Q.B

Discrete Mathematics

UNIT = 3

1) What is the cardinality of these sets:

(a) ϕ

→ Cardinality of ϕ is 0.

[Number of elements in a set is called cardinal number.]

It is an empty set, hence there are no or 0 elements present.

(b) $\{\phi\}$.

→ Cardinality of $\{\phi\}$ is 1.

It contains one element $\{\phi\}$, that is a set containing an empty set.

(c) $\{\phi, \{\phi\}\}$

→ Cardinality of $\{\phi, \{\phi\}\}$ is 2.

It contains two elements an empty set $\{\phi\}$, and a set containing an empty set $(\{\phi\})$.

(d) $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

→ Cardinality of $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ is 3.

It contains three elements an empty set (ϕ), a set containing an empty set $(\{\phi\})$, and a set containing $\{\phi, \{\phi\}\}$. In the their element, the full set $\{\phi, \{\phi\}\}$ is considered as one element.

2) Find power set of $x = \{\{\}, 1, \{b\}\}$

→ Power set of $x = \phi, \{\}, \{1\}, \{\{b\}\}, \{\{\}, 1\}, \{\{\}, \{b\}\}, \{1, \{b\}\}$
$\{\{\}, 1, \{b\}\}$

3) element of the set $P(P(P(\phi)))$

$\rightarrow$ $A = \{\phi\}$

$P(A) = 2^0 = 1$ element

$P(P(A)) = 2^1 = 2$ element

$P(P(P(A))) = 2^2 = 4$ element

$P(P(P(P(A)))) = 2^4 = 16$ element

$A = n$ element

powerset $P(A) = 2^n$

④ $A_i \{1, 2, 3, \ldots i\}$ for $i = 1, 2, 3, \ldots$ find

(a) $\bigcup_{i=1}^{n} A_i = A_n = \{1, 2, 3, \ldots, n\}$

(b) $\bigcap_{i=1}^{n} A_i = A_1 = \{1\}$ ☆☆☆

⑤ prove that

$(\overline{A \cap B}) = \bar{A} \cup \bar{B}$ & $(A \cup B \cup C)' = A' \cap B' C'$ ☆☆☆

⑥ Prove that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$\rightarrow$ Let $(x, y) \in A \times (B \cup C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\Rightarrow (x, y) \in (A \times B)$ or $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ —— (1)

Again let $(x, y) \in (A \times B) \cup (A \times C)$

$\Rightarrow (x, y) \in (A \times B)$ or $(x, y) \in (A \times C)$

$\Rightarrow (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow (x, y) \in A \times (B \cup C)$

$\therefore (A\times B)\cup(A\times C) \subseteq A\times(B\cup C)$ —— ②

from (1) and (2), we get

$A\times(B\cup C) = (A\times B)\cup(A\times C)$ Thus proved.

(ii) $A-(B\cup C) = (A-B)\cap(A-C)$

→ L.H.S. $A-(B\cup C)$

$\Rightarrow A\cap(B\cup C)'$

$\Rightarrow A\cap(B'\cap C')$

$\Rightarrow (A\cap B')\cap(A\cap C')$

$= (A\cup B)\cap(A\cup C)$

R.H.S $= (A-B)\cap(A-C)$

$= (A\cap B')\cap(A\cap C')$

$= (A\cup B)\cap(A\cup C)$

LHS = RHS Proved.

(iii) $A-(B\cap C) = (A-B)\cup(A-C)$

→ LHS $= A-(B\cap C)$

$\Rightarrow A\cap(B\cup C)'$

$\Rightarrow A\cap(B'\cap C')$

$\Rightarrow (A\cap B')\cap(A\cap C')$

$= (A-B)\cup(A-C)$ = RHS

(7) Multisets:- A multiset is an unordered collection of elements in which the multiplicity of an element may be one or more than one or zero, The multiplicity of an element is the number of times the element repeated in the multiset.

Ex $A = \{l, l, m, m, n, n, n, n\}$

$B = \{a, a, a, a, a, c\}$

Give the rules of finding Union, Intersection, & Differences.

→ The union of two sets contains all the elements contained in either set (or both sets).

The UNION IS notated $A\cup B$

- The intersection of two sets contains only the elements that are in both sets.
  The intersection is notated $A \cap B$.

- Difference :— The set A-B consists of elements that are in A but in B. For example if $A = \{1,2,3\}$ and $B = \{3,5\}$; then $A - B = \{1,2\}$.

(8) If a set has n elements then number of relations on A is given by ~~2^{n^2}~~ $2n^2$

(9) The number of reflexive relations on a set with the 'n' number of elements is given by $\boxed{N = 2^{n(n-1)}}$, where N is the number of reflexive relations and n is the number of elements in the set.

(10) $A = \{0,1,2,3,4\}$ to $B = \{0,1,2,3\}$ where $(a,b) \in R$

(a) $a = b$

→ Let $(a,b) \in R$ if and only if $a = b$. Then $R = \{(0,0), (1,1), (2,2), (3,3)\}$

(b) $a + b = 4$

→ Let $(a,b) \in R$ if and only if $a+b=4$. Then $R = \{(1,3), (2,2), (3,1), (4,0)\}$

(c) $a > b$

→ Let $(a,b) \in R$ if and only if $a > b$. Then $R = \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$

(d) $a | b$

→ Let $(a,b) \in R$ if and only if $a|b$. It follows that $R = \{(1,0), (1,1),$
$(1,2), (1,3), (2,0), (2,2), (3,0), (3,3), (4,0)\}$

(e) $\gcd(a,b) = 1$

→ Let $(a,b) \in R$ if any only if $\gcd(a,b) = 1$. Then $R = \{(0,1),$
$(1,0), (1,1), (1,2), (2,1), (1,3), (3,1), (4,1), (2,3), (3,2), (4,3)\}$

(f) $LCM(a,b) = 2$

→ Let $(a,b) \in R$ if and only if $LCM(a,b) = 2$. It follows that
$R = \{(2,2), (1,2), (2,1)\}$.

(11) Let R be the relation $R = \{(a,b) \mid a < b\}$ on the set of integers. find
(a) R^{-1} (b) $\overline{R}$

reflexive, symmetric, anti symmetric, transitive $\{1, 2, 3, 4\}$

(12) (a) $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$

→ relation R is not reflexive: $(1,1), (4,4) \notin R$
relation R is not symmetric: $(2,4) \in R$, $(4,2) \notin R$
relation R is not antisymmetric: $(2,3), (3,2) \in R$
relation R is transitive: $(2,2), (2,3) \in R \rightarrow (2,3) \in R;$
$(2,2), (2,4) \in R \rightarrow (2,4) \in R;$
$(2,3), (3,2) \in R \rightarrow (2,2) \in R; (2,3), (3,3) \in R \rightarrow (2,3) \in R;$
$(2,3), (3,4) \in R \rightarrow (2,4) \in R; (3,2), (2,2) \in R \rightarrow (3,2) \in R;$
$(3,2), (2,3) \in R \rightarrow (3,3) \in R; (3,2), (2,4) \in R \rightarrow (3,4) \in R;$
$(3,3), (3,2) \in R \rightarrow (3,2) \in R; (3,3), (3,4) \in R \rightarrow (3,4) \in R.$

(b) $\{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

$\rightarrow$ relation R is reflexive : $\{(1,1), (2,2), (3,3), (4,4)\} \in R$

relation R is symmetric : $\{(1,2), (2,1) \in R$

relation R is not symmetric : $(1,2), (2,1) \in R$

relation R is transitive : $(1,1), (1,2) \in R \rightarrow (1,2) \in R$; $(2,1), (1,2) \in R \rightarrow (2,2) \in R$; $(1,2), (2,1) \in R \rightarrow (1,1) \in R$; $(1,2), (2,2) \in R \rightarrow (1,2) \in R$; $(2,2), (2,1) \in R \rightarrow (2,1)$ R.

(c) $\{(2,4), (4,2)\}$

$\rightarrow$ relation R is not reflexive: $(1,1) \notin R$

relation R is symmetric : $(2,4), (4,2) \in R$

relation R is not antisymmetric : $(2,4), (4,2) \in R$

relation R is not transitive : $(2,4), (4,2) \in R, (2,2) \notin R$.

(13) $R = \{(1,2), (1,3), (2,3), (2,3), (2,4), (3,1)$ and $S = \{(2,1), (3,1), (3,2), (4,2)\}$

$A = \{1,2,3,4\}$

Doubt

find SoR and RoS.

$\rightarrow$

$R(1) = 2$	$S(R1) = S(2) = 1$
$R(1) = 3$	$S(R1) = S(3) = 1,2$
$R(2) = 3$	$S(R2) = S(3) = 1,2$
$R(2) = 3$	$S($
$R(2) = 4$	$S(R_2) = S(4) = 2$
$R(3) = 1$	

$SoR = \{(2,1), (3,1), (3,2)\}$

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14) $R1 = \{(a,b) : a \text{ divides } b\}$ and $R2 = \{(a,b) : a \text{ is a multiple of } b\}$.

(a) $R1 \cup R2$

(15) Let $R_1 = \{(a,b) : a >= b\}$

$R_1 = \{(1,1), (2,1), (2,2), (3,2), (3,3), (4,3), (4,4)\}$

$R_2 = \{(a,b) : a <= b\}$

$R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,3), (3,4), (4,4)\}$

(a) $R1 \cup R2$

$\Rightarrow \{(1,1),$

(16) (a)

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$$

$= \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

(b)

$$\begin{matrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{matrix}$$

$= \{(1,2), (2,2), (3,2)\}$

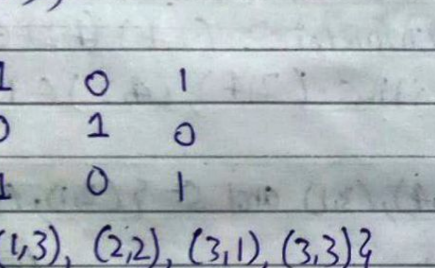
(17) $X = \{1,2,3,4\}$ and $R = \{(x,y) : x > y\}$

(a) Given the ordered pair of R.

~~$\Rightarrow R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$~~

$\Rightarrow R = \{(4,1), (4,2), (4,3), (3,2), (3,1), (2,1)\}$

(b) Draw the graph of R



(c) Give the relation matrix of R

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$(a,b)\ (b,c) \to (a,c)$

18. $R = \{(x,y) : |x-y| < 1\}$

19. (a) $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$

$\to$ Relation R is not reflexive: $(1,1), (4,4) \notin R$

Relation R is not symmetric: $(2,4) \in R, (4,2) \notin R$

Relation R is not antisymmetric: $(2,3), (3,2) \in R$

Relation R is transitive: $(2,2), (2,3) \in R \to (2,3) \in R$; ~~(2,2), (2,4~~

$(2,2), (2,4) \in R \to (2,4) \in R$; $(2,3), (3,2) \in R \to (2,2) \in R$;

$(2,3), (3,3) \in R \to (2,3) \in R$; $(2,3), (3,4) \in R \to (2,4) \in R$;

$(3,2), (2,2) \in R \to (3,2) \in R$; $(3,2), (2,3) \in R \to (3,3) \in R$;

$(3,2), (2,4) \in R \to (3,4) \in R$; $(3,3), (3,2) \in R \to (3,2) \in R$;

$(3,3), (3,4) \in R \to (3,4) \in R$; ...

(b) $\{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

$\to$ relation R is refloxive: $\{(1,1), (2,2), (3,3), (4,4)\}$

relation R is symmetric: $(1,2), (2,1)$.

relation R is not antisymmetric: $(1,2); (2,1) \in R$.

relation R is transitive: $(1,1), (1,2) \in R \to (1,2) \in R$;

$(1,2), (2,1) \in R \to (1,1) \in R$; $(1,2), (2,2) \in R \to (1,2) \in R$

$(2,1), (1,1) \in R \to (2,1) \in R$; $(2,1), (1,2) \in R \to (2,2) \in R$;

$(2,2), (2,1) \in R \to (2,1) \in R$;

(c) $\{(2,4), (4,2)\}$

relation R is not reflexive: $(1,1) \notin R$.

relation R is symmetric: $(1,2), (2,1) \notin R$.

relation R is antisymmetric: $(2,1), (3,2), (4,3) \notin R$.

relation R is not transitive: $(1,2), (2,3) \notin R \to (1,3) \notin R$.

(22) (23)

(e) $\{(1,1), (2,2), (3,3), (4,4)\}$

→ relation R is reflaxive : $\because (1,1), (2,2), (3,3), (4,4) \in R$

relation R is symmetric : $(1,1), (2,2), (3,3), (4,4) \in R$

Doubt relation R is antisymmetric : $(1,1), (2,2), (3,3), (4,4) \in R$

relation R is transitive : we an satisfy (a, b) and (b, c) while $a = b = c$.

(20) Doubt

(21) Let us consider $A = \{a, b, c\}$

$R: \{(a,a), (b,b), (c,c), (a,b), (b,a), (a,c), (c,a), (b,c), (c,b)\}$

$R^{-1}: \{(a,a), (b,b), (c,c), (b,a), (a,b), (c,a), (a,c), (c,b), (b,c)\}$

R^{-1} is reflexive, symmetric and transitive.

So, R^{-1} is an equivalence relation.

(15) $R_1 = \{(a,b) : a >= b\}$ and $R_2 = \{(a,b) : a <= b\}$

(a) $R1 \cup R2$ $= (a \geq b), (a \leq b)$

(b) $R1 \cap R2$ $\Rightarrow a = b$

(c) $R1 - R2$ $\Rightarrow a > b$

(d) $R2 - R1$ $\Rightarrow a < b$

(22) (23)

(24) set $A = \{1,2,3,4,5,6\}$

$R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}$

→ Let us find equivalence classes

$[1] = \{1,5\}$

$[2] = \{2,3,6\}$

$[3] = \{2,3,6\}$

$[4] = \{4\}$

$[5] = \{1,5,6\}$

$[6] = \{2,3,6\}$

$A = \{a,b,c\}$

$A_1 = \{a\}$ $A_2 = \{b,c\}$

$A_1 \cup A_2 = A$

$A_1 \cap A_2 = \phi$

$P_1 = \{1,5\}$ $\qquad$ $P_1 \cup P_2 \cup P_3 = A$

$P_2 = \{2,3,6\}$ $\qquad$ $P_1 \cap P_2 \cap P_3 = \Phi$

$P_3 = \{4\}$

(25) $S = \{1,2,3,4,5,6\}$

a) $P_1 = \{\{1,2,3\}, \{1,4,5,6\}]$

→ relation P_1 is not parartition of S.

(b) $P_2 [\{1,2\}, \{3,5,6\}]$

→ relation P_2 is partition of S.

(c) $P_3 = [\{1,3,5\}, \{2,4\}, \{6\}]$

→ relation P_3 is partition of S.

(d) $P4 = (\{1,3,5\})$, partition of S

→ relation P4 is partition of S.

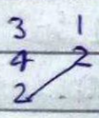
~~28~~ (27) Let $x = \{1,2,3,4\}$

(a) $f = \{(2,3), (1,4), (2,1), (3,2), (4,4)\}$

→ The element 2 have two images therefore f is not a function.

(b) $g = \{(3,1), (4,2), (2,1)\}$

→ Every element of x does not have an ~~unique~~ imge therefore g is



(c) $h = \{(2,1), (3,4), (1,4), (4,3)\}$

→ Every element of x has a unique image therefore h is a function.

8) 23, 31, 2, 5, 10

f() 0 1 | 0 1

1 — 3 | 1 2

Doubt

28 (a) $f(x) = 2x+1$

→ Here, $f(x) = 2x+1$

Let $x_1, x_2 \in \mathbb{R}$ and let us assume $f(x_1) = f(x_2)$

so,

$$f(x_1) = f(x_2)$$

$$\Rightarrow 2x_1 + 1 = 2x_2 + 1$$

$$\Rightarrow x_1 = x_2$$

Hence, we have $f(x_1) = f(x_2)$ implies $x_1 = x_2$

so, f is one-one (injective).

Also we know

$$-\infty < x < \infty$$

$$\Rightarrow -\infty < 2x < \infty$$

$$\Rightarrow -\infty < 2x+1 < \infty$$

$$\Rightarrow -\infty < f(x) < \infty$$

so, we clearly observe the co-Domain is the same as the Range,

so $f(x)$ is surjective.

And hence $f(x)$ is bijective.

(b) $f(x) = x^2+1$

Here we have

$$-\infty < x < \infty$$

$$\Rightarrow 0 \leq x^2 < \infty$$

$$\Rightarrow 1 \leq x^2+1 < \infty$$

$$\Rightarrow 1 \leq f(x) < \infty$$

so, the co-Domain of f is $\mathbb{R}$, but the range of f is $[1, \infty)$ so $f(x)$ is not surjective. Hence we conclude that $f(x)$ is not bijective.

$\frac{3500}{900}$ -video

(29) $f(x) = 2x+3$

$g(x) = 3x+2$

Given $f(x) = 2x+3$

$g(x) = 3x+2$

$(fog)(x) = f(g(x))$

$f(g(x)) = f(3x+2) = 2(3x+2)+3$

$= 6x+7$

$(gof)(x) = g(f(x)) = g(2x+3) = 3(2x+3)+2$

$= 6x+9+2$

$= 6x+11$

Hence, $6x+7$, $6x+11$

(30) $(gof)^{-1} = f^{-1}og^{-1}$ $\quad f: A \to B$ and $g: B \to C$

Sol $\Rightarrow$ $f: A \to B, g: B \to C$ are bijections

$\Rightarrow gof : A \to C$ is a bijection

Also $g^{-1}: C \to B$ and $f^{-1}: B \to A$ are bijections

$\Rightarrow f^{-1} o g^{-1} : C \to A$ is a bijections

Let c be any element of C.

Then $\exists$ an element $b \in B$ such that $g(b) = c$

$\Rightarrow b = g^{-1}(c)$

Also $\exists$ an element $a \in A$ such that $f(a) = b$

$\Rightarrow a = f^{-1}(b)$

Now $(gof)(a) = g(f(a)) = g(b) = c$

$\Rightarrow a = (gof)^{-1}(c) \Rightarrow (gof)^{-1}(c) = a$ — (1)

Also $(f^{-1}og^{-1})(c) = f^{-1}(g^{-1}(c)) = f^{-1}(b) = a$ — (2)

30, 31

from (1) and (2); $(g o f)^{-1}(c)$

$= (f^{-1} o g^{-1})(c)$

$\Rightarrow (g o f)^{-1} = f^{-1} o g^{-1}$.

(31) $$L(n) = \begin{cases} 0 & \text{if } n=1 \\ L\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1 & \text{if } n > 1 \end{cases}$$

(32) Size of a set :- The size of a set (also called its cardinality) is the number of elements in the set.

for ex; the size of the set $\{2, 4, 6\}$ is 3, while the size of the set E of positive even integers is infinity

(33) Show that the set of odd positive integers is a countable set.

33 34 35 36 37 38

(39) Show that

$|(0,1)| = |[0,1)|$

(41) $1+2+2^2+ \ldots + 2^n = 2^{n+1}-1$

Let $P(n): 1+2+2^2+ \ldots + 2^n = 2^{n+1}-1 \quad \forall n \in N$

Step 1: for $n=1$,

LHS = ~~1+2~~ $1+2^1 = 3$

RHS = $2^{1+1} - 1 = 4-1 = 3$

As, LHS = RHS

so, it is true for $n=1$

Step II: for $n=k$

Let $P(k): 1+2+2^2+\dots+2^k = 2^{k+1}-1$ be true $\forall k \in N$

Step III for $n=k+1$

$$LHS = 1+2+2^2+\dots+2^{k}+2^{k+1}$$
$$= 2^{k+1}-1+2^{k+1} \text{ (using step II)}$$
$$= 2\times 2^{k+1} - 1$$
$$= 2^{k+1+1}-1$$
$$= 2^{k+2}-1$$

$$RHS = 2^{(k+1)+1}-1 = 2^{k+2}-1$$

As LHS = RHS

So, it is also true for $n=k+1$

Hence, $1+2+2^2+\dots+2^n = 2^{n+1}-1$ for all $n \in N$.

(42) $f_0f_1 + f_1f_2 + f_2f_3 + \dots f_{2n-1}f_{2n} = f_{2n}^2$ for all $n>0$, with $f_0=0$, $f_1=1$

→ Let $n=1$

Then $f_0f_1 + f_1f_2 = (0)(1)+(1)(1) = 1$ and

$f_2^2 = (1)(1) = 1$, $f_0f_1 + f_1f_2 = f_2^2$

Inductive step: Assume $f_0f_1 + f_1f_2 + \dots + f_{2k-1}f_{2k} = f_{2k}^2$.

Show that $f_0f_1 + \dots + f_{2(k+1)-1}f_{2(k+1)} = f_{2(k+1)}^2$

$f_0f_1 + f_1f_2 + \dots + f_{2k+1}f_{2k+2} =$

Not complete.

(43) $2^n < n!$ n with $n \geq 4$

→ Let $P(n): 2^n > n$ for all positive n

for $n=1$

$L.H.S = 2^n = 2^1 = 2$

$RHS = n = 1$

Since $2 > 1$

$LHS > RHS$

$\therefore P(n)$ is true for $n=1$

Assume that $P(k)$ is true for all positive integers k.

i.e. $2^k > k$ --- (i)

we will prove that $P(k+1)$ is true.

i.e $2^{k+1} > k+1$

from (1)

$2^k > k$

multiplying by 2 on both sides.

$2^k \times 2 > 2 \times k$

$2 \cdot 2^k > 2k$

$2^{k+1} > k+k$ — (2)

Now

k is positive

we have proved $P(1)$ is true

so we have to prove for $k > 1$

$k > 1$

Adding k both side

$k+k > k+1$ —— (3)

from (2) and (3)

$2^{k+1} > k+k$ and $k+k > k+1$

Hence

$2^{k+1} > k+1$

$\therefore$ L.H.S > R.H.S

$\therefore$ P(K+1) is true whenever P(K) is true.

$\therefore$ By the principal of mathematical induction, p(n) is true for n, is a positive integer.

(44) $n \in Z^+$,

$$\sum_{i=1}^{n} i = 1+2+3+ \dots + n = \frac{n(n+1)}{2}$$

Let $P(n) = 1+2+3+ \dots + n = \frac{n(n+1)}{2}$

Step 2: prove for $n=1$

For $n=1$

LHS = 1

RHS = $\frac{n(n+1)}{2} = \frac{1 \times \cancel{2}}{\cancel{2}} = 1$.

Since, LHS = RHS

P(n) is true for n=1

Step 3: Assume P(K) to ~~to~~ be true and then prove P(K+1) is true

Assume that P(K) is true,

$P(K): 1+2+3+ \dots + K = \frac{K(K+1)}{2}$ --- (1)

we will prove that P(K+1) is true

$P(K+1): 1+2+3+ \dots + (K+1) = \frac{(K+1)((K+1)+1)}{2}$

$P(K+1): 1+2+3+ \dots + K+(K+1) = \frac{(K+1)(K+2)}{2}$.

we have to prove P(K+1) is true.

Solving LHS

$1+2+3+ \dots + k + (k+1)$

from (1): $1+2+3+\dots+k = \frac{k(k+1)}{2}$

$= \frac{k(k+1)}{2} + (k+1)$

$= \frac{k(k+1)+2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2} = RHS$

$P(k+1)$ is true when $P(k)$ is tree

(45) $\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{1}{2n+1}$

→ put $n = 1$

LHS $P(1) = \frac{1}{(2(1)-1)(2(1)+1)}$

$= \frac{1}{3}$

RHS $= \frac{1}{2\times1+1} = \frac{1}{3}$

$P(n)$ is true for $n=1$

Assumeng that true of $P(k)$

$P(k) = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \dots + \frac{1}{(2k-1)(2k+1)} - \frac{2}{2k+1}$

Let $P(k+1)$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2k-1)(2k+1)}+\frac{1}{(2(k+1)-1)(2(k+1)+1)}$$

$$=\frac{k}{(2k+1)}+\frac{1}{(2(k+1)-1)\cdot(2(k+1)+1)}$$

$$=\frac{k}{2k+1}+\frac{1}{(2k+2-1)(2k+2+1)}$$

$$=\frac{k}{2k+1}+\frac{1}{(2k+2-1)(2k+2+1)}$$

$$=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$$

$$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$=\frac{2k^2+2k+k+1}{(2k+1)(2k+3)}=\frac{2k(k+1)+(k+1)}{(2k+1)(2k+3)}$$

$$=\frac{(2k+1)(k+1)}{(2k+1)(2k+3)}=\frac{(k+1)}{(2k+3)}$$

Hence by using the principle of mathematical induction true for all $n\in N$.

46, 47

49 The prime factorizations of 100, 641, 999 and 1024 are given by

$100 = 2\cdot 2\cdot 5\cdot 5 = 2^2 5^2$

$641 = 641$

$999 = 3\cdot 3\cdot 3\cdot 37 = 3^3\cdot 37$

$1024 = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2 = 2^{10}$.

3	999
3	333
3	111
	37

(50)

(51) Prime factorization of 7007

→ 7007

The prime factors are: $7 \times 7 \times 11 \times 13$

(52) ~~Greatest~~ Greatest common divisor of 24 and 36

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

$24 = 2 \times 2 \times 2 \times 3$.

$36 = 2 \times 2 \times 3 \times 3$

$= 2 \times 2 \times 3 = 12$.