(1) Inherent Error

Error which are already present in the statement of a problem before its soln are called inherent error.

(2) Round-off error

To round off ~~to~~ a number to n - significant digits, discard all digit to the right of the nth digit and if the (n+1) digit is.

Case (1) (n+1) < 5 , n th. digit unchanged.

(2) (n+1) > 5 , increased the nth digit by +1

(3) (n+1) = 5

odd — increased by +1

even — ~~increase~~ unchanged.

(3) ~~(2)~~ Truncation Error

There error are caused by using a approximate result or replacing infinite process by a finite part

Ex:- $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$

(4) Absolute Error (E_A)

$E_A = |X_T - X_A|$

X_T = True value

X_A = Approximate value.

(5) Relative error

$E_R = \frac{E_A}{X_T} = \frac{|X_T - X_A|}{X_T}$

E_A - Absolute Error

$X_T \rightarrow$ True value.

(6) Percentage error

$E_P = E_R \times 100\% = \frac{|X_T - X_A|}{X_T} \times 100\%$

E_R → Relative Error

(7) 3.6250782

$(n+1) < 5$ no. is unchanged.

3.625 (3D)

(8) 35.00537 (7S)

(9) 56.487262

56.487

56.487262
56.487
00.000262

$E_r = |X_T - X_A| = |56.487262 - 56.487| = 0.000262$

(10)

$$f(x,y) = \frac{dy}{dx} \cdot \Delta x + \frac{dy}{dx} \cdot \Delta y.$$

(11) **Round off Error.**

i) may arises from the process of rounding off no. during computation.

Check $(n+1)$ digit

i) $(n+1) < 5 \to$ unchanged

$(n+1) > 5 \to$ increased by +1

$(n+1) = 5$

even — unchanged

odd — increased by +1

Ex:- 6.62525

Truncation Error

These error are caused by during approximate result or replacing infinite process by a finite part.

Ex:- e^x.

(12) $f(x) = 0$.

(13) An eqn. which contains some transcendential function such as exponential or trignometic function.

Ex:- $3x - \cos x = 0$

$3x - \cos x + e^x = 4$.

(14) **Polyn**

The eqn of the form $f(x) = 0$ where $f(x)$ is purely a polyn in x.

Ex:- $x^6 - x^5 - x = 0$ ii) $x - 2x + 5$

Transcendental eqn

The eqn which include trignometric, exponential or arthimatic is known as transcendental eqn.

Ex: i) $xe^x - 2 = 0$ (ii) $x\cos x + 2 = 0$.

(15) [illegible] ... is a theorem it explain about continuous function. it analyse the continuous function.

Page 3

Applying the Intermediate value theorem, we can say that graph must cross at same point b/w (0,2).
Hence there exist a solution to the eqn

$x^5-2x^3-2=0$ b/w interval [0,2)

(16) Bisection method is used to find the roots of a polynomial eqn. It separates the interval and subdivides the interval in which the root of eqn lies.

(18) $x^3-2x-5=0$

$f(0)=0-2\times0-5=-5<0$

$f(1)=1-2-5=-6<0$

$f(2)=8-4-5=-1<0$

$f(3)=27-6-5=16>0$] lies b/w f(2) & f(3)

$[a,b]=[2,3]$

(19) O.C of Regula falsi Method = 1

Order of Convergence of Newton Raphson = 2

" " " Secant method = 1.62

(20) $|f(x).f''(x)| < |f'(x)|^2$

(21) If we have don't have good starting point or interval then the secant method fail to converge.

(22) (23), (24), (25)

(26) 0.77729 (n+1)>5 increased by +1 ; 0.0022218 (n+1)<5 → unchanged

Round off

0.7773 ; 0.0022

(27) $\sqrt{3}=1.732$ Absolute Error $=|1.7320-1.732|+|2.2360-2.236|$

$\sqrt{5}=2.236$ $+|2.645-2.645|$

$\sqrt{7}=2.645$

$=0.0007$

Relative error $=\dfrac{0.007}{1.732+2.2360+2.645}$ $=0.0001058$

$X = 865250$

$X' = 865200$

$E_A = |X - X_a'| = |865250 - 865200|$
$= 50$

$E_R = \frac{50}{865250} = 0.0005778.$

$X = 37.46235$

$X' = 37.462$

$E_A = |X - X'| = |37.46235 - 37.462|$
$= 0.00235$

$E_R = \frac{E_A}{X_T} = \frac{0.00235}{37.46235} = 0.00004937.$

(29) $X = 0.0054\,5828$

(i) 0.005

$E_A = |0.0054\,5828 - 0.005|$
$= 0.0004\,5828$

(ii) Round off

x' 0.005

$E_A = |0.0054588 - 0.005| = 0.00045828$

(30)

- Regula falsi Method.
- Newton Raphson Method
- Seecant method
- Gauss jordon Method
- Gauss Seidal Method → Convergence of iterative method
- Bisection method.

working procedure

1. find the interval [a, b]
   s.t. $f(a) \cdot f(b) < 0$

2. find $c = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$

3. $f(a) \cdot f(c) < 0$ roots lies in (a, c)

4. Repeat step 1 & 2

Q root of $x^3 - 2x - 5 = 0$ up to four iteration

$f(x) = x^3 - 2x - 5$

$f(0) = 0 - 2\times 0 - 5$
$= -5 < 0$

$f(1) = 1 - 2 - 5 = -6 < 0$

$f(2) = 8 - 4 - 5 = -1 < 0$

$f(3) = 27 - 2\times 3 - 5 = 0$
$27 - 6 - 5 = 27 - 11 = 16$

Roots lies b/w 2 & 3.

I st iteration

$a = 2$, $b = 3$

$f(a) = -1$ $f(b) = 16$

$c = \dfrac{2\times 16 - 3\times(-1)}{16 + 1}$

$= \dfrac{32 + 3}{17} = \dfrac{35}{17} = 2.0588$

$f(c) = f(2.0588) = (2.0588)^3 - 2\times 0.588 - 5$

$= -0.3908 < 0$

II nd roots lies b/w 2.0588 & 3.

$a = 2.0588 \quad b = 3$

$f(a) = -0.3908 \quad f(b) = 16.$

$$c = \frac{a \cdot f(b) - b\,f(a)}{b-a}$$

$= 2.0812$

$f(c) = f(2.0812) = -0.1429 < 0.$

III rd iteration.

Roots lies b/w.

2.0812 & 3.

$a = 2.0812 \quad b = 3$

$f(a) = -0.1429 \quad f(b) = 16.$

$$c = \frac{a \cdot f(b) - b\,f(a)}{b-a} =$$

$= 2.0896$

$f(c) = f(2.0896) = -0.0551$

IV iteration

Roots lies b/w

2.0896 & 3.

$a = 2.0896 \quad b = 3$

$f(a) = -0.0551 \quad f(b) = 16.$

$$c = \frac{a\,f(b) - b\,f(a)}{b-a}$$

$= 2.0927$

Hence the required root is

2.0927.

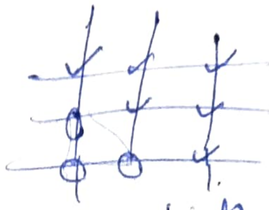
Unit - 1
Page - 1

Q solve the following eqn by gauss elimination method

$x - y + 2z = 3$

$x + 2y + 3z = 5$

$3x - 4y - 5z = -13$



Convert into upper triangular matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & -4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -13 \end{bmatrix}$$

$A \quad X \quad = B.$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 1 & 2 & 3 & 5 \\ 3 & -4 & -5 & -13 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & -11 & -22 \end{array}\right]$$

$R_3 \to 3R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & -32 & -64 \end{array}\right]$$

$x - y + 2z = 3$

$3y + z = 2$

$-32z = -64 \qquad \boxed{z = 2}$

$3y + z = 2$

$3y + 2 = 2$

$3y = 0$

$\boxed{y = 0}$

$x - y + 2z = 3$

$x - 0 + 2 \times 2 = 3$

$\boxed{x = -1}$

Solve the system

$2x+y+z=10$

$3x+2y+3z=18$ by using

$x+4y+9z=16$ (i) Gauss elimination method

(ii) Gauss Jordan method

Soln: (i) Gauss elimination method.

$x+4y+9z=16$

$2x+y+z=10$

$3x+2y+3z=18$.

$$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \\ 18 \end{bmatrix}$$

$AX = B$

$[A:B]$

$$\left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \end{array}\right] \quad R_2 \to R_2 - 2R_1, \; R_3 \to R_3 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -7 & -17 & -22 \\ 0 & -10 & -24 & -30 \end{array}\right]. \quad R_3 \to 7R_3 - 10R_2$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -7 & -17 & -22 \\ 0 & 0 & 2 & 10 \end{array}\right]$$

$x+4y+9z=16$ — (i)

$-7y-17z=-22$ — (ii)

$2z=10$ — (iii)

$2z=10$

$z=5$

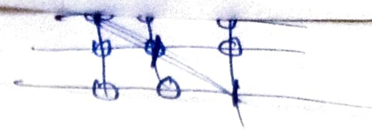
$-7y-17\times(5)=-22$

$y=-9$

$x=7$

(Page 3)

(Convert into diagonal matrix)



Q.

$$x + 4y + 9z = 16$$
$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$

$$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 16 \\ 10 \\ 18 \end{bmatrix}$$

$$A \quad X = B$$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \end{array}\right] \qquad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 7 & -17 & -22 \\ 0 & -10 & -24 & 30 \end{array}\right] \qquad \begin{array}{l} R_3 \to 7R_3 - 10R_2 \\ R_1 \to 7R_1 + 4R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 7 & 0 & -5 & 24 \\ 0 & -7 & -17 & -22 \\ 0 & 0 & 2 & 10 \end{array}\right] \qquad \begin{array}{l} R_1 \to 2R_1 + 5R_3 \\ R_2 \to 2R_2 + 17R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 14 & 0 & 0 & 98 \\ 0 & -14 & 0 & 126 \\ 0 & 0 & 2 & 10 \end{array}\right]$$

$$14x = 98 \to x = 7$$
$$-14y = 126 \to y = -9$$
$$2z = 10 \quad z = 5$$

Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad f'(x_n) \neq 0.$$

Q find by Newton Raphson Method a root of Equation $x^3 - 3x - 5 = 0$.

$f(0) = -5$

$f(2)\ 8 - 6 - 5 = -3$ → nearest to 0

$f(3) = 27 - 9 - 5 = 16.$

$x_0 = 2$

$f(x) = x^3 - 3x - 5$

$f'(x) = 3x^2 - 3$

$$x_{n+1} = x_n - \frac{x_n^3 - 3x_n - 5}{3x_n^2 - 3.}$$

put n=0

$$x_1 = x_0 - \frac{x_0^3 - 3x_0 - 5}{3 \times x_0^2 - 3}$$

$$= 2 - \frac{8 - 3\times 2 - 5}{3 \times 4 - 3} = 2.3333$$

put n=1

$$x_2 = x_1 - \frac{x_1^3 - 3x_1 - 5}{3x_1^2 - 3}$$

$$= 2.3333 - \frac{(2.3333)^3 - 3 \times (2.3333) - 5}{3 \times (2.3333)^2 - 3}$$

$$= 2.2805$$

put $n=2$ page 5

$$x_3 = x_2 - \frac{x_2^3 - 3x_2 - 5}{3x_2^2 - 3}$$

$$= (2.2805) - \frac{(2.2805)^3 - 3\times(2.2805) - 5}{3\times(2.2805)^2 - 3}$$

$$= 2.2790$$

put $n=3$

$$x_4 = x_3 - \frac{x_3^3 - 3x_3 - 5}{3x_3^2 - 3}$$

$$= 2.2790 - \frac{(2.2790)^3 - 3\times 2.2790 - 5}{3\times(2.2790)^2 - 3}$$

repeat

$$= 2.2790.$$

Q2 $\cos x - xe^x = 0$ correct up to four decimal place.

$f(x) = \cos x - xe^x$

$f(0) = 1$

$f(1) = \cos 1 - 1\cdot e^1 = -2.1779$

$f'(x) = -\sin x - e^x - xe^x.$

$= -\sin x - e^x(x+1)$

$x_0 = 0.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

put $n=0$ 0.5182 repeat.

put $n=1$ 0.5182

Unit-1

Secant Method (Chord method)

$x_0 f(x_1) - x_1 f(x_0)$

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Q A real root of the equation $x^3 - 5x + 1 = 0$ lies in the interval $(0,1)$. Perform four iteration of the secant Method.

$f(x) = x^3 - 5x + 1$

$f(0) = 0 - 5\times 0 + 1 = 1$

$f(1) = 1 - 5\times 1 + 1 = -3.$

$x_0 = 0$, $x_1 = 1$

$f(x_0) = 1$ $f(x_1) = -3$

put $n = 1$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{0\times(-3) - 1\times 1}{-3 - 1} = \frac{+1}{+4}$$

$x_2 = 0.25$

$f(x_2) =$

$f(0.5) = (0.25)^3 - 5\times 0.25 + 1$

$= -0.234375$

put $n = 2$

$x_3 = 0.18644$

$f(x_3) = 0.07428$

put $n = 3$. $\rightarrow x_4 = 0.20174$

$f(x_4) = 0.00048$

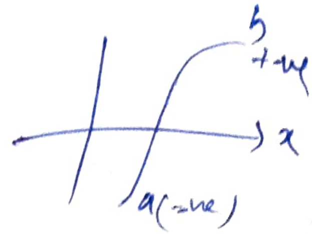
put $n = 4$

x_5

$x_5 = 0.20081$

Bisection Method . (Bolzano method) Unit - ①

This method is based on the repeated application of intermediate value property.



Q Find the real root of the equation

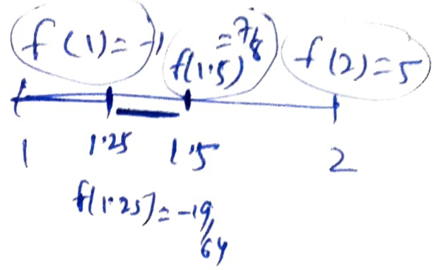
Nearest put 0 value

$$f(x) = x^3 - x - 1 = 0$$

$$f(-1) = -1 + 1 - 1 = -1$$

$$f(0) = -1$$

$$\left[\begin{array}{l} f(1) = -1 \\ f(2) = 5 \end{array}\right.$$

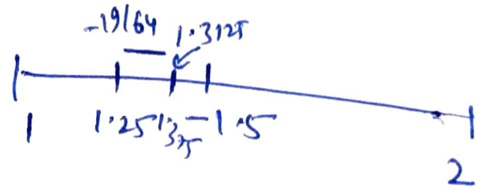


mid point

Root lies b/w 1 & 2

$$x_1 = \frac{1+2}{2} = \frac{3}{2} = 1.5$$

$$f(1.5) = (1.5)^3 - 1.5 - 1 = 7/8.$$



Roots lies b/w 1 & 1.5

$$x_2 = \frac{1+1.5}{2} = 1.25$$

$$f(1.25) = (1.25)^3 - 1.25 - 1 = -19/64$$

Root lies b/w 1.25 & 1.5

$$x_3 = \frac{1.25+1.5}{2} = 1.375$$

$$f(1.375) = (1.375)^3 - 1.375 - 1 = +ve$$

Root lies b/w 1.375 and 1.25

$$x_4 = \frac{1.375+1.25}{2} = \underline{\underline{1.3125}}$$

Convergence of iterative Method $x = \phi(x)$

Q. find a real root of equation

$f(x) = x^3 + x^2 - 1 = 0$

Solⁿ:-

$f(0) = -1$

$f(1) = 1$ $\quad x_0 = 0.5$

Method

Case I

$x^3 = 1 - x^2$

$x = \underbrace{(1-x^2)^{1/3}}_{\phi(x)}$

$\phi'(x) = \left(\frac{1}{3}\frac{2x}{(1-x^2)^{2/3}}\right)$

< 1

Method

Case (II)

$x^2 = 1 - x^3$

$x = \underbrace{(1-x^3)^{1/2}}_{\phi(x)}$

$\phi'(x)\ \frac{1}{2}\frac{3x^2}{(1-x^3)^{1/2}}$

at $x = 0.5 =$ < 1

$x^2(1+x) = 1$

$x^2 = \frac{1}{1+x}$

$x = \underbrace{\frac{1}{\sqrt{1+x}}}_{\phi(x)}$

$= \phi$

< 1

$\phi'(x)$ at $x = 0.5$

$|\phi'(x)|_{at\ x = x_0} < 1$

$\phi(x) = \frac{1}{\sqrt{1+x}}$

$x_n = \frac{1}{\sqrt{1+x_{n-1}}}$

put $n = 1$

$x_1 = \frac{1}{\sqrt{1+x_0}} = \frac{1}{\sqrt{1.5}} = 0.81649.$

$x_2 =$ ______

$x_3 =$ ______

x_4 ______

$x_5 =$ ______

Estimate the population in 1895 & 1925 from following

Unit - 2

Newton forward Interpolation

year	1891	1901	1911	1921	1931
population	46	66	81	93	101

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	46				
1901	66	$20 = \Delta y_0 = \Delta f(a)$			
1911	81	$15 = \Delta y_1$	$-5 = \Delta^2 y_0 = \Delta^2 f(a)$		
1921	93	$12 = \Delta y_2$	$-3 = \Delta^2 y_1$	$2 = \Delta^3 y_0 = \Delta^3 f(a)$	
1931	101	$8 = \Delta y_3$	$-4 = \Delta^2 y_2$	$-1 = \Delta^3 y_1$	$-3 = \Delta^4 y_0 = \Delta^4 f(a)$

Newton back

Newton forward backward formula

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$$

$$+ \frac{u(u-1)(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a) + \cdots$$

$f(1895)$:

$a = 1891$

$a + hu = $ $h = 10$

$1891 + 10(u) = 1895$

$u = 0.4$

Calcular:

$46 + 0.4 \times 20 + 0.4(0.4-1) \times -5 \div 2$
$+ 0.4(0.4-1)(0.4-2) \times 2 \div 6 + \cdots$

$$f(1895) = 46 + 0.4(20) + \frac{0.4(0.4-1)}{2}(-5) + \frac{0.4(0.4-1)(0.4-2)}{6}(2) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24} \times (-3)$$

$= 54.8528$

$=$

find number of men geeting wages between Rs 10 & Rs 15 from following Data.

wages	0-10	10-20	20-30	30-40
frequ.	9	30	35	42

x wages	$f(x)$ frequen	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
below 10	9			
		30		
below 20	30+9 = 39		5	
		35		2
below 30	39+35 = 74		7	
		42		
below 40	74+42 = 116			

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a).$$

$a + hu = 15$

$10 + 10(u) = 15$

$10 + 10u = 15$

$10u = 5$

$u = 0.5$

$$f(15) = 9 + (0.5)(30) + \frac{(0.5)(0.5-1)}{2!}(5) + \frac{(0.5)(0.5-1)(0.5-2)}{6}\times(2)$$

$= 23.5 \simeq 24.$

Hence no. of men. geeting wages. b/w 10 & 15 Rs is.

$24 - 9 = 15$

a ← 8 1925

Newton forward
Unit-2

find the lower degree polynomial $y(x)$ that find the data, find $y(5)$

x	0	2	4	6	8
y	5	9	61	209	501

$h = 2$
$a = 5$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	5				
		4			
2	9		48		
		52		48	
4	61		96		0.
		148		48	
6	209		144		
		292			
8	501				

Newton forward formula

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a)$$

$$\frac{u(u-1)(u-2)(u-3)}{6!}\Delta^4 f(a)$$

$a + hu = x$

$0 + 2(u) = x$

$u = x/2$

$f(x) = x^3 - 2x + 5$

$f(5) = 5^3 - 2 \times 5 + 5 = 125 - 10 + 5 = 120$

Newton forward Interpolation & 1925

Newton forward

Unit - 2

$$f'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 + \dots\right].$$

Q

x	0	1	2	3	4	5
$f(x)$	4	8	15	7	6	2

x	$f(x)$	$\Delta f(x)$ 1 D	$\Delta^2 f(x)$ 2 D	$\Delta^3 f(x)$ 3 D	$\Delta^4 f(x)$ 4 D	$\Delta^5 f(x)$ 5 D
0	4	$4 = \Delta y_0$	$3 = \Delta^2 y_0$	$-18 = \Delta^3 y_0$	$40 = \Delta^4 y_0$	$-77 \Delta^5 y_0$
1	8	$7 = \Delta y_1$	$-15 = \Delta^2 y_1$	$22 = \Delta^3 y_1$	$-33 \Delta^4 y_1$	
2	15	$-8 = \Delta y_2$	$7 = \Delta^2 y_2$	$-10 = \Delta^3 y_2$		
3	7	$-1 = \Delta y_3$	$-3 = \Delta^2 y_3$			
4	6	$-4 = \Delta y_4$				
5	2					

$a + hu = 1895$
$1891 + 10(4) = 4$
$u = 0.4$

$$\left.\frac{dy}{dx}\right)_{x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0\right]$$

$$= 1\left[4 - \frac{1}{2}(3) + \frac{1}{3}(-18) - \frac{1}{4}(40) + \frac{1}{5}(-77)\right)$$

$=$

2

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \dots$$

Unit-2

Newton Backward interpolation Unit-2

$$f'(x) = y' = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \cdots\right].$$

Interpolation

it is a method of deriving a simple function from the given discreate data set. such that the function passes through the provided data points.

polynomial interpolation is the interpolation of a given data set by the polyn of lowest possible degree that passes through the points of dataset.

inverse interpolation

The process of finding the value of the argument corresponding to a given value of the ~~argument~~ function lying b/w two tabulated function value.

Extrapolation

Extrapolation refers to estimating an unknown value based on extending a known sequence of value or fact.

Newton backward formula

unit 2

$$f(a+hu) = f(a) + \frac{u}{1!}\nabla f(a) + \frac{u(u+1)}{2!}\nabla^2 f(a) + \frac{u(u+1)(u+2)}{3!}\nabla^3 f(a) + \cdots$$

$$\nabla f(x) = f(x) - f(x-h)$$

Previous question

find 1925

$a = 1931$

$h = 10$

$a + hu = 1925$

$1931 + 10(u) = 1925$

$u = -0.6$

$$= 101 + (-0.6)(8) + (-0.6)\frac{((-0.6)+1)}{2}(-4) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{6}(-1)$$

$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{24} \times (-3)$$

$= \underline{96.83}$

Unit - 2

Lagrange's Interpolation

it is applicable for unequal interval

Q find the value of y when $x=10$ by Lagrange interpolation formula

x	5	6	9	11
$f(x)=y$	12	13	14	16

→ unequal.

$$f(x) = \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)} \times 13$$

$$+ \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)} \times 16.$$

put $x=10$.

$$f(10) = \frac{(10-6)(10-9)(10-11)}{\cancel{(10-6)(10-9)(10-11)}\ (-1)(-4)(-6)} \times 12 + \cdots$$

$= 14.66$

Newton Dividend difference (unequal interval).

Find the value of y when $x=10$ by Newton dividend formula.

$f(x)$	5	6	9	11
y	12	13	14	16

x		$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
		$f(x_0)$	$\Delta f(x_0)$	$\Delta^2 f(x_0)$	$\Delta^3 f(x_0)$
x_0	5	12	$\frac{13-12}{6-5}=1$	$\frac{1/3-1}{9-5}=-1/6$	$\frac{2/15+1/6}{11-5}=1/20$
x_1	6	13	$\frac{14-13}{9-6}=1/3$	$\frac{1-1/3}{11-6}=2/15$	
x_2	9	14	$\frac{16-14}{11-9}=2/2=1$		
x_3	11	16			

$f(x_0) = 12$

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0).$$

$$= 12 + (x-5)(1) + (x-5)(x-6)\left(\frac{-1}{6}\right) + (x-5)(x-6)(x-9)\left(\frac{1}{20}\right)$$

$$f(10) = 12 + (10-5)(1) + (10-5)(10-6)\left(\frac{-1}{6}\right) + (10-5)(10-6)(10-9)\left(\frac{1}{20}\right)$$

$$= 12 + 5 - \frac{20}{6} + 1$$

$$= 18 - 20/6 = 14.66$$

Use Lagrange's formula to fit polynomial to the following data Hence find $y(-2)$, $y(1)$, $y(4)$

Unit-2

x	-1	0	2	3
y	-8	3	1	2

$$f(x) = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)} \times -8 + \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)} \times 3$$

$$+ \frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)} \times 1 + \frac{(x+1)(x-0)(x-2)}{(3+1)(3)(3-2)} \times 2$$

$$= (x-2)(x-3)\left[\frac{-8x}{(-1)(-3)(-4)} + \frac{3(x+1)}{(1)(-2)(-3)}\right] +$$

$$(x+1)(x)\left[\frac{x-3}{3\times 2\times -1} + \frac{4(x-2)}{4(3)(1)}\right]$$

$$= x^2 - 2x - 3x + 6$$

$$= x^2 - 5x + 6$$

$$f(x) = \frac{7x^3 - 31x^2 + 28x + 18}{6}$$

$y(-2) = f(-2) = -36.83$

$y(1)$ $f(1) = 3.666$

$y(4)$ $f(4) = 13.666.$

Inverse Lagrange formula.

find value of x.

Q Apply Lagrange formula inversely to find the value of x when $y = 19$ when following.

x	0	1	2
y	0	1	20.

if y is given then find x we inverse Lagrange formula

$$x = \frac{(y-1)(y-20)}{(0-1)(0-20)} \times 0 + \frac{(y-0)(y-20)}{(1-0)(1-20)} \times 1$$

$$+ \frac{(y-0)(y-1)}{(20-0)(20-1)} \times 2$$

put $y = 19$.

$$x \text{ at } y=19 = 0 + \frac{(19)(-1)}{-19} + \frac{(19)(18)}{20(19)} \times 2$$

$$= 1 + 18/10 = 1+1.8$$

$$= 2.8$$

Formula Unit 3.

(I) Trapezoidal Rule

$$\int_a^b f(x)\,dx = h\left(\frac{y_0+y_n}{2} + y_1+y_2+y_3+ \cdots y_{n-1}\right)$$

it is applicable on any interval.

(II) Simson 1/3 Rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots) + 2(y_2+y_4+\cdots)\right]$$

it is applicable on 2 multiple.

(III) Simson 3/8 Rule

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+\cdots) + 2(y_3+y_6+\cdots)\right]$$

it is applicable on 3 multiple.

(IV) Boole's Rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{2h}{45}\left[7y_0+32y_1+12y_2+32y_3+7y_4+ 32y_5\ 12y_6\ 32y_7+\cdots\right]$$

This rule is applicable on 4 multiple : $n=4,8,12,16.$

$\int_0^1 \frac{dx}{1+x^2}$ also find value of π Unit-3

$$h = \frac{b-a}{n} = \frac{1-0}{6} = 1/6$$

$x_0 = 0$ $y_0 = \frac{1}{1+x_0^2} = \frac{1}{1+0} = 1$

$x_1 = x_0 + h = \frac{1}{6}$ $y_1 = \frac{1}{1+x_1^2} = \frac{1}{1+1/36} = \frac{36}{37}$.

$x_2 = x_1 + h = 2/6 = 1/3$ $y_2 = \frac{1}{1+x_2^2} = \frac{1}{1+1/9} = \frac{9}{10} = 0.9$.

$x_3 = x_2 + h = 3/6$ $y_3 = 0.8$

$x_4 = x_3 + h = 3/6 + 1/6 = 4/6$. $y_4 = 9/13$

$x_5 = x_4 + h = 4/6 + 1/6 = 5/6$ $y_5 = \frac{36}{61}$

$x_6 = x_5 + h = 5/6 + 1/6 = 1$ $y_6 = 0.5$

By Trapezoidal Rule

$$\int_0^1 \frac{dx}{1+x^2} dx = \frac{1}{6}\left(\frac{1+0.5}{2} + \frac{36}{37} = 0.9 + 0.8 + 9/13 + \frac{36}{61}\right)$$

$= 0.7842$.

By simson 1/3 Rule (It is applicable on 2 multiple).

$$\int_0^1 \frac{dx}{1+x^2} dx = \frac{1}{18}\left[(1+0.5) + 4\left(36/37 + 0.8 + \frac{36}{61}\right) + 2\left(0.9 + 9/13\right)\right)$$

$= 0.785396$.

By simson 3/8 Rule

$\frac{3 \times \frac{1}{6}}{8}$

$$\int_0^1 \frac{dx}{1+x^2} dx = \left(\frac{3}{8} \times \frac{1}{6}\right) = \left[(1+0.5) + 3\left(36/37 + 0.9 + 9/13 + 36/61\right) + 2\left(0.8\right)\right)$$

$= 0.785394$.

Direct integration

$$\int_0^1 \frac{1}{1+x^2}\,dx = \left(\tan^{-1}x\right)_0^1$$

$$= \tan^{-1}1 - \tan^{-1}0$$

$$= \pi/4 - 0$$

$$= \pi/4 .$$

By eq (I) & (IV)

$$\pi/4 = 0.78423.$$

$$\pi = 0.78423 \times 4$$

$$= 3.1369.$$

Boole's Rule

$$\int_0^4 \frac{dx}{1+x}$$ 4 interval.

$$h = \frac{4-0}{4} = 1$$

$x_0 = 0$
$x_1 = 1$
$x_2 = 2$
$x_3 = 3$
$x_4 = 4$

$y_0 = \frac{1}{1+x_0} = 1$
$y_1 = \frac{1}{1+x_1} = \frac{1}{1+1} = \frac{1}{2} = 0.5$
$y_2 = \frac{1}{1+x_2} = \frac{1}{1+2} = \frac{1}{3}$
$y_3 = 1/4 \quad y_5 = \frac{1}{5}$

x	0	1	2	3	4.
y	1	1/2	1/3	1/4	1/5

$$\int_0^4 \frac{1}{1+x}\,dx = \frac{2\times 1}{45}\left[\left(7\times 1\right) + 32\left(\tfrac{1}{2}\right) + 12\left(\tfrac{1}{3}\right) + 32\left(\tfrac{1}{4}\right) + 7\left(\tfrac{1}{5}\right)\right].$$

$$= 1.61778$$

Calculator

$$\int\left(1 \div (1+x), 0, 4\right).$$

exact value:

$$= 1.60994$$

Taylor Series Method

$$y = y(x_0) + y'(x_0)(x-x_0) + y''(x_0)\frac{(x-x_0)^2}{2!} + y'''(x_0)\frac{(x-x_0)^3}{3!} + \cdots$$

$\frac{dy}{dx} = x - y^2$, $y(0) = 1$

$x_0 = 0$, $y_0 = 1$

$\frac{dy}{dx} = x - y^2 = y'$

$y'(x_0) = x_0 - y_0^2 = 0 - (1)^2 = -1$

$y''(x_0) = 1 - \underset{u}{2y} \cdot \underset{v}{y'}$

$= 1 - 2 \times 1 \times (-1)$

$= 1 + 2 = 3.$

$y'''(x_0) = -2(y \cdot y'' + y' \cdot y'$ $\quad u \cdot \frac{dv}{dx} + v \cdot \frac{dy}{dx}.$

$= -2(1 \cdot 3 + (-1)(-1)$

$= -8$

$y^{IV} = -2[y \cdot y''' + y'' \cdot y' + y' \cdot y'' + y' y'']$

$= -2[1(-8) + 3 \times (-1) + (-1) \cdot (3) + (-1)(3)]$

$= -34$

$y = y(x_0) + y'(x_0)(x - x_0) + y''(x_0)\frac{(x-x_0)^2}{2!} + y'''(x_0)\frac{(x-x_0)^3}{3!} + y^{IV}(x_0)\frac{(x-x_0)^4}{4!}$

$= 1 + (-1)(x) + 3\frac{(x)^2}{2!} + (-8)\frac{(x)^3}{3!} + (-3)\frac{(x^4)}{4!}$

$y = 1 - x + \frac{3}{2}x^2 - \frac{4}{3}x^3 + \frac{17}{12}x^4.$

formula   Unit-4

Euler (it is Runge Kutta Method of 1st order
Euler Modified Method (it is R.K. Method of 2nd order).

→ **Euler Method** (R.K method of 1st order)

$$y_{n+1} = y_n + hf(x_n, y_n) \quad n = 0, 1, 2. \quad \left(h = \frac{b-a}{n}\right)$$

→ **Euler Modified Method** (R.K method of 2nd order)

$$y^*_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

→ **Runge Kutta Method.** (it is applicable on 4th order)

$$\frac{dy}{dx} = f(x)$$

where $y(x_0) = y_0$.

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf(x_n + h/2, y_n + K_1/2)$$

$$K_3 = hf(x_n + h/2, y_n + K_2/2)$$

$$K_4 = hf(x_n + h, y_n + K_3).$$

$$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$y_{n+1} = y_n + K.$$

→ **Taylor Series Method**

$$y = y(x_0) + y'(x_0)(x - x_0) + y''(x_0)\frac{(x-x_0)^2}{2!} + y'''(x_0)\frac{(x-x_0)^3}{3!} + y^{IV}(x_0)\frac{(x-x_0)^4}{4!} + \cdots$$

$= x+y$. boundary cond^n $y_1 = 1$ at $x=0$. $(n=5)$

$x = 0.1$

Euler Method

$$h = \frac{0.1-0}{5} = 0.02.$$

$x_0 = 0$

$x_1 = x_0 + h$

$= 0 + 0.02$

$x_2 = x_1 + h = 0.04$

$x_3 = 0.06.$

$x_4 = x_3 + h = 0.06 + 0.02 = 0.08.$

$x_5 = 1.0.$

$x_0 = 0, \ y_0 = 1$

$f(x,y) = x+y$

$y_0 = 1$ $\quad y_{n+1} = y_n + hf(x_n, y_n)$

$y_{n+1} = y_n + h(x_n + y_n)$

put $n=0$

$y_1 = y_0 + h(x_0 + y_0)$

$= 1 + 0.02(0+1)$

$y_1 = 1.02.$

put $n=1$

$y_2 = y_1 + h(x_1 + y_1)$

$= 1.0408$

put $n=2$

$y_3 = y_2 + h(x_2 + y_2)$

$= 1.0624$

put $n=3$

$y_4 = y_3 + h(x_3 + y_3)$

$= 1.0848$

put $n=4$

$y_5 = y_4 + h(x_4 + y_4)$

$= 1.1081$

Euler Method

Euler Modified Method

1.1.4.4

(it is Runge Kutta Method 2nd order)

$y^*_{n+1} = y_n + hf(x_n, y_n)$

$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})]$

Q $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$ (x_0)

$y_0 = 1$

$x_0 = 0$.

$y(0.02)$ & $y(0.04)$

Soln $x_0 = 0$ $\quad y_0 = 1$ $\quad$ put $n = 0$

$x_1 = 0.02$

$x_2 = 0.04$.

$y_1^* = y_0 + h(x_0^2 + y_0)$

$= 1 + 0.02(0^2 + 1)$

$= 1.02$

$h = 0.02$

put $n = 0$ in Euler Modified method.

put $n=0$

$y_1 = y_0 + \frac{h}{2}[(x_0^2 + y_0) + (x_1^2 + y_1^*)]$

$= 1 + \frac{0.02}{2}[(0^2 + 1) + ((0.02)^2 + 1.02)]$

$= 1.0202$

Ans
$y_1 = 1.0202$
$y_2 = 1.0408$

Again.

put $n = 1$

$y_2^* = y_1 + hf(x_1^2 + y_1)$

$= 1.0202 + 0.02((0.02)^2 + 1.0202)$

$= 1.0406$.

Again put $n=1$ in

$y_2 = y_1 + \frac{h}{2}[(x_1^2 + y_1) + (x_2^2 + y_2^*)]$

$= 1.0202 + \frac{0.02}{2}[((0.02)^2 + 1.0202) + ((0.04)^2 + 1.0406)]$

$= 1.0408$

R.K Method of 4th Order

11.1-4

$\frac{dy}{dx} = x + y^2 \quad y(0) = 1 \quad y(0.2) = ?$

where $h = 0.1$

$x_0 = 0 \quad y_0 = 1$

$x_1 = 0.1 \quad h = 0.1$

$x_2 = 0.2$

put $n = 0$

$K_1 = hf(x_0 + y_0^2)$

$= 0.1(0+1)$

$= 0.1$

$K_2 = hf[(x_0 + h/2) + (y_0 + K_1/2)^2]$

$= 0.1[(0 + 0.1/2) + (1 + 0.1/2)^2]$

$= 0.11525$

$K_3 = hf[(x_0 + h/2) + (y_0 + K_2/2)^2]$

$= 0.1[(0 + 0.1/2) + (1 + 0.11525/2)^2]$

$= \cancel{0.11525}\ 0.1169.$

$K_4 = hf[(x_0 + h) + (y_0 + K_3)^2]$

$= 0.1[(0 + 0.1) + (1 + 0.1169)^2]$

$= 0.1347.$

$K = 1/6[K_1 + 2K_2 + 2K_3 + K_4] = 1/6[0.1 + 2(0.11525) + 2(0.1169) + 0.1347)$

$= 0.1165$

put $n = 0$

$y_{n+1} = y_n + K$

$y_1 = y_0 + K = 1 + 0.1165 = 1.1165$

put $n=1$

$K_1 = hf(x_1 + y_1{}^2)$

$= 0.1\,(0.1 + (1.1165)^2)$

$= 0.1347$

$K_2 = hf[(x_1 + h/2) + (y_1 + K_1/2)^2]$

$= 0.1\,[(0.1 + 0.1/2) + (1.1165 + 0.1347/2)^2]$

$= 0.1552.$

$K_3 = hf[(x_1 + h/2) + (y_1 + K_2/2)^2]$

$= 0.1\,[(0.1 + 0.1/2) + (1.1165 + 0.1552/2)^2]$

$= 0.1576$

$K_4 = hf[(x_1 + h) + (y_1 + K_3)^2]$

$= 0.1\,[(0.1 + 0.1) + (1.1165 + 0.1576)^2]$

$= 0.1823$

$K = 1/6\,[K_1 + 2K_2 + 2K_3 + K_4]$

$= 1/6\,[0.1347 + 2(0.1552) + 2(0.1576) + 0.1823]$

$= 0.1572$

put $n=1$

$y_{n+1} = y_n + K$

$y_2 = y_1 + K$

$y_2 = 1.1165 + 0.1572$

$= 1.2737.$

Unit - 5

Bender-Schmidt Method

(page 1)

Q. Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$.

① $u(0,t) = 0$

② $u(5,t) = 0$

③ $u(x,0) = x^2(25 - x^2)$

$h = 1$

up to 3rd cord.

$x = 0, \; t = 0$

$x = 5, \; t = 0$

$x = x, \; t = x^2(25 - x^2)$

$x_0 = 0$

$x_1 = x_0 + h = 1$

$x_2 = x_1 + h = 2$

$x_3 = x_2 + h = 3$

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

By BSM

$$\boxed{\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}} \qquad \boxed{k = \frac{a}{2}h^2}$$

$a = 1$

$k = \frac{1}{2}(1)^2$

$= 0.5$

$\longrightarrow h.$

$\downarrow k$

t \ x	0	1	2	3	4	5
0	0	24	84	144	144	0
0.5	0	42	78	78	57	0
1	0	42	60	62.5	39	0
1.5	0	39	53.25	49.5	33.75	0
2	0	30	33	—	—	0
2.5	0	26.62	34.75	—	—	0
3	0	1.1905	—		—	0

$x = x, \; t = x^2(25 - x^2)$

$x = 1, \; t = 1(25 - 1)$

$= 24.$

$x = 2$ ——— 84

$x = 3$ ——— 144

$x = 4$ ——— 144

$a = \frac{c+b}{2}$

Unit-5

Crank-Nicholson Method

(28) (29) (30)

page 2

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0 \quad , 0 \le x \le 1$$

(i) $u(0,t) = 0$

(ii) $u(1,t) = 0$ $h = 0.25$

(iii) $u(x,0) = 100x(1-x)$ for one step.

$\boxed{x = 0 \\ x = 1}$ $, t = 0$ $, t = 0$

$x = x \quad , t = 100x(1-x)$

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

By CNM : $\boxed{\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}}$ $\boxed{K = ah^2}$

$\xrightarrow{h}$

$a = 1 \quad K = 1 \times (0.25)^2$
$= \boxed{0.0625}$

t \ x	0	0.25	0.5	0.75	1
0	0	18.75	25	18.75	0
0.06	0	a 9.82	b 14.28	c. 9.82	0

↓ k

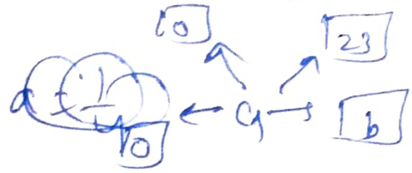
$x = x, \quad t = 100x(1-x)$

$x = 0.25 = t = (100(0.25)(1-0.25))$
$= 18.75$

$x = 0.5 = 25$

$x = 0.75 = 18.75$

$a = \frac{1}{4}[25 + b]$



$a = \frac{1}{4}[23 + b]$ —— (1)

$b = \frac{1}{4}[a + 18.75 + 18.75 + c]$ — (2)

$c = \frac{1}{4}[b + 25] = 0$

eq (1)

$4a = 25 + b$.

$4a - b = 25$ — (1)

eq 2)

$4b = a + \cancel{18}\ 37.5 + c$

$4b - a - c = 37.5$

$-a + 4b - c = 37.5$ — (2).

eq (3)

$4c = b + 25$

$4c - b = 25$

$0 - b + 4c = 25$ — (3)

$a = 9.82$

$b = 14.28$

$c = 9.82$

Alph

Mode

5

2

Unit 5

-ite difference approximation of partial derivatives.

Soln of Laplace's eqn (Elliptic) by Liebmann's iteration method.

3) Solution of ~~Laplace's equation~~ One dimensional heat eqn (parabolic) by Bender-Schmidt method. and Crank-Nicolson method.

4) soln of 1 Dimensional wave eqn (Hyperbolic)

5) CFL stability condition.

→ Bender-Schmidtt → heat eqn. ✓

→ Nicolson → ✓ heat eqn.

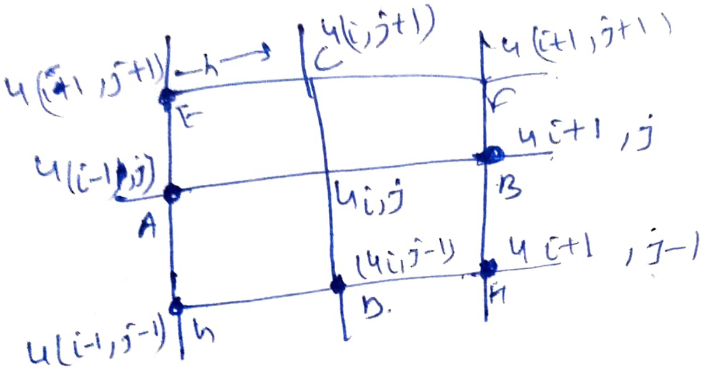
→ Laplace

→ wave eqn.

Laplace Eqn by Liebmann's iteration

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

or $u_{xx} + u_{yy} = 0$ or $\nabla^2 u = 0$



Standard five point formula

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right]$$

$$= \frac{1}{4}\left[A + B + D + C\right]$$

Diagonal five point formula

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j+1} + u_{i+1,j+1} + u_{i-1,j-1} + u_{i+1,j-1}\right]$$

$$= \frac{1}{4}\left[E + F + G + H\right]$$

Numerical Analyses. Analyse data.

Application,

Unit-8

i) Making weather Prediction (data is in the form of numerical form).

ii) Car Safety enhancement (air bags in Car use numerical analyses) (Care Company can improve the crash safety of their vehicle).

iii) Machine Learning (update use numerical analyse.) (Siri, Alexa).

iv) Artificial Intelligence (A.I) (Use in building)

v) Spacecraft dynamic. (move the space craft use trajectory to use numerical analyses)

vi) Price Estimation by Airline Companie (Ticket price estimation in use numerical analyses).

$6x^6+4x^5-\sin x+e^x-\cos x+x$ → to solve complex problem in everywhere.

→ Numerical Analysis is the study of algorithm that use numerical approximation for the problems of mathematical analyses.

→ the goal of the field of numerical analysis is the design and analysis of technique to give approximate but accurate sol^n to hard problems.

→ Numerical analysis provide fast and reliable sol^n.

→ create a model to use numerical analysis and analysis the data.

Unit 6

companies use numerical program for
financial analysis.

Page 2

computer and numerical analysis are interrelated to each other to solve the complex problem in a second or minute to get a soln.

Completed