Maths

question Bank

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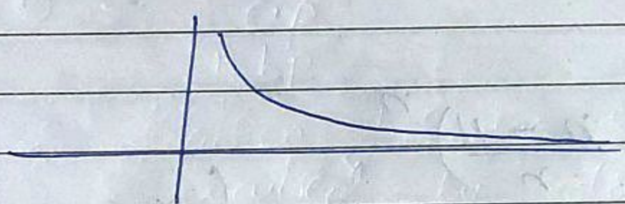
① Sequence :- It is ~~an~~ ~~expression~~ collection of data, which is placed in order, list.

② n^{th} term of the Sequence:

$1, -4, 9, -16, 25 \ldots$

$T_n = (-1)^{n+1} n^2$

③ Convergence of the Sequence → If the limits tends to the zero



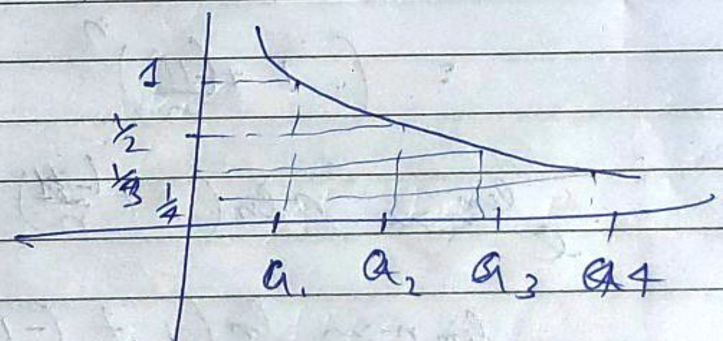
④ $a_n = \frac{1}{n}$

$a_1 = 1$

$a_2 = \frac{1}{2}$

$a_3 = \frac{1}{3}$

$a_4 = \frac{1}{4}$



⑤ $a_n = (-1)^{n+1}$

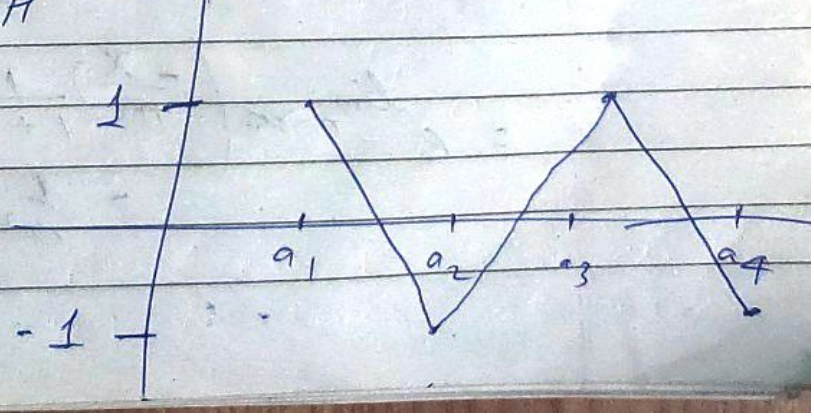
$a_1 = 1$

$a_2 = -1$

$a_3 = 1$

$a_4 = -1$

Curve do not converge



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(6) $\lim_{n\to\infty} \left(-\frac{1}{n}\right) = 0$ Convergence

(7) $\lim_{n\to\infty} \left(\frac{4-7n^6}{n^6+3n}\right)$

$= \lim_{n\to\infty} \left(\frac{\frac{4}{n^6} - 7}{1 + \frac{3}{n^5}}\right)$

$= \left(\frac{0-7}{1+0}\right) = -7$

(8) $\lim_{n\to\infty} \left(\frac{n-11}{n}\right)^n$ $(x = -11)$

$= \left(1 + \frac{(-11)}{n}\right)^n$

$e^{\lim_{n\to\infty} n \ln\left(1 + \frac{(-11)}{n}\right)}$

$e^{\lim n\to\infty \ln\left(\frac{1 + (-\frac{11}{n})}{\frac{1}{n}}\right)}$

$= e^{\lim_{n\to\infty} \left(\frac{\left(\frac{-x}{n^2}\right)\frac{1}{1+x/n}}{-\frac{1}{n^2}}\right)}$

$$= e^{\lim_{n\to\infty} \frac{x}{1+\frac{x}{n}}} = e^{x}$$

$$\left(1+\frac{x}{n}\right)^{n} \to e^{x}$$

Sol^n :- e^{-11} Ans

(9) Sequence of partial sum of series ;

An expression:- $a_1, a_2, a_3 \ldots\ldots a_n$

$S_1 = a_1$
$S_2 = a_1 + a_2$
$S_3 = a_1 + a_2 + a_3 \ldots$
$S_4 = a_1 + a_2 + a_3 + a_4$

$\langle S_1, S_2, S_3, S_4 \ldots\ldots \rangle$ → Sequence of partial Sum

(10) $S = a + ar + ar^2 + ar^3 \dots$

Ratio test,

$= \dfrac{ar^2}{ar}$

$= r \quad ; \; r < 1$

So, it is Convergence

$\because$ r is multiply in S;

$rS = ar + ar^2 + ar^3 + ar^4 \dots$

$rS - S = a$

$S(r-1) = a$

$S = \dfrac{-a}{(r-1)} = \left(\dfrac{a}{1-r}\right)$

(11) $GP = \dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} \dots$

$r = \dfrac{\frac{1}{\cancel{27}\,3}}{\frac{1}{\cancel{9}}} = \dfrac{1}{3} < 1$

So, it is Convergence

$S = \dfrac{a}{1-r} = \dfrac{\frac{1}{9}}{1 - \frac{1}{3}} = \dfrac{\frac{1}{\cancel{9}\,3}}{\frac{2}{\cancel{3}}} = \dfrac{2}{3}$ Ans

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(12) $\sum_{n=1}^{\infty} \left(\frac{-1}{4^n}\right)^n 5$

By using root test;

$\frac{-1}{4}$ Convergent.

(13) $5.2323 = 5 + \frac{23}{100} + \frac{23}{(100)^2} + \frac{23}{(100)^3} + \dots$

$= 5 + \frac{23}{100}\left(1 + \frac{1}{100} + \left(\frac{1}{100}\right)^2 \dots\right)$

$= 5 + \frac{23}{100}\left(\frac{1}{1 - \frac{1}{100}}\right)$

$= 5 + \frac{23}{\cancel{100}} \times \frac{\cancel{100}}{99} = \frac{518}{99}$

(14) $\sum_{n=1}^{\infty} \left(\frac{-n}{2n+5}\right)$

$a_n = \lim_{n \to \infty} \left(\frac{-n}{2n+5}\right)$

$a_n = \left(\frac{-1}{2}\right) \neq 0$

So, divergent

(15) Integral test- Any expression which can be solved by integration ~~can~~ ~~0~~ and value of intergal is finite then it is convergent otherwise divergent.

(16)

(17) $\sum_{n=1}^{\infty} ne^{-n^2}$ is convergent

$= \lim_{b\to\infty} \int_1^b ne^{-n^2} dn$

let $n^2 = x$

$2ndn = dx$

$dn = \frac{dx}{2\sqrt{x}}$

$= \lim_{c\to\infty} \int_1^c \sqrt{x} \times e^{-x} \times \frac{dx}{2\cancel{\sqrt{x}}}$

$= \frac{1}{2} \times (-e^{-x})^{\infty}$

$= \frac{1}{2}\left(-\frac{1}{e^{\infty}} + \frac{1}{e^1}\right)$

$= \frac{1}{2e}$ finite value Convergent

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(18) $\sum_{n=1}^{\infty} \frac{1}{2^{\ln 2}}$ is not Convergent.

$= \lim_{b \to \infty} \int_1^b \frac{1}{2^{\ln 2}} dx = \frac{1}{2^{\ln 2}} [x]_1^b = \infty$

So, divergent
not finite value.

(19) Absolutly Convergent Series;

↳ An expression have Convergent nature in both Case (+ve) & (-ve) (in modulus) after Solving limit then this type of series Called ACS.

(20) Ratio Test - This test used to find Convergent and divergent of Series

if $\frac{a_{n+1}}{a_n} < 1$ Convergent

> 1 divergent

$= 1$ in conclusive

(21) $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$ is Convergent and its sum is ~~1~~0.5

$$= \lim_{n\to\infty} \left| \frac{\frac{2^{n+1}+5}{3^{n+1}}}{\frac{2^n+5}{3^n}} \right| = \lim_{n\to\infty} \left| \frac{\frac{2^n.2+5}{\cancel{3^n}.3}}{\frac{2^n+5}{\cancel{3^n}}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2^n\left(2+\frac{5}{2^n}\right)}{3\times 2^n\left(1+\frac{5}{2^n}\right)} \right|$$

$$= \left(\frac{2}{3(1+0)}\right)$$

$$= \frac{2}{3} < 1 \text{ So, Convergent.}$$

$$\frac{2^n}{3^n} + \frac{5}{3^n}$$

$$\left(\frac{\frac{2}{3}}{1-2/3}\right) + 5\left(\frac{1}{3}+\frac{1}{3^2} \cdots\right)$$

$$\frac{\frac{2}{\cancel{3}}}{\frac{1}{\cancel{3}}} \quad 2 + 5\left(\frac{\frac{1}{3}}{1-1/3}\right)$$

$$4.5 = \quad 2 + 5\times\frac{1}{2} = 2+\frac{5}{2} = \frac{4+5}{2} = 9/2$$

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22) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ is not Convergent

$$\lim_{n\to\infty} \frac{\left(\frac{2(n+1)!}{((n+1)!)^2}\right)}{\frac{2n!}{(n!)^2}} = \frac{(2n+1)(2n+2)}{(n+1)(n+1)}$$

$$= \lim_{n\to\infty} \frac{4n^2+4n+2n+2}{n^2+2n+1}$$

$$= \lim_{n\to\infty} \frac{4+\frac{4}{n}+\frac{2}{n}+\frac{2}{n^2}}{1+\frac{2}{n}+\frac{1}{n^2}}$$

$$= 4 > 1$$

So, divergent.

23) Root test:-

We have the series $\sum a_n$

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |a_n|^{1/n}$$

$L < 1$ Convergent

$L > 1$ divergent

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(24) $\sum_{n=1}^{\infty} \frac{n^2}{(2)^n}$

By using root test;

(ignoring $n^2 \to \infty$)

ye merelko nhi pata, kyu ignore karna hn.

$$\frac{\sqrt[n]{1}}{\sqrt[n]{(2)^n}} = \frac{1}{2} < 1$$

so, Convergent.

(25) $\sum_{n=0}^{\infty} \left(\frac{1}{1+n}\right)^n$;

By root test;

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{1}{1+n}\right)^n}$$

$$\lim_{n\to\infty} \frac{1}{1+n}$$

$$\lim_{n\to\infty} \left(\frac{\frac{1}{n}}{1+\frac{1}{n}}\right) = 0 < 1$$

Convergence

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(26) $\sum_{n=1}^{\infty} \frac{2^n}{(n)^3}$ is not Convergent

By using root ts;

$$\frac{\sqrt[n]{2^n}}{\sqrt[n]{n^3}} = \frac{2}{1} > 1$$

→ So, divergent

? [27, 28, 29 → Altrnating Series nhi padhaya gaya hm]

(30) Power series :- It is a infinite series of express;

$$\sum_{n=0}^{\infty} c_n (x - x_0)^n = c_0 + (x - x_0) c_1 + c_2 (x - x_0)^2 + c_3 (x - x_0)^3 \ldots + c_n (x - x_0)^n$$

Where $c_1, c_2, c_3 \ldots$ → Coefficient of Series

$x_0 = a$.

$$\sum_{n=0}^{\infty} c_n (x - a)^n = c_0 + (x-a) c_1 + c_2 (x-a)^2 \ldots\ldots$$

(31) Power series;

$$1 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2 - \frac{1}{8}(x-2)^3 \cdots$$

$$\lim_{n\to\infty} a_n = \left(\frac{-1}{2}\right)^n (x-2)^n$$

By using ratio test;

$$\frac{a_{n+1}}{a_n} = \frac{\left(-\frac{1}{2}\right)^{n+1}(x-2)^{n+1}}{\left(-\frac{1}{2}\right)^n (x-2)^n}$$

$$= -\frac{1}{2}(x-2)$$

$$= 1 - \frac{x}{2}$$

for convergent;

$$1 - \frac{x}{2} < 1$$

$$0 < \frac{x}{2}$$

$$0 < x$$

(32) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{(n+1)}$ Converges for $(-1) < x \leq 1$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(-1)^{n+1+1} x^{n+1}}{n+1}}{\frac{(-1)^{n+1} x^n}{(n)}}$$ (By ratio test)

$$= \lim_{n\to\infty} \frac{(-1)\,x}{(n+1)} \times (n)$$

$$\lim_{n\to\infty} \frac{(-1)x \times \frac{n}{n}}{\left(1+\frac{1}{n}\right)}$$

$$= \frac{(-1)\,x}{1}$$

for Convergent

$-x < 1$

$x > -1$

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(33) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$

By using Ratio test;

$~~(-1)^{n-1} x^{2(n)-1+1}~~$

$$\frac{(-1)^{n+1-1} \frac{x^{2n+2-1}}{2n+2-1}}{(-1)^{n-1} \frac{x^{2n-1}}{2n-1}}$$

$$\lim_{n\to\infty} \frac{(-1) \times x^2 (2n-1)}{(2n+1)}$$

$$\lim_{n\to\infty} \frac{(-1) x^2 \left(2 - \frac{1}{n}\right)}{2 + \frac{1}{n}}$$

$$\frac{(-1) x^2 (\cancel{2})}{\cancel{2} + 0}$$

for Convergent

$-x^2 < 1$

$-x < \pm 1$

$-x < 1$ $\quad$ $-x < -1$

$x > -1$ $\quad$ $x < 1$

$-1 < x < 1$

~~for~~ Convergent

$-x^2 < 1$

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(37) Radius of Convergence

$$\sum_{n=0}^{\infty} \frac{n}{3^{n+1}} x^n$$

$$\frac{1}{R} = \frac{\frac{n+1}{3^{n+1+1}} x^{n+1}}{\frac{n}{3^{n+1}} x^n}$$

$$\frac{1}{R} = \lim_{n \to \infty} \frac{\frac{x(n+1)}{3}}{\frac{n}{1}}$$

$$\frac{1}{R} = \frac{x(n+1)}{3n}$$

$$\frac{1}{R} = \frac{x\left(1+\frac{1}{n}\right)}{3}$$

$$\frac{1}{R} = \frac{x}{3}$$

for Convergent

$$\frac{x}{3} < 1$$

$$x < 3$$

$$R = \frac{3}{x}$$

$$\frac{3}{R} < 3$$

$1 < R$ Ans

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(35) $\sum_{n=0}^{\infty} \frac{10^n}{n!} (x-1)^n$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{\frac{10^{n+1}}{(n+1)!}}{\frac{10^n}{n!}} \frac{10}{(n+1)} \frac{(x-1)^{n+1}}{(x-1)^n}$$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{10}{(n+1)} (x-1)$$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{\frac{10}{n}(x-1)}{1+\frac{1}{n}}$$

$$\frac{1}{R} = \frac{0}{1} \qquad 0 < 1$$

$$R = \infty \qquad \text{Convergent}$$

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(35) Taylor series;

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \ldots$$

$a = 0$ then;

Maclaurin series;

$$f(x) = f(0) + x f'(0) + x^2 f''(0) \ldots$$

(36) In Taylor's Theorem (An expression

$$f(a+h) = f(a) + \frac{h}{1!} f'(a) + \frac{h^2}{2!} f''(a) \ldots$$

$$\ldots + \frac{h^n}{n!} (f^n(a))$$

when we put $h = x-a$ & $n \to \infty$
then it become to Taylor's series

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a)$$

$$\ldots \ldots \infty$$

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(37) Maclaurin's Series is a special type of Taylor's series in which, we put

$a = 0;$

then the expression:-

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) \ldots \ldots \infty$$

(38) $f(x) = e^x$

Maclaurin Series;

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \ldots \ldots$$

$f(x) = e^x$
$f(0) = e^0 = 1$

$f'(x) = e^x$
$f'(0) = e^0 = 1$

$f''(x) = e^x$
$f''(0) = e^0 = 1$

$$f(x) = 1 + x \times 1 + \frac{x^2}{2!} \times 1 + \frac{x^3}{3!} \times 1 \ldots \infty$$

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$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \ldots \infty$$

(39) Maclaurin series $f(x) = \cos x$;

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \ldots \infty$$

(I) $f(x) = \cos x$; $f(0) = \cos x$

(II) $f'(x) = -\sin x$; $f'(0) = 0$

(III) $f''(x) = -\cos x$; $f''(0) = -1$

(IV) $f'''(x) = \sin x$; $f'''(0) = 0$

(V) $f^{IV}(x) = \cos x$; $f^{IV}(0) = 1$

(VI) $f^{V}(x) = \sin x$; $f^{V}(0) = 0$

$$f(x) = 1 + x \times 0 + \frac{x^2}{2!}(-1) + \frac{x^3}{3!} \times 0 + \frac{x^4}{4!} \times 1 + \frac{x^5}{5!} \times 0 \ldots \infty$$

$$f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \ldots \infty$$

$$\boxed{\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots \infty}$$

(40) Taylor Series; $f(x) = \frac{1}{x}$ at $x = 2$;

$$f(x) = f(a) + (x-a) f''(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) \ldots\ldots$$

Now;

$$f(x) = \frac{1}{x} \; ; \; f(2) = \frac{1}{2}$$

$$f'(x) = -\frac{1}{x^2} \; ; \; f(2) = -\frac{1}{4}$$

$$f''(x) = \frac{1}{x^3} \; ; \; f(2) = \frac{1}{8}$$

$$f'''(x) = -\frac{1}{x^4}$$

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(10) Taylor series, $f(x) = \frac{1}{x}$ at $x = 2$;

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) \dots\dots$$

$\rightarrow f(x) = \frac{1}{x}$; $f(2) = \frac{1}{2}$

$\rightarrow f'(x) = \frac{-1}{x^2}$; $f'(2) = \frac{-1}{4}$

$\rightarrow f''(x) = \frac{(+2)x}{x^3}$; $f''(2) = \frac{2}{8}$

$\rightarrow f'''(x) = 2 \times (-3) \times \frac{1}{x^4}$; $f'''(2) = \frac{-6}{16}$

$\rightarrow f^{iv}(x) = -6 \times -4 \times \frac{1}{x^5}$; $f^{iv}(2) = \frac{24}{32}$

$\rightarrow f^{v}(x) = \frac{24 \times (-5)}{x^6}$; $f^{v}(2) = \frac{-120}{64}$

$\cancel{f(x) = \frac{1}{2} + (x - 2)\left(\frac{-1}{4}\right) \cdot 1}$

$$f(x) = \frac{1}{2} + (x-2)\left(-\frac{1}{4}\right) + \frac{(x-2)^2}{2!} \times \frac{2}{8}$$
$$+ \frac{(x-2)^3}{3!} \times \frac{(-6)}{16} + \frac{(x-2)^4}{4!} \times \frac{24}{32}$$

$$\cdots\cdots$$

$$f(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{(x-2)^2}{8} - \frac{(x-2)^3}{16}$$
$$+ \frac{(x-2)^4}{32} - \frac{(x-2)^5}{64} + \frac{(x-2)^6}{128}$$

$$\cdots\cdots$$

$$\boxed{\frac{1}{x} = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{(x-2)^2}{8} - \frac{(x-2)^3}{16} + \frac{(x-2)^4}{32} - \frac{(x-2)^5}{64} + \frac{(x-2)^6}{128}}$$

(4) Maclaurin Series;

$$f(x) = \frac{1}{3}(2x + x\cos x)$$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \ldots$$

$\rightarrow$ $f(x) = \frac{1}{3}(2x + x\cos x)$; $f(0) = \frac{1}{3}(0) = 0$

$\rightarrow$ $f'(x) = \frac{1}{3}\left(2 + \cos x + x(-\sin x)\right) = \frac{1}{3}(2 + \cos x - x\sin x)$

$f'(0) = \frac{1}{3}(2 + 1 - 0) = 1$

$\rightarrow$ $f''(x) = \frac{1}{3}(-\sin x + x(-\cos x) - \sin x)$

$f''(0) = \frac{1}{3}(0 + 0 + 0) = 0$

$\rightarrow$ $f'''(x) = \frac{1}{3}(-\cos x + x\sin x - \cos x - \cos x)$

$f'''(0) = \frac{1}{3}(-1 - 1 - 1) = -1$

$\rightarrow$ $f^{iv}(x) = \frac{1}{3}(+\sin x + x\cos x + \sin x + \sin x + \sin x)$

$f^{iv}(0) = \frac{1}{3}(0 + 0 + 0 + 0 + 0) = 0$

~~$f^{v}(0) = \frac{1}{3}(\cos x + x\sin x$~~ $+ \cos x + \cos x + \cos x)$

$$f(x) = 0 + x \times 1 + \frac{x^2}{2!} \times 0 + \frac{x^3}{3!} \times (-1) + \frac{x^4}{4!} \times 0$$

$\ldots$

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$$

$$\boxed{\frac{1}{3}(2x + x\cos x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots}$$

(42) Maclaurin Series;

$$f(x) = e^x \cos x$$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \cdots$$
$$+ \frac{x^4}{4!} f^{iv}(0) \cdots \infty$$

$\rightarrow$ $f(x) = e^x \cos x$; $f(0) = e^0 \cos 0 = 1$

$\rightarrow$ $f'(x) = e^x \cos x - \sin x \, e^x$

$f'(0) = 1 \times 1 - 0 = 1$

$\rightarrow$ $f''(x) = e^x \cos x - \sin x \, e^x - (\sin x \, e^x + \cos x \, e^x)$

$f''(0) = 1 - 0 - (0 + 1)$

$= 0$

$f''(x) = \cancel{e^x \cos x} - \sin x \, e^x - \sin x \, e^x - \cancel{\cos x \, e^x}$

$f''(x) = -2 \sin x \, e^x$

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$\rightarrow f'''(x) = -2(\cos x e^{x} + \sin x e^{x})$

$f'''(0) = -2(1 + 0) = -2$

$f^{iv}(x) = -2(e^{x}\cos x - \cancel{\sin x} e^{x} + \cancel{\sin x} e^{x} + \cos x e^{x})$

$= -4(\cos x e^{x})$

$f^{iv}(0) = -4 = -4$

1st few term;

$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$

$+ f^{iv}(0)$

$f(x) = 1 + x \times 1 + \frac{x^2}{2!} \times 0 + \frac{x^3}{3!} \times (-2)$

$+ \frac{x^4}{4!} \times \cancel{f'} (-4)$

$$\boxed{f(x) = 1 + x + \frac{x^2}{2!} - \frac{2x^3}{3!} - \frac{4 \cdot x^4}{4!}}$$

$\rightarrow$ first few term

43, 44, 45, 46, 49

fourier series

(Padhaya gaya nhi hm)

$$\left(1+\frac{x}{n}\right)^n = e^x$$

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(47) $$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$$

$$\lim_{n\to\infty}\left(1+\frac{(-1)}{n}\right)^n\left(1+\frac{(-1)}{n}\right)^n$$

$$\rightarrow \quad e^{-1}\times e^{-1}$$

$$\rightarrow \quad e^{-1-1}$$

$$\rightarrow \quad e^{-2} \quad \underline{\text{Ans}}$$

(48) $$\sum_{n=1}^{\infty}\frac{(x+1)^n}{n\,2^n}$$

Ratio test;

$$= \frac{\dfrac{\cancel{(x+1)^{n+1}}\,(x+1)}{(n+1)\,\cancel{2^{n+1}}\,2}}{\dfrac{\cancel{(x+1)^n}}{n\,\cancel{2^n}}}$$

$$= \frac{\dfrac{x+1}{2(n+1)}}{\dfrac{1}{n}}$$

$$= \frac{n(x+1)}{2n+2}$$

$$= \lim_{n\to\infty} \frac{\frac{\cancel{n}}{\cancel{n}}(x+1)}{\frac{2\cancel{n}}{\cancel{n}} + \cancel{\frac{2}{n}}} \to 0$$

$$= \frac{x+1}{2}$$

for Convergent series,

$$\frac{x+1}{2} < 1$$

$$x+1 < 2$$

$$\boxed{x < 1} \to \text{interval}$$

(50) $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$

$$\Rightarrow 1 + \left[\frac{1}{(2)^2} + \frac{1}{(2)^4} + \frac{1}{(2)^6} + \frac{1}{(2)^8} \dots\right.$$

$$a = \frac{1}{(2)^2} \qquad r = \frac{1}{(2)^2}$$

$$\Rightarrow 1 + \frac{a}{1-r}$$

$$= 1 + \frac{\frac{1}{4}}{1-\frac{1}{4}} = 1 + \frac{\frac{1}{\cancel{4}}}{\frac{3}{\cancel{4}}} = 1 + \frac{1}{3} = \frac{4}{3} \text{ Ans}$$

(51) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{(n!)\,3^n}$

$\Rightarrow$ By using ratio test;

$$= \frac{\frac{(x-3)^{n+1}}{\cancel{(n+1)!}\,\cancel{3^{n+1}}}}{\frac{\cancel{(x-3)^n}}{\cancel{(n!)}\,\cancel{3^n}}} \quad \frac{(x-3)}{3\,(n+1)}$$

$$= \lim_{n\to\infty} \frac{x-3}{3(n+1)}$$

$$= \lim_{n\to\infty} \frac{\frac{x-3}{n}}{3\left(1+\frac{1}{n}\right)}$$

$$\frac{1}{R} = 0$$

$$R = \infty \quad \text{Ans}$$