DNF & CNF

→ We call any proposition p or its negation $\neg p$ a <u>literal</u>.

→ An <u>elementary disjunction</u> is a disjunction of literals.

→ An <u>elementary conjunction</u> is a conjunction of literals.

→ A <u>disjunctive normal form (DNF)</u> is a disjunction of elementary conjunctions

A <u>conjunctive normal form (CNF)</u> is a conjunction of elementary disjunctions.

<u>Construction to obtain DNF or CNF</u>:

<u>Step 1</u>: Eliminate $\rightarrow$ and $\leftrightarrow$ using $(p \rightarrow q) \equiv (\neg p \vee q)$ and $(p \leftrightarrow q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

<u>Step 2</u>: Use De Morgan's law to eliminate $\neg$ appearing before elem. disj. or conj.

<u>Step 3</u>: Apply distributive laws repeatedly to eliminate conj of disj or disj of conj

Ex. Obtain a dnf of $(p \wedge \neg(q \wedge r)) \vee (p \rightarrow q)$

$(p \wedge \neg(q \wedge r)) \vee (p \rightarrow q)$

$\equiv (p \wedge \neg(q \wedge r)) \vee (\neg p \vee q)$ $\quad [p \rightarrow q \equiv \neg p \vee q]$

$\equiv (p \wedge (\neg q \vee \neg r)) \vee (\neg p \vee q)$ $\quad$ [De Morgan's law]

$\equiv (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q)$ $\quad$ [Distributive law]

Ex. Obtain a cnf of $\neg(p \rightarrow q) \vee (r \rightarrow p)$

$\neg(p \rightarrow q) \vee (r \rightarrow p)$

$\equiv \neg(\neg p \vee q) \vee (\neg r \vee p)$ $\quad [\because p \rightarrow q \equiv \neg p \vee q]$

$\equiv (p \wedge \neg q) \vee (\neg r \vee p)$ $\quad$ [$\because$ De Morgan's law]

$\equiv (p \vee \neg r \vee p) \wedge (\neg q \vee \neg r \vee p)$ $\quad$ [Distributive law]

$\equiv (p \vee \neg r) \wedge (p \vee \neg q \vee \neg r)$ $\quad$ [Idempotent & commutative laws]

DNF and CNF form truth table :

- A <u>minterm</u> is a conjunction of literals in which each variable is represented exactly once
  - If a truth table has 3 variables p, q, r then $p \wedge \neg q \wedge r$ is a minterm but $p \wedge \neg q$ is not.
- Each minterm is true for exactly one assignment.

p	q	r	α
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

Suppose α is a logical expression (compound proposition) then α is true for three assignments

- p, q, r are all true, $(p \wedge q \wedge r)$
- $p, \neg q, r$ " , $(p \wedge \neg q \wedge r)$
- $\neg p, \neg q, r$ " , $(\neg p \wedge \neg q \wedge r)$

So, DNF of α : $(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$.

p	q	r	α	$\neg\alpha$
T	T	T	T	F
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

To Find cnf of α, first find $\neg\alpha$ then $\neg\alpha$ is true for 3 assignments.

$\therefore$ DNF of $\neg\alpha$ is

$(p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

$\therefore$ CNF of α is negation of dnf of $\neg\alpha$

$(\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee q \vee \neg r)$

Remember: Every literals, elementary disjuntion and elementary conjunction are DNF and CNF both.

e.g. p, $\neg p$, $p \vee q$, $p \wedge q$, $p \vee q \vee r$, $p \wedge q \wedge \neg r$ are CNF and DNF both.