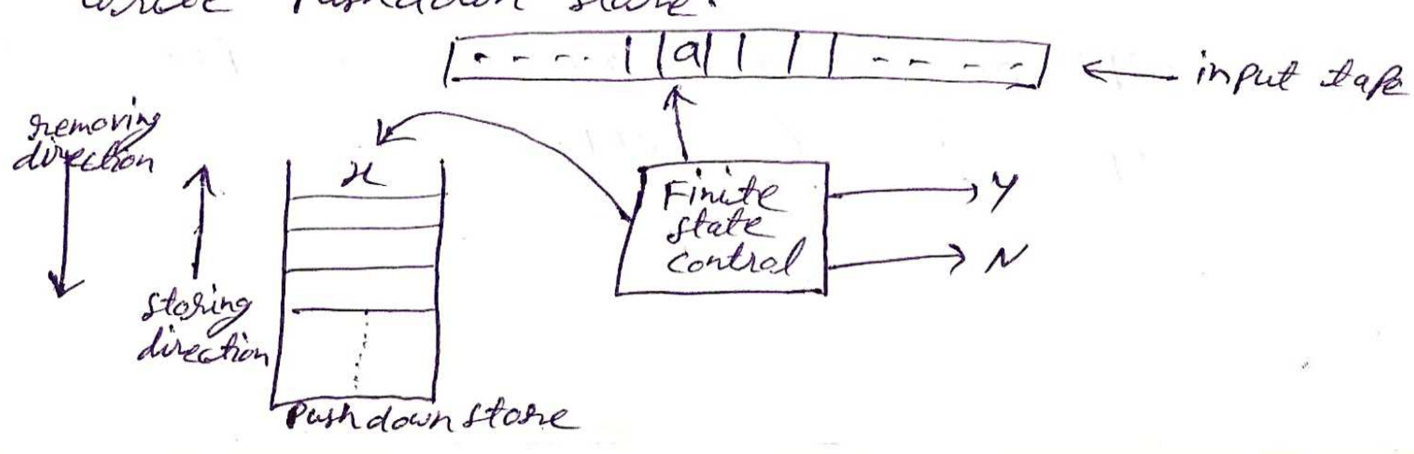
unit : 4

Push Down Automata (PDA)

if m is the finite automata accepting a language L, it is constructed in such a way that states act as a form of Primitive memory. Let us consider $L = \{a^n b^n : n \geq 1\}$. This is CFG but not regular. so a finite automata cannot accept L, ie it has to remember the no. of a's and no. of b's so it will require an infinite no. of states. This difficulty can be avoided by adding an auxiliary memory in the form of a stack. (operation on stack is in LIFO form)

so the components of a Pushdown Automaton are a read only input tape, input alphabets, a finite state control, a set of final state, initial state, and in addition to these, it has a stack called Pushdown store. it is a read-write Pushdown store.



so Pushdown Automata is a string accepter, generated by CFG.

a PDA consists seven tupple

$$A = (Q, \Sigma, \Gamma, \delta\, q_0, Z_0, F)$$

where Q is a nonempty finite set of states.
Σ is a nonempty finite set of input symbols.
Γ is a nonempty finite set of stack symbols.
q_0 is initial state
Z_0 is initial stack symbol
F is set of final states. it may be empty.
δ is a transection function. it is a mapping between $Q \times \Sigma \times \Gamma$ to $Q \times \Gamma^*$

it is three dimenshel so not represent in table so it is represent like as

$$\delta(q_0, 0, Z_0) = (q_0, xZ_0)$$

content free language (CFL):

set for which it is possible to CFG as a generater or PDA as a accepter is called CFL.

PDA's Instantaneous Description (ID):

A PDA has a configuration at any given instance: (q, w, α)

where q is current state

w is remainder of the input (unconsumed part)

α is current stack contents as a string from top to bottom of stack.

if $\delta(q, a, x) = (P, A)$ is a transition, then the following are also true.

* $(q, a, x) \vdash (P, \varepsilon, A)$

* $(q, aw, xB) \vdash (P, w, AB)$

where "$\vdash$" is called a "turnstile notation" and represent one move

$\vdash^*$ represents a sequence of moves.

There are two types of PDAs that one can design:

(i) accept by final state.

(ii) accept by empty state.

(i) PDAs that accept by final state:

For a PDA P, the language accepted by P denoted by L(P) by final state is:

$$\{w : (q_0, w, Z_0) \vdash^* (q_f, \epsilon, \alpha)\}$$

where q_f is final state & α is stack contents (as a string)

means any string is accepted only when input string is exhausted and finite state control reached on a final state.

(ii) PDAs that accept by empty stack:

For a PDA P, the language accepted by P denoted by L(P) by empty of stack is:

$$\{w : (q_0, w, Z_0) \vdash^* (q, \epsilon, \epsilon)\}$$

where $q \in Q$.

means any string is accepted only when input string is exhausted and stack is empty.

Q Design a PDA for $L = \{a^n b^n : n \geq 1\}$

Let PDA $P = (Q, \{a,b\} \{Z_0, x\}, \delta, q_0, Z_0, \phi)$

where δ defined as

$$\delta(q_0, a, Z_0) = (q_0, xZ_0)$$
$$\delta(q_0, a, x) = (q_0, xx)$$
$$\delta(q_0, b, x) = (q_1, \varepsilon)$$
$$\delta(q_1, b, x) = (q_1, \varepsilon)$$
$$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$$

and Q defined as $Q = \{q_0, q_1\}$

instantanous Description (ID) for a string "aaabbb"

$[q_0, aaabbb, Z_0] \vdash [q_0, aabbb, xZ_0]$
$\vdash [q_0, abbb, xxZ_0] \vdash [q_0, bbb, xxxZ_0]$
$\vdash [q_1, bb, xxZ_0] \vdash [q_1, b, xZ_0]$
$\vdash [q_1, \varepsilon, Z_0] \vdash [q_1, \varepsilon, \varepsilon]$

Note: in transection f^n, if transition $\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$ replace by $\delta(q_1, \varepsilon, Z_0) = (q_f, Z_0)$ then the PDA is accept by final state (where q_f is final state).

Q Design PDA for $L = a^n b^{2n} : n \geq 1$

$P = (\{q_0, q_1\}, \{a, b\}, \{x, Z_0\}, \delta, q_0, Z_0, \phi)$

where δ is defined as

$\delta(q_0, a, Z_0) = (q_0, xxZ_0)$
$\delta(q_0, a, x) = (q_0, xxx)$
$\delta(q_0, b, x) = (q_1, \varepsilon)$
$\delta(q_1, b, x) = (q_1, \varepsilon)$
$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

Q Design a PDA for $L = a^{2n} b^n : n \geq 1$

$P = (\{q_0, q_1, q_2\}, \{a, b\}, \{x, Z_0\}, \delta, q_0, Z_0, \phi)$

$\delta(q_0, a, Z_0) = (q_1, xZ_0)$
$\delta(q_1, a, x) = (q_0, x)$
$\delta(q_0, a, x) = (q_1, xx)$
$\delta(q_0, b, x) = (q_2, \varepsilon)$
$\delta(q_2, b, x) = (q_2, \varepsilon)$
$\delta(q_2, \varepsilon, Z_0) = (q_2, \varepsilon)$

write ID for a string "aaaabb"

Q Design PDA for $L = \{a^{2n} b^n : n \geq 1\}$ by using two states.

$P = (\{q_0, q_1\}, \{a, b\}, \{x, y, Z_0\}, \delta, q_0, Z_0, \phi)$

$\delta(q_0, a, Z_0) = (q_0, yxZ_0)$
$\delta(q_0, a, y) = (q_0, \varepsilon)$
$\delta(q_0, a, x) = (q_0, yxx)$
$\delta(q_0, b, x) = (q_1, \varepsilon)$
$\delta(q_1, b, x) = (q_1, \varepsilon)$
$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

Q Design a PDA for $L = \{a^n b^m c^m d^n : m, n \geq 1\}$

$m = \{Q, \{a, b, c, d\}, \{Z_0, x, y\}, \delta, q_0, Z_0, \phi\}$

$\delta(q_0, a, Z_0) = (q_0, xZ_0)$
$\delta(q_0, a, x) = (q_0, xx)$
$\delta(q_0, b, x) = (q_0, yx)$
$\delta(q_0, b, y) = (q_0, yy)$
$\delta(q_0, c, y) = (q_1, \varepsilon)$
$\delta(q_1, c, y) = (q_1, \varepsilon)$
$\delta(q_1, d, y) = (q_1, \varepsilon)$
$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

$Q = \{q_0, q_1\}$

Q Design a PDA for $L = wcw^R ; w \in (a,b)^+$, w^R is Reverse of w

$m = (\{q_0, q_1\}, \{a, b, c\}, \{x, y, Z_0\}, \delta, q_0, Z_0, \phi \}$

$\delta(q_0, a, Z_0) = (q_0, xZ_0)$

$\delta(q_0, a, x) = (q_0, xx)$

$\delta(q_0, b, Z_0) = (q_0, yZ_0)$

$\delta(q_0, b, y) = (q_0, yy)$

$\delta(q_0, b, x) = (q_0, yx)$

$\delta(q_0, a, y) = (q_0, xy)$

$\delta(q_0, c, x) = (q_1, x)$

$\delta(q_0, c, y) = (q_1, y)$

$\delta(q_1, a, x) = (q_1, \varepsilon)$

$\delta(q_1, b, y) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

write ID for a string "abbabcbabba"

Q. Design PDA for
$L = ww^R : w \in (a,b)^*$

$M = \{ \{q_0, q_1\}, \{a, b\}, \{x, y, Z_0\}, \delta, q_0, Z_0, \phi \}$

(i) $\delta(q_0, a, Z_0) = (q_0, xZ_0)$
(ii) $\delta(q_0, b, Z_0) = (q_0, yZ_0)$
(iii) $\delta(q_0, a, x) = (q_0, xx)$
(iv) $\delta(q_0, b, y) = (q_0, yy)$
(v) $\delta(q_0, a, y) = (q_0, xy)$
(vi) $\delta(q_0, b, x) = (q_0, yx)$
(vii) $\delta(q_0, a, x) = (q_1, \varepsilon)$
(viii) $\delta(q_0, b, y) = (q_1, \varepsilon)$
(ix) $\delta(q_1, a, x) = (q_1, \varepsilon)$
(x) $\delta(q_1, b, y) = (q_1, \varepsilon)$
(xi) $\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

here $\delta(q_0, a, x)$ have more then one different outputs (Transition f^n (iii) & (vii)) same as $\delta(q_0, b, y)$ (transitin f^n (iv) & (viii))

in this PDA transition f^ns have more than two different o/p so this PDA is Non Deterministic PDA.

here transition f^n (iii)&(vii), (iv)&(viii) can be written as
$\delta(q_0, a, x) = \{(q_0, xx), (q_1, \varepsilon)\}$
$\delta(q_0, b, y) = \{(q_0, yy), (q_1, \varepsilon)\}$

write ID for string "abbaabba"

in ww^R, last symbol of w & first symbol of w^R will ~~al~~ always same

$[q_0, abbaabba, Z_0] \vdash [q_0, bbaabba, xZ_0]$

$\vdash [q_0, baabba, yxZ_0] \vdash [q_0, aabba, yyxZ_0]$

$\vdash [q_1, aabba, xZ_0]$

$\vdash [q_1, abba, Z_0]$ X

$\vdash [q_0, bba, xxyyxZ_0]$

$\vdash [q_0, ba, yxxyyxZ_0]$

⋮

X

$\vdash [q_0, abba, xyyxZ_0]$

$\vdash [q_1, bba, yyxZ_0]$

$\vdash [q_1, ba, yxZ_0]$

$\vdash [q_1, a, xZ_0]$

$\vdash [q_1, \varepsilon, Z_0]$

$\vdash [q_1, \varepsilon, \varepsilon]$

Note: (i) it is not necessary that an unique Deterministic PDA exist for every non Deterministic PDA.

(ii) for all CFG theire exist PDA $\left\{ \begin{array}{l} \text{NPDA} \\ \text{or} \\ \text{DPDA.} \end{array} \right.$

(iii) the language for which NPDA is Design and not exist DPDA equivalent to it then the language is called inherently ambiguous.

Q Design a PDA for $\{L = a^n b^m c^n : m, n \geq 1\}$

$P = \{ \{q_0, q_1, q_2\}, \{a, b, c\}, \{x, Z_0\}, \delta, q_0, Z_0, \phi \}$

$\left.\begin{array}{l} \delta(q_0, a, Z_0) = (q_0, xZ_0) \\ \delta(q_0, a, x) = (q_0, xx) \end{array}\right\}$ Push x in stack for "a"

$\left.\begin{array}{l} \delta(q_0, b, x) = (q_1, x) \\ \delta(q_1, b, x) = (q_1, x) \end{array}\right\}$ nothing to do for "b"

$\left.\begin{array}{l} \delta(q_1, c, x) = (q_2, \varepsilon) \\ \delta(q_2, c, x) = (q_2, \varepsilon) \\ \delta(q_2, \varepsilon, Z_0) = (q_2, \varepsilon) \end{array}\right\}$ Pop x from stack for "c"

CFG to PDA conversion:

if the given CFG is not in GNF form then first convert is in GNF.

Let CFG in GNF is $G(V, T, P, S)$

then PDA P equivalent to it is

$$P = (\{q\}, T, V, \delta, q, S, \phi)$$

for each production in the grammar, there will be a transition in PDA as per the following rule

string of variables

$$A \to a\alpha \Rightarrow \delta(q, a, A) = (A, \alpha)$$

$$A \to a \Rightarrow \delta(q, a, A) = (A, \varepsilon)$$

Q convert following CFG into PDA.

$S \to aAA$

$A \to a / aS / bS$

Sol. the given CFG is in GNF form so PDA is

$P = (\{q\}, \{a,b\}, (S, A), \delta, q, S, \phi)$

$\delta(q, a, S) = (q, AA)$

$\delta(q, a, A) = (q, \varepsilon)$ } Due to this PDA is non-derministic

$\delta(q, a, A) = (q, S)$

$\delta(q, b, A) = (q, S)$

ID for string "abaaaa":

$[q, abaaaa, S] \vdash [q, baaaa, AA]$

$\vdash [q, aaaa, SA] \vdash [q, aaa, AAA]$

$\vdash [q, aa, AA] \vdash [q, a, A] \vdash [q_0, \varepsilon, \varepsilon]$

Q construct a PDA that accepts the ~~following~~ the language generated by the following CFG.

$S \rightarrow aB$
$B \rightarrow bA / b$
$A \rightarrow aB$

and show ane ID for string "abab".

solution: The given CFG is in GNF form so PDA is

$P = (\{q\}, \{a, b\}, \{S, A, B\}, \delta, q_0, S, \phi)$

where δ defined as.

for Production $S \rightarrow aB$

$\delta(q, a, S) = (q, B)$

for Production $B \rightarrow bA$

$\delta(q, b, B) = (q, A)$

for Production $B \rightarrow b$

$\delta(q, b, B) = (q, \varepsilon)$

for Production $A \rightarrow aB$

$\delta(q, a, A) = (q, B)$

ID for "abab":

$\delta[q, abab, S] \vdash [q, bab, B] \vdash [q, ab, A]$
$\vdash [q, b, B] \vdash [q, \varepsilon, \varepsilon]$

PDA to CFG conversion:

Let $G = (V, T, P, S)$ is converted CFG from a PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \phi)$

where $T = \Sigma$,

S is start variable of CFG

in this CFG any variable represent as

$$[\underset{\uparrow \atop \text{state}}{\_}\ \underset{\uparrow \atop \text{stack symbol}}{\_}\ \underset{\uparrow \atop \text{state}}{\_}]$$

* From start variable no. of production is equal to No. of states in the PDA M/C

$$S \rightarrow [q_0, Z_0, q_i] \quad \forall\, q_i \in Q.$$

* for one transition there will be $|Q|^{|\alpha|}$ Productions in Grammar where $|Q|$ is no. of states in PDA and $|\alpha|$ is no. of stack symbol push in stack for this specific transition.

Q convert PDA to CFG

$m = \{\{q_0, q_1\}, \{0,1\}, \{Z_0, x\}, \delta, q_0, Z_0, \phi\}$

$\delta(q_0, 1, Z_0) = (q_0\, xZ_0)$

$\delta(q_0, \varepsilon, Z_0) = (q_0, \varepsilon)$

$\delta(q_0, 1, x) = (q_0, xx)$

$\delta(q_0, 0, x) = (q_1, x)$

$\delta(q_1, 1, x) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_0, Z_0)$

in PDA No. of states = 2, so No. of Production from one transition f^n is 2^α where α is no. of element Push in stack.

Let converted CFG is $G = (V, T, P, S)$

$T = \{0, 1\}$

From start variable No. of Production, equal to No. of states in PDA m/c

$S \rightarrow [q_0, Z_0, q_0] \,/\, [q_0, Z_0, q_1]$

for $\delta(q_0, 1, Z_0) \Rightarrow (q_0, xZ_0)$ { Push 2 variable in stack so No. of Production is $2^2 = 4$

$[q_0, Z_0, q_0] \rightarrow 1\,[q_0, x, q_0]\,[q_0, Z_0, q_0]$

same　　same

$[q_0, Z_0, q_0] \rightarrow 1\,[q_0, x, q_1]\,[q_1, Z_0, q_0]$

$$[q_0, Z_0, q_1] \rightarrow 1\,[q_0, x, q_0][q_0, Z_0, q_1]$$
$$[q_0, Z_0, q_1] \rightarrow [q_0, x, q_1][q_1, Z_0, q_1]$$

for $\delta(q_0, 1, x) = (q_0, xx)$ { Push 2 variable in stack so $2^2 = 4$ Production generate

$$[q_0, x, q_0] \longrightarrow 1\,[q_0, x, q_0][q_0, x, q_0]$$
$$[q_0, x, q_0] \longrightarrow 1\,[q_0, x, q_1][q_1, x, q_0]$$
$$[q_0, x, q_1] \longrightarrow 1\,[q_0, x, q_0][q_0, x, q_1]$$
$$[q_0, x, q_1] \longrightarrow 1\,[q_0, x, q_1][q_1, x, q_1]$$

for $\delta(q_0, 0, x) = (q_1, x)$ { Push 1 variable in stack so $2^1 = 2$ Production Produce

$$[q_0, x, q_0] = 0\,[q_1, x, q_0]$$

same

$$[q_0, x, q_1] = 0\,[q_1, x\, q_1]$$

for $\delta(q_1, 1, x) = (q_1, \varepsilon)$ { Push 0 variable so $2^0 = 1$ Production Produce

$$[q_1, x, q_1] = 1$$

for $\delta(q_0, \varepsilon, Z_0) = (q_0, \varepsilon)$

$$[q_0, Z_0, q_0] \rightarrow \varepsilon$$

for $\delta(q_1, \varepsilon, Z_0) = (q_0, Z_0)$

$[q_1, Z_0, q_0] \rightarrow \varepsilon [q_0, Z_0, q_0]$

$[q_1, Z_0, q_1] \rightarrow \varepsilon [q_0, Z_0, q_1]$

Q. convert the following PDA into equivalent CFG

$\delta(q_0, a, Z_0) = (q_0, Z_1 Z_0)$
$\delta(q_0, a, Z_1) = (q_0, Z_1 Z_1)$
$\delta(q_0, b, Z_1) = (q_1, \varepsilon)$
$\delta(q_1, b, Z_1) = (q_1, \varepsilon)$
$\delta(q_1, b, Z_0) = (q_1, Z_2 Z_0)$
$\delta(q_1, b, Z_2) = (q_1, Z_2 Z_2)$
$\delta(q_1, c, Z_2) = (q_2, \varepsilon)$
$\delta(q_2, c, Z_2) = (q_2, \varepsilon)$
$\delta(q_2, \varepsilon, Z_0) = (q_2, \varepsilon)$

solution: there are 3 states in PDA so production from start variable is

- $S \rightarrow [q_0, Z_0, q_0] / [q_0, Z_0, q_1] / [q_0, Z_0, q_2]$

for $\delta(q_0, a, Z_0) = (q_0, Z_1 Z_0)$

$[q_0, Z_0, q_0] \rightarrow a[q_0, Z_1, q_0][q_0 Z_0 q_0]$
$[q_0, Z_0, q_0] \rightarrow a[q_0, Z_1, q_1][q_1 Z_0 q_0]$
$[q_0, Z_0, q_0] \rightarrow a[q_0, Z_1, q_2][q_2 Z_0 q_0]$

$[q_0 Z_0 q_1] \rightarrow a [q_0 Z_1 q_0][q_0 Z_0 q_1]$

$[q_0 Z_0 q_1] \rightarrow a [q_0 Z_1 q_1][q_1 Z_0 q_1]$

$[q_0 Z_0 q_1] \rightarrow a [q_0 Z_1 q_2][q_2 Z_0 q_1]$

$[q_0 Z_0 q_2] \rightarrow a [q_0 Z_1 q_0][q_0 Z_0 q_2]$

$[q_0 Z_0 q_2] \rightarrow a [q_0 Z_1 q_1][q_1 Z_0 q_2]$

$[q_0 Z_0 q_2] \rightarrow a [q_0 Z_1 q_2][q_2 Z_0 q_2]$

for $\delta(q_0, a, Z_1) = (q_0, Z_1 Z_1)$

$[q_0 Z_1 q_0] \rightarrow a [q_0, Z_1 q_0][q_0 Z_1 q_0]$

$[q_0 Z_1 q_0] \rightarrow a [q_0 Z_1 q_1][q_1 Z_1 q_0]$

$[q_0 Z_1 q_0] \rightarrow a [q_0 Z_1 q_2][q_2 Z_1 q_0]$

$[q_0 Z_1 q_1] \rightarrow a [q_0 Z_1 q_0][q_0 Z_1 q_1]$

$[q_0 Z_1 q_1] \rightarrow a [q_0 Z_1 q_1][q_1 Z_1 q_1]$

$[q_0 Z_1 q_1] \rightarrow a [q_0 Z_1 q_2][q_2 Z_1 q_1]$

$[q_0 Z_1 q_2] \rightarrow a [q_0 Z_1 q_0][q_0 Z_1 q_2]$

$[q_0 Z_1 q_2] \rightarrow a [q_0 Z_1 q_1][q_0 Z_1 q_2]$

$[q_0 Z_1 q_2] \rightarrow a [q_0 Z_1 q_2][q_0 Z_1 q_2]$

for $\delta(q_0, b, Z_1) = (q_1, \varepsilon)$

$[q_0, Z_1, q_1] \rightarrow b$

for $\delta(q_1, b, Z_1) = (q_1, \varepsilon)$

$[q_1, Z_1, q_1] \rightarrow b$

for $\delta(q_1, b, Z_0) = (q_1, Z_2 Z_0)$

$[q_1 Z_0 q_0] \rightarrow b\,[q_1 Z_2 q_0][q_0 Z_0 q_0]$

$[q_1 Z_0 q_0] \rightarrow b\,[q_1 Z_2 q_1][q_1 Z_0 q_0]$

$[q_1 Z_0 q_0] \rightarrow b\,[q_1 Z_2 q_2][q_2 Z_0 q_0]$

$[q_1 Z_0 q_1] \rightarrow b\,[q_1 Z_2 q_0][q_0 Z_0 q_1]$

$[q_1 Z_0 q_1] \rightarrow b\,[q_1 Z_2 q_1][q_1 Z_0 q_1]$

$[q_1 Z_0 q_1] \rightarrow b\,[q_1 Z_2 q_2][q_2 Z_0 q_1]$

$[q_1 Z_0 q_2] \rightarrow b\,[q_1 Z_2 q_0][q_0 Z_0 q_2]$

$[q_1 Z_0 q_2] \rightarrow b\,[q_1 Z_2 q_1][q_1 Z_0 q_2]$

$[q_1 Z_0 q_2] \rightarrow b\,[q_1 Z_2 q_2][q_2 Z_0 q_2]$

for $\delta(q_1, b, Z_2) = (q_1, Z_2 Z_2)$

$[q_1, Z_2, q_0] \rightarrow b\,[q_1 Z_2 q_0][q_0 Z_2 q_0]$

$[q_1, Z_2, q_0] \rightarrow b\,[q_1 Z_2 q_1][q_1 Z_2 q_0]$

$[q_1, Z_2, q_0] \rightarrow b\,[q_1 Z_2 q_2][q_2 Z_2 q_0]$

$[q_1, Z_2, q_1] \rightarrow b\,[q_1 Z_2 q_0][q_0 Z_2 q_1]$

$[q_1, Z_2, q_1] \rightarrow b\,[q_1 Z_2 q_1][q_1 Z_2 q_1]$

$[q_1, Z_2\ q_1] \rightarrow b\,[q_1 Z_2 q_2][q_2 Z_2 q_1]$

$[q_1, Z_2\ q_2] \rightarrow b\,[q_1 Z_2 q_0][q_0 Z_2 q_2]$

$[q_1, Z_2\ q_2] \rightarrow b\,[q_1 Z_2 q_1][q_1 Z_2 q_2]$

$[q_1, Z_2\ q_2] \rightarrow b\,[q_1 Z_2 q_2][q_2 Z_2 q_2]$

for $\delta(q_1, c, Z_2) = (q_2, \varepsilon)$

$[q_1, Z_2\ q_2] \rightarrow c$

for $\delta(q_2, c, Z_2) = (q_2, \varepsilon)$

$[q_2, Z_2, q_2] \rightarrow c$

for $\delta(q_2, \varepsilon, Z_0) = (q_2, \varepsilon)$

$[q_2, Z_0, q_2] \rightarrow \varepsilon$

Two stack PDA :

Finite Automata recognize regular languages. Adding one stack to a finite automata, it becomes PDA which can recognize CFL. in the case of content sensitive language such as $\{a^n b^n c^n, n \ge 1\}$ the PDA is helpless because of only one auxiliary storage. if more then one stack is added in the form of auxiliary storage with PDA, then the accepting power of PDA is increase. So the two stack PDA has come. Not only two stacks, but more then two stacks can be added to a PDA.

Two stack PDA is equivalent to the turing m/c

A two stack PDA consists of 9 tuple

$$m = (Q, \Sigma, \Gamma, \Gamma', \delta, q_0, Z_1, Z_2, F)$$

where Q is a finite set of states.
Σ is a finite set of input symbals.
Γ is a finite set of 1^{st} stack symbols.
Γ' is a finite set of 2^{nd} stack symbals.
δ is transition f^n as $(Q \times \Sigma \times \Gamma \times \Gamma') \to (Q, \Gamma^*, \Gamma'^*)$
q_0 is initial state.
Z_1 is initial stack symbal of stack 1^{st}
Z_2 is initial stack symbal of stack 2^{nd}
F is the final state of PDA.

Q construct a two stack PDA for $L = \{a^n b^n c^n, n \geq 0\}$.

Solution: while scanning "a" push x in stack 1. while scanning "b" push y in stack 2. while scanning "c" pop x and y from stack 1 and stack 2 Respectively.

the transition f^n as follows:

$$\delta(q_0, a, Z_1, Z_2) \rightarrow (q_0, xZ_1, Z_2)$$

$$\delta(q_0, a, x, Z_2) \rightarrow (q_0, xx, Z_2)$$

$$\delta(q_0, b, x, Z_2) \rightarrow (q_0, x, yZ_2)$$

$$\delta(q_0, b, x, y) \rightarrow (q_0, x, yy)$$

$$\delta(q_0, c, x, y) \rightarrow (q_1, \varepsilon, \varepsilon)$$

$$\delta(q_1, c, x, y) \rightarrow (q_1, \varepsilon, \varepsilon)$$

$$\left.\begin{array}{l} \delta(q_1, \varepsilon, Z_1, Z_2) \rightarrow (q_f, Z_1, Z_2) \\ \delta(q_0, \varepsilon, Z_1, Z_2) \rightarrow (q_f, Z_1, Z_2) \end{array}\right\} \text{accepted by final state.}$$

or

$$\left.\begin{array}{l} \delta(q_1, \varepsilon, Z_1, Z_2) \rightarrow (q_1, \varepsilon, \varepsilon) \\ \delta(q_0, \varepsilon, Z_1, Z_2) \rightarrow (q_0, \varepsilon, \varepsilon) \end{array}\right\} \text{accepted by empty stacks}$$