



**PES**  
UNIVERSITY

## **ENGINEERING MECHANICS- STATICS**

**UE24CV141A**

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*Offered by*  
**Department of Civil Engineering**



**PES**  
UNIVERSITY



## J S VISHWAS

C.Engg(IEI), M.Tech(Transportation Systems), B.E( Civil Engg.).

### Websites

- [staff.pes.edu/nm1060/](http://staff.pes.edu/nm1060/) (Company)
- [sites.google.com/view/jsvishwas/home](http://sites.google.com/view/jsvishwas/home) (Personal)



ID : J S Vishwas



# UE24CV141A

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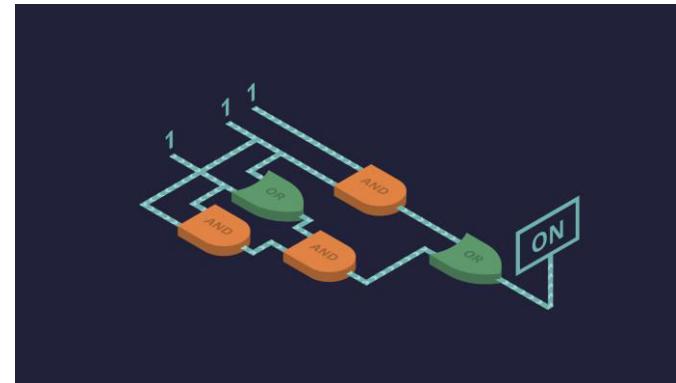
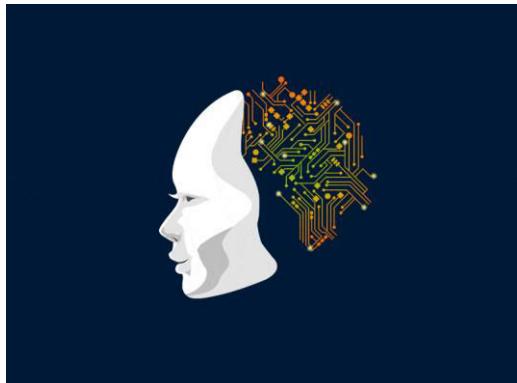
## Engineering Mechanics Statics

**Prof J S Vishwas**

Department of Civil Engineering

### What is Engineering ???

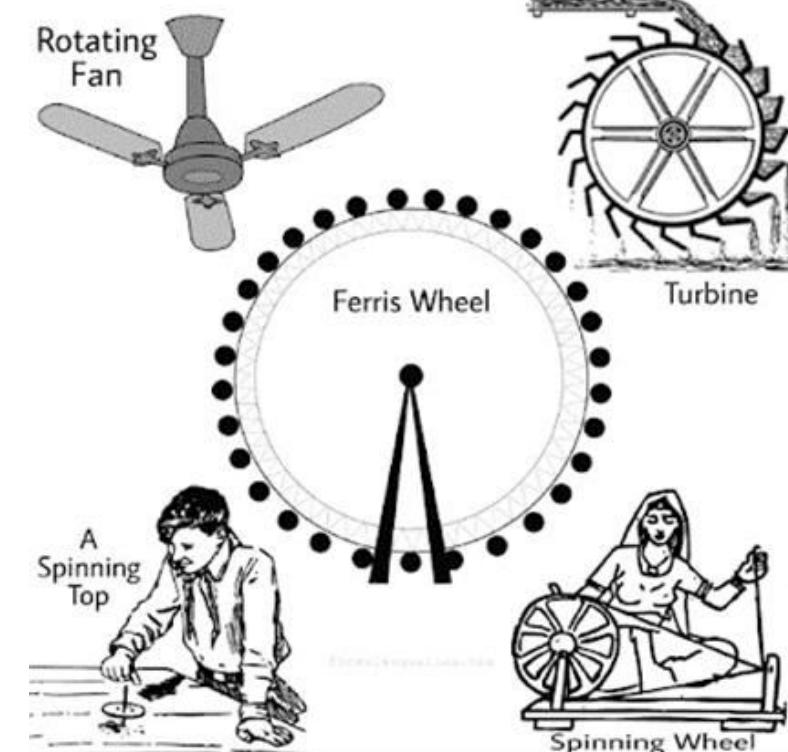
Engineering is the discipline of applying **technical, scientific knowledge and physical resources** to design and produce materials, structures, machines, devices, systems, and processes that meet a desired objective under specified criteria.



*What is Mechanics ???*

A branch of physical science that **deals with energy and forces and their effect on bodies.**

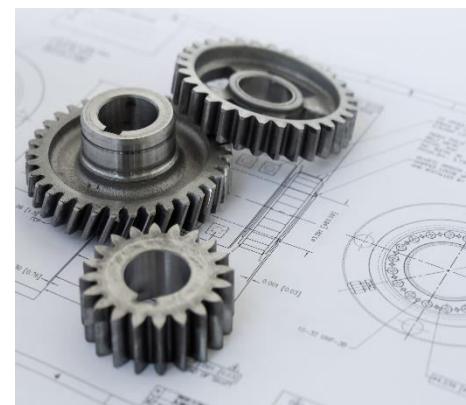
- the practical application of mechanics to the design, construction, or operation of machines or tools.
- As a branch of classical physics, mechanics deals with bodies that are either at rest or are moving with velocities significantly less than the speed of light.



### Course Introduction:

**Engineering mechanics is both a foundation and a framework for most of the branches of engineering.**

Many of the topics in such areas as civil, mechanical, aerospace, and agricultural engineering, and of course engineering mechanics itself, are based upon the subjects of statics and dynamics.

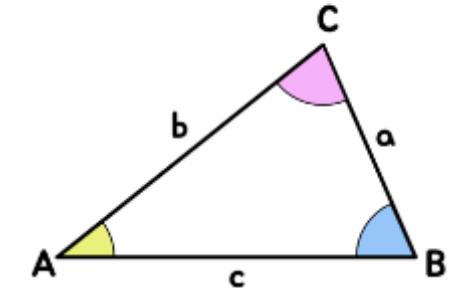


## Course Objectives:

This course will enable students to

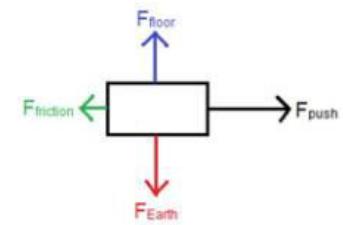
- Understand concepts of Engineering mechanics required for analysis of structures under static loads and predict the effect of loads.
- Understand the concept of Free body diagram to analyze the effect of forces on the structures.
- Analyze the distribution of forces acting on the structures by determining sectional properties and study the external effects of forces on structural members.
- To study and understand the effects of friction on bodies in contact for supporting loads.

### Law of Sines



The Law of Cosines

$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$
$$A + B + C = 180^\circ$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$



Free Body Diagram

# ENGINEERING MECHANICS

## Course Outcomes:

At the end of the course the students will be able to

1. Develop skill to determine resultants and apply conditions of static equilibrium to **plane force systems**.
2. Develop skill to identify and quantify all forces associated with a system.
3. Develop skill to identify, formulate and solve engineering problems.



### Unit 1: Introduction to statics

Mechanics, Basic Concepts, Scalars and Vectors Force Systems - Introduction, Force, Rectangular Components, Moment, Couple, Numerical. Force Systems - Resultants, Numerical

**13 Hours**

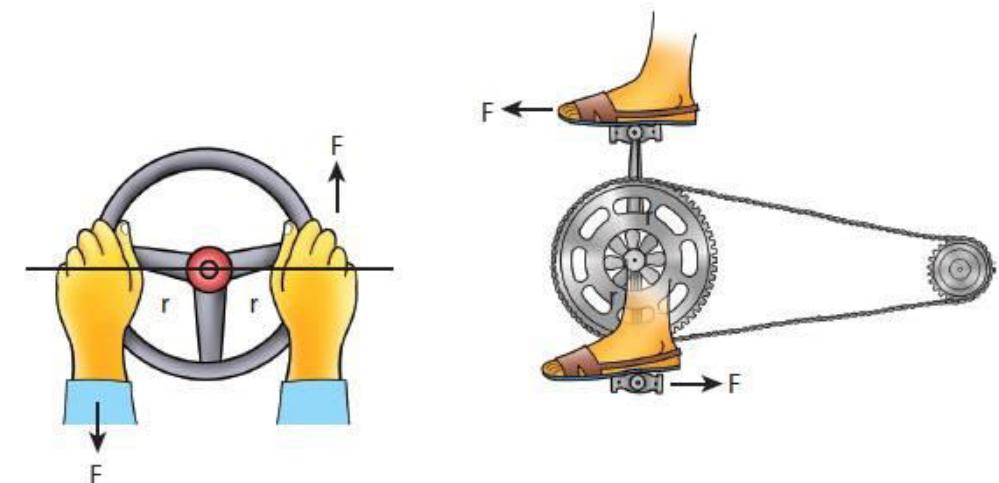
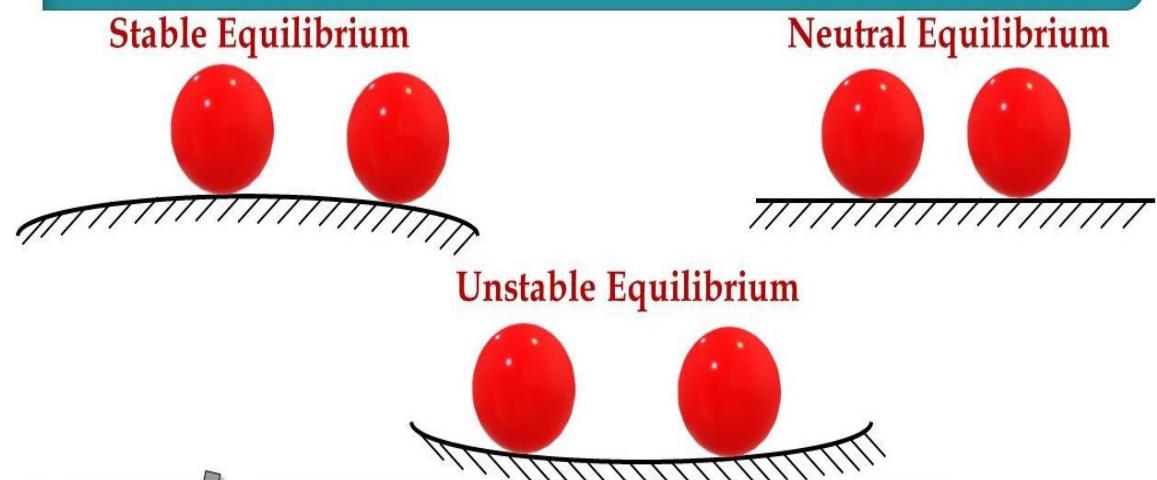


Figure 5.14. Turning effect of Couple

### Unit 2: Equilibrium & Structures

Introduction, Equilibrium in Two Dimensions - System Isolation and the Free-Body Diagram, Equilibrium conditions, Numerical problems.  
Introduction, Plane Trusses, Method of Joints, Numerical problems.

**15 Hours**

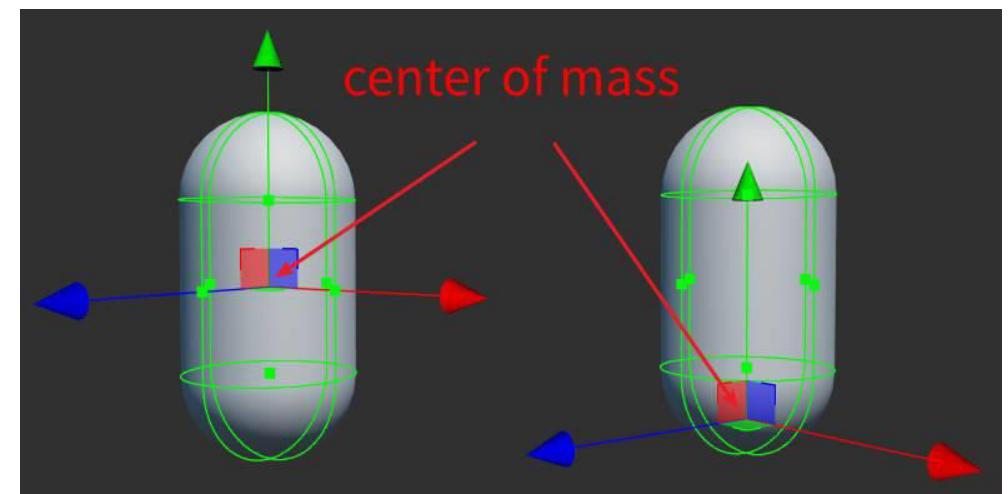


### Unit 3: Distributed Forces

Distributed Forces - Introduction to Centroids of Areas, Centroids of Composite Bodies and figures, Numerical. (Composite area method only).

Beams: External effects, Numerical

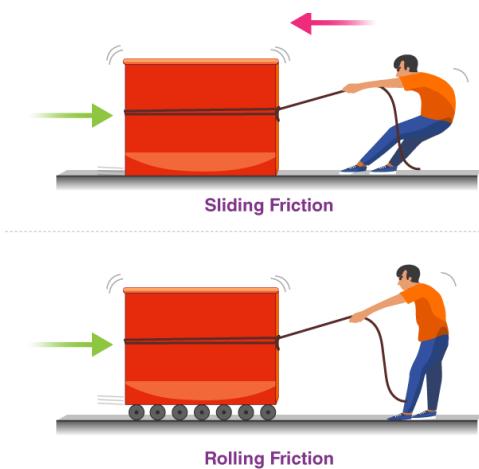
Area Moments of Inertia Introduction, Definitions, Composite areas, Numerical (Composite area method only). **10 Hours**



### Unit 4: Beams & Friction

Introduction, Frictional Phenomena - Types of Friction, Dry Friction, Fluid Friction, Internal Friction, Mechanism of Dry Friction, Static Friction, Kinetic Friction, Friction Angles, Factors Affecting Friction, Numerical problems involving bodies placed on Horizontal surfaces and inclined Surface, Application of Friction in Machines – Wedges, Numerical problems.

**11 Hours**



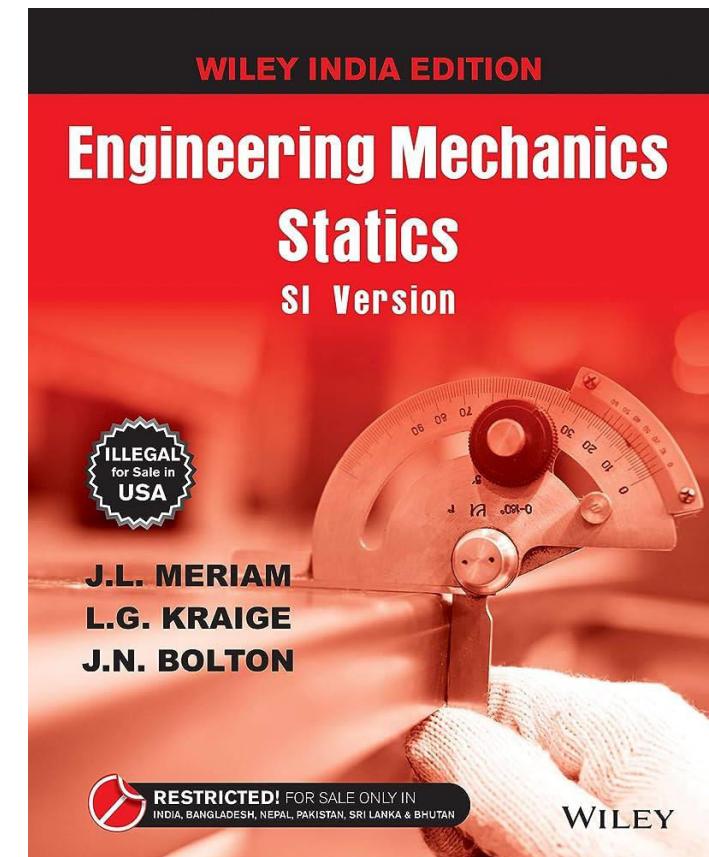
# ENGINEERING MECHANICS

Text Book:

1. "Engineering Mechanics Statics" SI Version J.L. Meriam, L.G. Kraige, J.N. Bolton, Wiley India Edition, eighth edition(original), Reprint: 2018, New Delhi.

Reference:

1. "Engineering Mechanics Statics and Dynamics" R C Hibbeler, Prentice Hall, 2010.
2. "Engineering Mechanics Statics and Dynamics" Irving Herman Shames, Prentice Hall, 1997.



# ENGINEERING MECHANICS

Text Book:

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Reference:

3. "Vector Mechanics for Engineers: Statics", Ferdinand Beer, E. Johnston and David Mazurek, McGraw-Hill Education; 11th edition, 2015.
  
4. Solving Statics Problems in MATLAB, sixth edition by J.L.Meriam and L.G. Kraige John Wiley & sons, Inc., 2007



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### UNIT I: Introduction to statics & Force Systems

CHAPTER 2: Problem No. 2/1 to 2/100 ,

- Excluding 2/29, 2/48, 2/56, 2/58, 2/88, 2/95, 2/99.

**Total– 93**

**47 Problems to be solved in class**

**46 Problems Assignment/Revision/Self study**

### UNIT II: Equilibrium & Structures

**CHAPTER 3:** Problem No. 3/1 to 3/57

- Excluding 3/22, 3/28, 3/29, 3/36, 3/38, 3/41, 3/46, 3/50, 3/51, 3/55, 3/56

**Total – 46**

**23 Problems to be solved in class**

**23 Problems Assignment/Revision/Self study**

**CHAPTER 4:** Problem No. 4/1 to 4/29

- Excluding 4/14, 4/18, 4/20, 4/25, 4/27, 4/28

**Total – 23**

**11 Problems to be solved in class**

**12 Problems Assignment/Revision/Self study**

### UNIT III: Distributed Forces

CHAPTER 5: Problem No. 5/47 to 5/61, 5/101 to 5/111, 5/116, 5/122

- **Excluding 5/54, 5/60, 5/108, 5/109**

**Total – 50**

**25 Problems to be solved in class**

**25 Problems Assignment/Revision/Self study**

ANNEXURE A: Problem No. A/1 to A/19 and A/35 to A/62

- **Excluding A/2, A/5, A/8, A/10, A/11, A/12, A/13, A/14, A/15, A/17, A/47, A/50, A/52, A/61.**

**Total – 33**

**17 Problems to be solved in class**

**16 Problems Assignment/Revision/Self study**

### UNIT IV: Friction

**CHAPTER 6:** Problem No. 6/1 to 6/33, 6/53, 6/55, 6/56, 6/63, 6/66, 6/67, 6/69

- Excluding 6/7, 6/10, 6/21, 6/22, 6/25, 6/26, 6/27, 6/29, 6/30, 6/31, 6/32.

**Total – 29**

15 Problems to be solved in class

14 Problems Assignment/Revision/Self study

# ENGINEERING MECHANICS

Assessment criteria:

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ISA - 50 Marks

Sl. No.	Component	Marks
1.	ISA - 1 – Hybrid for 1 & 2 Units $40/2 = 20$	20
2.	ISA - 2 – Hybrid for 3 & 4 Units $40/2 = 20$	20
3.	Experiential Component	05
4.	Supervised Assignment	05
TOTAL		50

Supervised Assignment

ESA - 50 Marks (100 scaled down to 50)

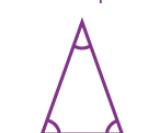
**BOJ**

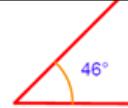
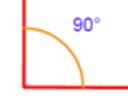


- Angles: Vertically opposite angles, Alternate angles &

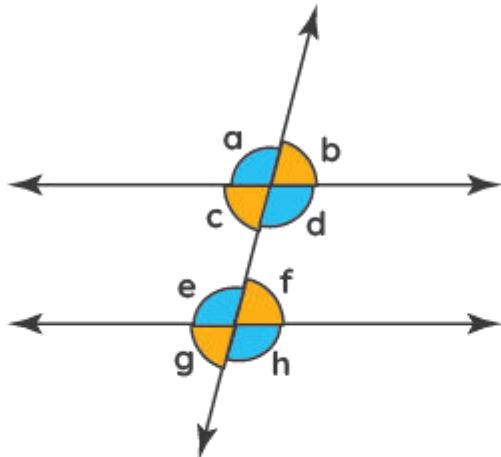
## Corresponding Angle

- Types of Triangles and Properties
- Sine rule and Cosine rule
- Law of Parallelogram properties
- Integration
- Differentiation



Type of Angle	Description	Example
Acute Angle	An angle that is less than $90^\circ$	
Right Angle	An angle that is exactly $90^\circ$	
Obtuse Angle	An angle that is greater than $90^\circ$ and less than $180^\circ$	
Straight Angle	An angle that is exactly $180^\circ$	
Reflex Angle	An angle that is greater than $180^\circ$ and less than $360^\circ$	
Full Angle	An angle that is exactly $360^\circ$	

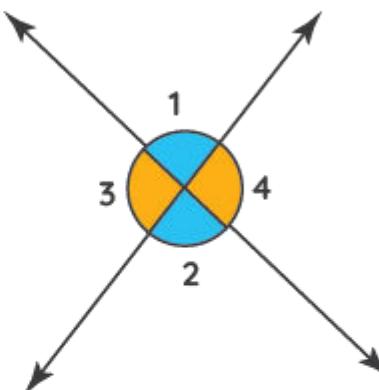
### Alternate, Corresponding and Vertical Angles



$\angle c = \angle f \Rightarrow$  Alternate Interior Angles Pair 1  
 $\angle e = \angle d \Rightarrow$  Alternate Interior Angles Pair 2

$\angle b = \angle g \Rightarrow$  Alternate Exterior Angles Pair 1  
 $\angle a = \angle h \Rightarrow$  Alternate Exterior Angles Pair 2

$\angle b = \angle f \Rightarrow$  Corresponding Angles Pair 1  
 $\angle a = \angle e \Rightarrow$  Corresponding Angles Pair 2  
 $\angle g = \angle c \Rightarrow$  Corresponding Angles Pair 3  
 $\angle h = \angle d \Rightarrow$  Corresponding Angles Pair 4

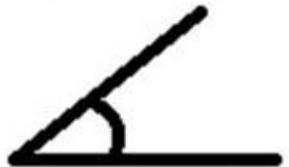


$\angle 1 = \angle 2 \Rightarrow$  Vertical Angles Pair 1  
 $\angle 3 = \angle 4 \Rightarrow$  Vertical Angles Pair 2

# ENGINEERING MECHANICS

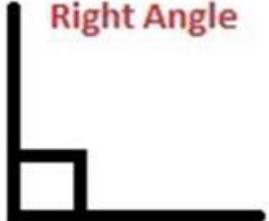
## Prerequisites:

Acute Angle



Less Than 90 Degree

Right Angle



Exact 90 Degree

Obtuse Angle



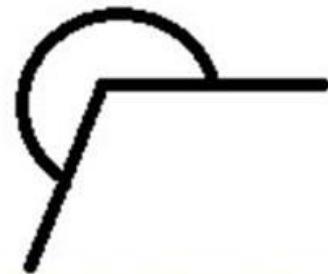
Greater than 90 degree and less than 180 degree.

Straight Angle



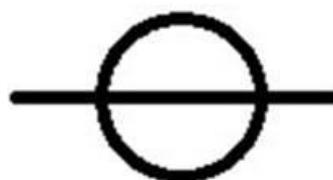
Exact 180 degree.

Reflex Angle



Greater than 180 degree.

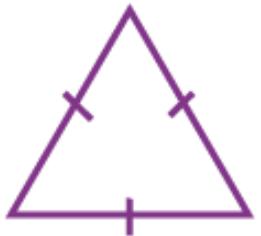
Full Rotation



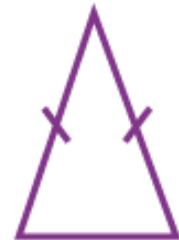
Exact 360 degree.

# ENGINEERING MECHANICS

## Prerequisites:



**Equilateral**  
All three sides  
are equal



**Isosceles**  
Two sides  
are equal



**Scalene**  
No sides are  
equal



**Acute**  
All angles acute



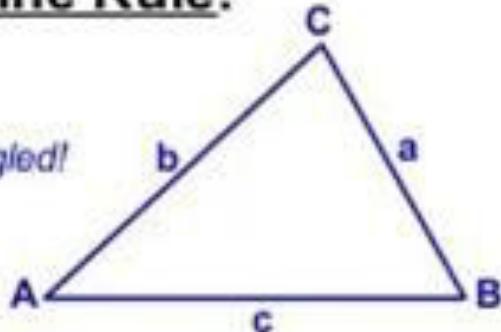
**Right**  
One right angle



**Obtuse**  
One obtuse angle

### The Sine Rule:

*Not right-angled!*



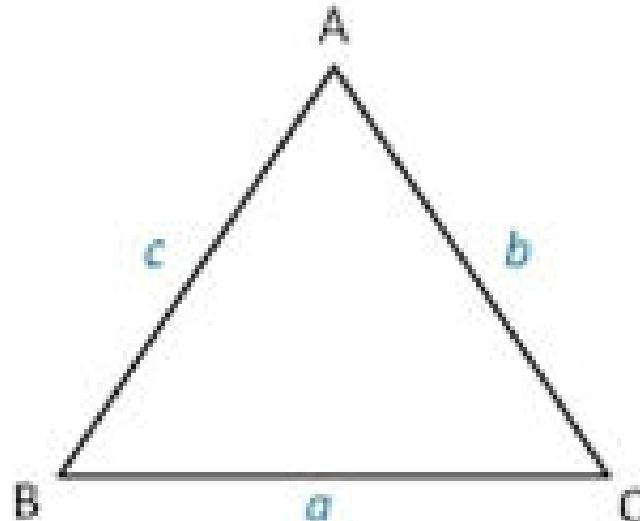
In any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### Cosine Law



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

# ENGINEERING MECHANICS

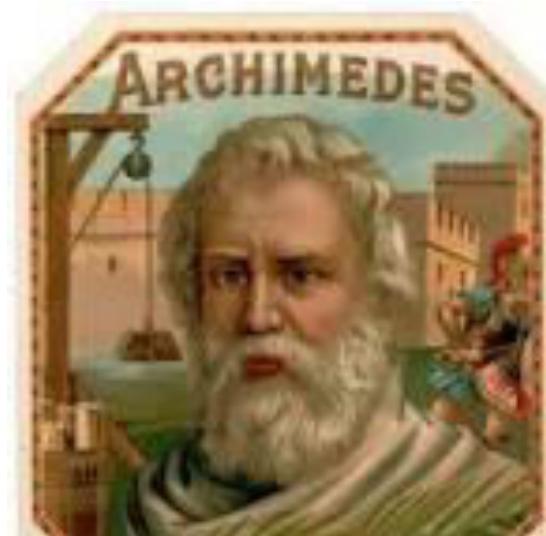
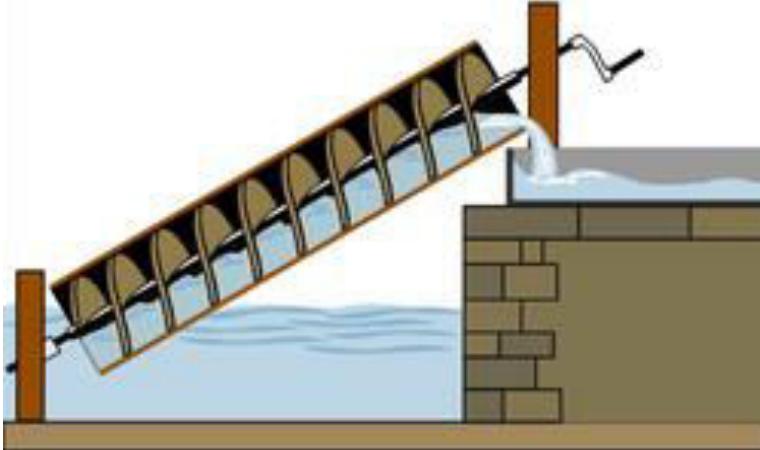
## Prerequisites:



*Mechanics is the oldest of the Physical science which deals with the effects of forces on objects.*

The earliest recorded writings in Mechanics

**Archimedes (287 -212 BC) Greek Mathematician : on the principle of lever and the principle of buoyancy.**



The earliest recorded writings in Mechanics

Stevinus (1548 – 1620)

A great Mathematician

*Laws of vector combination of forces*  
*Most of the principles of statics*



The earliest recorded writings in Mechanics

**Galileo Galilei (1564 – 1642)** An Italian Mathematician,  
Astronomer and Physicist

Father of Physics

*Major contribution to finding  
the motion of bodies and  
also developed telescope.*

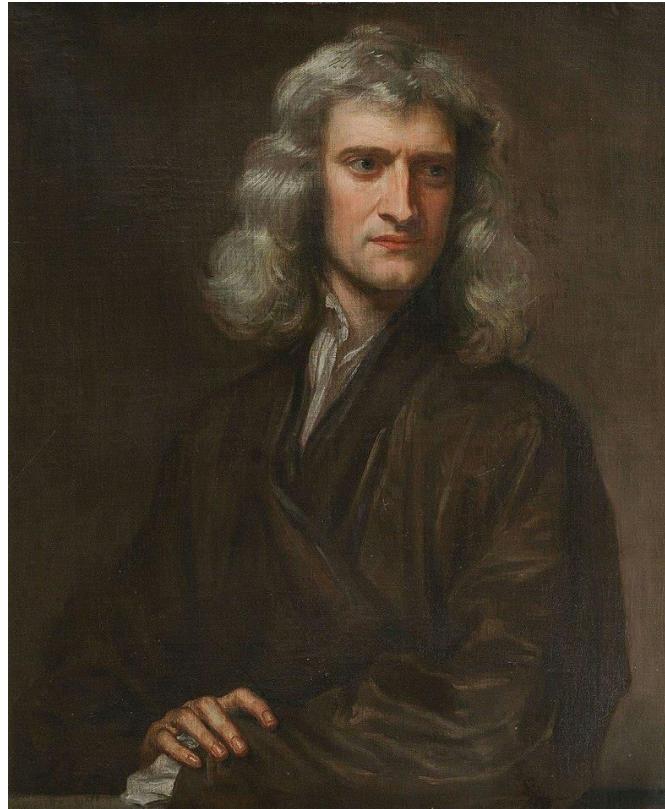


The earliest recorded writings in Mechanics

Sir Isaac Newton (1642 – 1727)

English Mathematician

*Formulation of the laws of motion,  
Law of gravitation*



<sup>^</sup>Da Vinci, Varignon, Euler, Dalembert, Lagrange, Laplace and others.

Mechanics can be logically divided into 2 parts.

- *Statics*
- *Dynamics*

***Statics deals with the equilibrium of bodies under the action of forces.***

***Dynamics which deals with the motion of bodies.***

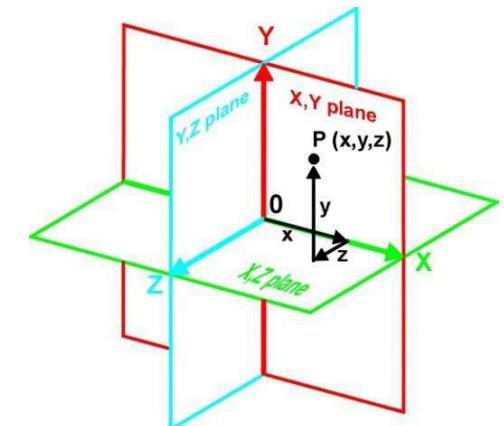
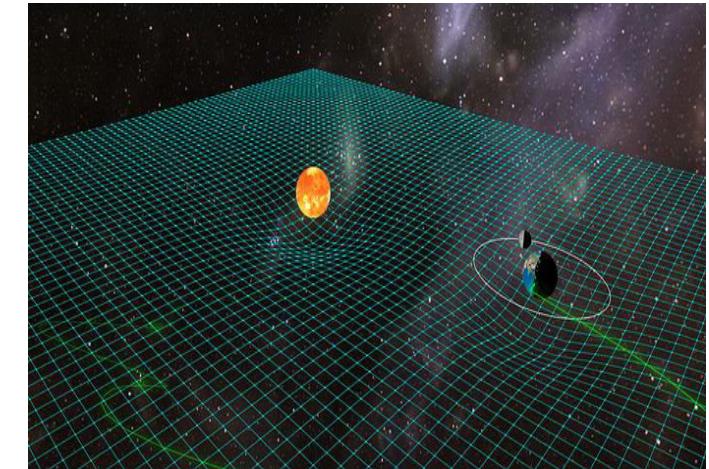
### Basic Concepts:

The following concepts and definitions are required to study Mechanics.

- Space:
- Time:
- Mass:
- Force:
- Rigid body:

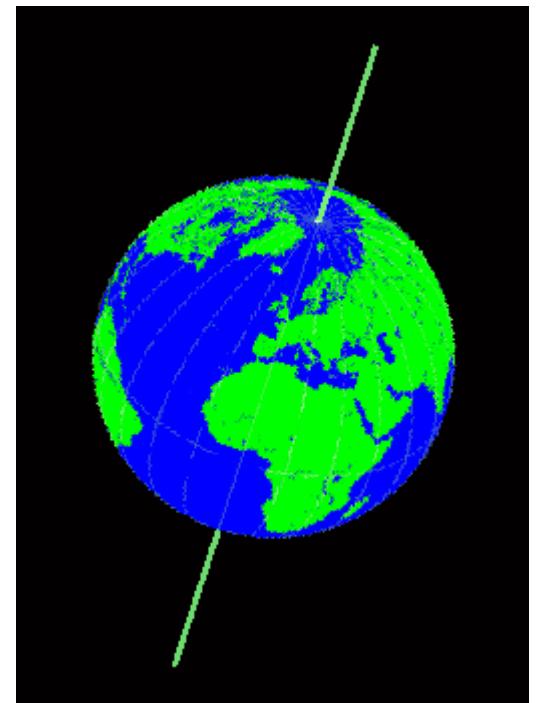
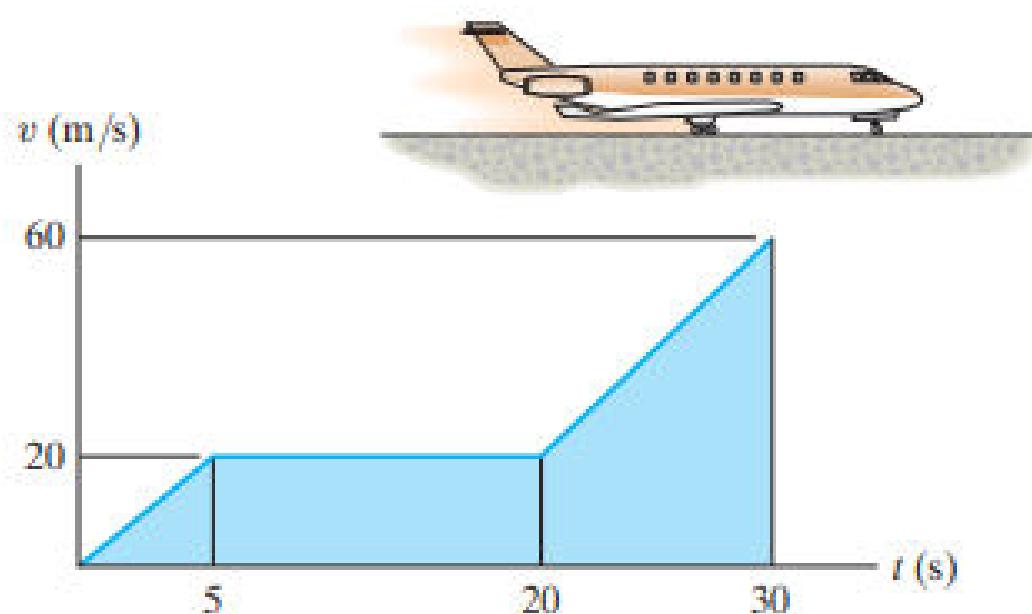
- **Space:**

- Space is the **geometric region occupied by bodies** whose positions are described by linear and angular measurements relative to a coordinate system.
- For **3 dimensional problems**, three independent co-ordinates are needed.
- For **2 dimensional problems**, two co-ordinates are required.

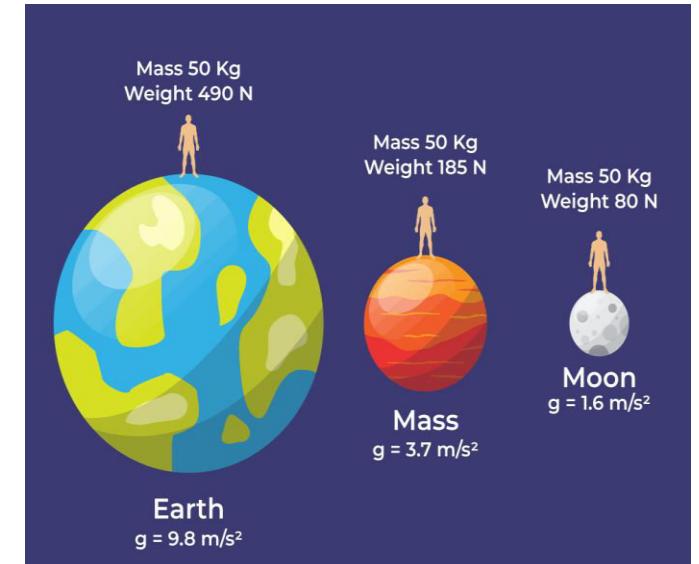


### Time:

Time is the **measure of the succession of events and is a basic quantity in dynamics.** *Time is not directly involved in the analysis of statics problems.*



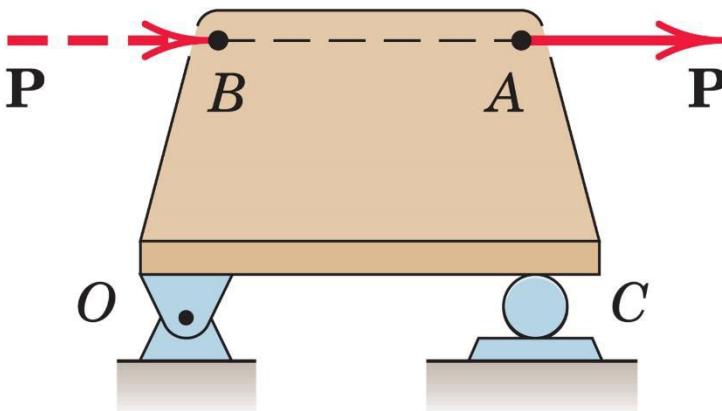
- Mass:
- Mass is a **measure of the inertia of a body, which is its resistance to a change of velocity.**
- Mass can also be thought of as the **quantity of matter in a body.**
- *The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in Statics.*



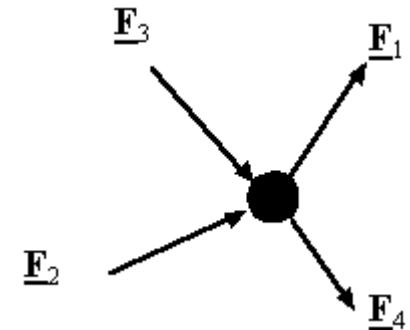
- Force:
- Force is the action of one body on another.
- A force tends to move a body in the direction of its action.

### Characteristics of force

1. Magnitude
2. Direction
3. Point of application
4. Line of action



- A particle:
- It is a body of negligible dimensions.
- In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point.



- **Rigid Body:**
- A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand.
- Example: The calculation of the tension in the cable which supports the boom of a mobile crane under load is unaffected by the small internal deformation in the structural member of the boom.



## Scalars and Vectors:

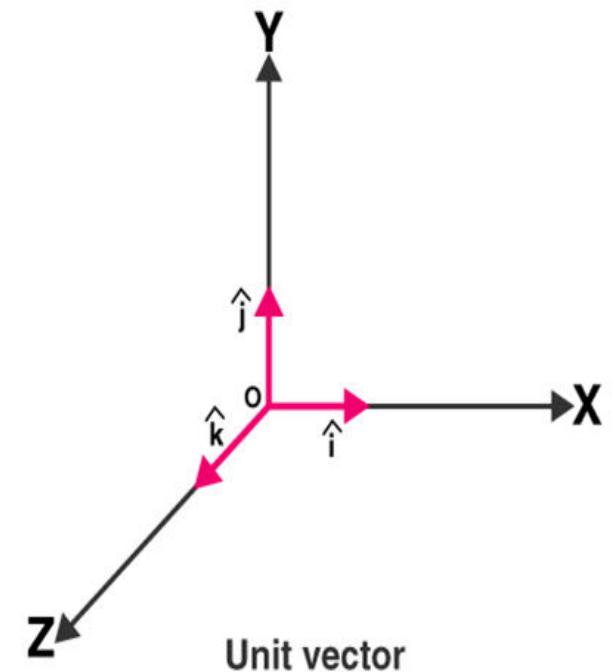
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Scalar quantities: are those with which only a magnitude is associated.

Example: Time, Volume, Density, Speed, Energy and Mass

Vector quantities: Are those which possess direction as well as magnitude and **must obey the parallelogram law of addition.**

Example: Displacement, Velocity, Acceleration, Force, Moment and Momentum.



A vector that has a magnitude of 1 is a unit vector

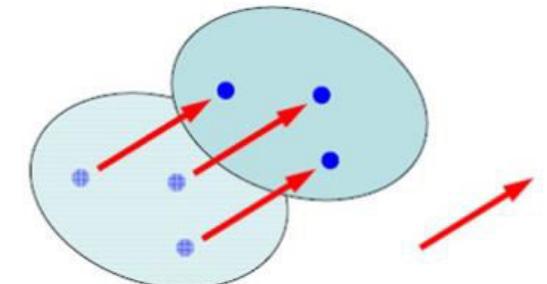
## Scalars and Vectors:

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Vectors representing physical quantities can be classified as free, sliding or fixed .

**A free Vector is one whose action is not confined to or associated with a unique line in space.**

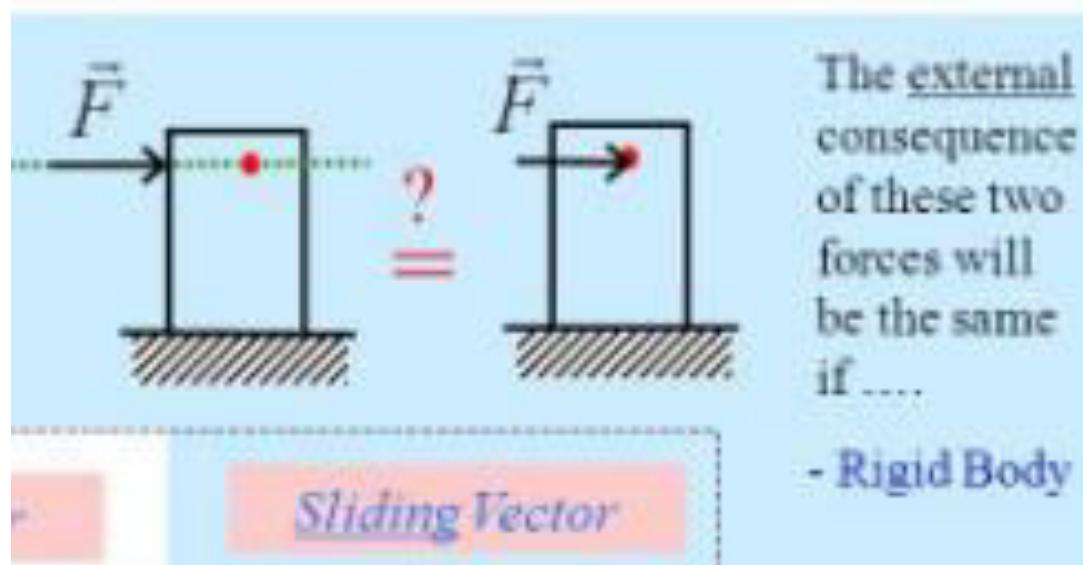
Example: If a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector.



## Scalars and Vectors:

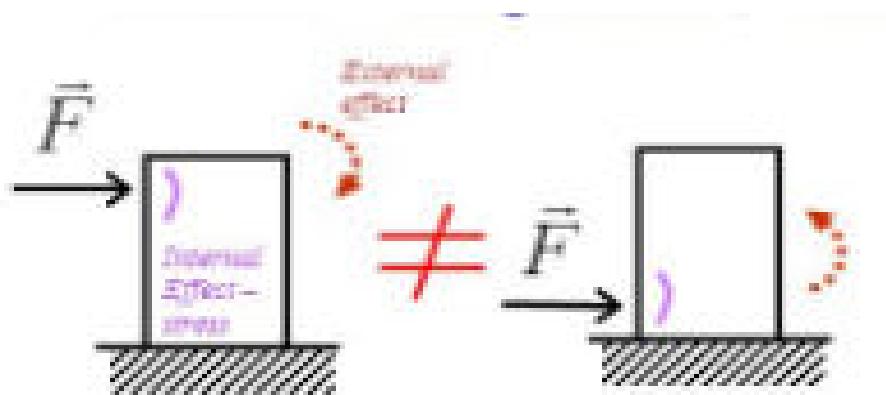
**A sliding vector:** has a unique line of action in space but not a unique point of application.

Example: when external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole.



## Scalars and Vectors:

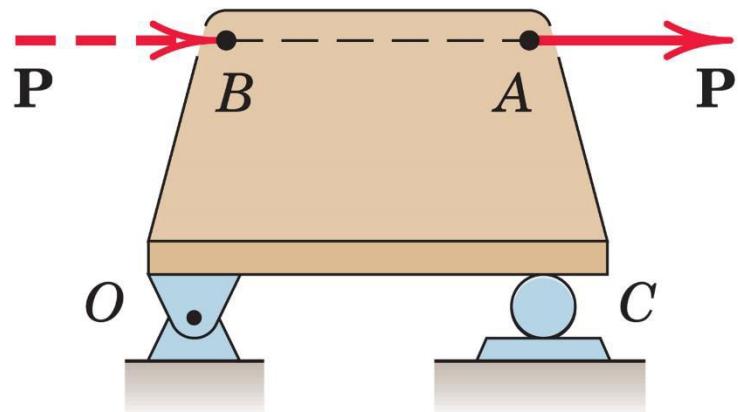
**A fixed Vector:** Is one for which a unique point of application is specified



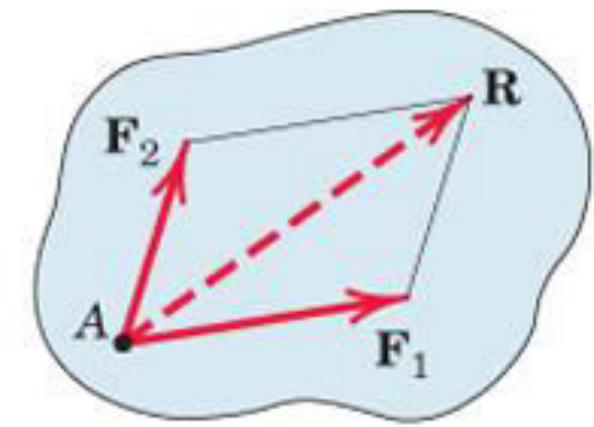
- Forces are classified as either **contact forces or body forces**.
- A **contact force is produced by direct physical contact**: example : the force exerted on a body by a supporting surface.
- **Body force is generated by virtue of the position of a body within a force field such as a gravitational, electric or magnetic field**. Example of body force : weight of a person

- Principle of Transmissibility:

*A force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts.*

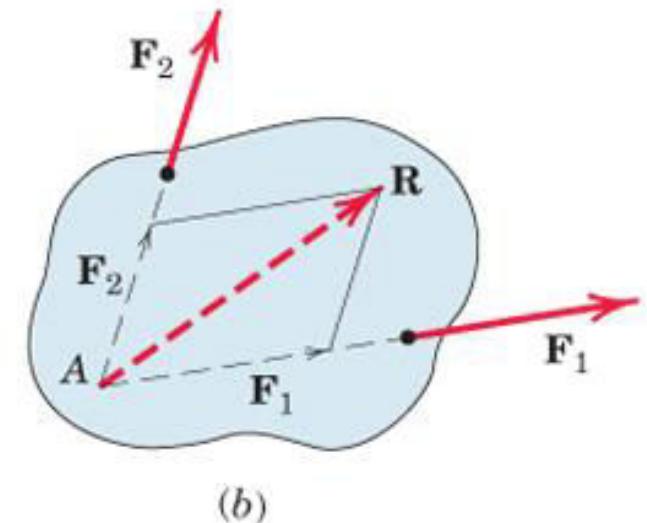


- Two or more forces are said to be concurrent at a point if their lines of action intersect at that point. **The forces F<sub>1</sub> and F<sub>2</sub> shown have a common point of application and are concurrent at the point A.**
- Thus they can be added using the parallelogram law in their common plane to obtain their resultant R. The resultant lies in the same plane as F<sub>1</sub> and F<sub>2</sub>.

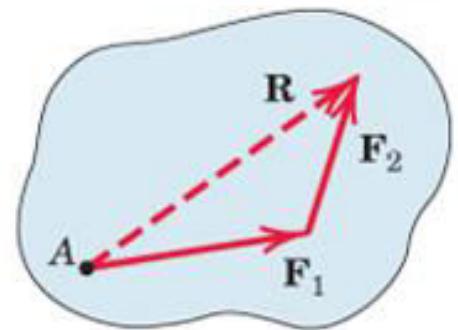


(a)

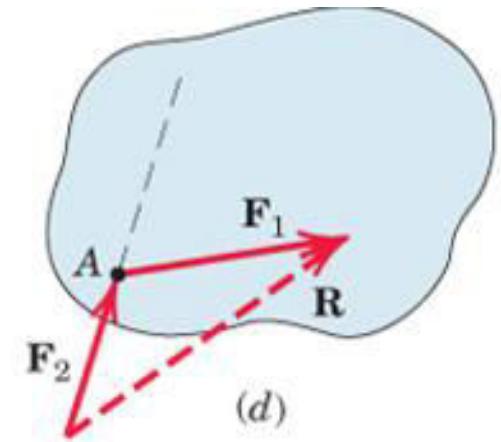
- Suppose the two concurrent forces lie in the same plane but are applied at two different points as in figure.
- By the principle of transmissibility, we can move them along their lines of action and complete their vector sum  $R$  at the point of concurrency A.
- We can replace  $F_1$  and  $F_2$  with the resultant  $R$  without altering the external effect on the body upon which they act.



- We can also use triangle law to obtain  $R$ , but we need to move the line of action of one of the forces, as shown in figure below.
- We can predict the magnitude and direction of  $R$ , but the correct line of action can not be determined. Resultant obtained so does not pass through A. So, this type of combination has limited application.



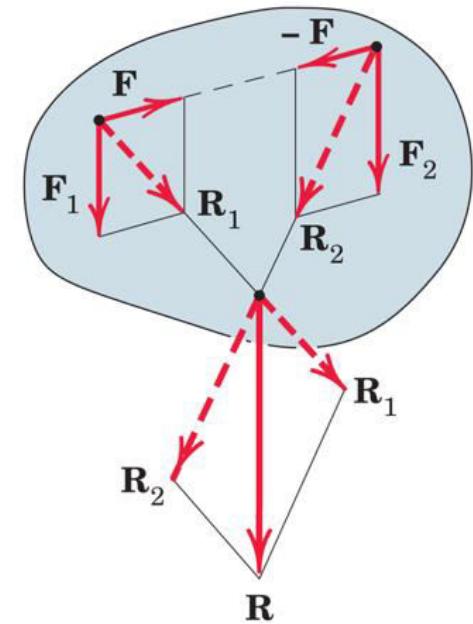
(c)

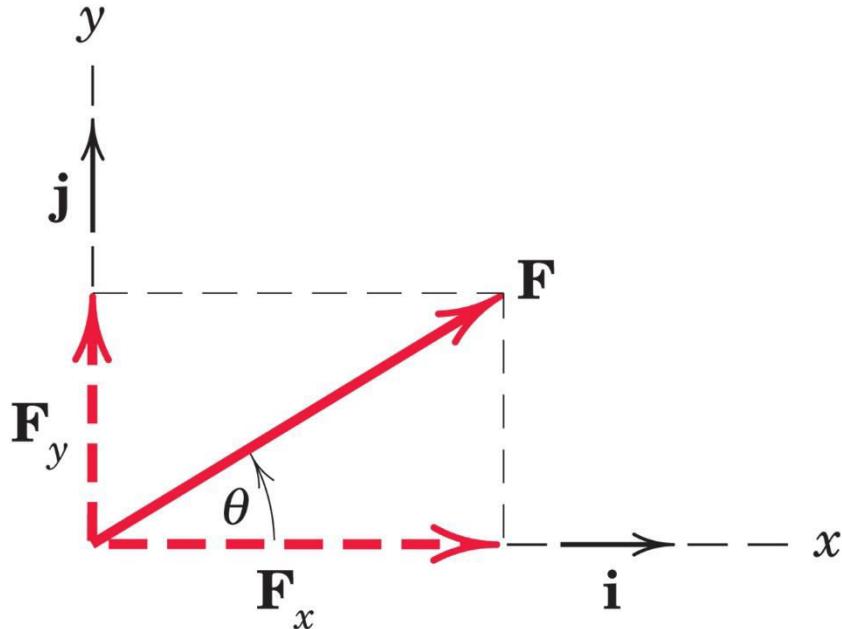


(d)

## Special case of Vector addition:

- To obtain the resultant when the **two forces  $F_1$  and  $F_2$  are parallel** as shown in figure below, we can use a special case of addition.
- Two vectors are combined by **first adding two equal, opposite and collinear forces  $F$  and  $-F$  of convenient magnitude.**
- This produces no external effect on the body.
- Adding  $F_1$  and  $F$  to produce  $R_1$  and combining with the other sum  $R_2$  of  $F_2$  and  $-F$  yields resultant  $R$  which is correct in magnitude, direction and line of action.





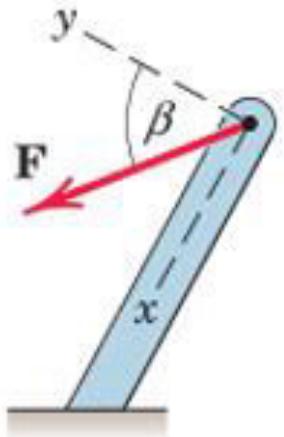
$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$

$$F_x = F \cos \theta \quad F = \sqrt{F_x^2 + F_y^2}$$

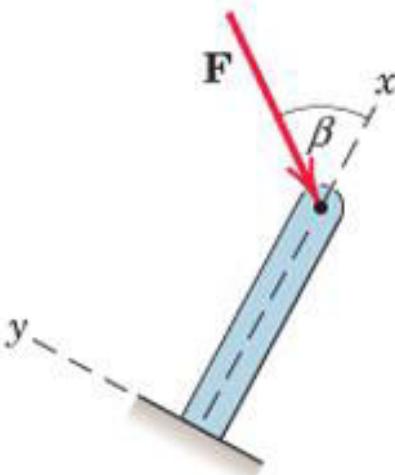
$$F_y = F \sin \theta \quad \theta = \tan^{-1} \frac{F_y}{F_x}$$

**F<sub>x</sub>** and **F<sub>y</sub>** are components of **F** in the x and y directions.



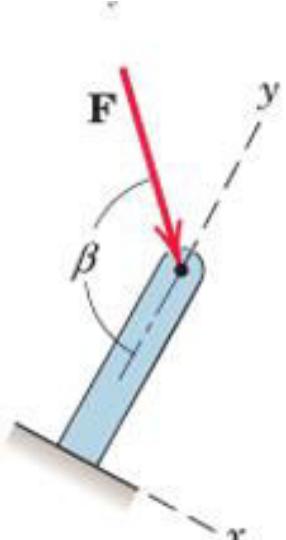
$$F_x = F \sin \beta$$

$$F_y = F \cos \beta$$



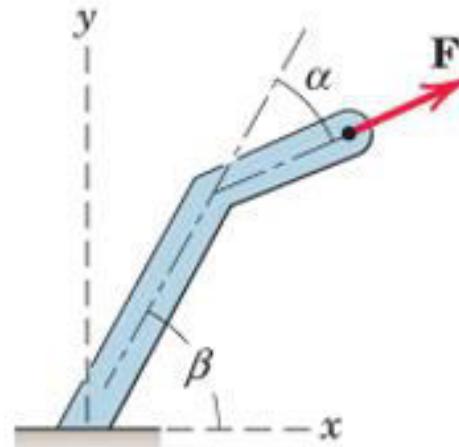
$$F_x = -F \cos \beta$$

$$F_y = -F \sin \beta$$



$$F_x = F \sin(\pi - \beta)$$

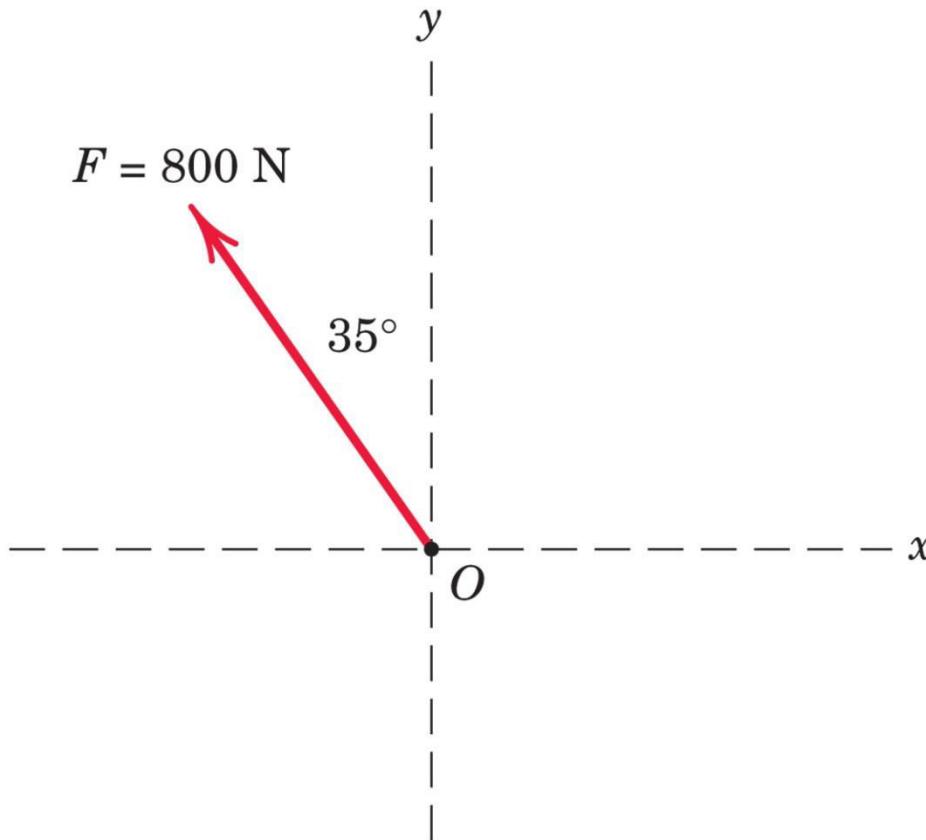
$$F_y = -F \cos(\pi - \beta)$$



$$F_x = F \cos(\beta - \alpha)$$

$$F_y = F \sin(\beta - \alpha)$$

2/1) The force  $\mathbf{F}$  has a magnitude of 800 N. Express  $\mathbf{F}$  as a vector in terms of the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ . Identify the  $x$  and  $y$  scalar components of  $\mathbf{F}$ .



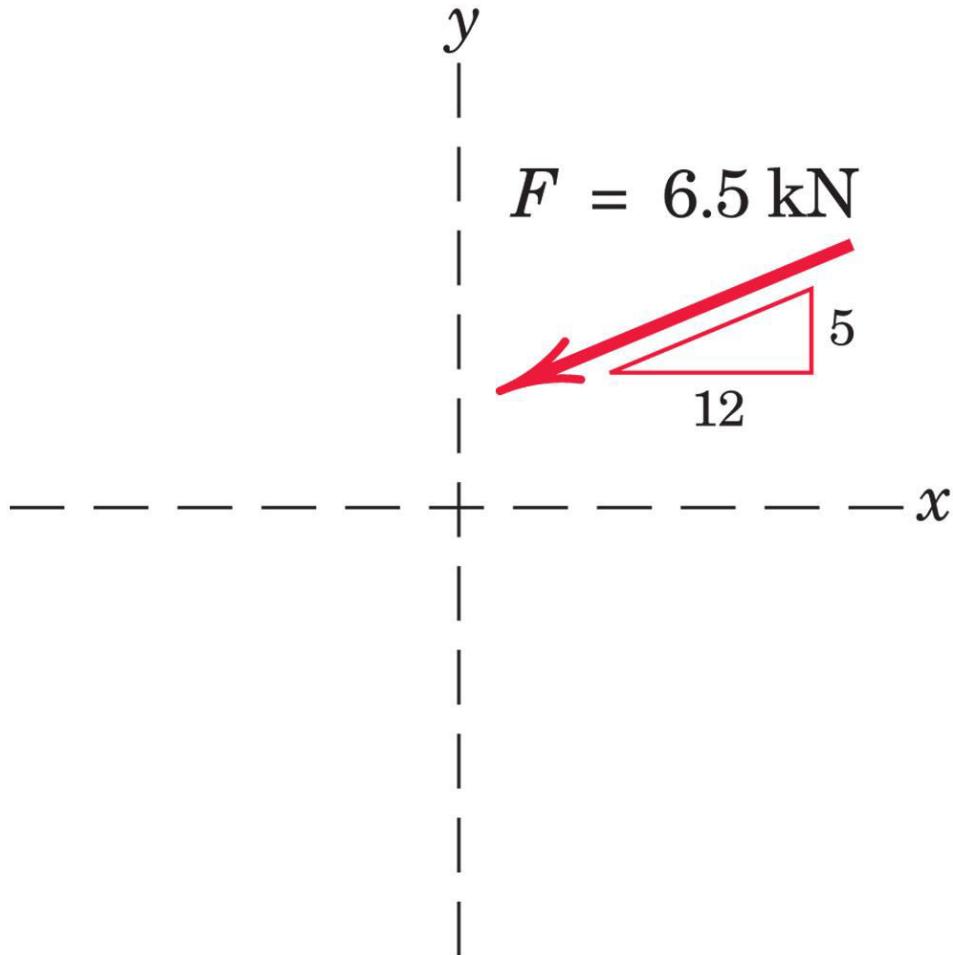
$$\begin{cases} F_x = -800 \sin 35^\circ = -459 \text{ N} \\ F_y = 800 \cos 35^\circ = 655 \text{ N} \end{cases}$$

$$\underline{\mathbf{F} = -459\mathbf{i} + 655\mathbf{j} \text{ N}}$$

# ENGINEERING MECHANICS

## Force Systems - Numerical

2/3) The slope of the 6.5-kN force  $F$  is specified as shown in the figure. Express  $F$  as a vector in terms of the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ .

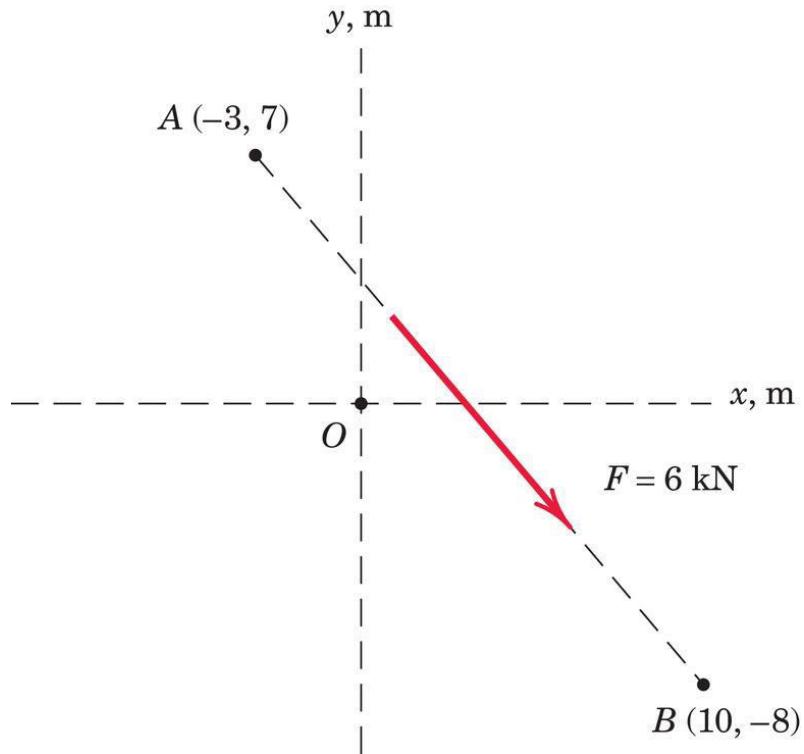


$$\begin{aligned} \mathbf{F} &= 6.5 \left( -\frac{12}{13} \mathbf{i} - \frac{5}{13} \mathbf{j} \right) \\ &= -4.6 \mathbf{i} - 2.5 \mathbf{j} \text{ kN} \end{aligned}$$

# ENGINEERING MECHANICS

## Force Systems - Numerical

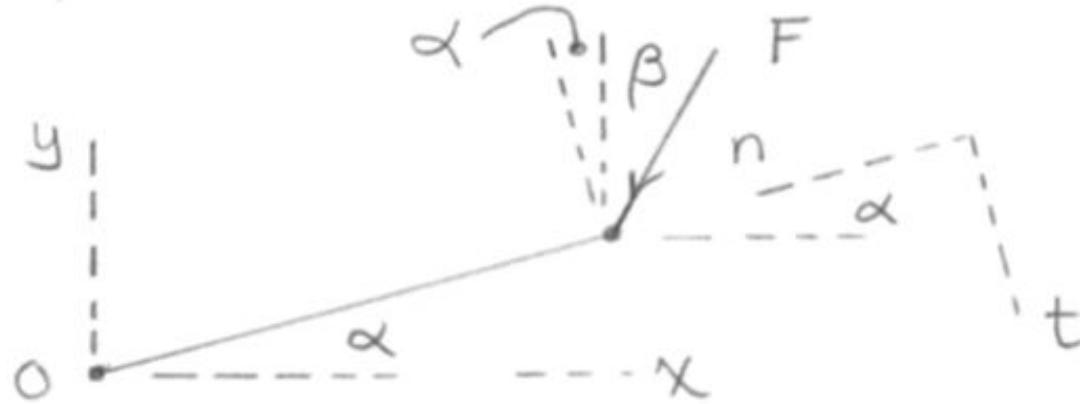
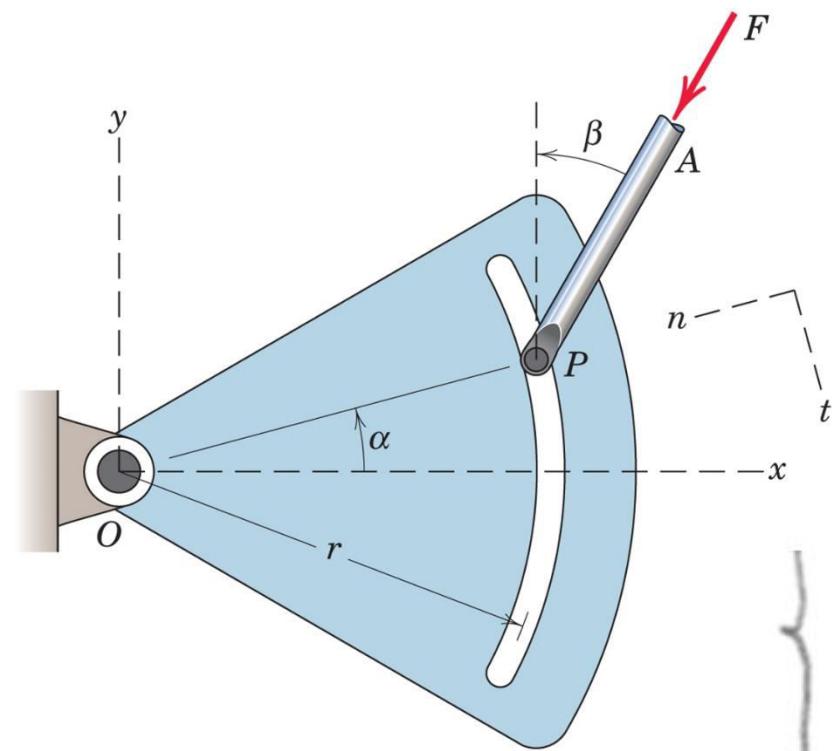
2/4) The force  $\mathbf{F}$  has a magnitude of 6 kN and has the indicated line of action. Write the unit vector  $\mathbf{n}$  associated with  $\mathbf{F}$  and use  $\mathbf{n}$  to determine the  $x$  and  $y$  scalar components of  $\mathbf{F}$ .



SCALAR COMPONENTS:

$$\begin{cases} F_x = F_{n_x} = 6(0.655) \rightarrow \underline{F_x = 3.93 \text{ kN}} \\ F_y = F_{n_y} = 6(-0.756) \rightarrow \underline{F_y = -4.53 \text{ kN}} \end{cases}$$

2/5) The control rod AP exerts a force  $F$  on the sector as shown. Determine both the x-y and the n-t components of the force.

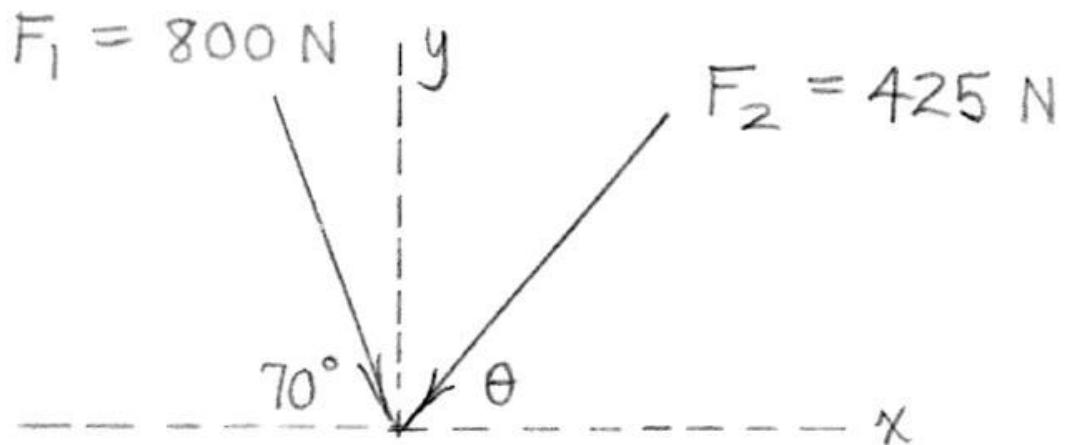
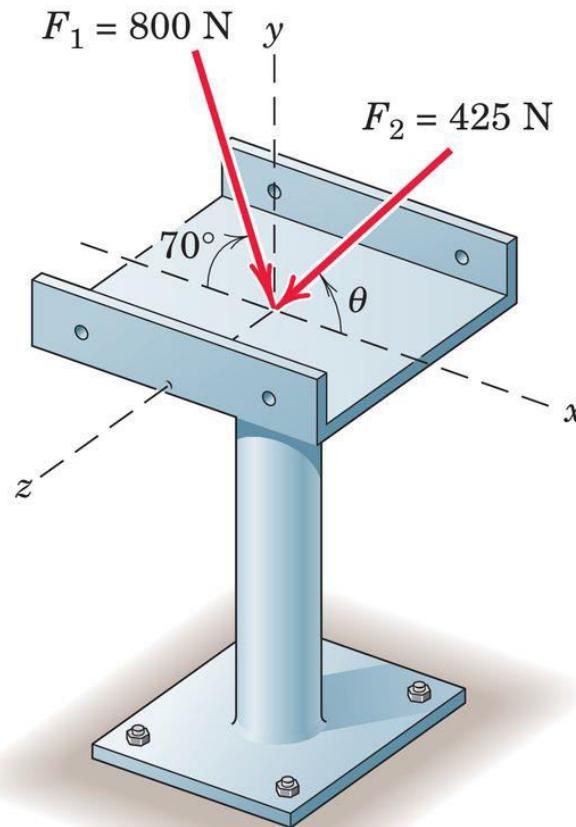


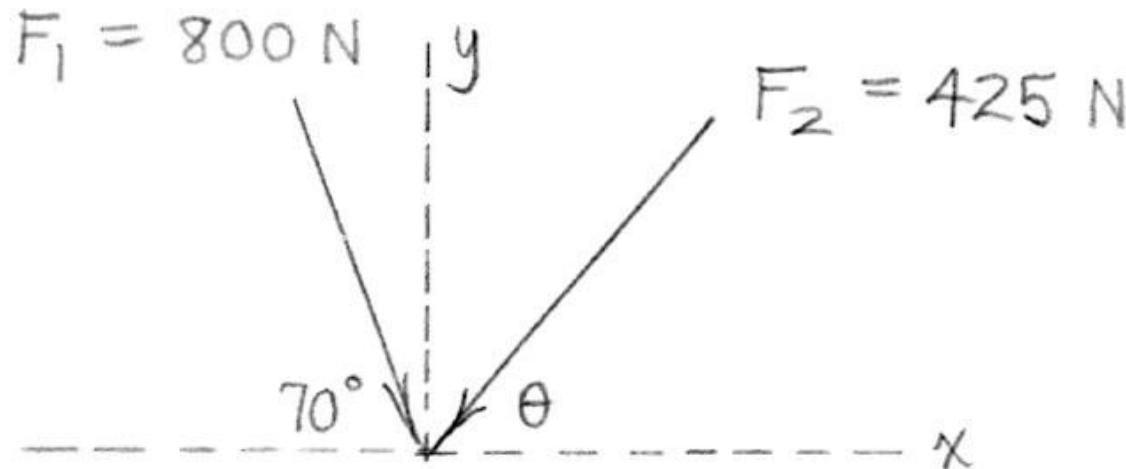
$$\begin{cases} F_x = -F \sin \beta \\ F_y = -F \cos \beta \end{cases} \quad \begin{cases} F_n = F \sin(\alpha + \beta) \\ F_t = F \cos(\alpha + \beta) \end{cases}$$

# ENGINEERING MECHANICS

## Force Systems - Numerical

2/6) Two forces are applied to the construction bracket as shown. Determine the angle which makes the **resultant of the two forces vertical**. Determine the magnitude R of the resultant.





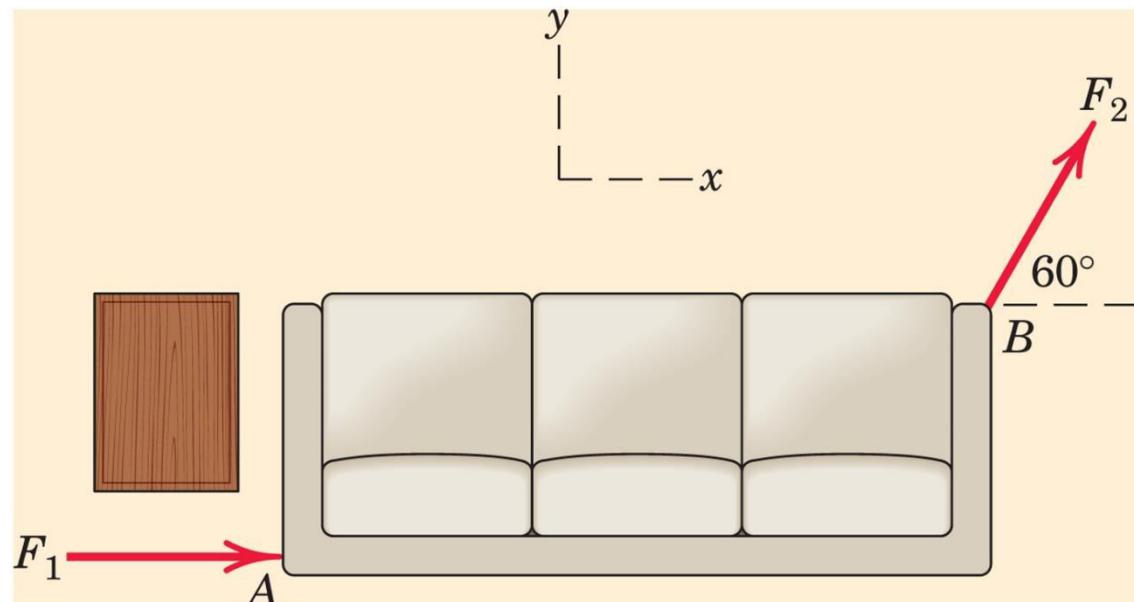
$$R_x = \sum F_x = 800 \cos 70^\circ - 425 \cos \theta = 0$$
$$\underline{\theta = 49.9^\circ}$$

$$R_y = \sum F_y = -800 \sin 70^\circ - 425 \sin 49.9^\circ$$
$$= -1077 \text{ N}$$

$$\text{So } R = 1077 \text{ N}$$

2/7) Two individuals are attempting to relocate a sofa by applying forces in the indicated directions. If  $F_1$  is 500 N and  $F_2$  is 350 N, determine the vector expression for the resultant  $R$  of the two forces.

Then determine the magnitude of the resultant and the angle which it makes with the positive x-axis.



2/7)

$$\left\{ \begin{array}{l} \underline{R} = (500 + 350 \cos 60^\circ) \underline{i} + 350 \sin 60^\circ \underline{j} \\ \underline{R} = 675 \underline{i} + 303 \underline{j} \text{ N} \end{array} \right.$$

$$R = \sqrt{675^2 + 303^2} \longrightarrow \underline{R} = 740 \text{ N}$$

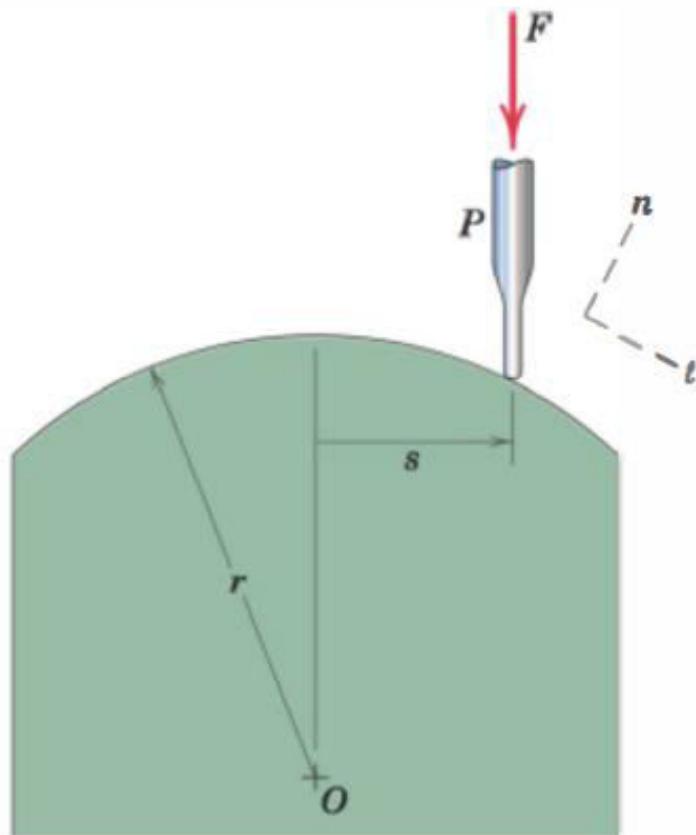
$$\theta_k = \cos^{-1} \left( \frac{R_x}{R} \right) = \cos^{-1} \left( \frac{675}{740} \right)$$

$$\underline{\theta_x = 24.2^\circ \text{ ABOVE } +k \text{ AXIS}}$$

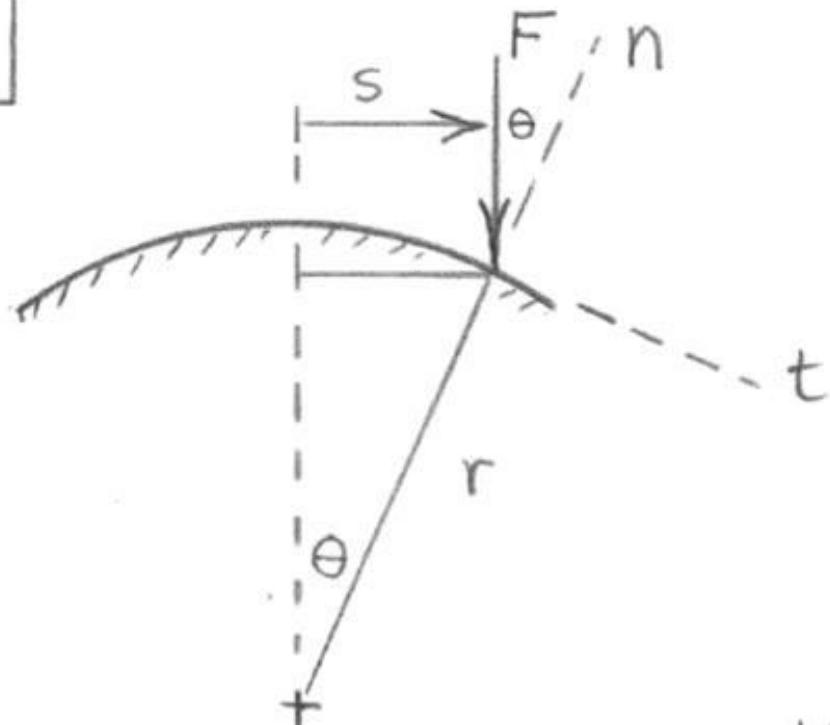
# ENGINEERING MECHANICS

## Force Systems - Numerical

2/8 A small probe P is gently forced against the circular surface with a vertical force F as shown. Determine the n- and t-components of this force as functions of the horizontal positions.

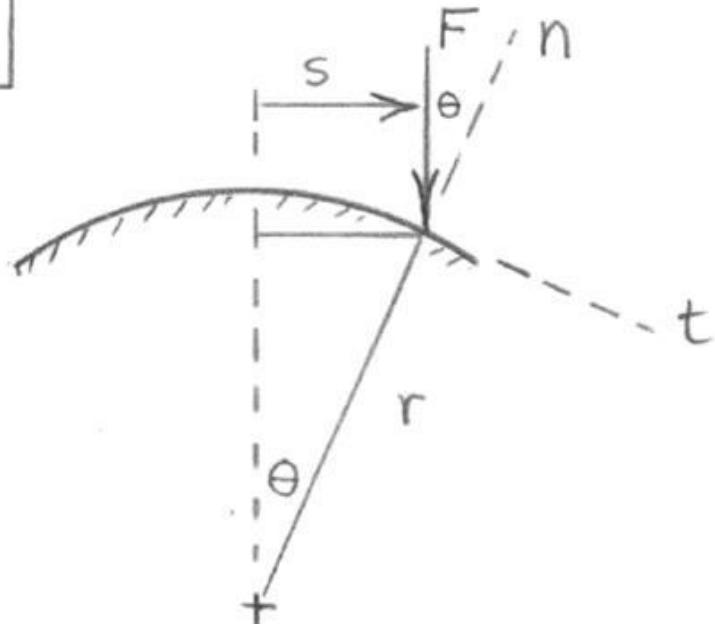


2/8



$$\theta = \sin^{-1}\left(\frac{s}{r}\right)$$

2/8

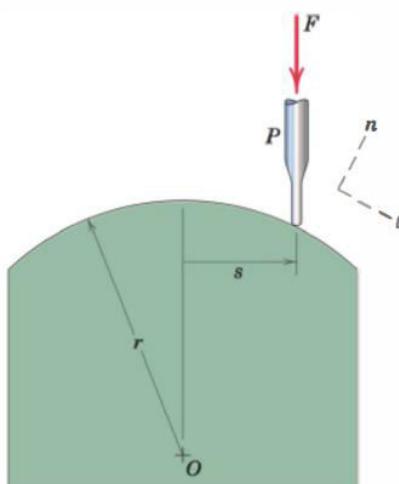


$$\theta = \sin^{-1}\left(\frac{s}{r}\right)$$

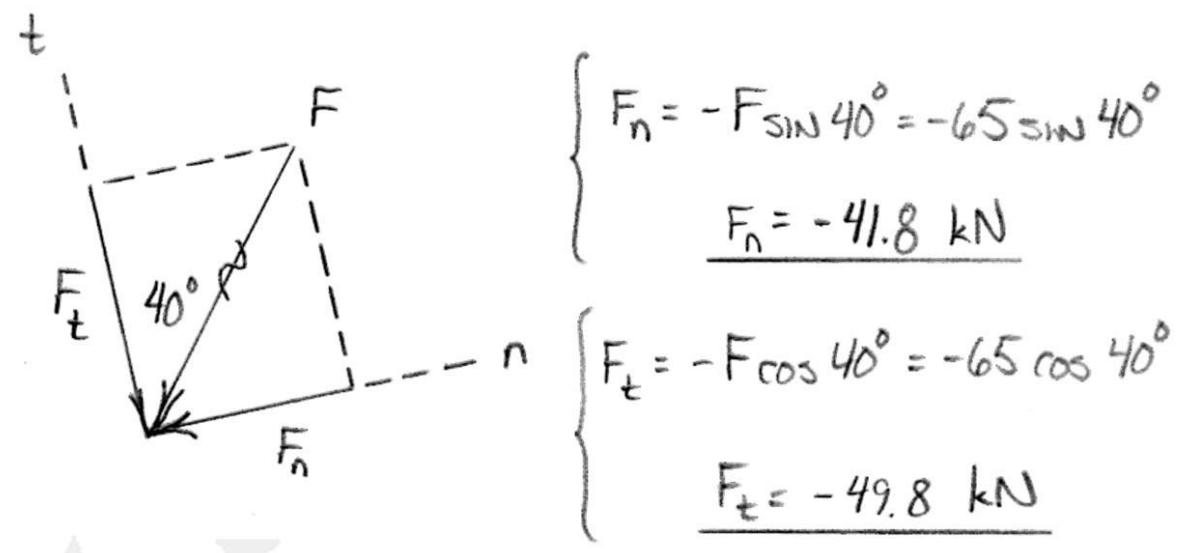
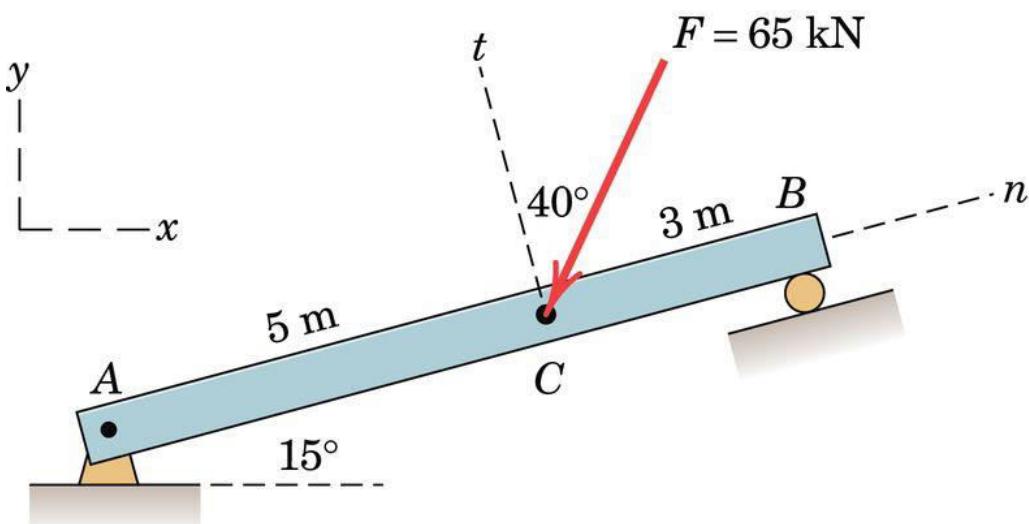
$$F_t = F \sin \theta = F \sin \left[ \sin^{-1} \left( \frac{s}{r} \right) \right] = \frac{Fs}{r}$$

$$F_n = -F \cos \theta = -F \cos \left[ \sin^{-1} \left( \frac{s}{r} \right) \right]$$

$$= -\frac{F \sqrt{r^2 - s^2}}{r}$$

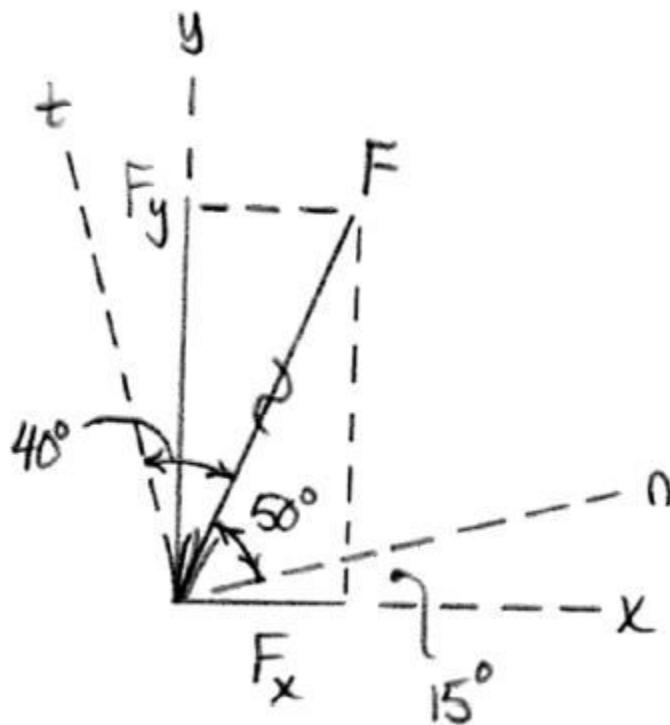
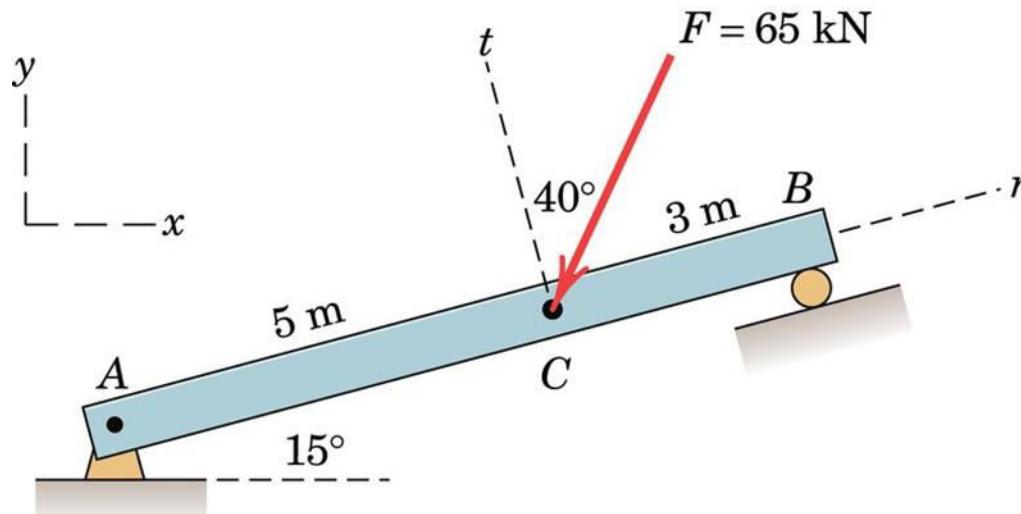


2/10) Determine the x-y and n-t components of the 65-kN force F acting on the simply-supported beam.



# ENGINEERING MECHANICS

## Force Systems - Numerical



$$\left\{ \begin{array}{l} F_x = -F \cos 65^\circ = -65 \cos 65^\circ \\ F_x = -27.5 \text{ kN} \end{array} \right.$$

$$\underline{F_x = -27.5 \text{ kN}}$$

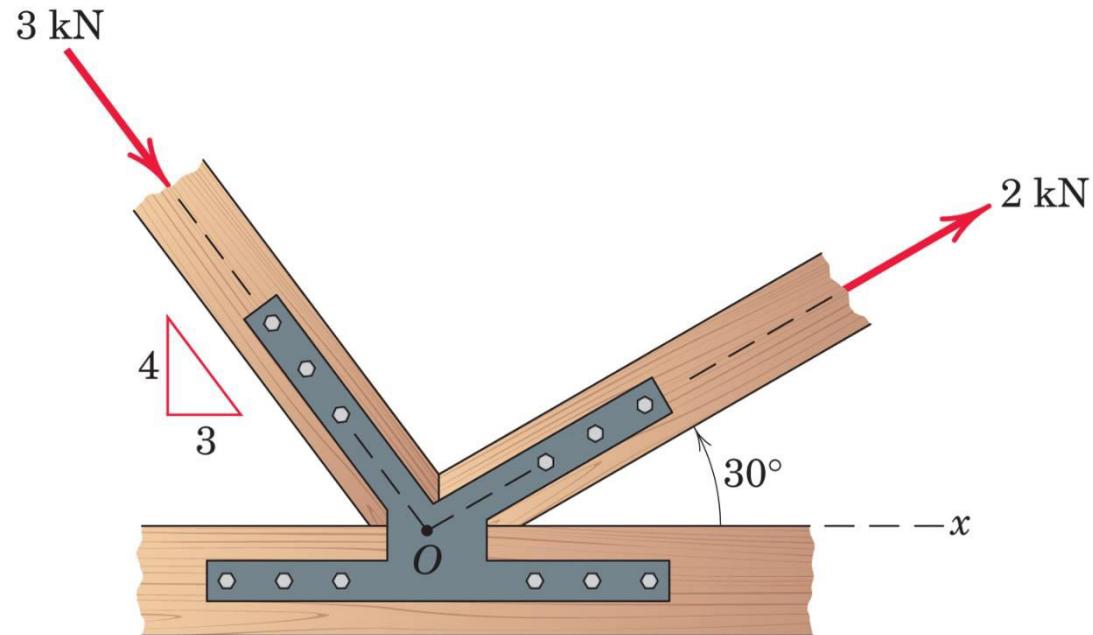
$$\left\{ \begin{array}{l} F_y = -F \sin 65^\circ = -65 \sin 65^\circ \\ F_y = -58.9 \text{ kN} \end{array} \right.$$

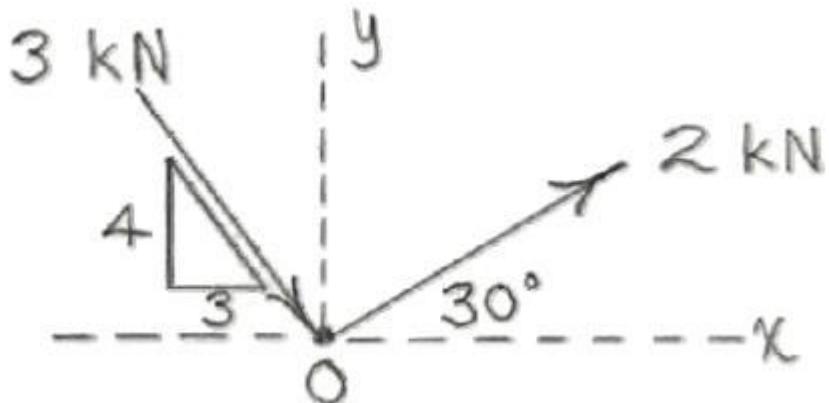
$$\underline{F_y = -58.9 \text{ kN}}$$

# ENGINEERING MECHANICS

## Force Systems - Numerical

2/11) The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant R of the two forces and the angle  $\theta$  which R makes with the positive x-axis.





$$\left\{ \begin{array}{l} R_x = \sum F_x = +3\left(\frac{3}{5}\right) + 2 \cos 30^\circ = 3.53 \text{ kN} \\ R_y = \sum F_y = -3\left(\frac{4}{5}\right) + 2 \sin 30^\circ = -1.4 \text{ kN} \end{array} \right.$$

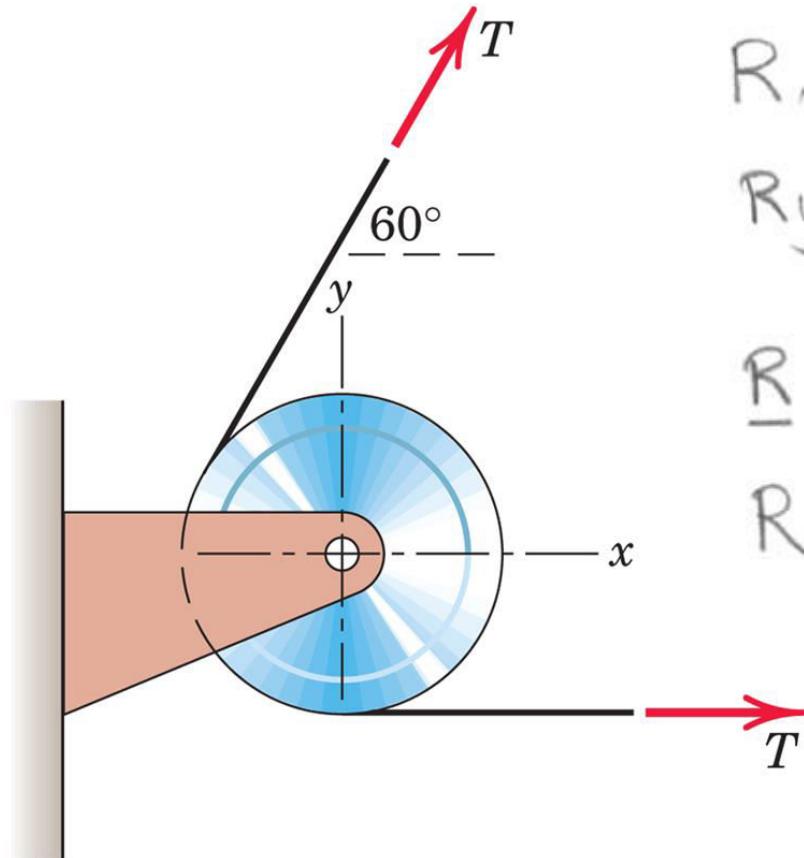
$$R = \sqrt{R_x^2 + R_y^2} = 3.80 \text{ kN}$$

$$\theta = \tan^{-1} \left( \frac{R_y}{R_x} \right) = \tan^{-1} \left( \frac{-1.4}{3.53} \right) = 338^\circ \quad (\text{or } -21.6^\circ)$$

# ENGINEERING MECHANICS

## Force Systems - Numerical

2/13 If the equal tensions T in the pulley cable are 400 N, express in vector notation the force R exerted on the pulley by the two tensions. Determine the magnitude of R.



$$R_x = \sum F_x = 400 + 400 \cos 60^\circ = 600 \text{ N}$$

$$R_y = \sum F_y = 400 \sin 60^\circ = 346 \text{ N}$$

$$\underline{R} = \underline{600i} + \underline{346j} \text{ N}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$

# ENGINEERING MECHANICS

## Force Systems - Numerical

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2/14, 2/16 and 2/19 Try to solve on your own



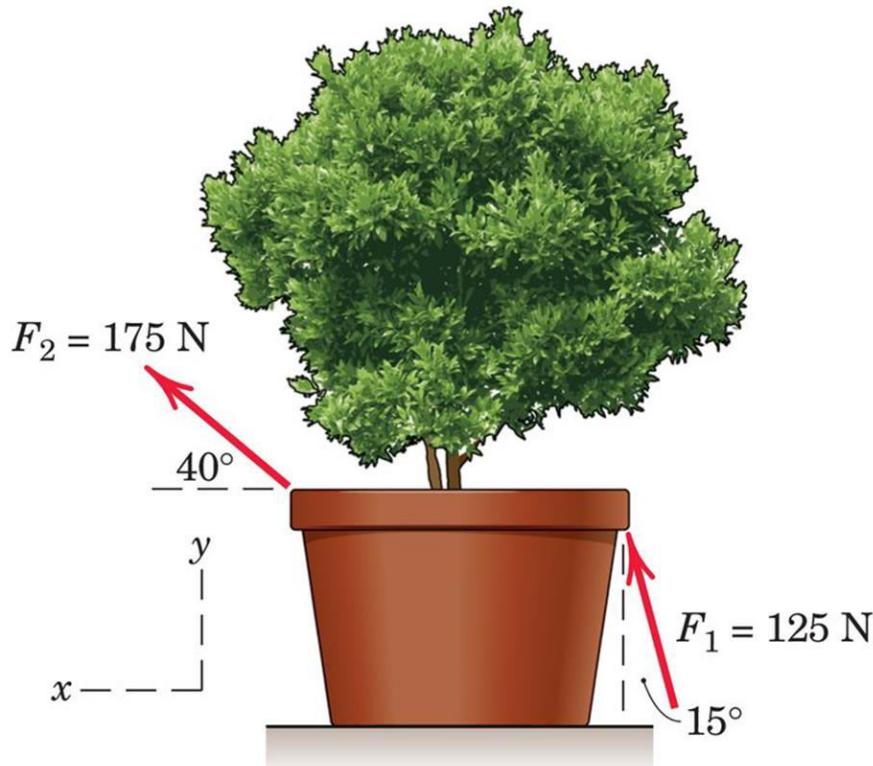
# ENGINEERING MECHANICS

## Force Systems - Numerical



**PES**  
UNIVERSITY  
ONLINE

**2/14** Two people exert the forces shown on the potted shrub. Determine the vector expression for the resultant  $R$  of the forces and determine the angle which the resultant makes with the positive  $y$ -axis.



# ENGINEERING MECHANICS

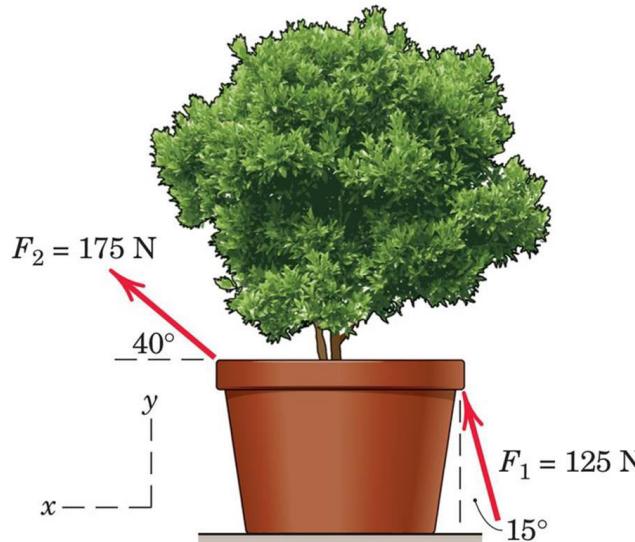
## Force Systems - Numerical



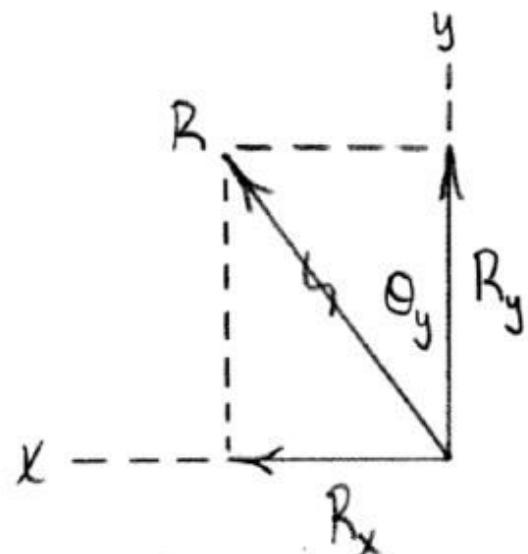
PES  
UNIVERSITY  
ONLINE

2/14

$$\underline{R} = (175 \cos 40^\circ + 125 \sin 15^\circ) \underline{i} + (175 \sin 40^\circ + 125 \cos 15^\circ) \underline{j}$$



$$\underline{R} = 166.4 \underline{i} + 233 \underline{j} \text{ N}$$



$$\Theta_y = \tan^{-1} \left( \frac{R_x}{R_y} \right) = \tan^{-1} \left( \frac{166.4}{233} \right)$$

$$\underline{\Theta_y = 35.5^\circ \text{ CCW OFF } +y\text{-Axis}}$$

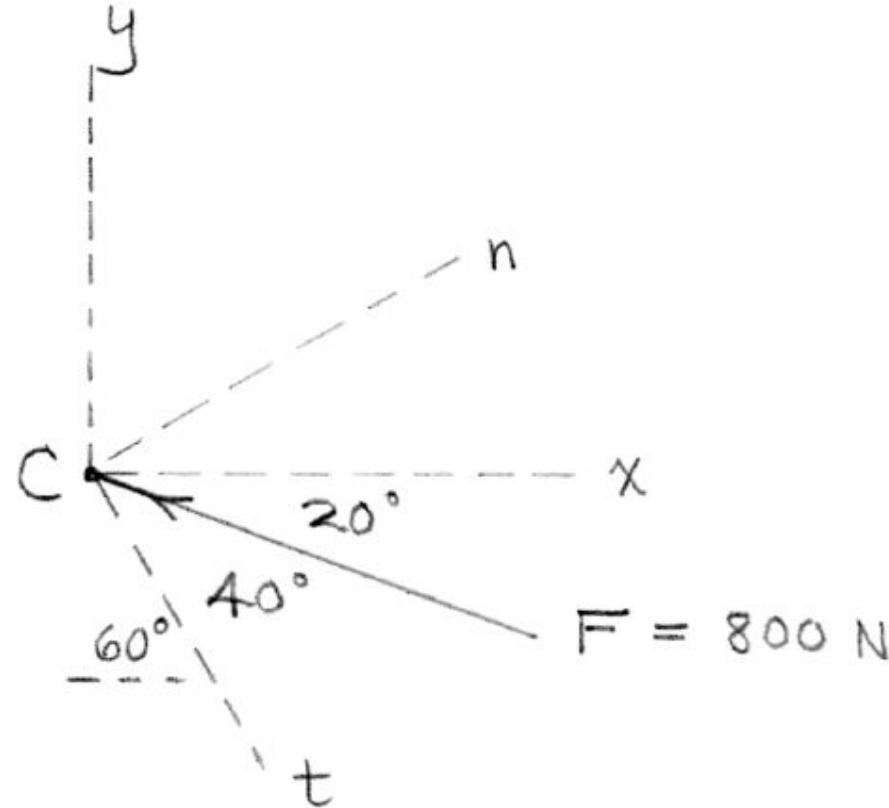
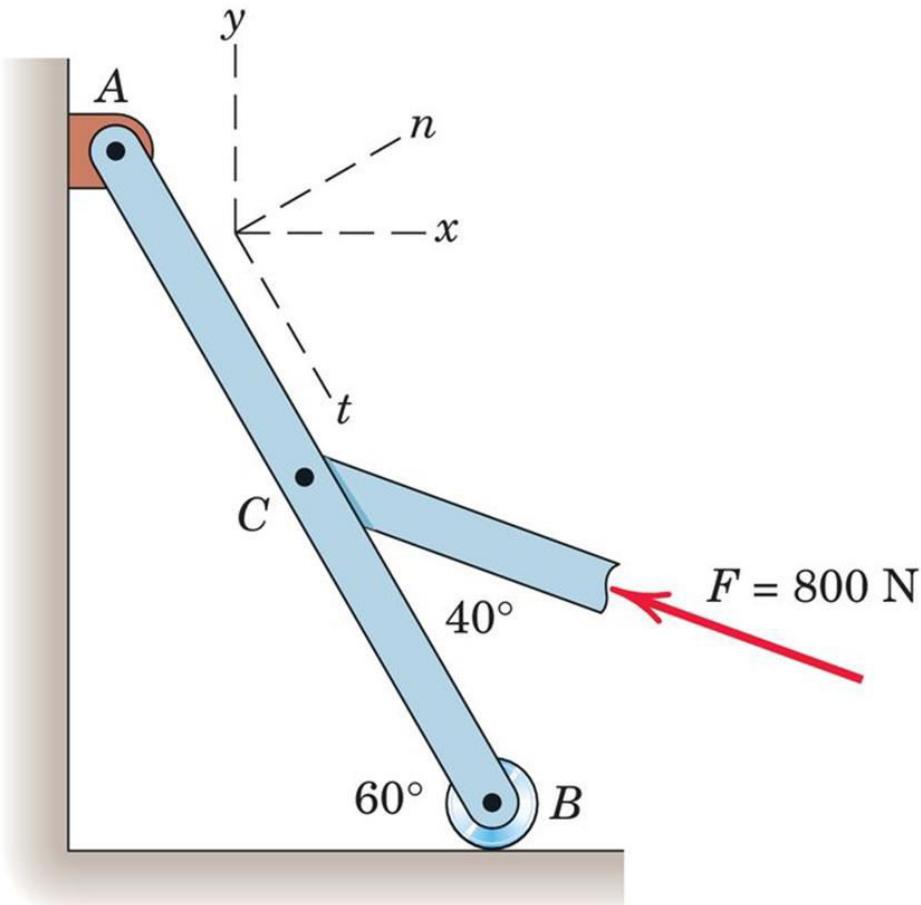
# ENGINEERING MECHANICS

## Force Systems - Numerical



PES  
UNIVERSITY  
ONLINE

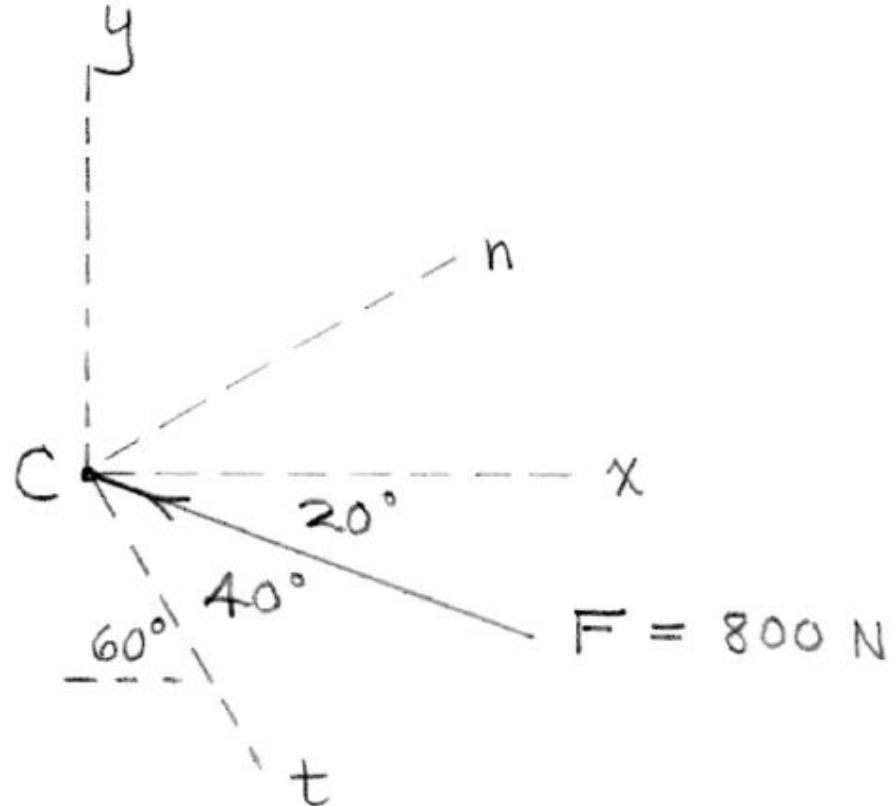
2/16) A force  $F$  of magnitude 800 N is applied to point C of the bar AB as shown. Determine both the X-Y and the n-t components of  $F$ .



# ENGINEERING MECHANICS

## Force Systems - Numerical

2/16)

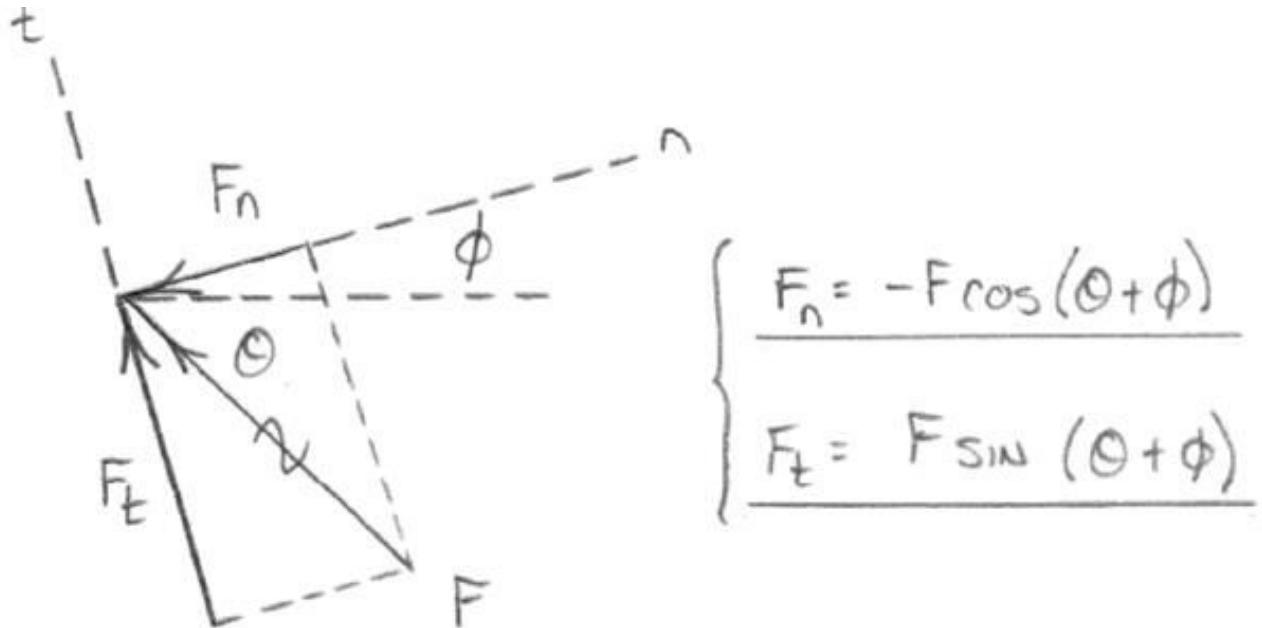
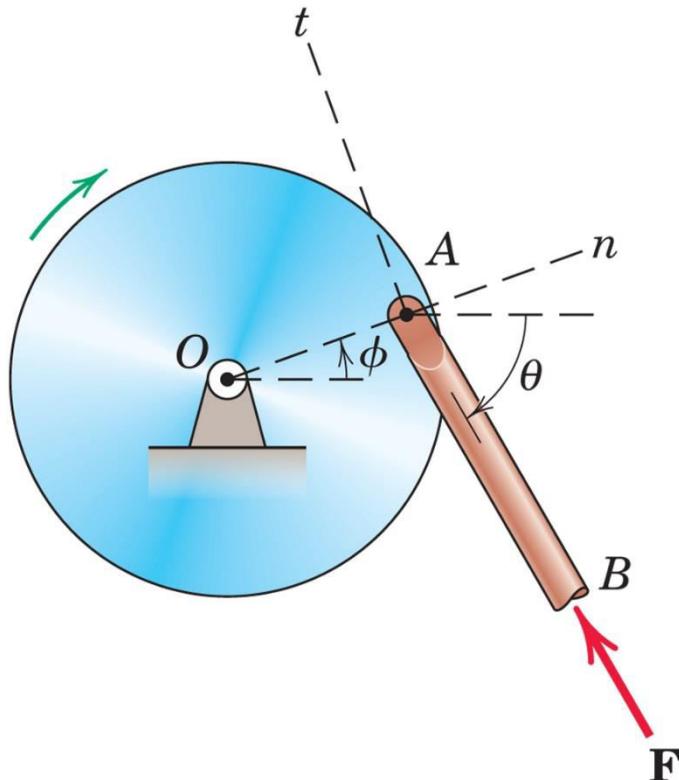


$$\begin{cases} F_x = -800 \cos 20^\circ = -752 \text{ N} \\ F_y = 800 \sin 20^\circ = 274 \text{ N} \end{cases}$$
$$\begin{cases} F_n = -800 \sin 40^\circ = -514 \text{ N} \\ F_t = -800 \cos 40^\circ = -613 \text{ N} \end{cases}$$

# ENGINEERING MECHANICS

## Force Systems - Numerical

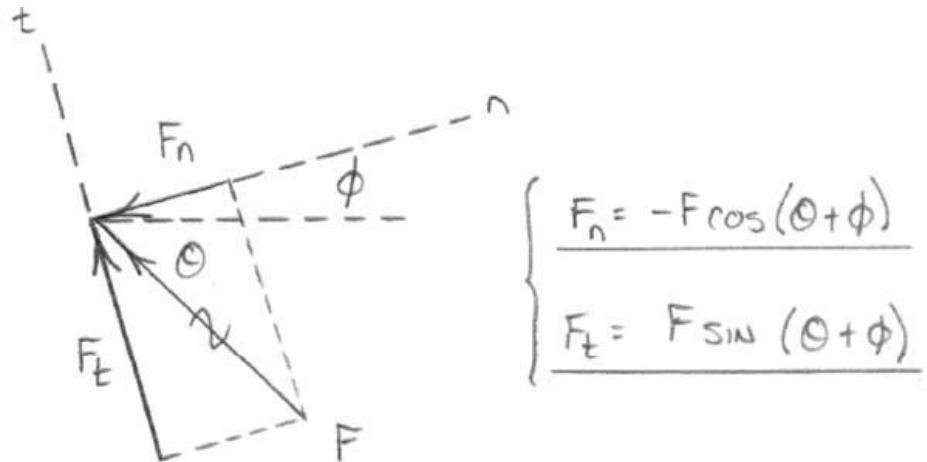
2/15) A compressive force  $F$  is transmitted via the coupler arm AB to disk OA. Develop the general expression for the n-and t-components of  $F$  as they act on the disk. Evaluate your expression for (a)  $F = 500 \text{ N}$ ,  $\theta = 60^\circ$  and  $\phi = 20^\circ$   
(b)  $F = 800 \text{ N}$ ,  $\theta = 45^\circ$  and  $\phi = 150^\circ$



$$\left\{ \begin{array}{l} F_n = -F \cos(\theta + \phi) \\ F_t = F \sin(\theta + \phi) \end{array} \right.$$



2/15)



$$\left\{ \begin{array}{l} F_n = -F \cos(\theta + \phi) \\ F_t = F \sin(\theta + \phi) \end{array} \right.$$

a)  $F = 500 \text{ N}$ ,  $\theta = 60^\circ$ ,  $\phi = 20^\circ$

$F_n = -86.8 \text{ N}$

$F_t = 492 \text{ N}$

b)  $F = 800 \text{ N}$ ,  $\theta = 45^\circ$ ,  $\phi = 150^\circ$

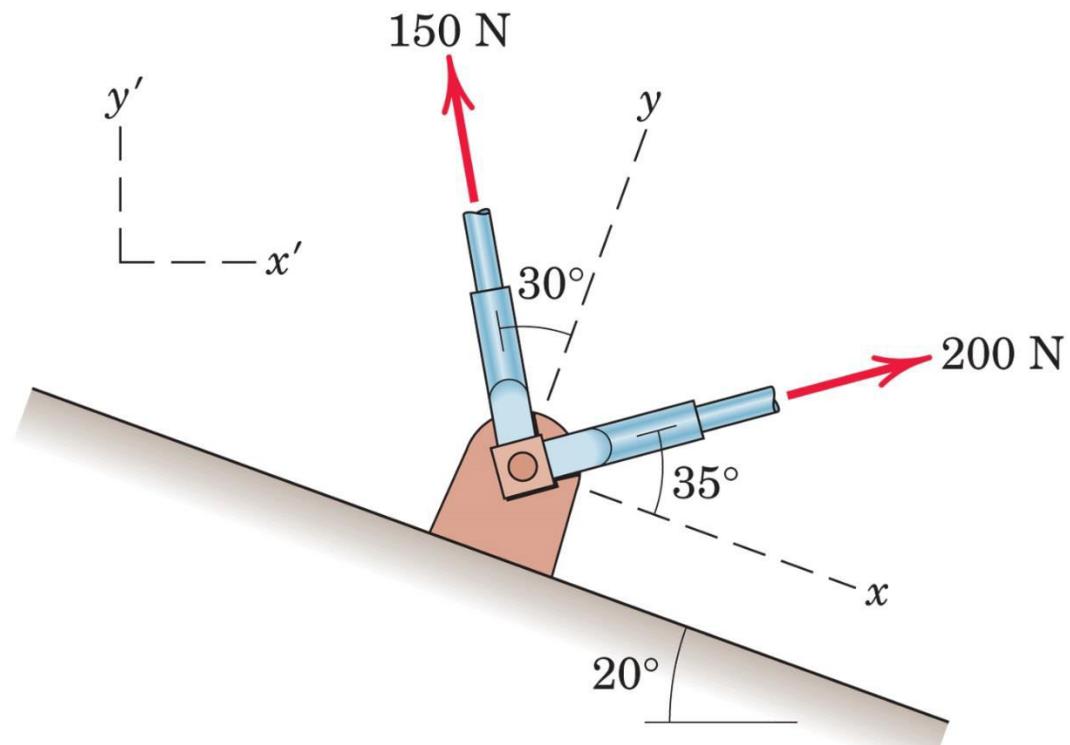
$F_n = 773 \text{ N}$

$F_t = -207 \text{ N}$

# ENGINEERING MECHANICS

## Force Systems - Numerical

2/21) Determine the resultant R of the two forces applied to the bracket. Write R in terms of unit vectors along the x- and y-axes shown.

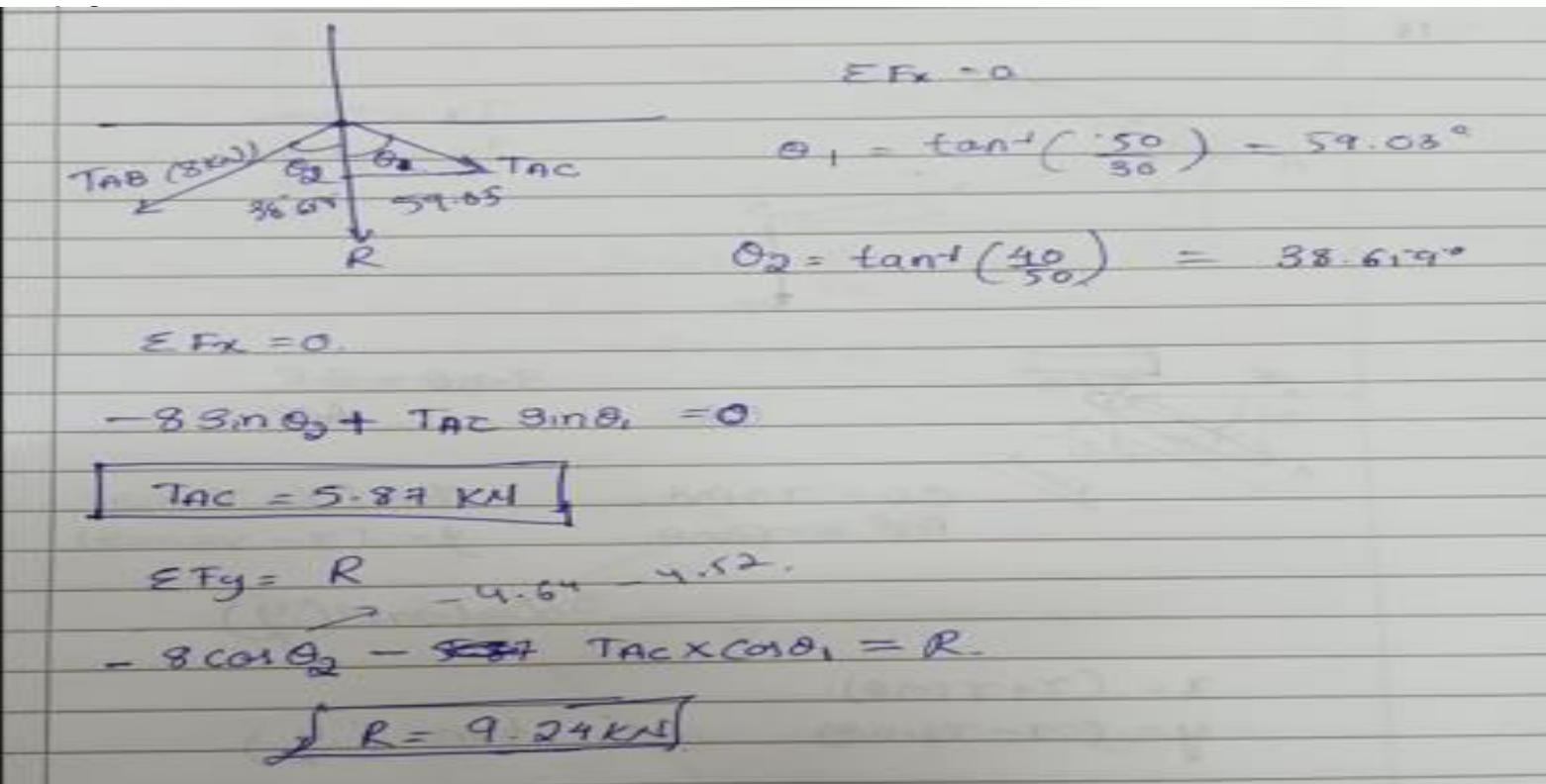
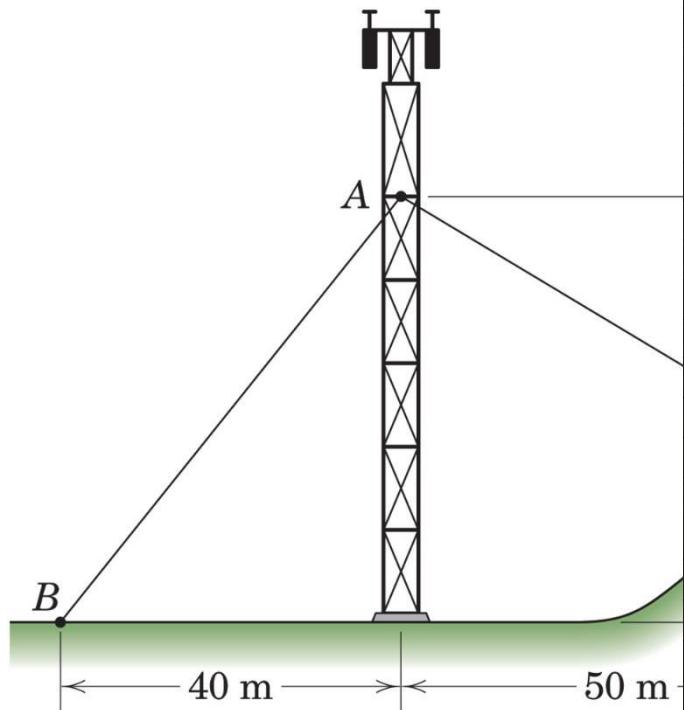


$$\begin{aligned}R_x &= \sum F_x = 200 \cos 35^\circ - 150 \sin 30^\circ \\&= 88.8 \text{ N}\end{aligned}$$

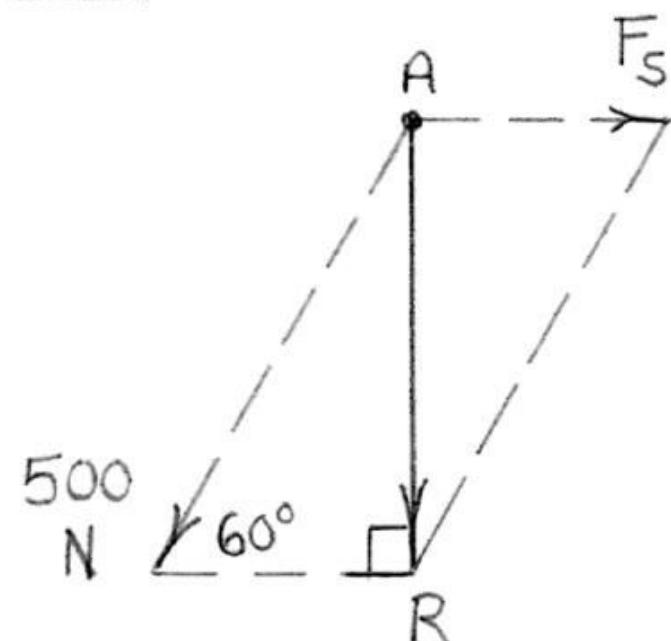
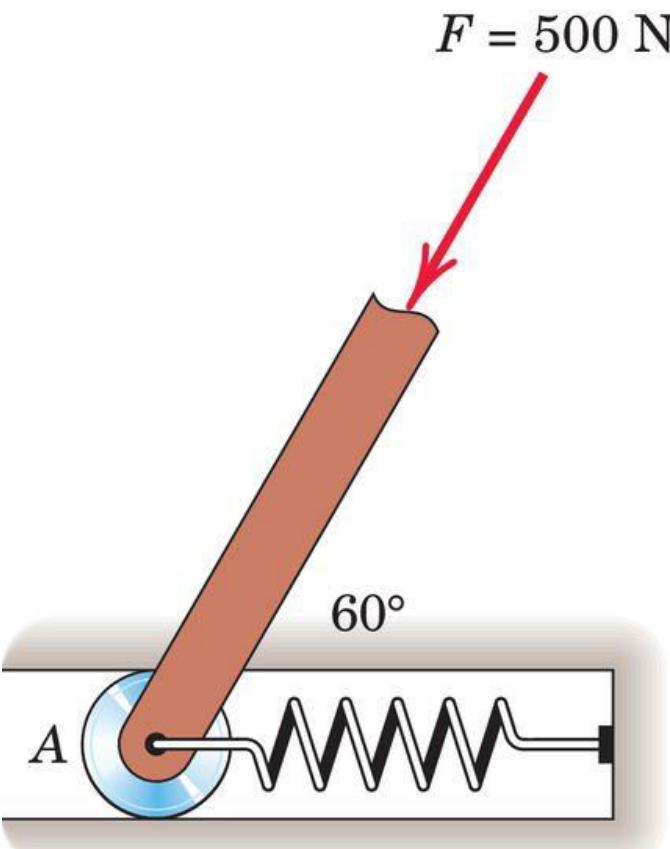
$$\begin{aligned}R_y &= \sum F_y = 200 \sin 35^\circ + 150 \cos 30^\circ \\&= 245 \text{ N}\end{aligned}$$

$$\therefore \underline{R} = 88.8 \underline{i} + 245 \underline{j} \text{ N}$$

2/12) The guy cables AB and AC are attached to the top of the transmission tower. The tension in cable AB is 8 kN. Determine the required tension T in cable AC such that the **net effect of the two cable tensions is a downward force at point A**. Determine the magnitude R of this downward



2/20) Determine the magnitude  $F_s$  of the tensile spring force in order that the resultant of  $F_s$  and  $F$  is a vertical force. Determine the magnitude  $R$  of this vertical resultant force.



$$\cos 60^\circ = \frac{F_s}{500}$$

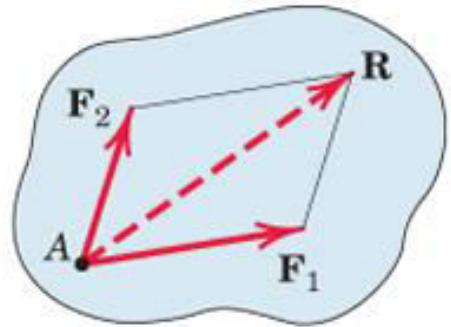
$$F_s = 250 \text{ N}$$

$$\sin 60^\circ = \frac{R}{500}$$

$$R = 433 \text{ N}$$

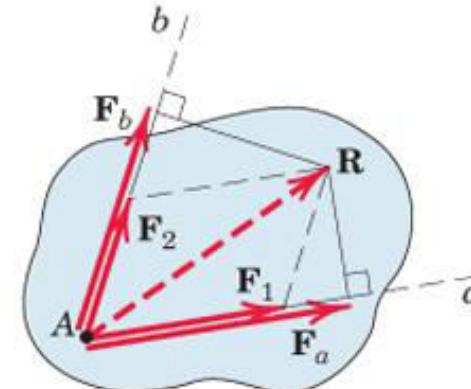
# ENGINEERING MECHANICS

## Projections



(a)

The force  $\mathbf{R}$  in Fig. a may be replaced by, or resolved into, two vector components  **$F_1$  and  $F_2$**  with the specified directions by completing the parallelogram as shown to obtain the magnitudes of  **$F_1$  and  $F_2$** .

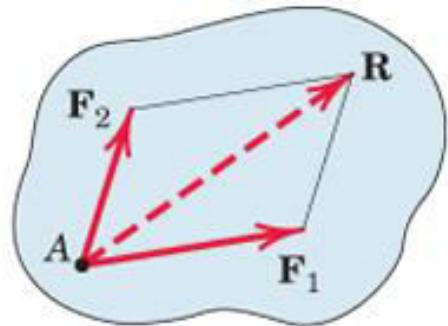


(e)

The relationship between a force and its vector components along given axes must not be confused with the relationship between a force and its perpendicular\* projections onto the same axes.

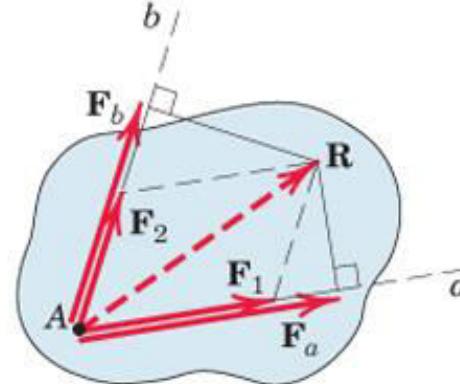
# ENGINEERING MECHANICS

## Projections



(a)

Figure e shows the perpendicular projections  $F_a$  and  $F_b$  of the given force  $R$  onto axes  $a$  and  $b$ , which are parallel to the vector components  $F_1$  and  $F_2$  of Fig.a.



(e)

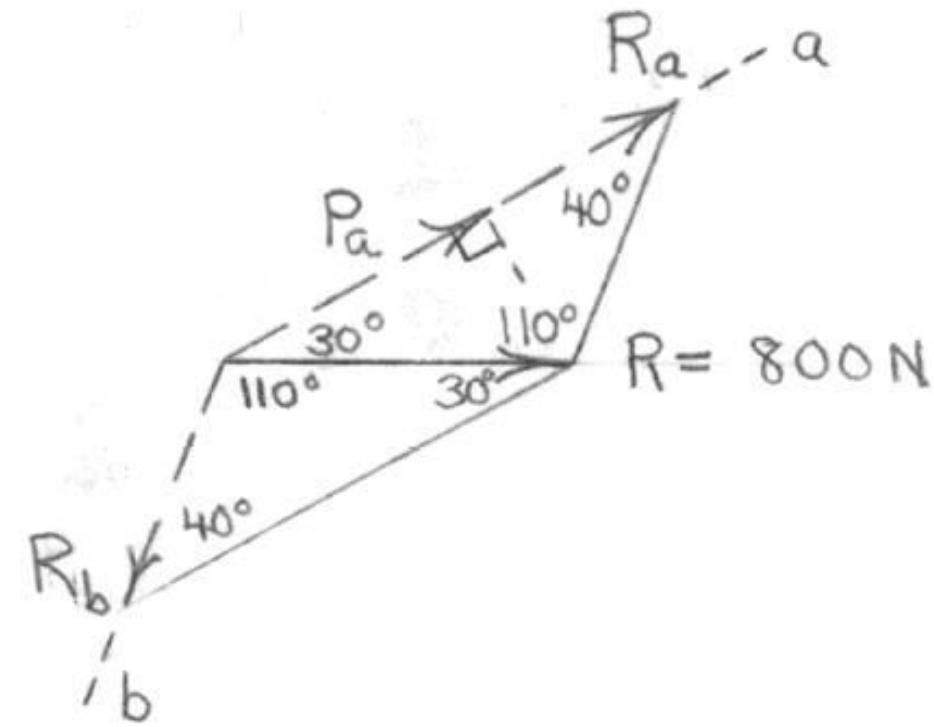
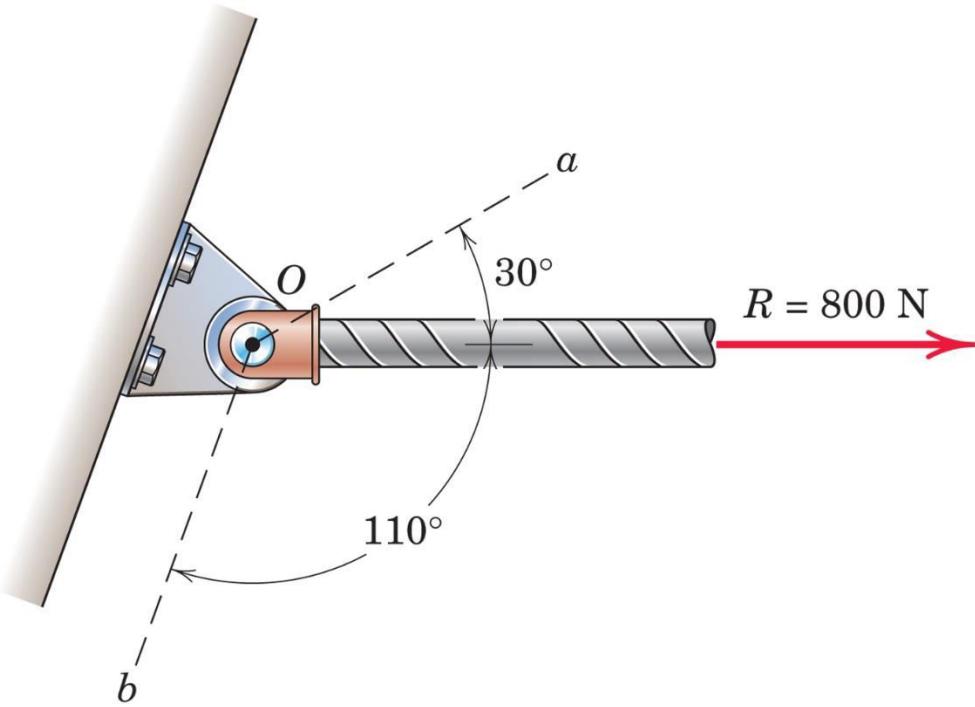
Figure e shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections  $F_a$  and  $F_b$  is not the vector  $R$ , because the parallelogram law of vector addition must be used to form the sum.

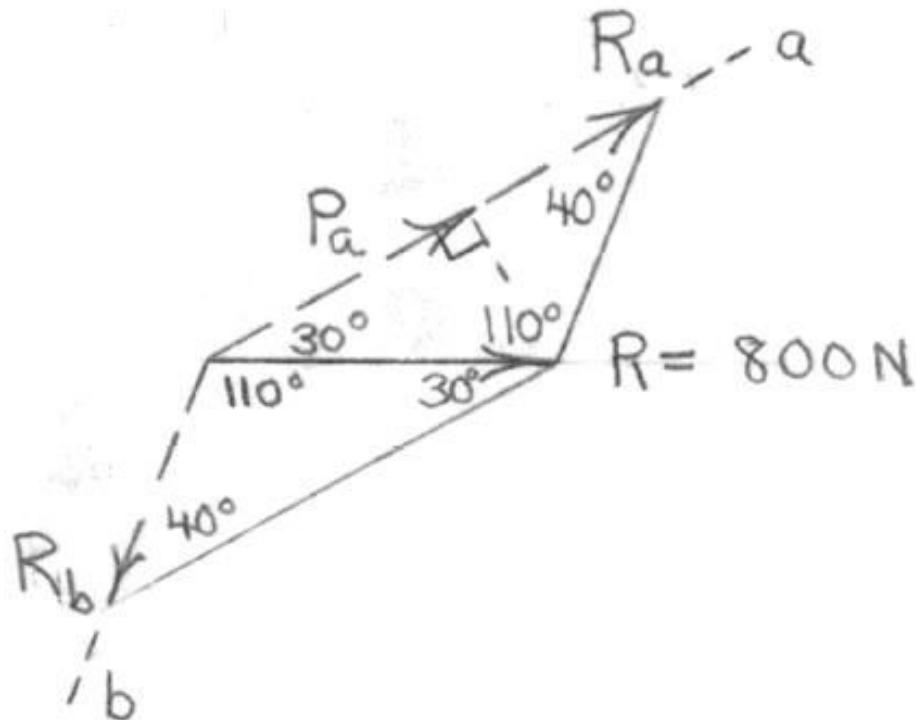
The components and projections of  $R$  are equal only when the axes  $a$  and  $b$  are perpendicular.

# ENGINEERING MECHANICS

## Projections - Numerical

2/23) Determine the scalar components  $R_a$  and  $R_b$  of the force  $R$  along the nonrectangular axes  $a$  and  $b$ . Also determine the orthogonal projection  $P_a$  of  $R$  onto axis  $a$ .





Law of sines :

$$\frac{800}{\sin 40^\circ} = \frac{R_a}{\sin 110^\circ} = \frac{R_b}{\sin 30^\circ}$$

$$\underline{R_a = 1170 \text{ N}}$$

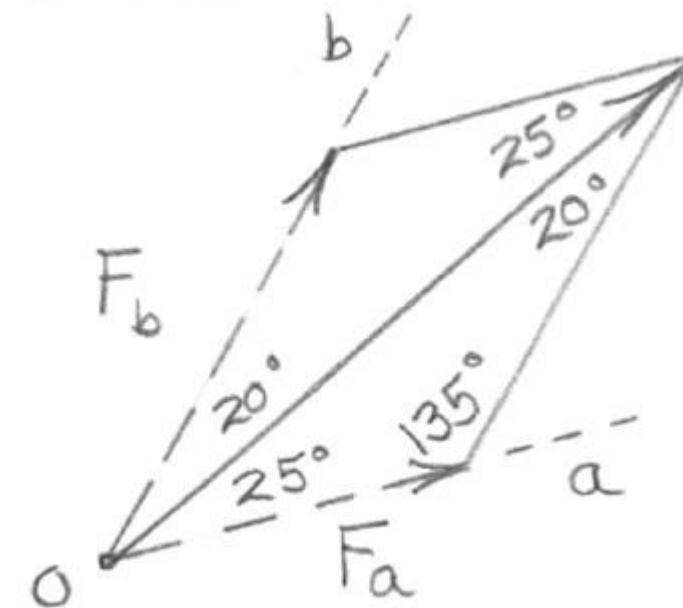
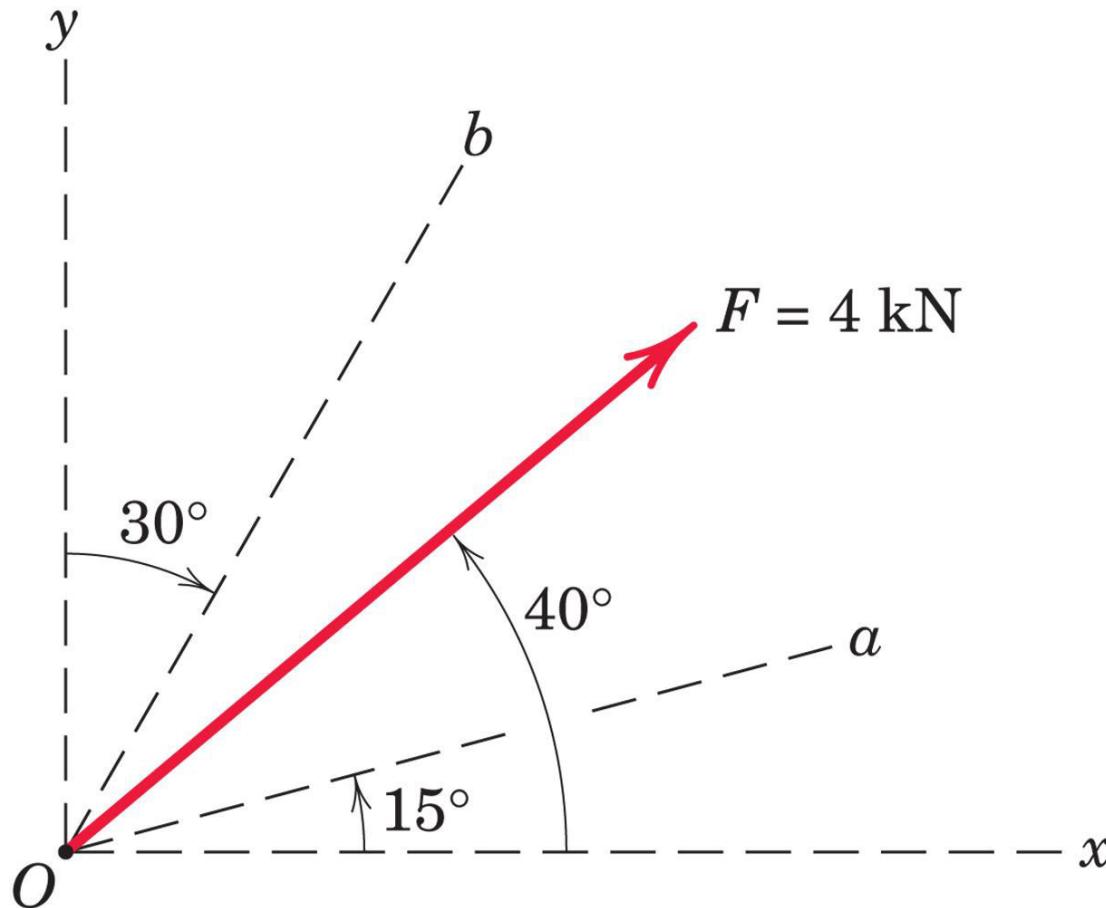
$$\underline{R_b = 622 \text{ N}}$$

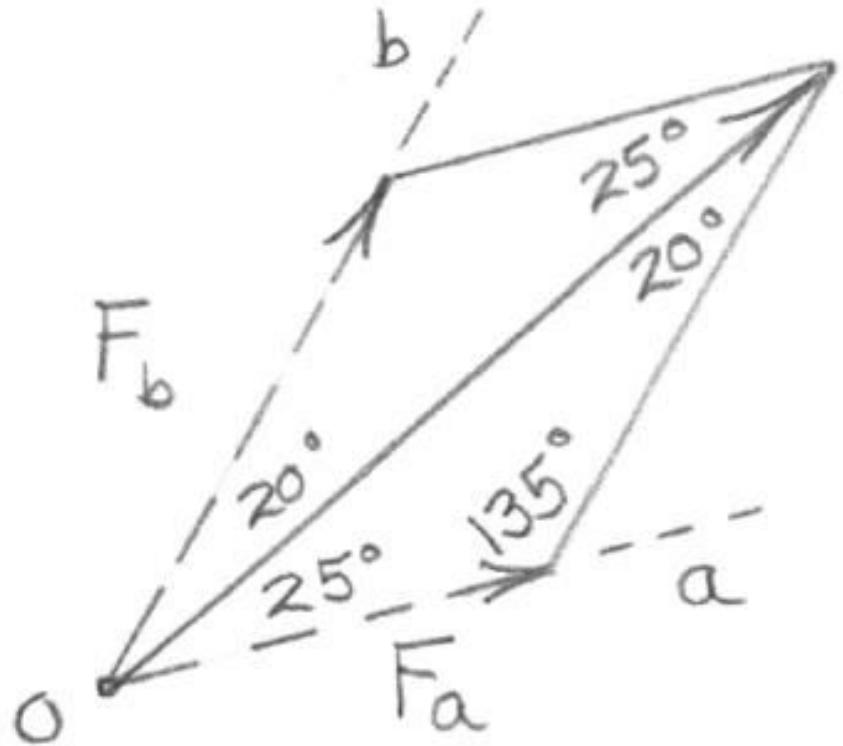
$$\text{Projection } P_a = R \cos 30^\circ = 800 \cos 30^\circ = \underline{693 \text{ N}}$$

# ENGINEERING MECHANICS

## Projections - Numerical

2/24) Determine the components  $F_a$  and  $F_b$  of the 4-kN force along the oblique axes a and b. Determine the projections  $P_a$  and  $P_b$  of F onto the a- and b-axes.

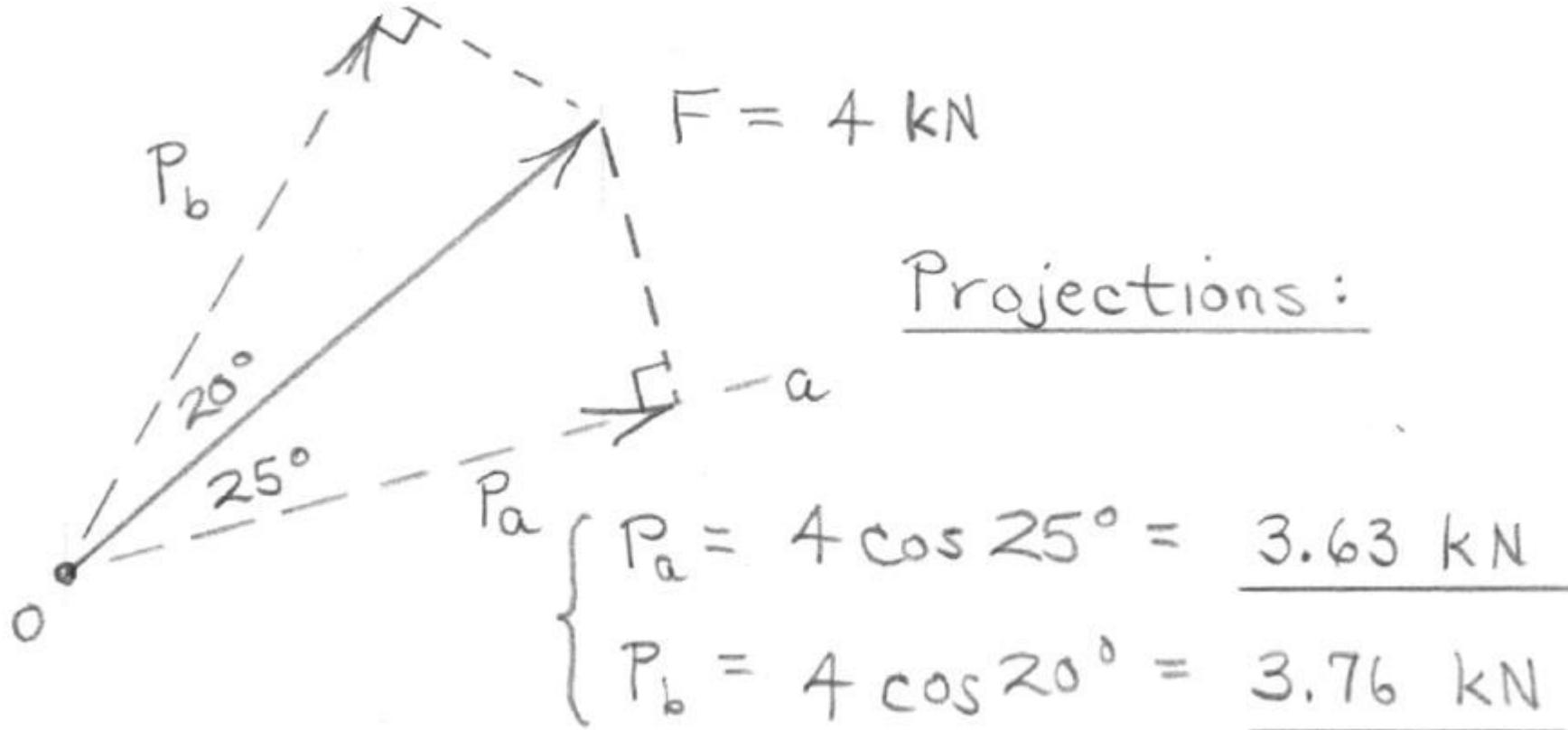




Components:

$$\frac{\sin 135^\circ}{4} = \frac{\sin 20^\circ}{F_a} \quad ) \quad \underline{F_a = 1.935 \text{ kN}}$$

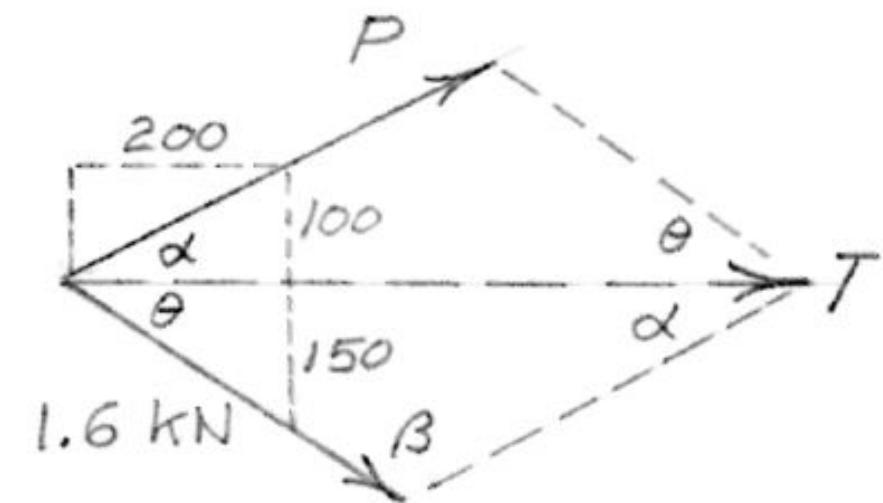
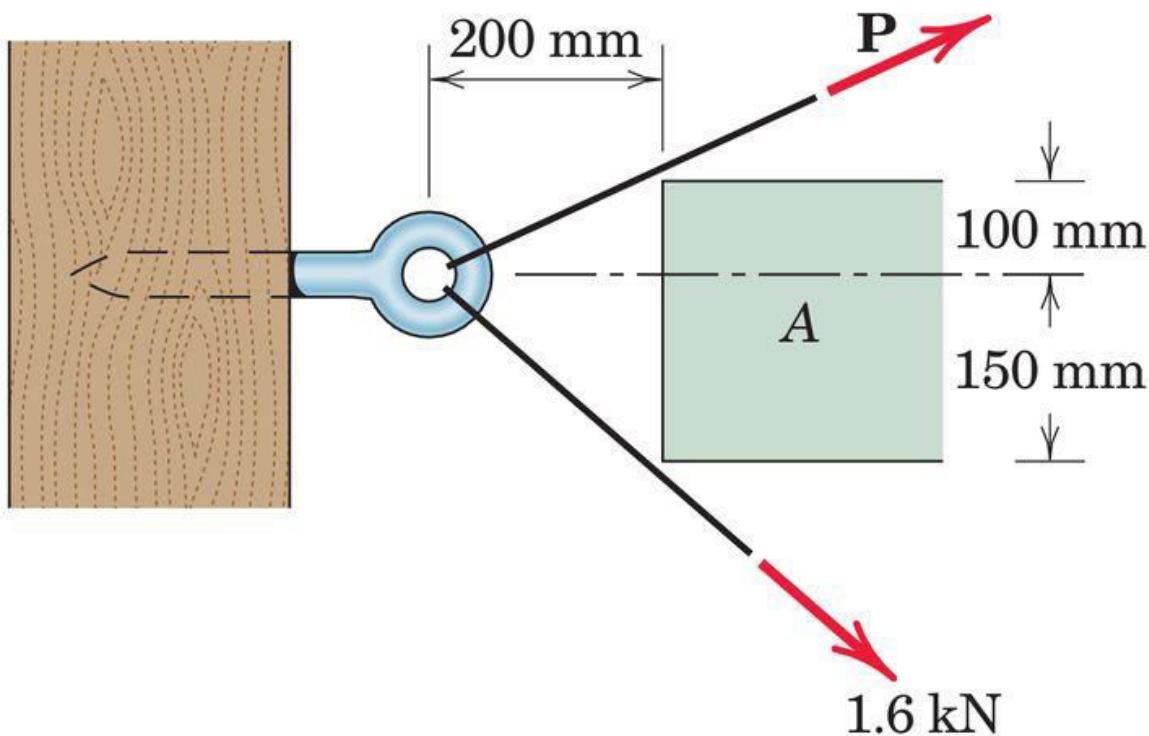
$$\frac{\sin 135^\circ}{4} = \frac{\sin 25^\circ}{F_b} \quad , \quad \underline{F_b = 2.39 \text{ kN}}$$

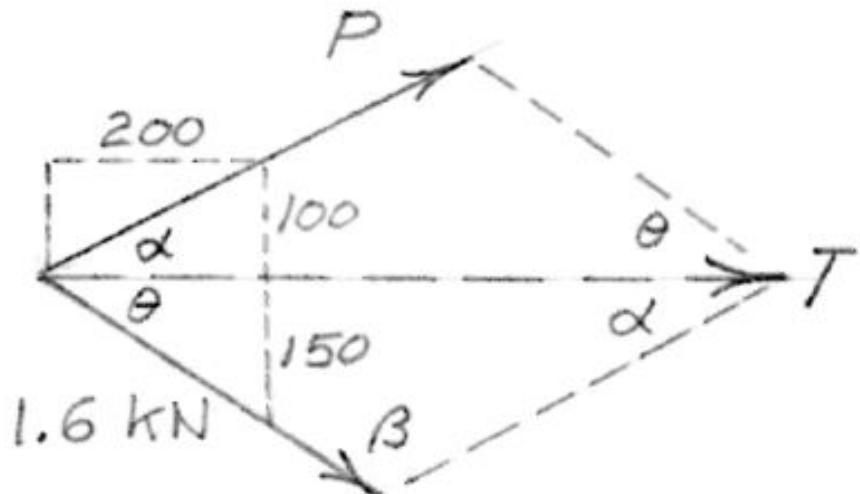


# ENGINEERING MECHANICS

## Force Systems - Numerical

2/26) It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction A prevents direct access, so that two forces, one 1.6 kN and the other P, are applied by cables as shown. Compute the magnitude of P necessary to ensure a resultant T directed along the spike. Also find T.





$$\alpha = \tan^{-1} \frac{100}{200} = 26.57^\circ$$

$$\theta = \tan^{-1} \frac{150}{200} = 36.87^\circ$$

$$\beta = 180 - (\alpha + \theta) = 116.57^\circ$$

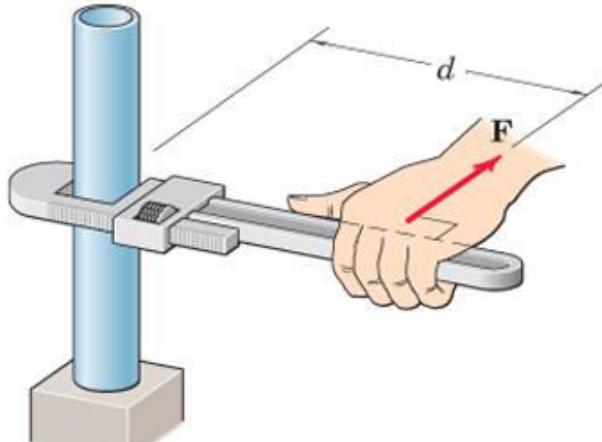
$$\frac{P}{\sin 36.87^\circ} = \frac{1.6}{\sin 26.57}$$

$$P = 1.6 \frac{0.6}{0.4472} = \underline{2.15 \text{ kN}}$$

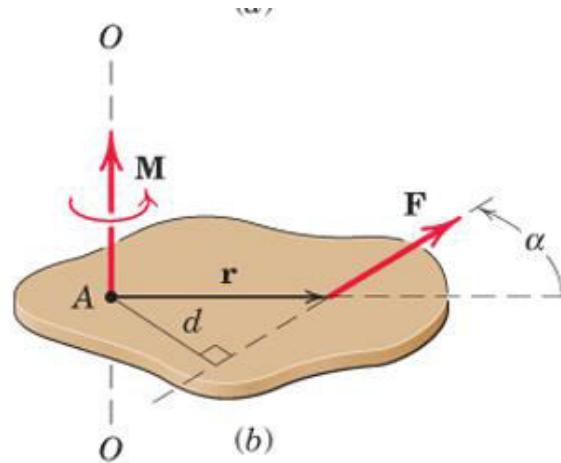
$$\frac{T}{\sin 116.57^\circ} = \frac{1.6}{\sin 26.57^\circ}, \quad T = 1.6 \frac{0.8944}{0.4472} = \underline{3.20 \text{ kN}}$$

# ENGINEERING MECHANICS

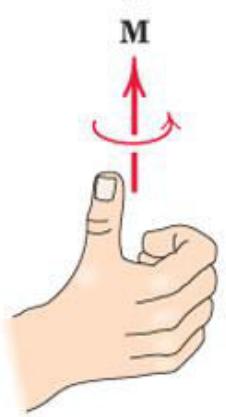
## Moment



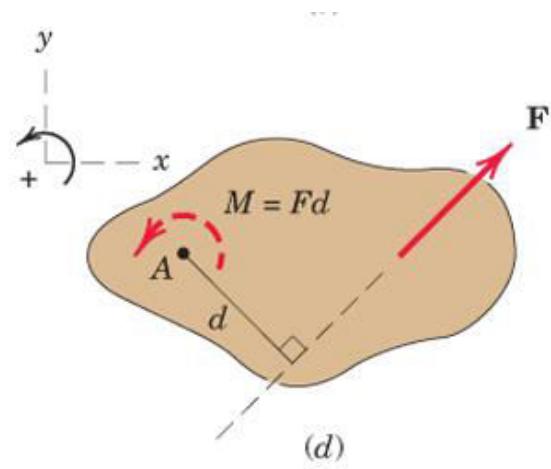
(a)



(b)



(c)



(d)

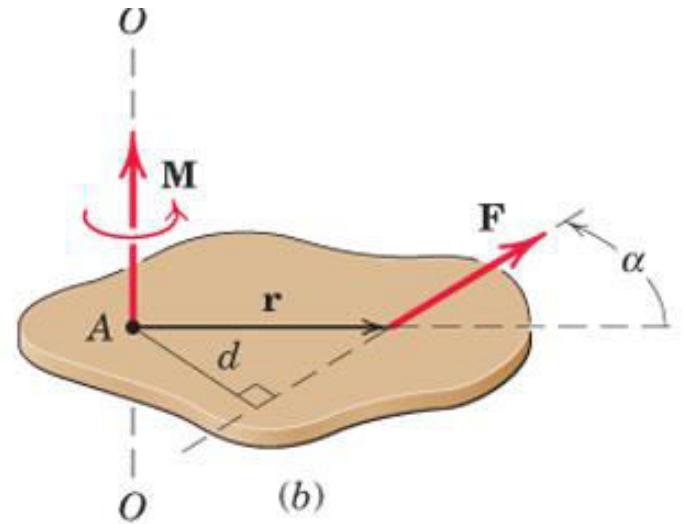
- In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis.
- The axis may be any line which neither intersects nor is parallel to the line of action of the force.
- This rotational tendency is known as the moment  $M$  of the force. Moment is also referred to as torque.

# ENGINEERING MECHANICS

## Cross Product

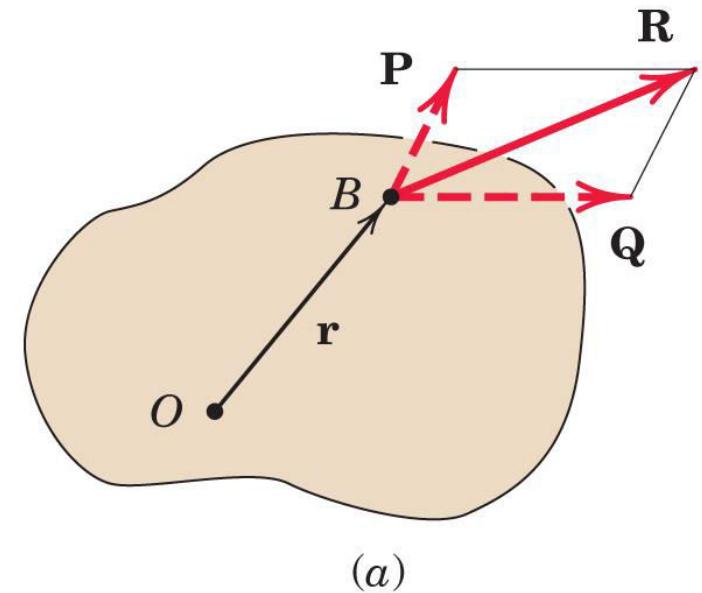
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Where  $\mathbf{r}$  is a position vector which runs from the moment reference point A to any point on the line of action of  $\mathbf{F}$ .  
The magnitude of this expression is given by\*



Varignon's theorem, which states that the *moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.*

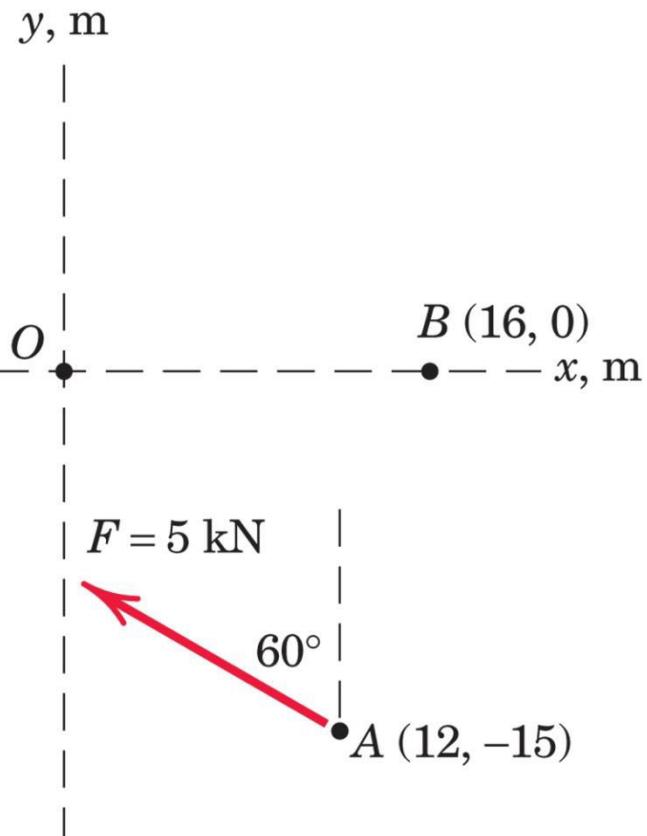
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$



# ENGINEERING MECHANICS

## Moment - Numerical

2/31) Determine the moments of the 5-kN force about point O and about point B.



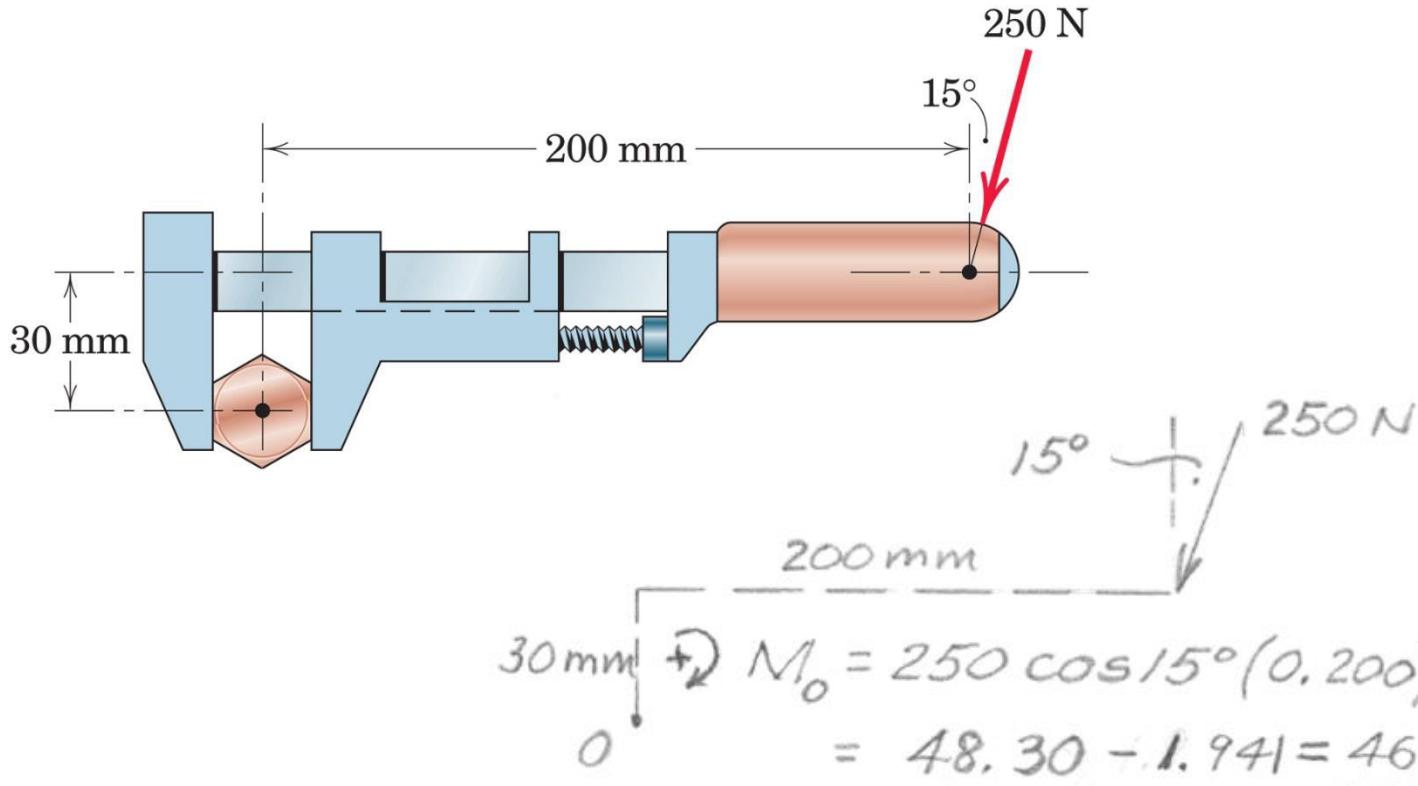
$$M_O = 5 \cos 60^\circ (12) - 5 \sin 60^\circ (15) = -35.0$$

$$\therefore M_O = 35.0 \text{ kN}\cdot\text{m} \text{ CW}$$

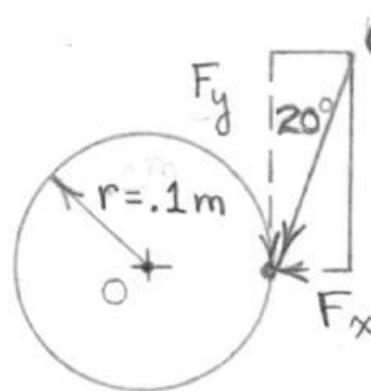
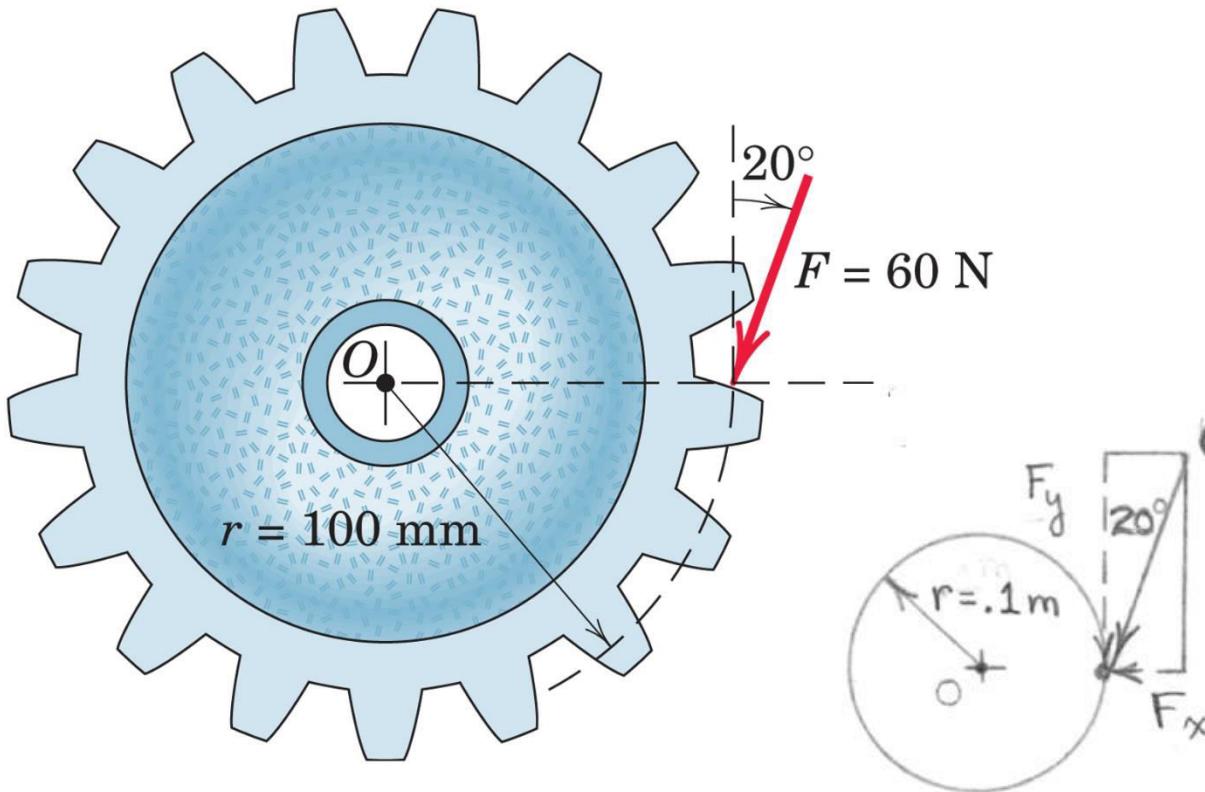
$$M_B = -5 \cos 60^\circ (4) - 5 \sin 60^\circ (15) = -75.0$$

$$\therefore M_B = 75.0 \text{ kN}\cdot\text{m} \text{ CW}$$

2/34) Calculate the moment of the 250-N force on the handle of the monkey wrench about the center of the bolt.



2/36) A force  $F$  of magnitude 60 N is applied to the gear.  
Determine the moment of  $F$  about point 0.

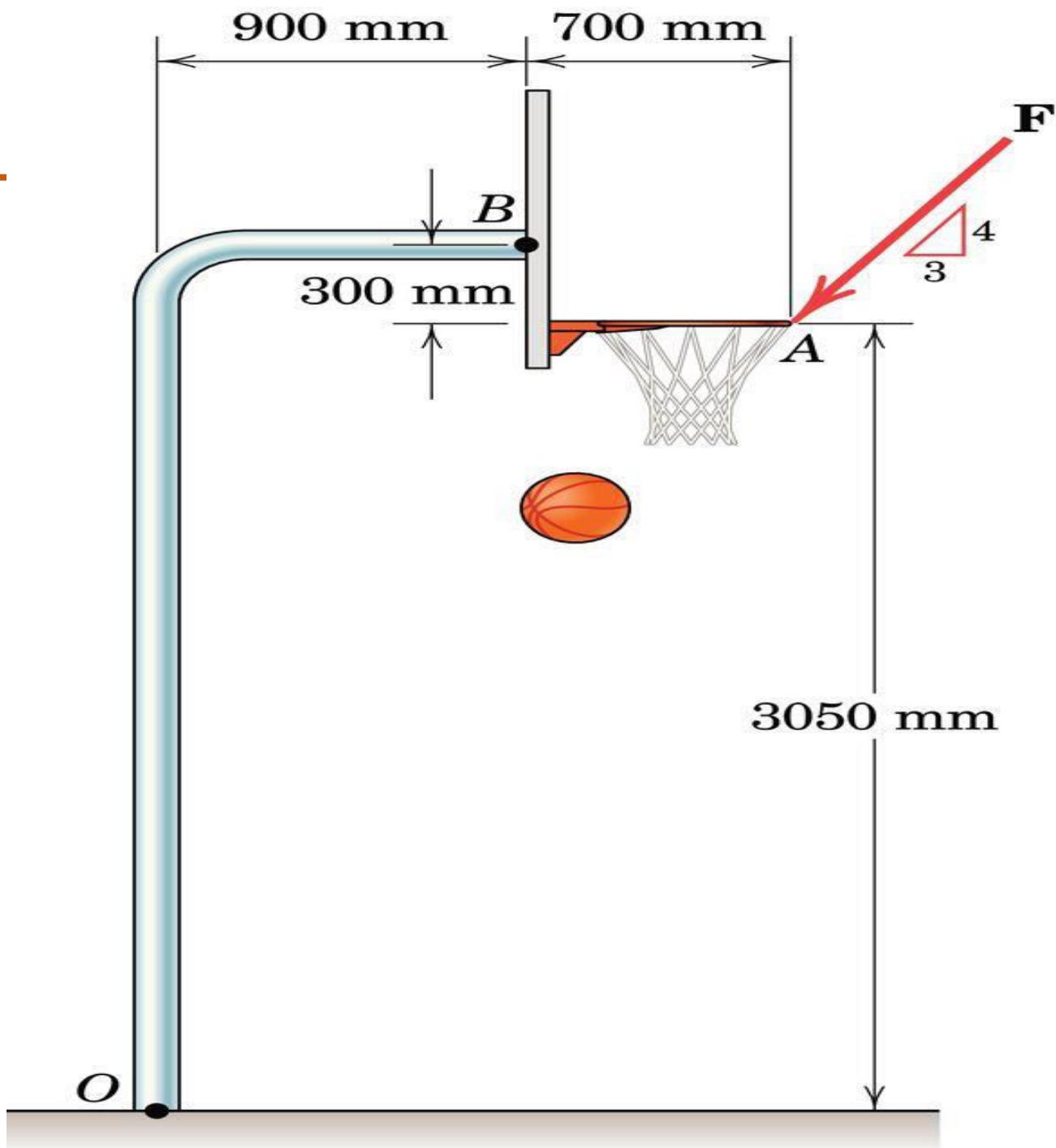


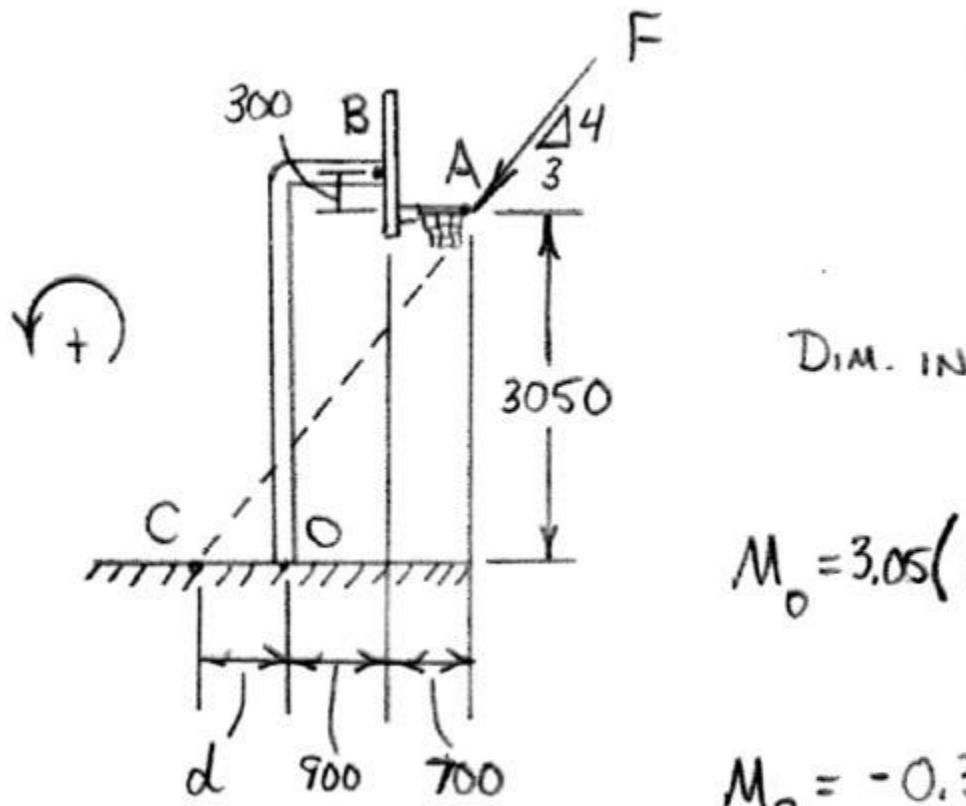
$$\begin{aligned}60 \text{ N} + 2 M_0 &= r F_y \\&= (0.1) (60 \cos 20^\circ) \\&= 5.64 \text{ N}\cdot\text{m}\end{aligned}$$

# ENGINEERING MECHANICS

## Moment - Numerical

2/35) An experimental device imparts a force of magnitude  $F = 225 \text{ N}$  to the front edge of the rim at A to simulate the effect of a slam dunk. Determine the moments of the force F about point O and about point B. Finally, locate, from the base at O, a point C on the ground where the force imparts zero moment.





$$F = 225 \text{ N}$$

DIM. IN mm

$$M_0 = 3.05 \left( \frac{3}{5} F \right) - 1.6 \left( \frac{4}{5} F \right) = 123.8 \rightarrow \underline{\underline{M_0 = 123.8 \text{ N}\cdot\text{m CCW}}}$$

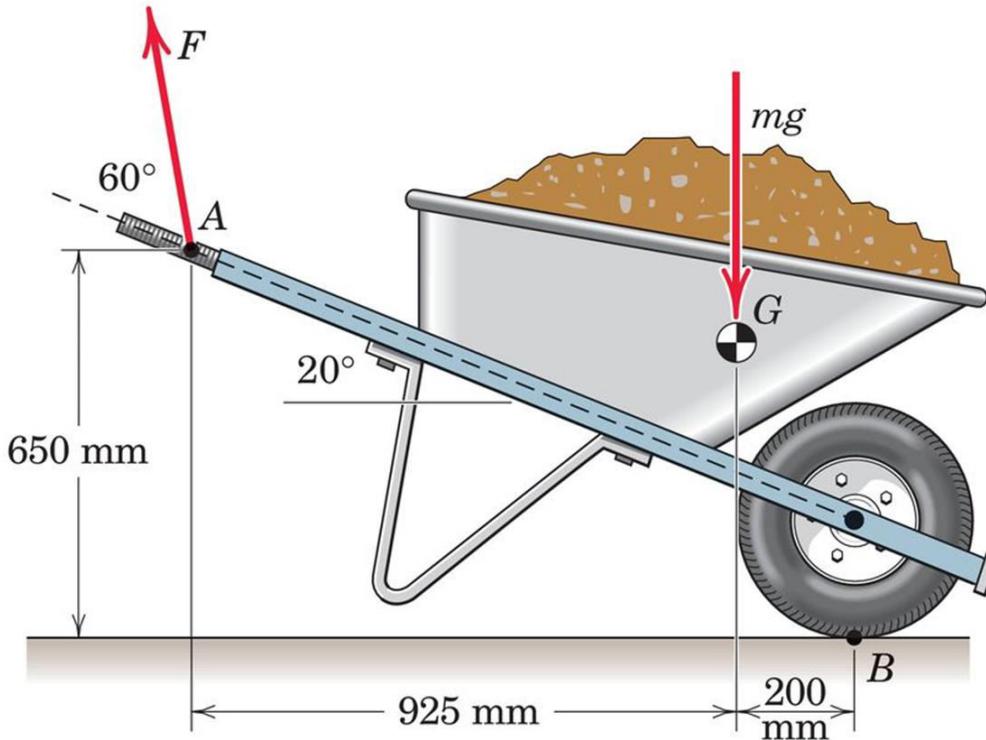
$$M_B = -0.3 \left( \frac{3}{5} F \right) - 0.7 \left( \frac{4}{5} F \right) = -166.5 \rightarrow \underline{\underline{M_B = 166.5 \text{ N}\cdot\text{m CW}}}$$

$$M_C = 0 = 3.05 \left( \frac{3}{5} F \right) - (1.6 + d) \left( \frac{4}{5} F \right) \rightarrow d = 0.688 \text{ m}$$

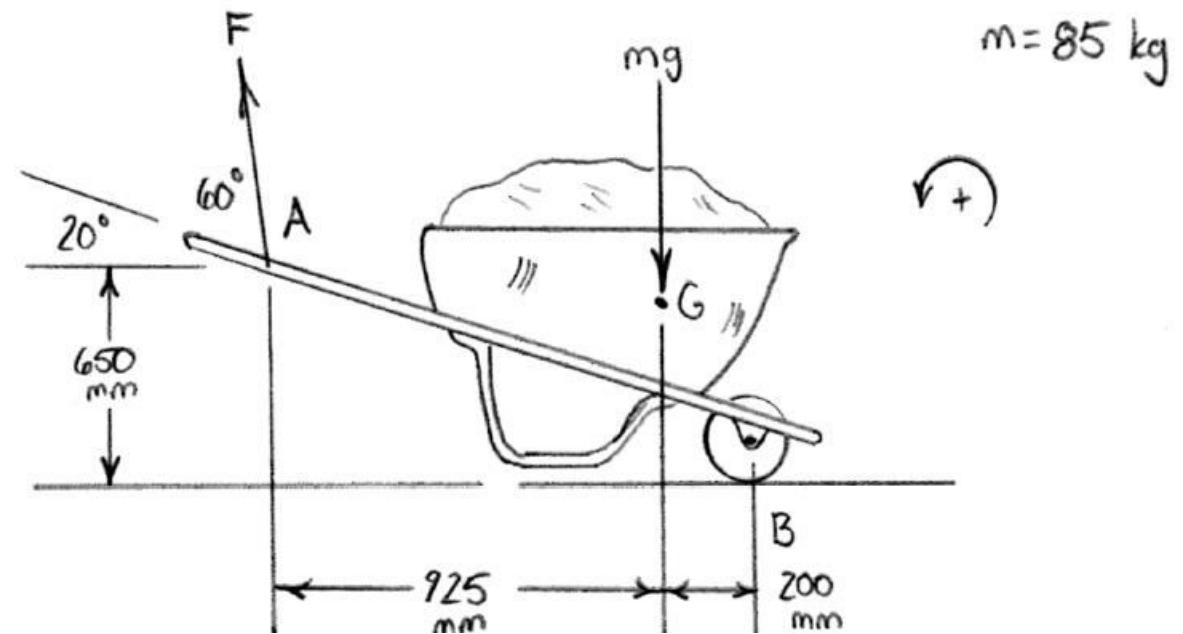
so...

$d = 688 \text{ mm LEFT OF O}$

2/40) A man exerts a force  $F$  on the handle of the stationary wheelbarrow at A. The mass of the wheelbarrow along with its load of dirt is 85 kg with center of mass at G. For the configuration shown, what force  $F$  must the man apply at A to make the net moment about the tire contact point B equal to zero?



2/40

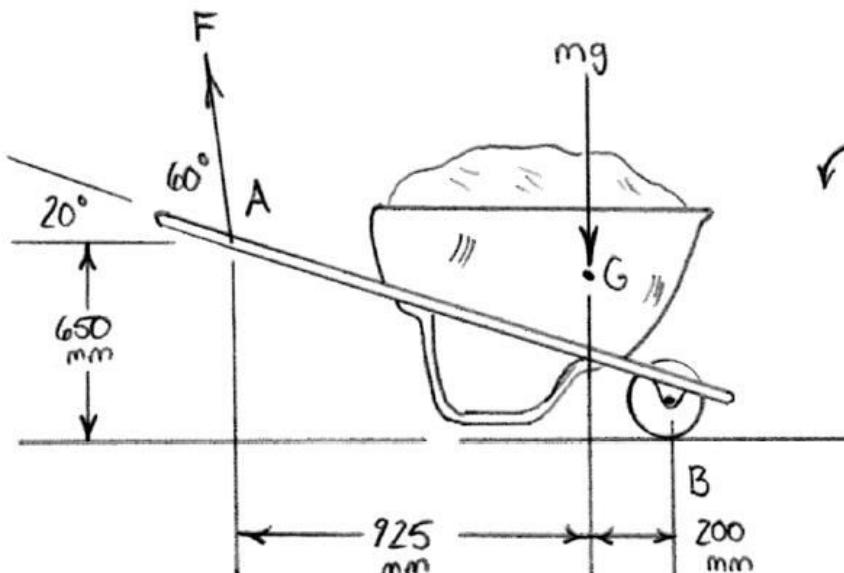
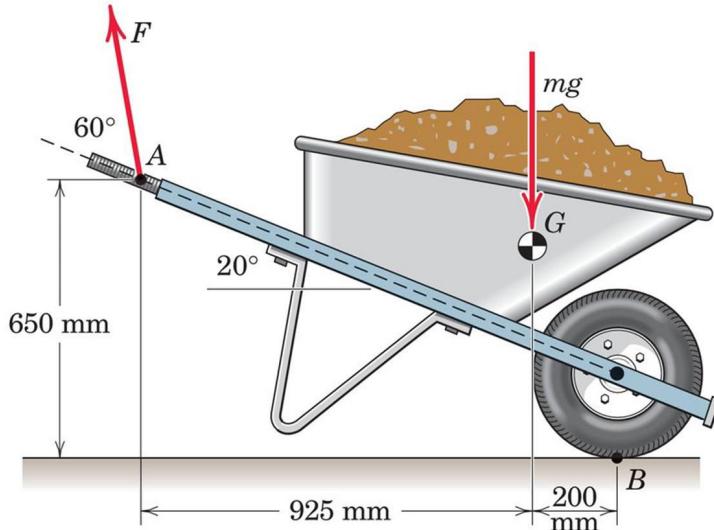


# ENGINEERING MECHANICS

## Moment - Numerical

2/40)

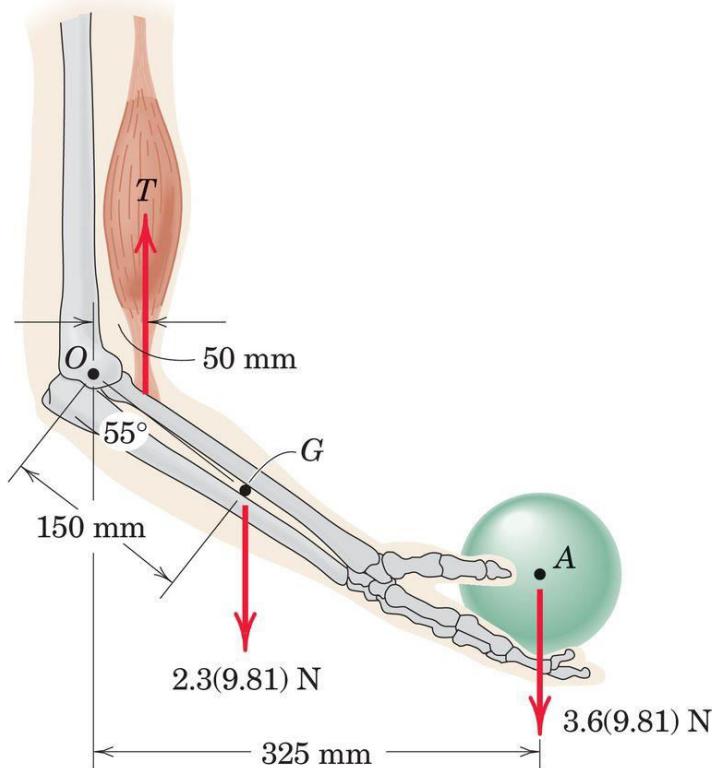
2/40

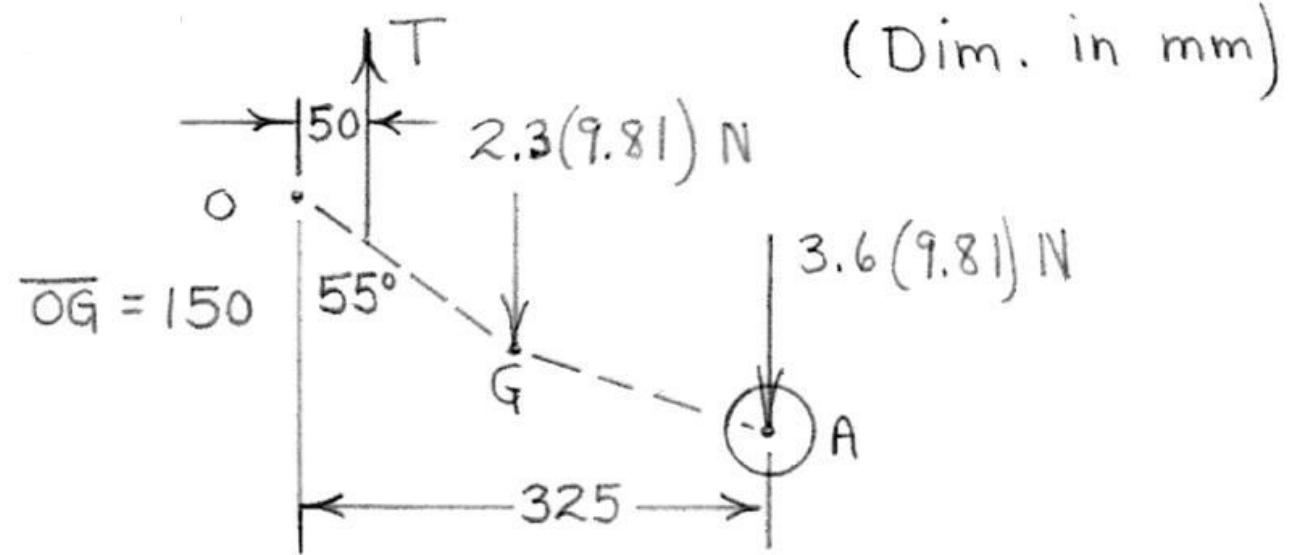


$$M_B = 0.2(85)(9.81) - F \sin 80^\circ \left(\frac{925+200}{1000}\right) + F \cos 80^\circ \left(\frac{650}{1000}\right) = 0$$

$$F = 167.6 \text{ N}$$

2/50) Elements of the lower arm are shown in the figure. The mass of the forearm is 2.3 kg with center of mass at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the sphere. What must the biceps tension force be so that the overall moment about O is zero?

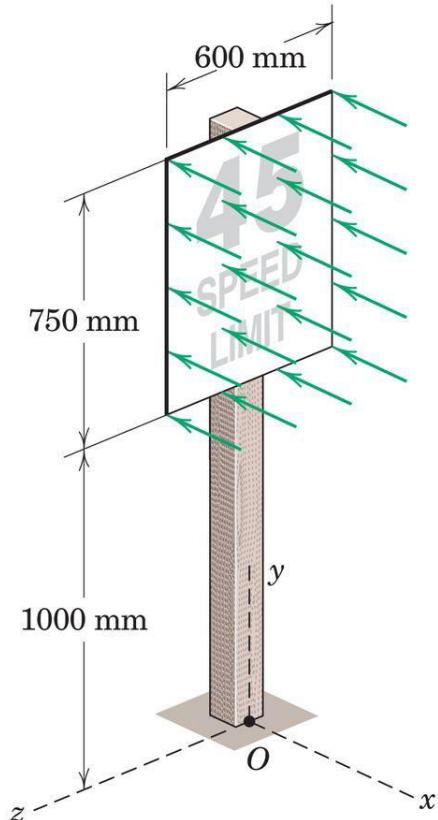


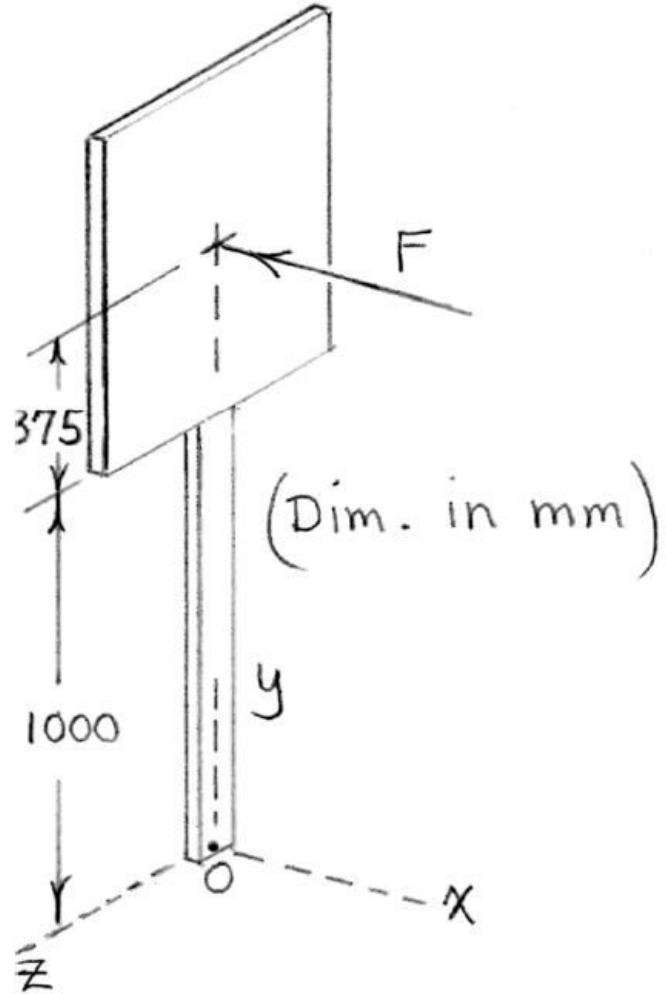


$$\begin{aligned}\therefore M_O &= 2.3(9.81)(0.150 \sin 55^\circ) + 3.6(9.81)(0.325) \\ &= \underline{14.25 \text{ N}\cdot\text{m} \text{ (CW)}}\end{aligned}$$

$$\begin{aligned}\therefore \sum M_O &= 0 : -T(0.050) + 14.25 = 0 \\ T &= 285 \text{ N}\end{aligned}$$

2/51) As the result of a wind blowing normal to the plane of the rectangular sign, a uniform pressure of  $175 \text{ N/m}^2$  is exerted in the direction shown in the figure. Determine the moment of the resulting force about point O. Express your result as a vector using the coordinates shown.





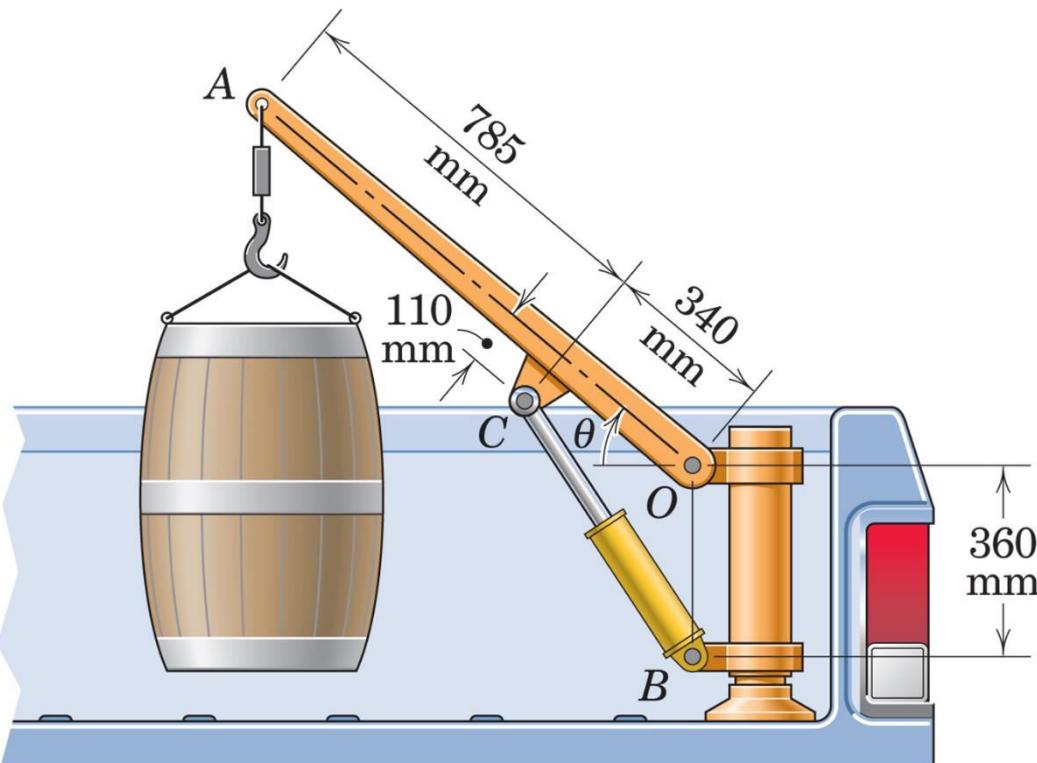
$$\begin{aligned} F &= pA = 175(0.750)(0.6) \\ &= 78.8 \text{ N} \end{aligned}$$

$$\begin{aligned} \underline{M}_o &= \underline{r} \times \underline{F} = 1.375\hat{j} \times (-78.8\hat{i}) \\ &= 108.3\hat{k} \text{ N}\cdot\text{m} \end{aligned}$$

# ENGINEERING MECHANICS

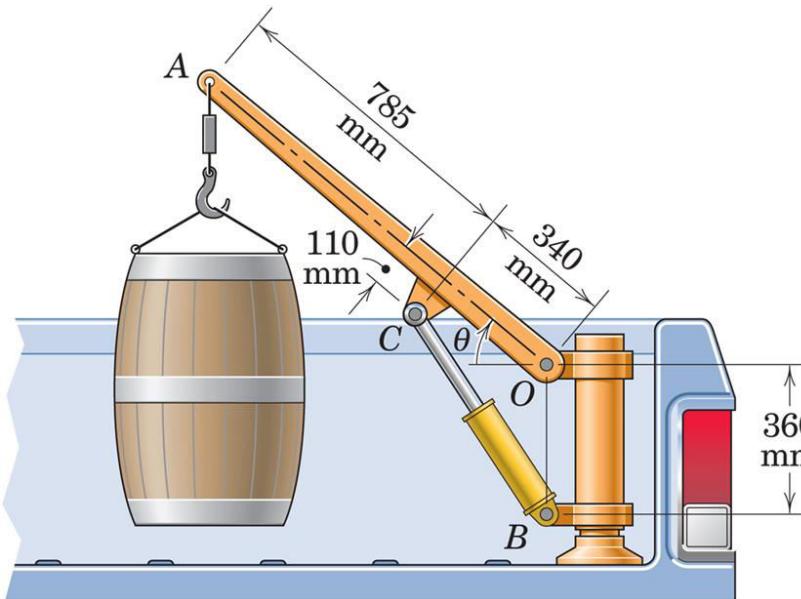
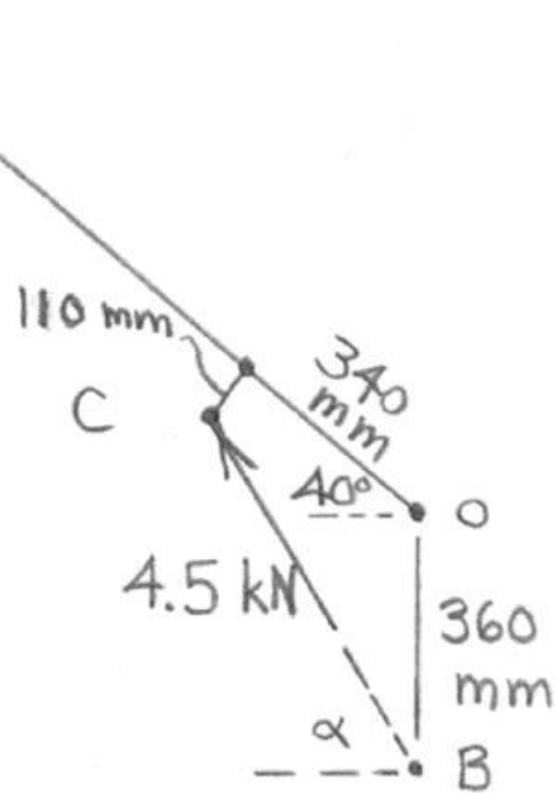
## Moment - Numerical

2/53) The small crane is mounted along the side of a pickup bed and facilitates the handling of heavy loads. When the boom elevation angle is  $\theta = 40^\circ$ , the force in the hydraulic cylinder BC is 4.5 k.N, and this force applied at point C is in the direction from B to C (the cylinder is in compression). Determine the moment of this 4.5-kN force about the boom pivot point O.



# ENGINEERING MECHANICS

## Moment - Numerical

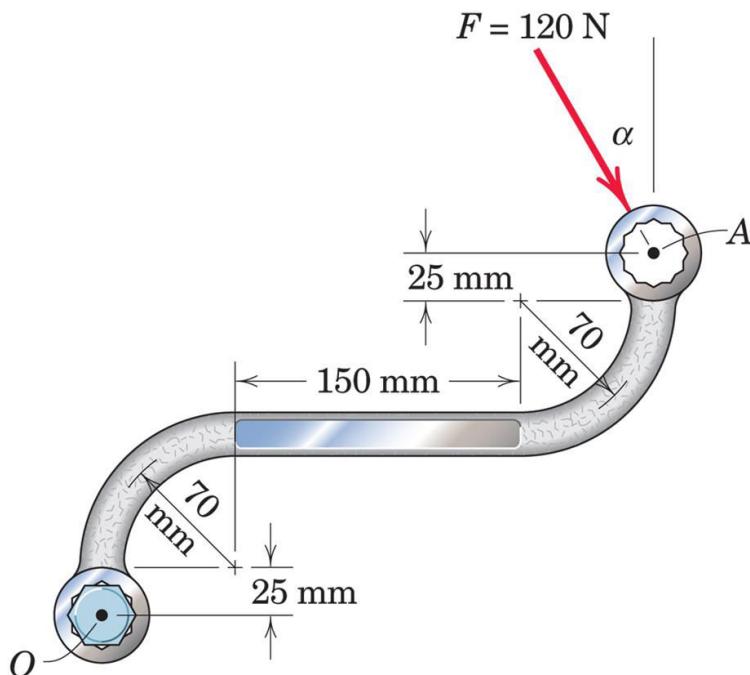


$$\alpha = \tan^{-1} \left[ \frac{360 + 340 \sin 40^\circ - 110 \sin 50^\circ}{340 \cos 40^\circ + 110 \cos 50^\circ} \right]$$

$$= 56.2^\circ$$

$$\therefore M_o = 4.5 (0.360 \cos 56.2^\circ) = \underline{\underline{0.902 \text{ kN}\cdot\text{m CW}}}$$

2/54) The 120-N force is applied as shown to one end of the curved wrench. If  $a = 30"$ , calculate the moment of  $F$  about the center 0 of the bolt. Determine the value of  $a$  which would maximize the moment about 0; state the value of this maximum moment.



$$\begin{aligned} (M_O)_{\max} &= 120 \sqrt{(25+70+25+70)^2 + (70+150+70)^2} \\ &= 41600 \text{ N}\cdot\text{mm} \text{ or } \underline{\underline{41.6 \text{ N}\cdot\text{m CW}}} \end{aligned}$$

$$\begin{aligned} M_O &= 120 \cos 30^\circ [70 + 150 + 70] \\ &\quad + 120 \sin 30^\circ [25 + 70 + 70 + 25] = 41500 \text{ N}\cdot\text{mm} \\ \text{or } M_O &= \underline{\underline{41.5 \text{ N}\cdot\text{m CW}}} \end{aligned}$$

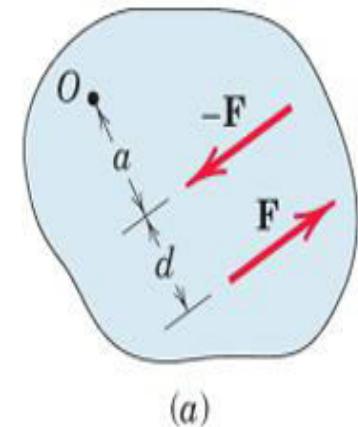
# ENGINEERING MECHANICS

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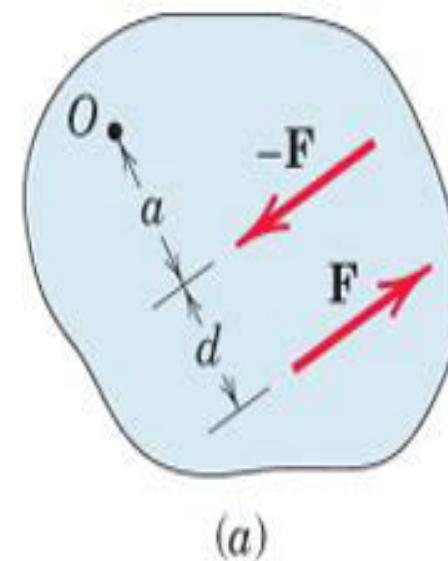
## Couple

Department of Civil Engineering

- The moment produced by two equal, opposite, and noncollinear forces is called a **couple**.
- **Couples** have certain unique properties and have important applications in Mechanics.
- Consider the action of two equal and opposite forces  $F$  and  $-F$  at a distance  $d$  apart as shown in figure.
- These two forces can not be combined into a single force, as their sum in every direction is zero.

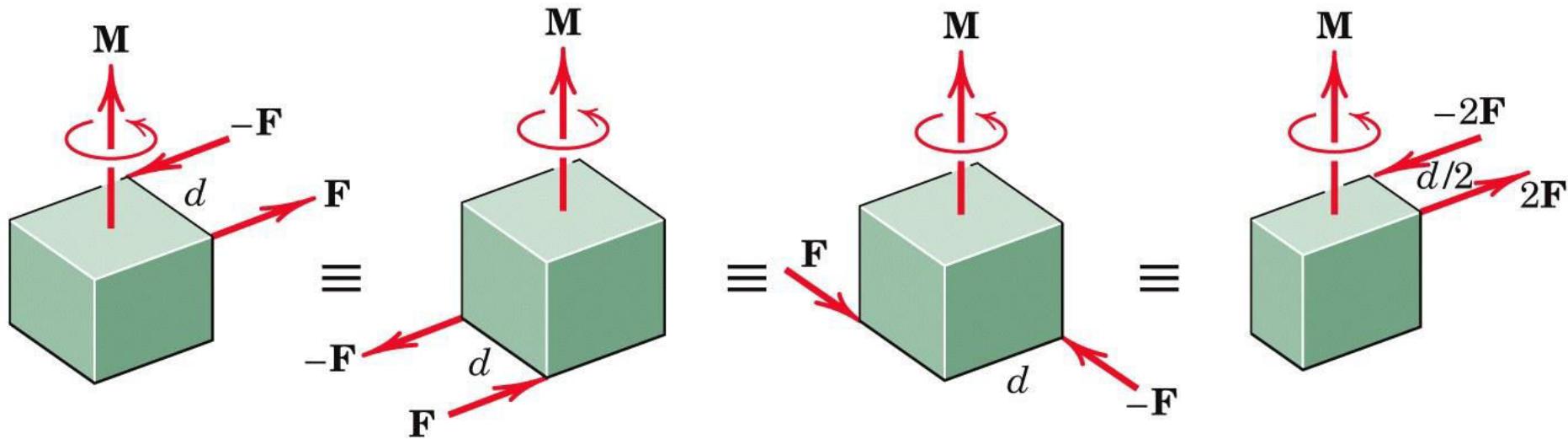


- Their only effect is to produce a tendency of rotation.
- *The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple M.*
- The couple has a magnitude
- $M = F(a+d) - Fa$
- $M = Fd$



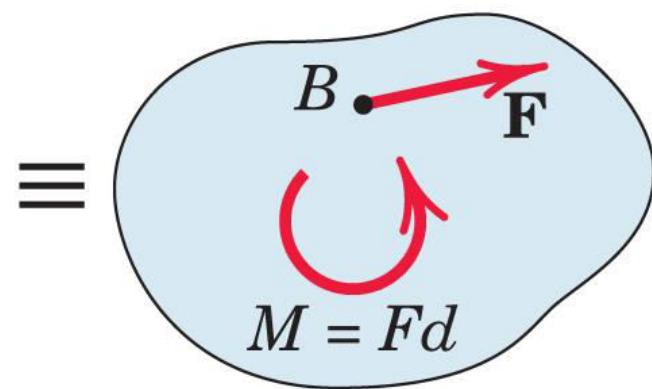
**Equivalent Couples:** Changing the values of  $F$  and  $d$  does not change a given couple as long as the product  $Fd$  remains the same.

Similarly, a **couple is not affected** if the forces act in a **different but parallel plane**.



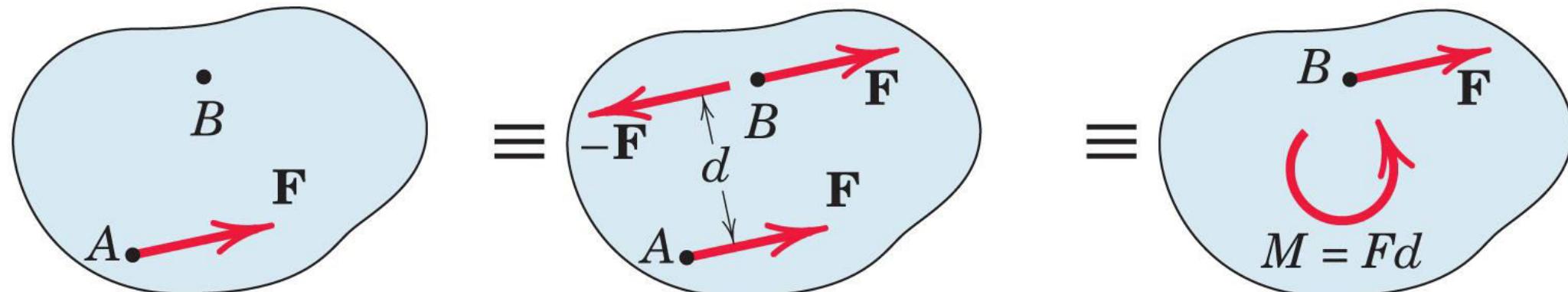
### Force Couple System:

- The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force and to rotate the body about any fixed axis which does not intersect the line of the force.

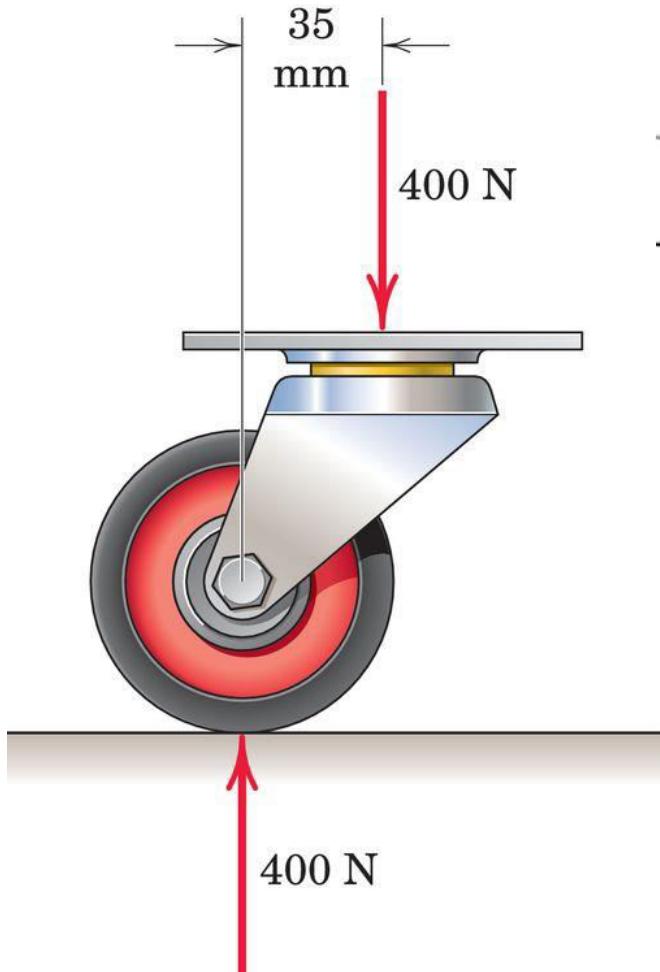


### Force Couple System:

- Consider a force  $F$  which is acting at point A.
- This can be replaced by an equal force  $F$  at some other point B and counter clockwise couple  $M = Fd$ .
- This combination of the force and couple in the diagram below is referred to as the *force-couple system*.



2/59) The caster unit is subjected to the pair of 400-N forces shown. Determine the moment associated with these forces.

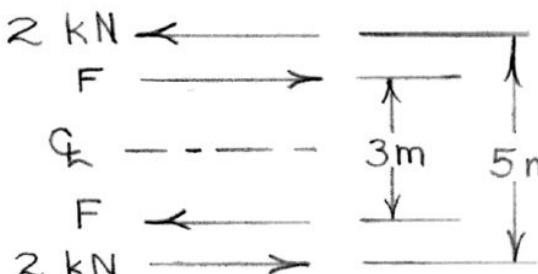


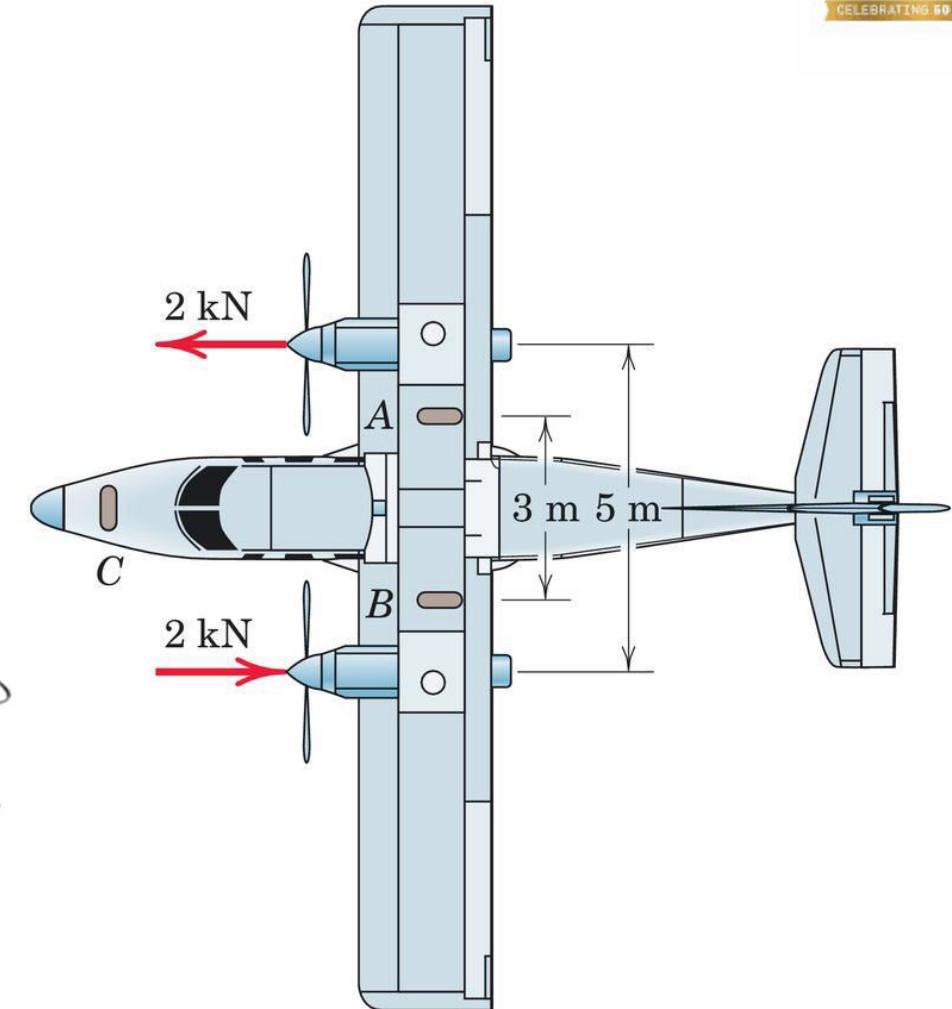
$$\begin{array}{l} \boxed{2/59} \rightarrow M = Fd = 400(0.035) \\ = \underline{\underline{14 \text{ N}\cdot\text{m CW}}} \end{array}$$

# ENGINEERING MECHANICS

## Couple - Numerical

2/63) As part of a test, the two aircraft engines are revved up and the propeller pitches are adjusted so as to result in the fore and aft thrusts shown. What force F must be exerted by the ground on each of the main braked wheels at A and B to counteract the turning effect of the two propeller thrusts? Neglect any effects of the nose wheel C, which is turned 90° and unbraked.

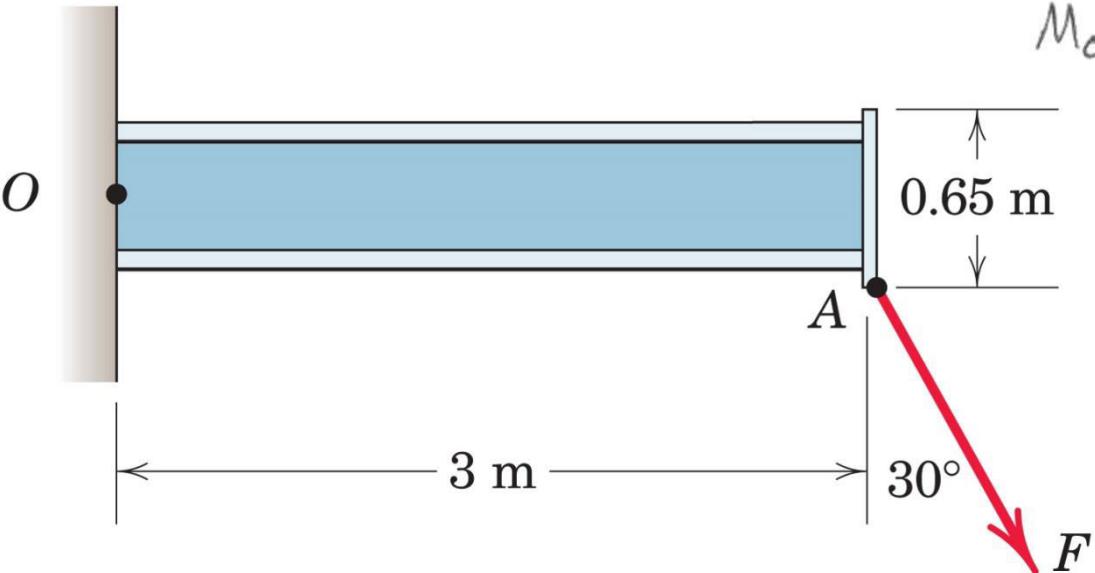

$$\text{At } C: \sum F_x = 0 \Rightarrow F - 2\text{ kN} = 0 \Rightarrow F = 2\text{ kN}$$
$$\text{At } A: \sum M_C = 0 \Rightarrow 2(5) - F(3) = 0 \Rightarrow F = 3.33\text{ kN}$$



# ENGINEERING MECHANICS

## Couple - Numerical

2/64) The cantilevered W530 X 150 beam shown is subjected to an 8-kN force F applied by means of a welded plate at A. Determine the equivalent force couple system at the centroid of the beam cross section at the cantilever O.

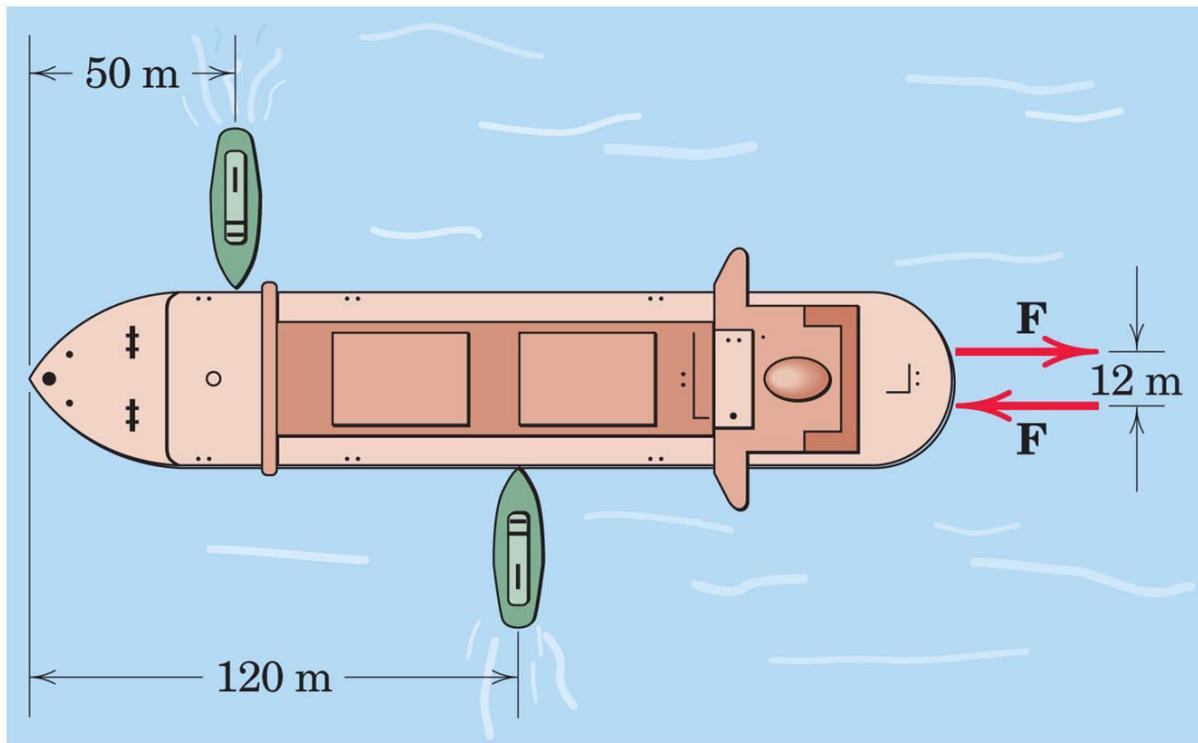


$$M_O = 8 \cos 30^\circ (3) - 8 \sin 30^\circ \left( \frac{0.65}{2} \right) \rightarrow M_O = 19.48 \text{ kN}\cdot\text{m CW}$$

# ENGINEERING MECHANICS

## Couple - Numerical

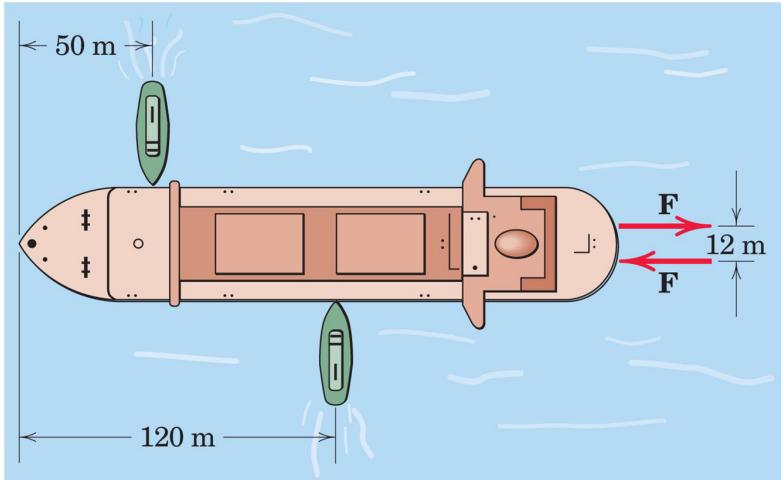
2/65) Each propeller of the twin-screw ship develops a full-speed thrust of 300 kN. In maneuvering the ship, one propeller is turning full speed ahead and the other full speed in reverse. What thrust  $P$  must each tug exert on the ship to counteract the effect of the ship's propellers?



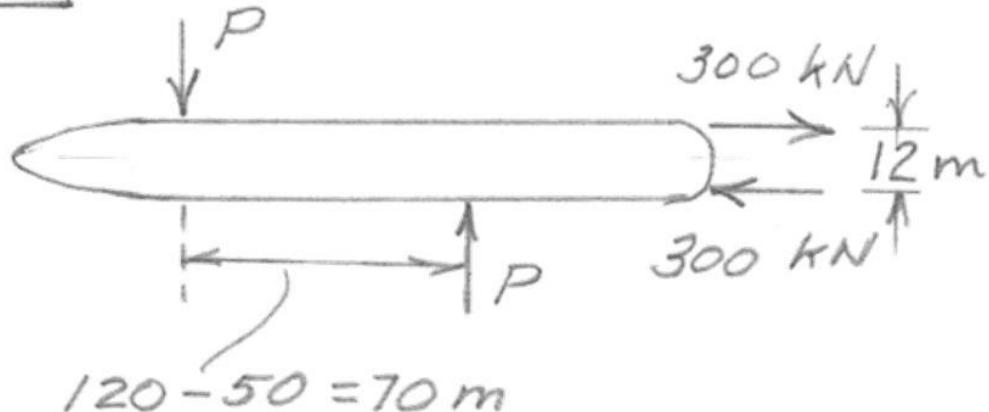
# ENGINEERING MECHANICS

## Couple - Numerical

2/65)



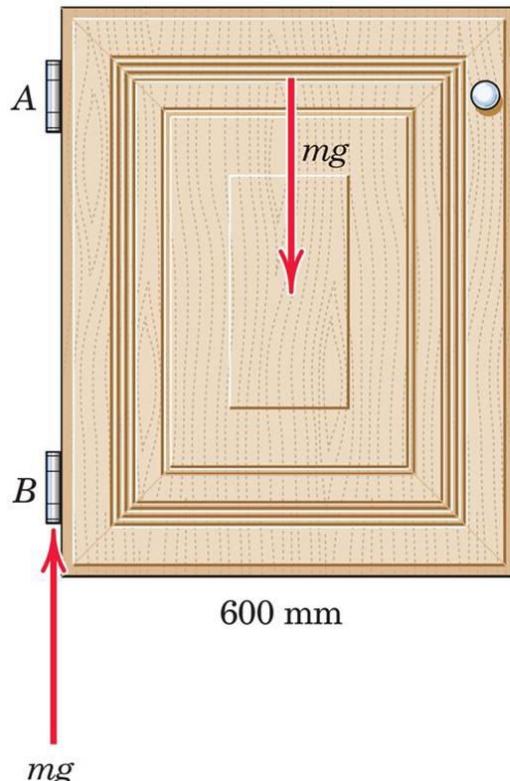
2/65



$$70P = 300(12)$$

$$\underline{P = 51.4 \text{ kN}}$$

2/66) The upper hinge A of the uniform cabinet door has malfunctioned, causing the entire weight  $mg$  of the 5-kg door to be carried by the lower hinge B . Determine the couple associated with these two forces. You may neglect the slight offSet from the edge of the cabinet door to the hinge centerline.

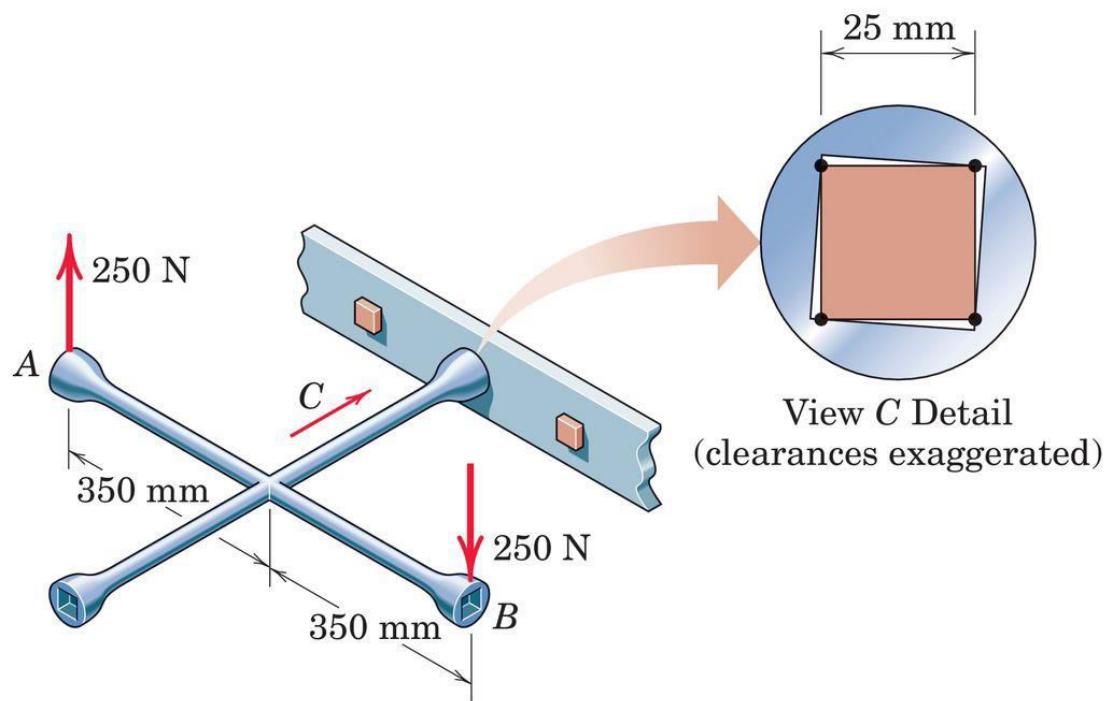


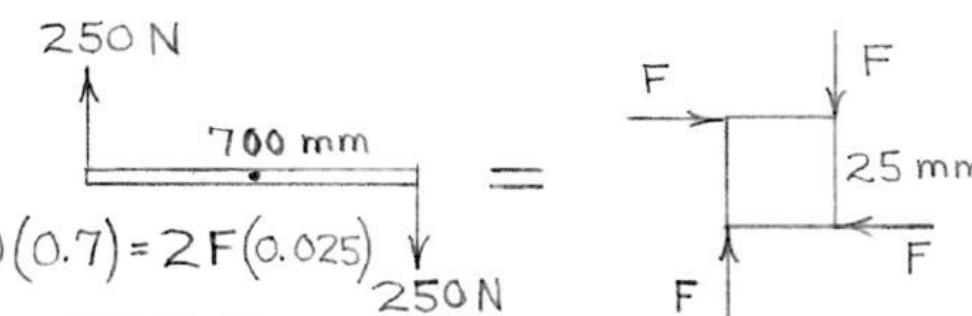
$$M = \frac{600/2}{1000} (5)(9.8) \rightarrow M = 14.72 \text{ N}\cdot\text{m CW}$$

# ENGINEERING MECHANICS

## Couple - Numerical

2/67) A lug wrench is used to tighten a square-head bolt. If 250-N forces are applied to the wrench as shown, determine the magnitude  $F$  of the equal forces exerted on the four contact points on the 25-mm bolt head so that their external effect on the bolt is equivalent to that of the two 250-N forces. Assume that the forces are perpendicular to the flats of the bolt head.



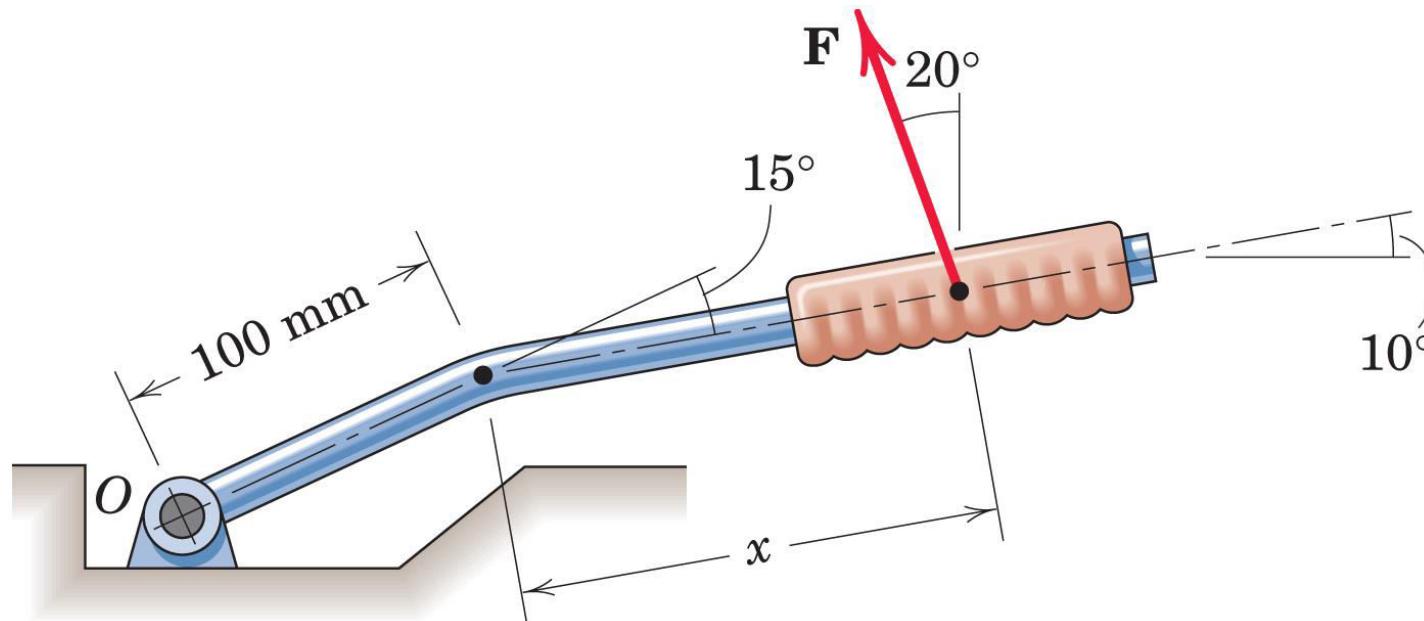
$$M = 250(0.7) = 2F(0.025)$$
$$\underline{F = 3500 \text{ N}}$$


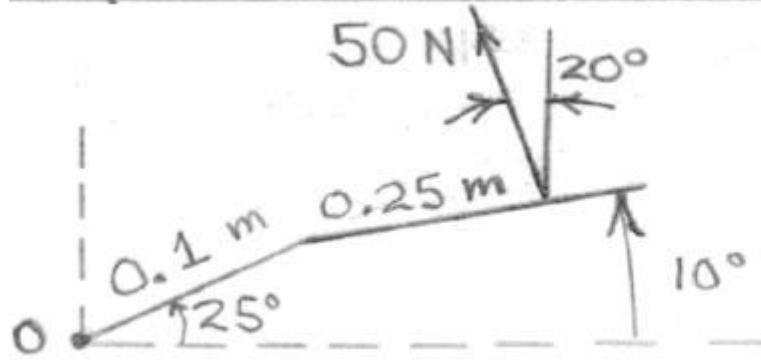
A free body diagram of the bolt head is shown. It consists of a square with a side length of 25 mm. Four forces, each of magnitude  $F$ , are applied perpendicular to the sides of the square. The top force is directed upwards, the bottom force downwards, the left force to the right, and the right force to the left. The center of the square is marked with a dot. A horizontal double-headed arrow below the square indicates a distance of 700 mm from the center to the point where the forces are applied. Above the square, a vertical double-headed arrow indicates a height of 250 mm from the base to the top edge. An equals sign follows the free body diagram, indicating that the moment produced by the wrench's forces is equivalent to the moment produced by the four forces on the bolt head.

# ENGINEERING MECHANICS

## Couple - Numerical

2/69) A force  $F$  of magnitude 50 N is exerted on the automobile parking-brake lever at the position  $x = 250$  mm. Replace the force by an equivalent force-couple system at the pivot point 0.





Use principle of moments.

$$\begin{aligned}
 \sum M_O = & 50 \cos 20^\circ [0.1 \cos 25^\circ + 0.25 \cos 10^\circ] \\
 & + 50 \sin 20^\circ [0.1 \sin 25^\circ + 0.25 \sin 10^\circ] \\
 = & 17.29 \text{ N}\cdot\text{m}
 \end{aligned}$$

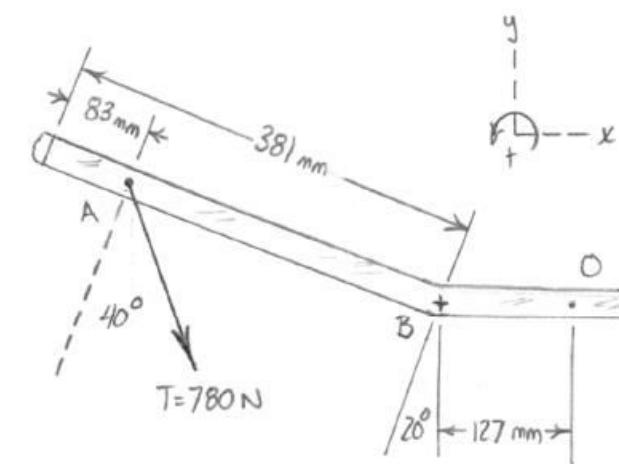
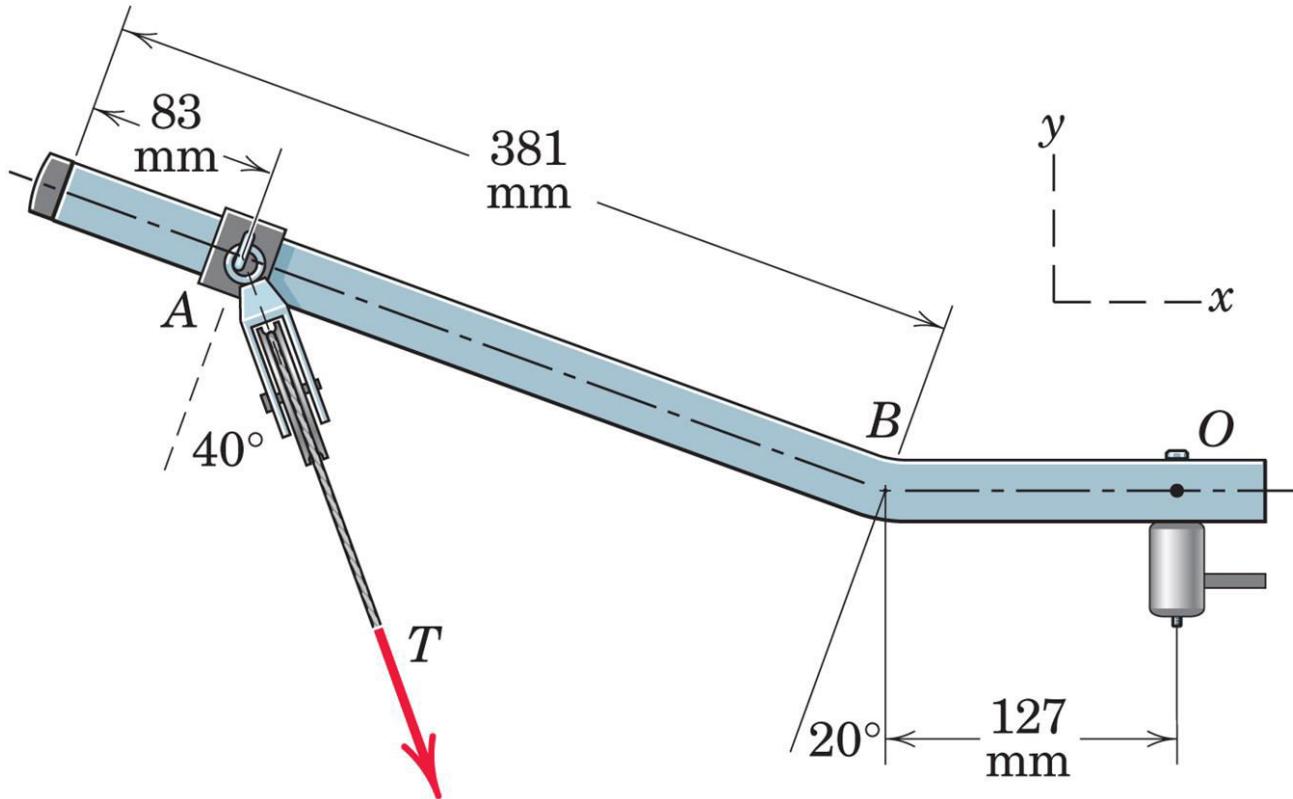
Force - Couple System at O

$$\left\{
 \begin{array}{l}
 R = 50 \text{ N} \quad \nearrow 110^\circ \\
 M_O = 17.29 \text{ N}\cdot\text{m}
 \end{array}
 \right.$$

# ENGINEERING MECHANICS

## Couple - Numerical

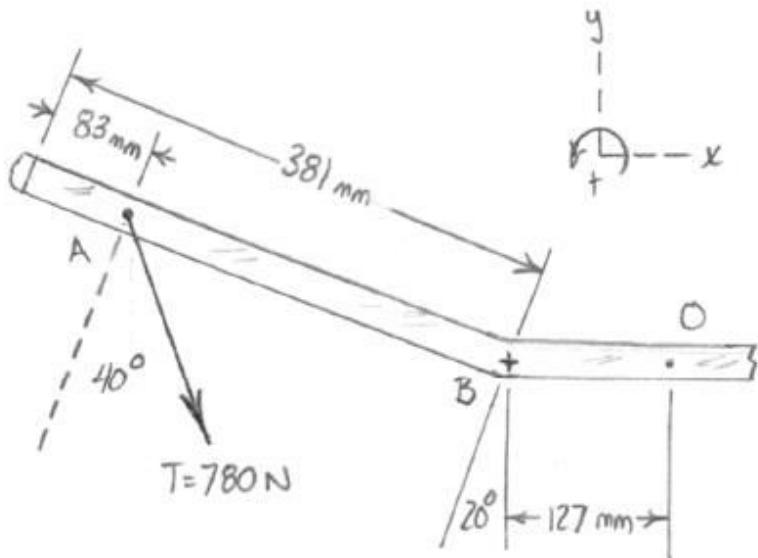
2/70) An overhead view of a portion of an exercise machine is shown. If the tension in the cable is  $T = 780$  , determine the equivalent force-couple system at (a) point B and at (b) point O. Record your answers in vector format.



# ENGINEERING MECHANICS

## Couple - Numerical

2/70)

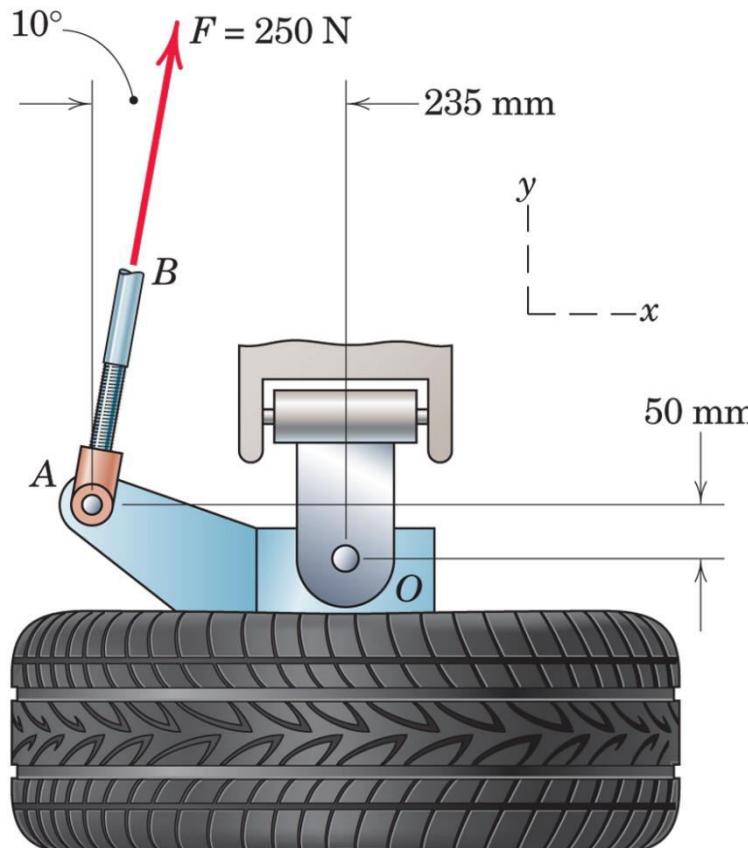


$$M_B = 780 \cos 40^\circ \left( \frac{381 - 83}{1000} \right) \text{ k} \rightarrow M_B = 178.1 \text{ k N}\cdot\text{m}$$

$$M_O = M_B + T \cos 20^\circ (\overline{OB}) = \left( 178.1 + 780 \cos 20^\circ \left( \frac{127}{1000} \right) \right) \text{k}$$

$$\therefore M_O = 271 \text{ k N}\cdot\text{m}$$

2/71) The tie-rod AB exerts the 250-N force on the steering knuckle AO as shown. Replace this force by an equivalent force-couple system at O.

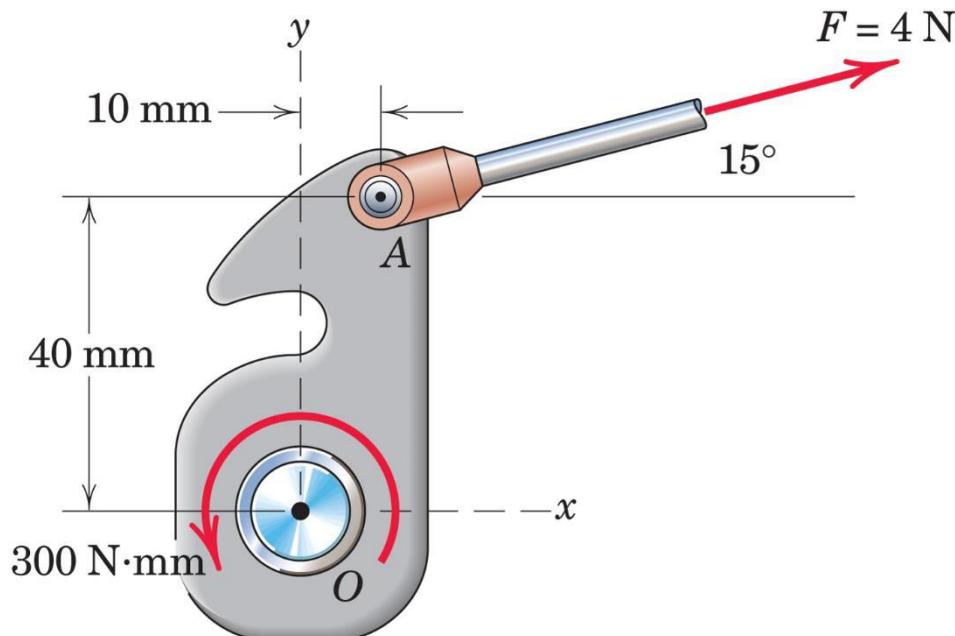


$$\begin{aligned} \therefore M_O &= 250 [\cos 10^\circ (0.235) + \sin 10^\circ (0.050)] \\ &= \underline{\underline{60.0 \text{ N}\cdot\text{m} \text{ CW}}} \end{aligned}$$

# ENGINEERING MECHANICS

## Couple - Numerical

2/76) The device shown is a part of an automobile seatback- release mechanism. The part is subjected to the 4-N force exerted at A and a 300-N · mm restoring moment exerted by a hidden torsional spring. Determine the y-intercept of the line of action of the single equivalent force.

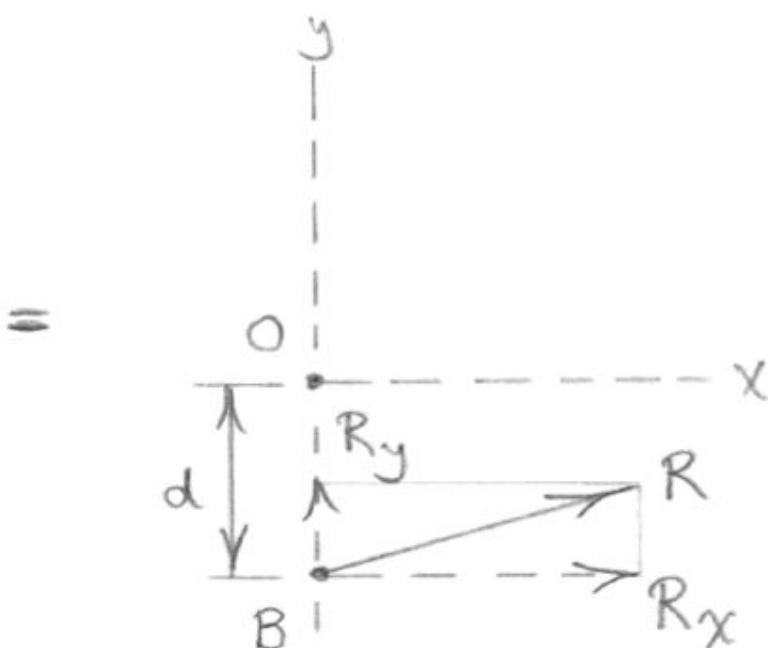
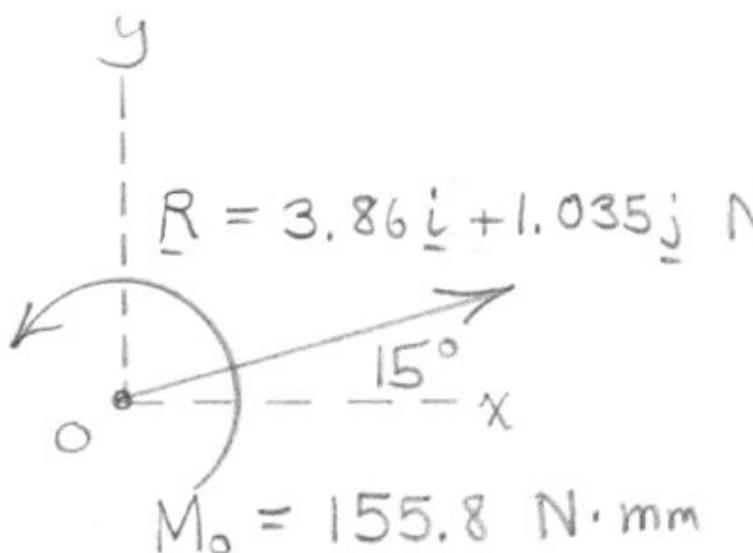




At O :

$$\underline{R} = 4 (\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j}) = 3.86 \underline{i} + 1.035 \underline{j} \text{ N}$$

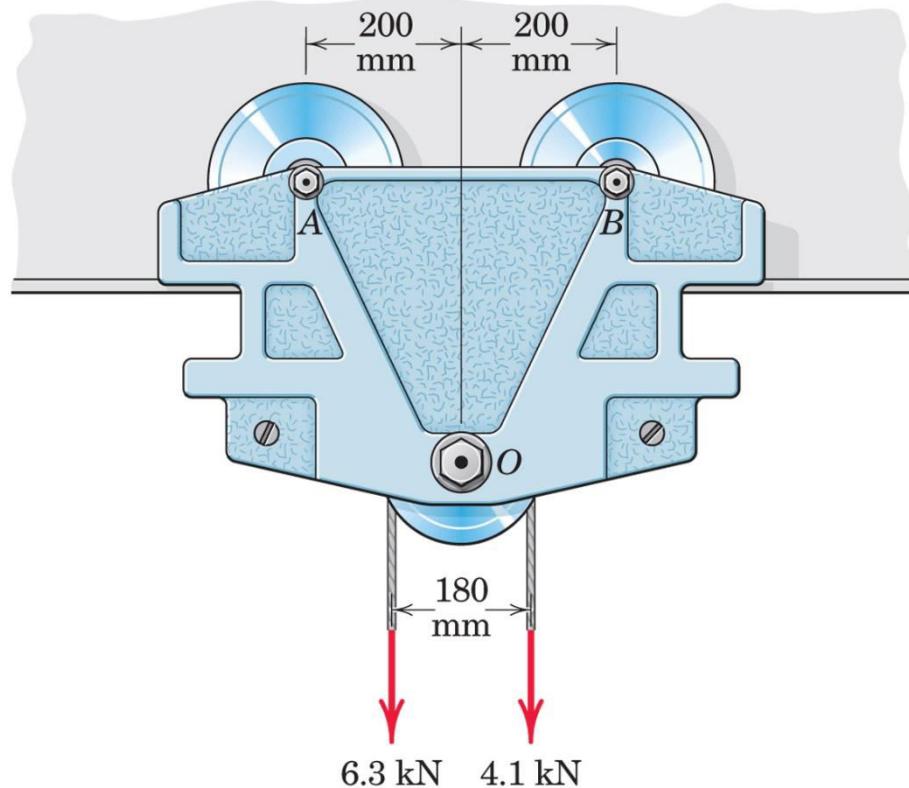
$$\begin{aligned}\text{At } M_O &= 300 - 4 \cos 15^\circ (40) + 4 \sin 15^\circ (10) \\ &= 155.8 \text{ N-mm CCW}\end{aligned}$$



Condition:  $R_x d = M_o$

$$3.86 d = 155.8, \quad d = 40.3 \text{ mm}$$

2/77) Replace the two cable tensions which act on the pulley at O of the beam trolley by two parallel forces which act at the track-wheel connections A and B.



# ENGINEERING MECHANICS

## Resultants - Numerical

$$\begin{cases} \sum F_y: & F_A + F_B = 6.3 + 4.1 \\ \sum M_O: & 200 F_A - 200 F_B = 90(6.3) - 90(4.1) \end{cases}$$

SOLVING...

$$\begin{cases} F_A = 5.70 \text{ kN} \\ F_B = 4.70 \text{ kN} \end{cases}$$

(Both Down As Shown)

# ENGINEERING MECHANICS

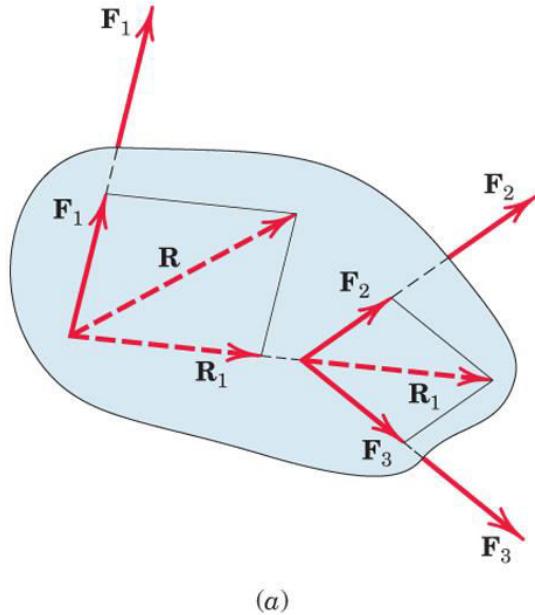
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## Resultants

Department of Civil Engineering

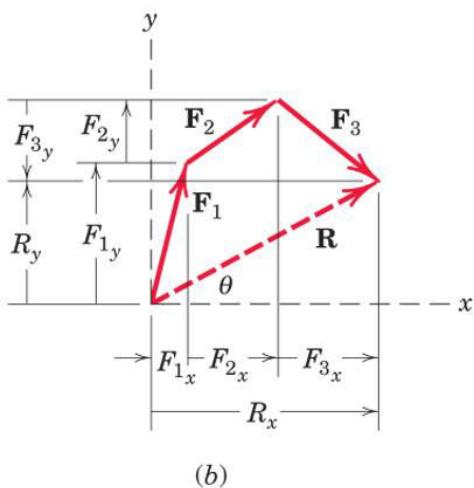
# ENGINEERING MECHANICS

## Resultants - Numerical



(a)

The resultant of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied



(b)

# ENGINEERING MECHANICS

## Resultants - Numerical

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots = \Sigma \mathbf{F}$$

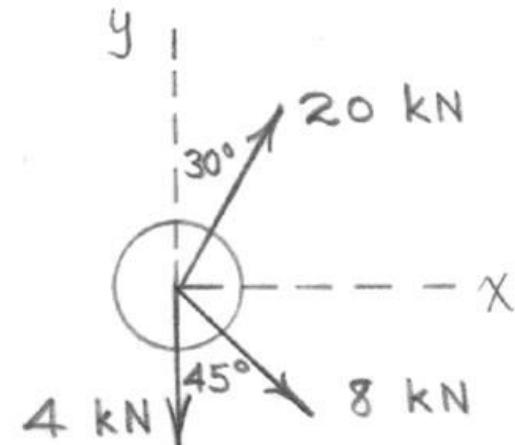
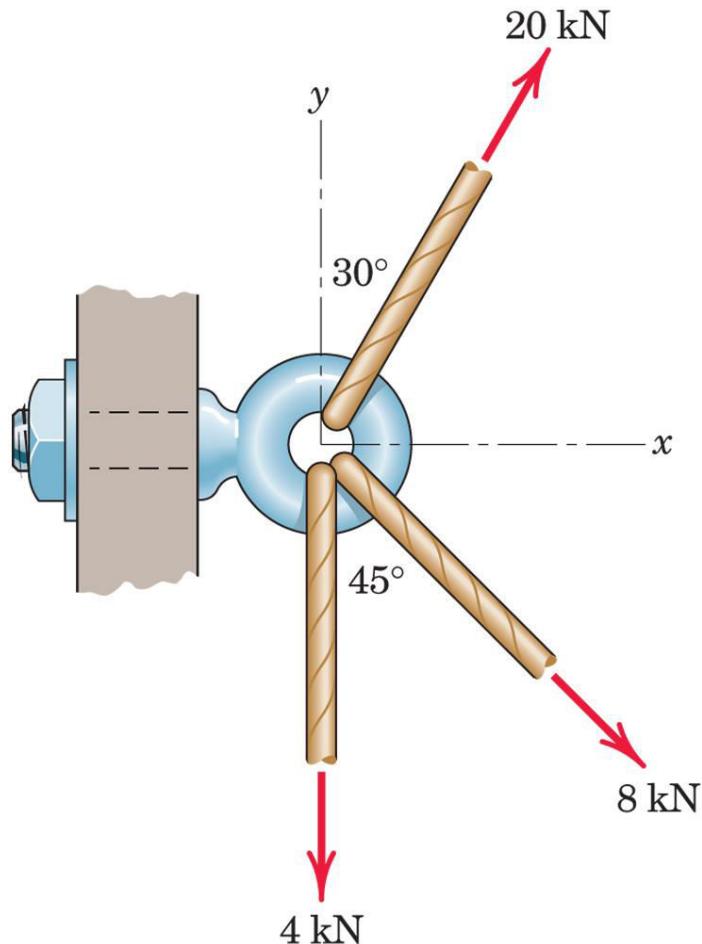
$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \quad (2/9)$$

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

# ENGINEERING MECHANICS

## Resultants - Numerical

2/79) Determine the resultant R of the three tension forces acting on the eye bolt. Find the magnitude of R and the angle  $\theta$  which R makes with the positive x-axis.



$$R_x = \sum F_x = 20 \sin 30^\circ + 8 \sin 45^\circ = 15.66 \text{ kN}$$

$$R_y = \sum F_y = 20 \cos 30^\circ - 8 \cos 45^\circ - 4 = 7.66 \text{ kN}$$

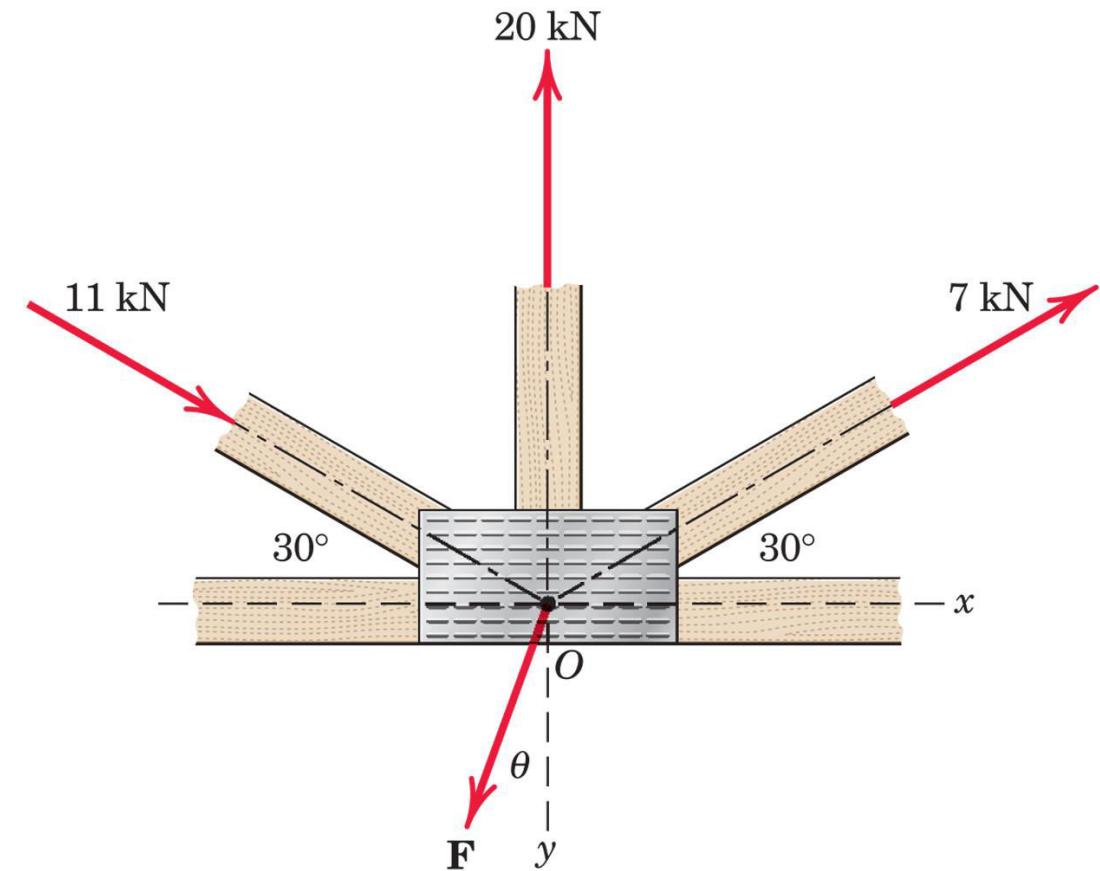
$$R = \sqrt{R_x^2 + R_y^2} = 17.43 \text{ kN}$$

$$\theta_x = \tan^{-1}(R_y/R_x) = 26.1^\circ$$

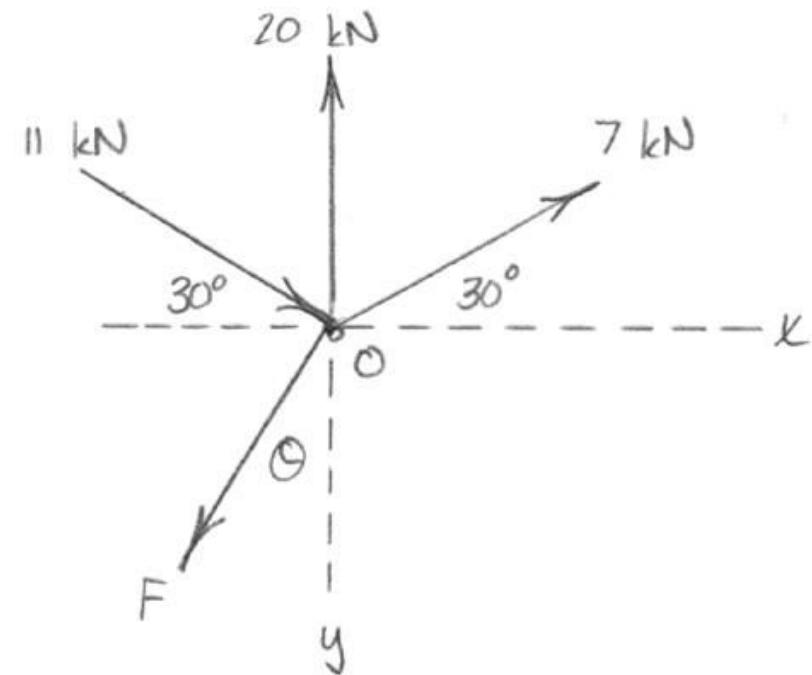
# ENGINEERING MECHANICS

## Resultants - Numerical

2/80) Determine the force magnitude  $F$  and direction  $\theta$  (measured clockwise from the positive  $y$ -axis) that will cause the resultant  $R$  of the four applied forces to be directed to the right with a magnitude of 9 kN.



$$R = 9 \text{ kN, RIGHTWARD}$$



# ENGINEERING MECHANICS

## Resultants - Numerical



$$\begin{cases} R_x = 9 = 11\cos 30^\circ + 7\cos 30^\circ - F\sin \theta \\ R_y = 0 = 11\sin 30^\circ - 7\sin 30^\circ + F\cos \theta - 20 \end{cases}$$

SOLVING ..

$$\underline{F = 19.17 \text{ kN}} \quad \text{AND} \quad \underline{\theta = 20.1^\circ}$$

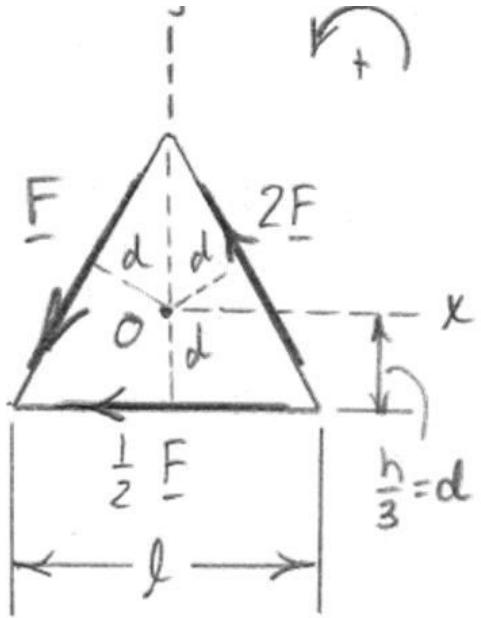
# ENGINEERING MECHANICS

## Resultants – Numerical 2/83)



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b)

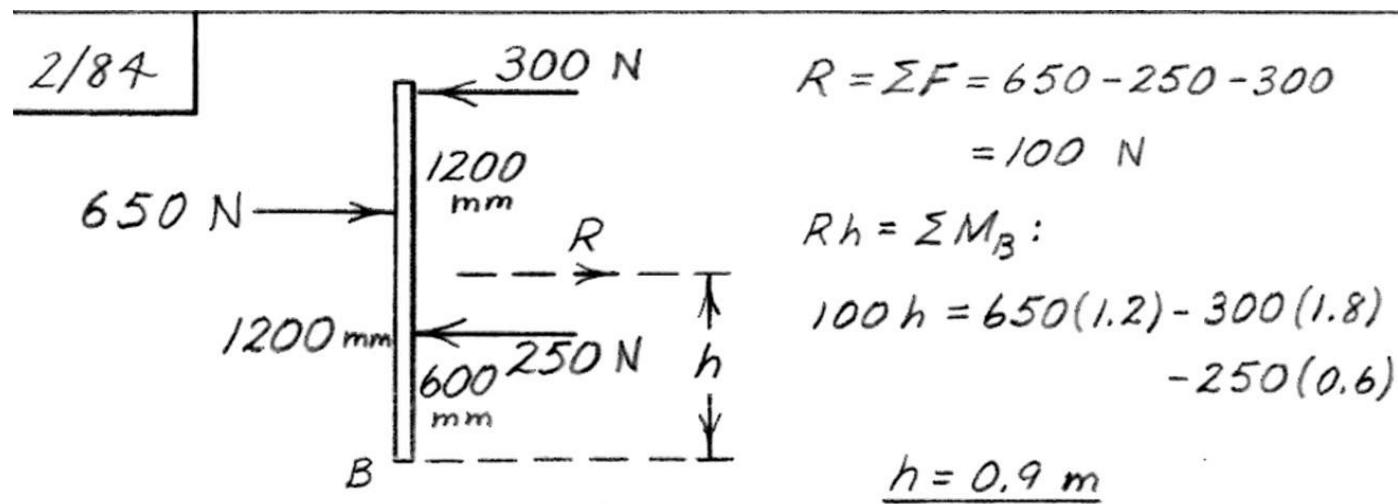
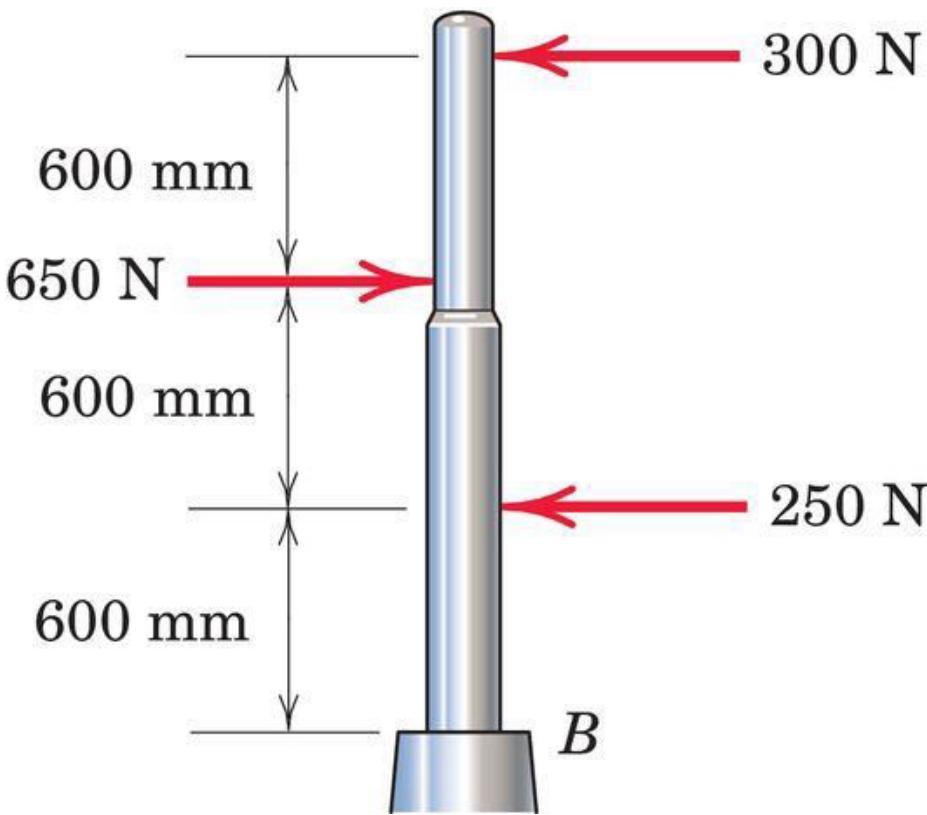


$$\left\{ \begin{array}{l} R = (-2F\cos 60^\circ - F\cos 60^\circ - \frac{1}{2}F)\hat{i} + (2F\sin 60^\circ - F\sin 60^\circ)\hat{j} \\ R = -2F\hat{i} + \frac{\sqrt{3}}{2}F\hat{j} \\ M_0 = 2Fd + Fd - \frac{1}{2}Fd \rightarrow M_0 = \frac{5\sqrt{3}}{12}Fl \text{ CCW} \end{array} \right.$$

To produce Accw moment at O with negative  $R_x$ , R is placed above O.

$$R_x y = M_0 \rightarrow 2Fy = \frac{5\sqrt{3}}{12}Fl \rightarrow y = \frac{5\sqrt{3}}{24}l \text{ above O}$$

2/84) Determine the height  $h$  above the base B at which the resultant of the three forces acts.

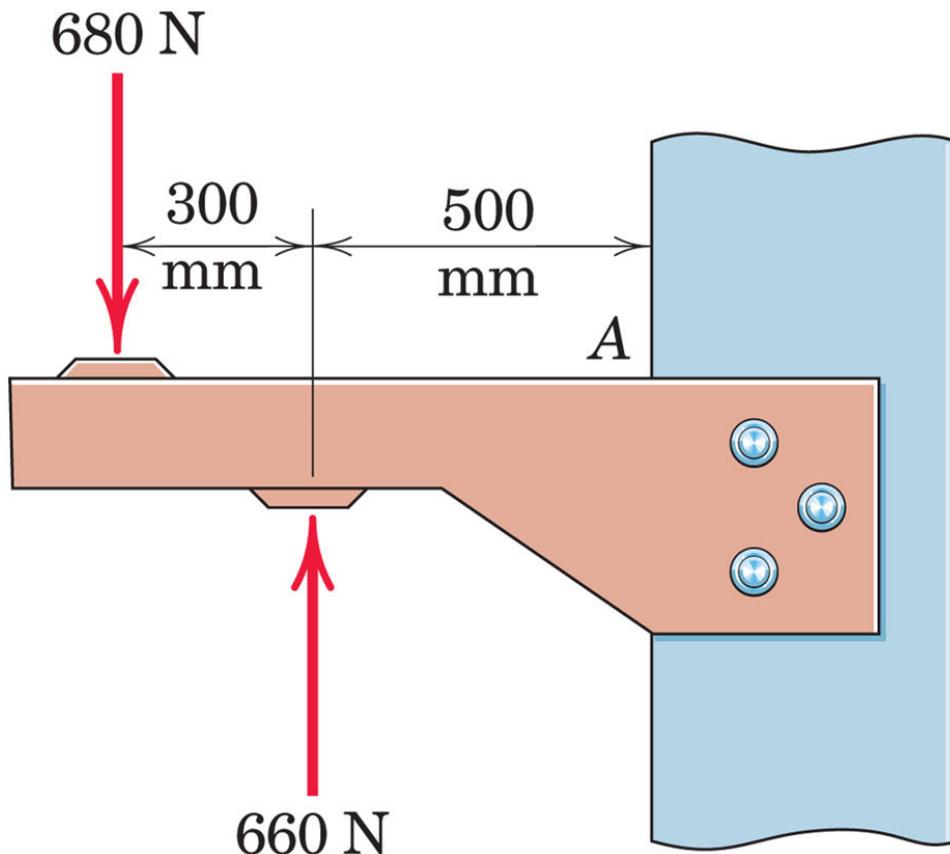


$$R = \sum F = 650 - 250 - 300 \\ = 100 \text{ N}$$

$$Rh = \sum M_B : \\ 100h = 650(1.2) - 300(1.8) \\ - 250(0.6)$$

$$\underline{h = 0.9 \text{ m}}$$

2/85) Where does the resultant of the two forces act?



$$R = \Sigma F = 680 - 660 = 20 \text{ N}$$

$$R_d = \Sigma M_A$$

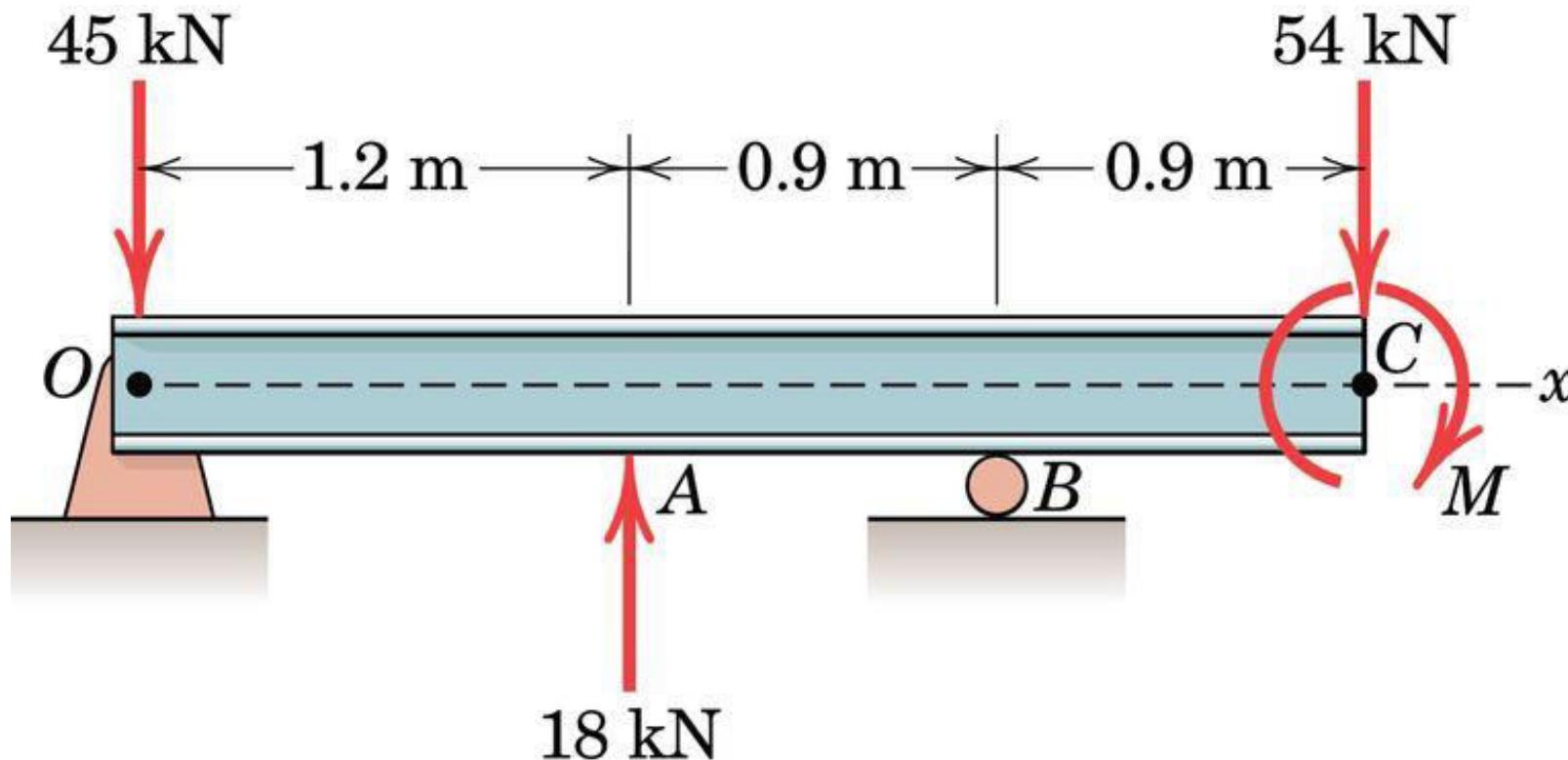
$$20d = 680(0.8) - 660(0.5)$$

$$\underline{d = 10.70 \text{ m to the left of } A}$$

# ENGINEERING MECHANICS

## Resultants - Numerical

2/86) If the resultant of the loads shown passes through point B, determine the equivalent force-couple system at O.



$$\underline{R = 81 \text{ kN down}}$$

$$\sum M_B = 0: 45(2.1) - 18(0.9) - 0.9(54) - M = 0$$

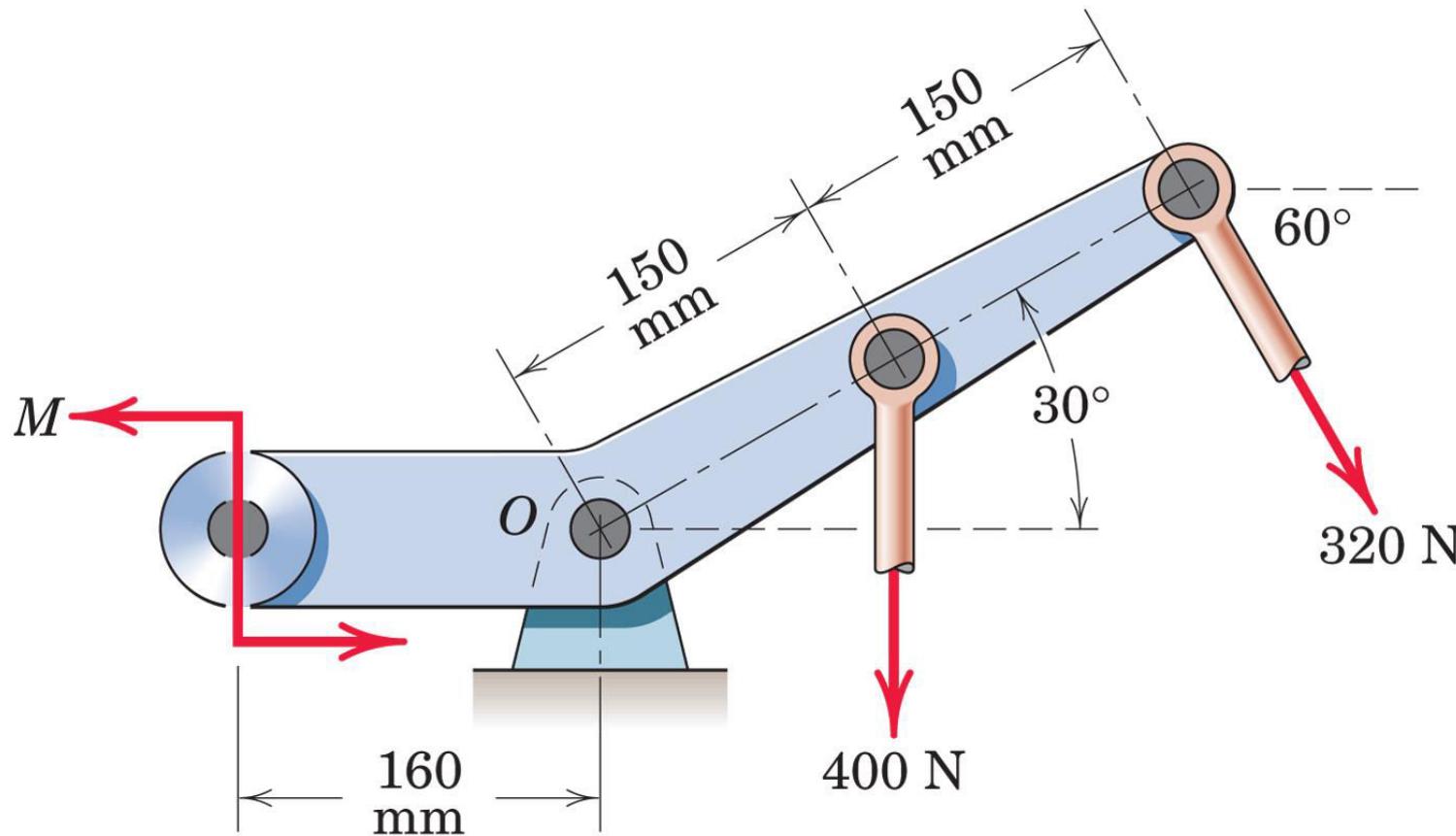
$$\text{so... } M = 29.7 \text{ kN}\cdot\text{m CW}$$

$$M_o = 18(1.2) - 54(3) - 29.7 = -170.1 \quad \text{so... } \underline{M_o = 170.1 \text{ kN}\cdot\text{m CW}}$$

# ENGINEERING MECHANICS

## Resultants - Numerical

2/87) If the resultant of the two forces and couple M passes through point O, determine M.

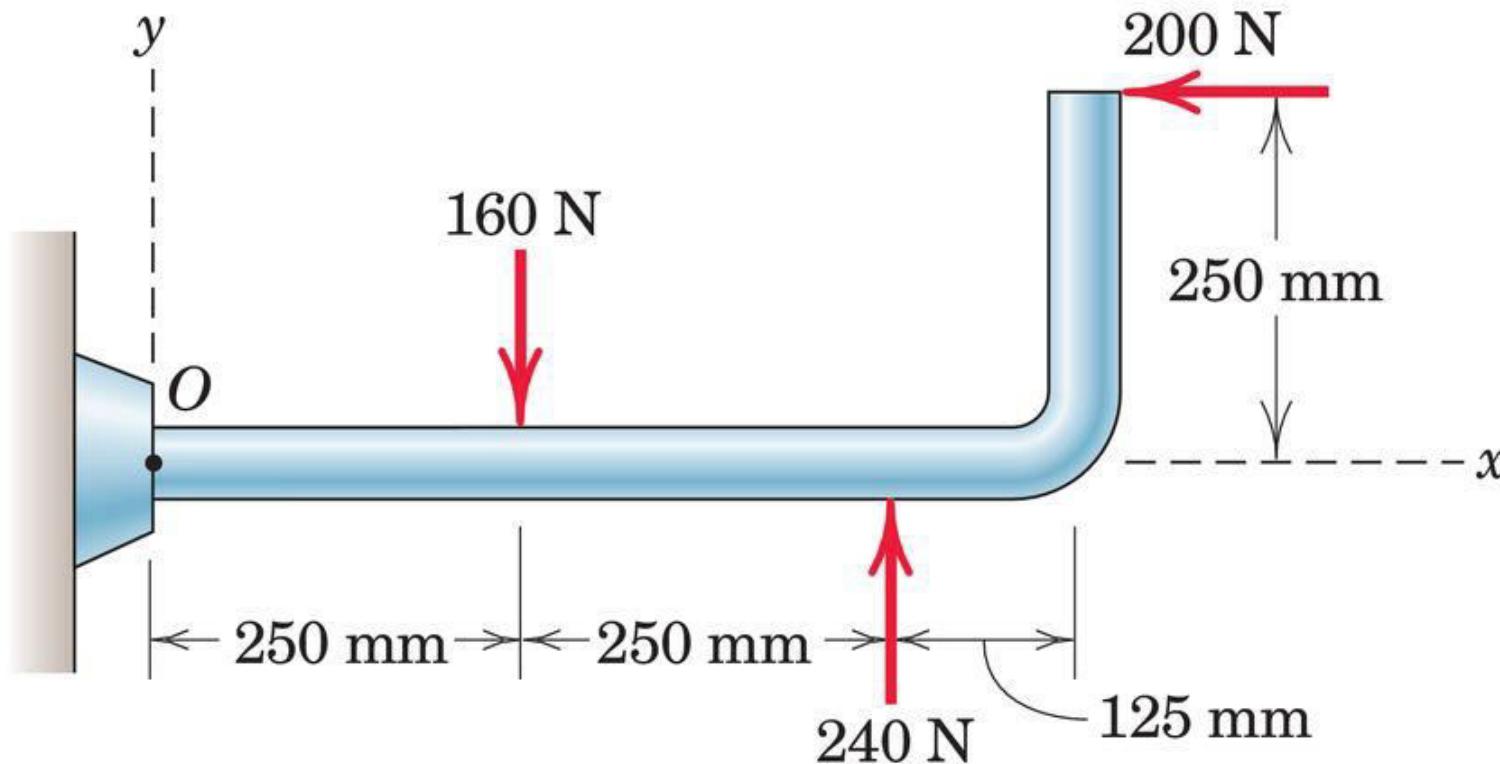


2/87  $M_o = 0$ , so

$$\sum M - 400(0.150 \cos 30^\circ) - 320(0.300) = 0$$

$$\underline{M = 148.0 \text{ N}\cdot\text{m}}$$

2/89) Replace the three forces acting on the bent pipe by a single equivalent force R. Specify the distance x from point O to the point on the x-axis through which the line of action of R passes.



# ENGINEERING MECHANICS

## Resultants - Numerical

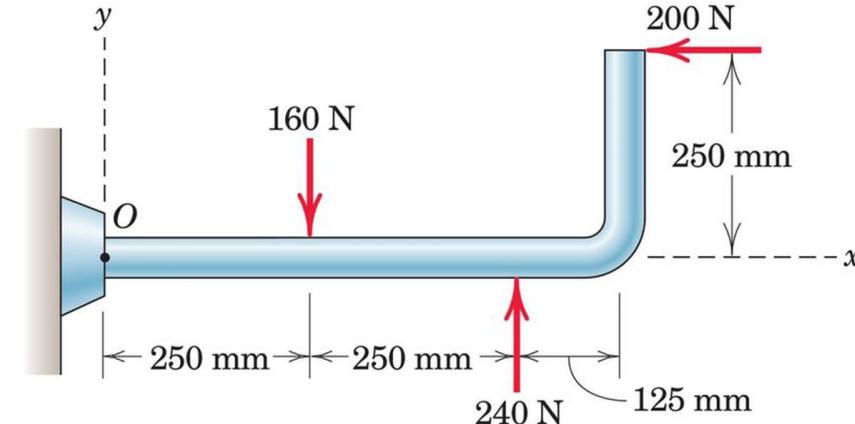
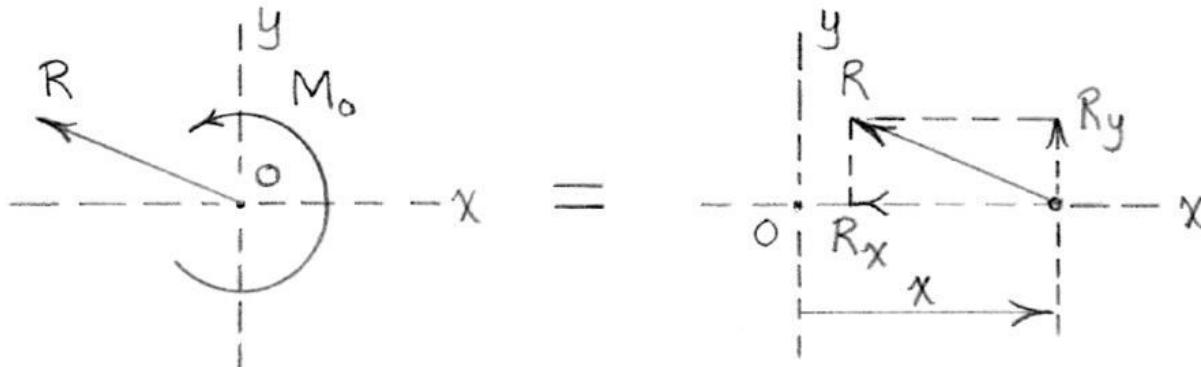


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2/89

$$R = -200i + 80j \text{ N}$$

$$\text{And } M_o = -160(0.25) + 240(0.50) + 200(0.25) = 130 \text{ N}\cdot\text{m}$$

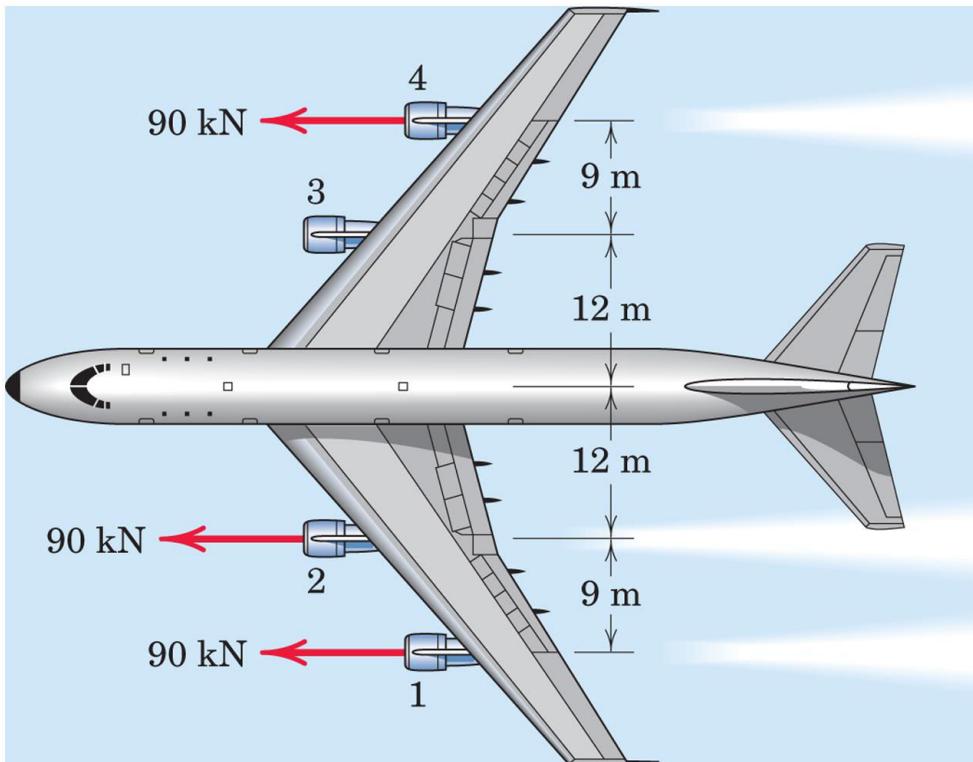


$$R_y x = M_o, \quad x = \frac{130}{80} = 1.625 \text{ m} \quad (\text{off pipe})$$

# ENGINEERING MECHANICS

## Resultants - Numerical

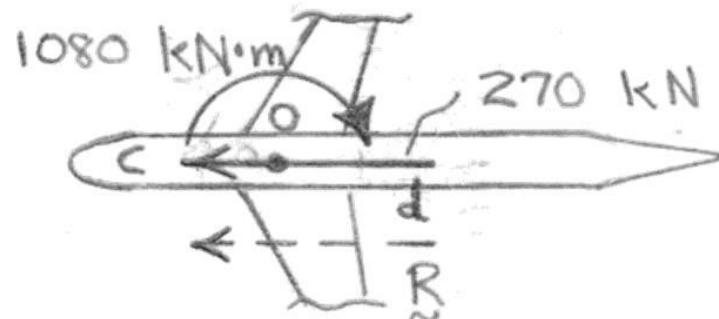
2/93) A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two dimensional problem.



2/93

Force - Couple system at point O:

$$\left\{ \begin{array}{l} R = 3(90) = 270 \text{ kN} \quad (\leftarrow) \\ +\circlearrowright M_O = 12(90) = 1080 \text{ kN}\cdot\text{m} \end{array} \right.$$



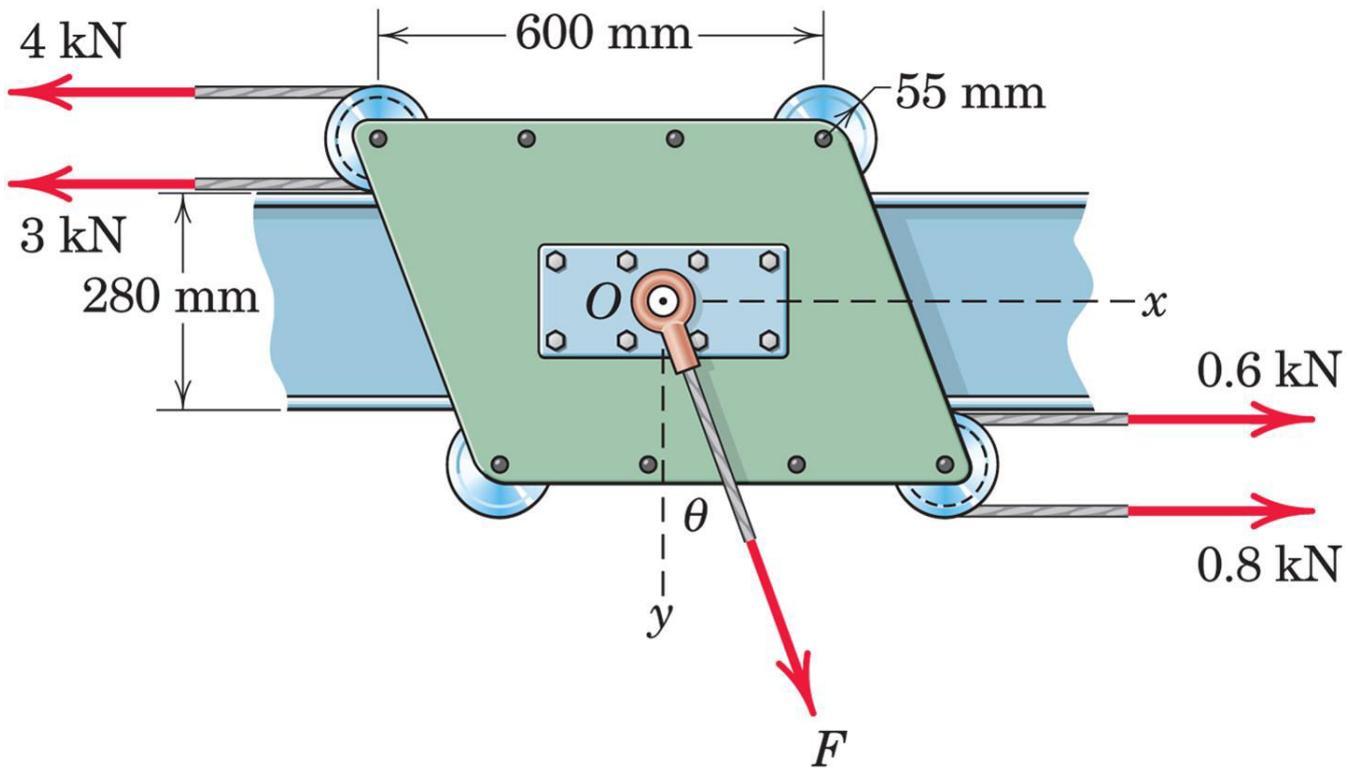
$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= 4 \text{ m}$$

# ENGINEERING MECHANICS

## Resultants - Numerical

2/98) Five forces are applied to the beam trolley as shown. Determine the coordinates of the point on the y-axis through which the stand-alone resultant R must pass if  $F = 5 \text{ kN}$  and  $\theta = 30^\circ$ .

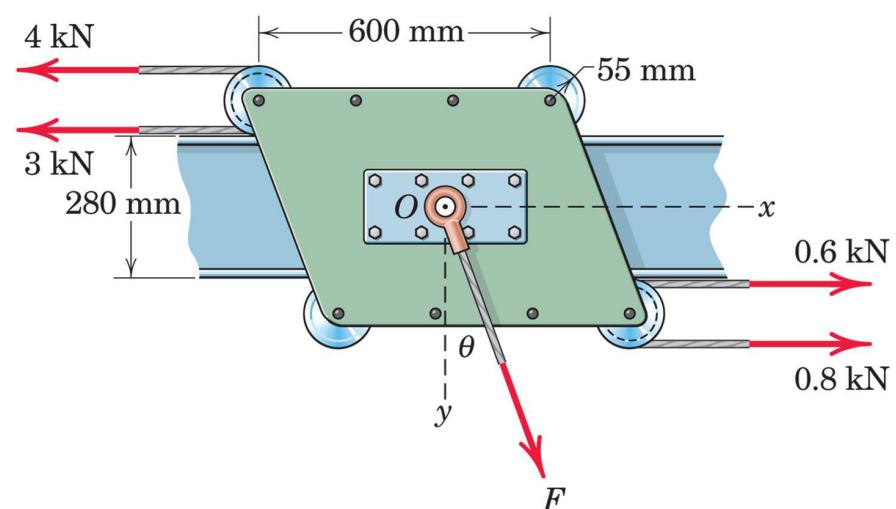


# ENGINEERING MECHANICS

## Resultants - Numerical

2/98)

$$\left\{ \begin{array}{l} \underline{R} = (0.8 + 0.6 + 5 \sin 30^\circ - 4 - 3) \underline{i} + 5 \cos 30^\circ \underline{j} \rightarrow \underline{R} = -3.10 \underline{i} + 4.33 \underline{j} \text{ kN} \\ \sum M_O = 0.6 \left( \frac{140}{1000} \right) + 0.8 \left( \frac{140 + 110}{1000} \right) + 3 \left( \frac{140}{1000} \right) + 4 \left( \frac{140 + 110}{1000} \right) = \\ \therefore \sum M_O = 1.704 \text{ kNm CCW} \end{array} \right.$$



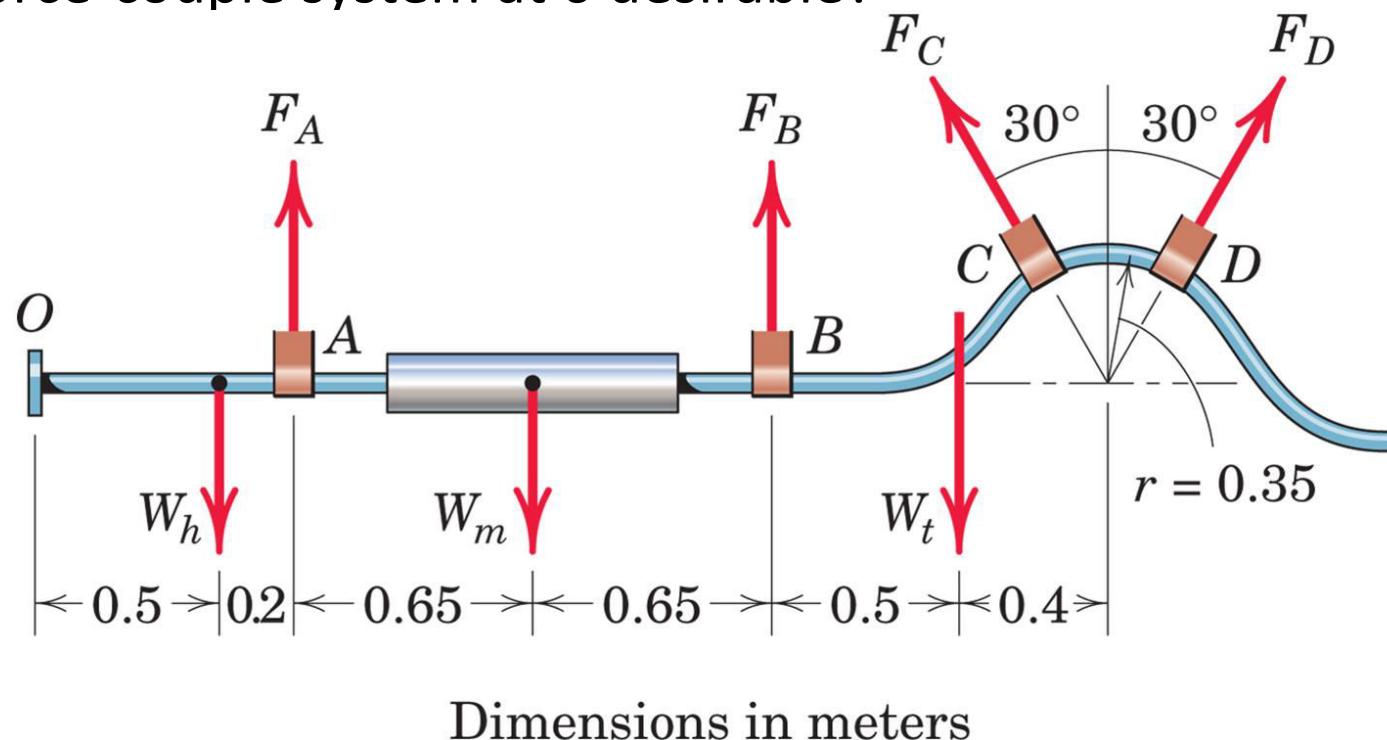
For A CCW  $M_O$  With NEGATIVE  $R_x$ ,  $R$  is PLACED ABOVE O IN MINUS y.

$$\left\{ \begin{array}{l} R_x y = M_O \rightarrow 3.10 |y| = 1.704 \rightarrow |y| = 0.550 \\ \therefore |y| = 550 \text{ mm ABOVE O or } (0, -550) \text{ (mm)} \end{array} \right.$$

# ENGINEERING MECHANICS

## Resultants - Numerical

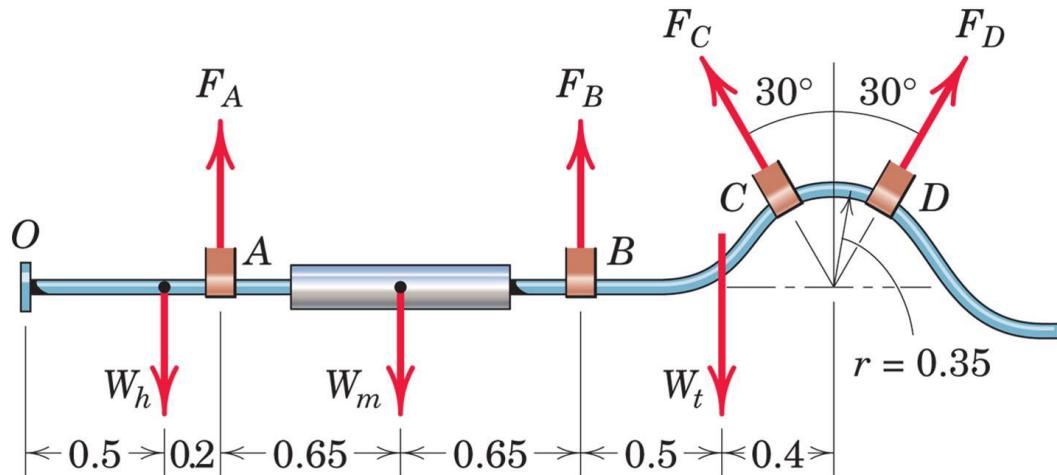
2/100) An exhaust system for a pickup truck is shown in the figure. The weights  $W_h$ ,  $W_m$  and  $W_t$  of the head pipe, muffler, and tailpipe are 10, 100, and 50 , respectively, and act at the indicated points. If the exhaust-pipe hanger at point A is adjusted so that its tension  $F_A$  is 50 N, determine the required forces in the hangers at points B, C, and D so that the force-couple system at point O is zero. Why is a zero force-couple system at O desirable?



# ENGINEERING MECHANICS

## Resultants - Numerical

2/100)



Dimensions in meters

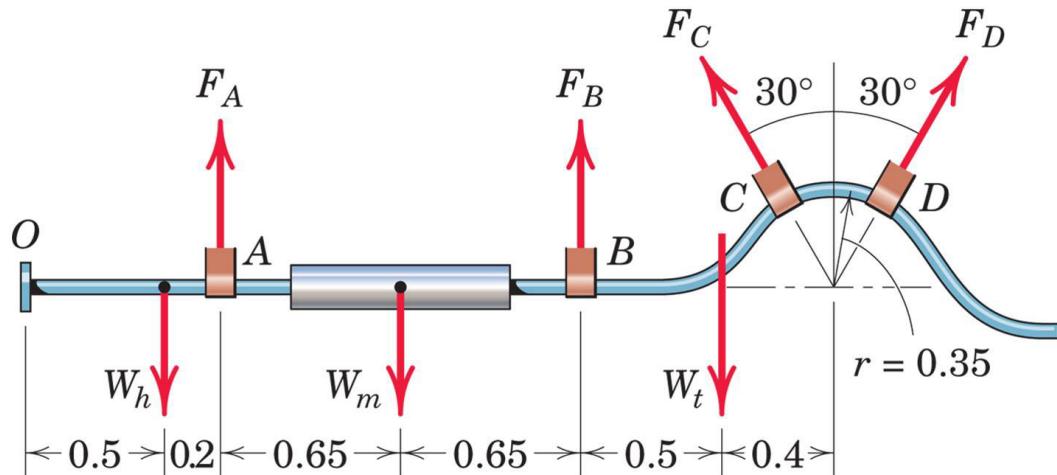
2/100] For a zero force - couple system  
at point O:

$$\begin{aligned} R = \sum F &= (-F_C \sin 30^\circ + F_D \sin 30^\circ) i \\ &+ (50 - 10 - 100 - 50 + F_B \\ &+ F_C \cos 30^\circ + F_D \cos 30^\circ) j = 0 \end{aligned}$$

# ENGINEERING MECHANICS

## Resultants - Numerical

2/100)



Dimensions in meters

$$\Rightarrow F_C = F_D = F$$

$$\text{At } O: -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) \\ - 50(2.5) + 2F \cos 30^\circ (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}} \quad ; \quad \underline{F_B = 98.9 \text{ N}}$$



**THANK YOU**

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