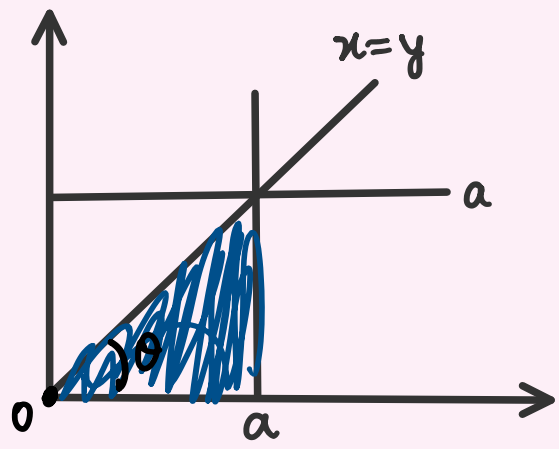
Unit-1 class-3

1. $\int_0^a \int_y^a \frac{xdxdy}{x^2+y^2}$ ans: $\frac{\pi a}{4}$

x varies from y to a
y varies from 0 to a

θ varies from 0 to π/4
r varies from 0 to asecθ



$\cos\theta = \frac{a}{r}$

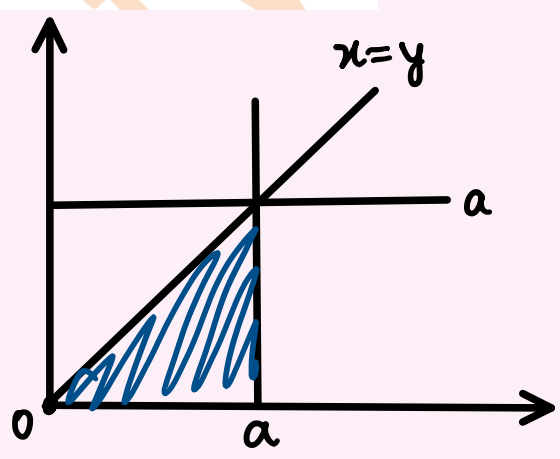
$$\int_0^{\pi/4}\int_0^{a\sec\theta} \frac{r\cos\theta \cdot r\,dr\,d\theta}{x^2}$$

$$= \int_0^{\pi/4} \left[\frac{r}{2}\right]_0^{a\sec\theta} \cos\theta\, d\theta = a\sec\theta \cdot \cos\theta \int_0^{\pi/4} d\theta = \frac{a\pi}{4}$$

2. $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\,dxdy$ ans: $\frac{a^3}{3}\log(\sqrt{2}+1)$

x varies from y to a
y varies from 0 to a

r varies from 0 to asecθ
θ varies from 0 to π/4



$$\int_0^{\pi/4}\int_0^{a\sec\theta} \frac{r^2\cos^2\theta}{x}\,dr\,d\theta$$

$$= \int_0^{\pi/4} \left[\frac{r^3}{3}\right]_0^{a\sec\theta} \cos^2\theta\, d\theta$$

$$= \frac{a^3}{3}\int_0^{\pi/4} \sec\theta\, d\theta$$

$$= \frac{a^3}{3}\left[\log(\sec\theta+\tan\theta)\right]_0^{\pi/4}$$

$$= \frac{a^3}{2}\log(\sqrt{2}+1)$$

$= \pi\frac{x^2}{8} = \frac{\pi a^2}{8}$