Computer Representation of Sets:

Let $U = \{\overset{a_1}{1}, \overset{a_2}{2}, \overset{a_3}{3}, \overset{a_4}{4}, \overset{a_5}{5}, 6, 7, 8, 9, \overset{a_{10}}{10}\}$, a subset

$A = \{\underset{a_1}{1}, 3, 5, 7, 9\}$, $B = \{1, 3, 4, 5, 6, 7\}$

Represent a subset A of U with bit string of length n, where n is cardinal number of U, the ith bit in this string is 1 if a_i belongs to A and is 0 if a_i does not belong to A.

The bit string for subset A is 10 10 10 10 10

The " B is 10 11 11 10 00

$A \cup B = \{1, 3, 4, 5, 6, 7, 9\}$ - Bit string 1011111010

$A \cap B = \{1, 3, 5, 7\}$ - string 10 10 10 10 00

$\bar{A} = \{2, 4, 6, 8, 10\}$ - String 01 01 01 01 01

$A - B = \{9\}$ - string 00 00 00 00 10

Multi-Sets and Multiplicity

Multisets are unordered collection of elements where an ~~elements~~ where an element can occur as a member more than once.

$A = \{m_1 \cdot a_1, m_2 \cdot a_2, \ldots, m_r \cdot a_r\}$ denotes the multiset with element a_1 occurring m_1 times, element a_2 occurring m_2 times, and so on. The numbers m_i, $i = 1, 2, \ldots r$ called the multiplicities of the elements a_i

Let P and Q be multisets. The union $P \cup Q$ is the multiset where the multiplicty of an element is the maximum of its multiplicities in P and Q.

The intersection $P \cap Q$ = multiset where multiplicty is mini.

The difference P-Q is multiset when the multiplicity of an element is the multiplicity of the element in P less its multiplicity in Q unless this difference is negative in which case the multicity is zero.
The sum P+Q is multiset where the multiplicity of an element is the sum of multiplicities in P and Q.

Generalized Unions and Intersections

Suppose there are n sets $A_1, A_2, \ldots, A_n$ then union of these sets is again a set and notation is,

$$A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$$

The intersection of these sets is a set ~~which~~ contains those elements that are members of all sets and notation

$$A_1 \cap A_2 \cap \ldots A_n = \bigcap_{i=1}^{n} A_i$$

Ex. For $i = 1, 2, \ldots$, let $A_i = \{i, i+1, i+2, \ldots\}$

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$
$$= \{1, 2, 3, \ldots\} \cup \{2, 3, 4, \ldots\} \cup \ldots \cup \{n, n+1, n+2, \ldots\}$$
$$= \{1, 2, 3, \ldots\} = A_1$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$
$$= \{n, n+1, n+2, \ldots\} = A_n.$$

Symmetric Difference of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B.

$$A \oplus B = (A-B) \cup (B-A) = (A \cup B) - (A \cap B)$$