

Activation Functions

itemize

If we use identity activation function, i.e. $z^{(i)}(x) = x$, then no matter how many layers are there, the fully connected neural network falls back to a linear regression.

Proof: The output of the first layer, with identity activation function, will be $z^{(1)}(w^{(1)}x + b^{(1)}) = w^{(1)}x + b^{(1)}$. Then the output will be the input of the next layer, and hence the output of the second layer will be $z^{(2)}(w^{(2)}(w^{(1)}x + b^{(1)}) + b^{(2)}) = w^{(2)}w^{(1)}x + w^{(2)}b^{(1)} + b^{(2)}$. Similar induction can be obtained for multiple layers. Hence if we use identity activation function, the fully connected neural network falls back to a linear regression.

With ReLU (Rectified Linear Unit) as activation function i.e. $z^{(i)}(x) = \max(0, x)$, then the fully connected neural network falls back to a linear regression.

Proof: The output with ReLU activation function, will be $z^{(1)}(w^{(1)}x + b^{(1)}) = \max(w^{(1)}x + b^{(1)}, 0)$. Then the output will be $z^{(2)}(w^{(2)}\max(w^{(1)}x + b^{(1)}, 0) + b^{(2)}) = \max(w^{(2)}w^{(1)}x + w^{(2)}b^{(1)} + b^{(2)}, 0)$. Similar induction can be obtained for multiple layers. Hence if we use identity activation function, the fully connected neural network falls back to a linear regression.