Moment Generating Function of Binomial Dist

(1) About Origin

$$M_x(t) = E(e^{tx}) = \sum_{x=0}^{n} e^{tx} \cdot {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x\, (pe^t)^x q^{n-x}$$

$$= (q+pe^t)^n$$

(2) About Mean

$$M_{x-np}(t) = E\left(e^{t(x-np)}\right)$$

$$= e^{-npt} E(e^{tx})$$

$$= e^{-npt} (q+pe^t)^n = \left(e^{-pt} q + pe^{qt}\right)^n$$

Ex1 If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts choosen at random
(1) 1 (2) None (3) At most 2 bolts will be defective

Sol:-

$$p(\text{defective}) = \frac{10}{100} = \frac{1}{10}$$

$$q(\text{non-defective}) = 1 - \frac{1}{10} = \frac{9}{10}$$

Also, $n = 10$

$$P(r) = {}^nC_r\, p^r q^{n-r}$$

(1) $r = 1$ $\quad P(1) = {}^{10}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^9 = 0.3874$

(2) $r = 0$ $\quad P(0) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} = 0.3486$

(3) $P(r \le 2) = P(0) + P(1) + P(2)$

$P(2) = {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{10-2} = 0.1937$

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$P(r \le 2) = 0.9297$

Fit a Binomial Distribution to the following frequency data:

x:	0	1	2	3	4
f:	30	62	46	10	2

Sol^n:-

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{192}{150} = 1.28$$

$np = 1.28 \qquad n = 4$ (No. of Trials)

$p = 0.32$

$q = 1 - 0.32 = 0.68$

$N = 150$

Hence, the Binomial Distribution is $= N(p+q)^n$

$= 150(0.68 + 0.32)^4$

Ex 3 If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit at least once?

Sol^n:-

$p = \frac{10}{100} = \frac{1}{10} \qquad q = \frac{9}{10} \qquad n = 10$

Prob that the target will be hit at least once $= P(r \geq 1) = 1 - P(r=0)$

$= 1 - ({}^nC_0 p^0 q^n) = 1 - {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10}$

$= 0.6513$

Ex 4 6 dice are thrown 729 times. How many times do you expect at least three dice to show a five or six?

Sol^n:- p = chance of getting a 5 or 6 with one dice $= \frac{2}{6} = \frac{1}{3}$

$q = 2/3 \qquad n = 6 \qquad N = 729$

Expected No of times at least 3 dice showing 5 or 6

$= 729 \cdot P(r \geq 3) = 729\left[{}^6C_3 \left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + {}^6C_4 \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_5 \frac{2}{3}\left(\frac{1}{3}\right)^5 + {}^6C_6 \left(\frac{1}{3}\right)^6\right]$

Ans

Binomial Probability Distribution (1)

$P(X=r)$ = Prob of r successes in n trials

$$= {}^nC_r\, p^r q^{n-r}$$

Mean and Variance of the binomial Distribution

$$\text{Mean } \mu = \sum_{r=0}^{n} r\, P(r) = \sum_{r=0}^{n} r\, {}^nC_r\, q^{n-r} p^r$$

$$= 0 + 1.\, {}^nC_1\, q^{n-1} p + 2.\, {}^nC_2\, q^{n-2} p^2 + \cdots + n.\, {}^nC_n\, p^n$$

$$= nq^{n-1}p + n(n-1)\, q^{n-2}p^2 + \frac{n(n-1)(n-2)}{2}\, q^{n-3}p^3 + \cdots + np^n$$

$$= np\left[{}^{n-1}C_0\, q^{n-1} + {}^{n-1}C_1\, q^{n-2} p + \cdots + {}^{n-1}C_{n-1}\, p^{n-1}\right]$$

$$= np\,(q+p)^{n-1} = np.$$

$$\sigma^2 = \sum_{r=0}^{n} r^2 P(r) - \left[\sum r P(r)\right]^2$$

$$\text{Variance } \sigma^2 = \sum_{r=0}^{n} r^2 P(r) - \mu^2$$

$$= \sum_{r=0}^{n} \left[r + r(r-1)\right] P(r) - \mu^2$$

$$= \mu + \sum_{r=2}^{n} r(r-1)\, P(r) - \mu^2$$

$$= \mu + \left[2.1 * {}^nC_2\, q^{n-2} p^2 + 3.2.\, {}^nC_3\, q^{n-3} p^3 + \cdots + n(n-1)\, {}^nC_n\, p^n\right] - \mu^2$$

$\mu = np$

$$= \mu + \left\{n(n-1)\, q^{n-2}p^2 + n(n-1)(n-2)\, q^{n-3}p^3 + \cdots + n(n-1)p^n\right\} - \mu^2$$