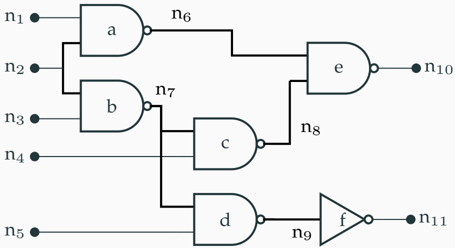
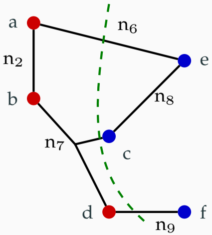
# Fiduccia-Mattheyses Algorithm

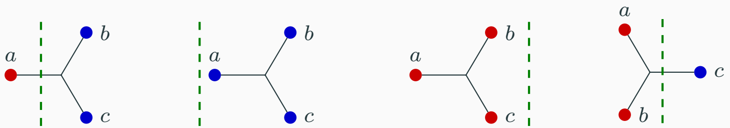
- Moves single cells and handles hypergraphs
- Linear runtime (vs $O(N^3)$ for KL)
- Hypergraph $H = (V, E)$; $area(v)$ is the area of vertex $v \in V$
- A hyperedge (net) is *cut* if connected vertices occupy more than one partition; *uncut* otherwise
- *Cutset* of a partition is the set of all nets that are cut



# Fiduccia-Mattheyses Algorithm - Terminology

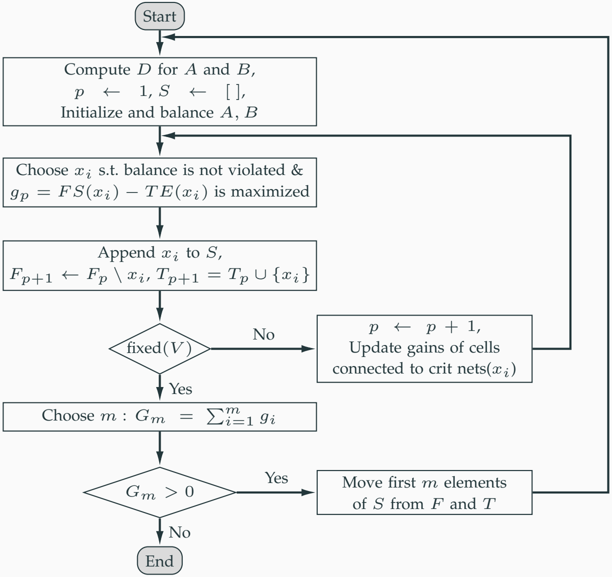
- Gain of vertex $v$ ($g_i(v)$): change in cutset size on moving vertex between partitions at $i^{th}$ pass
  - $FS(v)$ is the number of cut hyperedges connected only to $v$
  - $TE(v)$ is the number of uncut hyperedges connected to $v$
- Maximum positive gain $G_m$ is the sum of individual gains of $m$ moves that minimizes the cut cost: $G_m = \sum_{i=1}^{m} g_i$
- A hyperedge $e$ is *critical* if it is connected to a cell $v$ whose move will affect its cut status
  - All vertices connected to $e$ are in same partition
  - Exactly one vertex connected to $e$ is in a different partition



# Fiduccia-Mattheyses Algorithm - Terminology

- Balance criterion : ratio $r = \frac{area(A)}{area(A)+area(B)} = \frac{\sum\limits_{v \in A} area(v)}{\sum\limits_{v \in V} area(v)} = \frac{area(A)}{area(V)}$
  - Tolerance : $area_{\max}(V) = \max(\{area(v) : v \in V\})$
  - $r \cdot area(V) - area_{\max}(V) \leq area(A) \leq r \cdot area(V) + area_{\max}(V)$
- *Base cell*: cell with maximum gain $g$ and does not violate the balance criterion

# Fiduccia-Mattheyses Algorithm

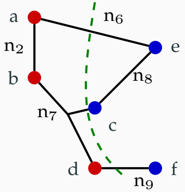


# Fiduccia-Mattheyses Algorithm

| $v$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| Area | 3 | 2 | 2 | 2 | 3 | 1 |

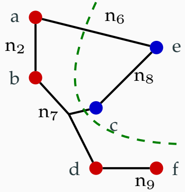
Ratio : $r = 0.5 \implies$ Balance criterion:

$$3.5 \leq \ area(A) \ \leq 9.5$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | 1 | 1 | 0 |
| $b$ | 0 | 1 | -1 |
| $c$ | 1 | 1 | 0 |
| $d$ | 1 | 0 | 1 |
| $e$ | 1 | 1 | 0 |
| **f** | **1** | **0** | **1** |

$$area(A) = 7$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| **a** | **1** | **1** | **0** |
| $b$ | 0 | 1 | -1 |
| $c$ | 1 | 1 | 0 |
| $d$ | 0 | 1 | -1 |
| $e$ | 1 | 1 | 0 |
| $f$ | – | – | – |

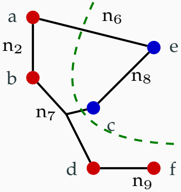
$$area(A) = 8$$

# Fiduccia-Mattheyses Algorithm

| $v$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| Area | 3 | 2 | 2 | 2 | 3 | 1 |

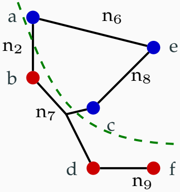
Ratio : $r = 0.5 \implies$ Balance criterion:

$$3.5 \leq \ area(A) \ \leq 9.5$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| **a** | **1** | **1** | **0** |
| $b$ | 0 | 1 | -1 |
| $c$ | 1 | 1 | 0 |
| $d$ | 0 | 1 | -1 |
| $e$ | 1 | 1 | 0 |
| $f$ | – | – | – |

$$area(A) = 8$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| $b$ | 1 | 0 | 1 |
| **c** | **1** | **1** | **0** |
| $d$ | 0 | 1 | -1 |
| $e$ | 0 | 2 | -2 |
| $f$ | – | – | – |

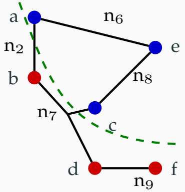
$$area(A) = 5$$

# Fiduccia-Mattheyses Algorithm

| $v$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| Area | 3 | 2 | 2 | 2 | 3 | 1 |

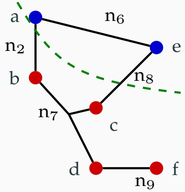
Ratio : $r = 0.5 \implies$ Balance criterion:

$$3.5 \leq \ area(A) \ \leq 9.5$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| $b$ | 1 | 0 | 1 |
| **c** | **1** | **1** | **0** |
| $d$ | 0 | 1 | -1 |
| $e$ | 0 | 2 | -2 |
| $f$ | – | – | – |

$$area(A) = 5$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| **b** | **1** | **1** | **0** |
| $c$ | – | – | – |
| $d$ | 0 | 2 | -2 |
| $e$ | 1 | 1 | 0 |
| $f$ | – | – | – |

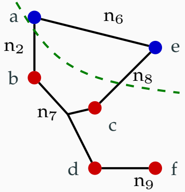
$$area(A) = 7$$

# Fiduccia-Mattheyses Algorithm

| $v$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| Area | 3 | 2 | 2 | 2 | 3 | 1 |

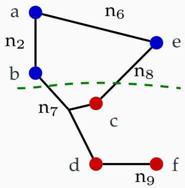
Ratio : $r = 0.5 \implies$ Balance criterion:

$$3.5 \leq \; area(A) \; \leq 9.5$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| **b** | **1** | **1** | **0** |
| $c$ | – | – | – |
| $d$ | 0 | 2 | -2 |
| $e$ | 1 | 1 | 0 |
| $f$ | – | – | – |

$$area(A) = 7$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| $b$ | – | – | – |
| $c$ | – | – | – |
| $d$ | 0 | 1 | -1 |
| **e** | **1** | **1** | **0** |
| $f$ | – | – | – |

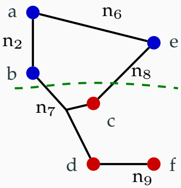
$$area(A) = 5$$

# Fiduccia-Mattheyses Algorithm

| $v$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| Area | 3 | 2 | 2 | 2 | 3 | 1 |

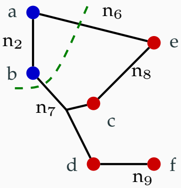
Ratio : $r = 0.5 \implies$ Balance criterion:

$$3.5 \leq \; area(A) \; \leq 9.5$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| $b$ | – | – | – |
| $c$ | – | – | – |
| $d$ | 0 | 1 | -1 |
| **e** | **1** | **1** | **0** |
| $f$ | – | – | – |

$area(A) = 5$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| $b$ | – | – | – |
| $c$ | – | – | – |
| **d** | **0** | **1** | **-1** |
| $e$ | – | – | – |
| $f$ | – | – | – |

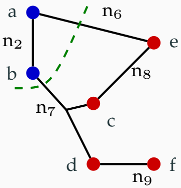
$area(A) = 8$

# Fiduccia-Mattheyses Algorithm

| $v$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| Area | 3 | 2 | 2 | 2 | 3 | 1 |

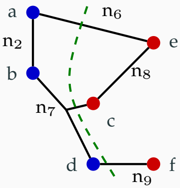
Ratio : $r = 0.5 \implies$ Balance criterion:

$$3.5 \leq \ area(A) \ \leq 9.5$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| $b$ | – | – | – |
| $c$ | – | – | – |
| **d** | **0** | **1** | **-1** |
| $e$ | – | – | – |
| $f$ | – | – | – |

$$area(A) = 8$$




| $v$ | $FS(v)$ | $TE(v)$ | $g(v)$ |
|---|---|---|---|
| $a$ | – | – | – |
| $b$ | – | – | – |
| $c$ | – | – | – |
| $d$ | – | – | – |
| $e$ | – | – | – |
| $f$ | – | – | – |

$$area(A) = 6$$

# Multilevel partitioning

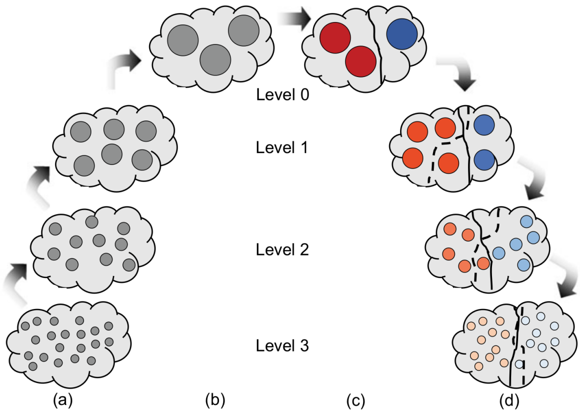
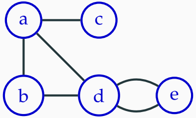



**Fig. 2.6** An illustration of multilevel partitioning. (**a**) The original graph is first coarsened through several levels. (**b**) The graph after coarsening. (**c**) After coarsening, a heuristic partition is found of the most coarsened graph. (**d**) That partition is then projected onto the next coarsest graph (*dotted line*) and then refined (*solid line*). Projection and refinement continue until a partitioning solution for the original graph is found

# Multilevel partitioning




Initial graph


Coarsened