Direct Proof of a condn statement $p \to q$ is constructed by assuming p is true, & then showing q is true, by using thms, rules of inference

Indirect Proof:- are proofs that do not start with the hypothesis and end with the conclusion.

(1) Proof by contraposition:-
This proof technique make use of the fact that the conditional statement $p \to q$ is equivalent to $\neg q \to \neg p$.

This means that condn statement $p \to q$ can be proved by showing $\neg q \to \neg p$

Q. (2) P.T if n is Integer & $3n+2$ is odd, then n is odd.

Soln:- Direct Technique
Let $(3n+2)$ be odd
ie $3n+2 = 2k+1$ f.s. Integer k
$\Rightarrow$ $3n+1 = 2k$
$\Rightarrow$ n is odd. (If n is even $\Rightarrow n = 2k \Rightarrow$ $3n+1 = 6k+1 \Rightarrow$ odd)

By Contraposition method
Assume n is not odd.
i.e n is even $\Rightarrow$ $n = 2k$

$\Rightarrow (3n+2) = 3(2k)+2 = 6k+2$
$= 2(3k+1)$
$=$ even

Hence, $\neg q = n$ is not odd
$\Rightarrow (3n+2) =$ even $\Rightarrow \neg(3n+2) = \neg p$

$\therefore \neg q \to \neg p \Rightarrow \underline{\underline{p \to q}}.$

Proof by Contradiction

Suppose we want to prove that a statement p is true [T.S p is true]

Assume p is false ie $\neg p$ is true.

$\neg p \to q$, q is a contradiction.

& $\underset{F}{\neg p} \to \overset{F}{q}$ is true

$\neg p$ is false $\Rightarrow$ p is true.

Q. (4.) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

Soln:- p: $\sqrt{2}$ is irrational.

Suppose that $\neg p$ is true.

ie $\sqrt{2}$ is ~~rational~~ not irrational

or $\sqrt{2}$ is rational.

If $\sqrt{2}$ is rational nb. then,

$\sqrt{2} = \frac{a}{b}$ ($b \neq 0$, a & b have no common ~~divisor~~ factor)

$\Rightarrow (\sqrt{2})^2 = \frac{a^2}{b^2} \Rightarrow 2 = \frac{a^2}{b^2}$.

$2b^2 = a^2$ — (1)
even.

$\Rightarrow$ a^2 is an even no

a^2 is even $\Rightarrow$ a is even.

Let $a = 2c$ s.t. integer c.

from (1) $2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$] even

$\Rightarrow$ b^2 is even $\Rightarrow$ b is even

So, both a & b are even, then a & b have common factor.

So, $\neg p$ leads to the eqn $\sqrt{2} = \frac{a}{b}$, where (a & b have no common factors)

But both a & b are even, that is. 2 divides both a & b

$\Rightarrow$ Therefore, our assumption must be false

Hence, "$p = \sqrt{2}$ is Irrational" is true

(Contraposition)

(3.) P.t if n is an integer & n^2 is odd, then n is odd.

Soln:- We'll prove it by contra (i.e ~~p→q~~ $\neg q \to \neg p$)

Suppose that n is not odd

ie n is even

$\Rightarrow$ $n = 2k$ for some integer k.

$\Rightarrow$ $n^2 = 4k^2 = 2(2k^2)$

$\Rightarrow$ n^2 is even.

ie n^2 is not odd.

$\therefore$ By contra n is odd.

Direct Proof

(1) Give a direct proof of the theorem,
" If n is an odd integer, then n^2 is odd. $(p \to q)$

Ans:- Assume that n is odd

$\Rightarrow n = 2k+1$, f.s. integer k.

$\Rightarrow n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

By defn of an odd integer,
n^2 is odd. integer.