Density of states: 1D (Quantum wire) and 0D (Quantum dot)

Outlines:

1. Density of state of 1D and 0D
2. Applications of 1D and 0D

Prerequisite/Recapitulations

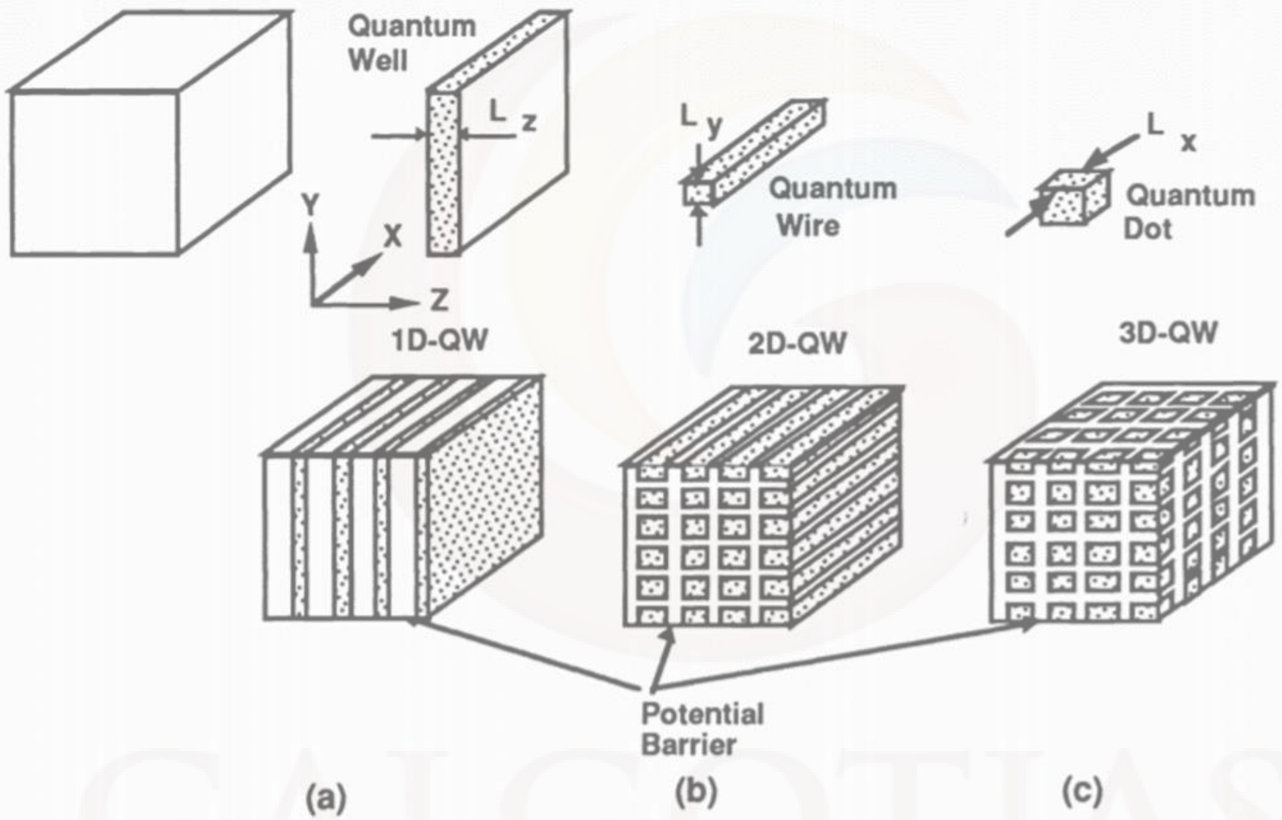
Three-dimensional (3D) structure or bulk structure: No quantization of the particle motion occurs, i.e., the particle is free.

Two-dimensional (2D) structure or quantum well: Quantization of the particle motion occurs in one direction, while the particle is free to move in the other two directions.

One-dimensional (1D) structure or quantum wire: Quantization occurs in two directions, leading to free movement along only one direction.

Zero-dimensional (0D) structure or quantum dot (sometimes called “quantum box”): Quantization occurs in all three directions

Prerequisite/Recapitulations



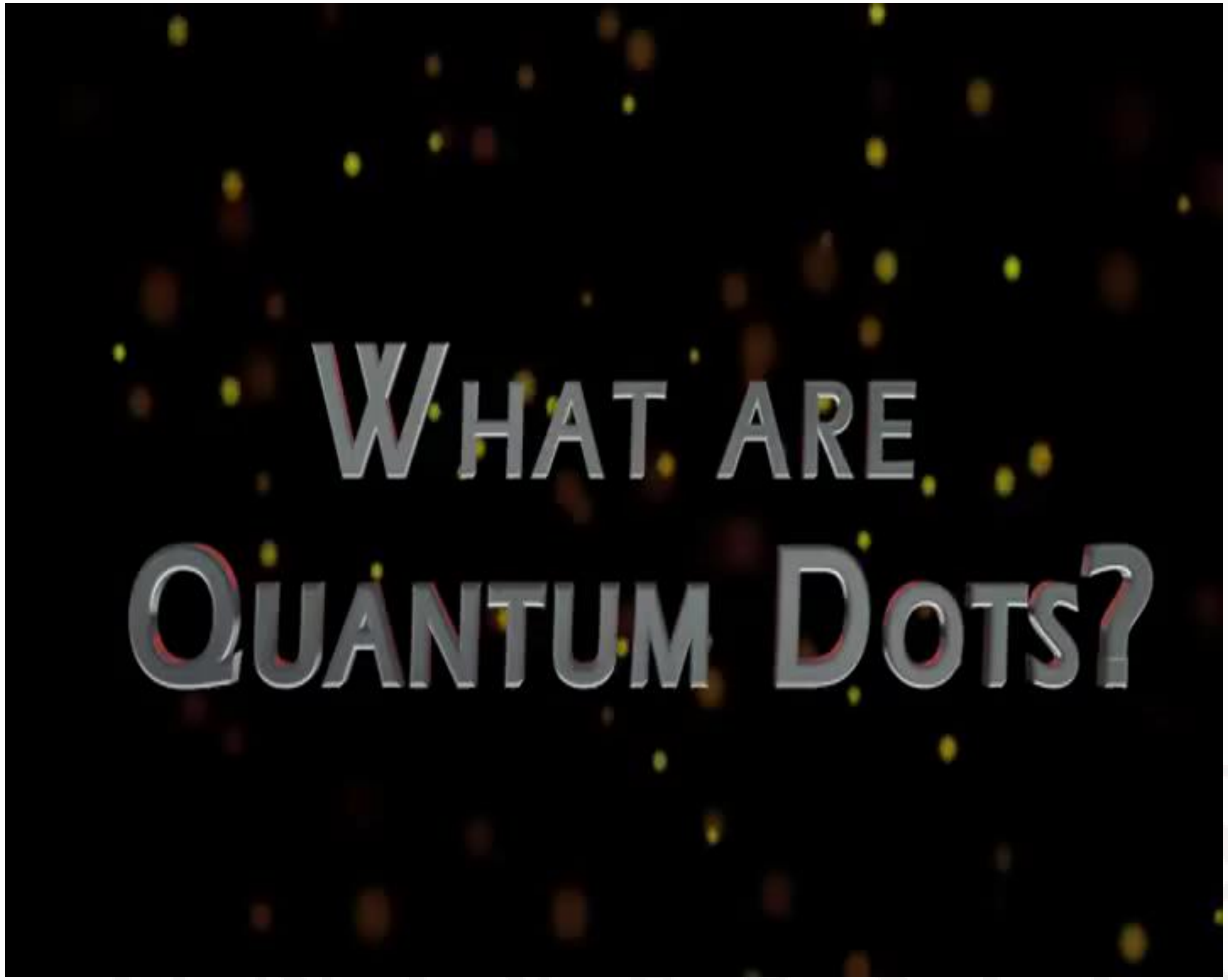
Quantum wells, wires and dots showing the successive dimensions of confinement .

Learning outcome

After the completion of the lecture you will be able to:

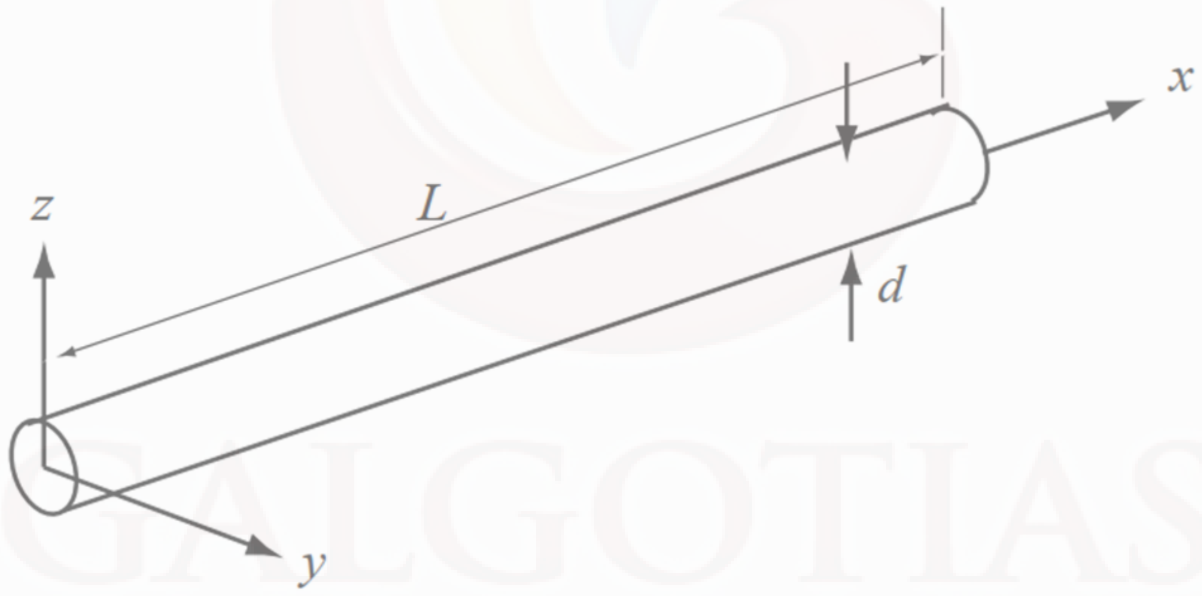
1. **Explain the Density of state of 1D and 0D**
2. **Understand the different applications of 1D and 0D**

Conceptualization



Introduction: 1D (Quantum wire)

Quantum effects in systems which confine electrons to regions comparable to their de Broglie wavelength. When such confinement occurs in two dimensions only (say, by two restrictions on the motion of the electron in the z- and y-directions), with free motion in the x-direction, a one dimensional electron is created, which is shown in Figure



Schematic presentation of a quantum wire

1D (Quantum wire)

In a one-dimensional system for a quantum wire, the total energy is

$$E = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\pi^2 \hbar^2 n^2}{2m_y d^2} + \frac{\pi^2 \hbar^2 l^2}{2m_z d^2}$$

where $n = 1, 2\ldots..$ and $l = 1, 2, \ldots$

Density of States in 1D

For calculating the density of states for a 1D structure (i.e. quantum wire), we can use a approach as used in 3D.

The equations for 3D crystal ------------------------------ $V_{single-state} = \left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)\left(\frac{\pi}{c}\right) = \left(\frac{\pi^3}{V}\right) = \left(\frac{\pi^3}{L^3}\right)$

change to the following:

k-space volume of single state cube in k-space: single-state = $V_{single-state} = \left(\frac{\pi}{a}\right) = \left(\frac{\pi}{V}\right) = \left(\frac{\pi}{L}\right)$

K-space Volume of single state cube in k-space $V_{line} = k$

1D (Quantum wire)

Number of filled state in K-space (N):

$$N = \frac{V_{line}}{V_{\sin\ gle-state}} \times 2 \times \left(\frac{1}{2}\right)$$

$$N = \frac{k}{\frac{\pi}{L}} = \frac{kL}{\pi}$$

Substituting $k = \sqrt{\frac{2mE}{\hbar^2}}$ yields

$$N = \frac{\sqrt{\frac{2mE}{\hbar^2}}L}{\pi} = \sqrt{2mE}\frac{L}{\hbar\pi}$$

$$N = (2mE)^{1/2}\frac{L}{\hbar\pi}$$

1D (Quantum wire)

The density per unit energy is then obtained by using the chain rule:

$$\frac{dN}{dE} = \frac{dN}{dk}\frac{dk}{dE} = \frac{\frac{1}{2}(2mE)^{-1/2}\cdot 2mL}{\hbar\pi} = \frac{(2mE)^{-1/2}\cdot mL}{\hbar\pi}$$

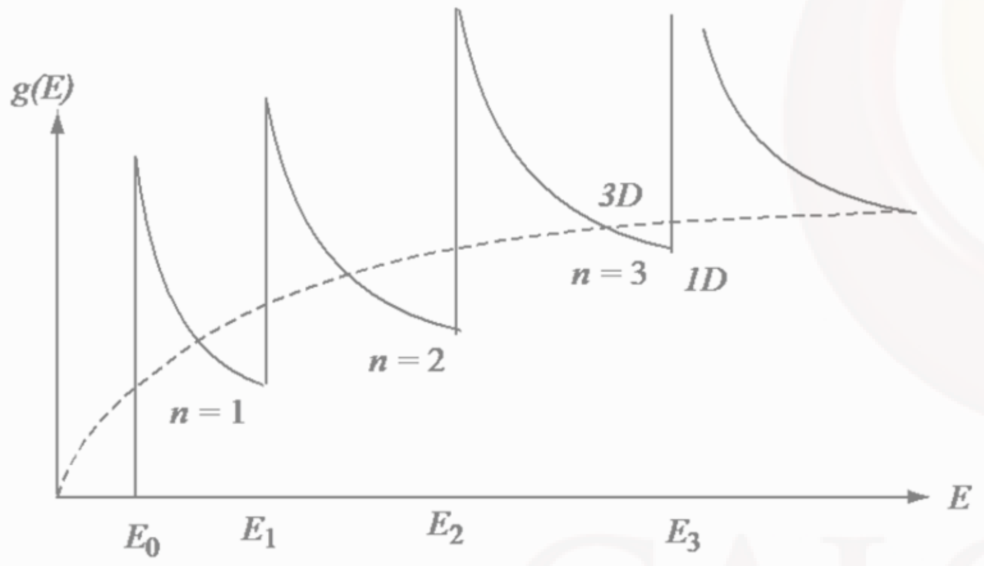
The density of states per unit length, per unit energy is found by dividing by L (length of the crystal).

$$g(E)_{1D} = \frac{\frac{(2mE)^{-1/2}\cdot mL}{\hbar\pi}}{L} = \frac{(2mE)^{-1/2}\cdot m}{\hbar\pi} = \frac{m}{\hbar\pi\sqrt{2mE}}$$

1D (Quantum wire)

Final expression for the density of state of 1D:

$$g(E)_{1D=}\frac{1}{\hbar\pi}\cdot\sqrt{\frac{m}{2E}}$$



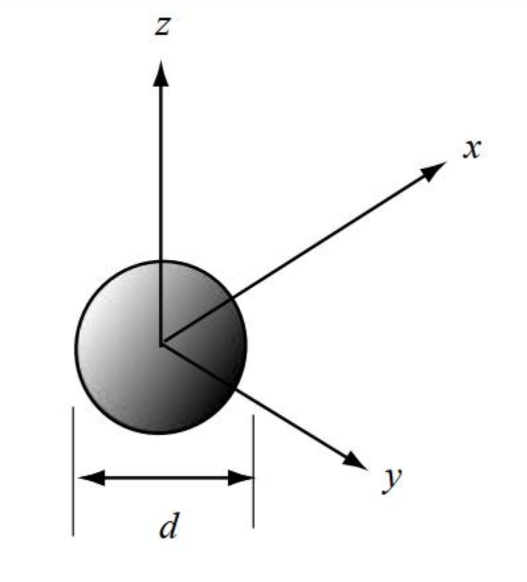
Density of states for a quantum wire. The corresponding density of states for an unconfined 3D system is also shown for comparison (broken line).

Figure shows the density of states for an ideal quantum wire, showing the characteristic singularity in E $^{-1/2}$ which was derived for 1D as shown in Equation In a quantum wire, such a singularity will occur at each energy of quantization in the x-direction

0D (Quantum dot)

Electrons can be confined in all three dimensions in a dot. The situation is analogous to that of a hydrogen atom: only discrete energy levels are possible for electrons trapped by such a zero-dimensional potential. The spacing of these levels depends on the precise shape of the potential. The development and application of quantum dot systems is an increasingly important research topic for a number of reasons, both technological and theoretical.

Quantum dots, where a confinement potential replaces the potential of the nucleus, are fascinating objects. On the other hand, these systems are thought to have vast potential for future technological applications, such as possible applications in memory chips, quantum computation, quantum-dot lasers, and so on.



Schematic presentation of a quantum dot

0D (Quantum dot)

When an electron motion is confined in all directions, one gets a zero-dimensional system as

$$E_n = \frac{\hbar^2\pi^2 n^2}{2d^2}\left(\frac{1}{m_x}+\frac{1}{m_y}+\frac{1}{m_z}\right)$$

which, for the lowest subband energy n = 1, leads to

$$E_0 = \frac{\hbar^2\pi^2}{2m_c^* d^2}$$

where

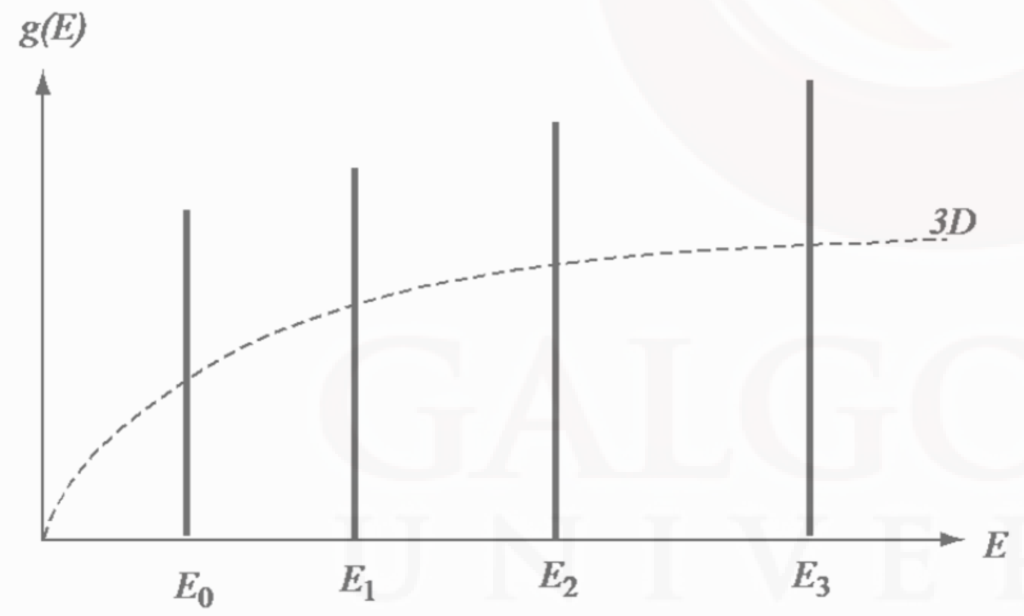
$$\frac{1}{m_c^*} = \frac{1}{3}\left(\frac{1}{m_x}+\frac{1}{m_y}+\frac{1}{m_z}\right)$$

which is called the *conductivity effective mass*

Density of state of 0D (Quantum dot)

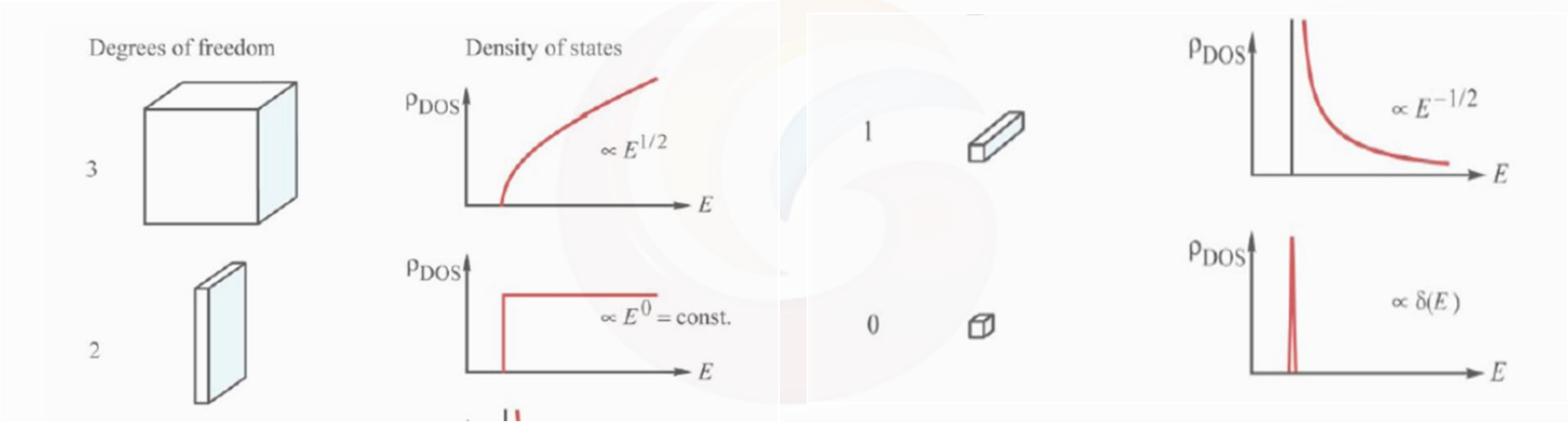
When considering the density of states for a 0D structure (i.e. quantum dot), no free motion is possible. Because there is no k-space to be filled with electrons and all available states exist only at discrete energies, we describe the density of states for 0D with the delta function.

The density of state for 0D is given by: $g_{0D}(E) = \delta(E - E_0)$



Plot of Density of states for a quantum dot

Comparison of density of states of 3D, 2D ,1D and 0D

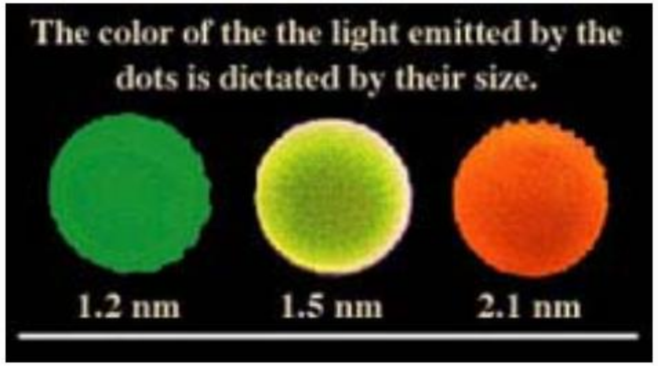


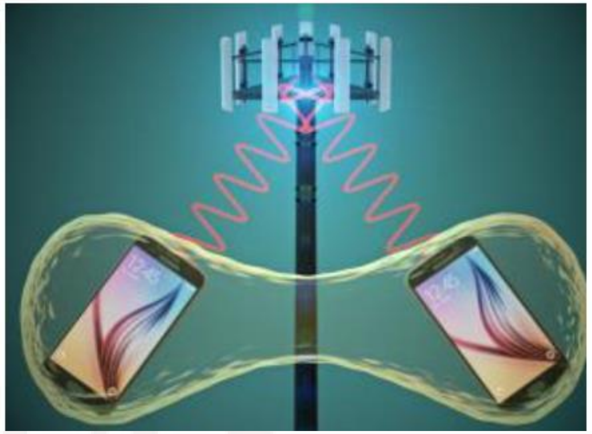
Electronic density of states of semiconductors with 3, 2, 1, and 0 degrees of freedom for electron propagation. Systems with 2, 1, and 0 degrees of freedom are referred to as quantum wells, quantum wires, and quantum boxes, respectively.

Applications of 1D and 0D structures

Possible applications of such structures are in :

1. Memory chips
2. Quantum computation
3. Quantum-dot lasers
4. Quantum communications
5. Color coded dots for fast DNA testing
6. 3-D imaging inside living organism





Quantum entanglement for quantum communications

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