

\Rightarrow Irreflexive Relation: A relation R on the set A is irreflexive if for every $a \in A$, $(a, a) \notin R$.

eg' $R = \{(1, 1), (1, 2)\}$ — Not Irreflexive

$R = \{(1, 2), (1, 3), (2, 1)\}$ — Irreflexive

\rightarrow Non-Reflexive Relation: A relation R on the set A is non-reflexive if for some $a \in A$, $(a, a) \in R$, but not all.

$R = \{(1, 1), (1, 2)\}$ — Non-Reflexive

\rightarrow Asymmetric Relation: A relation R on the set A is called asymmetric if $(a, b) \in R \Rightarrow (b, a) \notin R$.

* Combining Relations (Operation)

Because relations from A to B are subsets of $A \times B$, two relations from A to B can be combined in any way two sets can be combined.

Ex. $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

$$R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$$

$$R_1 \cup R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\}$$

$$R_1 \cap R_2 = \{(1, 1)\}$$

$$R_1 - R_2 = \{(2, 2), (3, 3)\}$$

$$R_2 - R_1 = \{(1, 2), (1, 3), (1, 4)\}$$

$$R_1 \oplus R_2 = \{(1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\}$$

Composite Relation

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→ Let R be a relation from a set A to a set B and S a relation from B to set C . The composite of $(S \circ R)$ R and S is the relation consisting of ordered pair (a, c) , where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

$$\text{eg. } R: \{1, 2, 3\} \rightarrow \{1, 2, 3, 4\}$$

$$R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$$

$$S: \{1, 2, 3, 4\} \rightarrow \{0, 1, 2\}$$

$$S = \{(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)\}$$

$$S \circ R = \{(1, 0), (1, 1), (2, 1), (2, 2), (3, 0), (3, 1)\}$$

→ The powers of a transitive relation R are subsets of this relation; that is, $R^m \subseteq R$, $m = 1, 2, 3, \dots$