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Question: Bank :- 4

(Q.1) **Ordinary Differential Equation:-**
An ordinary ~~differential~~ differential equation in mathematics is an equation which consists of one or more function of one independent variable with their derivatives

ex:- $\boxed{\frac{\partial^2 x}{\partial^2 y} = (x+y)^6}$

(Q.2) The Order of a differential equation is defined to be that of the highest order derivative it contains. The degree of a differential equation is defined as the power to which the highest order derivative is raised.

(3) $\frac{\partial^2 y}{\partial x^3} - 5x\frac{\partial y}{\partial x} = e^x + 1$

order = 3
Degree = 1

(4) $y\left(\frac{dy}{\partial x}\right)^2 = x^2 + 1$

Order = 1
degree = 2

(4) **General Solution of ODE :-**
A soln which have no of arbitary Constants is equal to order of differential eq.
ex:- $\frac{\partial^2 y}{\partial x^2} + 2\frac{\partial y}{\partial x} + y = 0$

Particular Solution of ODE :-
A Particular Solution of Diff. equn is a solution of the form $y = f(x)$, which do not have any

Q.3
Soln:-
Q.4)

arbitary constants.

6. given
$y = e^{5x}$ -- (1)
$y' - 5y = 0$ -- (2)
] to show particular Solution

form (1)
$y = e^{5x}$
diff w.r.t x
$y' = 5e^{5x}$

Put y' and y value in eqn (2)

$y' - 5y = 0$

$= 5e^{5x} - 5e^{5x}$
$= 5(e^{5x} - e^{5x})$
$= \underline{\underline{0}}$

Ordinary differential equation

7. Given $y = ce^{t}$ and
Now
$\frac{dy}{dt} = \frac{d\,ce^t}{dt} = c\frac{de^t}{dt} = ce^t = y$

i.e. $\frac{dy}{dt} = y = ce^t$

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(8) The differential equation $M(x,y)\,dx + N(x,y)\,dy = 0$ is said to be an exact differential equn. If their exist an function of u of x and y such that $Mdx + Ndy = du$.

The necessary & sufficient condition for differential equation $Mdx + Ndy = 0$ to be an

(9) $y\,dx + x\,dy = 0$

Here $M = y$
$N = x$

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x}$

hence, given differential equation is exact.

Now for solution

$\frac{\partial u}{\partial x}\,dx + \frac{\partial u}{\partial y}\,dy = 0$

$\frac{\partial u}{\partial x} = M$ & $\frac{\partial u}{\partial y} = N$

Now to find u

$u = \int M\,dx + f(y)$ --- (i)

so, $\frac{\partial u}{\partial x} = y$

$u = \int y\,dx + f(y)$

$u = xy + f(y)$

differential w.r.t y

$\frac{\partial u}{\partial y} = x + f'(y)$

$x = x + f'(y)$

$f'(y) = 0$

$f(y) = 0$

Now on putting in eq(i) $u = xy$.

(Q10) Given $(D^2 - 4)y = 0$

AE $= m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

Here $m = +2, -2$

(distinct value of m).

$\therefore$ $\boxed{y = c_1 e^{2x} + c_2 e^{-2x}}$

(Q.11) Given $\frac{dy}{dx} = \sin(x)$

$dy = \sin(x)\, dx$

Now Integrating both side

$\int dy = \int \sin(x) dx$

$y = -\cos(x) + c$

Now at $y(0) = 0$

$y(0) = -\cos(0) + c$

$0 = -1 + c$

$\boxed{c = 1}$

Now

$y(\pi/3) = -\cos(\pi/3) + 1$

$= -\frac{1}{2} + 1 = \frac{1}{2}$ Ans

(Q13) given $y'' + 4y = 0$

$m^2 + 4 = 0$

$m^2 = -4$

$\cancel{m^2} = \sqrt{-4} = \pm 2i$

$m = \pm 2i \Rightarrow \boxed{a = 0, \ b = 2}$

$y = e^{ax} (c_1 \cos bx + c_2 \sin bx)$

$y = e^{0x} (c_1 \cos bx + c_2 \sin bx)$

$y = c_1 \cos bx + c_2 \sin bx$

$\boxed{b = 2}$

$y(x) = c_1 \cos 2x + c_2 \sin 2x$

(15) Given $(D^2 - 2D)y = 5$

$$PI = \frac{5}{D^2-2D} = \frac{5\times 1}{D^2-2D} \quad \left[\begin{array}{l}\text{as we know} \\ e^{0x} = 1\end{array}\right]$$

$$= \frac{5\times e^{0x}}{D^2-2D} = \frac{5\times e^{0x}}{0^2-2\times 0} = \frac{5}{0}$$

So we differentiate F(D) and multiply x in Numerator

$$= \frac{x.5}{2D-2} = \frac{x.5.e^{0x}}{2\times 0-2} = \frac{x.5}{0-2}$$

$$= \frac{x.5}{-2} = -\frac{x5}{2}$$

(16) given $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$

A.E = $m^2 + 2m + 1 = 0$

$\Rightarrow m^2+m+m+1=0 \Rightarrow (m+1)(m+1) = 0$

$m = -1, -1$

Here m values are equals

So $y = (c_1 + xc_2)e^{mx}$

$$\boxed{y = (c_1 + xc_2)e^{-x}}$$

(17) given $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 25y = 0$

A·E $\quad m^2 - 6m + 25 = 0$

$a = 1, \quad b = -6, \quad c = 25$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 100}}{2 \times 1}$

$m = \frac{6 \pm \sqrt{-64}}{2} = \frac{6 \pm \sqrt{-1}\sqrt{64}}{2}$

$m = \frac{6 \pm i8}{2} = \frac{2(3 \pm 4i)}{2}$

$\boxed{m = 3 \pm 4i}$ → Here $a = 3$ $\quad$ ($\because a \pm bi$)

and $b = 4$

Now

$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

$y = e^{3x}(c_1 \cos 4x + c_2 \sin 4x)$.

(18) Given

$\cos(x+y)\,dx + (3y^2 + 2y + \cos(x+y))\,dy = 0$

Now $M = \cos(x+y)$

and $N = (3y^2 + 2y + \cos(x+y))\,\cancel{dy}$

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Now $\frac{\partial M}{\partial y} = \frac{\partial \cos(x+y)}{\partial y} = -\sin(x+y)$

$\Rightarrow \boxed{\frac{\partial M}{\partial y} = -\sin(x+y)}$

again for

$\frac{\partial N}{\partial x} = \frac{\partial (3y^2+2y+\cos(x+y))}{\partial x}$

$= 0+0-\sin(x+y)$

$\boxed{\frac{\partial N}{\partial x} = -\sin(x+y)}$

$\because \frac{\partial N}{\partial x} = -\sin(x+y) = \frac{\partial M}{\partial y}$

$\therefore$ It is an exact differential equation

Now $u = \int_{y\ const} M\,dx + \int_{x\ ignore} N\,dy$

$= \int \cos(x+y)\,dx + \int 3y^2+2y+\cos(x+$

$= \sin(x+y) +$

(Q.19) Given $x^3 dx + y^3 dy = 0$

$M = x^3$, $N = y^3$

Now $\frac{\partial M}{\partial y} = 0$, $\frac{\partial N}{\partial x} = 0$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 0$

$\therefore$ It is an exact differential equation

Now

$u = \int M dx + \int N dy$

$= \int x^3 dx + \int y^3 dy$

$= \frac{x^4}{4} + \frac{y^4}{4}$

$= \frac{x^4 + y^4}{4}$

(Q.20) Given $(2x \tan y) dx + \sec^2 y \, dy = 0$

$M = 2x \tan y$

$N = \sec^2 y$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial 2x \tan y}{\partial y} = 2x \frac{\partial \tan y}{\partial y} = 2x \sec^2 y$

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$$\frac{\partial N}{\partial x} = \frac{\partial \sec^2 y}{\partial x} = 0$$

$\therefore \quad \frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial x}$

$\therefore$ It is not exact differential

(21) given $e^{x^2}(2xy\,dx + dy) = 0$

Now $e^{x^2}2xy\,dx + e^{x^2}dy = 0$

$M = e^{x^2}2xy$ | $N = e^{x^2}$

$\frac{\partial M}{\partial y} = \frac{\partial e^{x^2}2xy}{\partial y}$ | $\frac{\partial N}{\partial y} = \frac{\partial e^{x^2}}{\partial y} =$

$= e^{x^2}2x$ | $2xe^{x^2}$

$\frac{\partial M}{\partial y} = 2xe^{x^2}$ $\qquad \frac{\partial N}{\partial y} = 2xe^{x^2}$

$\therefore \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial y} = 2xe^{x^2}$

Now $\therefore$ It is an exact differential eqn

$u = \int M dx + \int N dy$

$= \underset{(y = \text{constant})}{\int e^{x^2}2xy\,dx} + \underset{x \text{ Not contain}}{\int e^{x^2}dy}$

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$u = 2y\int x e^{x^2} dx + 0$

$u = 2y\int \cancel{x} e^{t} \frac{dt}{2\cancel{x}} + 0$

$u = y\int e^{t} dt + c$

$u = y e^{t} + c$

$u = y e^{x^2} + c$ $\{as\ t = x^2\}$

let $x^2 = t$

$2x = \frac{dt}{dx}$

$\frac{dt}{2x} = (dx)$

(21)

(22) Given $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0$

A.E $\Rightarrow$ $m^2 + 2m - 5 = 0$

Now $a = 1$, $b = 2$, $c = -5$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2\times 1} = \frac{-2 \pm \sqrt{4+20}}{2}$

$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm 2\sqrt{6}}{2}$

$= \frac{2(-1 \pm \sqrt{6})}{2} = -1 \pm \sqrt{6}$

$y = e$

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(Q.23) Given $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$

$y(0) = 0$ $\quad y' = 3e^{-1}$

A.E. = $m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$(m+1)(m+1) = 0$

$m = -1, -1$

$y = (c_1 + xc_2)e^{mx}$

$y = (c_1 + xc_2)e^{-x}$ ---- (i)

Now given $y = 0$ so $(x = 0)$

$0 = (c_1 + 0 c_2) e^{-0}$

$0 = c_1$

$\boxed{c_1 = 0}$

Now put $c_1 = 0$ in eqn (i)

So $y = xc_2 e^{-x}$

Now $y' = c_2 \left[\frac{d}{dx} xe^{-x}\right]$

$y' = c_2 \left[e^{-x} - xe^{-x}\right]$

$y' = c_2 (1-x) e^{-x}$

Now $y' = 3e^{-1}$ (at $x=0$)

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$3e^{-1} = C_2 (1-0) e^{0}$

$\boxed{3e^{-1} = C_2}$

$\therefore \quad C_2 = 3e^{-1}$ put eqn (1)

$y = (C_1 + x C_2) e^{-x}$

$y = (0 + x3e^{-1}) e^{-x}$

$= x3e^{-1} \cdot e^{-x}$

$\boxed{y = x \cdot 3 \cdot e^{(-1-x)}}$

Now for $y(2)$ $\quad (x=2)$

$y(2) = 2 \cdot 3 \cdot e^{(-1-2)}$

$\boxed{y(2) = 6 \cdot e^{-3}}$

(24) Given $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$

$y(0) = 1 \quad \& \quad y'(0) = 0$

Now

A E $\Rightarrow$ $m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$(m+1)(m+1) = 0$

$\boxed{m = -1, -1}$

Now $y = (c_1 + xc_2)e^{-x}$ — (1)

Now $y(0) = 0$

$(c_1 + 0c_2)e^0 = 0$

$\boxed{c_1 = 0}$

Put $c_1 = 0$ in eqn (1)

so $y = xc_2 e^{-x}$

$\Rightarrow y' = c_2 [e^{-x} - xe^{-x}]$

$\Rightarrow y' = c_2 (1-x)e^{-x}$

Now $y'(0) = 0$

$0 = c_2(1-0)e^0$

$0 = c_2$

$\therefore$ $c_2 = 0$ & $c_1 = 0$

$\therefore$ $y = 0$

(25) Given $\frac{d^2y}{dx^2} + \frac{3dy}{dx} = 0$ $y(0) = 0$ and $y'(0) = 1$

Now

$m^2 + 3m = 0$

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$m(m+3)=0$

$m = 0$ & -3

Now

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$y = c_1 e^{0x} + c_2 e^{-3x}$

$y = c_1 + c_2 e^{-3x}$

Now $y(0) = 0$

$0 = c_1 + c_2$

$\boxed{c_1 = -c_2}$

Now $y = c_1 e^{m_1 x} + (-c_1) e^{m_2 x}$

$y = c_1 e^{m_1 x} - c_1 e^{m_2 x}$

$y = c_1 (e^{m_1 x} - e^{m_2 x})$

$y' = c_1 [m_1 e^{m_1 x} - m_2 e^{m_2 x}]$

$y'(0) = 1.$ $[m_1 = 0, \ m_2 = -3]$

$1 = c_1 [m_1 - m_2]$

$1 = c_1 [0 - (-3)]$

$\boxed{c_1 = \frac{1}{3}}$ and $\boxed{c_2 = -\frac{1}{3}}$

25

$$y = \frac{1}{3} + \left(\frac{-1}{3}\right)e^{-3x}$$

$$y = \frac{1}{3}\left[1 - e^{-3x}\right]$$

(26) Given $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0$ $\quad y(0) = 0$, $y'(0) = 1$

Now

A.E $\Rightarrow m^2 - 2m + 1 = 0$

$\Rightarrow m^2 - m - m + 1 = 0$

$\Rightarrow (m-1)(m-1) = 0$

$\Rightarrow m = 1, 1$

Now

$y = \cancel{c_1 + xc_2} \ (c_1 + \cancel{x}c_2)e^{m\cancel{x}}$ — (1) $\quad t$ x

$y = (c_1 + \cancel{x}c_2)e^{\cancel{x}}$ $\quad t$ x

Now $y(t=0) = 0$

$\cancel{0} = (c_1 + 0)1$

$\boxed{c_1 = 0}$

Now putting $c_1 = 0$ in eqn (1)

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$y = t c_2 e^t$

$y' = c_2 [e^t + t e^t]$

$y' = c_2 [1 + t] e^t$

$y'(0) = c_2 [(1 + 0) 1$

$0 = c_2$

$\therefore \boxed{c_2 = 0}$

Now $c_2 = 0$ & $c_1 = 0$

So $y = 0$

So $y(t=1) = 0$

(27) Given $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

$y(0) = 1$

$y'(0) = 1$

Now

$m^2 - 4m + 4 = 0$

$a = 1, \quad b = -4, \quad c = 4$

$m = \frac{-(-4) \pm \sqrt{16 - 16}}{2} = \frac{4 \pm 0}{2}$

$m = \frac{4}{2} = 2$

$\boxed{m = 2}$

$$y = (c_1 + x c_2) e^{mx} \quad -(1)$$

$$y = [c_1 + x c_2] e^{2x}$$

Now $y(0) = 1$

$$1 = (c_1 + 0 c_2) 1$$

$$\boxed{c_1 = 1}$$

Putting $c_1 = 1$ in equation $-(1)$

$$y = (1 + x c_2) e^{mx}$$

$$y' = e^{mx} + x c_2 e^{mx}$$

$$y' = m e^{mx} + c_2 [e^{mx} + mx e^{mx}]$$

$$y' = 2 e^{2x} + c_2 [e^{2x} + 2x e^{2x}]$$

$$y' = 2 e^{2x} + c_2 (1 + 2x) e^{2x}$$

$$y' = [2 + c_2 (1 + 2x)] e^{2x}$$

$$y'(0) = 1$$

$$1 = [2 + c_2 (1 + 0)] 1$$

$$\boxed{c_2 = -1}$$

$$y = (1-x)e^{2x}$$

Now $y(1) = (1-1)e^{2\times 1}$

$$\boxed{y(1) = 0}$$

(28) Polynomial function

(29) given $y'' + y' - 2y = 0$ $\quad y(0) = 4$

$y'(0) = -5$

A.E $\Rightarrow m^2 + m - 2 = 0$

$\Rightarrow m^2 + 2m - m - 2 = 0$

$\Rightarrow (m+2)(m-1) = 0$

$\Rightarrow m = -2, +1$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$\boxed{y = c_1 e^{-2x} + c_2 e^{x}} \quad \text{--- (i)}$$

$y(0) = 4$

$4 = c_1 + c_2$

$$\boxed{c_1 = 4 - c_2}$$

Now putting $c_1 = 4 - c_2$ in eqn (i)

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$y = (4-c_2)e^{-2x} + c_2 e^{x}$

$y = 4e^{-2x} - c_2 e^{-2x} + c_2 e^{x}$

$y' = -8e^{-2x} + 2c_2 e^{-2x} + c_2 e^{x}$

$y'(0) = -5$

$-5 = -8 + 2c_2 + c_2$

$-5 = -8 + 3c_2$

$3c_2 = 3$

$\boxed{c_2 = 1}$

Now $c_1 = 4 - c_2$

$\therefore \boxed{c_1 = 3}$

so $\boxed{y = 3e^{-2x} + e^{x}}$ Ans

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(3a) Now given $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 10e^{-3x}$

Now

A E =

$m^2 + 5m + 4 = 0$

$m^2 + 4m + m + 4 = 0$

$(m+4)(m+1) = 0$

$$\boxed{m = -4, -1}$$

$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$$\boxed{y_c = c_1 e^{-4x} + c_2 e^{-x}}$$

$y_p = \frac{10e^{-3x}}{D^2 + 5D + 4} = \frac{10e^{-3x}}{(-3)^2 + 5(-3) + 4}$

$= \frac{10e^{-3x}}{9 + 15 + 4} = \frac{10e^{-3x}}{9 - 15 + 4} = \frac{10e^{-3x}}{-2}$

$$\boxed{y_p = -5e^{-3x}}$$

$y = y_c + y_p$

$$\boxed{y = c_1 e^{-4x} + c_2 e^{-x} - 5e^{-3x}}$$

Ans

Q.35) Given $\frac{d^2y}{dx^2} - 2y = 6e^{2x} - 4e^{-2x}$

A.E $\Rightarrow m^2 - 2 = 0$

$\Rightarrow m^2 = 2$

$\Rightarrow m = \pm\sqrt{2}$

Now

$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$\Rightarrow \boxed{y_c = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}}$

Now

$y_p = \frac{6e^{2x} - 4e^{-2x}}{D^2 - 2} = \frac{6e^{2x}}{D^2-2} - \frac{4e^{-2x}}{D^2-2}$

$= \frac{6e^{2x}}{4-2} - \frac{4e^{-2x}}{4-2} \Rightarrow \frac{6e^{2x}}{2} - \frac{4e^{-2x}}{2}$

$\Rightarrow \boxed{y_p = 3e^{2x} - 2e^{-2x}}$

$y = y_c + y_p$

$\boxed{y = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x} + 3e^{2x} - 2e^{-2x}}$

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34) given

$$3\frac{d^2y}{dx^2} + 27y = 3\cos x + \cos 3x$$

$AE = 3m^2 + 27 = 0$

$3m^2 = -27$

$m^2 = -9$

$m = \pm 3i \qquad a = 0, \quad b = 3$

Now

$y_c = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

$y_c = e^{0x}[c_1 \cos 3x + c_2 \sin 3x]$

$$\boxed{y_c = [c_1 \cos 3x + c_2 \sin 3x]}$$

$$y_p = \frac{3\cos x + \cos 3x}{3D^2 + 27} = \frac{3\cos x}{3D^2+27} + \frac{\cos 3x}{3D^2+27}$$

$$= \frac{3\cos x}{-3+27} + \frac{\cos 3x}{-27+27} \qquad \left\{ \frac{3\cos x}{-3\times(1)+27} \text{ and similar} \right.$$

$$= \frac{3\cos x}{24} + \frac{\cos 3x}{0}$$

$$= \frac{3\cos x}{24} + \frac{x \cdot \cos 3x}{6D}$$

$$= \frac{\cos x}{8} + \frac{x}{6} D^{-1}\cos 3x$$

$$= \frac{\cos x}{8} + \frac{x}{6} \cdot \frac{\sin 3x}{3}$$

$$= \frac{\cos x}{8} + \frac{x \sin 3x}{18}$$

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$$y = y_c + y_p$$

$$= c_1 \cos 3x + c_2 \sin 3x + \frac{\cos x}{8} + \frac{x \sin 3x}{18}$$