Set Theory
Unit-1

Proposition (Statement) logic–

Definition– A proposition is a declarative sentence which is either true or false. But not both. These statements are called proposition. for example–

(a) I am a Doctor.
(b) Paris is in England.
(c) 2 + 2 = 4
(d) Bangalore is in India.
(e) 5 is greater than 4.

Proposition(✓) (either true or false)

(a) How nice!
(b) Where are you going?
(c) Put the homework on the blackboard.
(d) What do you mean?
(e) This statement is false.

Proposition (X)

* The sentences which include Questions, exclaimation & expressions of opinions are not statements (proposition)

* We usually denote the statements or proposition by lower case letter like p, q, r, s -------

eg p: Paris is in England.

Here p is false.

Basic Connectives, Truth Table, Tautology, logical equivalance-

Basic Connectives-

(i) 'and' is denoted by $\wedge$. → Conjuctive

(ii) 'or' is denoted by $\vee$. → Dis-conjective

(iii) 'not' is denoted by $\sim$. → negation

(iv) 'if ---- then' is denoted by $\rightarrow$ or $\Rightarrow$

(v) 'if and only if' is denoted by $\leftrightarrow$ or $\Leftrightarrow$

The implication of $p \rightarrow q$ can also read as-

(a) if p then q
(b) p is sufficient for q.
(c) q is necessary for p.
(d) q if p.
(e) p implies q.
(f) q is implied by p.

Q Translate the following Compound Statements into symbolic form.

(1) I am a doctor and I am a professor.
p: I am a doctor. q: I am a professor.

$$\boxed{p \wedge q}$$

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(2) He studies hard and does not understand logic.

q: He understand logic {not को हमेशा negation में लेंगे}

$\cancel{p \wedge q}$ $\boxed{p \wedge \sim q}$

(3) Ram and shyam went to college.

$\boxed{p \wedge q}$

(4) Govind is clever and handsome.

$\boxed{p \wedge q}$

(5) If all odd integers are primes and 4 divides 6 then it will not rain.

p: All odd integers are prime

q: 4 divides 6

r: it will rain.

$$\boxed{(p \wedge q) \rightarrow (\sim r)}$$

Truth Table –

(a) Conjuction ($\wedge$) –

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(b) Disjunction ($\vee$)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(c) Negation ($\sim$)

p	$\sim p$
T	F
F	T

(d) Conditional ($\rightarrow$)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(e) Biconditional ($\leftrightarrow$)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautology -

A proposition P is a tautology if it is true under all circumstances. It means it contains Only 'T' in the find colounn of its truth table.

example

p	$p \vee p$	$p \vee p \leftrightarrow p$
T	T	T
F	F	T

Q Which of the following are tautology.

(a) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p) = X$

p	q	$p \rightarrow q$	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$	X
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

(b) $p \vee p \leftrightarrow p$

p	$p \vee p$	$p \vee p \Leftrightarrow p$
T	T	T
F	F	T

(c) $(p \wedge \sim q) \vee (\sim p \wedge q) = X$ (Let)

p	q	~p	~q	p∧~q	~p∧q	X
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

(d) $\sim(p \vee q) \vee (\sim p \vee \sim q)$

p	q	~p	~q	p∨q	~(p∨q)	(~p∨~q)	X
T	T	F	F	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T

(e) $\underbrace{(p \to q) \wedge (q \to r)}_{X} \to \underbrace{(p \to r)}_{Y}$

p	q	r	p→q	q→r	p→r	X	X→Y
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	F	F	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	T	T

Contradiction -

A compound statement (proposition) is said to be contradiction if it is always False for all possible Combination of truth values of its components.

That is the final coloumn in truth table are always False.

Ex.

p	$\sim p$	$(p \wedge \sim p)$
T	F	F
F	T	F

→ Contradiction ✓

Q Determine whether the following One is a Tatology or Contradiction.

① $(p \vee q) \vee (\sim p \vee \sim q) = X$

p	q	$\sim p$	$\sim q$	$p \vee q$	$(\sim p \vee \sim q)$	X
T	T	F	F	T	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	T

↓ Tautology

② $(q \wedge \sim p) \leftrightarrow r = X$

p	q	r	$\sim p$	$q \wedge \sim p$	X
T	T	T	F	F	F
T	T	F	F	F	T
T	F	T	F	F	F T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	F
F	F	F	T	F	T

neither Tautology nor Contradiction

Inverse, Converse & Contrapositive of a Statement –

Consider an implication $p \to q$

(i) The implication $\sim p \to \sim q$ is called the inverse of $p \to q$

(ii) The implication $q \to p$ is called the converse of $p \to q$

(iii) The implication $p\ q \to \sim p$ is called the contrapositive of $p \to q$.

ex. ① If n is an even integer then $2n$ is divisible by 4.

Inverse – If n is not an even integer then $2n$ is not divisible by 4.

Converse → If $2n$ is divisible by 4 then n is an even integer.

Contrapositive → If $2n$ is not divisible by 4 then n is not even integer.

Q If I do not go to cinema then I will study.

Inverse → If I go to cinema then I will not study.

Converse → If I will study then I do not go to cinema.

Contrapositive → If I will not study then I go to cinema.

Logical equivalance - In logical equivalance if & only if two statements have same logical truth values then those statements are logical equivalance. it is denoted by '≡'.

eg p: 8 is prime number.
q: $3^2 + 2^2 = 3^2$

So Both are false then $(p \equiv q)$

Or

$\left[\begin{array}{c}\sim \Rightarrow \top \\ \text{(negation)}\end{array}\right]$

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Any two simple statement (proposition) p & q are said to be logical equivalance if & only if they have same truth values.

eg ⇒ p : 2 is even number.
q : 3 + 2 = 5

Both are true ⇒ $p \equiv q$

eg ⇒ p : 8 is prime number
q : 2 + 3 = 5

Here $p \not\equiv q$ bcoz p is false & q is true.

Theorem (1)

$$\boxed{\sim(\sim p) \equiv p}$$

Theorem (2)

$$\boxed{\sim(p \vee q) \equiv \sim p \wedge \sim q}$$

Theorem (3)

$$\boxed{\sim(p \wedge q) \equiv \sim p \vee \sim q}$$

Theorem (4)

$$\boxed{\sim(p \to q) \equiv p \wedge \sim q}$$

The Law of Logic -

① De-Morgan's Law -

(a) $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

(b) $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

② Associative law -

(a) $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

(b) $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

③ Distributive Law -

(a) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

(b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

④ Absorption Law -

(a) $p \vee (q \wedge r) \Leftrightarrow p$

(b) $p \wedge (q \vee r) \Leftrightarrow p$

Theorem ⑤

Let $P(P_1, P_2, \text{------} P_n)$ & $Q(Q_1, Q_2, \text{----} Q_n)$ be two propositions. The following conditions are equivalent -

(1) $\sim P(P_1, P_2 ----) \vee q(q_1, q_2 ----)$ is a tautology.

(2) $P(P_1, P_2, ----) \wedge q(q_1, q_2 ---)$ is a contradiction.

(3) $P(P_1, P_2 ----) \rightarrow q(q_1, q_2 ---)$ is a tautology.

Q Write negation of -

(i) $q \vee \sim(p \wedge r)$
(ii) $(p \rightarrow r) \wedge (q \rightarrow p)$

Q Prove the logical equivalance -

(i) $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$
(ii) $p \vee (p \wedge q) \equiv p$

Sol^n (i)

$\sim(q \vee \sim(p \wedge r))$ (use of Theorem ②)
$= \sim q \wedge p \wedge r$

(ii)

$\sim((p \Rightarrow r) \wedge (q \rightarrow p))$

(use of theorem ③ & ④)

$= \sim(p \rightarrow r) \vee \sim(q \rightarrow p)$
$= p \wedge \sim r \vee q \wedge \sim p$

L.H.S.

(1) $(p \vee q) \wedge \sim p \equiv \sim p \wedge (p \vee q)$ {Commutative}
$\equiv (\sim p \wedge p) \vee (\sim p \wedge q)$ {Distributive}
$\equiv f \vee (\sim p \wedge q)$
$= \sim p \wedge q$ (Identity law)

$\{\sim p \wedge p \Rightarrow F\}$

$f \Rightarrow$ false

Identity Law

① $p \vee f \Leftrightarrow p$
② $p \wedge t \Leftrightarrow p$
③ $p \vee t \Leftrightarrow t$
④ $p \wedge f \Leftrightarrow f$
⑤ $p \wedge \sim p \Leftrightarrow f$

(ii) $p \vee (p \wedge q) \equiv p$

L.H.S.

$$p \vee (p \wedge q) \equiv (p \wedge t) \vee (p \wedge q) \quad \{\text{Identity}\}$$
$$\equiv p \wedge (t \vee q) \quad \{\text{Distributive}\}$$
$$\equiv p \wedge t \quad \{\text{identity}\}$$
$$\equiv p = \text{R.H.S.}$$

Rule of Inference -

ⓐ Law of Detachment (or Modus ~~Modds~~ Ponens) -
(In German)

The form of the arguement is -

$$\begin{array}{l} p \rightarrow q \\ \underline{p} \\ q \end{array}$$

Here the premises are

P1 : $p \rightarrow q$ ("p implies q")
P2 : p ("p is assumed to be true")

Conclusion : q ("so q is true")

i.e;

$p \rightarrow q$ (premise) is true
$\therefore q$ (Conclusion) is true

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Q Supposing the following propositions are true

p : two triangle are similar

$p \to q$: if two triangle are similar then their corresponding sides are proportional.

Ans-

q : The corresponding sides are proportional is true.

* Let us the truth table for $[(p \to q) \wedge p] \to q$

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(b) Law of Contraposition (or Modus Tollens)

The form of the arguement is

$$p \to q$$
$$\sim q$$
$$\therefore \sim p$$

The logical expression like as
$[(p \to q) \wedge (\sim q)] \to \sim p$

Truth table –

p	q	$\sim p$	$(p \to q)$	$(\sim q)$	$(p \to q) \wedge (\sim q)$	$[(p \to q) \wedge (\sim q) \to (\sim p)]$
T	T	F	T	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	T
F	F	T	T	T	T	T

eg ① If $\underbrace{\text{it snows today}}_{p}$ then $\underbrace{\text{we will go for skiing}}_{q}$.

② We $\underbrace{\text{will not go}}_{\sim q}$ for skiing.

① $p \to q$

② $\sim q$

Ⓒ **Disjunctive syllogism –**

The form of the arguement is

$$p \vee q$$

$$\therefore \frac{\sim p}{q}$$

logical expression

$$[(p \vee q) \wedge (\sim p)] \to q$$

(Truth table से Proof हो जायेगा)

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eg 1: Either it is below freezing or raining now.
2: It is not below freezing.

Conclusion - It is raining now.

(d) Hypothetical syllogism -

The form of the arguement is

$$p \to q$$

$$\therefore \frac{q \to r}{p \to r}$$

logical expression $[(p \to q) \wedge (q \to r)] \to (p \to r)$

eg 1: if the weather is good then we will go for an outing.
2: if we go for an outing, we will play a game.

Conclusion -
if the weather is good, we will play a game.

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Normal form -

(i) Disjunctive Normal form (DNF)

(ii) Conjunctive Normal form (CNF)

Q If the advertisement is successfull then the sales of the product will ~~grow~~ up.
OR
Either the advertisement is successfull then production of Product will stop. The sales of the product will not go up. therefore the production of the product will be stop. So check the validity of product.

Soln p : The advertisement is successfull
q : The sales of the product will go up
r : The production of the product will be stop.

$$\left[\begin{array}{l} p \rightarrow q \text{ --- } (1) \\ p \vee r \text{ --- } (2) \end{array}\right] \rightarrow \sim p \quad (4)$$

$\sim q$ --- (3)

r

from (1)

$$p \rightarrow q \equiv \sim q \rightarrow \sim p \text{ --- } (4)$$

from (3) & (2)

$$\sim q \rightarrow \sim p$$
$$\sim q$$
$$\overline{\qquad\qquad}$$
$$\sim p \text{ --- } (5)$$

So

$\sim p$ is True

$\Rightarrow p$ is false

from $p \vee r \Rightarrow r$ is True.

① Disjunctive normal form (DNF) –

A conjunction of statement variables & (or) their negations is called as fundamental conjunction.

eg $p, \sim p, \sim p \wedge q, p \wedge q$ are called fundamental conjuction.

A statement form which consists of disjunction of fundamental conjuctions This is called as disjunctive normal form.

eg ① $(p \wedge q \wedge r) \vee (p \wedge r) \vee (q \wedge r)$

② $(p \wedge \sim q) \vee (p \wedge r)$

③ $(p \wedge q \wedge r) \vee \sim r$

④ $(p \wedge q) \vee \sim q$

Disjunction मतलब OR ($\wedge$) लगना चाहिये।

Q. Obtain the DNF of the form $(p \to q) \wedge (\sim p \wedge q)$

$$(p \to q) \wedge (\sim p \wedge q)$$

$$(\sim p \vee q) \wedge (\sim p \wedge q)$$

$$\equiv (\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$$

$$(\sim p \wedge q) \vee (\sim p \wedge q)$$

Q. Obtain the DNF of the form $\sim (p \to (q \wedge r))$

Q. Find the DNF of $(\sim p \to r) \wedge (p \leftrightarrow q)$ by truth table.

	p	q	r	$\sim p$	$(\sim p \to r)$	$(p \leftrightarrow q)$	$(\sim p \to r) \wedge (p \leftrightarrow q)$
1 →	T	T	T	F	T	T	T
2 →	T	T	F	F	T	T	T
	T	F	T	F	T	F	F
	T	F	F	F	T	F	F
	F	T	T	T	T	F	F
	F	T	F	T	F	F	F
3 →	F	F	T	T	T	T	T
	F	F	F	T	F	T	F

DNF is –

$$\underset{①}{(p \wedge q \wedge r)} \vee \underset{②}{(p \wedge q \wedge \sim r)} \vee \underset{③}{(\sim p \wedge \sim q \wedge r)}$$

or $(p \cdot q \cdot r) + (p \cdot q \cdot \bar{r}) + (\bar{p} \cdot \bar{q} \cdot r)$ = sum of products

(समझाने के लिये)

CNF ⇒ False statement ⇒ Conclusion ⇒ True
DNF ⇒ True statement

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② Conjunctive normal form (CNF) –

A disjunction of statement variables and (or) their negation is called fundamental disjunction. for eg – $p, \sim p, \sim p \vee q, p \vee q, p \vee \sim p \vee q$ are fundamental disjunction.

The statement form, which consist of conjunction of fundamental disjunction is called conjunctive normal form.

Note – it is noted that CNF is a tautology if & only if every fundamental disjunction content if it is a tautology.

for eg –

Conjunctive and $(\wedge)$

(i) $p \wedge r$

(ii) $\sim p \wedge (p \vee r)$

(iii) $(p \vee q \vee r) \wedge (\sim p \vee r)$

Q. obtain the CNF of $(p \wedge q) \vee (\sim p \wedge q \wedge r)$.

$$(p \wedge q) \vee (\sim p \wedge q \wedge r)$$

$$\equiv [p \vee (\sim p \wedge q \wedge r)] \wedge [q \vee (\sim p \wedge q \wedge r)]$$ By distributive law

$$\equiv [(p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)]$$

$$\equiv [T \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q) \wedge (q \vee r)]$$

$$\equiv (p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p) \wedge (q) \wedge (q \vee r)$$

$$\{ \text{or} \Rightarrow (+) \quad \text{and} \Rightarrow (\cdot) \}$$

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Q $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

$\equiv (\sim p \rightarrow r) \wedge [(p \rightarrow q) \wedge (q \rightarrow p)]$

$\equiv (\sim(\sim p) \vee r) \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$

$\equiv (p \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p)$

Truth table of this example is in example of DNF & there all false statements are for CNF -

This negation is for change all statements from false to true

$\equiv \sim [(p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r)$

$\vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)]$

now by the demorgan's law all sign will be interchange.

$\equiv [(\sim p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (p \vee q \vee \sim r) \wedge$

$(p \vee \sim q \vee r) \wedge (p \vee q \vee r)]$

$\equiv [(\sim p + q + \sim r) \cdot (\sim p + q + r) \cdot (p + \sim q + \sim r) \cdot$

$(p + \sim q + r) \cdot (p + q + r)]$

$=$ Product of Sum.

समझाने के लिये।

Predicate logic -

Let A be a given set. A propotional function [or open sentence or condition or predicate logic] defined on A is an expression

Which has the property that $P(a)$ is true or false for each $a \in A$.

That is $P(x)$ becomes a statement (with a truth value) whenever any element $a \in A$ is substituted for the variable x.

In other words

$$T_p = \text{collection of all elements of A then}$$

$$T_p = \{ x : x \in A ,\ P(x) \text{ is true} \}$$

$$\text{Or } T_p = \{ x : P(x) \}$$

Q find the truth set of each propositional function $P(x)$ defined on the set N of (+)ve integers

(a) Let $P(x) = x + 2 > 7$ Its truth set is

$$T_p = \{ x ; x \in N ,\ x+2 > 7 \} = \{ 6, 7, 8 ---- \}$$

consisting of all integers greater than 5.

(b) Let $P(x) = x+5 < 3$ Its truth set is $T_P = \{x : x \in N, x+5<3\} = \phi$ the empty set.

(c) Let $P(x) = x+5 > 1$ Its truth set is $T_P = \{x : x \in N, x+5>1\} = N$ is true for every element in $\underline{N}$

Quantifiers-

Certain proposition involving a specified number of objects. for example-

(a) All squares are rectangle.
$= \forall x \, [x \text{ is rectangle}]$

(b) Some men are short.
$= \exists y \, [y \rightarrow \text{men are short}]$

(c) for every real no. of x, $x^2 \geq 0$
$= \forall x \, [x^2 \geq 0]$

(d) At least one student is interested in logic.
$= (\exists)$

(e) Ram or Mohan none of there stood first.
$= (\exists)$

(f) There exists a function whose derivative is t^3
$= (\exists)$

The words 'all', 'some', 'for every', 'or' 'at least one', 'exists' indicate quantity. The words expression is called quantifiers.

There are two type of quantifiers-

(1) <u>Universal</u>

⇓

'for all'
'for every'
'for ~~some~~ each'

These all are denoted by ($\forall$).

(2) <u>Existential</u>

⇓

'for some'
'there exists'
'or'
'at least'

It is denoted by ($\exists$)

Skolemization-

The procedure for systematic elemination of the existential quantifiers is logic [prenex form] is called skolemization by introducing new constant & function symbols.

That new constant is called skolem constant.

That new function is called skolem function.

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① for simple case -

The result of skolenization of the formula

$\exists x \ \forall y \ \forall z \ (A)$ ↪ prenex form

$= \forall y \ \forall z \ (c/x)$ where c is new (skolen) Constant

⑧ for instance -

The result of skolenization of the formula

$\exists x \ \forall y \ \forall z \ (P(x,y) \rightarrow Q(x,z))$ is

$= \forall y \ \forall z \ ((P(c,y) \rightarrow Q(c,y))$

Therefore the general formula result is -

$\exists x_1 \ ---- \exists x_K \ \forall y_1 \ ---- \ \forall y_n \ A$

$= \forall y_1 \ ----- \ \forall y_n \ A \ [c_1/x_1, \ ----- \ c_n/x_n]$

Where $c_1, ------ c_n$ are skolen constant.

Note- It is noted that resulting formula is not equivalent to the original statement, But ~~this~~ is equally satified by it.

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Q every philospher writes at least one book.

$= \forall x\,[\text{philospher}(x) \rightarrow \exists y\,[\text{Book}(y) \wedge \text{Write}(x,y)]]$

now eliminate implication

$= \forall x\,[\sim \text{philospher}(x) \vee \exists y\,[\text{Book}(y) \wedge \text{Write}(x,y)]]$

Skoleminaze ⇒ substitute y by g(x)

$= \forall x\,[\sim \text{philospher}(x) \vee [\text{Book}(g(x)) \wedge \text{Write}(x, g(x))]]$

* Proof By Contraposition

Indirect proof is known as poroof by contraposition.

Proofs By contraposition make use of the fact that the conditional statement $p \rightarrow q$ is equivalent to $\sim q \rightarrow \sim p$

This means the conditional statement $p \rightarrow q$ showing that its contrapositive by $\sim q \rightarrow \sim p$

Q Proove that if n is integer & $3n+2$ is odd then n is odd.

Soln Direct –

Let $(3n+2) = (2K+1)$ for some integer K

or

$$\underbrace{3n}_{\text{odd}}+1 = \underbrace{2K}_{\text{even}}$$

odd
क्योंकि 1 में
add करके even
बनाना है।

by contraposition method –

$p = (3n+2)$ is odd

$q = n$ is odd

Assume –

$q = n$ is not odd

i.e;

n is even $\Rightarrow \boxed{n = 2K}$

$$\begin{aligned}(3n+2) &= (3 \times 2K) + 2 \\ &= 6K + 2 \\ &= \underbrace{2(3K+1)}_{\text{even}}\end{aligned}$$

Hence

$\sim q = n$ is not odd

$(3n+2) = $ even no. $= \sim(3n+2) = \sim p$

Therefore it satisfies

$$\boxed{\sim q \rightarrow \sim p} \equiv \boxed{p \rightarrow q}$$

* Proof by Contradiction -

Suppose we want to proove that a statement p is true. further more, suppose that we can find a contradiction q such that $\sim p \longrightarrow q$ is true. Because q is false but $\sim p \rightarrow q$ is true, we can find conclusion of that statement $\sim p$ is false. which means that p is true.

Q Proove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

p : $\sqrt{2}$ is irrational

We suppose

$\sim p$ is true

i.e;

$\sim p = \sqrt{2}$ is not irrational

or $\sqrt{2}$ is rational.

if $\sqrt{2}$ is rational no. then

$$\sqrt{2} = \frac{a}{b} \quad \left(b \neq 0, \ a \ \& \ b \text{ have not common factor} \right)$$

$$(\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$$

$$2 = \frac{a^2}{b^2}$$

$$\boxed{\underbrace{2b^2}_{\text{even}} = a^2} \text{ --- } (1)$$

Here a^2 is an even number (By the definition of even integer)

$a^2 \rightarrow$ even

So $a \rightarrow$ even

$a = 2c$ $\quad c \rightarrow$ integer

from (1)

$$2b^2 = 4c^2$$

$$b^2 = 2c^2 \rightarrow \text{even}$$

$b^2 \rightarrow$ even

$b \rightarrow$ even

So both are even (a & b) then a & b have common factor

So up leads to the equation $\sqrt{2} = \frac{a}{b}$ Where (a & b have no common factor)

But both a & b are even that is 2 divides both a & b.

Therefore our assumption ~p must be false. Hence statement p "$\sqrt{2}$ is irrational" is true.

Q Proof that if $(3n+2)$ is odd then n is odd by proof by contradiction.