# Practical 4

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# Define the partially ordered set P as a set of elements and a \
    relation
P = {1, 2, 3, 4, 5}
R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3), (3, 4),\
     (4, 5)}  # A relation for partial order

# Define subset S
S = {2, 3}

# Check if x is an upper bound of S
def is_upper_bound(x, S, R):
    for s in S:
        if (s, x) not in R and s != x:  # Check if s <= x for all s \
   in S
            return False
    return True

# Check if x is a lower bound of S
def is_lower_bound(x, S, R):
    for s in S:
        if (x, s) not in R and x != s:  # Check if x <= s for all s \
   in S
            return False
    return True

# Get all upper bounds of S in P
def get_upper_bounds(S, P, R):
    return [x for x in P if is_upper_bound(x, S, R)]

# Get all lower bounds of S in P
def get_lower_bounds(S, P, R):
    return [x for x in P if is_lower_bound(x, S, R)]

# Get the least upper bound (if it exists)
def get_least_upper_bound(S, P, R):
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    upper_bounds = get_upper_bounds(S, P, R)
    if not upper_bounds:
        return None
    lub = min(upper_bounds, key=lambda x: [y for y in P if (y, x) in\
    R])
    return lub if is_upper_bound(lub, S, R) else None

# Get the greatest lower bound (if it exists)
def get_greatest_lower_bound(S, P, R):
    lower_bounds = get_lower_bounds(S, P, R)
    if not lower_bounds:
        return None
    glb = max(lower_bounds, key=lambda x: [y for y in P if (x, y) in\
    R])
    return glb if is_lower_bound(glb, S, R) else None

# Example usage
x = 4  # Example element to check if it is an upper or lower bound \
   of S

print("Is", x, "an upper bound of S:", is_upper_bound(x, S, R))
print("Is", x, "a lower bound of S:", is_lower_bound(x, S, R))
print("Set of all upper bounds of S:", get_upper_bounds(S, P, R))
print("Set of all lower bounds of S:", get_lower_bounds(S, P, R))
print("Least upper bound of S:", get_least_upper_bound(S, P, R))
print("Greatest lower bound of S:", get_greatest_lower_bound(S, P, R\
   ))
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Is 4 an upper bound of S: False
Is 4 a lower bound of S: False
Set of all upper bounds of S: [3]
Set of all lower bounds of S: [2]
Least upper bound of S: 3
Greatest lower bound of S: 2
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