A <u>differential equation</u> is an equation which contains one or more variables and their derivatives.

<u>Variables</u> means dependent and independent variables.

<u>e.g.</u> $e^x dx + e^y dy = 0$, $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} - \frac{d^2y}{dx^2} = c$

$y = x\frac{dy}{dx} + \frac{x}{\frac{dy}{dx}}$, $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

These are differential Equations (D. E.) and

$\frac{dy}{dx}$ means y is dependent variable and x is independent variable.

The <u>Order</u> of a D.E. is the order of the highest derivative. That is, how many times it is differentiated.

e.g. $e^x dx + e^y dy = 0$ It is one-time differentiated so Order is 1.

$\frac{d^2x}{dt^2} + n^2 x = 0$ It is 2-times differentiated So Order is 2.

$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} - \frac{d^2y}{dx^2} = c$ It is 2-times differentiated So order is 2.

The <u>degree</u> of a D.E. is the <u>power</u> (or <u>Exponent</u>) of the highest derivatives provided the equation must be free from <u>radicals</u> and <u>fractions</u> of derivatives.

e.g.

1. $\frac{d^2x}{dt^2} + \frac{dx}{dt} = 0$ It has degree 1.

2. $\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = 0$ It has degree 1 because highest derivative is $\frac{d^2x}{dt^2}$ and its power is 1.

3. $y = x\frac{dy}{dx} + \frac{x}{dy/dx}$ Here, dy/dx is in denominator so first remove it

$\Rightarrow y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + x$ Now, no fraction. So, degree is 2.

4. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ $\Rightarrow$ degree is 2. (How?)

An Ordinary differential equation (ODE) is an D.E. which consists of derivatives of one dependent variable with respect to one independent variable.

e.g. $\frac{dy}{dx} = \cos x$

$\frac{dy}{dx}\frac{d^2y}{dx^2} = 3\left(\frac{dy}{dx}\right)^2$

A partial differential equation (PDE) is an D.E. which have partial derivatives (∂) of one or more dependent variables w.r.t. 2 or more independent variables.

e.g. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$

$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$

A solution (or Integral) of a D.E. is a relation between the variables (Independent and dependent variables) which satisfies the given D.E.

e.g. $x = A\cos(nt+\alpha)$ is a solution of $\frac{d^2x}{dt^2} + n^2x = 0$

The general (or complete) solution of a D.E. is that in which the number of arbitrary constants is equal to the order of D.E.

The above example, $x = A\cos(nt+\alpha)$ is general solution of $\frac{d^2x}{dt^2} + n^2x = 0$ because x has two constants A and α as order of $\frac{d^2x}{dt^2} + n^2x = 0$ is 2.

The particular solution is what can be obtained from the general solution by giving particular values to the arbitrary constants.

e.g. $x = 2\cos(nt + \pi/4)$ is the particular solution of

$$\frac{d^2x}{dt^2} + n^2x = 0.$$