Question Banks 5

A=1, B=0, C=3. More, A=1, B=0, C=3 is given. $B^2 - 4AC = (0)^2 - 4(1)(3)$ 2 0 -12 So, B2-4AC <0 : 10 the gluen equation 1s Elliptic. $02 \quad A = -2, \quad B = 3, \quad C = -4.$ More, A = -2, B = 3 and C = -4 is given, $B^2 - 4AC = (3)^2 - 4(-2)(-4)$

So, B= 4AC < 0. i. it 15 €lliptre.

A = -4, B = -2, C = -1

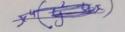
Here, A = -4, B = -2, C = -1 is given.

 $B^2 - 4AC = (-2)^2 - 4(-4)(-1)$ = 4-16

1, e -12 = 0 So, B2- 4AC LO, i, it is Elliptie.

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A = 2, B = 2, C = 2
 Here, A = 2, B = 2 and C = 2 is given
    B2-4AC = (2)2-4(2)(2)
 1. e -12 <0

So, B²-4AC <0, : it is Elliptre.
B A = -5, B = -5, C = -5
 Mere, A = -5, B = -5 and C = -5 15 given
      B2-4AC = (-5)2-4(-5)(-5)
   1.e -75 = 0
So, B²-4AC ZO, :. it is Elliptic
A = 100, B = 10, E = 10
   Hore, A=100, B=10 and C=10 is given.
     B2-4AC 2 (10) -4(100)(10)
            = 100 - 4000
      1.e -3900 <0
  So, B2-4AC LO, ... it is Elliptre.
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 $OF A = x^2, B = x^2y, C = x^3$ Here, $9A=x^2$, $B=x^2y$ and $C=x^3$ is given. B2-4AC = (x2y)2-4(x2)(x3) $= x^4y^2 - 4x^5$ Case 1: ij x 20 0 9 20 $\Rightarrow B^{2}-4AC = x^{4}y^{2}-4x^{5} = 0,$ $\Rightarrow B^{2}-4AC = 0,$ if will be then Elliptic parabolic. Case 2: if y = 0By then there are 3 possible values of x.

(i) x = 0then it will be parabolic

(ii) x > 0if will be elliptic (as $B^2 - 4AC \ge 0$). it will be hyperbolice (as B= 4AC>0). Case 3: if x>0, y>0 \Rightarrow $B^2-4Ae = x^4y^2-4x^5$ i.e $x^4y^2-4x^5 \ge 0$ if well be then elliptic.

Case 4: if x > 0, y < 0 of $B^2 - 4AC = x^4(y^2 - 4x)$ i.e $x^4y^2 - 4x^5 < 0$ it will be then elliptie.

(ase 5: 1/3 x < 0, y 0 > 0 > B^-4AC = x4y^- - 4x5

i. e x4y^2 - 4x5 > 0

it will be then hyperbolic. Case 6: abet. $B^{2}-4AC = x^{4}y^{2}-4x^{5}$ 1. e $x^{4}y^{2}-4x^{5} > 0$ it will be then hyperbolic. $B = \chi^2$ $A = \chi^2$ $B = \chi^2$ $C = \chi^5$ Here, A = 23y, B=y2 and C=x5 is given B2-4AC = (y2)2-4(x2y)(x5) $= y^4 - 4x^7y$ $= y(y^3 - 4x^7)$ Cases > $09 A = y^3, B = x^2, C = y^2$ B2-4AC = (22)2-4(43)(92) = 2c4 - 4y5 (ases \Rightarrow $A = 2i^3$ $B = (xy)^{-3/2}$ $C = y^{-3}$ 060 B2-4Acz[(xy)3/2]2-4(x3)(y-3) = (xy)-3 - 4xy-3 = 2(xy)-4(xy)=

$$= (xy)^{-3} \begin{bmatrix} 1 - 4 \end{bmatrix}$$

$$= (xy)^{-3} \begin{bmatrix} 1 - 4 \end{bmatrix}$$

$$= 3(xy)^{-3}$$

$$Cases \Rightarrow \frac{2}{3} \quad B = x^{2}, \quad C = y^{2}$$

$$B^{2} - 4AC = (x^{2})^{2} - 4(x^{3})(y^{2})$$

$$= x^{4} - 4x^{3}y^{2}$$

$$Cases \Rightarrow \frac{2}{3} \quad B = x^{2}, \quad C = x^{2}$$

$$B^{2} - 4AC = (x^{2})^{2} - 4(x^{2})(x^{2})$$

$$= x^{4} - 4x^{4}$$

$$= x^{4} - 4x$$

Question Bank -5 $\frac{\partial \phi}{\partial t^2} + \frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2} \rightarrow 0$ Let the complete solution of 1 be. $u(x,t) = \chi(x). T(t) \rightarrow ②$ Now, $\frac{\partial u}{\partial t^2} = XT''$ and $\frac{\partial u}{\partial x^2} = X''T$ Put them in (1) $XT'' = c^2 X''T$ $T'' = c^2 X'' \rightarrow 3$ $T \qquad X$ Smee in 3 RHS is a function of x and LHS is a function of the only. So, we have-T'' = X'' = X (constant) $\frac{T''}{C^2T} = K \text{ and } X'' = K$ Now, T'- KcT = 0 and x"-KX = 0 -> 9 -> 5 Here, there are 3 cases aruses for value of K.

TIME

Case I when
$$K = 0$$
, then we have

in "In 9

 $T' = 0$

Integrating w. M. t. t.

 $T' = C_1 \Rightarrow \int T' = \int C_1 dt + C_2$
 $\Rightarrow T = C_1 t + C_2$

In 5

 $X'' = 0$

Integrating w. M. t. x.

 $X' = C_3 \Rightarrow \int X' = \int C_3 dx + C_4$
 $\Rightarrow X' = C_3 x + C_4$
 $\Rightarrow X' = C_3 x + C_4$
 $\Rightarrow X' = C_3 x + C_4$

Case I when $X = C_1 t + C_2 = C_2 t + C_4 = C_3 t + C_4$

Case I when $X = C_1 t + C_2 = C_2 t + C_4 = C_3 t + C_4$
 $T'' = \int_0^2 T = 0 \Rightarrow \int_0^2 (C_1 - P_1^2)^2 = 0 \Rightarrow C_1 t + C_2 = C_2 t + C_3 t + C_4$

In 6

 $X'' - P_1^2 x = 0 \Rightarrow (D_1^2 - P_1^2)^2 x = 0 \Rightarrow C_1 t + C_2 = C_2 t + C_3 t + C_4$

where, D = d and O' = d dx

Auxiliany Eq" bor (8) $m^2 - p^2c^2 = 0$ $m = \pm pc$ T = C, e Pct + c2e Pct

Auxiliary Eq for (7) $m^2 - \rho^2 = 0$ $m = \pm \rho$ 1 x = Czepx + cyepx

: Complete Sol" is_ $u(x,t) = (c_1e^{pct} + c_2e^{-pct})(c_3e^{px} + c_4e^{-px}).$

Case III cuhen K 18 -ve 1.e $K = -p^2$ (where p is then we have, any no.)

I'm (9)

T" + $p^2c^2T = 0 \Rightarrow (D^2 + p^2c^2)T = 0 \Rightarrow (8)$

 $\frac{1}{x} + p^{2}x = 0 \Rightarrow (\dot{p}^{2} + p^{2})x = 0 \Rightarrow 9$

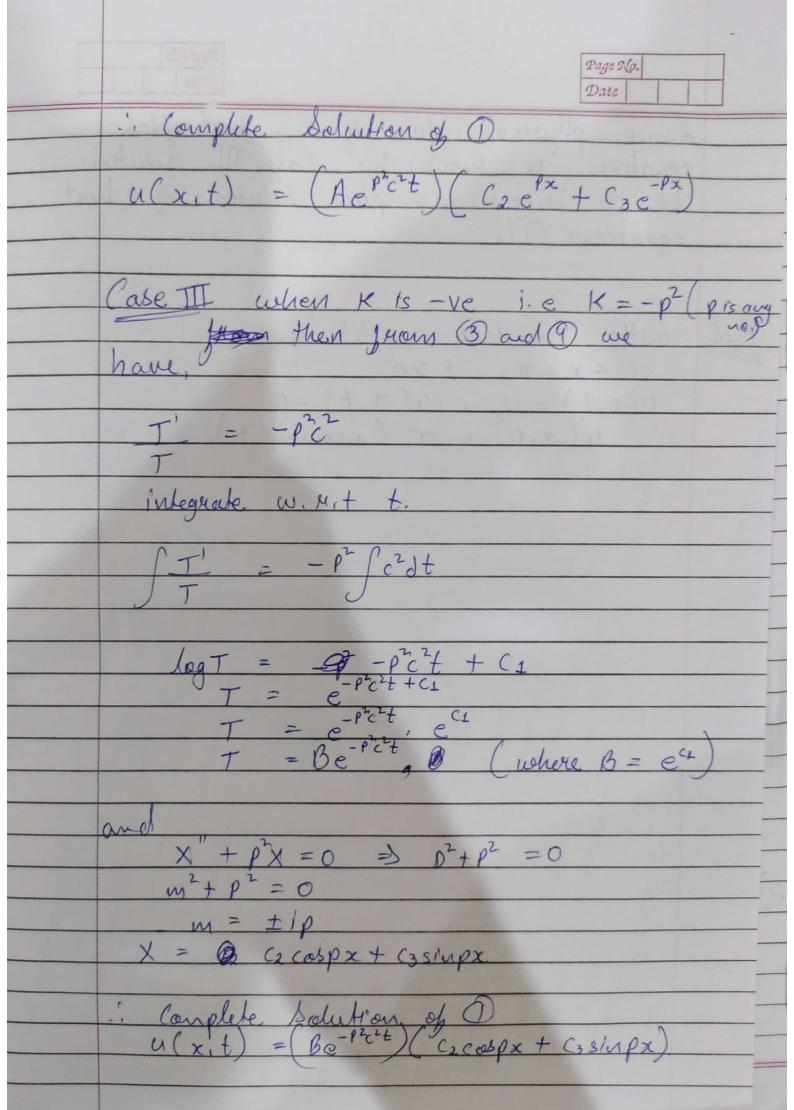
A. E for 8 $m^2 + p^2c^2 = 0$ $m = \pm ipc$ $T = c_1 cospet + c_2 simpet$ $X = C_3 cospx + C_4 simpx$

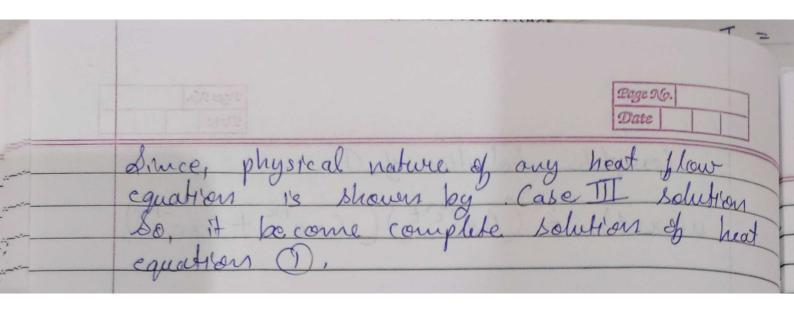
'. Complete Sol" is.

u(x,t) = (c,cospet + C2 slupet) (c3 cospx + C4 slupx)

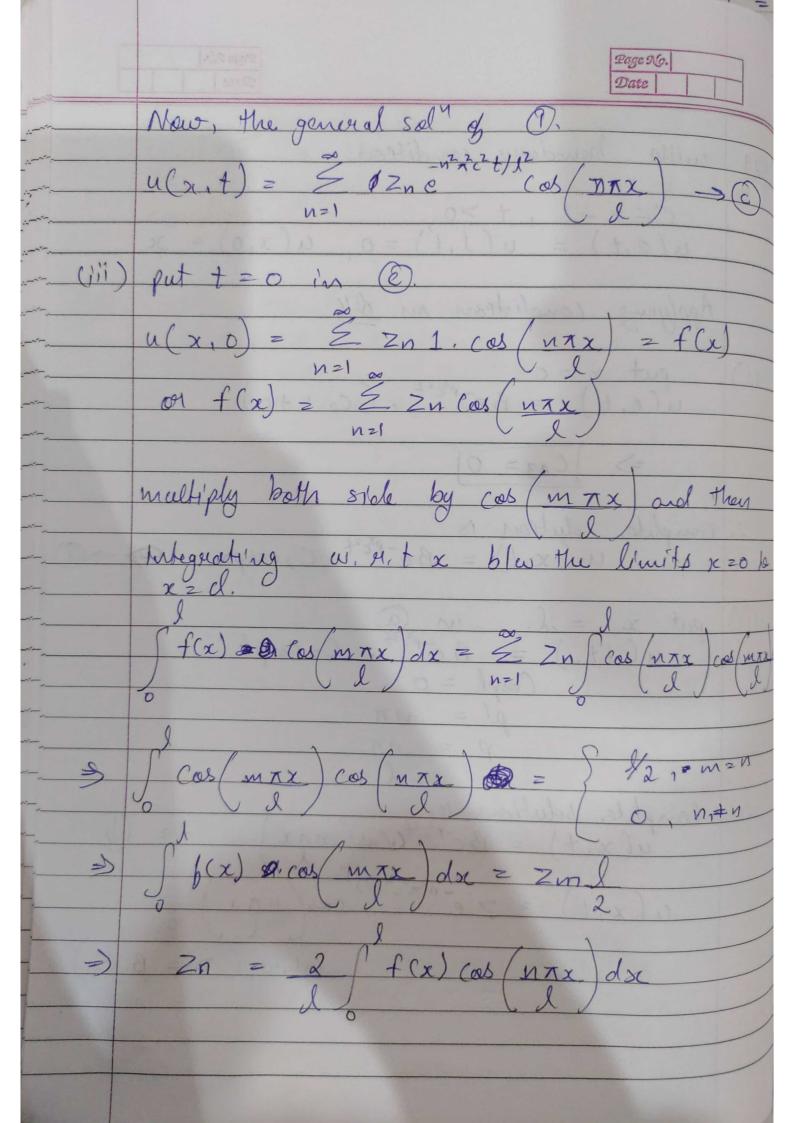
Since periodice function are present in Case III solution and it shows the physical nature of wave, So, it is only solution which become the complete solution of T, $\frac{O16}{Ot} = \frac{C^2 J^2 u}{Ot} \rightarrow 0$ Let the complete of 1 be u(x,t) = X(x).T(t).Now, du = XT' and 324
put them in 1 T' = X' = K (constant) # T'- KCT = 0 and X"- KX = 0 → 3 9 Here, there are 3 cases arises for value of k.

Case I when K = 0, then we have, brown (3) and (9) T'=0 > T = CID (Integrating w. xit t) $X'' = 0 \Rightarrow X' = C_2 \Rightarrow \int X' = \int c_2 dx + C_3$ => X = C2x + C3) (Integration w. Hit x) (ase I when K is the, i.e K = p² (where p is any then we have from 3 and 9 no.) $T' - p^2c^2T = 0 \Rightarrow T' = p^2c^2$ integrating w. H. t t $\int \frac{1}{T} = p^2 \int c^2 dt$ => logT = pet + C1 $T = e^{p^{2}c^{2}t} + c_{1}$ $T = e^{p^{2}c^{2}t} \cdot e^{c_{1}}$ $T = A e^{p^{2}c^{2}t} \quad (\text{where } A = e^{c_{1}}).$ Integliating w. 4, t x. m2-p2 =0 $m^2 = p^2$ \Rightarrow $X = C_2 e^{Px} + C_3 e^{Px}$ mtp





E	Page No.
029	with boundary condition.
78	
	0 < x < l, t > 0 $u(0,t) = u(l,t) = 0, u(x,0) = 0$
	Applying condition on 016
(i)	$put \times 20$ $u(0,t) = Be^{-p^{2}c^{2}t} \cdot (c_{2}\cdot 1+0)$
1.370	=> Co3 = 0
	complete. solution is u(x,t) = Be-P22t, C2cospx > 3
(11)	put $x = l$. in (a) $u(lt,) = Be^{p^2c^2t} \cdot C_2 \cosh l$
	Cospl = 0 $pl = NT$
	$p = n\pi$
	Conglete, solution 15. u(x,t) = Be-P2c2tacos(ngx) -> (b)
	$u(x,t) = ze^{-n^2\pi^2 t/2^2} \cos(n\pi x)$
	where, (2 = B. C2.)



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Hence, the final general solution $b = 0$ () $u(x,t) = \sum_{n=1}^{\infty} \frac{-n^2 x^2 t / 4^2}{2n^2} \left(cos \left(\frac{n \pi x}{n} \right) \right)$ where, $2n = 2$ of $f(x) cos \left(\frac{n \pi x}{n} \right) dx$.
$\frac{1}{100} = \frac{1}{100} = \frac{1}$
3 = 0/3 (018) + 1/2 (c + p ² y8) - 1/2 2 2 2 2 2 2 2 2 2
The state of the s