Relations

Cartesian Product of two set – Let A & B be two sets. The set of all ordered pair (a, b) where $a \in A$ & $b \in B$ is called cartesian Product of A & B.

for eg –

$A = \{1, 2, 3\}$ $B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

But $\Rightarrow$ $[A \times B \neq B \times A]$

Relation –

Let A & B be two non-empty sets then any subset of $A \times B$ is called "a relation from A into B".

i.e

Let $a \in A$ & $b \in B$ then Pair (a, b) is called ordered pair of $(a, b) \in R$ then we write aRb which is read as "a" is related to "b".

for example-

① Let $A = \{3, 6, 9\}$, $B = \{4, 8, 12\}$

then

$R = \{(3,4), (3,8), (4,12)\}$ is relation A to B.

Domain & Range of a relation-

Domain ⇒ If R is a relation from A & B then set of all first elements of the ordered Pair (x, y), which belongs to R is called domain of R and written as $D(R)$ or $Dom(R)$.

Range ⇒ The set of all objects y such that for some x, $(x,y) \in R$

Types of Relation-

① Reflexive Relation-

A relation R on a set A is reflexive if aRa for every $a \in A$.

i.e;

if $(a,a) \in R$ for every $a \in A$ / Thus R is not reflexive if there exist an $a \in A$ such that $(a,a) \notin R$.

eg

Set $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$

$R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

$R_1 \Rightarrow$ reflexive $\Rightarrow$ ✗

$R_2 \Rightarrow$ reflexive $\Rightarrow$ ✓

* $R_3 = \emptyset \Rightarrow$ reflexive ✗

* $R_4 = A \times A$, the universal relation $\Rightarrow$ reflexive ✓

Q Determine the reflexive relation for following example.

(1) Relation $\leq$ (less than or equal) on the set Z of integers

(2) Set inclusion $\subseteq$ on a collection C of sets

(3) Relation $\perp$ on the set L of lines in the plane

(4) Relation // on the set L of lines in the plane.

Sol^n R_3 is not reflexive since no lines is $\perp$ to itself. Similarly R_4 is not reflexi

Since no lines are parallel to itself.
& R_1 & R_2 are reflexive.

Parallel & Perpendicular होने के लिए दो different lines होना Compulsory है।

(2) Irreflexive relation -

A relation R on a set A is irreflexive if (a, a) does not comes to R for every $a \in A$.
i.e.
R is not irreflexive if there exist atleast one $a \in A$ such that $(a, a) \in A$

eg
Let R on set A such that $R = \{(a,b) | a \neq b\}$

(3) Symmetric relation -

A relation R on a set A is symmetric if whenever aRb then bRa.

Thus R is not symmetric if there exist $a, b \in A$ such that $(a, b) \in R$ but $(b, a) \notin R$

eg Set $A = \{1, 2, 3, 4\}$
$R_1 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$
$R_2 = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$

$R_3 = \{(1,3), (2,1)\}$

$R_4 = \phi$

$R_5 = A \times A$, the universal set

$R_1 \Rightarrow$ Not symmetric

$R_2 \Rightarrow$ symmetric

$R_3 \Rightarrow$ Not symmetric

$R_4 \Rightarrow$ symmetric

$R_5 \Rightarrow$ symmetric

Q ① The relation $\perp$ on the set L of lines in the plane $\longrightarrow$ Symmetric

② The relation // on the set L of lines in the plan $\longrightarrow$ Symmetric

③ The relation inequalities, subsets are not symmetric.

④ Anti Symmetric -

A relation R on a set A is anti-symmetric if whenever aRb and bRa then $a=b$

i.e.

if whenever $(a,b), (b,a) \in R$ then $a=b$

Thus R is not antisymmetric if there exist $a, b \in A$ such that (a,b) and $(b,a) \in R$ but $a \neq b$

{ ϕ के लिये ⇒ Reflexive को छोड़कर ϕ सारे relation follow करेगा }

eg

$A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$

$R_2 = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$

$R_3 = \{(1,3), (2,1)\}$

R_1 ⇒ anti symmetric ✓

R_2 ⇒ not anti symmetric

R_3 ⇒ anti symmetric

$R_4 = \phi$ ⇒ Antisymmetric

$R_5 = A \times A$ ⇒ universal set ⇒ Not antisymmetric

★ The properties of being symmetric & being anti symmetric are not negative of each other.

eg Let * $R = \{(1,3), (3,1), (2,3)\}$

R is neither symmetric nor anti symmetric.

* $R' = \{(1,1), (2,2)\}$

R' is both symmetric & anti symmetric.

(5) Transitive relation -

A relation R on set A is transitive if whenever aRb & bRc the aRc.

i.e.

if whenever $(a,b), (b,c) \in R$ then $(a,c) \in R$

Thus R is not transitive if there exist $a, b, c \in A$ such that $(a,b), (b,c) \in R$ but $(a,c) \notin R$

eg in last example -

$R_1 \Rightarrow$ transitive
$R_2 \Rightarrow$ transitive
$R_3 \Rightarrow$ not transitive

(6) Equivalance relation -

A relation R on set S is an equivalence relation if R is reflexive, symmetric & transitive.

i.e.
(i) for every $a \in S$, aRa
(ii) if aRb then bRa
(iii) if aRb, bRc then aRc

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eg ① in last example

$R_2 \Rightarrow$ symmetric ✓, reflexive ✓, transitive ✓

$\Downarrow$

$\boxed{\text{equivalance ✓}}$

② equality of numbers ~~of~~ on a set of real number

③ equality of subsets of universal sets.

④ Similarity of trigle on the set of triangles.