Notation :- $a * a * a * \dots * a$ (n times) $= a^n$. $[a^0 = \text{identity}]$; $a \in G$.

$$a^n * a^m = \underbrace{a * a * \dots * a}_{n \text{ times}} * \underbrace{a * a * a * \dots * a}_{m \text{ times}} = a^{n+m}$$

$$(a^n)^m = a^{nm} = \underbrace{a^n * a^n * \dots * a^n}_{m\text{-times}} = \underbrace{a * a * \dots * a}_{n\text{-times}} * \underbrace{\quad}_{n\text{-times}} * \dots * \underbrace{\quad}_{n\text{-times}}$$

$$a^{-n} = (a^{-1})^n = \underbrace{a^{-1} * a^{-1} * \dots * a^{-1}}_{n\text{-times}}$$

Order of an element : The order of an element a in a group G is the smallest positive integer n such that $a^n = e$. where 'e' is the identity of G.
If no such integer exists, say a has <u>infinite order</u>.
Notation is $|a|$ or $o(a)$.

Ex 1. $\{Z_4, \oplus_4\}$ is a group of order 4. The order of each element of this group is as. [0 is the identity and order of identity is always 1]

Order of 0 is 1.

Order of 1 is $[1 \oplus_4 1 \oplus_4 1 \oplus 1 = 0 \Rightarrow 1^4 = e] = 4$.

Order of 2 is $[2^2 = 2 \oplus_4 2 = 0 \Rightarrow 2^2 = e] = 2$

Order of 3 is $[3^4 = 3 \oplus_4 3 \oplus_4 3 \oplus_4 3 = 0 \Rightarrow 3^4 = e] = 4$.

We can see the order of 1 and 3 is same and the reason is both are inverse of each other in this group.

$$\boxed{\therefore\ |a| = |a^{-1}|,\ \forall a \in G}$$

Note :- Every element of a finite group has order finite.
But order of element of infinite group may be finite or infinite.