→ The <u>ordered n-tuple</u> $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, and a_n as its n^{th} element.

In particular, ordered 2-tuples is called ordered pairs.

The ordered pairs (a,b) and (c,d) are equal iff $a=c$ and $b=d$.

<u>Note</u>: $(a,b) = (b,a)$ iff $a=b$.

→ Let A and B be sets. The <u>Cartesian Product</u> of A and B denoted by $A \times B$, is the set of all ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a,b) \mid a \in A \ \& \ b \in B\}$$

The cartesian product $A \times B$ and $B \times A$ are not equal unless $A = \phi$ or $B = \phi$ or $A = B$.

The Cartesian product of the sets $A_1, A_2, \ldots, A_n$, denoted by $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots, a_n)$, where $a_i \in A_i$ for $i = 1, 2, \ldots n$.

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i, \ \forall i = 1, 2, \ldots n\}$$

<u>Ex.</u> $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

$$\begin{aligned} A \times B \times C = \{ &(0,1,0), (0,1,1), (0,1,2), (0,2,0) \\ &(0,2,1), (0,2,2), (1,1,0), (1,1,1) \\ &(1,1,2), (1,2,0), (1,2,1), (1,2,2) \} \end{aligned}$$

<u>Note</u>: $(A \times B) \times C \neq A \times B \times C$

$A^2 = A \times A$, $A^n = A \times A \times \cdots \times A$ (n times)

Relation

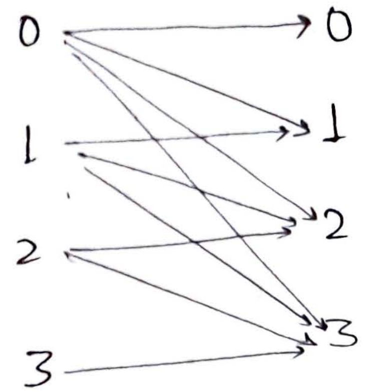
A subset R of the Cartesian Product $A \times B$ is called a relation from the set A to the set B.
A relation from a set A to itself is called a relation on A.

Ex. $A = \{0, 1, 2, 3\}$

$R = \{ (a, b) \mid a \le b;\ a, b \in A \}$

$= \{(0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

When $(a,b) \in R$, a is said to be related to b by R then written as aRb and $(a,b) \notin R \rightarrow a \not R b$.
Relation can be represented graphically as



R	0	1	2	3
0	X	X	X	X
1		X	X	X
2			X	X
3				X

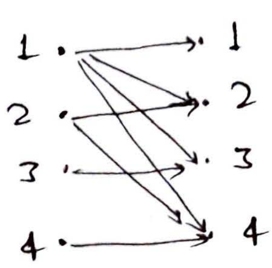
Table

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix representation of Relation

Ex. $R = \{ (a, b) \mid a \text{ divides } b \}$ on $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$



R	1	2	3	4
1	X	X	X	X
2		X		X
3			X	
4				X

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$