

(1)

Proposition :- is a sentence that declares a fact that is either true or false but not both.

- Ex:-
- (1)  $1+1=2$  (prop) (T)
  - (2)  $2+3=65$  (prop) (F)
  - (3)  $2+1=2$  (not a prop)
  - (4) What time is it? (")
  - (5) Delhi is the capital of India (prop) (T)
  - (6) 5 is an odd no. (P)

(7) The truth value of a proposition is true denoted by T, if it is a true proposition & the truth value of a prop. is false, denoted by F, if it is a false proposition.

Ex:- What is the truth value of prop  $2+3=5$ ?

Compound Proposition:- are formed from existing proposition using logical operators.

Logical operators are

- (1) Negation
- (2) Conjunction
- (3) Disjunction
- (4) Exclusive OR
- (5) Implication
- (6) Biconditional.

(7) Negation

Let  $p$  be a prop. The negation of  $p$ , denoted by  $\neg p$  ( $\bar{p}$ ) is the statement "It is not the case that  $p$ ".



$p$  is true, then  $\neg p$  is false

$p$  is false, then  $\neg p$  is true.

Ex:- (1) My pc runs linux  
 $\neg p$  It is not the case that my pc runs linux  
(2) My ~~phone~~ phone has atleast 32gb of memory

Truth Table for Negation

$p$	$\neg p$
T	F
F	T

## Conjunction

Let  $p$  &  $q$  be two propositions. Conjunction of  $p$  &  $q$ , is denoted by  $p \wedge q$ , is the propn "p and q"

The conjunction  $p \wedge q$  is true when both  $p$  &  $q$  are true & is false otherwise.

Ex:-  $p \rightarrow 15$  is divisible by 3  $\rightarrow T$   
 $q \rightarrow 3$  is a prime number.  $\rightarrow T$

$p \wedge q \rightarrow 15$  is divisible by 3 & 3 is a prime number  
 $\rightarrow T$

Ex:-  $p \rightarrow$  Squares are rectangles  
 $q \rightarrow$  rectangles have 4 sides

$p \wedge q \rightarrow$  Squares are rectangles & rec. have 4 sides



# Truth Table for the Conjunction of Two Prop

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

## Disjunction

Let  $p$  &  $q$  be propositions. The disjunction of  $p$  &  $q$ , denoted by  $p \vee q$ , is the proposition "p or q".

The disjunction  $p \vee q$  is false when both  $p$  &  $q$  are false & is true otherwise.

Ex: - (1)  $\sqrt{30} > 6$  or  $\sqrt{30} < 5 \rightarrow F$   
 $\quad \quad \quad \downarrow \quad \quad \quad \downarrow$   
 $\quad \quad \quad F \quad \quad \quad F$

(2)  $16 - 4 = 10$  or  $4$  is an even no.  $\rightarrow T$   
 $\quad \quad \quad \downarrow \quad \quad \quad \downarrow$   
 $\quad \quad \quad F \quad \quad \quad T$

## Truth Table for the Disjunction of a Two Prop

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F



## Exclusive or

Let  $p$  &  $q$  be two prop. The exclusive of  $p$  &  $q$ , denoted by  $p \oplus q$ , is the prop that is true when exactly one of  $p$  &  $q$  is true & is false otherwise. (but both cannot be true)

Ex: - (1) When you purchase a item, you will get

₹ 200 cashback or a voucher worth ₹ 200.

$p \rightarrow$  When you will get a cashback of ₹ 200

$q \rightarrow$  " " " " " voucher worth ₹ 200

Truth Table:

$p$	$q$	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

( $p$  &  $q$  both cannot be true)

## (5.) Conditional Statement

Let  $p$  &  $q$  be proposition. The proposition "if  $p$  then  $q$ ", denoted by  $p \rightarrow q$  is called implication.

In the cond<sup>y</sup> statement,  $p$  is called the hypothesis &  $q$  is called the conclusion.

Ex: - If you try hard for your exam, then you will succeed.  $p \rightarrow q$   
 $p \rightarrow$  you try hard for your exam  
 $q \rightarrow$  you will succeed.

Case 1: - You tried hard for your exam & Succeeded.  $\rightarrow q \rightarrow T$   $\overbrace{p \rightarrow T}$

$(p \rightarrow q) \rightarrow T$



II - You tried hard for your exam but you failed.  
 $p \rightarrow T, q \rightarrow F$

$$p \rightarrow q \rightarrow F$$

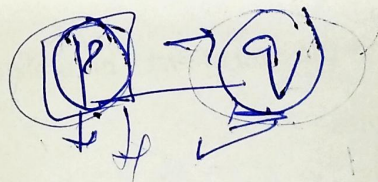
III - You haven't tried hard for your exam & you succeeded.

$$p \rightarrow F, q \rightarrow T, p \rightarrow q \rightarrow T \checkmark$$

$p \rightarrow q$  is false only when  $p$  is satisfied  
 here,  $p$  is not satisfied, so we cannot make  
 the compound propn false]

IV - You haven't tried hard for your exam & you failed.  
 $p \rightarrow \text{false}, q \rightarrow \text{false}, p \rightarrow q \rightarrow T$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T





## Biconditional Statement

Let  $p$  &  $q$  be propositions. The biconditional statement  $p \leftrightarrow q$  is the proposition "p if and only if q".  
 $p \leftrightarrow q$  is equivalent to  $p \rightarrow q$  and  $q \rightarrow p$ .

Ex -  $p$ :  $\rightarrow$  You can take the flight  
 $q$ :  $\rightarrow$  You buy a ticket

$p \leftrightarrow q$ :  $\rightarrow$  You can take the flight if and only if you buy a ticket.

$p \rightarrow q$  [If you buy a ticket, then you can take.]  
 $q \rightarrow p$  [If you have to take the flight, ...]

The biconditional statement  $p \leftrightarrow q$  is true when  $p$  &  $q$  have the same truth values, & is false otherwise.

Truth Table

$p$	$q$	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Q<sub>2</sub> Construct the truth table of the compound proposition:  $p \vee \neg q$  &  $p \wedge q$  &  $(p \vee \neg q) \leftrightarrow (p \wedge q)$

$p$	$q$	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \leftrightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F