



## **ENGINEERING MECHANICS**

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Department of Civil Engineering



**UE23CV131A**

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## **Engineering Mechanics Statics**

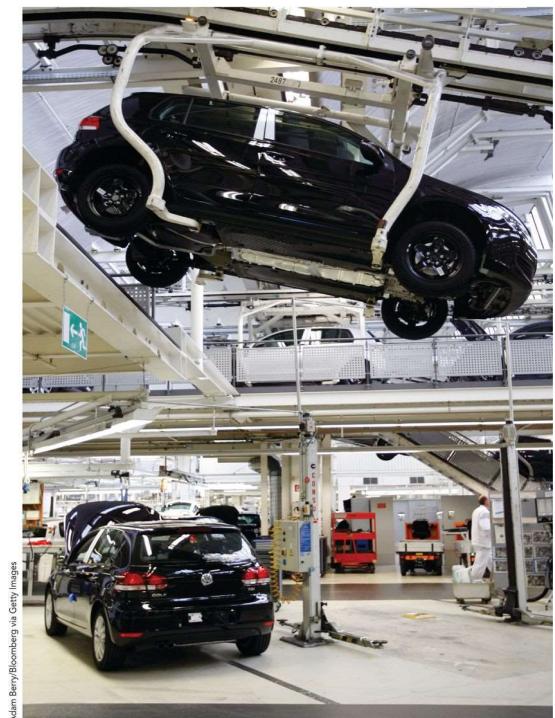
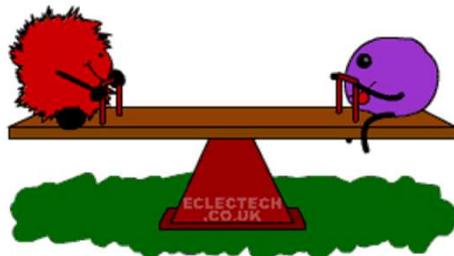
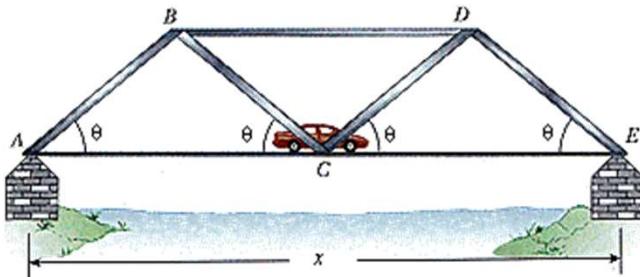
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# ENGINEERING MECHANICS

## Unit -2

### Part -1 - Equilibrium



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Adam Berry/Bloomberg via Getty Images

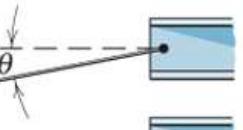
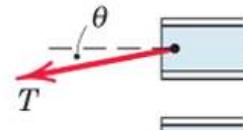
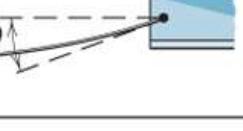
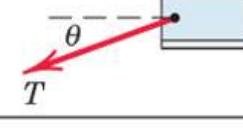
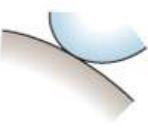
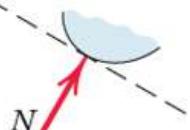


# ENGINEERING MECHANICS

## Equilibrium



### MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

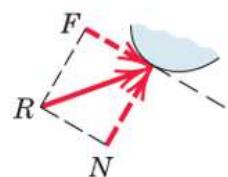
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible	  Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
Weight of cable not negligible	 
2. Smooth surfaces	  Contact force is compressive and is normal to the surface.

# ENGINEERING MECHANICS

## Equilibrium

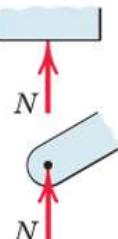
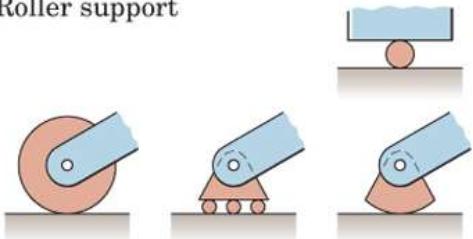


### 3. Rough surfaces



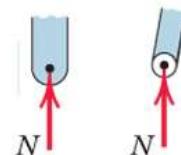
Rough surfaces are capable of supporting a tangential component  $F$  (frictional force) as well as a normal component  $N$  of the resultant contact force  $R$ .

### 4. Roller support



Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

### 5. Freely sliding guide



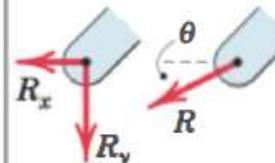
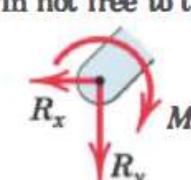
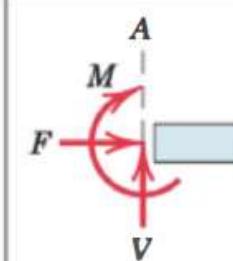
Collar or slider free to move along smooth guides; can support force normal to guide only.

# ENGINEERING MECHANICS

## Equilibrium



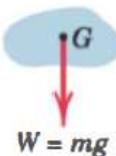
### MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)

Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection  	<p><b>Pin free to turn</b></p>  <p>A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components <math>R_x</math> and <math>R_y</math>, or a magnitude <math>R</math> and direction <math>\theta</math>. A pin not free to turn also supports a couple <math>M</math>.</p> <p><b>Pin not free to turn</b></p> 
7. Built-in or fixed support  	 <p>A built-in or fixed support is capable of supporting an axial force <math>F</math>, a transverse force <math>V</math> (shear force), and a couple <math>M</math> (bending moment) to prevent rotation.</p>

# ENGINEERING MECHANICS

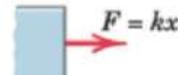
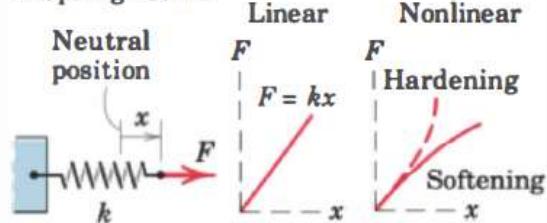
## Equilibrium

### 8. Gravitational attraction



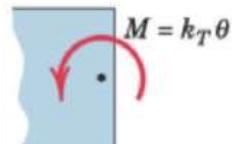
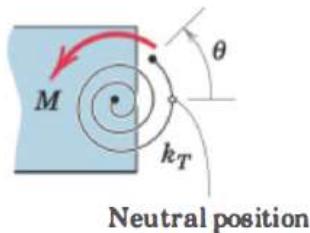
The resultant of gravitational attraction on all elements of a body of mass  $m$  is the weight  $W = mg$  and acts toward the center of the earth through the center of gravity  $G$ .

### 9. Spring action



Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness  $k$  is the force required to deform the spring a unit distance.

### 10. Torsional spring action

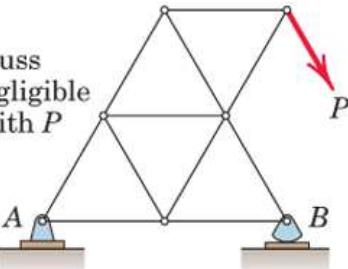
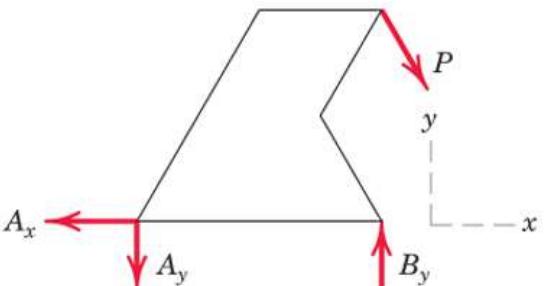
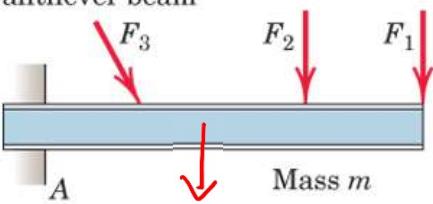
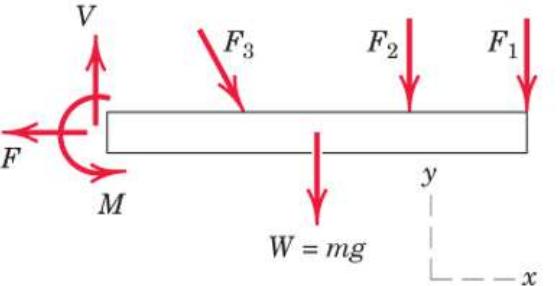


For a linear torsional spring, the applied moment  $M$  is proportional to the angular deflection  $\theta$  from the neutral position. The stiffness  $k_T$  is the moment required to deform the spring one radian.

# ENGINEERING MECHANICS

## Equilibrium

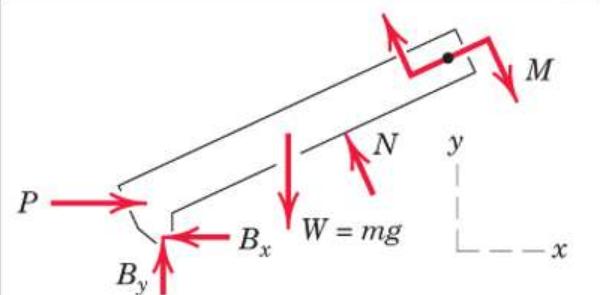
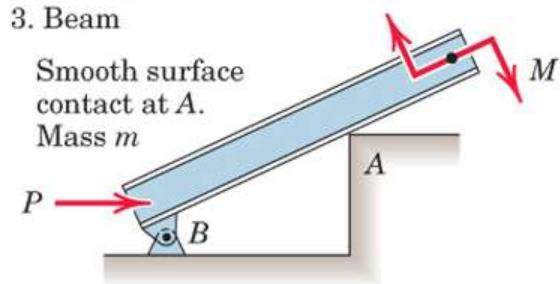
SAMPLE FREE-BODY DIAGRAMS

Mechanical System	Free-Body Diagram of Isolated Body
<p>1. Plane truss</p> <p>Weight of truss assumed negligible compared with <math>P</math></p> 	
<p>2. Cantilever beam</p> 	

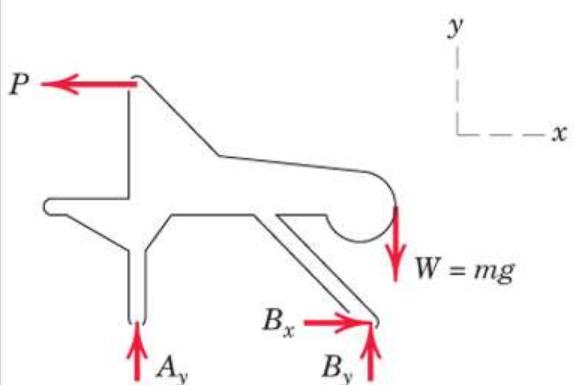
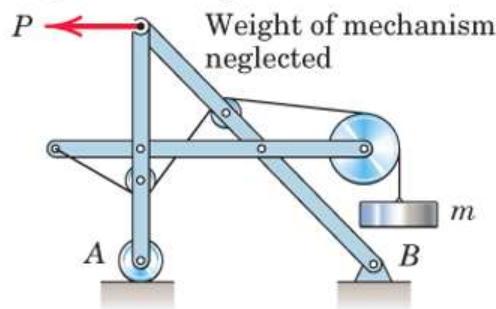
# ENGINEERING MECHANICS

## Equilibrium

3. Beam

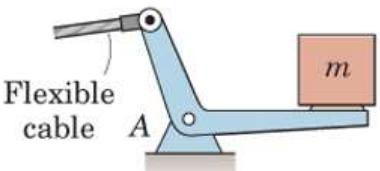
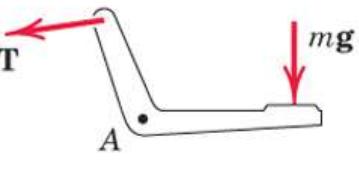
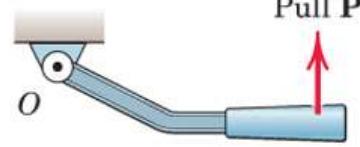
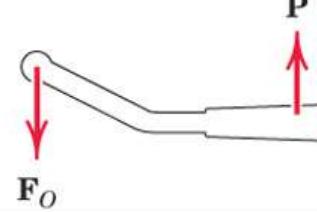


4. Rigid system of interconnected bodies analyzed as a single unit



# ENGINEERING MECHANICS

## Equilibrium

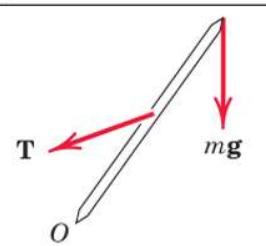
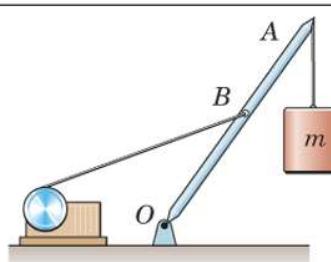
	Body	Incomplete FBD
1. Bell crank supporting mass $m$ with pin support at $A$ .		
2. Control lever applying torque to shaft at $O$ .		

# ENGINEERING MECHANICS

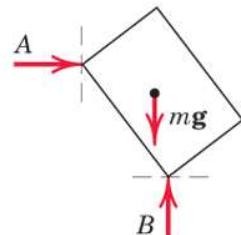
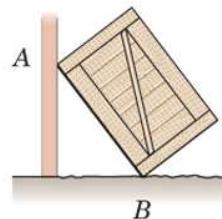
## Equilibrium



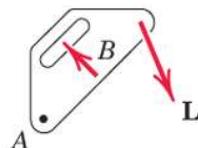
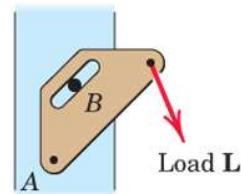
3. Boom  $OA$ , of negligible mass compared with mass  $m$ . Boom hinged at  $O$  and supported by hoisting cable at  $B$ .



4. Uniform crate of mass  $m$  leaning against smooth vertical wall and supported on a rough horizontal surface.



5. Loaded bracket supported by pin connection at  $A$  and fixed pin in smooth slot at  $B$ .

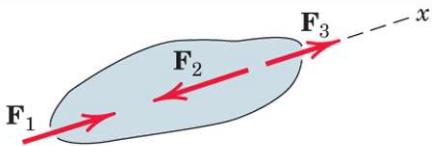
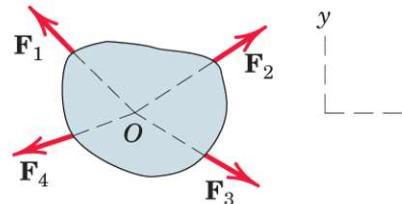
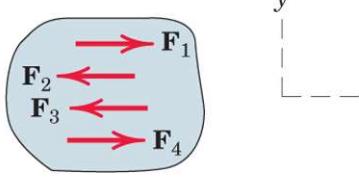
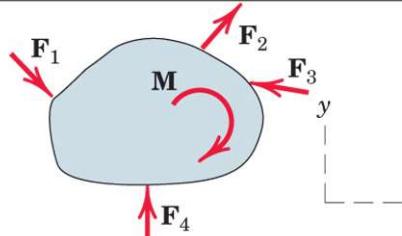


# ENGINEERING MECHANICS

## Equilibrium



CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

# ENGINEERING MECHANICS

## Equilibrium

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### CONSTRUCTION OF FREE-BODY DIAGRAMS

The full procedure for drawing a free-body diagram which isolates a body or system consists of the following steps.

#### Step 1.

- Decide which system to isolate. The system chosen should usually involve one or more of the desired unknown quantities.

#### Step 2.

- Next isolate the chosen system by drawing a diagram which represents its complete external boundary.
- This boundary defines the isolation of the system from all other attracting or contacting bodies, which are considered removed.
- This step is often the most crucial of all. Make certain that you have completely isolated the system before proceeding with the next step.



# ENGINEERING MECHANICS

## Equilibrium

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### CONSTRUCTION OF F R E E -BODY DIAGRAMS

#### Step 3.

- Each unknown force should be represented by a vector arrow with the unknown magnitude or direction indicated by symbol. If the sense of the vector is also unknown, you must arbitrarily assign a sense.
- The subsequent calculations with the equilibrium equations will yield a positive quantity if the correct sense was assumed and a negative quantity if the incorrect sense was assumed.
- It is necessary to be consistent with the assigned characteristics of unknown forces throughout all of the calculations.
- If you are consistent, the solution of the equilibrium equations will reveal the correct senses.



# ENGINEERING MECHANICS

## Equilibrium

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### CONSTRUCTION OF F R E E -BODY DIAGRAMS

#### Step 3.

- Identify all forces which act on the isolated system as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system.
- Make a systematic traverse of the entire boundary to identify all contact forces. Include body forces such as weights, where appreciable.
- Represent all known forces by vector arrows, each with its proper magnitude, direction, and sense indicated.



# ENGINEERING MECHANICS

## Equilibrium

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### CONSTRUCTION OF F R E E -BODY DIAGRAMS

#### Step 4.

- Show the choice of coordinate axes directly on the diagram. Pertinent dimensions may also be represented for convenience.
- Note, however, that the free-body diagram serves the purpose of focusing attention on the action of the external forces, and therefore the diagram should not be cluttered with excessive extraneous information.
- Clearly distinguish force arrows from arrows representing quantities other than forces. For this purpose a colored pencil may be used.

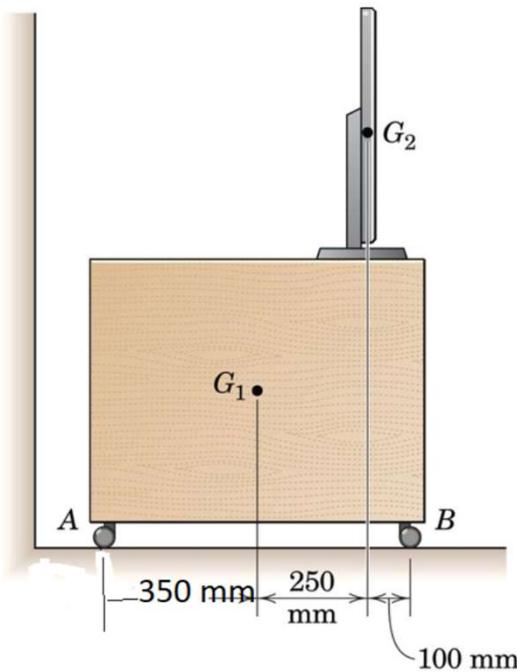


# ENGINEERING MECHANICS

## Equilibrium - Numerical

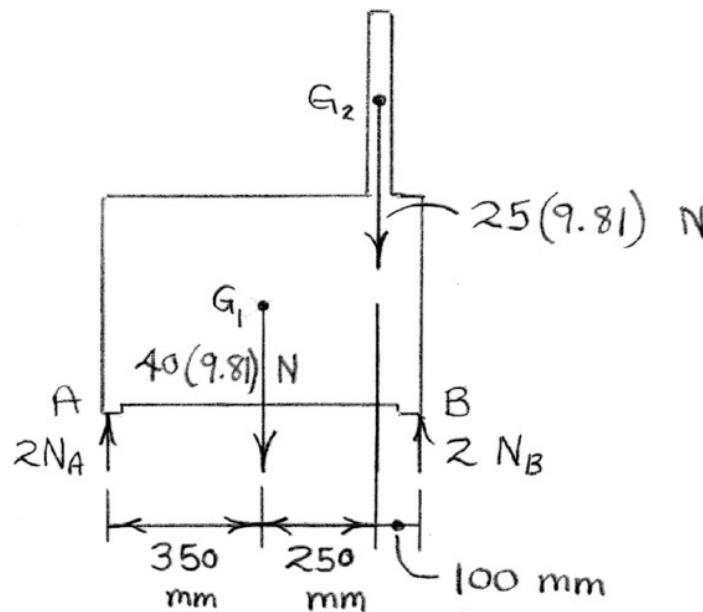


3/1) In the side view of a 25-kg flat-screen television resting on a 40-kg cabinet, the respective centers of mass are labeled  $G_2$  and  $G_1$ . Assume symmetry into the paper and calculate the normal reaction force at each of the four casters.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$+\uparrow \sum F = 0 : 2N_A + 2N_B - (25 + 40)(9.81) = 0$$

$$F_f \sum M_A = 0 : -40(9.81)(350) - 25(9.81)(600) \\ + 2N_B (700) = 0$$

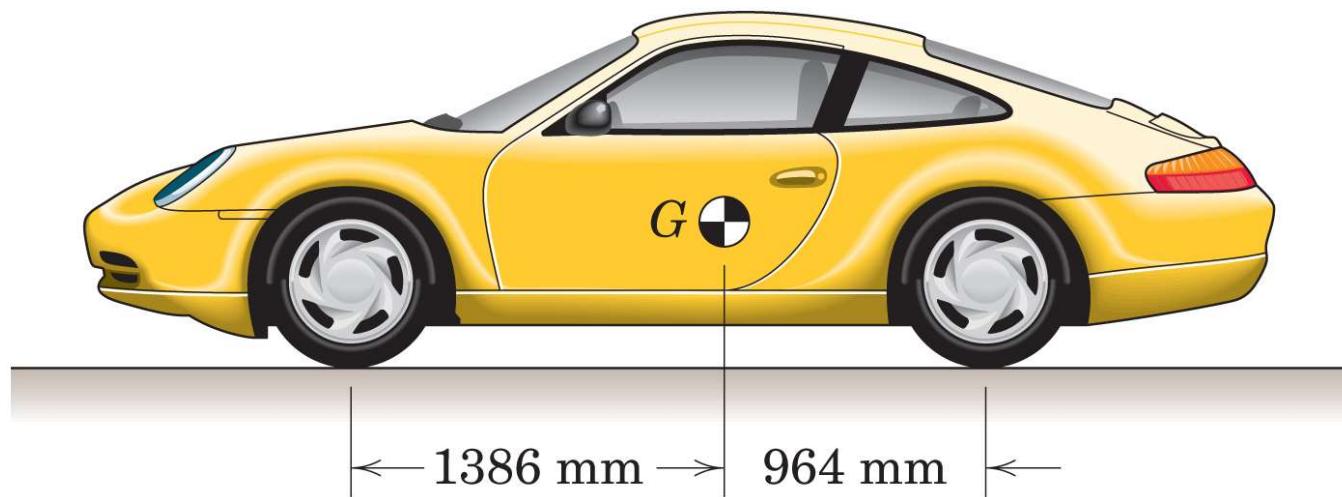
$$\begin{cases} N_A = 115.6 \text{ N} \\ N_B = 203 \text{ N} \end{cases}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

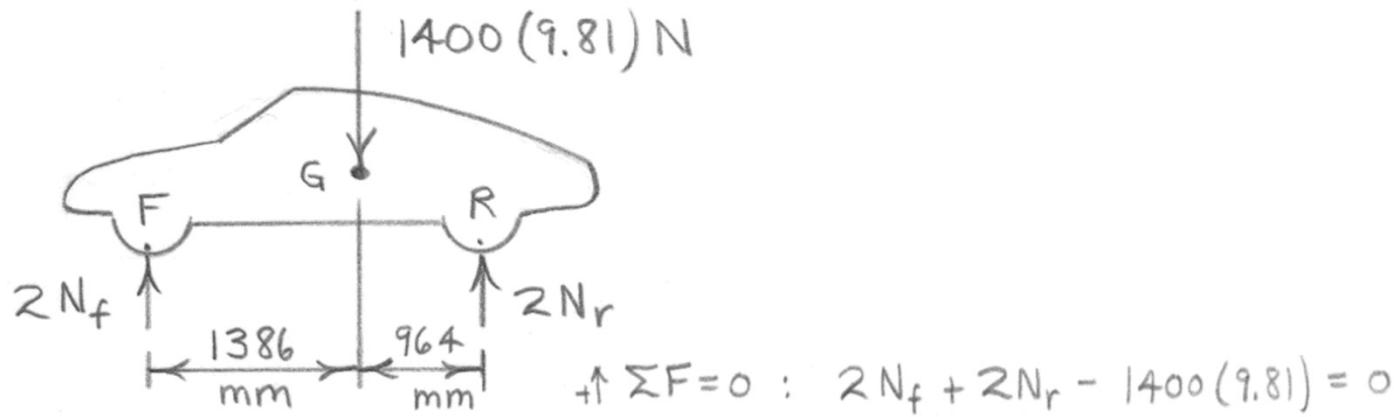


3/2) The mass center G of the 1400-kg rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\curvearrowleft \sum M_F = 0 : -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$$

Solution :

$$\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$$

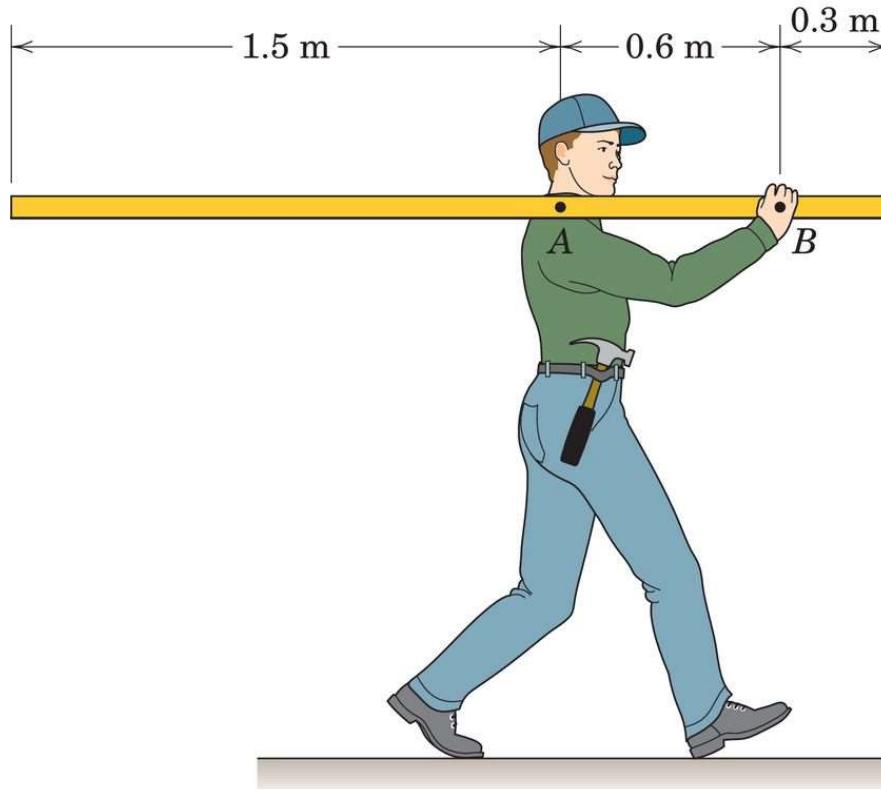
Assumes G midway between left and right wheels.

# ENGINEERING MECHANICS

## Equilibrium - Numerical

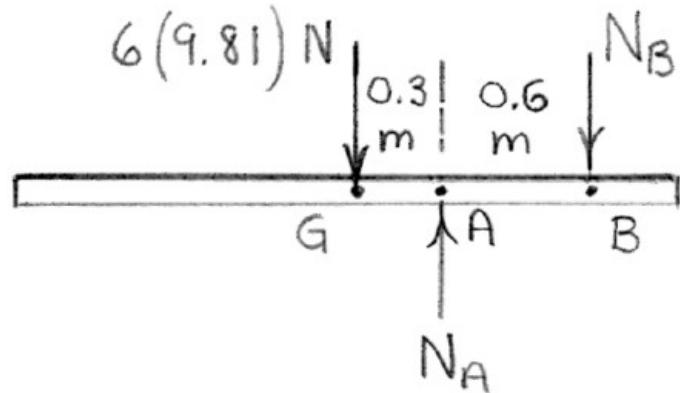


3/3) A carpenter carries a 6-kg uniform board as shown. What downward force does he feel on his shoulder at A?



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\text{↶ } \sum M_B = 0: 6(9.81)(0.9) - N_A (0.6) = 0$$

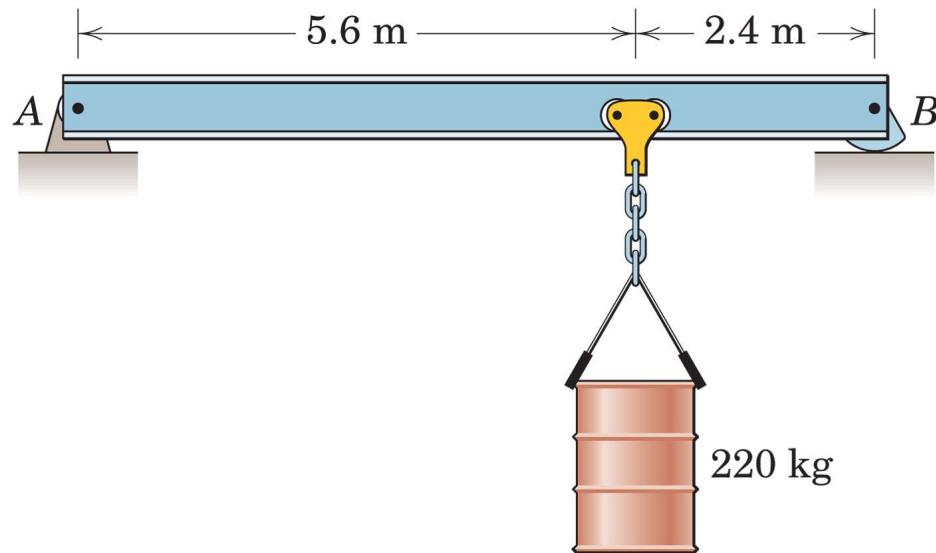
$$\underline{N_A = 88.3 \text{ N}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

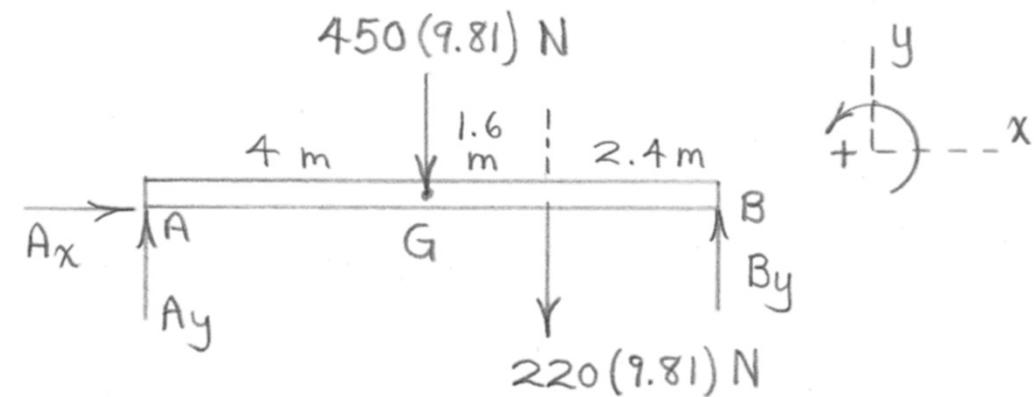


3/4) The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\text{From } \sum F_x = 0, \quad A_x = 0$$

$$\sum M_A = 0 : -450(9.81)4 - 220(9.81)(5.6)$$

$$+ B_y(8) = 0, \quad \underline{B_y = 3720 \text{ N}}$$

$$\sum F_y = 0 : A_y - 450(9.81) - 220(9.81) + 3720 = 0$$

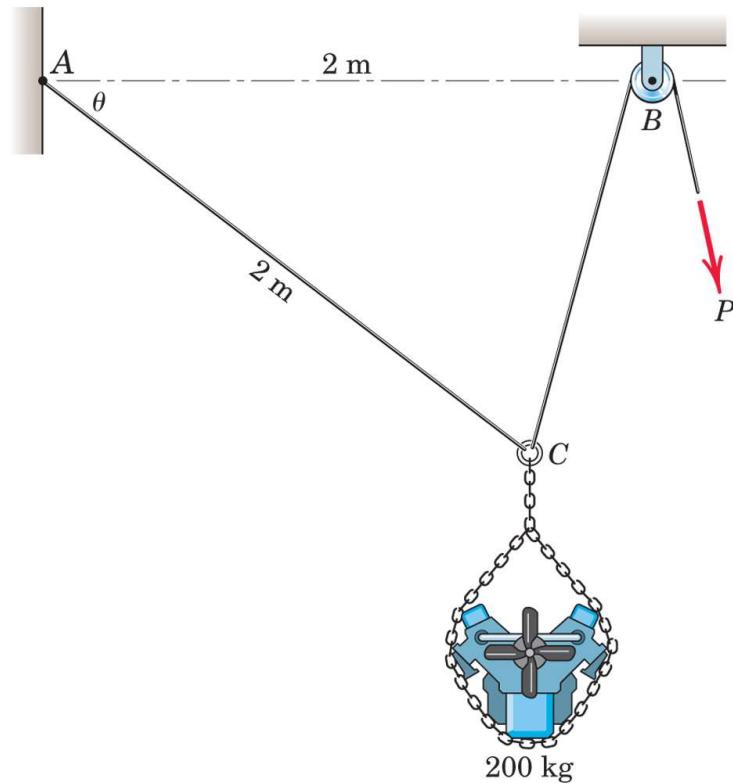
$$\underline{\underline{A_y = 2850 \text{ N}}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical



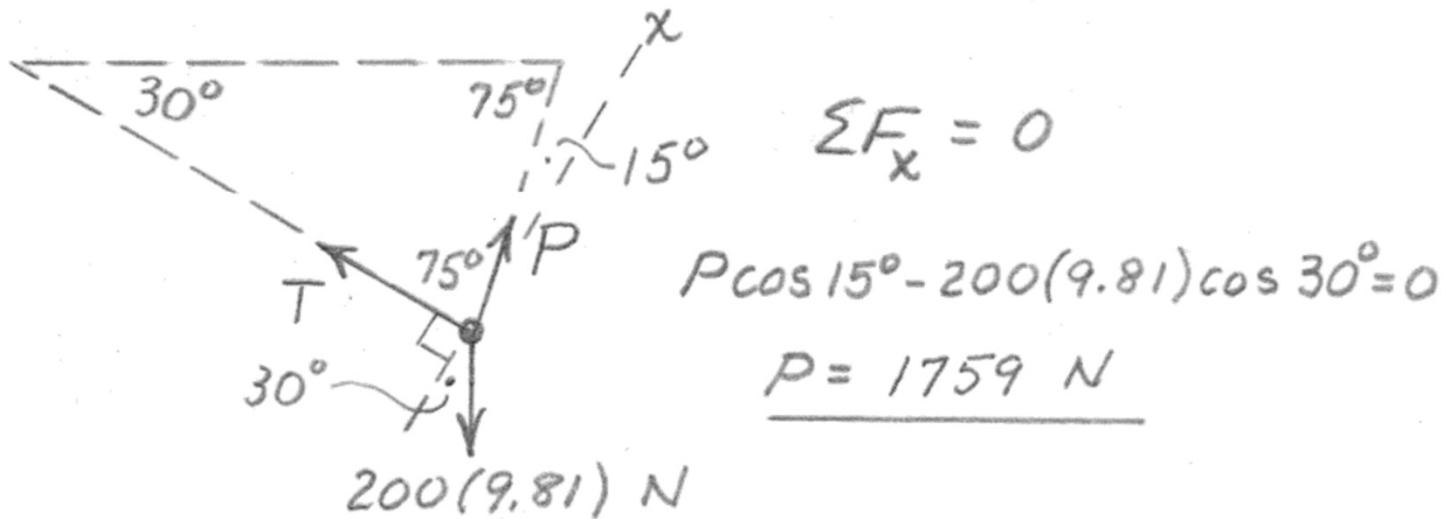
3/5) Determine the force  $P$  required to maintain the 200-kg engine in the position for which  $\theta = 30^\circ$ . The diameter of the pulley at B is negligible.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

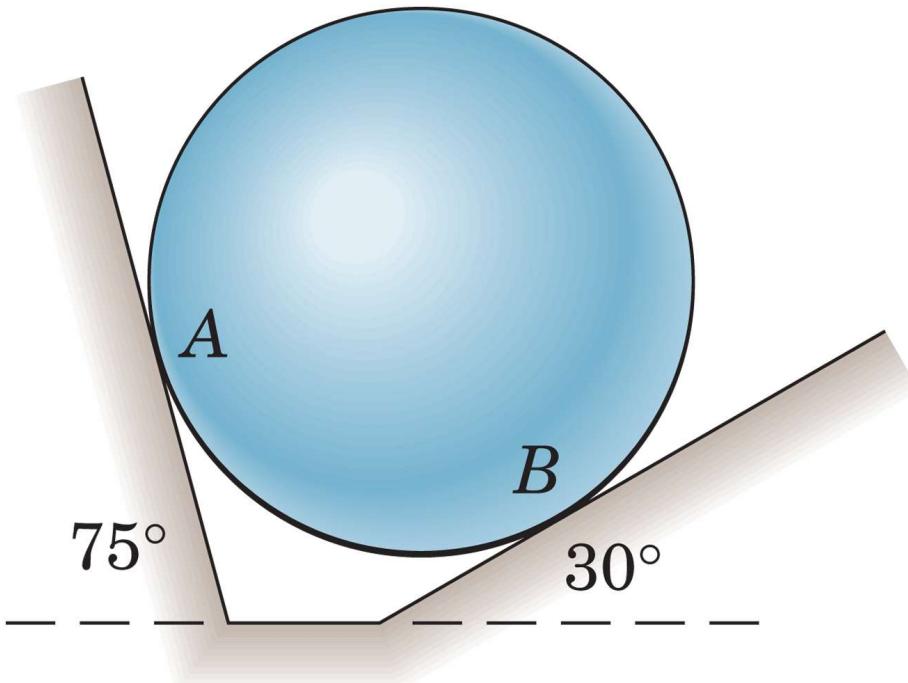
3/5



## ENGINEERING MECHANICS

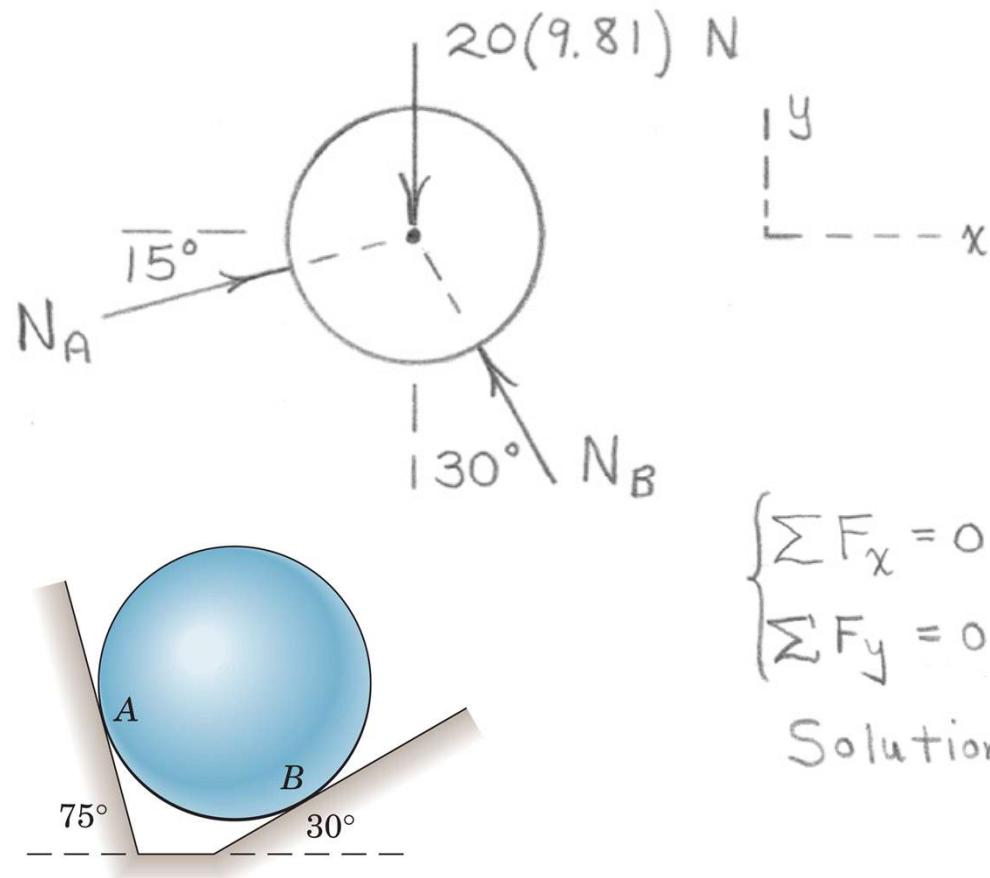
### Equilibrium - Numerical

3/6) The 20-kg homogeneous smooth sphere rests on the two inclines as shown.  
Determine the contact forces at A and B.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\left\{ \begin{array}{l} \sum F_x = 0 : N_A \cos 15^\circ - N_B \sin 30^\circ = 0 \\ \sum F_y = 0 : N_A \sin 15^\circ + N_B \cos 30^\circ - 20(9.81) = 0 \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} N_A \cos 15^\circ - N_B \sin 30^\circ = 0 \\ N_A \sin 15^\circ + N_B \cos 30^\circ - 20(9.81) = 0 \end{array} \right. \quad (2)$$

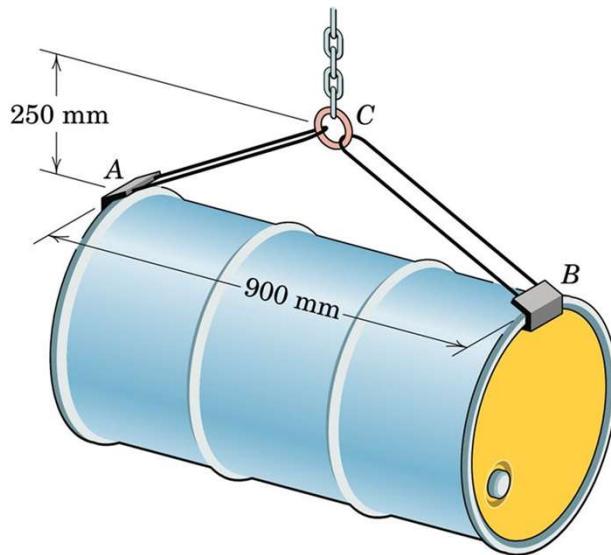
Solution :

$$\left\{ \begin{array}{l} N_A = 101.6 \text{ N} \\ N_B = 196.2 \text{ N} \end{array} \right.$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/7) The 275-kg drum is being hoisted by the lifting device which hooks over the end lips of the drum. Determine the tension  $T$  in each of the equal-length rods which form the two U-shaped members of the device.

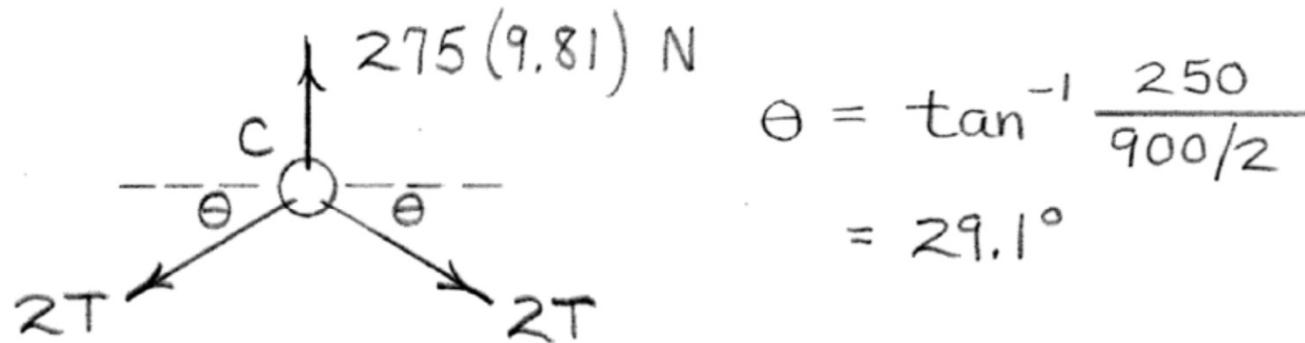


## ENGINEERING MECHANICS

### Equilibrium - Numerical



3/7 FBD of junction ring C:



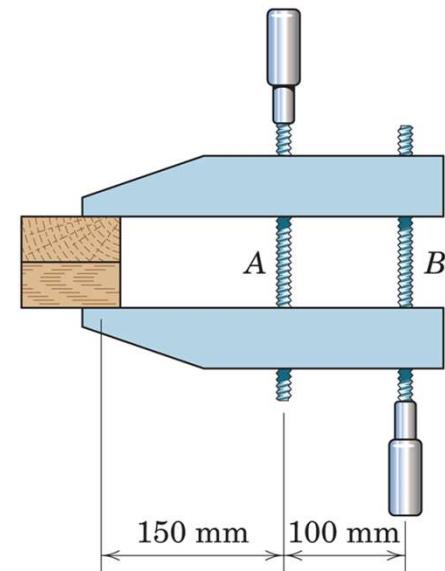
$$\uparrow + \sum F = 0: 275(9.81) - 4T \sin 29.1^\circ = 0$$

$$T = 1389 \text{ N}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

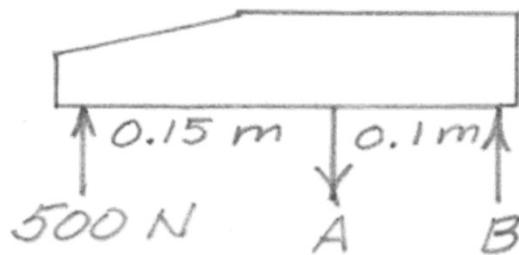
3/8) If the screw B of the wood clamp is tightened so that the two blocks are under a compression of 500 N, determine the force in screw A. (Note: The force supported by each screw may be taken in the direction of the screw. )



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/8



$$\sum M_B = 0; 500(0.25) - 0.1A = 0$$
$$\underline{A = 1250 \text{ N}}$$

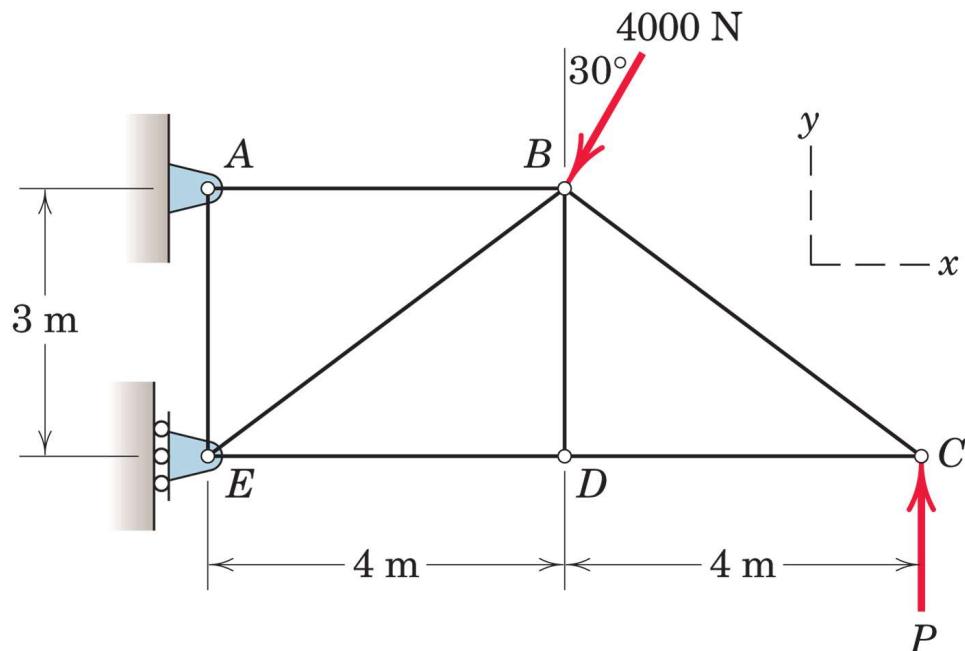


# ENGINEERING MECHANICS

## Equilibrium - Numerical

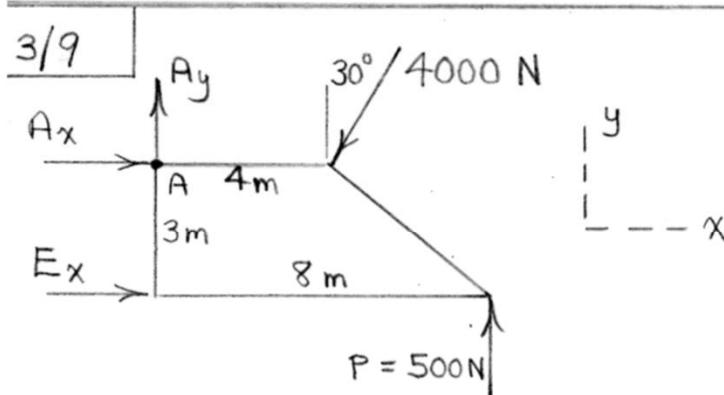


3/9) Determine the reactions at A and E if  $P = 500 \text{ N}$ . What is the maximum value which  $P$  may have for static equilibrium? Neglect the weight of the structure compared with the applied loads.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum F_x = 0: A_x + E_x - 4000 \sin 30^\circ = 0$$

$$\sum F_y = 0: A_y - 4000 \cos 30^\circ + 500 = 0$$

$$\sum M_A = 0: E_x(3) + 500(8) - 4000 \cos 30^\circ (4) = 0$$

$$\Rightarrow A_x = -1285 \text{ N}, \quad A_y = 2960 \text{ N}, \quad E_x = 3290 \text{ N}$$

For maximum  $P$ :  $E_x = 0$  and  $\sum M_A = 0$ :

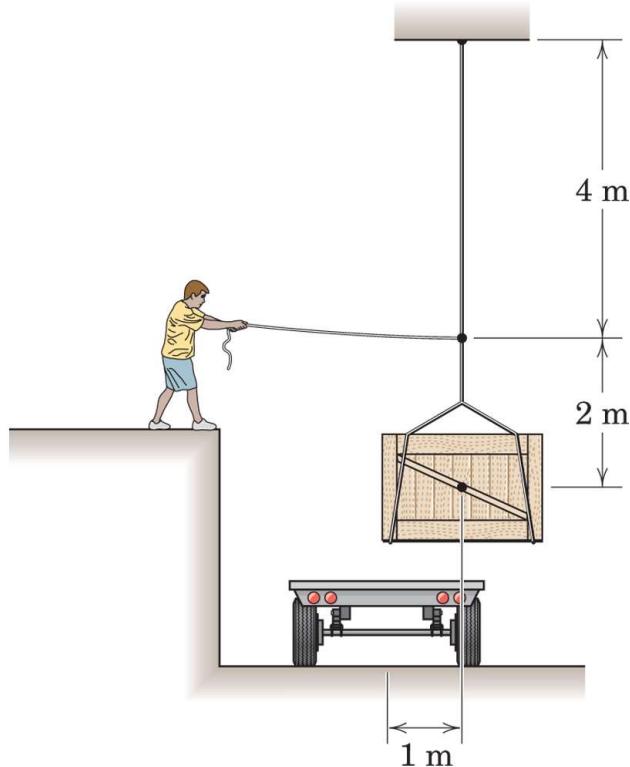
$$P(8) - 4000 \cos 30^\circ (4) = 0, \quad P = 1732 \text{ N}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical



3/10) What horizontal force  $P$  must worker exerts on the rope to position the 50 kg weight directly over the trailer

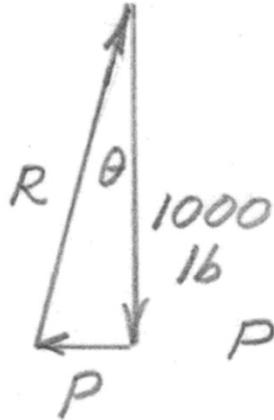
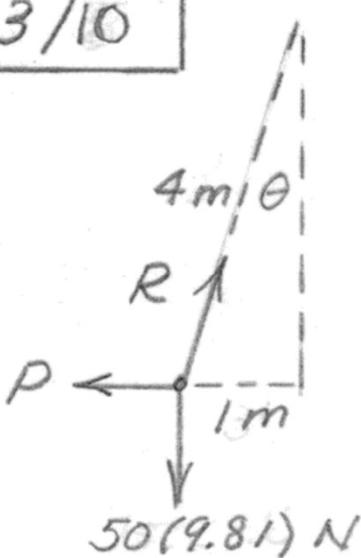


# ENGINEERING MECHANICS

## Equilibrium - Numerical



3/10



$$P = 50(9.81) \tan \theta$$

$$\sin \theta = 1/4$$

$$\tan \theta = 1/\sqrt{4^2 - 1^2} = 0.258$$

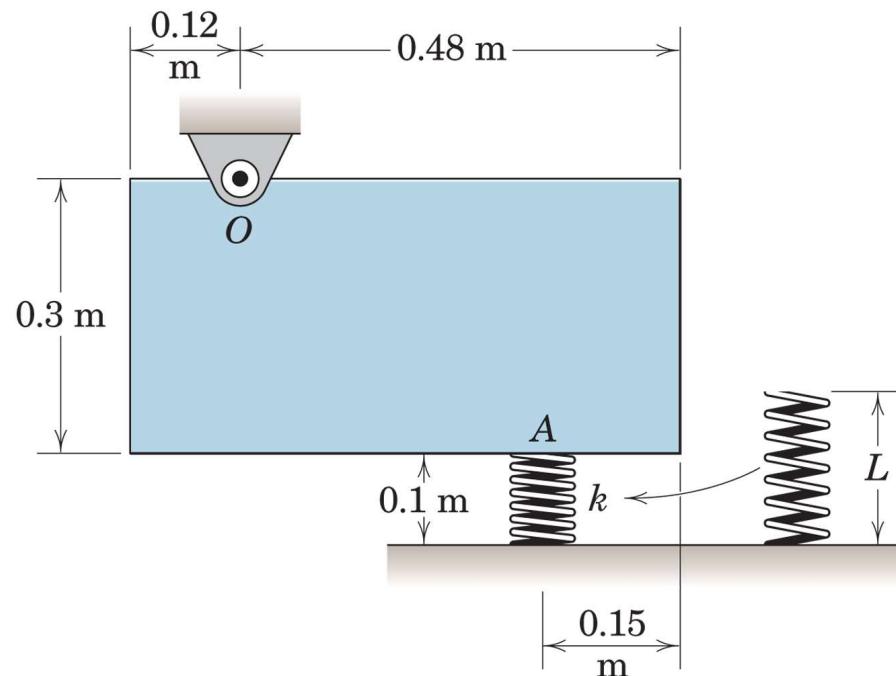
$$P = 50(9.81)(0.258) = \underline{126.6 \text{ N}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

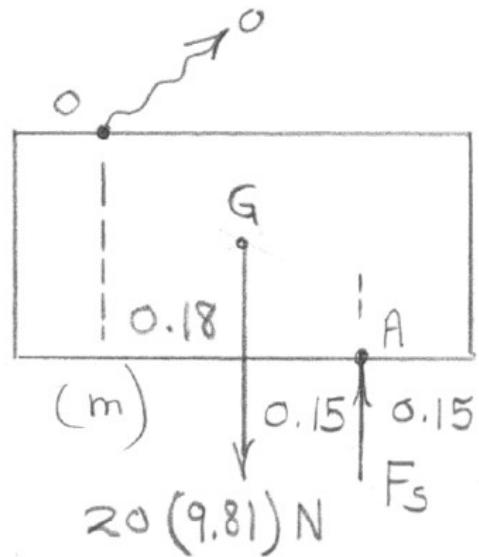


3/11) The 20-kg uniform rectangular plate is supported by an ideal pivot at O and a spring which must be compressed prior to being slipped into place at point A. If the modulus of the spring is  $k = 2 \text{ kN/m}$ , what must be its un-deformed length L?



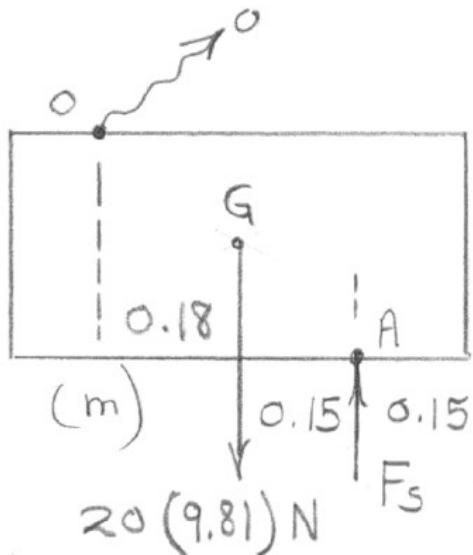
# ENGINEERING MECHANICS

## Equilibrium - Numerical



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\text{At } \sum M_O = 0 : -20(9.81)(0.18) + F_s(0.18 + 0.15) = 0$$

$$F_s = 107.0 \text{ N}$$

$$F_s = k\delta : 107.0 = 2000\delta, \quad \delta = 0.0535 \text{ m}$$

$$\text{or } \delta = 53.5 \text{ mm}$$

$$L = 0.1 + \delta = 0.1 + 0.0535 = 0.1535 \text{ m}$$

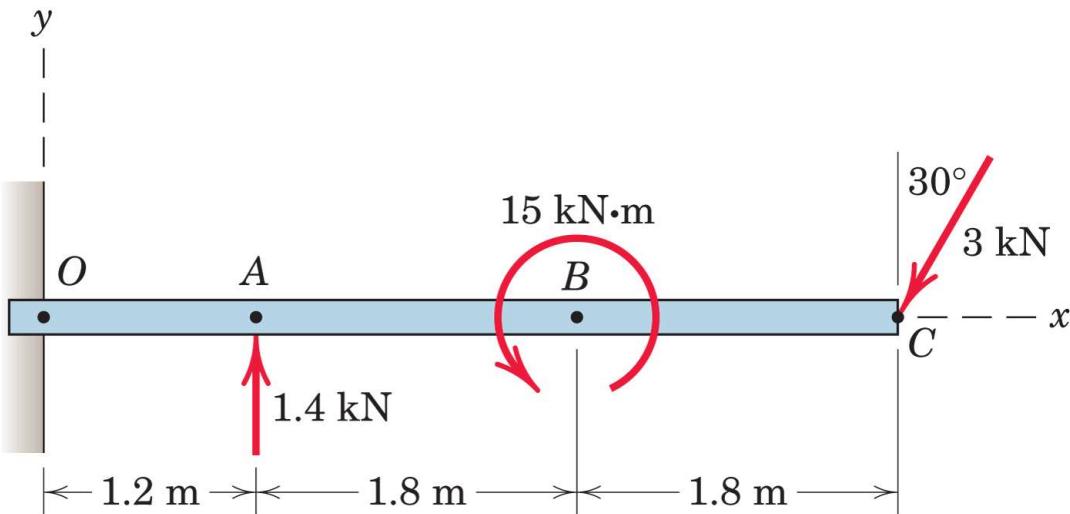
$$\text{or } \underline{\underline{153.5 \text{ mm}}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

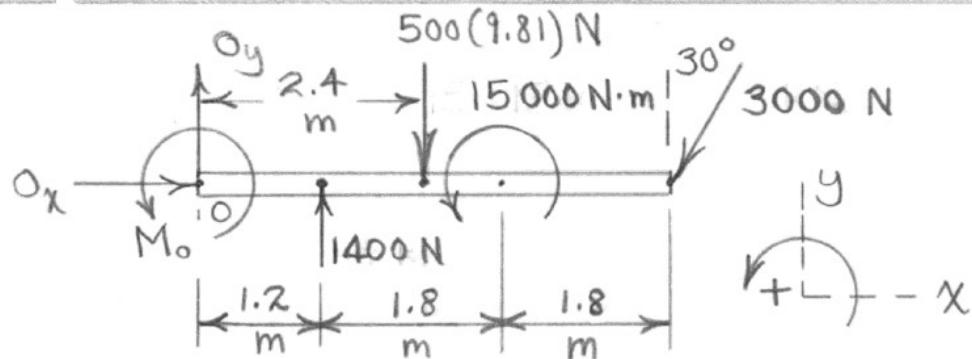


3/12) The 500-kg uniform beam is subjected to the three external loads shown . Compute the reactions at the support point O. The x-y plane is vertical.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum F_x = 0 : O_x - 3000 \sin 30^\circ = 0 , \quad O_x = 1500 \text{ N}$$

$$\sum F_y = 0 : O_y + 1400 - 500(9.81) - 3000 \cos 30^\circ = 0$$

$$O_y = 6100 \text{ N}$$

$$\begin{aligned} \sum M_o = 0 : & M_o + 1400(1.2) - 500(9.81)(2.4) \\ & + 15000 - (3000 \cos 30^\circ)(4.8) = 0 \end{aligned}$$

$$M_o = 7560 \text{ N}\cdot\text{m}$$

# ENGINEERING MECHANICS

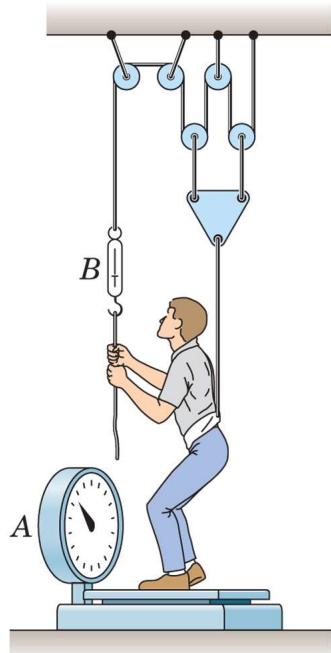
## Equilibrium - Numerical



PES

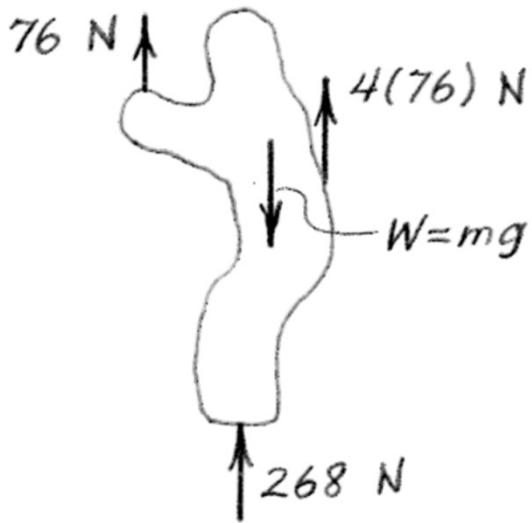
UNIVERSITY

3/13) A former student of mechanics wishes to weigh himself but has access only to a scale A with capacity limited to 400 N and a small 80-N spring dynamometer B. With the rig shown he discovers that when he exerts a pull on the rope so that B registers 76 N, the scale A reads 268 N. What are his correct weight W and mass m?



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum F = 0: 76 + 4(76) + 268 - W = 0$$

$$\underline{W = 648 \text{ N}}$$

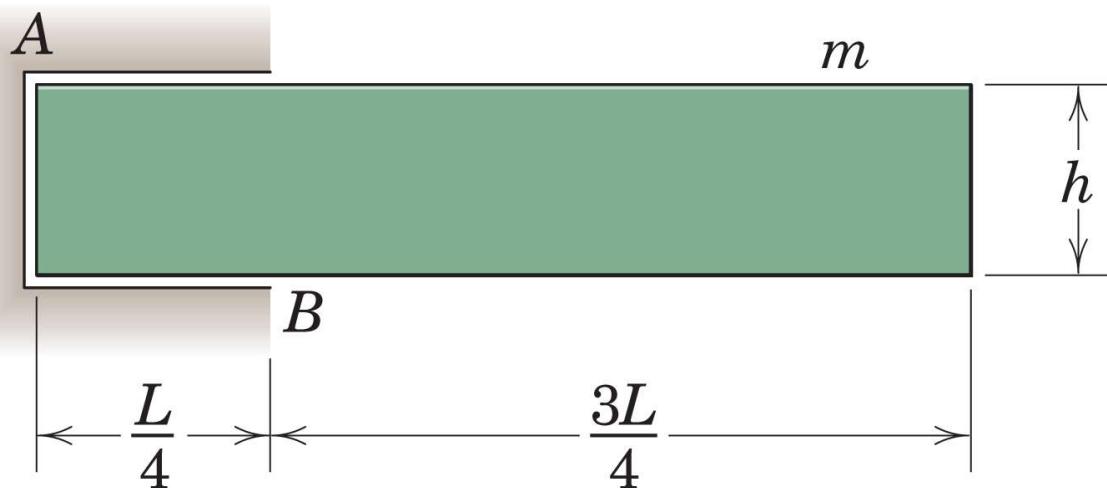
$$m = \frac{W}{g} = \frac{648}{9.81} = \underline{66.1 \text{ kg}}$$

## ENGINEERING MECHANICS

### Equilibrium - Numerical



3/14) The uniform rectangular body of mass  $m$  is placed into a fixed opening with slight clearances as shown. Determine the forces at the contact points A and B. Do your results depend on the height  $h$ ?

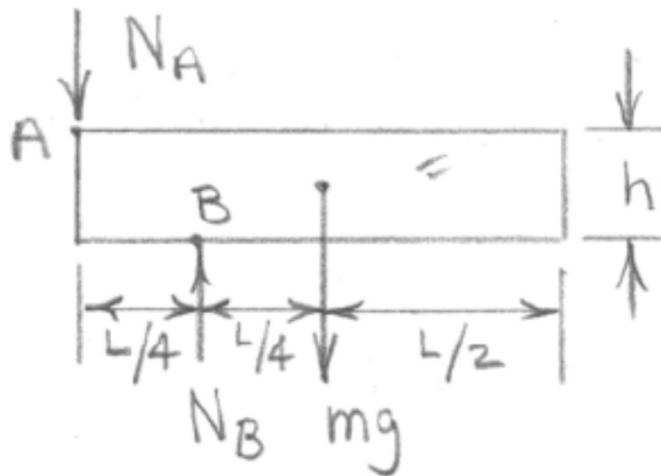


# ENGINEERING MECHANICS

## Equilibrium - Numerical



3/14)



$$\begin{aligned}\uparrow \sum F &= 0 : N_B - N_A - mg = 0 \\ \leftarrow \sum M_A &= 0 : N_B \left(\frac{L}{4}\right) - mg \left(\frac{L}{2}\right) = 0\end{aligned}$$

Solution :  $\begin{cases} N_A = mg & (\text{down}) \\ N_B = 2mg & (\text{up}) \end{cases}$

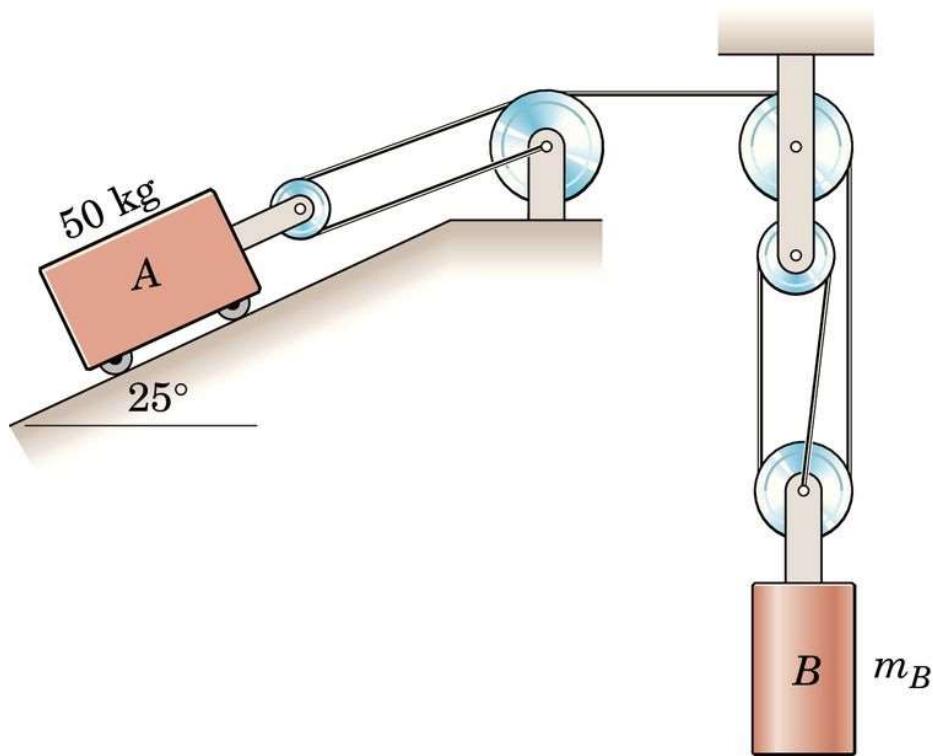
The height  $h$  has no bearing on the above results, assuming no friction at A and B.

# ENGINEERING MECHANICS

## Equilibrium - Numerical

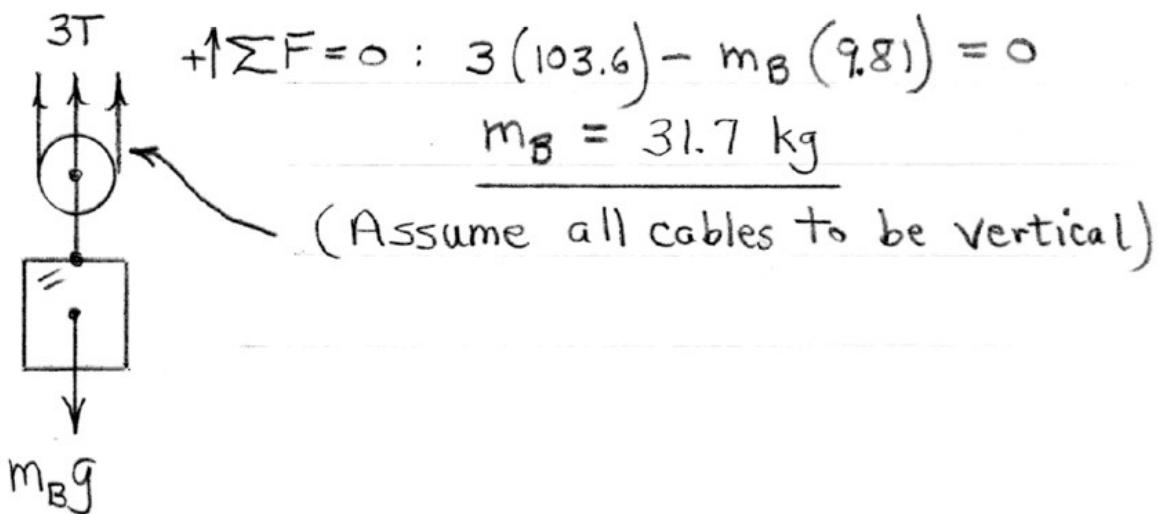
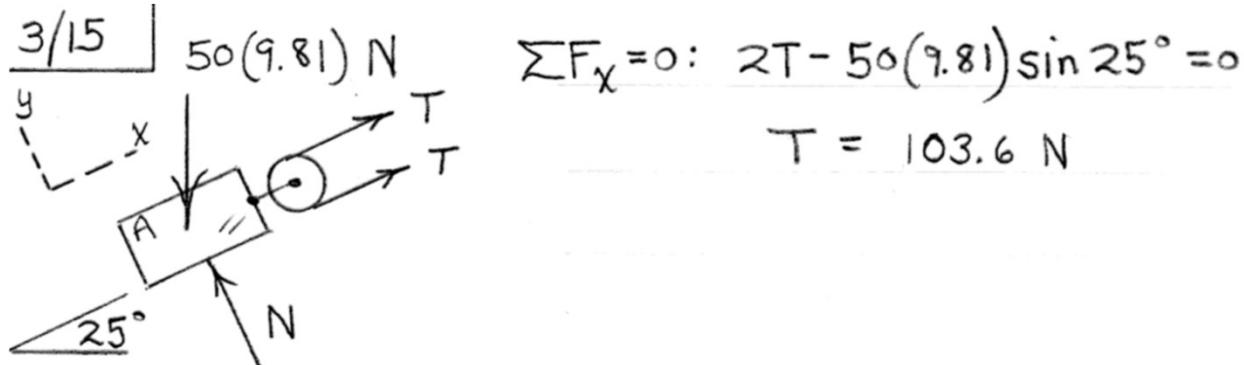


3/15) What mass  $m_B$  will cause the system to be in equilibrium? Neglect all friction, and state any other assumptions.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

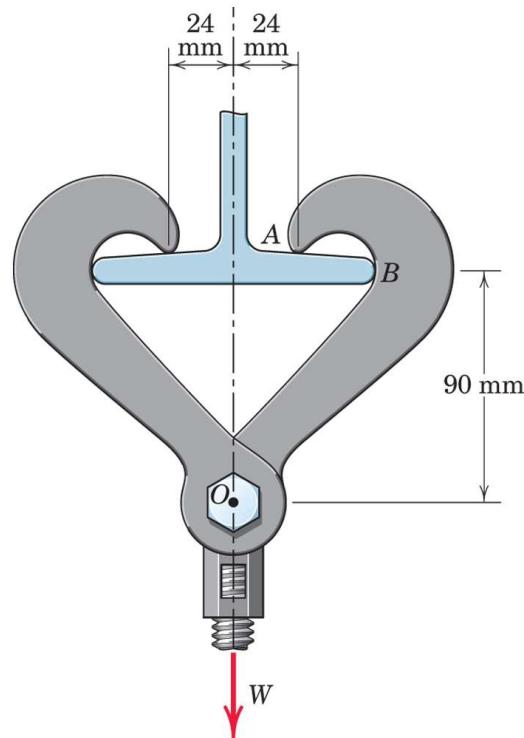


# ENGINEERING MECHANICS

## Equilibrium - Numerical

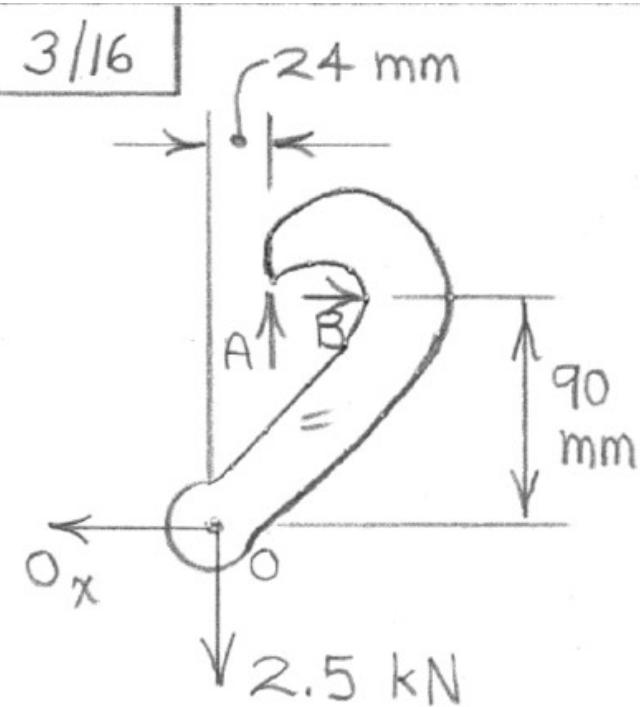


3/16) The pair of hooks is designed for the hanging of loads from horizontal I-beams. If the load  $W = 5 \text{ kN}$ , estimate the contact forces at A and B. Neglect all friction.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



From vertical equilibrium,

$$A = 2.5 \text{ kN}$$

$$\sum M_o = 0 :$$

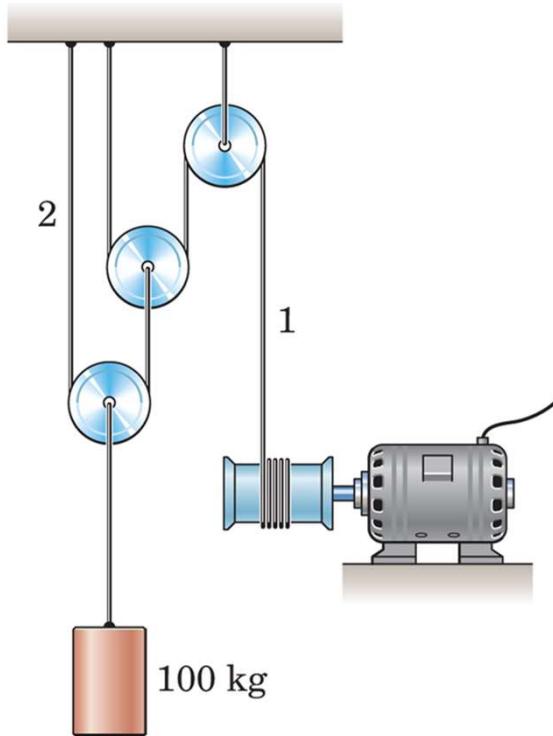
$$2.5(24) - B(90) = 0$$

$$B = 0.667 \text{ kN}$$

## ENGINEERING MECHANICS

### Equilibrium - Numerical

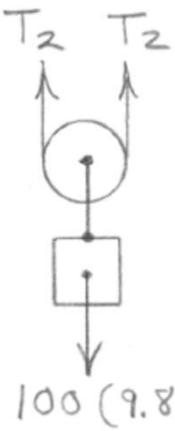
3/17) The winch takes in cable at the constant rate of 200 mm/s. If the cylinder mass is 100 kg, determine the tension in cable 1. Neglect all friction.



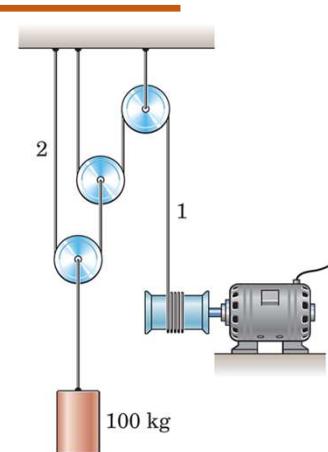
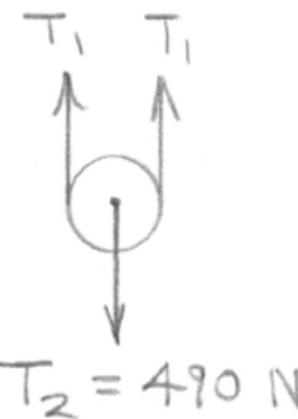
# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/17



From  $\sum F = 0$ ,  $T_2 = \frac{100(9.81)}{2}$   
 $T_2 = 490 \text{ N}$



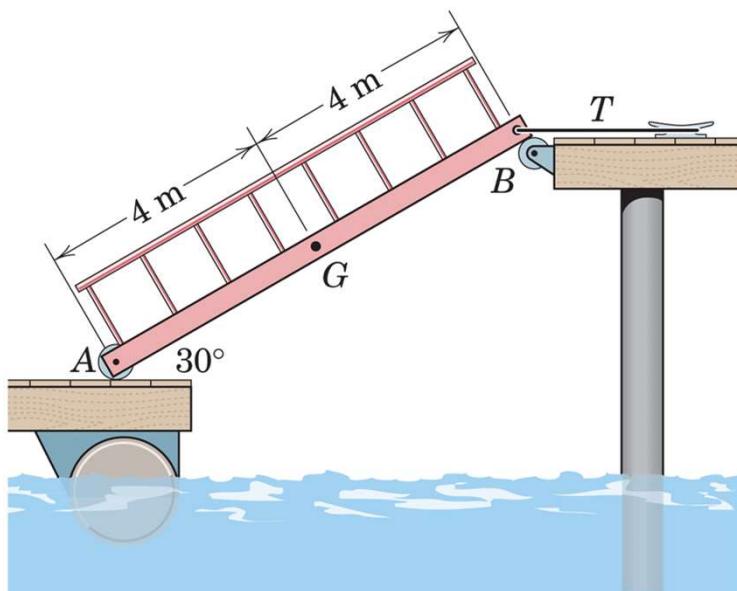
Similarly,  $T_1 = \frac{1}{2}T_2$   
 $= \frac{1}{2}(490) = \underline{\underline{245 \text{ N}}}$

# ENGINEERING MECHANICS

## Equilibrium - Numerical



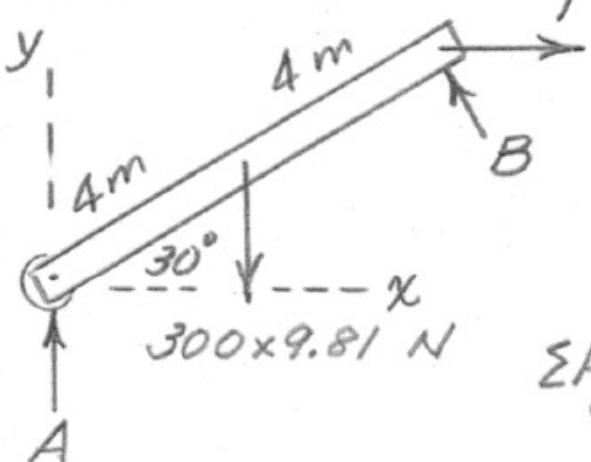
3/18) To accommodate the rise and fall of the tide, a walkway from a pier to a float is supported by two rollers as shown. If the mass center of the 300-kg walkway is at G, calculate the tension T in the horizontal cable which is attached to the cleat and find the force under the roller at A.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/18



$$\sum M_B = 0;$$

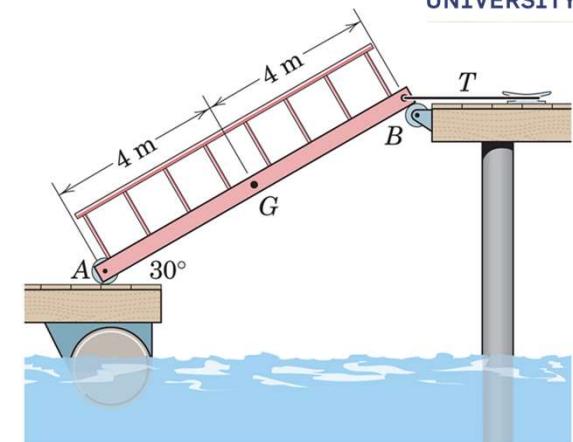
$$300(9.81) 4 \cos 30^\circ - 8A \cos 30^\circ = 0,$$

$$\underline{A = 1472 \text{ N}}$$

$$\sum F_y = 0; B \cos 30^\circ + 1472 - 300(9.81) = 0$$

$$B = 1699 \text{ N}$$

$$\sum F_x = 0; T - 1699 \sin 30^\circ = 0, \quad \underline{T = 850 \text{ N}}$$



# ENGINEERING MECHANICS

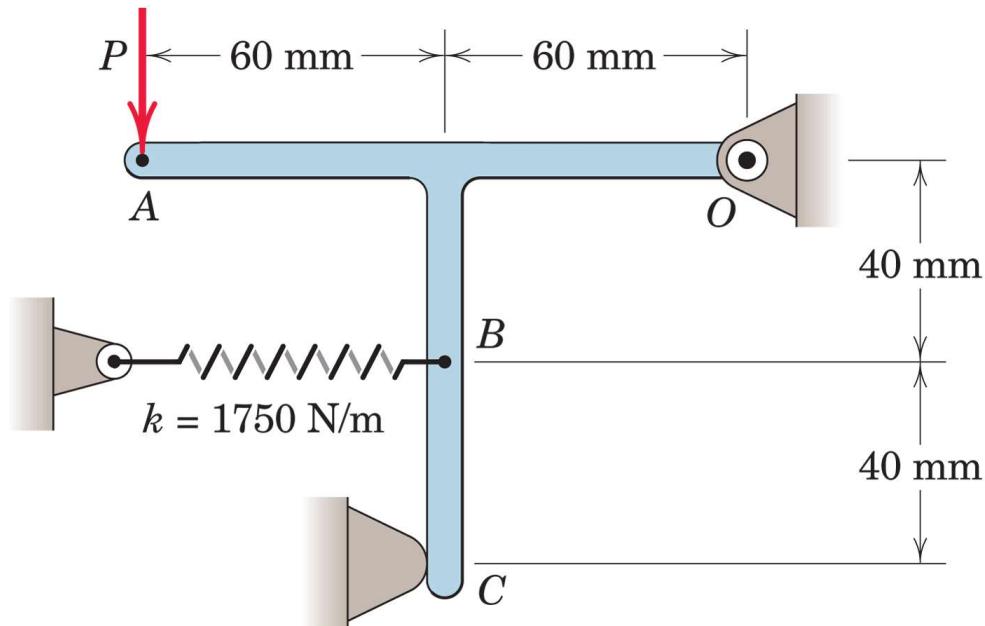
## Equilibrium - Numerical



PES

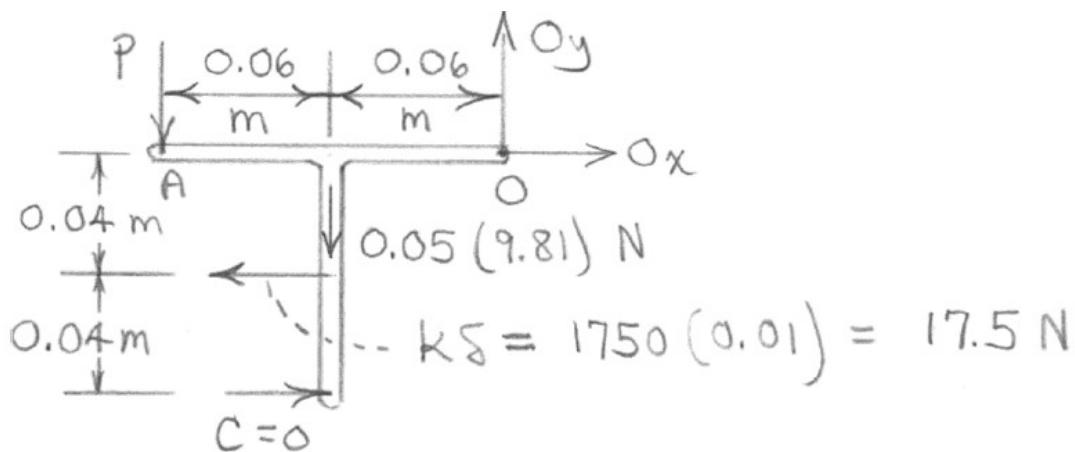
UNIVERSITY

3/19) When the 0.05-kg body is in the position shown, the linear spring is stretched 10 mm. Determine the force  $P$  required to break contact at C. Complete solutions for (a) including the effect of the weight and (b) neglecting the weight.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

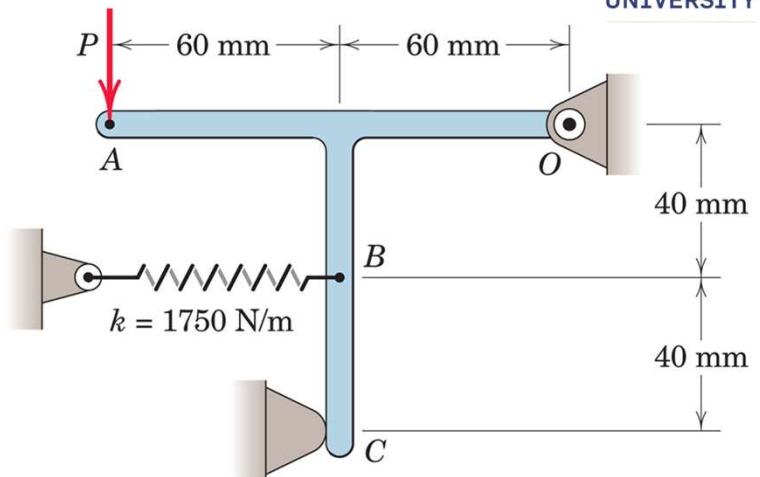


$$(a) \nabla \sum M_O = 0: P(0.12) + 0.05(9.81)(0.06) - 17.5(0.04) = 0$$

$$P = 5.59 \text{ N}$$

$$(b) \nabla \sum M_O = 0: P(0.12) - 17.5(0.04) = 0$$

$$\underline{P = 5.83 \text{ N}}$$

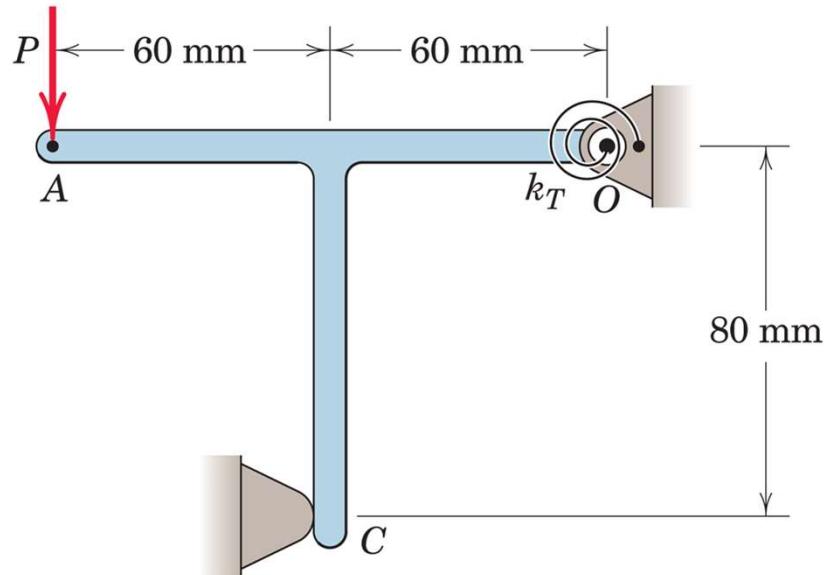


## ENGINEERING MECHANICS

### Equilibrium - Numerical

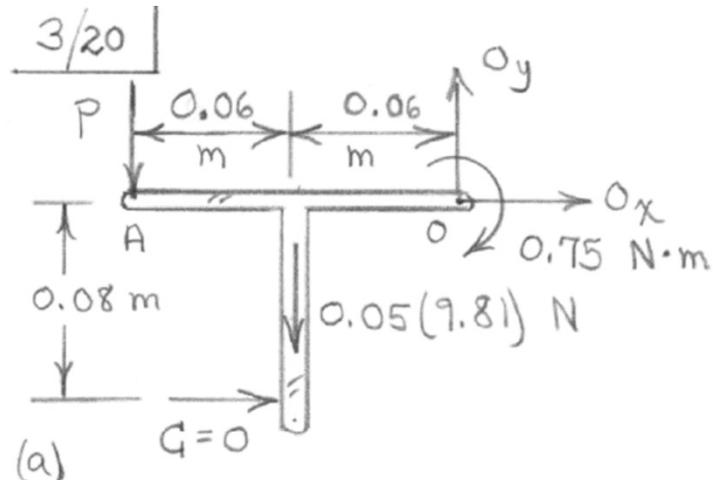


3/20) When the 0.05-kg body is in the position shown, the torsional spring at O pretensioned so as to exert a 0.75-N.m clockwise moment on the body. Determine the force P required to break contact at c. Complete solutions for (a) including the effect of the weight and(b) neglecting the weight.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



(a)  $\sum M_o = 0 : P(0.12) + 0.05(9.81)(0.06) - 0.75 = 0$

$$P = 6.00 \text{ N}$$

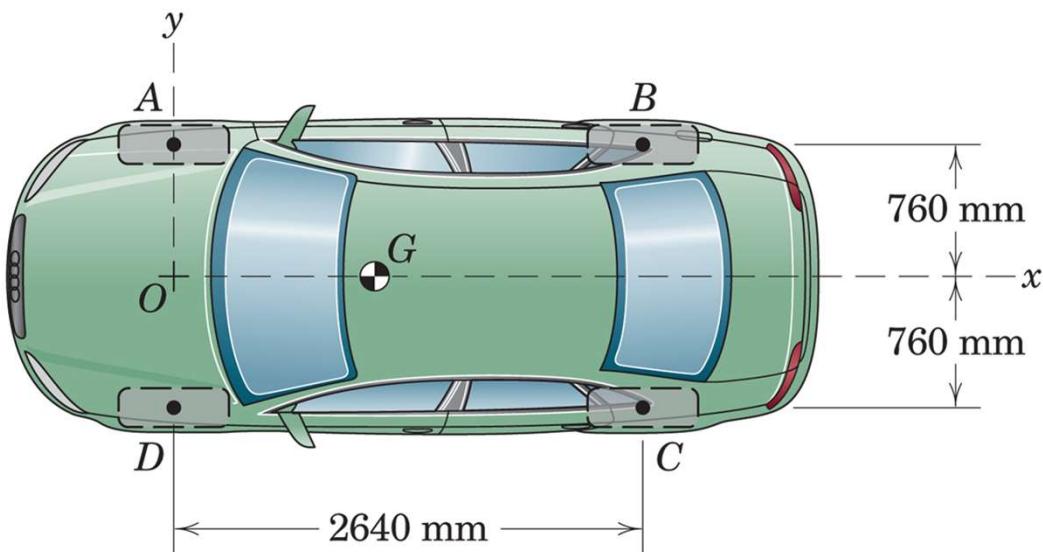
(b)

$$\sum M_o = 0 : P(0.12) - 0.75 = 0$$
$$P = 6.25 \text{ N}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/21) When on level ground, the car is placed on four individual scales-one under each tire. The scale readings are 4450 N at each front wheel and 2950 N at each rear wheel. Determine the x-coordinate of the mass center G and the mass of the car.

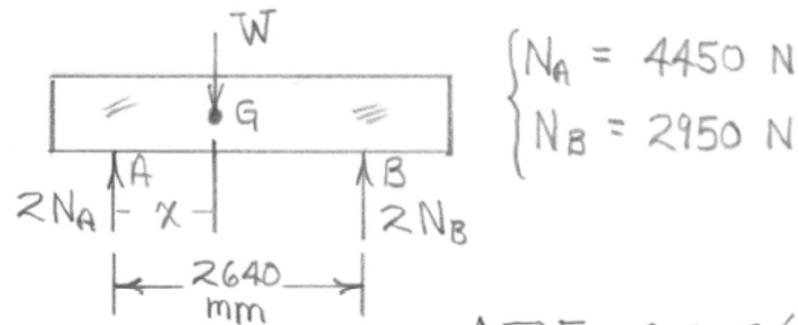


# ENGINEERING MECHANICS

## Equilibrium - Numerical



3/21 Car modeled as a slab and viewed from the driver's (left) side :



$$\begin{cases} N_A = 4450 \text{ N} \\ N_B = 2950 \text{ N} \end{cases}$$

$$\uparrow \sum F = 0 : 2(4450) + 2(2950) - W = 0$$

$$W = 14800 \text{ N}$$

$$m = \frac{W}{g} = \frac{14800}{9.81} = \underline{\underline{1509 \text{ kg}}}$$

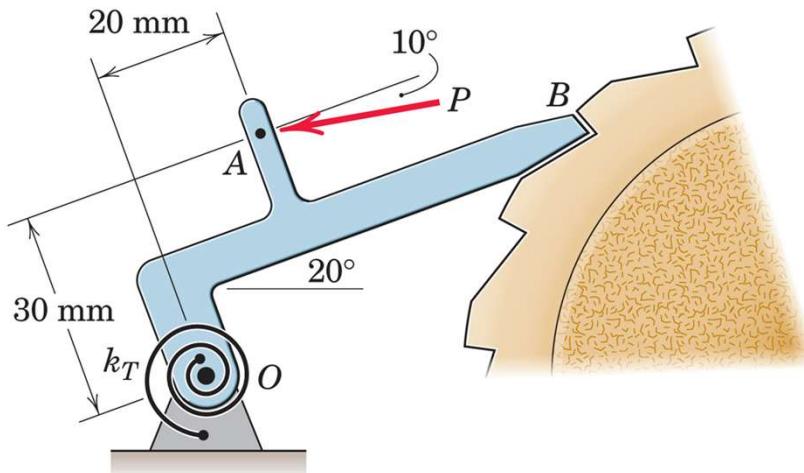
$$\nabla \sum M_A = 0 : - 14800x + 2(2950)(2640) = 0$$

$$\underline{\underline{x = 1052 \text{ mm}}}$$

# ENGINEERING MECHANICS

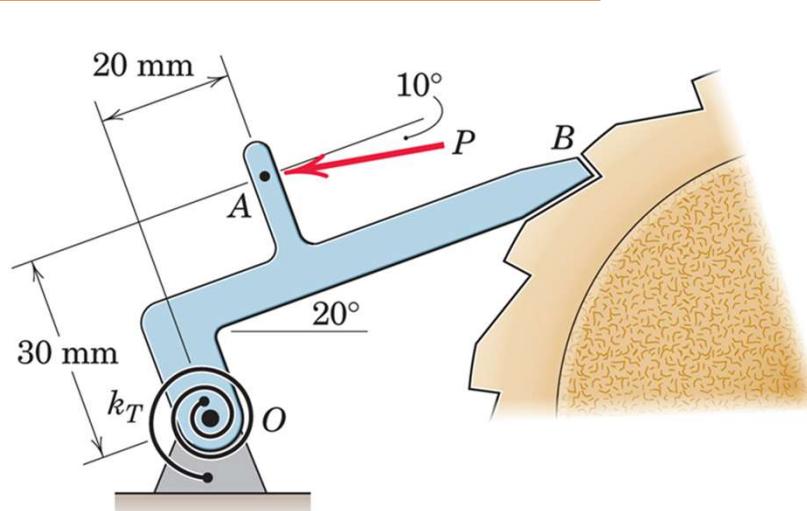
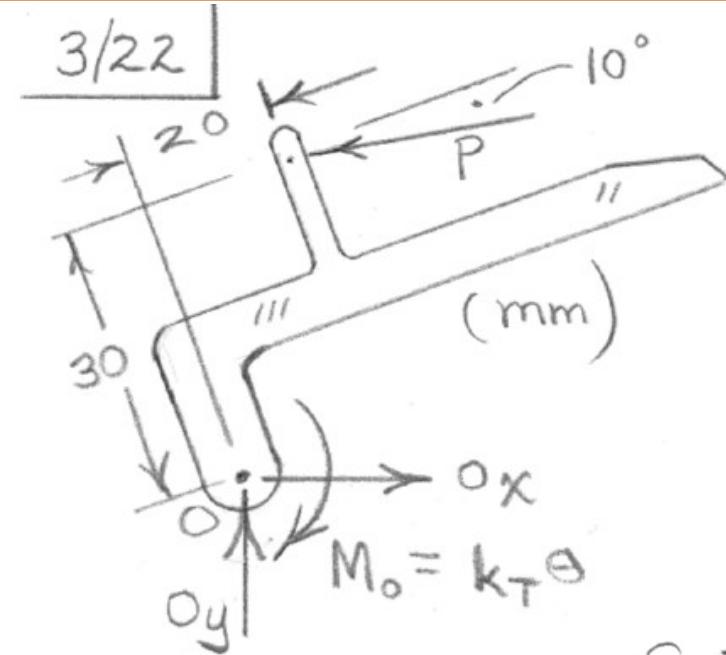
## Equilibrium - Numerical

3/22) Determine the magnitude P of the force required to rotate the release pawl OB counterclockwise from its locked position. The torsional spring constant is  $k_T = 3.4 \text{ N} \cdot \text{m/rad}$  and the pawl end of the spring has been deflected  $25^\circ$  counterclockwise from the neutral position in the configuration shown. Neglect any forces at the contact point B.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\text{At } \sum M_o = 0 : P \cos 10^\circ (30) + P \sin 10^\circ (20) - 3400 \left( 25 \frac{\pi}{180} \right) = 0$$

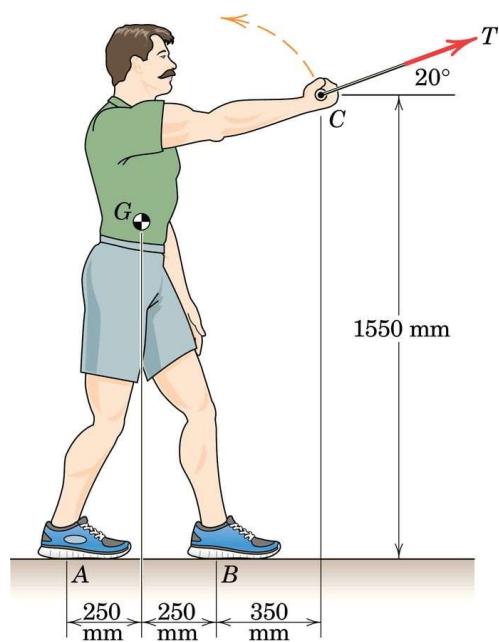
$$P = 44.9 \text{ N}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

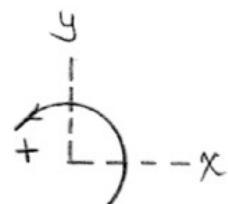
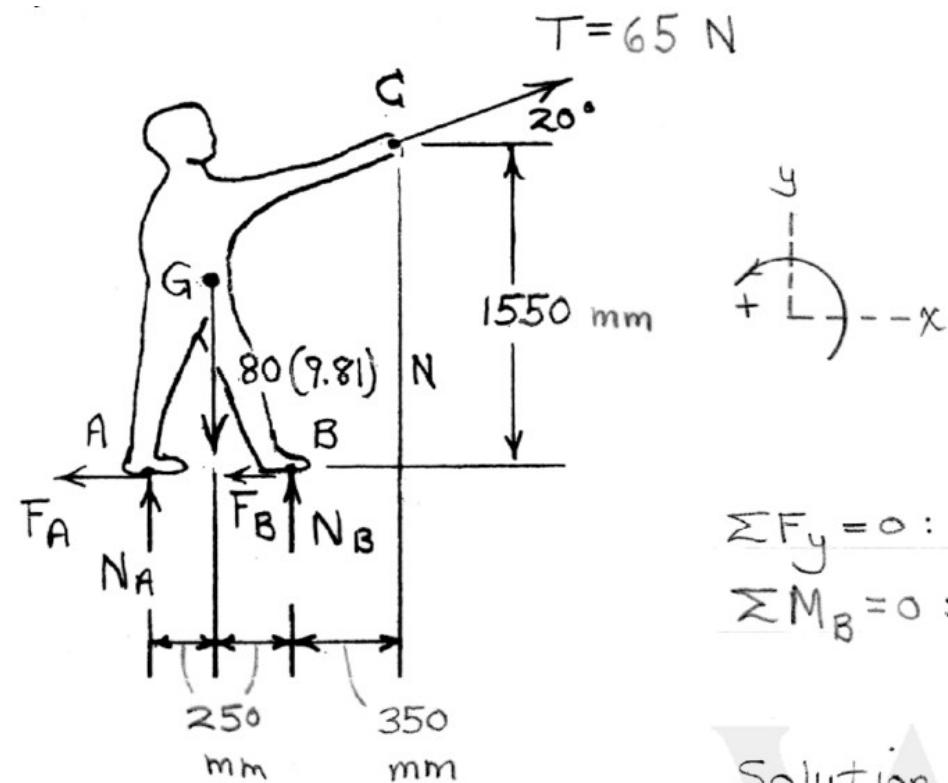


3/23) The 80-kg exerciser is beginning to execute some slow, steady bicep curls. As the tension  $T = 65 \text{ N}$  is developed against an exercise machine (not shown), determine the normal reaction forces at the feet A and B. Friction is sufficient to prevent slipping, and the exerciser maintains the position shown with center of mass at G.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum F_y = 0 : N_A + N_B - 80(9.81) + 65 \sin 20^\circ = 0$$

$$\sum M_B = 0 : 80(9.81)(250) - N_A(500) - 65[1550 \cos 20^\circ - 350 \sin 20^\circ] = 0$$

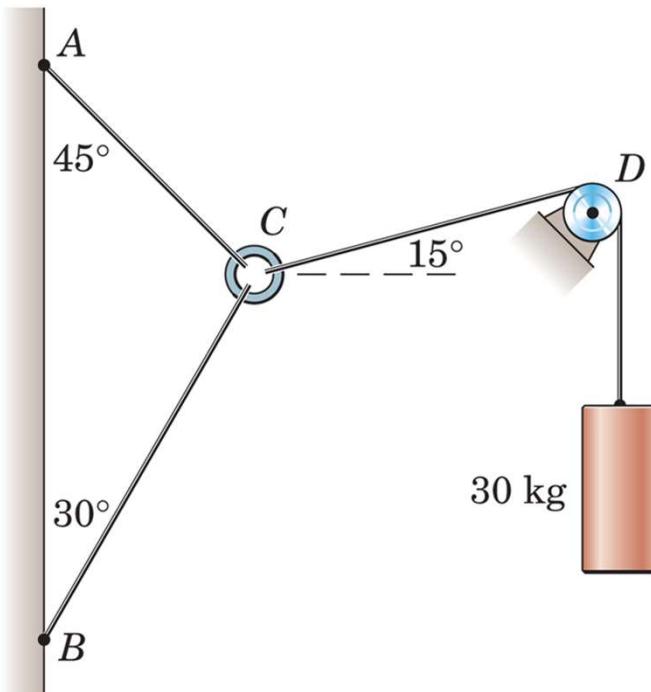
Solution :  $\begin{cases} N_A = 219 \text{ N} \\ N_B = 544 \text{ N} \end{cases}$

## ENGINEERING MECHANICS

### Equilibrium - Numerical



3/24) Three cables are joined at the junction ring C. Determine the tensions in cables AC and BC caused by the weight of the 30 kg cylinder.

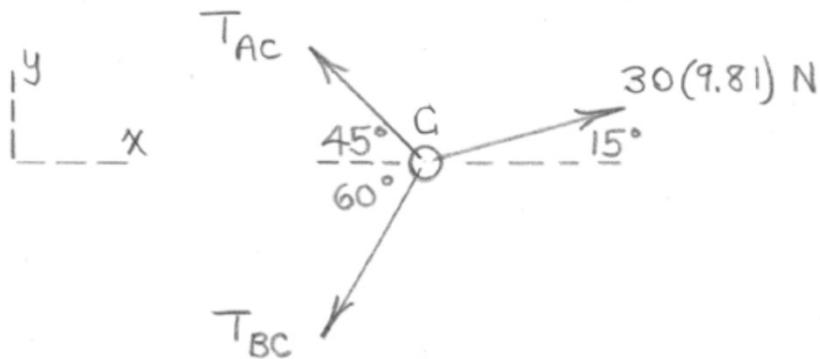


# ENGINEERING MECHANICS

## Equilibrium - Numerical



3/24 FBD of junction ring G:



$$\begin{cases} \sum F_x = 0 : -T_{AC} \cos 45^\circ - T_{BC} \cos 60^\circ + 30(9.81) \cos 15^\circ = 0 \\ \sum F_y = 0 : T_{AC} \sin 45^\circ - T_{BC} \sin 60^\circ + 30(9.81) \sin 15^\circ = 0 \end{cases}$$

Solve simultaneously to obtain

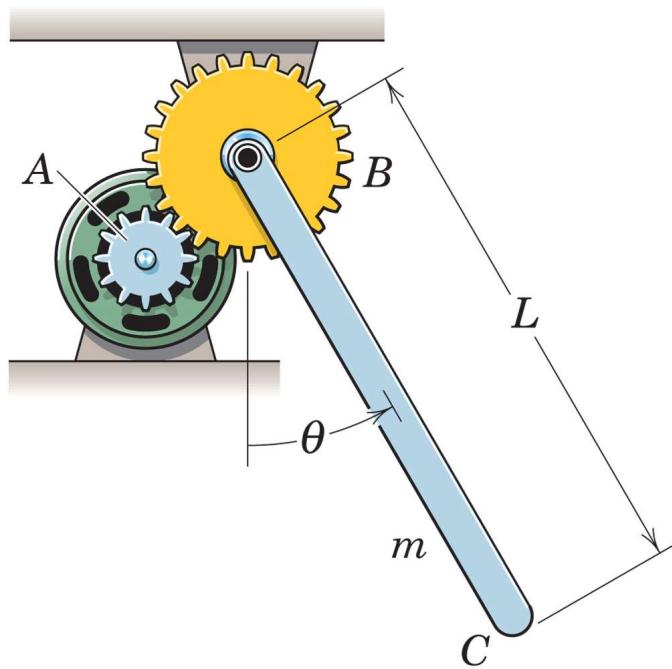
$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

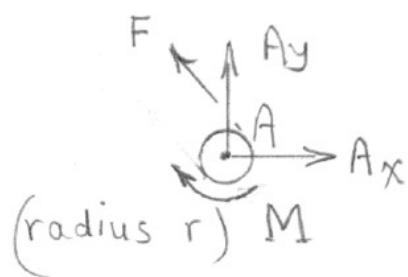
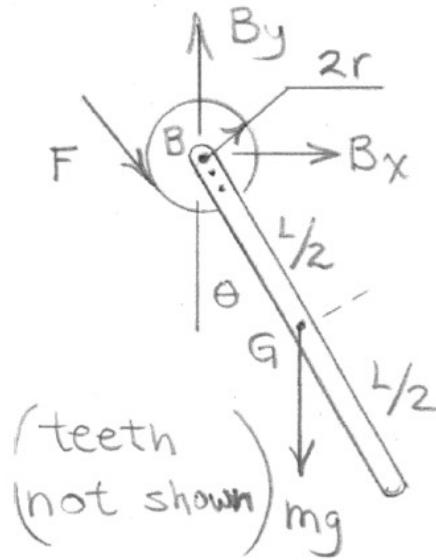


3/25) Determine the moment  $M$  which the motor must exert in order to position the uniform slender bar of mass  $m$  and length  $L$  in the arbitrary position  $\theta$ . The ratio of the radius of the gear wheel  $B$  attached to the bar to that of the gear wheel  $A$  attached to the motor shaft is 2.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\curvearrowleft \sum M_B = 0 :$$

$$F(2r) - mg \frac{L}{2} \sin \theta = 0$$

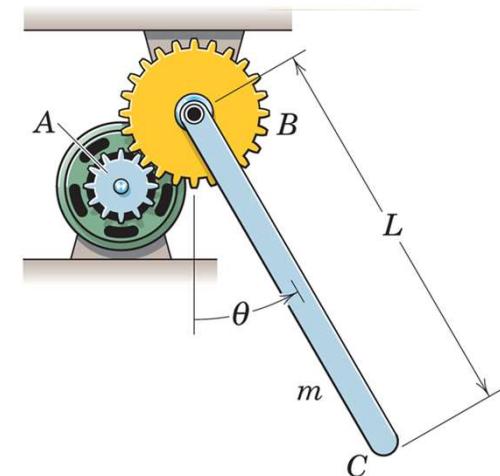
$$F = \frac{1}{4r} mg L \sin \theta$$

$$\curvearrowleft \sum M_A = 0 :$$

$$-M + Fr = 0$$

$$M = \frac{1}{4} mg L \sin \theta$$

(CW)

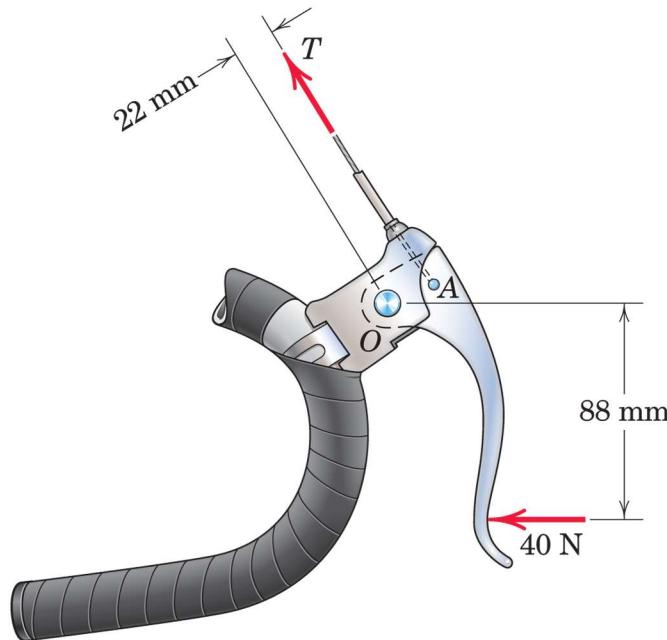


## ENGINEERING MECHANICS

### Equilibrium - Numerical



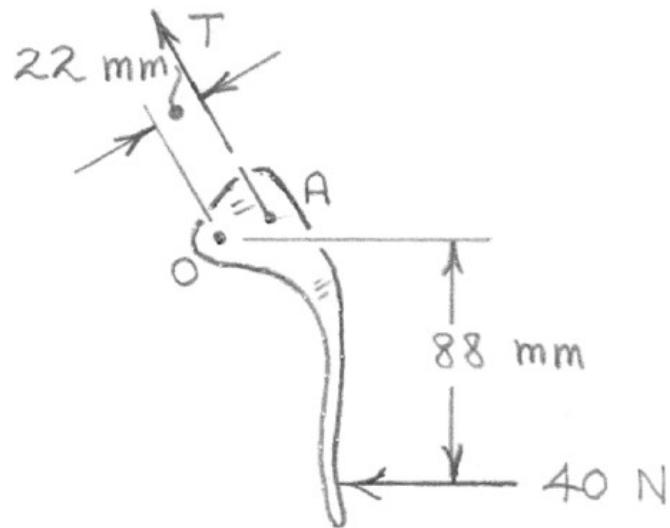
3/26) A bicyclist applies a 40-N force to the brake lever of her bicycle as shown. Determine the corresponding tension  $T$  transmitted to the brake cable. Neglect friction at the pivot 0.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

FBD of brake lever:

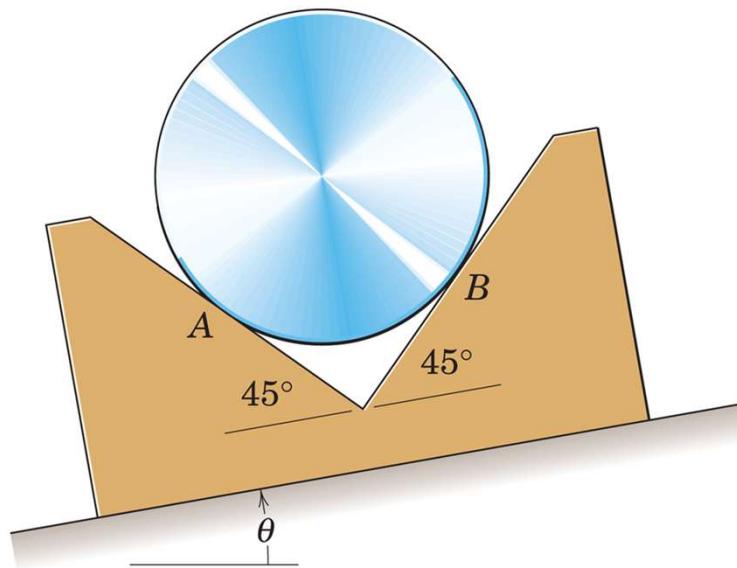


$$\therefore \sum M_A = 0 : T(22) - 40(88) = 0$$
$$\underline{T = 160 \text{ N}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

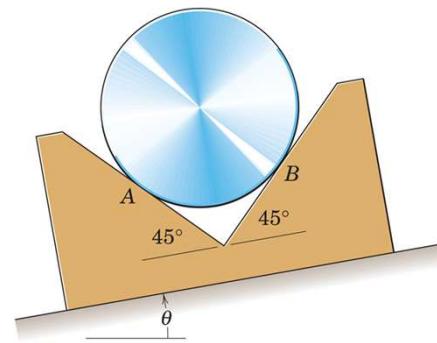
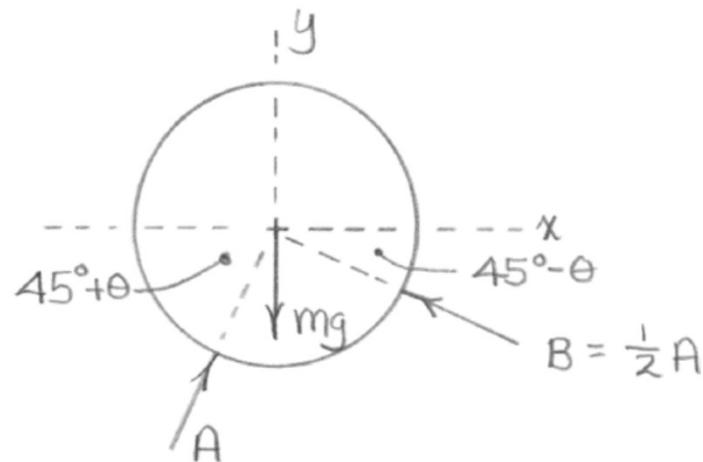
3/27) Find the angle of tilt  $\theta$  with the horizontal so that the contact force at B will be one-half that at A for the smooth cylinder.



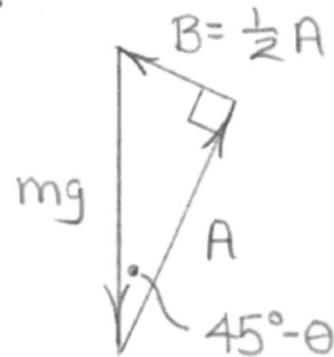
# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/27



$$\sum F = 0 :$$



$$\tan(45^\circ - \theta) = \frac{A/z}{A} = \frac{1}{2}$$

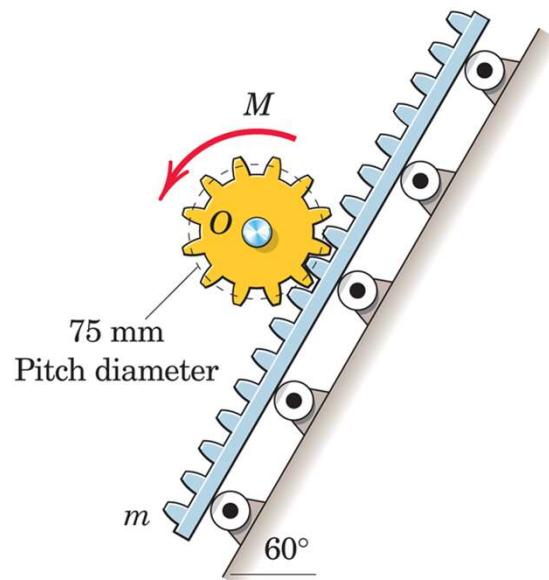
$$45^\circ - \theta = 26.6^\circ$$

$$\underline{\theta = 18.43^\circ}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

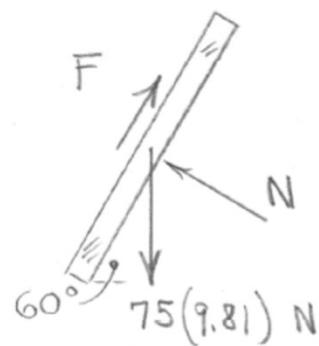
3/28) The rack has a mass  $m = 75 \text{ kg}$ . What moment  $M$  must be exerted on the gear wheel by the motor in order to lower the rack at a slow steady speed down the  $60^\circ$  incline? Neglect all friction. The fixed motor which drives the gear wheel via the shaft at O is not shown.



# ENGINEERING MECHANICS

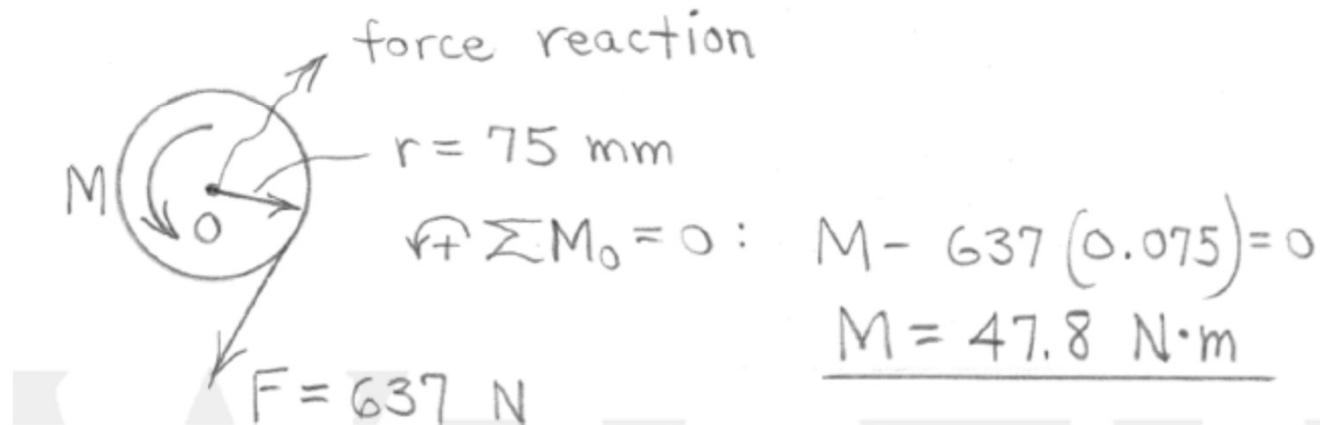
## Equilibrium - Numerical

3/28 | Rack :



$$+ \uparrow \sum F = 0 : F - 75(9.81) \sin 60^\circ = 0 \\ F = 637 \text{ N}$$

Gear wheel :

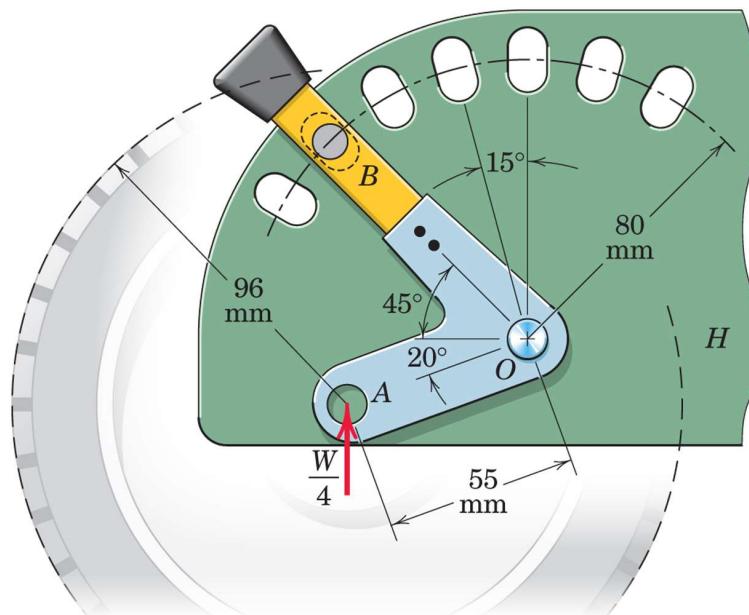


# ENGINEERING MECHANICS

## Equilibrium - Numerical

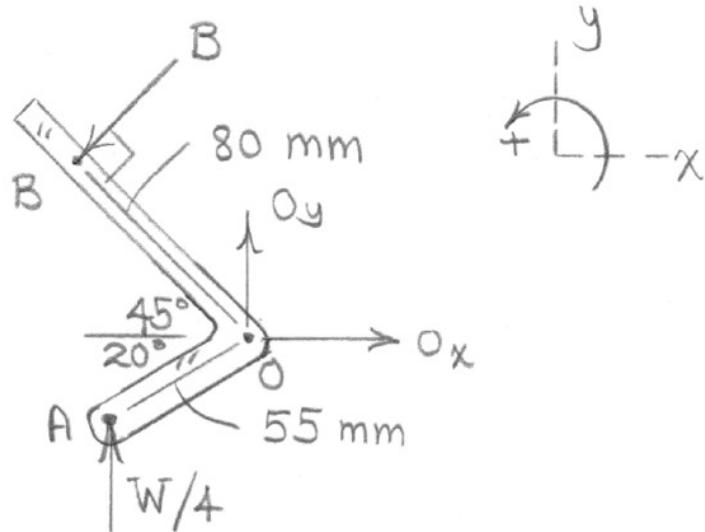


3/29) The elements of a wheel-height adjuster for a lawn mower are shown. The wheel (partial outline shown dashed for clarity) bolts through the hole at A, which goes through the bracket but not the housing H. A pin fixed to the back of the bracket at B fits into one of the seven elongated holes of the housing. For the position shown, determine the force at the pin B and the magnitude of the reaction at the pivot O. The wheel supports a force of magnitude  $W/4$ , where W is the weight of the entire mower.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum M_O = 0 : B(80) - \frac{W}{4} (55 \cos 20^\circ) = 0$$

$$B = 0.1615W$$

$$\sum F_x = 0 : -0.1615W \cos 45^\circ + O_x = 0$$

$$O_x = 0.1142W$$

$$\sum F_y = 0 : O_y - 0.1615W \sin 45^\circ + \frac{W}{4} = 0$$

$$O_y = -0.1358W$$

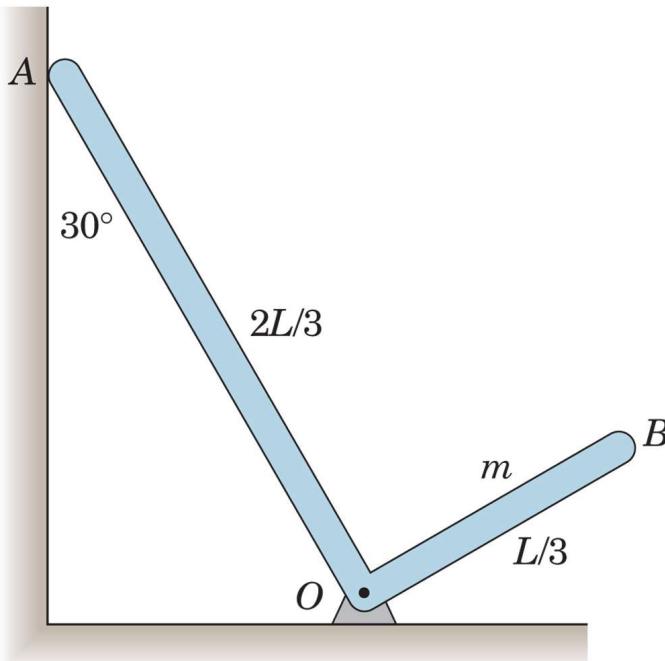
$$O = \sqrt{O_x^2 + O_y^2} = \underline{0.1774W}$$

## ENGINEERING MECHANICS

### Equilibrium - Numerical

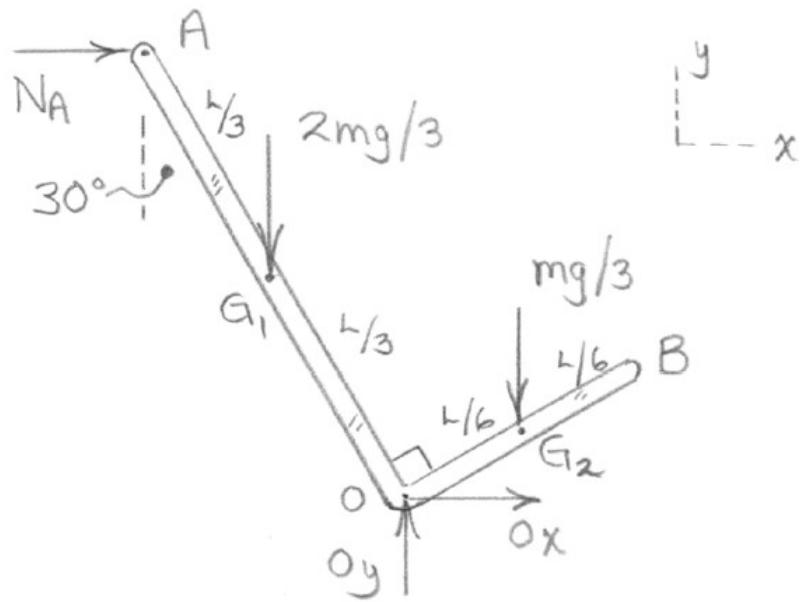


3/30) The right-angle uniform slender bar AOB has mass  $m$ . If friction at the pivot  $O$  is neglected, determine the magnitude of the normal force at  $A$  and the magnitude of the pin reaction at  $O$ .



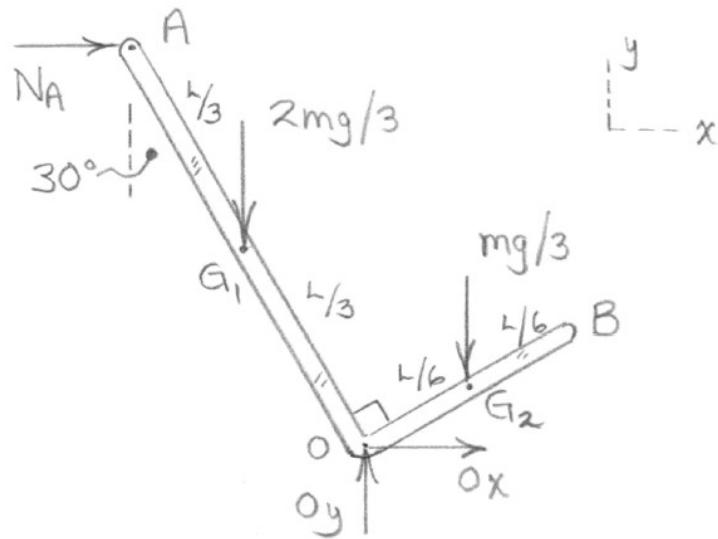
# ENGINEERING MECHANICS

## Equilibrium - Numerical



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\text{At } \sum M_O = 0 : -N_A \left( \frac{2L}{3} \cos 30^\circ \right) + \frac{2mg}{3} \left( \frac{L}{3} \sin 30^\circ \right) - \frac{mg}{3} \left( \frac{L}{6} \cos 30^\circ \right) = 0$$

$$\underline{N_A = 0.1091 mg}$$

$$\sum F_y = 0 \Rightarrow Oy = mg$$

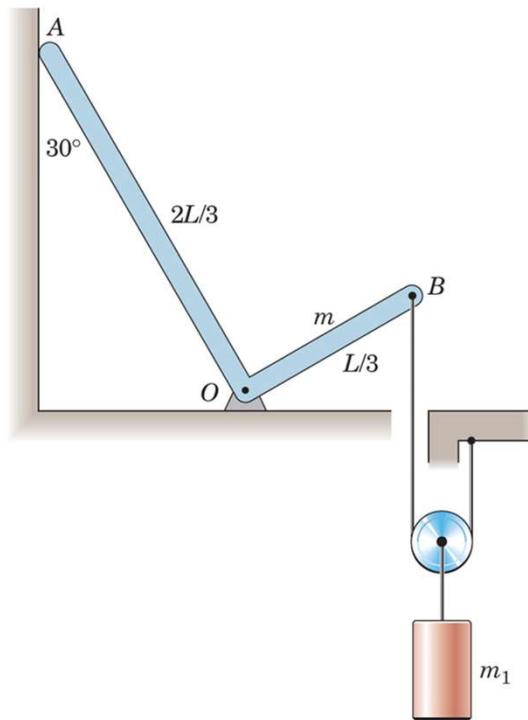
$$\sum F_x = 0 \Rightarrow Ox = -0.1091 mg$$

$$O = \sqrt{O_x^2 + O_y^2} = \underline{1.006 mg}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

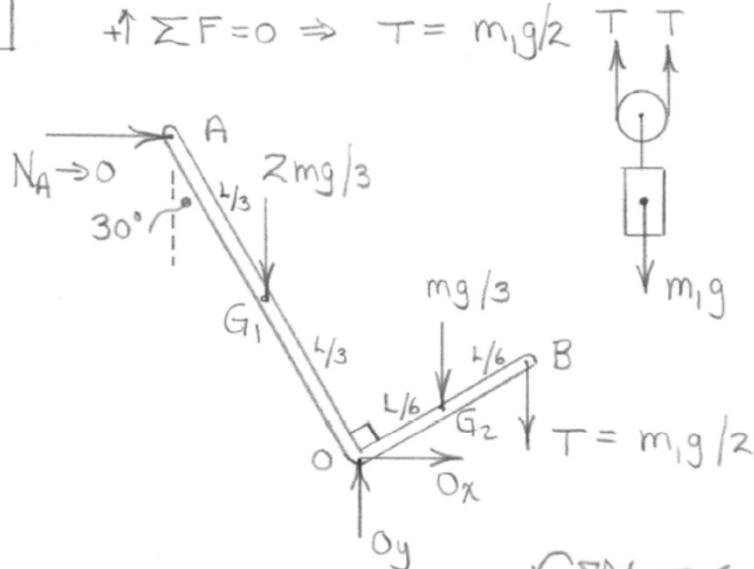
3/31) Determine the minimum cylinder mass  $m_1$  required to cause loss of contact at A



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/31  $\uparrow \sum F = 0 \Rightarrow T = m_1 g / 2$



$$\text{At } O: \sum M_O = 0 : \frac{2mg}{3} \left( \frac{L}{3} \sin 30^\circ \right) - \frac{mg}{3} \left( \frac{L}{6} \cos 30^\circ \right) - \frac{m_1 g}{2} \left( \frac{L}{3} \cos 30^\circ \right)$$

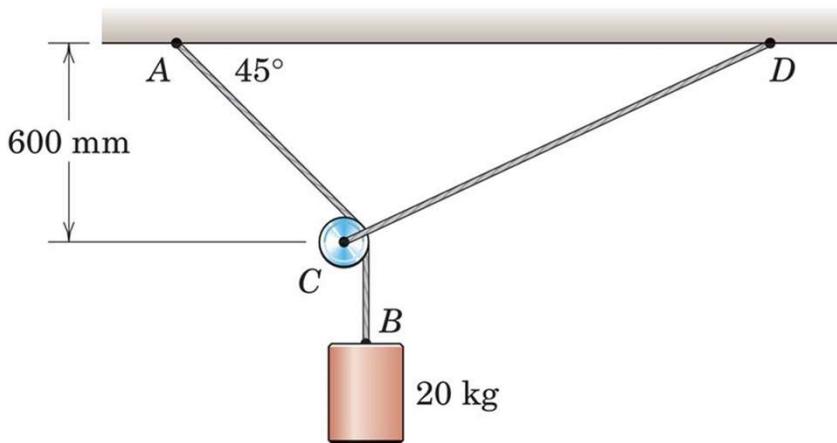
$m_1 = 0.436 \text{ m}$

## ENGINEERING MECHANICS

### Equilibrium - Numerical



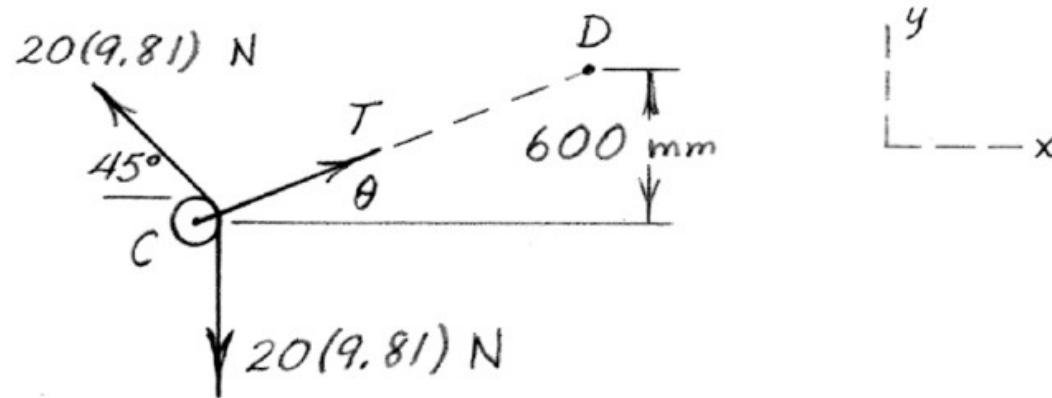
3/32) Cable AB passes over the small ideal pulley C without a change in its tension. What length of cable CD is required for static equilibrium in the position shown? What is the tension T in cable CD?



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/32



$$\sum F_x = 0: T \cos \theta - 20(9.81) \cos 45^\circ = 0$$

$$\sum F_y = 0: T \sin \theta + 20(9.81) \sin 45^\circ - 20(9.81) = 0$$

Solve to obtain  $\theta = 22.5^\circ$ ,  $T = 150.2 \text{ N}$

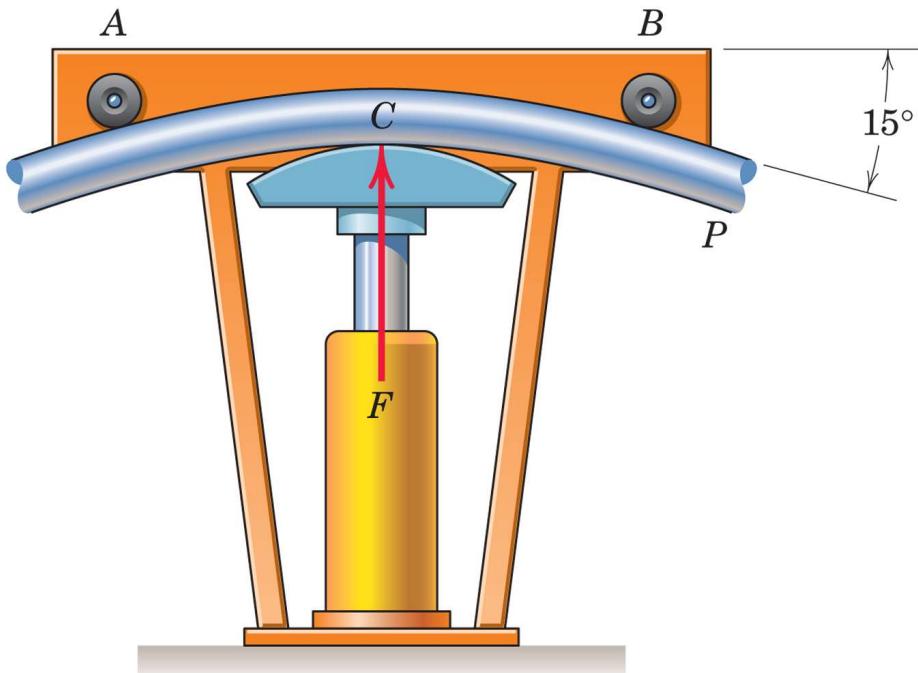
$$\frac{600}{CD} = \sin \theta = \sin 22.5^\circ, \quad \underline{\overline{CD} = 1568 \text{ mm}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

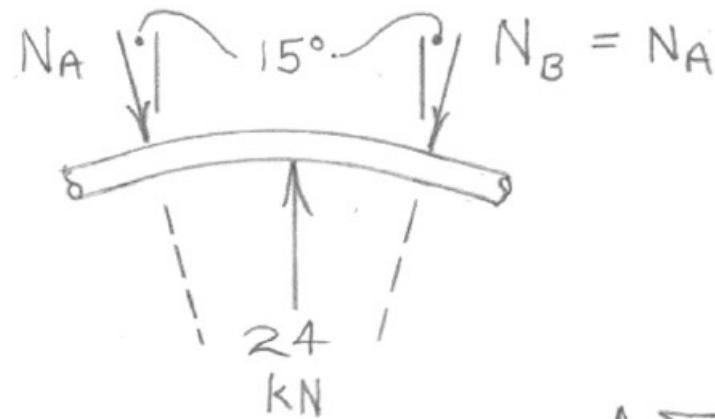


3/33) A pipe P is being bent by the pipe bender as shown. If the hydraulic cylinder applies a force of magnitude  $F = 24 \text{ kN}$  to the pipe at C, determine the magnitude of the roller reactions at A and B



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\uparrow \sum F = 0: 24 - 2N_A \cos 15^\circ = 0$$

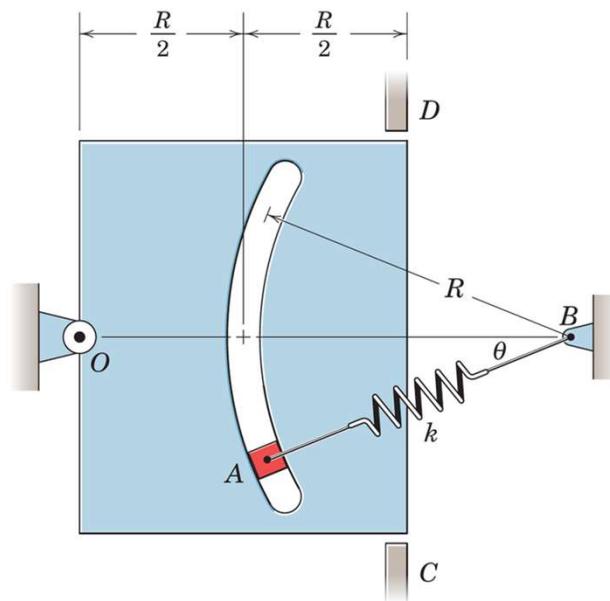
$$N_A = N_B = 12.42 \text{ kN}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical



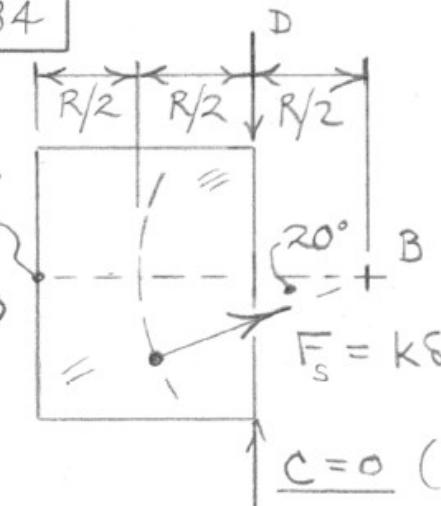
3/34) The small slider A is moved along the circular slot by a mechanism attached to the back side of the rectangular plate. For the slider position  $\theta = 20^\circ$  shown, determine the normal forces exerted at the small stops C and D. The unstretched length of the spring of constant  $k = 1.6 \text{ kN/m}$  is  $R/3$ . The value of R is 25 mm, and the plate lies in a horizontal plane. Neglect all friction.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/34



$$R = 0.025 \text{ m}$$

$$\begin{aligned} F_s &= k\delta \\ &= k[R - \frac{R}{3}] \\ &= 1600 \left(\frac{2}{3} \cdot 0.025\right) \\ &= 26.7 \text{ N} \end{aligned}$$

C = 0 (by inspection)

$$\begin{aligned} \text{Sum of moments about } C &= 0 : 26.7 \sin 20^\circ \left(\frac{3}{2} \cdot 0.025\right) - D(0.025) \\ &= 0 \end{aligned}$$

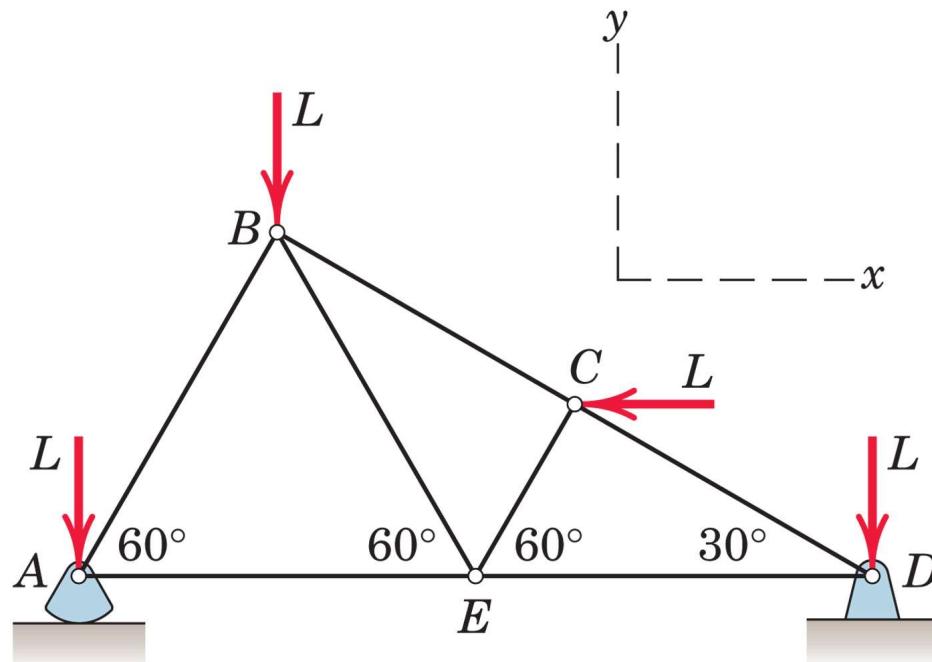
$$D = 13.68 \text{ N}$$

## ENGINEERING MECHANICS

### Equilibrium - Numerical

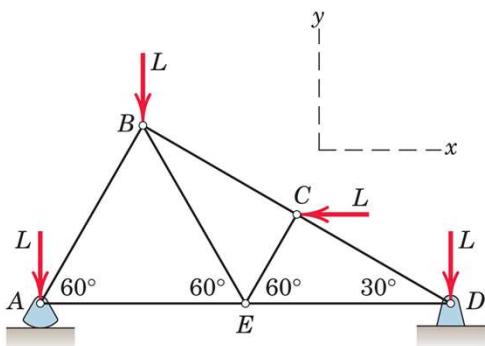
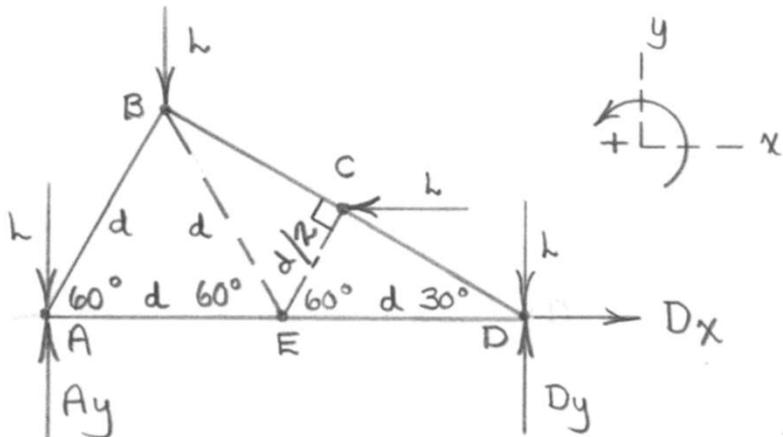


3/35) The asymmetric simple truss is loaded as shown. Determine the reactions at A and D. Neglect the weight of the structure compared with the applied loads. Is knowledge of the size of the structure necessary?



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum F_x = 0 : D_x - L = 0 , \quad D_x = L$$

$$\sum F_y = 0 : A_y + D_y - 3L = 0$$

$$\sum M_A = 0 : D_y(2d) + L\left(\frac{d}{2}\frac{\sqrt{3}}{2}\right) - L\left(\frac{d}{2}\right) - L(2d) = 0$$

Solving the last 2 equations :  $A_y = \frac{L}{4}(7 + \frac{\sqrt{3}}{2})$

$$D_y = \frac{L}{4}(5 - \frac{\sqrt{3}}{2})$$

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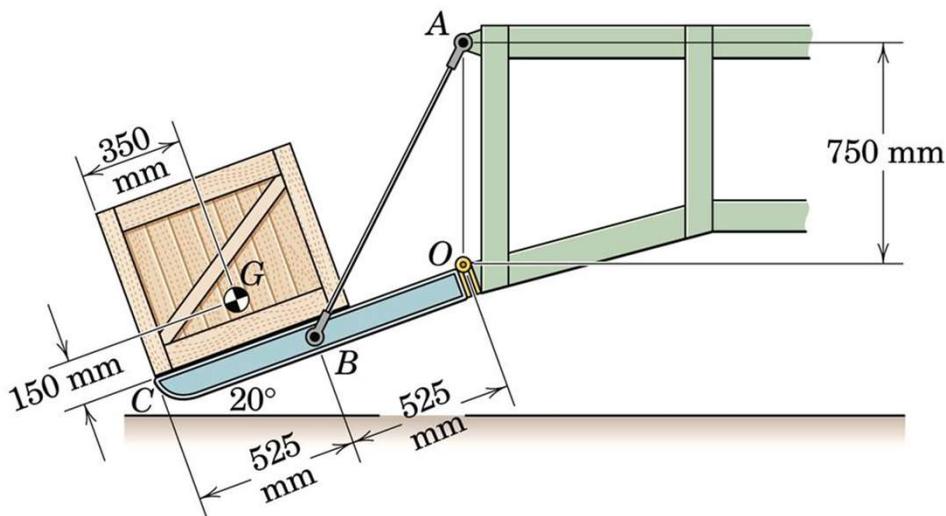
(or  $A_y = 1.967L$ ,  $D_y = 1.033L$ )

# **ENGINEERING MECHANICS**

## **Equilibrium - Numerical**



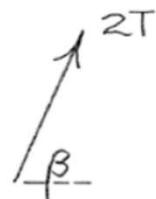
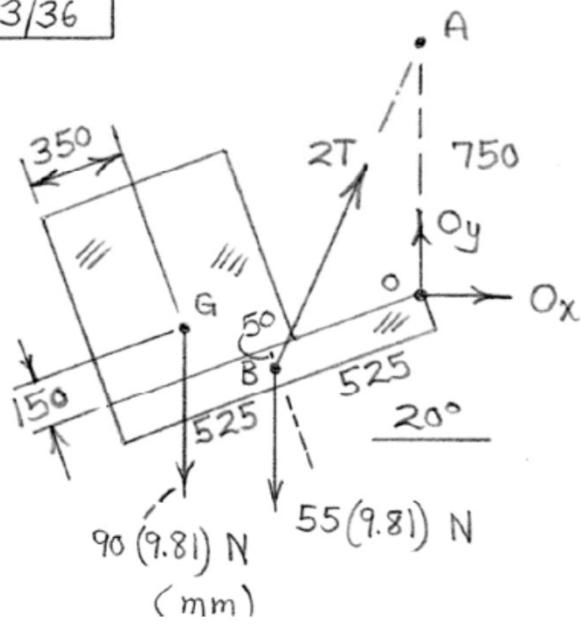
**3/36**) The tailgate OBC is attached to the rear of a trailer via hinges at O and two restraining cables AB. The 55-kg tailgate is 100 mm thick with center of mass at B, which is at mid thickness. The crate is centered between the two cables and has a mass of 90 kg with center of mass at G. Determine the tension T in each cable.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/36



$$\beta = \cot^{-1} \left[ \frac{525 \cos 20^\circ - 50 \sin 20^\circ}{750 + 525 \sin 20^\circ + 50 \cos 20^\circ} \right]$$

$$= 64.0^\circ$$

For  $\sum M_o = 0$  :

$$55(9.81)(525 \cos 20^\circ - 50 \sin 20^\circ)$$

$$+ 90(9.81)[(1050 - 350) \cos 20^\circ + 150 \sin 20^\circ]$$

$$- 2T \cos \beta (750) = 0$$

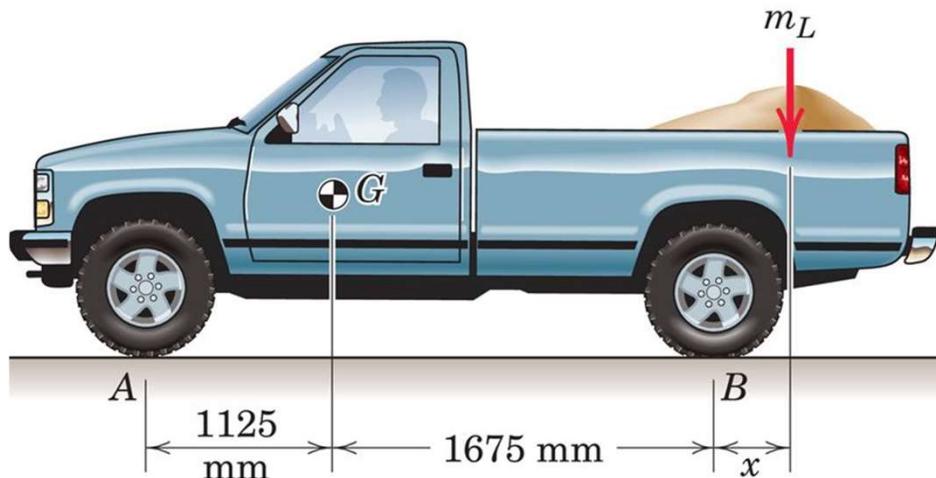
(Transmissibility used  
on  $2T \rightarrow$  point A)

# ENGINEERING MECHANICS

## Equilibrium - Numerical



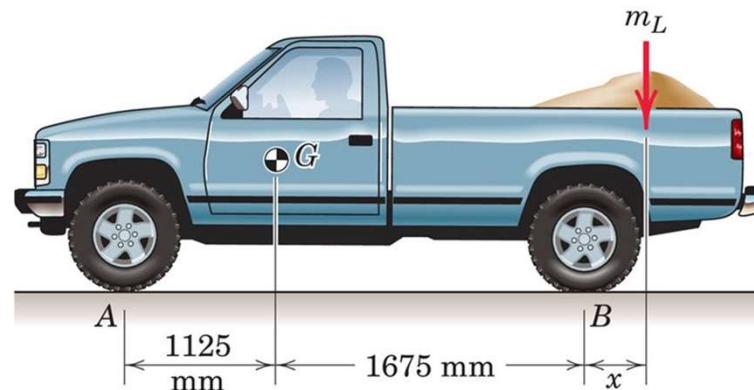
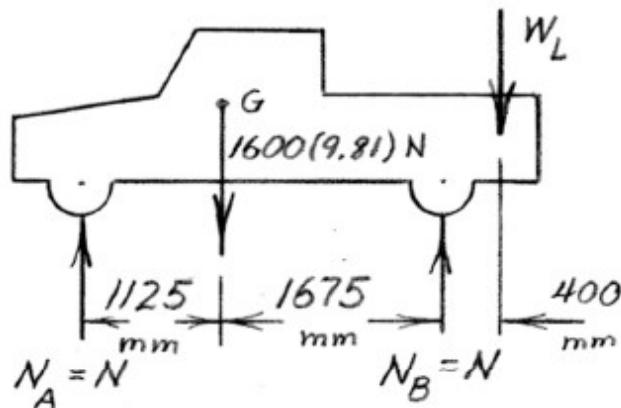
3/37) The indicated location of the center of mass of the 1600-kg pickup truck is for the unladen condition. If a load whose center of mass is  $x = 400$  mm behind the rear axle is added to the truck, determine the load mass  $m_L$  for which the normal forces under the front and rear wheels are equal.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/37



$$\textcircled{+} \sum M_A = 0: 1600(9.81)1.125 - N(2.80) + W_L(3.20) = 0.$$

$$\uparrow \sum F = 0: 2N - 1600(9.81) - W_L = 0$$

Solve to obtain  $N = 9050 \text{ N}$

$$W_L = 2400 \text{ N}$$

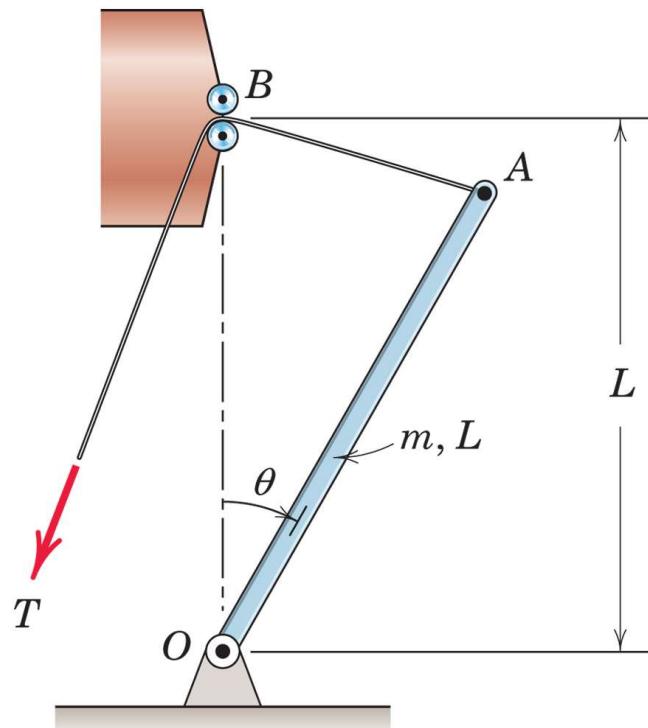
$$m_L = \frac{W_L}{g} = \frac{2400}{9.81} = \underline{\underline{244 \text{ kg}}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

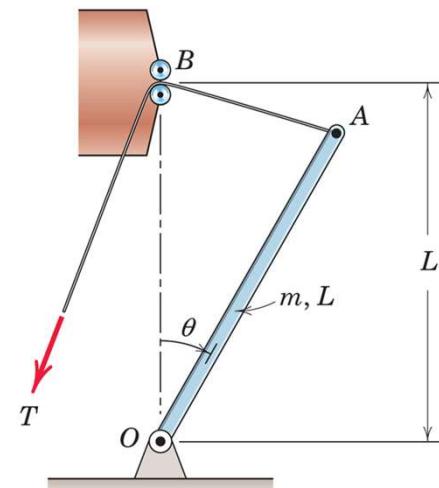
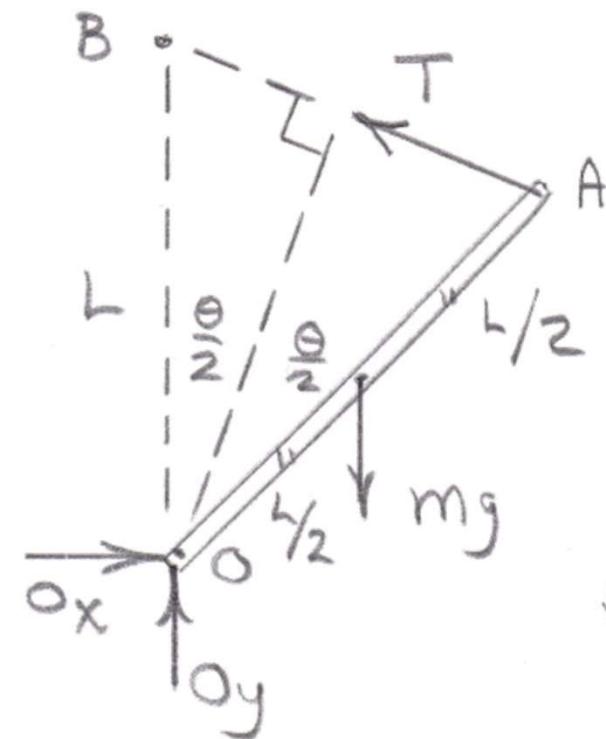


3/39) Determine the force  $T$  required to hold the uniform bar of mass  $m$  and length  $L$  in an arbitrary angular position  $\theta$ . Plot your result over the range  $0 \leq \theta \leq 90^\circ$ , and state the value of  $T$  for  $\theta = 40^\circ$ .



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum M_O = 0 : T(L \cos \theta) - mg \left( \frac{L}{2} \sin \theta \right) = 0$$

$$T = \frac{mg \sin \theta}{2 \cos \frac{\theta}{2}}$$

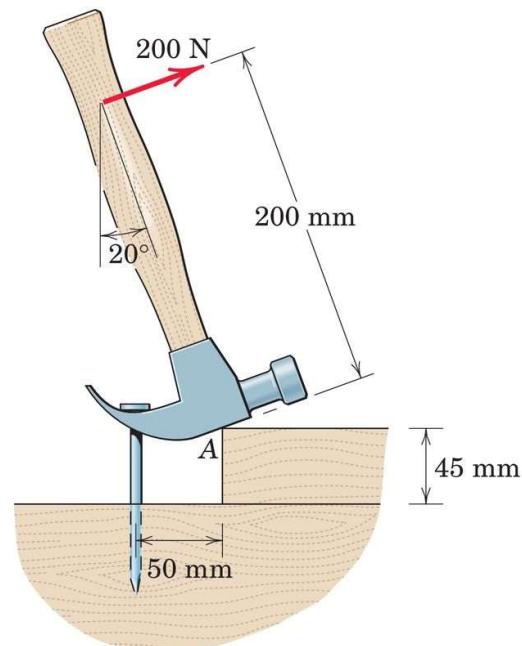
$$\underline{T_{40^\circ} = 0.342 mg}$$

## ENGINEERING MECHANICS

### Equilibrium - Numerical

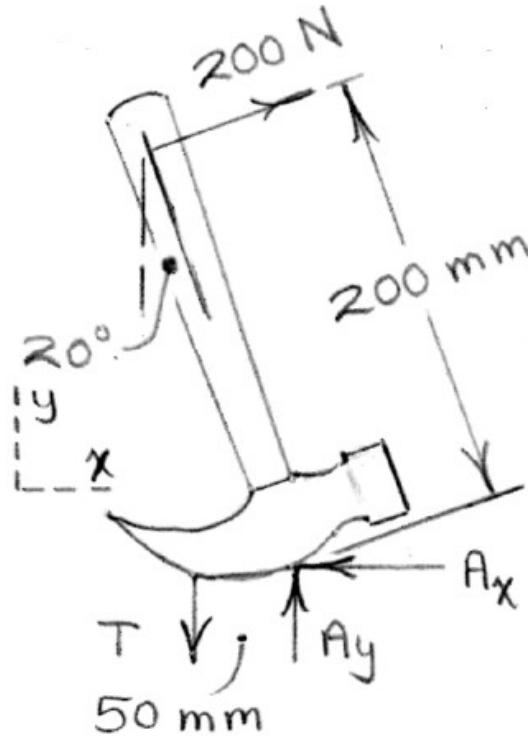


3/40) A block placed under the head of the claw hammer as shown greatly facilitates the extraction of the nail. If a 200-N pull on the handle is required to pull the nail, calculate the tension  $T$  in the nail and the magnitude  $A$  of the force exerted by the hammer head on the block. The contacting surfaces at A are sufficiently rough to prevent slipping.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum M_A = 0 : 200(200) - 50T = 0, T = 800 \text{ N}$$

$$\sum F_x = 0 : 200 \cos 20^\circ - A_x = 0$$

$$A_x = 187.9 \text{ N}$$

$$\sum F_y = 0 : A_y + 200 \sin 20^\circ - 800 = 0$$

$$A_y = 732 \text{ N}$$

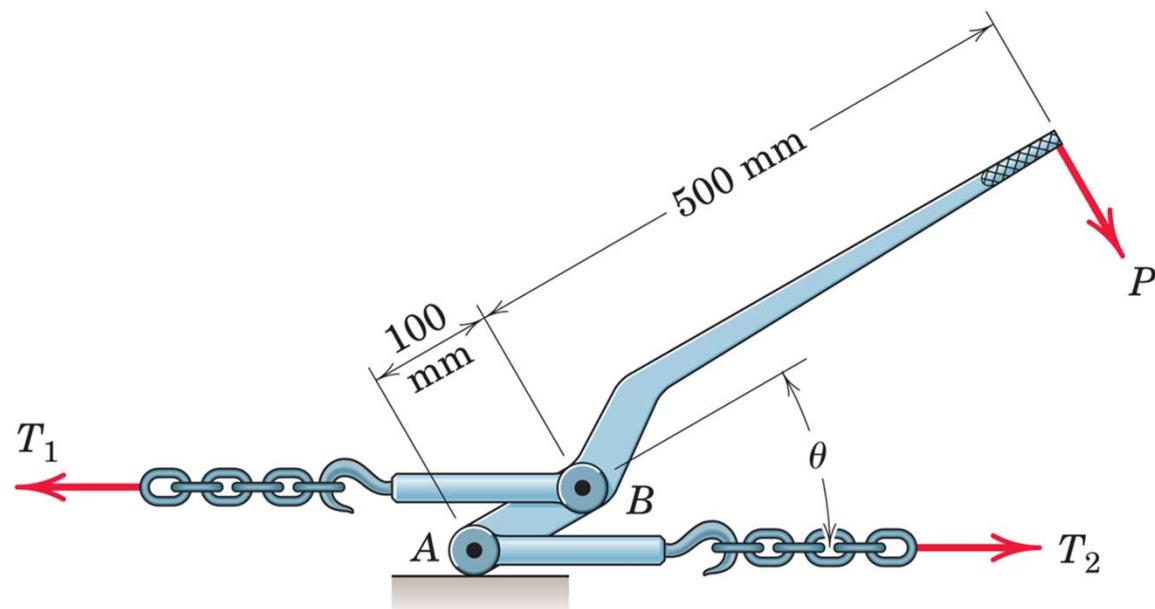
$$A = \sqrt{A_x^2 + A_y^2} = 755 \text{ N}$$

## ENGINEERING MECHANICS

### Equilibrium - Numerical



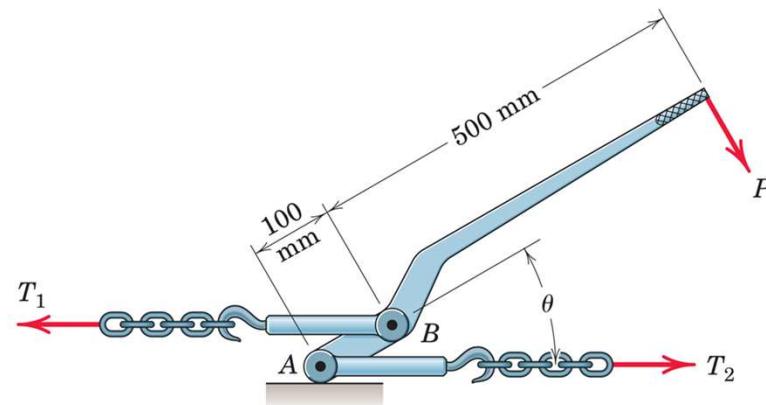
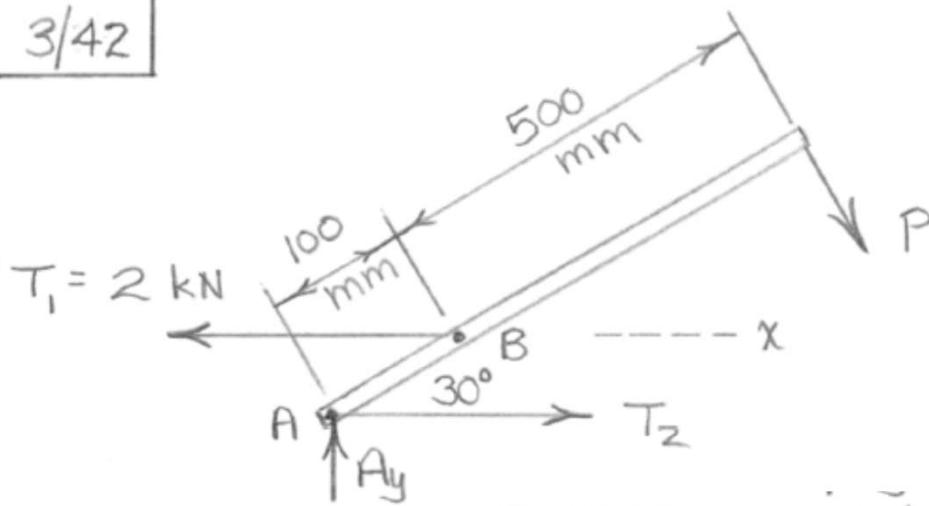
3/42) The chain binder is used to secure loads of logs, lumber, pipe, and the like. If the tension  $T_1$  is 2 kN when  $\theta = 30^\circ$ , determine the force  $P$  required on the lever and the corresponding tension  $T_2$  for this position. Assume that the surface under A is perfectly smooth.



# ENGINEERING MECHANICS

## Equilibrium - Numerical

3/42



$$\sum M_A = 0 : P(500 + 100) - 2(100 \sin 30^\circ) = 0$$

$$P = 0.1667 \text{ kN} \quad \text{or} \quad \underline{\underline{P = 166.7 \text{ N}}}$$

$$\sum F_x = 0 : 0.1667 \sin 30^\circ + T_2 - 2 = 0$$

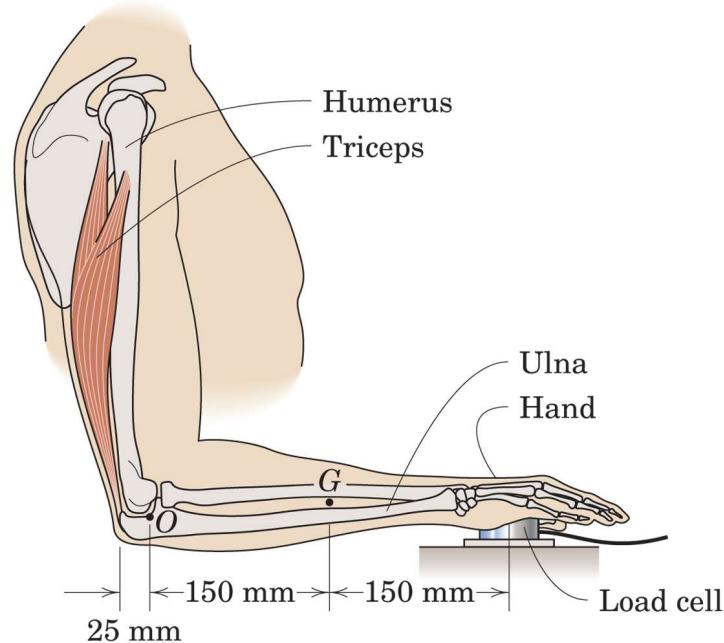
$$\underline{\underline{T_2 = 1.917 \text{ kN}}}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

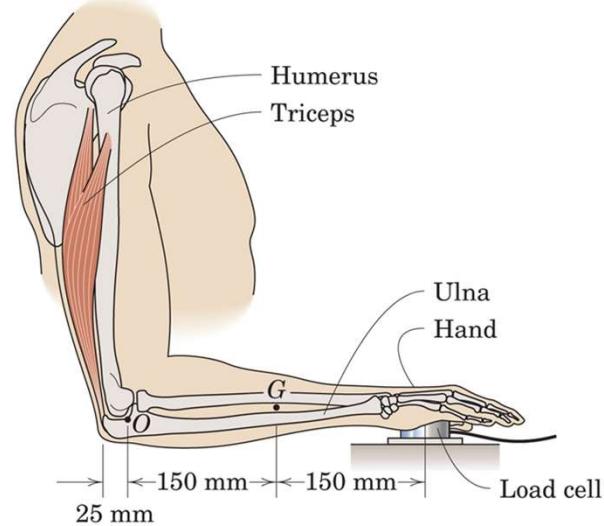
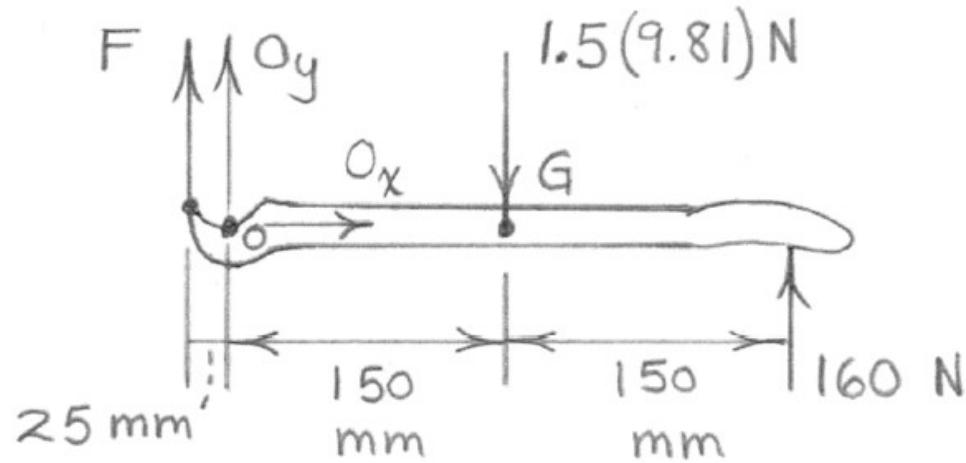


3/43) In a procedure to evaluate the strength of the triceps muscle, a person pushes down on a load cell with the palm of his hand as indicated in the figure. If the load-cell reading is 160 N, determine the vertical tensile force  $F$  generated by the triceps muscle. The mass of the lower arm is 1.5 kg with mass center at G. State any assumptions.



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\sum M_O = 0 : -F(25) - 1.5(9.81)(150) + 160(300) = 0$$
$$F = 1832 \text{ N}$$

## ENGINEERING MECHANICS

### Equilibrium - Numerical



**3/44)** A woman is holding a 3.6-kg sphere in her hand with the entire arm held horizontally as shown in the figure. A tensile force in the deltoid muscle prevents the arm from rotating about the shoulder joint 0; this force acts at the  $21^\circ$  angle shown.

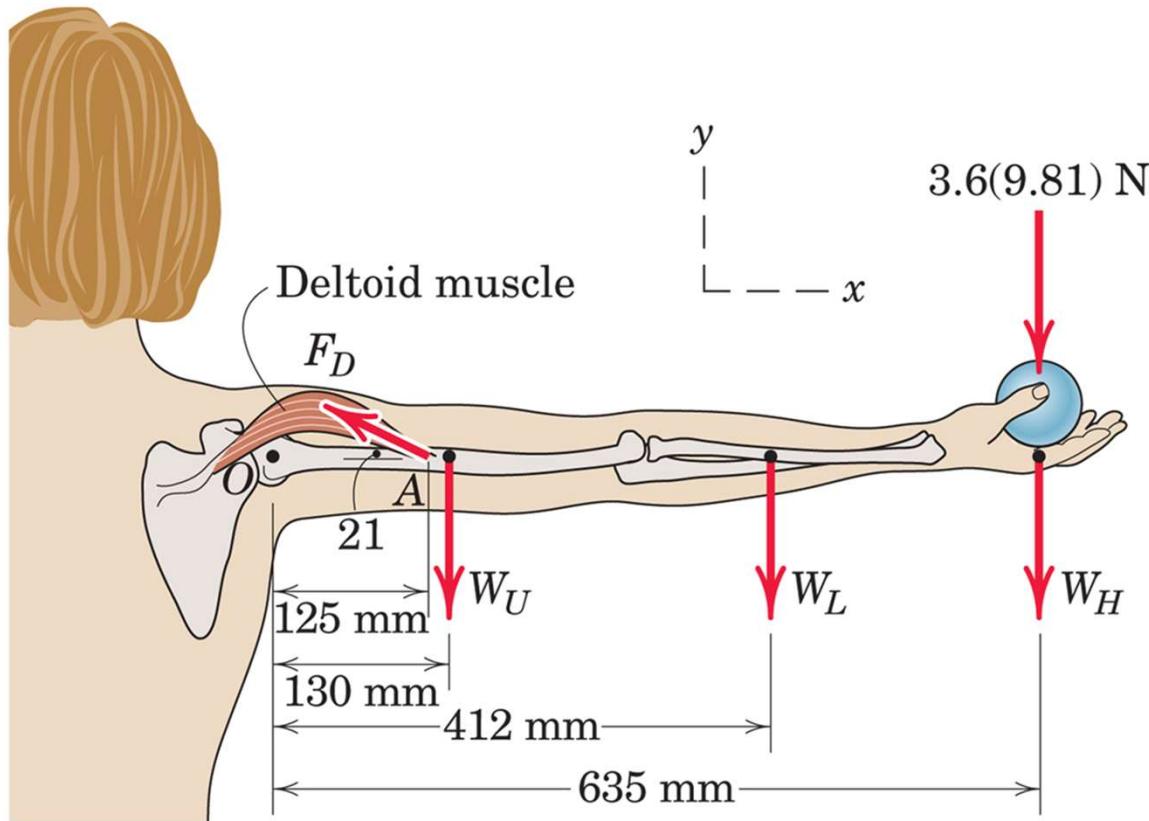
Determine the force exerted by the deltoid muscle on the upper arm at A and the x- and y-components of the force reaction at the shoulder joint 0. The mass

of the upper arm is  $m_u = 1.9$  kg, the mass of the lower arm is  $m_L = 1.1$  kg, and the mass of the hand is  $m_H = 0.4$  kg; all the corresponding weights act at the locations shown in the figure.

# ENGINEERING MECHANICS

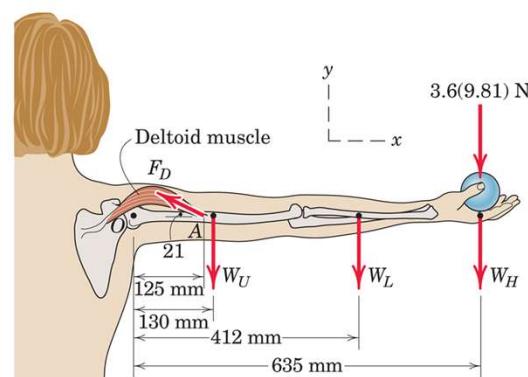
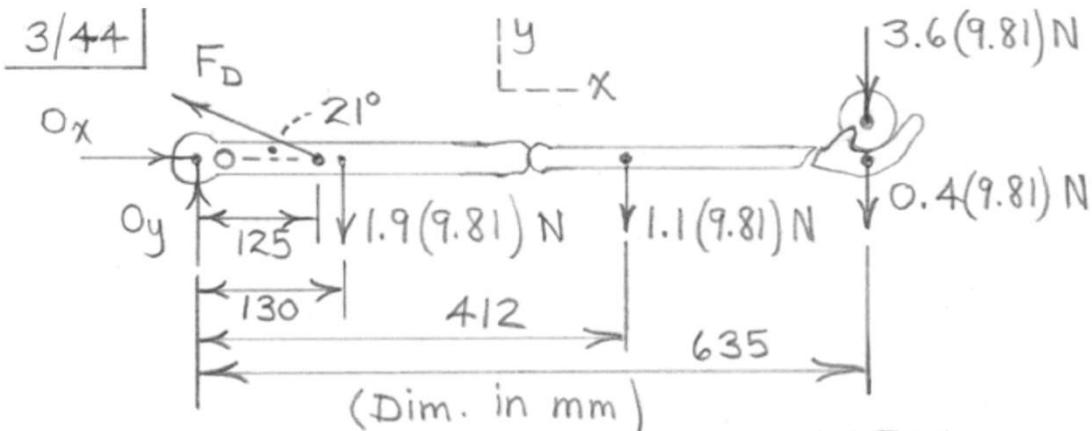
## Equilibrium - Numerical

3/44)



# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\begin{aligned} \text{For } \sum M_O = 0: & F_D \sin 21^\circ (125) - 1.9(9.81)(130) \\ & - 1.1(9.81)(412) - (3.6+0.4)(9.81)(635) = 0 \\ F_D &= 710 \text{ N} \end{aligned}$$

$$\therefore \sum F_x = 0: O_x - 710 \cos 21^\circ = 0, \quad O_x = 662 \text{ N}$$

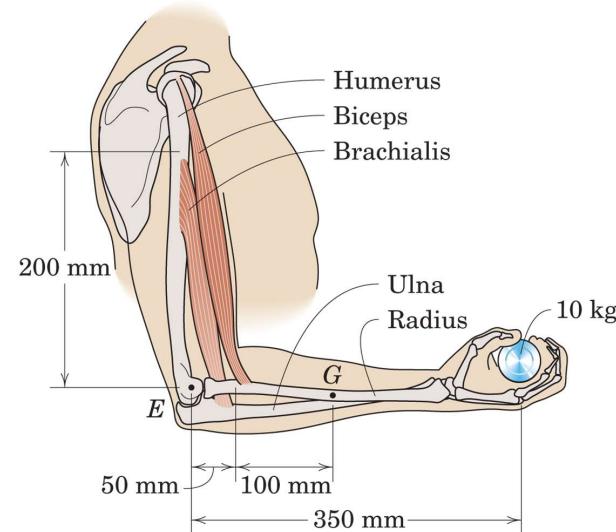
$$\begin{aligned} +\uparrow \sum F_y = 0: & O_y + 710 \sin 21^\circ - (1.9+1.1+3.6+0.4)9.81 \\ & = 0, \quad O_y = -185.6 \text{ N} \end{aligned}$$

# ENGINEERING MECHANICS

## Equilibrium - Numerical

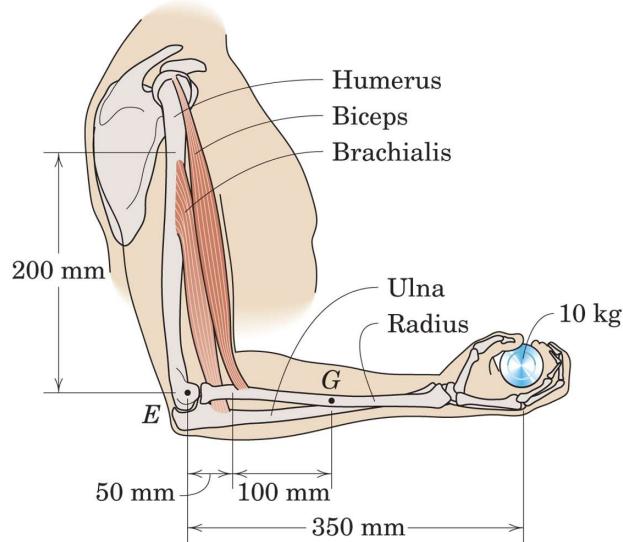


**3/45)** A person is performing slow arm curls with a 10-kg weight as indicated in the figure. The brachialis muscle group (consisting of the biceps and brachialis muscles) is the major factor in this exercise. Determine the magnitude F of the brachialis-muscle group force and the magnitude E of the elbow joint reaction at point E for the forearm position shown in the figure. Take the dimensions shown to locate the effective points of application of the two muscle groups; these points are 200 mm directly above E and 50 mm directly to the right of E. Include the effect of the 1.5-kg forearm mass with mass center at point G. State any assumptions.

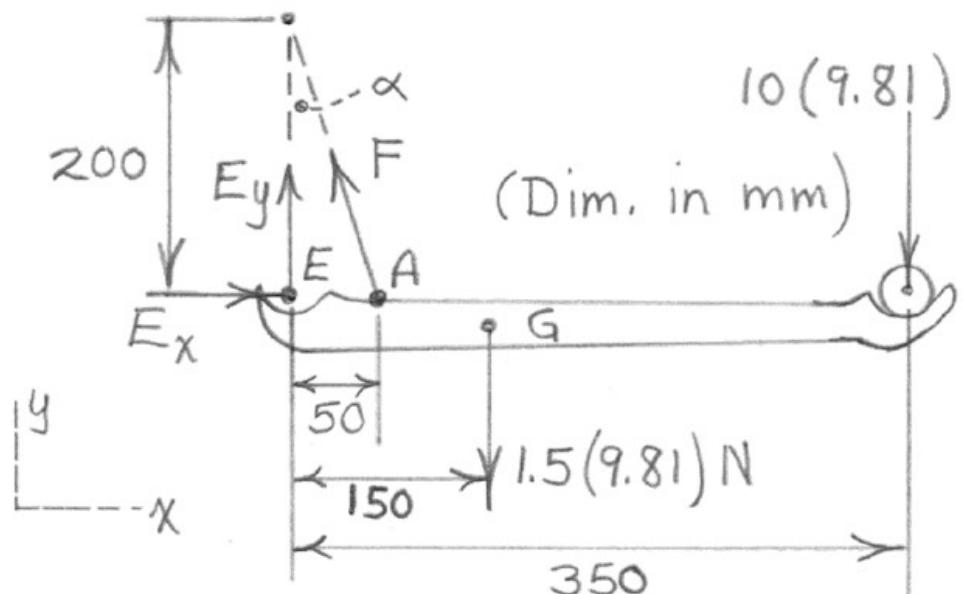


# ENGINEERING MECHANICS

## Equilibrium - Numerical



$$\alpha = \tan^{-1} \left( \frac{50}{200} \right) = 14.04^\circ$$

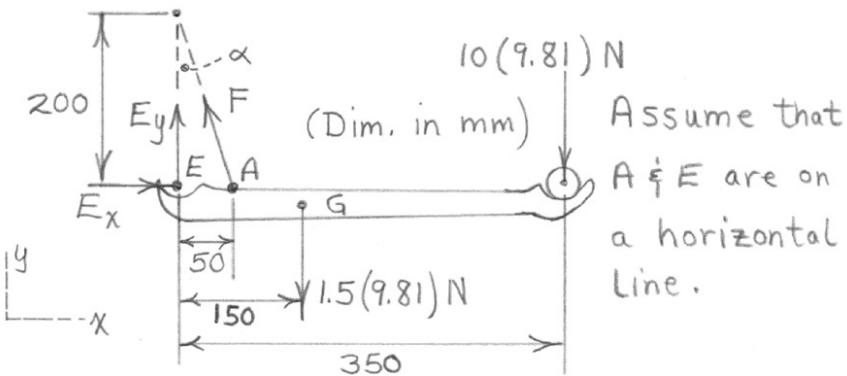


Assume that  
 $A \notin E$  are on  
a horizontal  
line.

# ENGINEERING MECHANICS

## Equilibrium - Numerical

$$\alpha = \tan^{-1} \left( \frac{50}{200} \right) = 14.04^\circ$$



$$\sum F_x = 0 : -753 \sin 14.04^\circ + E_x = 0$$

$$E_x = 182.7 \text{ N}$$

$$\sum F_y = 0 : 753 \cos 14.04^\circ - (10+1.5)(9.81) + E_y = 0$$

$$E_y = -618 \text{ N}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{182.7^2 + 618^2} = 644 \text{ N}$$

$$\text{For } \sum M_E = 0 : F \cos 14.04^\circ (50) - 1.5(9.81)(150) - 10(9.81)(350) = 0, \quad F = 753 \text{ N}$$



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UNIVERSITY

**THANK YOU**

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Department of Civil Engineering

[r@pes.edu](mailto:r@pes.edu)

+91 78i87