(Question 1)

Inherent errors are those error which are already present in the statements of a problem before its solution.

Example :- π, e and   i.e. $\pi = 3.141592654$ to
$e = 2.7182818$

for better approximation we will minimize the data for better answer, we will take $\pi = 3.14$ and $e = 2.718$

(Question :-2)

Round off error are those error occur due to Rounding of number while process of computing the values.

Example :- $x = 7.5846712$ $x' = 7.585$

$\therefore$ Round off error $= |x - x'| = |7.5846712 - 7.585|$
$= 0.0003288$

$$\boxed{\text{Round off error} = |(\text{Original value}) - (\text{Round off Value})|}$$

X   X'

Question:-3

Truncaation error are those error in which we Truncating (Breaking down) The infinite Series to the finite number of series for better approximation.

$$\boxed{\text{Truncation error} = \left|\begin{pmatrix}\text{original}\\\text{value}\end{pmatrix} - \begin{pmatrix}\text{Truncated}\\\text{value}\end{pmatrix}\right|}$$

Example :- $e^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$$e^{x'} = 1 + \frac{x}{1!} + \frac{x^2}{2!}$$

Truncation Error $= e^x - e^{x'}$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots\right) - \left(1 + x + \frac{x^2}{2!}\right)$$

$$= \frac{x^3}{3!} + \frac{x^4}{4!} \ldots\ldots$$

Ans

Question:-4

Absolute error is equal to Modulus of original value minus approximated value

$$\boxed{E_A = |x - x'|}$$

where
x = original value
x' = Approximated value

Question:-5

Relative error = $E_R = \left|\frac{x - x'}{x}\right|$

where x = Original value
x' = Approximated value

Question:-6

Percentage error = $E_P = \left|\frac{x - x'}{x}\right| \times 100$

where x = Original value
x' = Approximated value

Question:-7

Given that = $x = 3.6252782$
we have to Round off approx. 3 decimal place
so
$\therefore$ $\boxed{x \approx 3.625}$ Ans.

Question:- 8

Given $x = 35.00537$

So There are 7 significant digits in the number 35.00537.

Question:- 9

Given $x = 56.487262$

Now
$x' = 56.487$ {up to 3 decimal place}

Max^m Absolute error $= |x - x'|$
$= |56.487262 - 56.487|$

$E_A = 0.000262$ Ans

Question:-10

let $u = f(x, y)$

i.e.

Max^m absolute error $\boxed{\Delta u = \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y}$ done

Question:-11

See Question no:- 2 and Question no:- 3 for difference.

Question:-12

Question no:- 13

An equation which contains polynomials, trigono-metric function, logarithmic functions, exponential functions etc. is called a Transcendental equation.

The equation is in the form of $f(x) = 0$

Ex → $x\log x - 1 = 0$, $xe^{x} + 10x - \log x = 0$

Question :- 14

- Transcendental Equation → same as Q13
- Polynomial Equation → The expression which is of the form $f(x) = a_0 x^{n} + a_1 x^{n-1} + \cdots + a_n$ is called a Polynomial in x of degree n, where $a_0 \neq 0$ and $a_0, a_1, a_2 \ldots a_n$ are constants and n is +ve.

example:- $8x^4 + 17x^3 + 45x^2 + 9x + 14$

Question:-16

Suppose we have ~~free~~ equation $f(x) = 0$ where

(i) $f(x)$ is contineous in $[a, b]$ and

(ii) $f(a) \cdot f(b) < 0$ then

by interdimate value theoram there exist atleast one point $c \in (a, b)$ such $f(c) = 0$

Now for bisection method
taking 1st approximate value $x_1 = \frac{a+b}{2}$

then finding the $f(x_1)$ and check it value is negative or positive and replace by ~~upper~~ lower value. or upper value respectively.

Question-18

Soln:- $f(x) = x^3 - 2x - 5 = 0$

Now at $x = 0 \Rightarrow f(0) = -5$
at $x = 1 \Rightarrow f(1) = 1 - 2 - 5 = -6$
at $x = 2 \Rightarrow f(2) = 8 - 4 - 5 = -1$ (-ve)
at $x = 3 \Rightarrow f(3) = 27 - 6 - 5 = 16$ (+ve)

So the root must lies between $(2, 3)$ as the $f(x)$ sign changes from $f(2)$ to $f(3)$.

Question: 26

$x = 0.77729$ and $y = 0.0022218$

Now

$x = 0.7773$ {4 significant figure}

$y = 0.002222$ {4 significant figure}

Question: 27

$$\left.\begin{aligned}\sqrt{3} &= 1.73205080757\\ \sqrt{5} &= 2.2360679775\\ \sqrt{7} &= 2.64575131106\end{aligned}\right\}\text{True values}$$

$$\left.\begin{aligned}\sqrt{3} &= 1.732\\ \sqrt{5} &= 2.236\\ \sqrt{7} &= 2.646\end{aligned}\right\}\text{approximate value upto 4 SF}$$

Now

$X =$ True value $= \sqrt{3} + \sqrt{5} + \sqrt{7}$

$= 6.61387009613$

$X' =$ Approximate value $= 1.732 + 2.236 + 2.646$

$= 6.614$

Now

Absolute error $= |X - X'| =$

$= -0.00012990387$

Relative error $= \left|\frac{x-x'}{x}\right| = \frac{|x-x'|}{|x|}$

$= \frac{0.00012990387}{6.61387009613}$

$= 0.0000196 4113$. Ans

Question 28 Given number, 865250, 37.46235] true value

8652, 37.46] approximate value upto 4 significant fig

~~for 8652~~

Absolute error $= |x - x'|$

$x = 865250 \quad x' = 8652$

$E_A = |865250 - 8652| = 856598$

Relative error $= \left|\frac{x-x'}{x}\right| = \frac{856598}{865250} = 0.9900005$

percentage error $= \left|\frac{x-x'}{x}\right| \times 100$

$= 0.9900005 \times 100 = 99.00005$

for 37.46235

Absolute Error $= |X - X'|$

$= |37.46235 - 37.46| = 0.00235$

Relative Error $= \left|\frac{X - X'}{X}\right| = \frac{0.00235}{37.46235} = 0.00006273$

Percentage Error $= \left|\frac{X - X'}{X}\right| \times 100 = 0.00006273 \times 100$
$= 0.006273$

Question 29 Given $X = 0.00545828$

(i) Truncated to three decimal places

$X' = 0.005$ (3 decimal place truncated)

for Absolute error $= |X - X'|$

$= |0.00545828 - 0.005|$
$= 0.00045828$

(ii) Roundoff to 3 decimal places

$X' = 0.005$ (Roundoff to 3 decimal places)

Absolute error $= |X - X'|$
$= |0.00545828 - 0.005|$
$= 0.00045828$

* Question:- 30

Given error $\Delta x = \Delta y = \Delta z = 0.001$

$x = y = z = 1$

Given

$$u = \frac{4x^2y^3}{z^4}$$

$$\Delta u = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z$$

$$\Delta u = 8x\frac{y^3}{z^4}\Delta x + \frac{12y^2x^2}{z^4}\cdot\Delta y + (-16)\frac{x^2y^3}{z^5}\Delta z$$

$$\Delta u = 8\times 0.001 + 12\times 0.001 - 16\times 0.001$$

$$\Delta u = 0.008 + 0.012 - 0.016$$

$$\Delta u = 0.004 \text{ (maximum Absolute)}$$

Now for u we get

$u = 4$ {By putting the value of x, y, z}

Now $\left[\frac{\Delta u}{u}\right]$

$\left|\frac{x-x'}{x}\right|$ = Relative error $= \frac{0.004}{4} = 0.001$

Now Percentage error $= \left|\frac{\Delta u}{u}\right| \times 100$

$= 0.1\%$ Ans

Question 31 Given area of circle πr^2 (let r be radius)

$$\frac{\Delta u}{u} \times 100 = 0.1\% \quad (u = \pi r^2)$$

$$\frac{\Delta u}{u} = 0.001 \quad \ldots\ldots (1)$$

Now, $\Delta u = \frac{\partial f}{\partial r}\Delta r = 2\pi r \Delta r \quad \ldots\ldots (11)$

from (1) and (11)

$$\frac{\Delta u}{u} = \frac{2\cancel{\pi} \cancel{r} \Delta r}{\cancel{\pi} r^{\cancel{2}}} = \frac{2\Delta r}{r}$$

Comparing from (1)

$$2\frac{\Delta r}{r} = 0.001$$

$$\frac{\Delta r}{r} = \frac{1 \times 0.001}{2} = 0.0005$$

$$\boxed{\frac{\Delta r}{r} \times 100 = 0.05\%}$$

Question:- 33

Given

$f(x) = 3x + \sin x - e^x$

$\therefore f(x) = 3x + \sin x - e^x = 0$

$f(0) = 0 + 0 - 1 = -1$

$f(1) = 3 + 0.8415 - 2.7183 = 1.1232$

So the sign of $f(x)$ changes from $f(0)$ to $f(1)$

$\therefore$ root must lies between $(0, 1)$.

Now

(Step 1.) :- Let $x_0 = 0$ $x_1 = 1$

$f(x_0) = -1$ $f(x_1) = 1.1232$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{0 + 1}{1.1232 + 1} = \frac{1}{2.1232} = 0.4710$$

Now at $x = 0.4710$

$f(0.4710) =$

$= -0.9604$ (-ve)

Now The root must lies between $(0.4710, 1.1232)$

(Step-2) let $x_0 = 0.4710$ $x_1 = 1$

$f(x_0) = -0.9604$ $f(x_1) = 1.1232$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(0.4710)(1.1232) + (0.9604)}{1.1232 + 0.9604}$$

$$= \frac{1.4894}{2.0836} = 0.7148$$

Now at $x = 0.7148$

$f(0.7148) =$

$= 0.7560$ (+ve)

So the root must be lies between $(0.4710, 0.7148)$

→ (Step:-3)

Now let $x_0 = 0.4710$ $x_1 = 0.7148$

$f(x_0) = -0.9604$ $f(x_1) = \cancel{1.1232}$ 0.7560

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(0.4710)(\overset{0.7560}{\cancel{1.1232}}) + (0.7148)(0.9604)}{\underset{0.7560}{\cancel{1.1232}} + 0.9604}$$

$$= \frac{1.0426}{1.7164} = 0.6074$$

Question:- 34

Given that $f(x) = x \cdot \log_{10} x = 1.9$

Now $f(x) = x \cdot \log_{10} x - 1.9 = 0$

at $x=0 \Rightarrow f(0) =$ Not valid

at $x=1 \Rightarrow f(1) = -1.9$

$x=2 \Rightarrow f(2) = -1.2979$

$x=3 \Rightarrow f(3) = -0.4686$ (-ve)

$x=4 \Rightarrow f(4) = 0.5082$ (+ve)

Here the $f(x)$ sign changes from $f(3)$ to $f(4)$

$\therefore$ The root must lies between $(3,4)$.

Now

(Step 1) let $x_0 = 3$ $x_1 = 4$

$f(x_0) = -0.4686$ $f(x_1) = 0.5082$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{3 \times 0.5082 + 4 \times 0.4686}{0.5082 + 0.4686}$$

$$= \frac{3.399}{0.9768}$$

$$= 3.4797$$

Now at $x = 3.4797$

$f(3.4797) = -0.0156$ (-ve)

Now

The root must lies between $(3.4797, 4)$

(Step 2) $x_2 = 4$ $\quad x_1 = 3.4797$

$f(x_2) = +0.5082 \quad = -0.0156$

Now

$$x_3 = \frac{x_1 f(x_2) - f(x_1) x_2}{f(x_2) - f(x_1)}$$

$$x_3 = \frac{3.4797(0.5082) + 4 \times 0.0156}{0.5082 + 0.0156}$$

$$= \frac{1.8308}{0.5238} = 3.4952$$

Now at $x = 3.4952$

$f(3.4952) = -0.00046$

Roots must lies between $(3.4952 \; 4)$

(Step 3) $x_2 = 3.4952$ $\quad x_3 = 4$

$f(x_2) = -0.00046 \quad f(x_3) = +0.5082$

Now

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

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$$x_4 = \frac{3.4952 \times 0.5082 + 4 \times 0.00046}{0.5082 + 0.00046}$$

$$x_4 = \frac{1.7781}{0.50866} = 3.4956$$

Now at $x = 3.4956$

$f(3.4956) = -0.00007.$

the root must lies between $(3.4956, 4)$

Step:-4 $\quad x_3 = 3.4956 \quad x_4 = 4$

$f(x_3) = -0.00007 \quad f(x_4) = 0.5082$

$$x_5 = \frac{0.5082 \times 3.4956 + 4 \times 0.00007}{0.5082 + 0.00007}$$

$$= \frac{1.7767}{}$$

$$= 3.49558$$

Ques 35 Given $f(x) = x^3 - x^2 - 1 = 0$

$f'(x) = 3x^2 - 2x - 0 = 0 \quad 3x^2 - 2x$

Now take $x = 0 \quad f(0) = -1$

$x = 1 \quad f(1) = -1$

$x = 2 \quad f(2) = 3$

Roots must lies b/w $(1, 2)$

Now taking $x_0 = 2$

$f(2) = 3 \qquad f'(2) = 8$

1st approx So, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{3}{8} = 1.625$

$f(x_1) = f(1.625) = 0.6503$

$f'(x_1) = f'(1.625) = 4.6718$

2nd approx $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.625 - \frac{0.6503}{4.6718}$

$= 1.4858$

$f(1.4858) = 0.0725$

$f'(1.4858) = 3.6512$

3rd approx $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.4858 - \frac{0.0725}{3.6512}$

$= 1.4659$

Question:- 37.

Given $f(x) = x^3 - 9x + 1 = 0$

let At $x = 0 \Rightarrow f(0) = 1$
$x = 1 \Rightarrow f(1) = -7$
$x = 2 \Rightarrow f(2) = -9$ (-ve)
$x = 3 \Rightarrow f(3) = 1$ (+ve)

So The sign of $f(x)$ changes from $f(2)$ to $f(3)$

$\therefore$ The root must lies between (2 3)

Now We take $x_0 = 3$

Now (case - I) $\Rightarrow$ $x^3 - 9x + 1 = 0$
$x(x^2 - 9) + 1 = 0$
$\boxed{x = \frac{-1}{x^2 - 9}} = \phi(x)$

Now $\phi'(x) = \frac{-2x}{(x^2 - 9)^2} = \frac{-6}{0} > 1$

$\therefore$ This case is not satisfy.

(Case - II) = $x^3 = 9x - 1$
$x = (9x - 1)^{1/3} = \phi(x)$

Now $\phi'(x) = \frac{3}{(9x - 1)^{2/3}}$

$$\phi'(3) = \frac{3}{(26)^{2/3}} = 0.3418 < 1$$

This case is satisfying. So It may be consider

Now (Case - III) = $9x = x^3+1$

$$x = \frac{x^3+1}{9} = \phi(x)$$

$$\phi'(x) = x = \frac{3x^2}{9} = \frac{x^2}{3} = \frac{9}{3} = 3$$

which is greater than 1

So This case does not satisfying.

∴ The only one case is satisfying i.e (Case - II)

So we will consider case - II

$$x_1 = (9x_0 - 1)^{1/3} = (9\times3-1)^{1/3} = 2.9625$$

$$x_2 = (9x_1 - 1)^{1/3} = \cancel{2.9496}$$
$$= (9\times 2.9625 - 1)^{1/3} = 2.9496$$

$$x_3 = (9x_2 - 1)^{1/3}$$
$$= (9\times 2.9496 - 1)^{1/3} = 2.9451$$

★ Q39

$$2x + 2y + z = 6$$
$$4x + 2y + 3z = 4$$
$$x - y + z = 0$$

Now $AX = B$

$$\begin{bmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

augmented matrix $C = [A : B]$

$$\left[\begin{array}{ccc|c} 2 & 2 & 1 & 6 \\ 4 & 2 & 3 & 4 \\ 1 & -1 & 1 & 0 \end{array}\right] \quad \text{Apply } R_3 \leftrightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & 2 & 3 & 4 \\ 2 & 2 & 1 & 6 \end{array}\right] \quad \text{Apply } R_2 \to R_2 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 6 & -1 & 4 \\ 2 & 2 & 1 & 6 \end{array}\right] \quad \text{Apply } R_3 \to R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 6 & -1 & 4 \\ 0 & 4 & -1 & -2 \end{array}\right] \quad \text{Apply } R_3 \to 6R_3 - 4R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 6 & -1 & 4 \\ 0 & 0 & -2 & -28 \end{array}\right]$$

Now on comparing

* $\Rightarrow -2z = -28$

$\boxed{z = 14}$

* $\Rightarrow 6y - z = 4$

$\Rightarrow 6y - 14 = 4$

$\Rightarrow y = 18/6 = 3 \quad \Rightarrow \boxed{y = 3}$

$\Rightarrow x - y + z = 0$

$\Rightarrow x - 3 + 14 = 0$

$\Rightarrow \boxed{x = -11}$

Question 40

Given. $f(x) = \cos x = x e^x$

$$f(x) = \cos x - x e^x = 0$$

Now at $x = 0 \Rightarrow f(0) = 1$ (+ve)

$x = 1 \Rightarrow f(1) = -2.1779$ (-ve)

$x = 2 \Rightarrow f(2) = -15.$ (-ve)

Now The sign changes from $f(0)$ to $f(1)$

So The root must lies between $(0, 1)$

Step $\Rightarrow x = \frac{0+1}{2} = 0.5$

Now at $x = 0.5$

$f(0.5) = 0.0532$

So The root lies between (0.5, 1)

$$x = \frac{0.5+1}{2} = \frac{1.5}{2} = 0.75$$

Now at $x = 0.75$

$f(0.75) = -0.8560$ (-ve)

root (0.5, 0.75)

* (Step :- 3)

$$\frac{0.5+0.75}{2} = \frac{1.25}{2} = 0.625$$

Now at $x = 0.625$

$f(0.625) = -0.3566$

root = (0.5, 0.625)

* Step :- 4

$$\frac{0.5+0.625}{2} = \frac{1.125}{2} = 0.5625$$

Now at $x = 0.5625$

$f(0.5625) = -0.14129$

* Step :- 5

$$\frac{0.5+0.5625}{2} = \frac{1.0625}{2} = 0.53125$$

at $x = 0.53125$

$f(0.53125) = -0.06415$

* Step-6

$$\frac{0.5+0.53125}{2} = \frac{1.03125}{2} = 0.515625$$

$f(0.515625) = 0.006475$

Question 41

$$f(x) = x^3 - 4x + 9 = 0$$

at $x = 0 \Rightarrow f(0) = 9$

$x = 1 \Rightarrow f(1) = 6$

$x = 2 \Rightarrow f(2) = 9$

$x = 3 \Rightarrow f(3) = 24$

$x = -1 \Rightarrow f(-1) = 12$

$x = -2 \Rightarrow f(-2) = 9$] (+ve)

$x = -3 \Rightarrow f(-3) = -6$] (-ve)

Root must lies beetween $(-3, -2)$

(step 1) $x = \frac{-2-3}{2} = \frac{-5}{2} = -2.5$

$f(-2.5) = 3.375$

Root mus lies between $(-3, -2.5)$

(Step-2) = $x = \frac{-3-2.5}{2} = \frac{-5.5}{2} = -2.75$

$f(-2.75) = -0.796875$

Root must lies between $(-2.75, -2.5)$

(Step 3) = $x = \frac{-2.75-2.5}{2} = \frac{-5.25}{2}$

$x = -2.625$

$f(-2.625) = 1.41210$

Root must lies between $(-2.75, -2.625)$

(Step :- 4) $x = \frac{-2.75 - 2.625}{2} = \frac{-5.375}{2} = -2.687$

$f(-2.6875) = 0.339111$

Root must lies b/w $(-2.75, -2.6875)$

(Step :- 5)

$x = \frac{-2.75 - 2.6875}{2} = \frac{-5.4375}{2}$

$x = -2.71875$

$f(-2.71875) = -0.2209.$

Root must lies between $(-2.71875, -2.6875)$

Step(6) $= \frac{-5.40625}{2} = -2.703125$

$f(-2.703125) = 0.06107711$

Question 43 Given $x = e^{-x}$ Here $x_0 = 0.5$

let $f(x) = x - e^{-x} = 0$

at $x = 0.5$ $f(0.5) = 0.5 - e^{-0.5}$
$= -0.10653$

at $x = 0$ $f(0) = -1$
at $x = 1$ $f(1) = 0.6321$

Roots must lies b/w at $x(0,1)$
we will consider $x_0 = 0.5$

Case I $f(x) = 0$
$x - e^{-x} = 0$
$x = e^{-x} = \phi(x)$

$\phi'(x) = -e^{-x}$
$\phi'(0.5) = -e^{-0.5} = -0.3032 < 1$

Since case I satisfied the condition $|\phi'(x)| < 1|$

$x_1 = e^{-x_0} = e^{-0.5} = 0.6065$
$x_2 = e^{-x_1} = e^{-6065} = 0.5452$
$x_3 = e^{-x_2} = e^{-0.5452} = 0.5797$
$x_4 = e^{-x_3} = e^{-0.5797}$ 0.5600
$x_5 = e^{-x_4} = 0.5712$
$x_6 = e^{-x_5} = 0.5648$
$x_7 = e^{-x_6} = 0.5684$
$x_8 = e^{-x_7} = 0.5666$
$x_9 = e^{-x_8} = 0.5674$
$x_{10} = e^{-x_9} = 0.5669$

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Question - 45

$$f(x) = x^4 - x = 10 \qquad f'(x) = 4x^3 - 1$$

$$f(x) = x^4 - x - 10 = 0$$

at $x=0 \Rightarrow f(0) = -10$

at $x=1 \Rightarrow f(1) = 1-1-10 = -10$

at $x=2 \Rightarrow f(2) = 4$

So The $f(x)$ sign changes from $f(1), f(2)$

$\therefore$ The root must lies b/w $(1, 2)$.

Step 1) let. $\boxed{x_0 = 2}$

Now $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{4}{31}$

$x_1 = 1.8709$

(Step - 2) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \cancel{1.8709758...}$

$= 1.85577$

Step 3 $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.85558$

Here correct upto 3 decimal place

1.855

(46) $f(x) = 3x = \cos x + 1$ | $f'(x) = 3 + \sin x$

$f(x) = \cancel{3x} \; 3x - \cos x - 1$

Now = At $x=0 \Rightarrow f(0) = -2$ (-ve)

$x=1 \Rightarrow f(1) = 1.4596$ (+ve)

$x=2 \Rightarrow f(2) = 5.4161$ (+ve)

So The sign of $f(x)$ changes from $f(0)$ to $f(1)$

∴ The root must lies between $(0,1)$.

Now let $\boxed{x_0 = 1}$

(Step 1) $\Rightarrow$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 0.62001$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} \Rightarrow 0.60712$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 0.60710$

∴ Correct upto 4-decimal place is

$\boxed{0.6071}$ Ans

[Question - 47]

Given $f(x) = x^3 - 2x - 5 = 0$

Now at $x=0 \Rightarrow f(0) = -5$
at $x=1 \Rightarrow f(1) = -6$
at $x=2 \Rightarrow f(2) = -1$ (-ve)
at $x=3 \Rightarrow f(3) = 16$ (+ve)

So The sign of The $f(x)$ changes $f(2)$ to $f(3)$
Now
The root must be lies between $(2,3)$

(Step:-1) $x_0 = 2$ $x_1 = 3$
$f(x_0) = -1$ $f(x_1) = 16$

$$x_2 = \frac{x_0(f(x_1)) - x_1 f(x_0)}{f(x_1) - f(x_0)} = 2.0588$$

$f(2.0588) = -0.391052$

(Step:-2) $x_1 = 3$ $x_2 = 2.058$
$f(x_1) = +16$ $f(x_2) = -0.391052$

$$x_3 = \frac{x_2 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = 2.08047$$

$f(2.08047) = -0.15592$

Step:-3 $x_2 = 2.08047$; $x_3 = 3$
$f(x_2) = -0.15592$; $f(x_3) = 16$

$$x_4 = \frac{x_2 f(x_3) - f(x_2)\, x_3}{f(x_3) - f(x_2)} = 2.089344$$

$$f(2.089344) = -0.05794$$

(Step:-4) = $x_3 = 2.089344$ $x_4 = 3$
$f(x_3) = -0.05794$ $f(x_4) = 16$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)} = 2.092630$$

$$f(2.092630) = -0.0214233$$

(Step:-5)

$x_5 = 3$ $x_4 = 2.092630$
$f(x_5) = 16$ $f(x_4) = -0.0214233$

$$x_6 = \frac{x_4 f(x_5) - f(x_4)\, x_5}{f(x_5) - f(x_4)} = 2.09384$$

$$f(x_6) = -0.00790$$

Step:-6 = $x_5 = 2.09384$ $x_6 = 3$
$f(x_5) = -0.00790$ $f(x_6) = 16$

$$x_7 = \frac{x_5(f(x_6)) - x_6 f(x_5)}{f(x_6) - f(x_5)} = 2.0942$$

$\therefore$ $x_6 = 2.0998 \approx 2.094$

$x_7 = 2.0942 \approx 2.094$

(Question 4.8) $x = \sqrt[4]{32}$

$f(x) = x^4 - 32 = 0$

Now at $x = 0 \Rightarrow f(0) = -32$

at $x = 1 \Rightarrow f(1) = -31$

at $x = 2 \Rightarrow f(2) = -16$ (−ve)

at $x = 3 \Rightarrow f(3) = 49$ (+ve)

The root must lies b/w (2, 3)

(Step-1) let $x_0 = 2$ $\quad x_1 = 3$

$f(x_0) = -16$ $\quad f(x_1) = 49$

$$x_2 = \frac{2 \times 49 + 3 \times 16}{16 + 49} = 2.24615$$

$f(2.24615) = -6.54605$