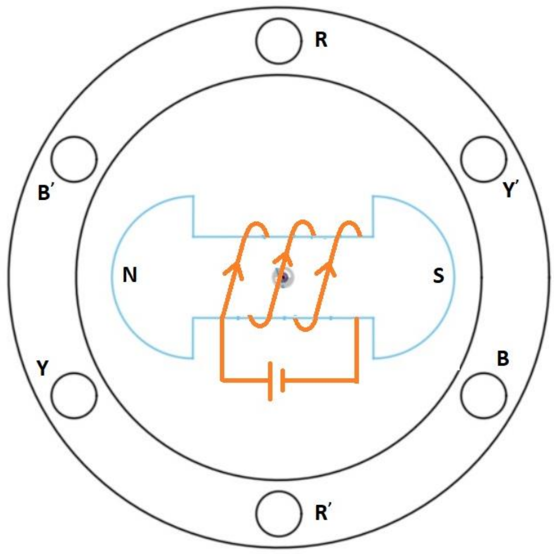
Principle of Operation:

Let us consider Position 1: Coil R facing centre of North Pole



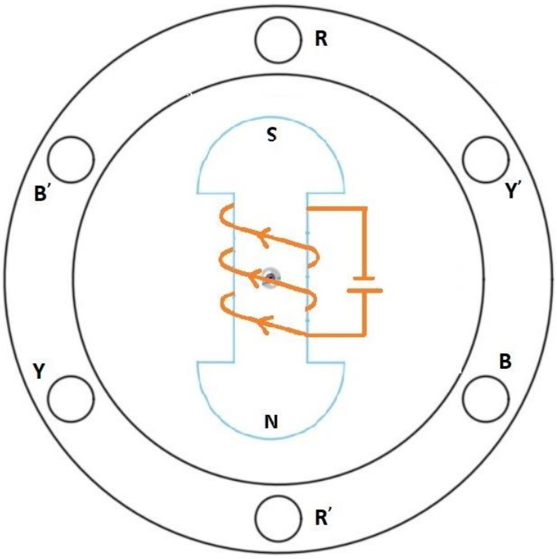
In this position, R-coil will have maximum rate of change of flux linkages hence, maximum positive EMF is induced in it in this position.

Let us consider Position 2: Coil R facing interpolar axis



In this position, R-coil will have minimum rate of change of flux linkages hence, Zero EMF is induced in it in this position.

Let us consider Position 3: Coil R facing centre of South Pole



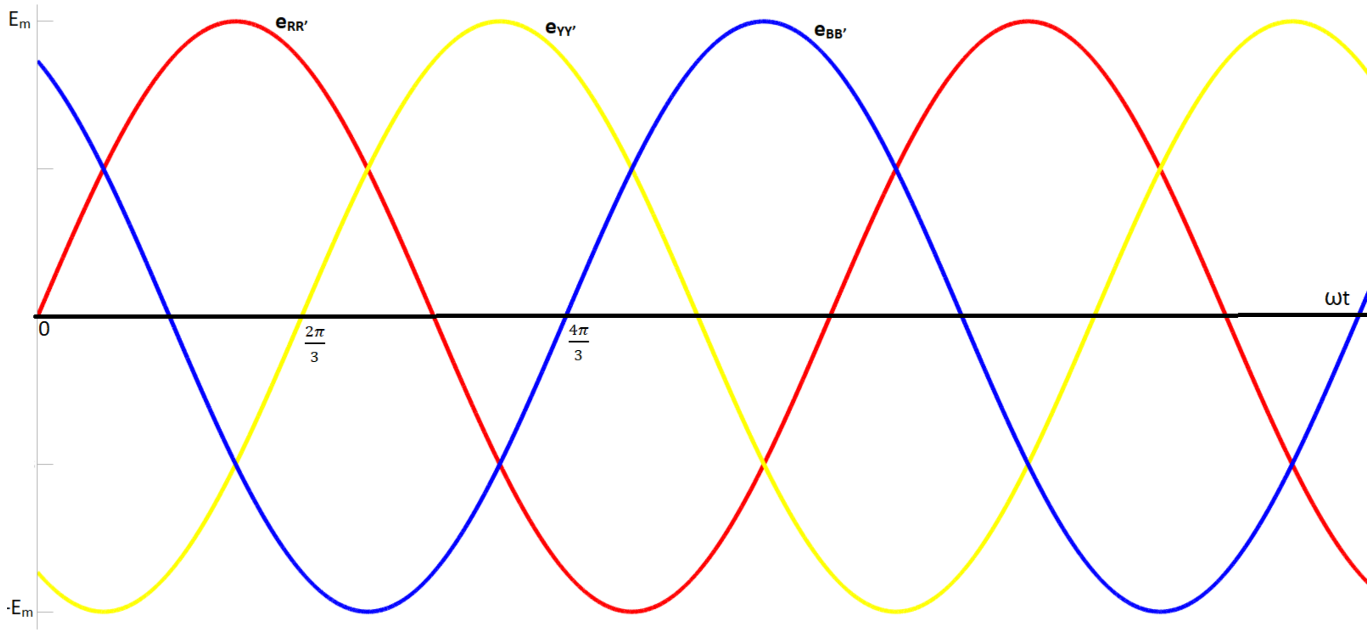
In this position, R-coil will have maximum rate of change of flux linkages and since facing south pole, will have maximum negative EMF induced in it.

As rotor completes one revolution, one cycle of sinusoidal EMF is induced in R – coil. In Y – coil, another sinusoidal EMF is induced which has same amplitude and frequency as that in R – coil but it lags R – coil EMF by 120°. Similarly in B – coil, another sinusoidal EMF is induced which has same amplitude and frequency as that in R – coil but it lags R – coil EMF by 240°.

Thus, As rotor completes one revolution, balanced three phase system of EMFs are generated in the three coils placed in the stator.

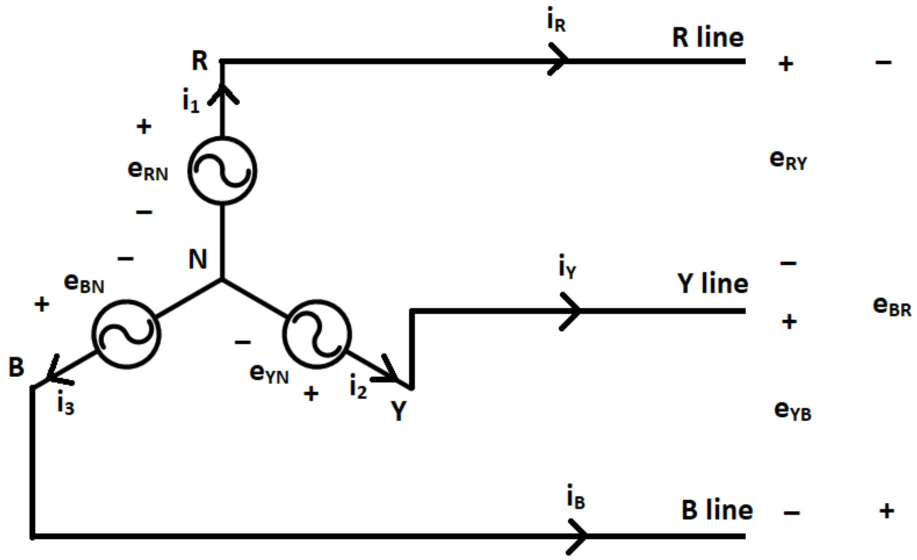
Phase Sequence:

Phase Sequence of a Three Phase System is defined as the sequence in which the three phase EMFs attain peak value. Usual phase sequence is RYB.


Em
-Em
0
eRR'
eYY'
eBB'
2π/3
4π/3
ωt

Balanced Star (or) WYE Connected Three Phase System

To make a star connected three phase system, similar terminals such as R' , Y' and B' are connected together to make 'Neutral' point (N) of the three phase system. And lines are run from the other three terminals i.e., R, Y and B.



Phase Voltage:

The voltage across the terminals of a phase is called the Phase Voltage.

Here, e_{RN} , e_{YN} & e_{BN} represent phase voltages.

Line Voltage:

The voltage across any two lines is called the Line Voltage.

Here, e_{RY} , e_{YB} & e_{BR} represent Line (or) Line to line voltages.

Phase Current:

The current flowing through a phase is called the Phase Current.

Here, i_1 , i_2 & i_3 represent phase currents.

Line Current:

The current flowing through a line is called the Line Current.

Here, i_R , i_Y & i_B represent line currents.

Relation between Line & Phase currents – Balanced Star System:

In a balanced star connected three phase system, it can observed that

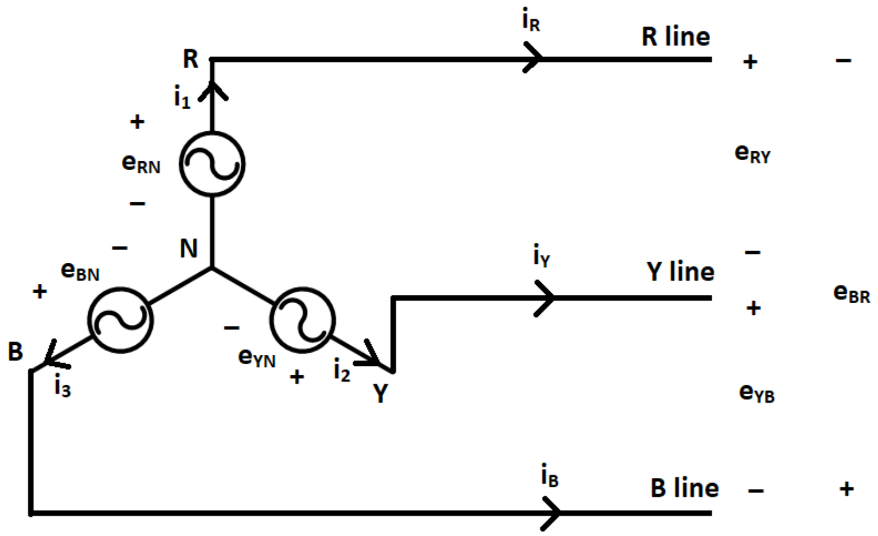
Line current = Phase current

i.e., $i_1 = i_R$

$i_2 = i_Y$

$i_3 = i_B$

Relation between Line & Phase voltages – Balanced Star System:



By KVL in the path RYNR, $- e_{RY} - e_{YN} + e_{RN} = 0$

Hence, $e_{RY} = e_{RN} - e_{YN}$

$$\overline{E_{RY}} = \overline{E_{RN}} - \overline{E_{YN}}$$

$$\overline{E_{RN}} = \frac{E_m}{\sqrt{2}} \angle 0^\circ = E_{ph} \angle 0^\circ$$

where, E_{ph} is the RMS value of phase voltage.

$$\overline{E_{YN}} = \frac{E_m}{\sqrt{2}} \angle -120^\circ = E_{ph} \angle -120^\circ$$

$$\overline{E_{BN}} = \frac{E_m}{\sqrt{2}}\angle -240^{\circ} = E_{ph}\angle -240^{\circ}$$

$$\overline{E_{RY}} = E_{ph}\angle 0^{\circ} - E_{ph}\angle -120^{\circ}$$

$$= E_{ph}(1-(\cos 120^{\circ} - \mathrm{j}\sin 120^{\circ}))$$

$$= E_{ph}(\frac{3}{2} + j\frac{\sqrt{3}}{2})$$

$$= \sqrt{3}E_{ph}(\cos 30° + \mathrm{j}\sin 30°)$$

$$= \sqrt{3}E_{ph}\angle 30°$$

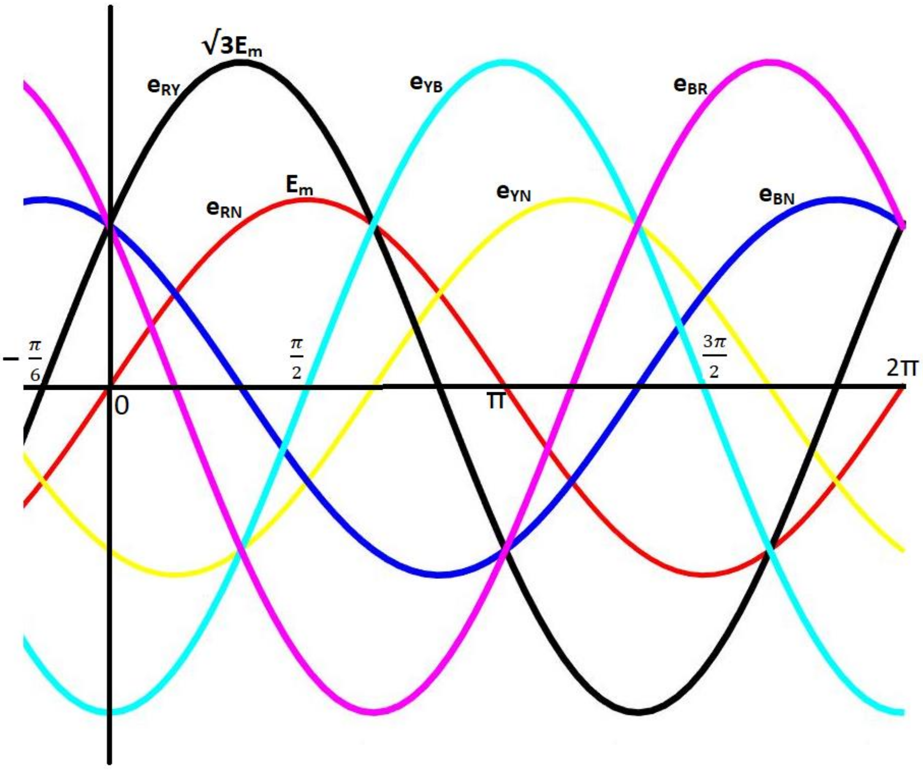
$$\overline{E_{YB}} = \overline{E_{YN}} - \overline{E_{BN}} = \sqrt{3}E_{ph}\angle -90^{\circ}$$

$$\overline{E_{BR}} = \overline{E_{BN}} - \overline{E_{RN}} = \sqrt{3}E_{ph}\angle -210^{\circ}$$

Thus, in a balanced star connected three phase system,

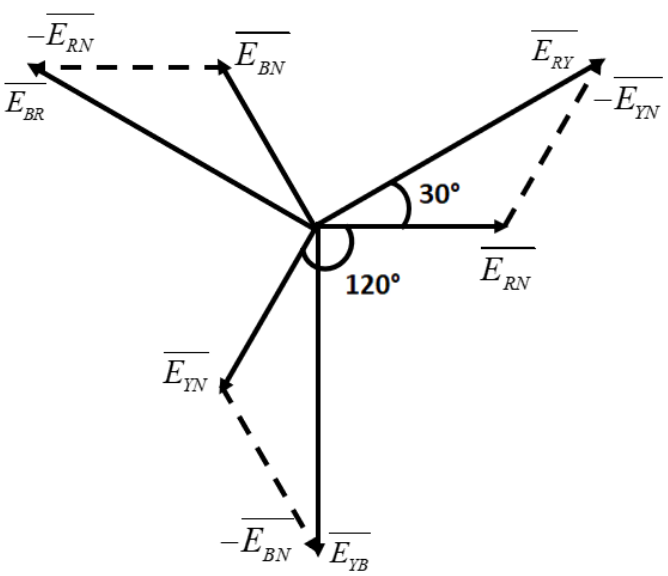
(i) Magnitude (RMS value) of Line Voltage = √3*(Magnitude of Phase Voltage)
(ii) Each line voltage leads the corresponding phase voltage by 30°

Balanced Star System – Line and Phase Voltage Waveforms:



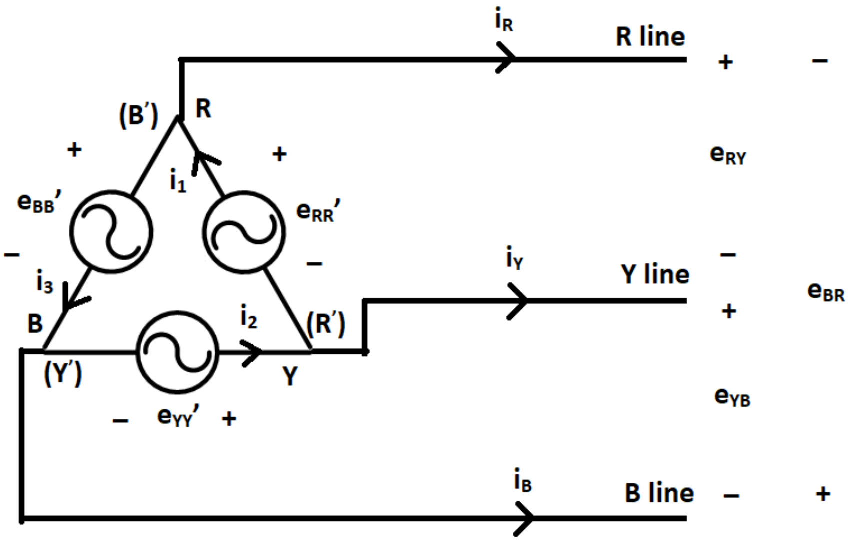
Balanced Star System – Phasor diagram:

$$\overline{E_{RY}} = \overline{E_{RN}} - \overline{E_{YN}}$$



Balanced Delta (or) Mesh Connected Three Phase System

To make a delta connected three phase system, the three coils R, Y & B are connected back to back (or) end to end. And lines are run from the junction points R, Y and B.



Phase Voltage:

The voltage across the terminals of a phase is called the Phase Voltage.

Here, $e_{RR'}$, $e_{YY'}$ & $e_{BB'}$ represent phase voltages.

Line Voltage:

The voltage across any two lines is called the Line Voltage.

Here, e_{RY} , e_{YB} & e_{BR} represent Line (or) Line to line voltages.

Phase Current:

The current flowing through a phase is called the Phase Current.

Here, i_1 , i_2 & i_3 represent phase currents.

Line Current:

The current flowing through a line is called the Line Current.

Here, i_R , i_Y & i_B represent line currents.

Relation between Line & Phase Voltages – Balanced Delta System:

In a balanced delta connected three phase system, it can observed that

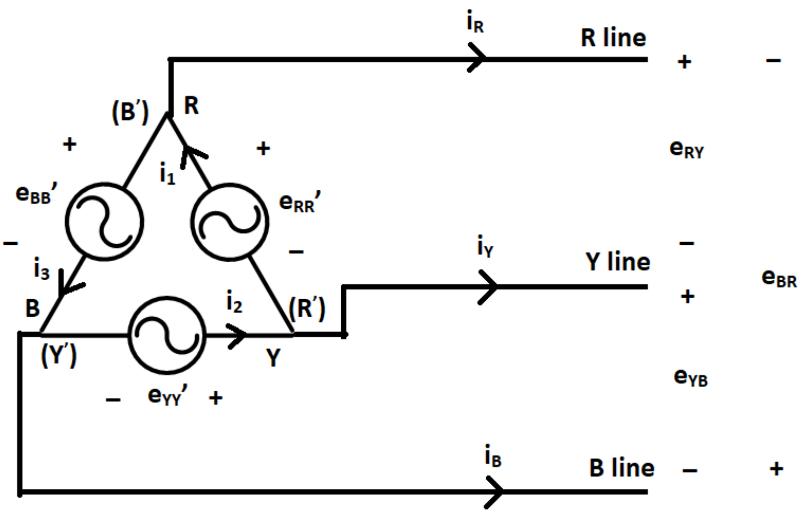
Line Voltage = Phase Voltage

i.e., $e_{RY} = e_{RR'}$

$e_{YB} = e_{YY'}$

$e_{BR} = e_{BB'}$

Relation between Line & Phase Currents – Balanced Delta System:



By KCL at R, $i_1 = i_R + i_3$

Hence, $i_R = i_1 - i_3$

$$\overline{I_R} = \overline{I_1} - \overline{I_3}$$

$$\overline{I_1} = \frac{I_m}{\sqrt{2}}\angle 0^\circ = I_{ph}\angle 0^\circ$$

where, I_{ph} is the RMS value of phase current.

$$\overline{I_2} = \frac{I_m}{\sqrt{2}}\angle -120^\circ = I_{ph}\angle -120^\circ$$

$$\overline{I_3} = \frac{I_m}{\sqrt{2}}\angle -240^{\circ} = I_{ph}\angle -240^{\circ}$$

$$\overline{I_R} = I_{ph}\angle 0^{\circ} - I_{ph}\angle -240^{\circ}$$

$$= I_{ph}(1-(\cos 240^{\circ} - \mathrm{j}\sin 240^{\circ}))$$

$$= I_{ph}(\frac{3}{2} - j\frac{\sqrt{3}}{2})$$

$$= \sqrt{3}I_{ph}(\cos 30° - \mathrm{j}\sin 30°)$$

$$= \sqrt{3}I_{ph}\angle -30°$$

$$\overline{I_Y} = \overline{I_2} - \overline{I_1} = \sqrt{3}I_{ph}\angle -150^{\circ}$$

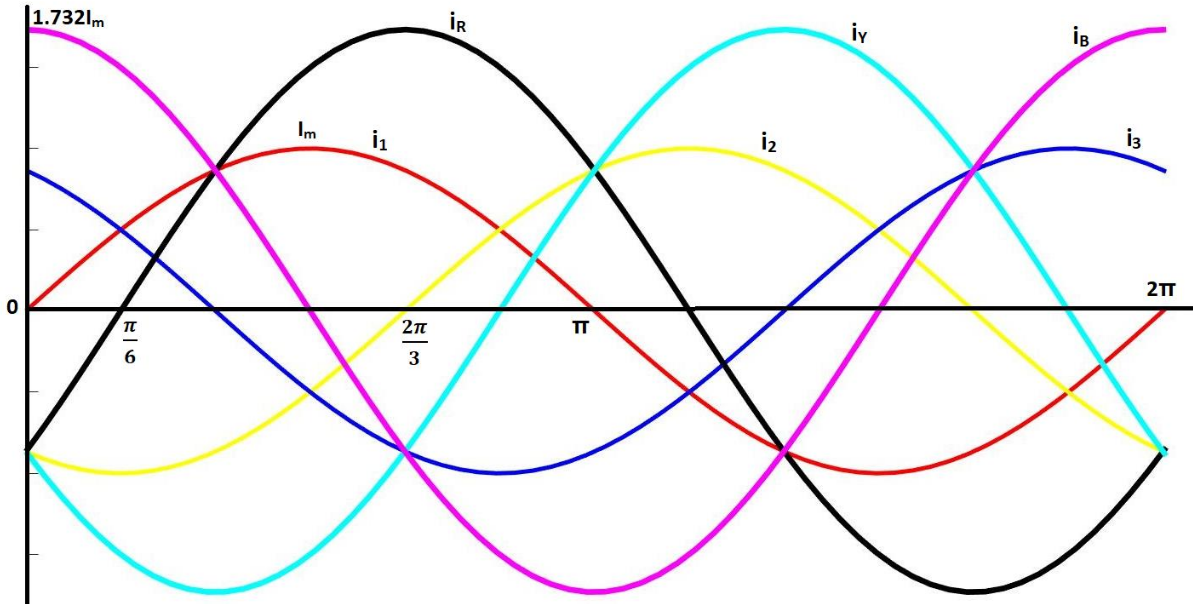
$$\overline{I_B} = \overline{I_3} - \overline{I_2} = \sqrt{3}I_{ph}\angle -270^{\circ}$$

Thus, in a balanced delta connected three phase system,

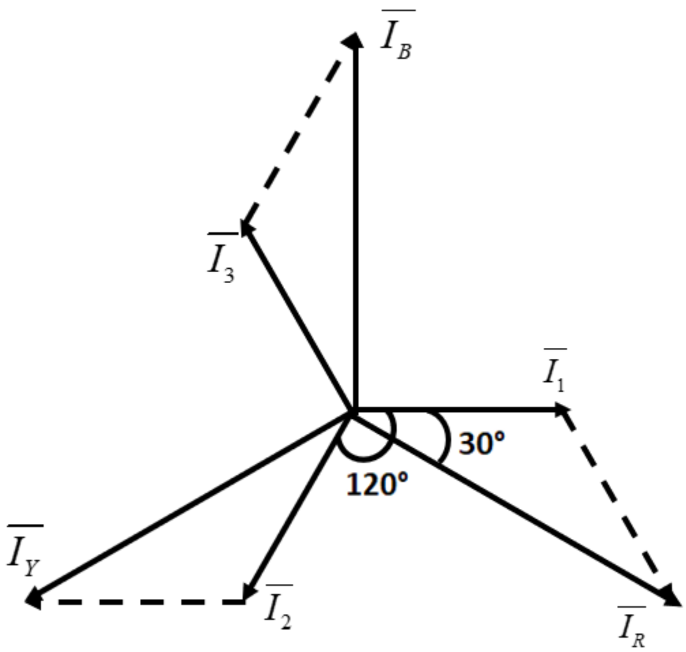
(i) Magnitude (RMS value) of Line current = √3*(Magnitude of Phase current)

(ii) Each line current lags the corresponding phase current by 30°

Balanced Delta System – Line and Phase current Waveforms:



Balanced Delta System – Phasor diagram:



$$\overline{I_R} = \overline{I_1} - \overline{I_3}$$

Balanced Three Phase Supply:

A Three phase supply is said to be balanced if

i) The three EMFs are equal in magnitude

ii) Phase displaced from one another by 120°

For example, $e_1 = 100\sin(\omega t)$

$e_2 = 100\sin(\omega t-120°)$

$e_3 = 100\sin(\omega t+120°)$

represents balanced three phase supply.

Examples of unbalanced three phase supply systems are:

i) $e_1 = 100\sin(\omega t)$, $e_2 = 110\sin(\omega t-120°)$, $e_3 = 100\sin(\omega t+120°)$

ii) $e_1 = 100\sin(\omega t)$, $e_2 = 100\sin(\omega t-125°)$, $e_3 = 100\sin(\omega t+120°)$

Balanced Three Phase Load:

A Three phase Load can be either star connected type or delta connected type.

A Three phase Load is said to be balanced if in each phase of the load both resistance and reactance are exactly same.

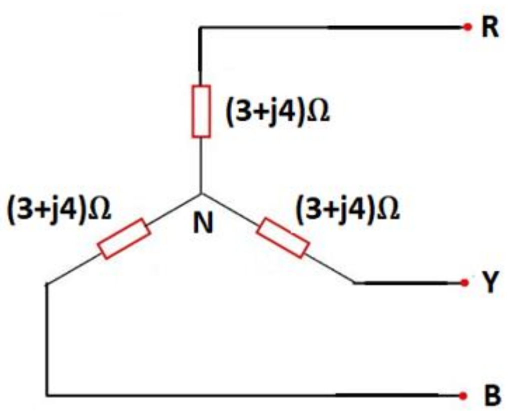


Fig 1: Balanced Star connected Load

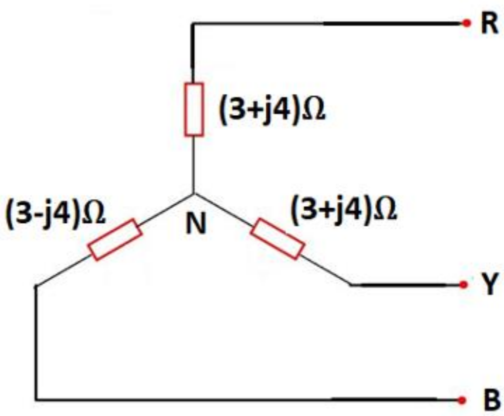


Fig 2: Unbalanced Star connected Load

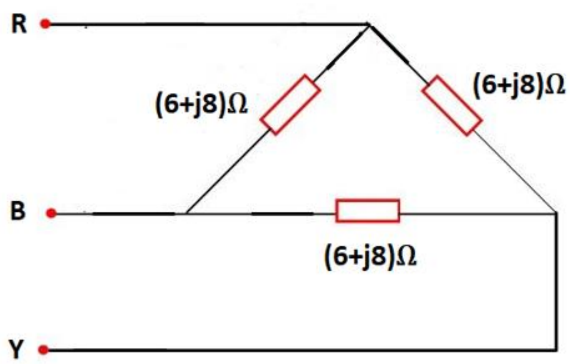


Fig 1: Balanced Delta connected Load

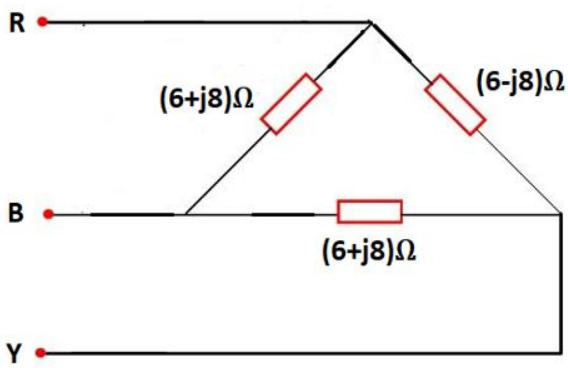


Fig 2: Unbalanced Delta connected Load

When a balanced three phase load is connected across a balanced three phase supply, the three phase currents drawn will also be balanced in nature.

Power Relations in Balanced Three Phase Systems:

When a balanced three phase load is connected across a balanced three phase supply, the three phase currents drawn will also be balanced in nature.

This leads to an advantage that balanced three phase systems can be analysed per phase basis and the results can be extended to all three phases.

Thus, in balanced three phase systems, power can be calculated for any one phase and the total three phase power will be thrice that of any one phase.

Active Power in Balanced Three Phase Systems:

Three phase Active Power, $P_{3\text{-phase}} = 3*P_{1\text{-phase}} = 3*(V_{ph}*I_{ph}*\cos\phi)$

For a balanced star connected three phase system,

$P_{3\text{-phase}} = 3*V_{ph}*I_{ph}*\cos\phi = \sqrt{3}*(\sqrt{3}*V_{ph})*I_{ph}*\cos\phi = \sqrt{3}*(V_L)*I_L*\cos\phi$

For a balanced delta connected three phase system,

$P_{3\text{-phase}} = 3*V_{ph}*I_{ph}*\cos\phi = \sqrt{3}*V_{ph}*(\sqrt{3}*I_{ph})*\cos\phi = \sqrt{3}*V_L*(I_L)*\cos\phi$

Thus, alternatively either for star (or) delta system, $P_{3\text{-phase}} = \sqrt{3}*V_L*I_L*\cos\phi$

Also, Alternatively, $P_{3\text{-phase}} = 3*(I_{ph}^2*R)$

Reactive Power in Balanced Three Phase Systems:

Three phase Reactive Power, $Q_{3\text{-phase}} = 3*Q_{1\text{-phase}} = 3*(V_{ph}*I_{ph}*\sin\phi)$

Alternatively, $Q_{3\text{-phase}} = \sqrt{3}*V_L*I_L*\sin\phi$

Also, Alternatively,

$Q_{3\text{-phase}} = 3*(I_{ph}^2*X_L)$ for inductive loads

(or)

$Q_{3\text{-phase}} = -3*(I_{ph}^2*X_C)$ for capacitive loads

(or)

$Q_{3\text{-phase}} = 3*(I_{ph}^2*(X_L - X_C))$ for series RLC type of loads

Apparent Power in Balanced Three Phase Systems:

Three phase Apparent Power, $S_{3\text{-phase}} = 3*S_{1\text{-phase}} = 3*(V_{ph}*I_{ph})$

Alternatively, $S_{3\text{-phase}} = \sqrt{3}*V_L*I_L$

Also, Alternatively,

$S_{3\text{-phase}} = 3*(I_{ph}^2*|Z|)$

Apparent power can also be found as

$$S_{3\text{-phase}} = \sqrt{{P_{3-phase}}^2 + {Q_{3-phase}}^2}$$

Power factor of a balanced three phase system is

$$\cos\phi = \frac{P_{3-phase}}{S_{3-phase}}$$

Numerical Examples on Balanced Star connected Three Phase System:

Numerical Example 1:

Question:

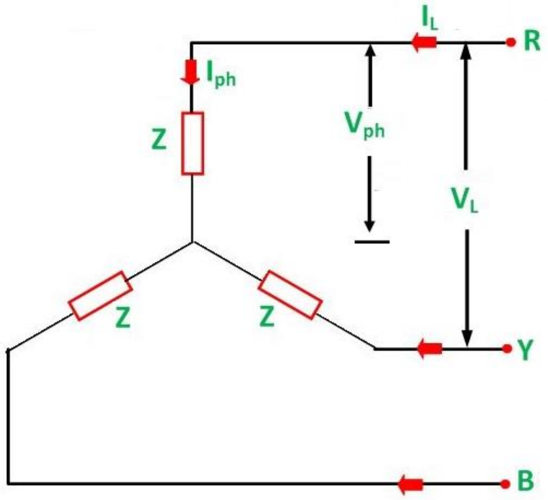
A balanced 3 phase load consists of three coils, each of 4Ω resistance and 0.02H inductance. Determine the total active and reactive power when the coils are connected in star, if the supply voltage is 400V, 50 Hz.

Solution :

Note:

In three phase systems, by default, the voltages and currents given are line voltage and line current respectively.

Similarly, power given is three phase power by default.



Given Data:

Line voltage, V_L = 400V ; f = 50Hz

Resistance per phase, R = 4Ω

Inductance per phase, L = 0.02H

Calculations:

Inductive reactance per phase, $X_L = 2\pi fL = 6.28\Omega$

Impedance per phase, $Z = R+jX_L = (4+j6.28)\Omega$

Hence, $|Z| = 7.45\Omega$; Phase Angle, $\phi = 57.5°$

Since star connected system, Phase voltage, $V_{ph} = \frac{V_L}{\sqrt{3}} = 230.94V$

Hence, Phase current, $I_{ph} = \frac{V_{Ph}}{|Z|} = 31A$

Therefore, Line current, $I_L = I_{ph} = 31A$

Three phase Active Power, $P_{3\text{-phase}} = 3*P_{1\text{-phase}} = 3*V_{ph}*I_{ph}*\cos(\phi) = 11.54KW$

Alternatively, $P_{3\text{-phase}} = \sqrt{3}*V_L*I_L*\cos(\phi) = 11.54KW$

Alternatively, $P_{3\text{-phase}} = 3*I_{ph}^2*R = 11.54KW$

Similarly, Three phase Reactive Power,

$Q_{3\text{-phase}} = 3*Q_{1\text{-phase}} = 3*V_{ph}*I_{ph}*\sin(\phi) = 18.11KVAR$

Alternatively, $Q_{3\text{-phase}} = \sqrt{3}*V_L*I_L*\sin(\phi) = 18.11KVAR$

Alternatively, $Q_{3\text{-phase}} = 3*I_{ph}^2*X_L = 18.11KVAR$

Numerical Example 2

Question:

A balanced 3Φ, star connected load of 100KW takes a leading current of 80A when connected to a 3Φ, 1.1KV, 50Hz supply. Find the resistance, impedance, and the capacitance of the load per phase. Also calculate the power factor of the load.

Solution :

Given Data:

Line voltage, V_L = 1.1KV ; f = 50Hz

Line current, I_L = 80A

Three phase Active Power, $P_{3\text{-phase}}$ = 100KW

Calculations:

Since star connected system, Phase current, $I_{ph} = I_L$ = 80A

And Phase voltage, $V_{ph} = \frac{V_L}{\sqrt{3}}$ = 635.08V

Impedance per phase, Z $= \frac{V_{ph}}{I_{ph}}$ = 7.94Ω

$P_{3\text{-phase}}$ = 100KW = $3*I_{ph}^2*R$; Hence, R = 5.21Ω

Capacitive reactance per phase, $X_C = \sqrt{(Z^2 - R^2)}$ = 5.99Ω

Hence, Capacitance per phase, C = $\frac{1}{\omega X_C}$ = 531.25μF

Powerfactor of the load, cosϕ $= \frac{R}{Z}$ = 0.656 Lead

Alternative solution:

Since star connected system, Phase current, $I_{ph} = I_L$ = 80A

And Phase voltage, $V_{ph} = \frac{V_L}{\sqrt{3}}$ = 635.08V

Impedance per phase, Z $= \frac{V_{ph}}{I_{ph}}$ = 7.94Ω

$P_{3\text{-phase}}$ = 100KW = $\sqrt{3}*V_L*I_L*\cos(\phi)$

Hence, Powerfactor of the load, cosϕ = 0.656 Lead

Resistance per phase, R = Z*cosϕ = 5.21Ω

Capacitive reactance per phase, $X_C = \sqrt{(Z^2 - R^2)}$ = 5.99Ω

Hence, Capacitance per phase, C = $\frac{1}{\omega X_C}$ = 531.25μF

Numerical Examples on Balanced Delta connected Three Phase System:

Numerical Example 1:

Question:

A balanced delta connected load consumes 2 KW of power when connected to a three phase, 400 V, 50Hz supply. The same load when connected to a three phase 230V , 50 Hz supply, draws a current of 2 A at lagging power factor. Determine the load power factor and resistance and inductance per phase.

Solution :

Case1: Delta connected Load

Given Data:

Line voltage, V_L = 400V ; f = 50Hz

Three phase Active Power, $P_{3\text{-phase}}$ = 2KW

Therefore, $3 * V_{ph} * I_{ph} * \cos\phi$ = 2000W

i.e., $3 * V_{ph} * \frac{V_{ph}}{Z} * \frac{R}{Z}$ = 2000W

Since, delta connected system, $V_{ph} = V_L$ = 400V

Therefore, $3 * 400^2 * \frac{R}{Z^2}$ = 2000W ; Hence, $\frac{R}{Z^2}$ = 0.004167 ------ (1)

Case2: Same Delta connected Load

Given Data:

Line voltage, V_L = 230V ; f = 50Hz

Line current, I_L = 2A

Since same load, Z is same in both the cases.

Since, delta connected system, $V_{ph} = V_L$ = 230V

And, $I_{ph} = \frac{I_L}{\sqrt{3}}$ = 1.155A

Therefore, Z = $\frac{V_{ph}}{I_{ph}}$ = 199.13Ω

Substituting 'Z' value in equation (1) above,

Resistance per phase, R = 165.24Ω

Inductive reactance per phase, X_L = $\sqrt{Z^2 - R^2}$ = 111.12Ω

Inductance per phase, L = $\frac{X_L}{\omega}$ = 0.354H

Power factor of the Load, cosϕ = $\frac{R}{Z}$ = 0.83 Lag

Numerical Example 2

Question:

The load connected to a three phase supply comprises three similar coils connected in star. The line current is 25 A, the real and apparent powers are 11KW, 20 KVA. Find the line voltage, resistance, and reactance of each coil. If the coils are connected in delta, find the line current and power taken.

Solution :

Case1: Balanced Star connected Load

Given Data:

Line current, I_L = 25A

Three phase Active Power, $P_{3\text{-phase}}$ = 11KW

Three phase Apparent Power, $S_{3\text{-phase}}$ = 20KVA

Calculations:

$S_{3\text{-phase}}$ = √3*V_L*I_L = 20KVA

Hence, V_L = 461.88V

Since balanced star connected load,

I_{ph} = I_L = 25A ; V_{ph} = $\frac{V_L}{\sqrt{3}}$ = 266.66V

Impedance per phase, $Z = \frac{V_{ph}}{I_{ph}} = 10.66\Omega$

$P_{3\text{-phase}} = 11KW = 3*I_{ph}^2*R$; Hence, $R = 5.86\Omega$

Inductive reactance per phase, $X_L = \sqrt{(Z^2 - R^2)} = 8.905\Omega$

Case2: Same Load reconnected as Delta Load across same supply

Since same supply, V_L remains same and since same load, Z, R & L in each phase remain same.

Since balanced delta connected load, $V_{ph} = V_L = 461.88V$

Phase current, $I_{ph} = \frac{V_{ph}}{Z} = 43.33A$

Line current, $I_L = \sqrt{3}*I_{ph} = 75.04A$

Power drawn, $P_{3\text{-phase}} = 3*I_{ph}^2*R = 33KW$

Apparent Power, $S_{3\text{-phase}} = \sqrt{3}*V_L*I_L = 60KVA$

Measurement of Power and Power factor using Two Wattmeter Method:

Wattmeter:

Wattmeter is a power measuring instrument. It consists of a fixed coil called 'Current Coil (CC) and a moving coil called 'Pressure Coil' (PC). Pressure coil is also called 'Voltage coil' (VC).

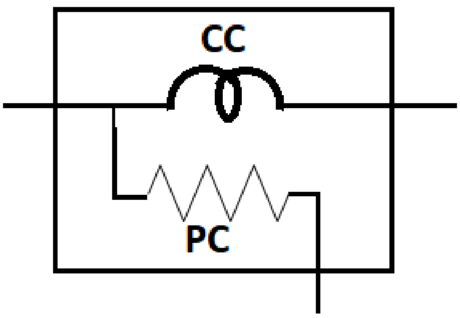


Fig: Wattmeter symbol

PC carries the pointer of the instrument and indicates the average power.

Two Wattmeter Method:

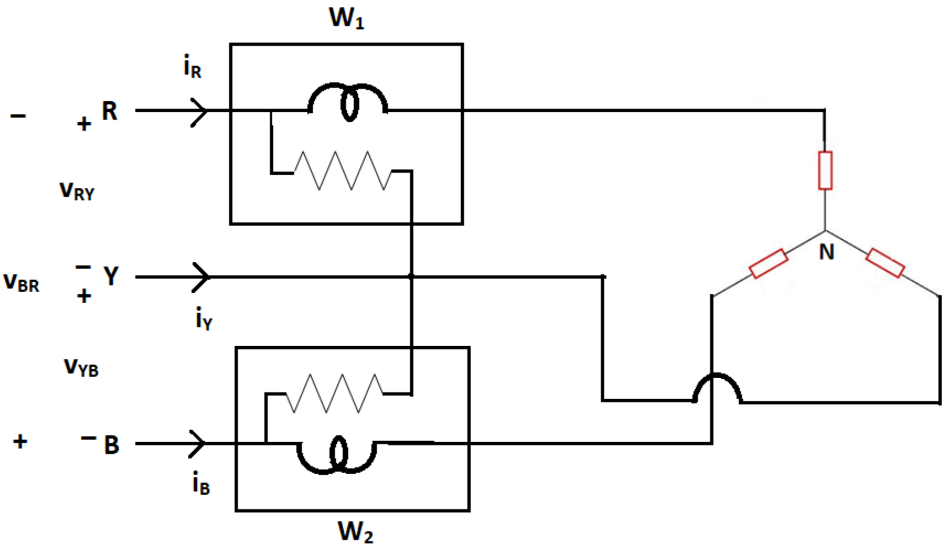


Fig: Two Wattmeter Method circuit diagram

Two wattmeters are sufficient to measure three phase active power irrespective of whether the load is star connected or delta connected and irrespective of whether the load is balanced or unbalanced.

First Wattmeter reading, $W_1 = V_{RY}*I_R*\cos(\angle(V_{RY}, I_R))$

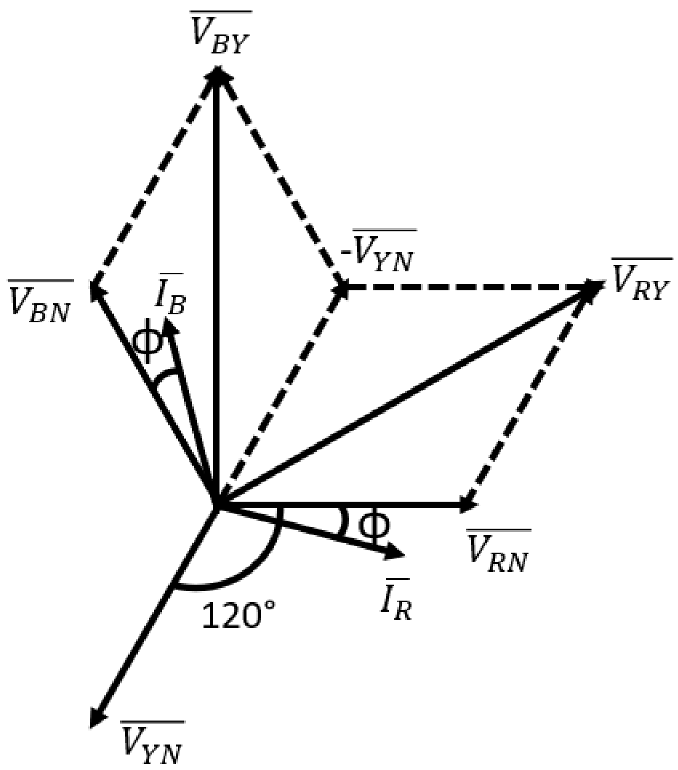
Second Wattmeter reading, $W_2 = V_{BY}*I_B*\cos(\angle(V_{BY}, I_B))$

To obtain the angles, let us draw the phasor diagram.

Consider inductive load. Then, phase current lags phase voltage.

$$\overline{V_{RY}} = \overline{V_{RN}} - \overline{V_{YN}}$$

$$\overline{V_{BY}} = \overline{V_{BN}} - \overline{V_{YN}}$$



Therefore, $W_1 = V_{RY}*I_R*\cos(30+\phi)$

$W_2 = V_{BY}*I_B*\cos(30-\phi)$

Hence, $W_1 = V_L * I_L * \cos(30+\phi)$ & $W_2 = V_L * I_L * \cos(30-\phi)$

Therefore, $W_1 + W_2 = \sqrt{3} * V_L * I_L * \cos\phi = P_{3-\text{phase}}$ --- (1)

Similarly, $\sqrt{3} * (W_2 - W_1) = \sqrt{3} * V_L * I_L * \sin\phi = Q_{3-\text{phase}}$ --- (2)

Therefore, $\frac{(2)}{(1)}$ gives,

$$\frac{Q_{3-\text{phase}}}{P_{3-\text{phase}}} = \frac{\sqrt{3} * (W_2 - W_1)}{W_1 + W_2} = \text{Tan}\phi$$

Hence, power factor of the system is,

$$\cos\phi = \cos\left(Tan^{-1}\left(\frac{\sqrt{3} * (W_2 - W_1)}{W_1 + W_2}\right)\right)$$

Variation in Wattmeter readings with power factor of the load

Variation in wattmeter readings – Inductive load

For inductive loads, $W_1 = V_L*I_L*\cos(30+\phi)$ & $W_2 = V_L*I_L*\cos(30-\phi)$

As phase angle ϕ increases, load power factor decreases.

With an increase in the phase angle ϕ, $W_1 = V_L*I_L*\cos(30+\phi)$ decreases &

$W_2 = V_L*I_L*\cos(30-\phi)$ increases.

Phase Angle, ϕ	**Load Power factor, $\cos\phi$**	**$W_1 = V_LI_L\cos(30+\phi)$**	**$W_2 = V_LI_L\cos(30-\phi)$**	**Comments**
0°	1	$\frac{\sqrt{3}V_LI_L}{2}$	$\frac{\sqrt{3}V_LI_L}{2}$	$W_1 = W_2$
30°	0.866 Lag	$\frac{V_LI_L}{2}$	V_LI_L	$W_1 = \frac{W_2}{2}$
60°	0.5 Lag	0	$\frac{\sqrt{3}V_LI_L}{2}$	$W_1 = 0$; $W_2 = P_{3-\text{phase}}$
>60°	< 0.5 Lag	Negative	Positive	W_1 = -ve; W_2 = +ve

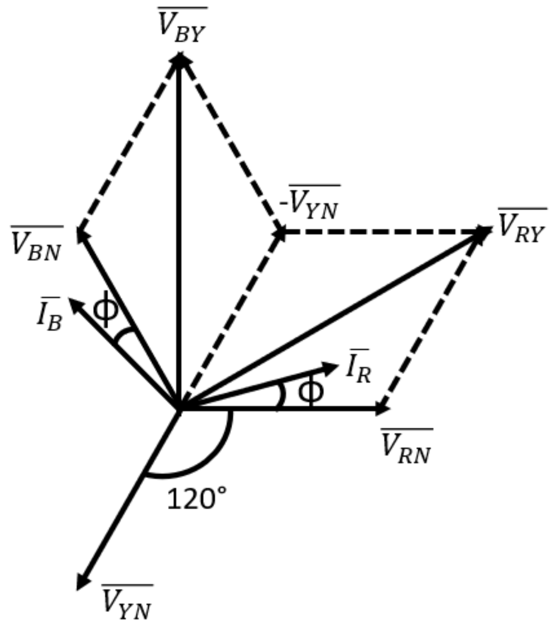
Important observations:

- When phase angle is <60° (or) power factor of the load is > 0.5 Lag, both the wattmeters read positive readings.
- When phase angle = 60° (or) power factor of the load is = 0.5 Lag, one of the wattmeters reads zero and the other reads the total three phase active power.
- When phase angle is >60° (or) power factor of the load is < 0.5 Lag, one of the wattmeters reads negative i.e., its pointer moves behind zero. To

record its reading, either reverse its CC connections or PC connections (not both) & record this value with a negative sign.

Variation in wattmeter readings – Capacitive load

Now, consider capacitive Loads



For this case, $W_1 = V_{RY}*I_R*\cos(30-\phi)$ & $W_2 = V_{BY}*I_B*\cos(30+\phi)$

Phase Angle, ϕ	**Load Power factor, cosϕ**	**$W_1 = V_LI_L\cos(30-\phi)$**	**$W_2 = V_LI_L\cos(30+\phi)$**	**Comments**
0°	1	$\frac{\sqrt{3}V_LI_L}{2}$	$\frac{\sqrt{3}V_LI_L}{2}$	$W_1 = W_2$
30°	0.866 Lead	V_LI_L	$\frac{V_LI_L}{2}$	$W_2 = \frac{W1}{2}$
60°	0.5 Lead	$\frac{\sqrt{3}V_LI_L}{2}$	0	$W_1 = P_{3-phase}$; $W_2 = 0$
>60°	< 0.5 Lead	Positive	Negative	W_1 = +ve; W_2 = -ve

Thus, the readings become just the opposite to that of inductive load case.

Numerical Example 1

Question:

In a two wattmeter method of measuring three phase power, it is observed that the wattmeter readings are in the ratio of 3:1. Determine the power factor of the Load.

Solution:

Given Data:

$W_1 : W_2 = 3:1$

Power factor = cosϕ = $\cos(Tan^{-1}(\sqrt{3}*\frac{(W_1 - W_2)}{(W_1 + W_2)}))$

Therefore, Power factor = 0.756

Numerical Example 2

Question:

Two wattmeters are connected to measure input to a balanced three phase circuit indicate 2000W and 500W respectively. Find the power factor when

i) **Both readings are positive**

ii) **Latter reading is obtained after reversing its CC**

Solution:

Case 1: $W_1 = 2000W$; $W_2 = 500W$

Power factor = $\cos(Tan^{-1}(\sqrt{3}*\frac{(W_1 - W_2)}{(W_1 + W_2)}))$ = 0.693

Case 2: $W_1 = 2000W$; $W_2 = -500W$

Power factor = $\cos(Tan^{-1}(\sqrt{3}*\frac{(W_1 - W_2)}{(W_1 + W_2)}))$ = 0.327

Numerical Examples on Two Wattmeter Method

Numerical Example 1

Question:

Three coils each having a resistance of 20Ω and a reactance of 15Ω are connected in star across a three phase 400V, 50Hz supply. Calculate the readings of the two wattmeters connected to measure the power input.

If the coils are now connected in delta across the same supply, calculate the new wattmeter readings.

Solution:

Case 1: Balanced Star connected Load

Given Data:

Line voltage, V_L = 400V, f = 50Hz

Resistance per phase, R = 20Ω

Reactance per phase, X_L = 15Ω

Calculations:

Impedance per phase, Z = (20+j15)Ω

Therefore, |Z| = 25Ω & Phase Angle, ϕ = 36.87°

Phase voltage, $V_{ph} = \frac{V_L}{\sqrt{3}}$ = 230.94V

Phase current, $I_{ph} = \frac{V_{ph}}{|Z|}$ = 9.24A = I_L (since star system)

Therefore, $W_1 = V_L I_L \cos(30+\phi)$ = 1.451KW

Similarly, $W_2 = V_L I_L \cos(30-\phi)$ = 3.67KW

Case 2: Same Load reconnected as Delta Load

Since same supply, V_L remains same and since same load, Z remains same

Phase voltage, $V_{ph} = V_L$ = 400V

Phase current, $I_{ph} = \frac{V_{ph}}{|Z|}$ = 16A

Line current, $I_L = \sqrt{3} * I_{ph}$ = 27.71A

Therefore, $W_1 = V_L I_L \cos(30+\phi)$ = 4.354KW

Similarly, $W_2 = V_L I_L \cos(30-\phi)$ = 11KW

Numerical Example 2

Question:

Two wattmeters are connected to measure power in a three phase circuit. The reading of one of the wattmeters is 5KW when the load power factor is unity. If the power factor of the load is changed to 0.707 lag without changing the total input power, calculate the new readings of the wattmeters.

Solution:

Case 1: Load Power factor is unity

Given Data:

W_1 = 5KW

Calculations:

Since, power factor is unity, $W_1 = W_2$

Therefore, $P_{3\text{-phase}} = W_1 + W_2$ = 10KW

Case 2: Load Power factor is changed to 0.707 Lag with total input power unchanged.

Since total input power is same, $P_{3\text{-phase}} = W_1 + W_2$ = 10KW ---- (1)

Power factor = $\cos(Tan^{-1}(\sqrt{3} * \frac{(W_2 - W_1)}{(W_1 + W_2)}))$ = 0.707 ---- (2)

Solving (1) & (2), W_1 = 2.12KW ; W_2 = 7.88KW

Numerical Example 3

Question:

Two wattmeters connected to measure three phase power for star connected load read 3KW and 1KW respectively. The line current is 10A. Calculate

a) **Line and Phase Voltage**

b) **Resistance and reactance per phase**

Solution:

Given Data: Star connected Load

W_1 = 3KW & W_2 = 1KW

Line current, I_L = 10A

Calculations:

Power factor = $\cos(Tan^{-1}(\sqrt{3}*\frac{(W_1 - W_2)}{(W_1 + W_2)}))$ = 0.756

$P_{3\text{-phase}} = W_1 + W_2 = 4KW = \sqrt{3}*V_L*I_L*\cos\phi$

Therefore, Line voltage, V_L = 305.48V

Phase voltage, $V_{ph} = \frac{V_L}{\sqrt{3}}$ =176.37V

Impedance per phase, Z = $\frac{V_{ph}}{I_{ph}}$ = 17.64Ω

Resistance per phase, R = Z*cosϕ = 13.33Ω

Reactance per phase, X = $\sqrt{Z^2 - R^2}$ = 11.55Ω

Advantages of Three Phase Systems over Single Phase Systems

1. For certain amount of power to be transmitted over certain distance, a three phase system requires less conductor material compared to single phase system.

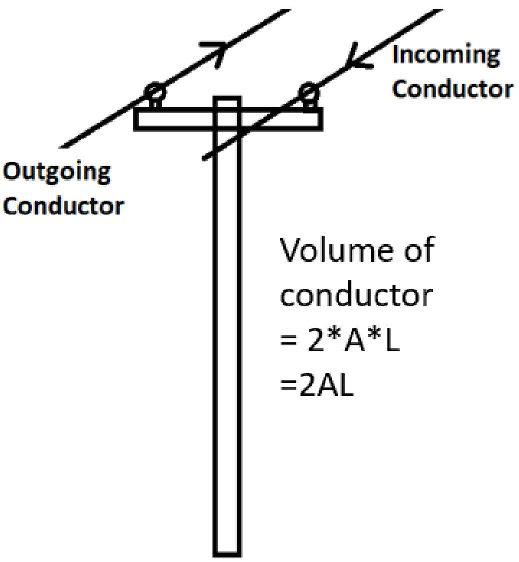


Fig: Single Phase Transmission Line

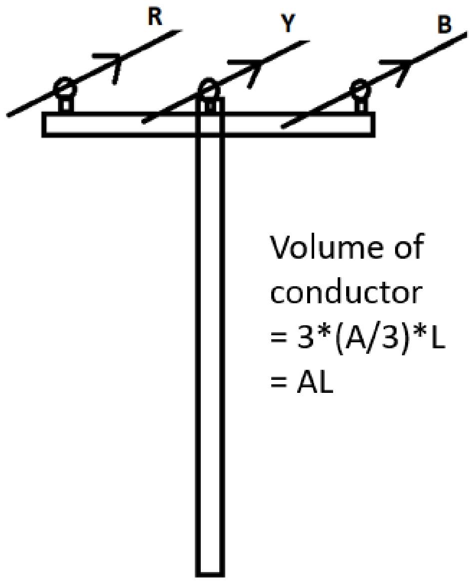


Fig: Three Phase Transmission Line

2. For the same frame size, a three phase machine can handle higher amount of power compared to its single phase counterpart.
3. Single Phase Power is pulsating in nature, whereas three phase power flowing into a three phase motor is almost constant at every instant. Hence, three phase motors run smoother and less noisy & hence, have better lifespan than a single phase motor.
4. Three Phase Induction Motors are self-starting whereas single phase induction motors are not self-starting in nature which makes three phase induction motors widely popular in industrial drives.

Numerical Example on Wattmeter Method

Question:

A 3-phase Y-connected, balanced load with a lagging power factor is supplied at 400 V (between the lines). A wattmeter when connected with its current coil in the R-line and voltage coil between R and Y lines gives a reading of

6kW. When the same terminals of the voltage coil are switched over to Y- and B-lines, the current coil connections remaining the same, the reading of the wattmeter remains unchanged. Calculate the line current and power factor of the load. Phase sequence is RYB.

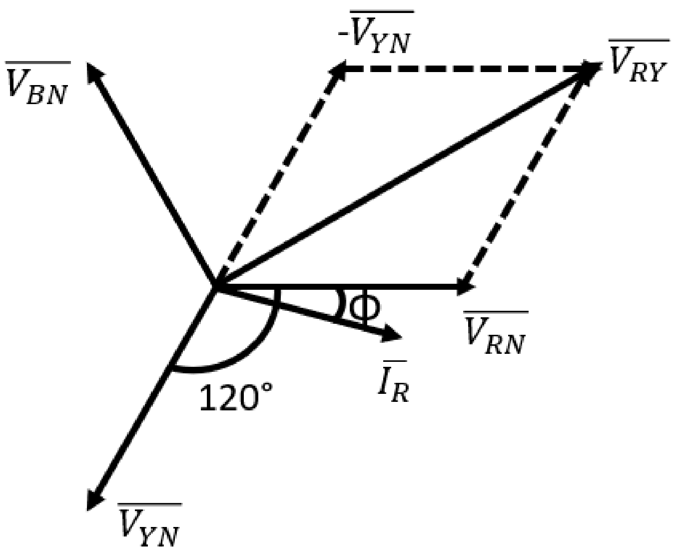
Solution:

Case 1: CC in R-line and PC between R & Y lines

Wattmeter reading,

$W_1 = V_{RY} * I_R * \cos(\angle(V_{RY}, I_R))$

$$\overline{V_{RY}} = \overline{V_{RN}} - \overline{V_{YN}}$$



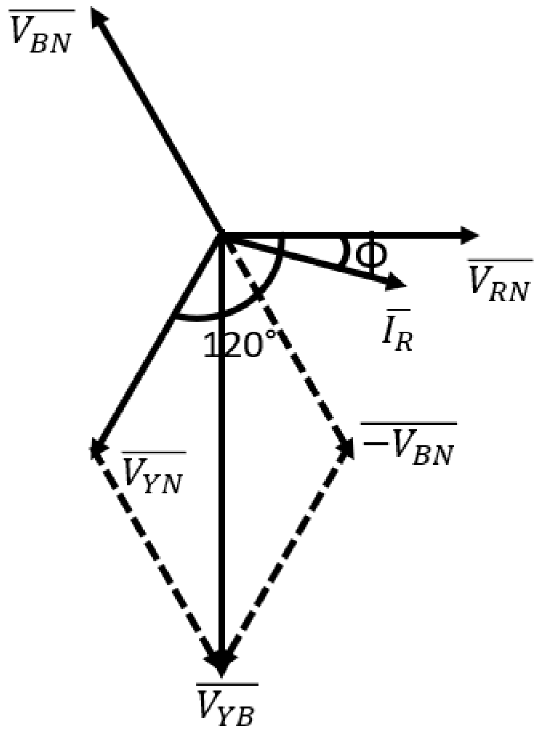
Therefore, $W_1 = V_L * I_L * \cos(30+\phi) = 6000W$ (given)

Case 2: CC in R-line and PC between Y & B lines

Wattmeter reading,

$W_2 = V_{YB} * I_R * \cos(\angle(V_{YB}, I_R))$

$$\overline{V_{YB}} = \overline{V_{YN}} - \overline{V_{BN}}$$



Therefore, $W_2 = V_L*I_L*\cos(90-\phi) = 6000W$ (given)

Since it is given that wattmeter reading is same in both the cases,

$W_1 = W_2$

i.e., $V_L*I_L*\cos(30+\phi) = V_L*I_L*\cos(90-\phi)$

Therefore, $\cos(30+\phi) = \sin\phi$

Hence, $\phi = 30°$

Therefore, Power factor = $\cos\phi = 0.866$ Lag

To find Line current I_L , substitute V_L & ϕ values either in W_1 equation or W_2 equation above.

Therefore, $W_1 = V_L*I_L*\cos(30+\phi) = 6000W$

Hence, Line current, $I_L = 30A$

Calculation of Energy Consumption

Step 1

- Calculate Watts per day consumption.

Step 2

- Convert to KilloWatts per day consumption.

Step 3

- Calculate energy uses over a month period.

Step 4

- Figure out the cost considering the rate.

Example:

Sl. No.	Item	Wattage	No. of Hours Working per Day	KWh per day
1	Lights	280	5	1.4
2	Fans	60 x 2	10	1.2
3	Geyser	2000	1/2	1
4	Mixer- grinder	750	1/3	0.25
5	Refrigerator	100	15	1.5
6	Freezer	300	4	1.2
7	Washing Machine	325	1	0.325
8	Iron	1000	1/2	0.5
9	Motor – 1 hp	746	1	0.746
10	Television	100	10	1
11	Laptop Charging	120	2	0.24
12	Mobile Charger	15	2	0.03
13	Internet Modem	8	24	0.192
14	Microwave	900	1	0.9
15	Vacuum Cleaner	350	1	0.35

Total Energy Consumption per day = 10.833 KWh
Total Energy Consumption in a month =10.833 x 30= 325 KWh

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Total Energy Consumption per day = 10.833 KWh
Total Energy Consumption in a month =10.833 x 30= 325 KWh

Tariff Calculation:

Effective from 1st April'2018	Sanctioned Load Fixed Charge (FC)	Tariff
Domestic - Applicable to Areas under Bruhat Bangalore Mahangara Palike (BBMP),Municipal Corporations and all Urban Local Bodies	FC For 1st KW	Rs.50
	FC For addl. KW	Rs.60
	Consumption Energy Charges (EC)	
	0 to 30 KWH	350 Ps
	31 to 100 KWH	495 Ps
	101 to 200 KWH	650 Ps
	201 to 300 KWH	755 Ps
	301 to 400 KWH	760 Ps
	Above 400 KWH	765 Ps
	Fuel Adjustment Charges (FAC)	
	Unit consumed KWH	14 Ps

Fixed Charge			
Unit	Rate (Rs)	Amount (Rs)	Total Amount (Rs)
1	50	50 x 1	50
2	60	60 x 2	120
Energy Charges			
Unit	Rate (Rs)	Amount (Rs)	Total Amount (Rs)
30	3.5	30 x 3.5	105
100	4.95	70 x 4.95	346.5
200	6.5	100 x 6.5	650
300	7.55	100 x 7.55	755
325	7.6	25 x 7.6	190
Fuel Adjustment Charge			
Unit	Rate (Rs)	Amount (Rs)	Total Amount (Rs)

325	0.14	325 x 0.14	45.5

Total amount: Rs.2262
Tax amount (7.3%) = 2262 x 0.073= Rs.165.126
Total billing amount = Rs. 2427.126