

Carrier concentration and Fermi Level of intrinsic and extrinsic semiconductor

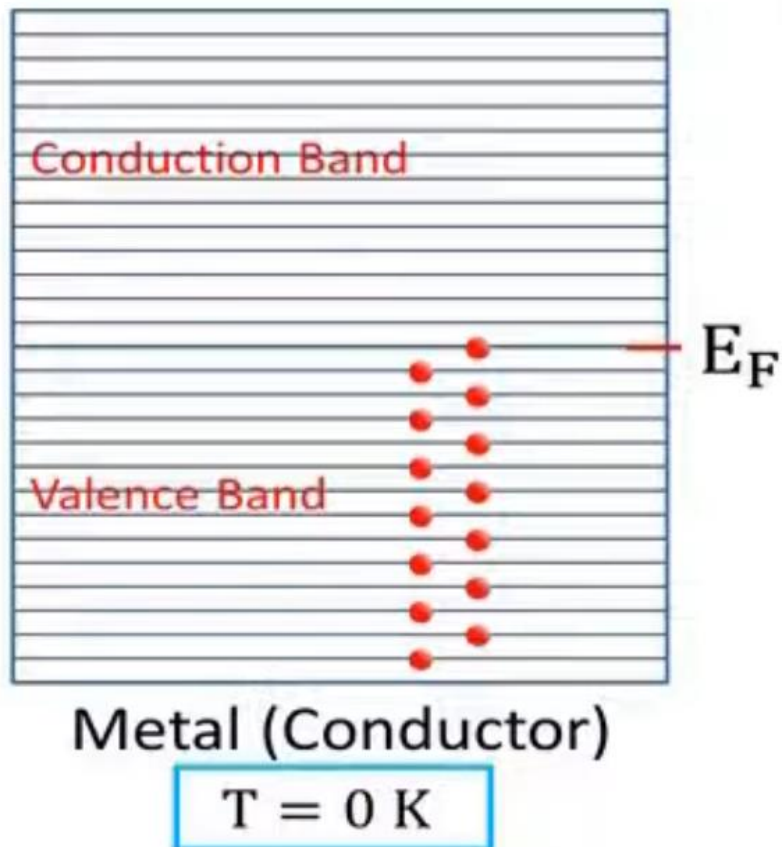
Direct and Indirect band gap Semiconductors

After the completion of this lecture you will be able to:

- **Understand Fermi Energy**
- **Understand the Fermi Dirac distribution function and its variation with temperature**
- **Density of states**
- **Carrier concentration in intrinsic semiconductor**
- **Carrier concentration in extrinsic semiconductor**

The Fermi Level and Fermi Energy

Fermi Level



- In metal, there is one partially filled band which is a result of conduction band overlapping with valence band.
- In this band lowest energy levels are filled first.
- The highest occupied energy level at absolute zero temperature (0K) is called as Fermi level.
- Energy corresponding to it is called as **Fermi energy denoted by E_F**

The Fermi-Dirac Distribution: The Fermi-Dirac

distribution applies to Fermions

- particles with half-integer spin
- obey the Pauli Exclusion Principle.

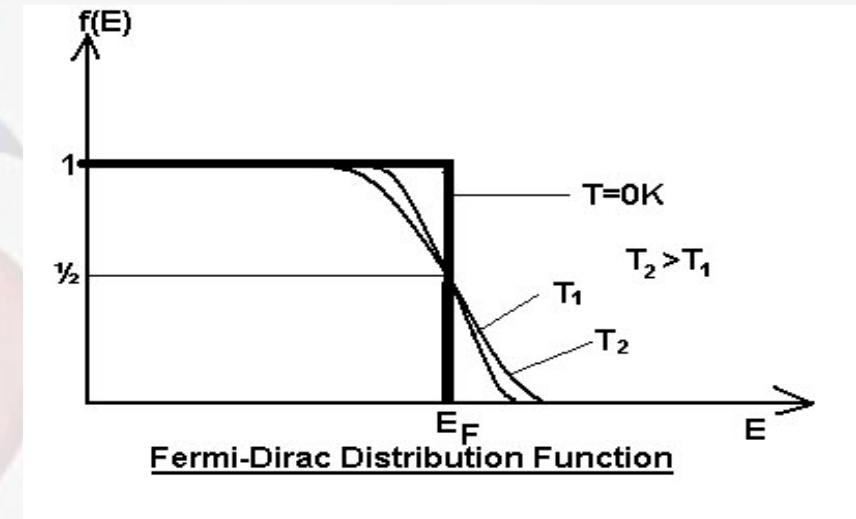
The Fermi-Dirac Distribution $f(E) = \frac{1}{1 + \exp[(E - E_F)/k_B T]}$

$f(E)$ the probability for the occupation of a particular energy level E by an electron then

where k_B is Boltzmann's constant, T is the absolute temperature, E is the energy of the particular energy level E , and E_F is the Fermi energy, the energy of the highest filled level at absolute zero.

At $T = 0K$,

For $E < E_F$, $f(E) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + 0} = 1$ **and for $E > E_F$,** $f(E) = \frac{1}{1 + e^{\infty}} = \frac{1}{1 + \infty} = 0$



Course Code : BBS01T10

1. The allowed energy for 1D potential box, $E_n = \frac{h^2 n^2}{8mL^2}$

where m is mass of particle, L is the length of potential box and n are positive integers like 1, 2, 3, 4, 5....

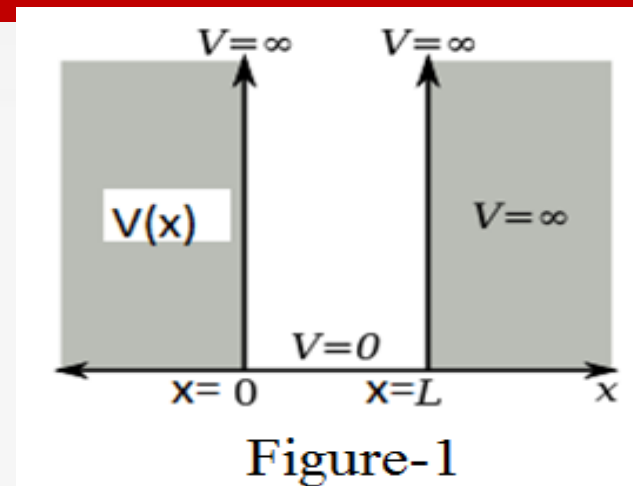
The allowed energy for 3D cubical potential box

$$E = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2) \quad \text{-----(1)}$$

where n_x , n_y and n_z are three quantum numbers which are only positive integer value.

where $n^2 = n_x^2 + n_y^2 + n_z^2 \quad \text{-----(2)}$

$$E = \frac{h^2}{8mL^2} n^2$$



The potential energy within the 1D crystal or box is

$$V(x) = 0 \quad \text{for } 0 < x < L$$

$$V(x) = \infty \quad \text{for } x \leq 0 \text{ and } x \geq L$$

- Density of energy states is defined by the number of allowed energy states present in unit volume at a given energy.
- Since even at highest energy, the difference between neighbouring energy levels is as small as 10^{-6} eV, in a macroscopically small energy interval dE there are still many discrete energy levels. So the concept of density of energy states is introduced.
- The Fermi energy, E_F is the energy of the highest filled level at absolute zero.

Definition: The number of energy states available per unit volume in unit energy range

Let, $Z(E)dE$ represents density of states,
i.e., the density of states between energy E and $E + dE$

$$Z(E)dE = \frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

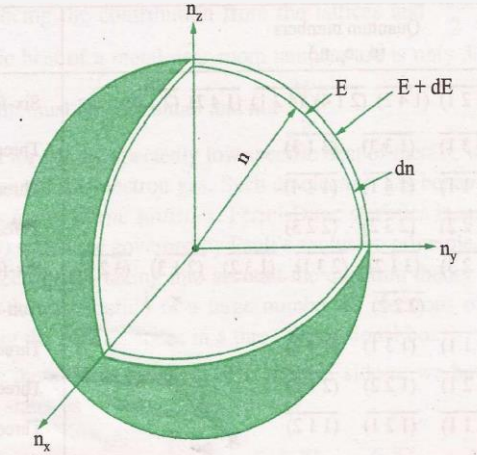
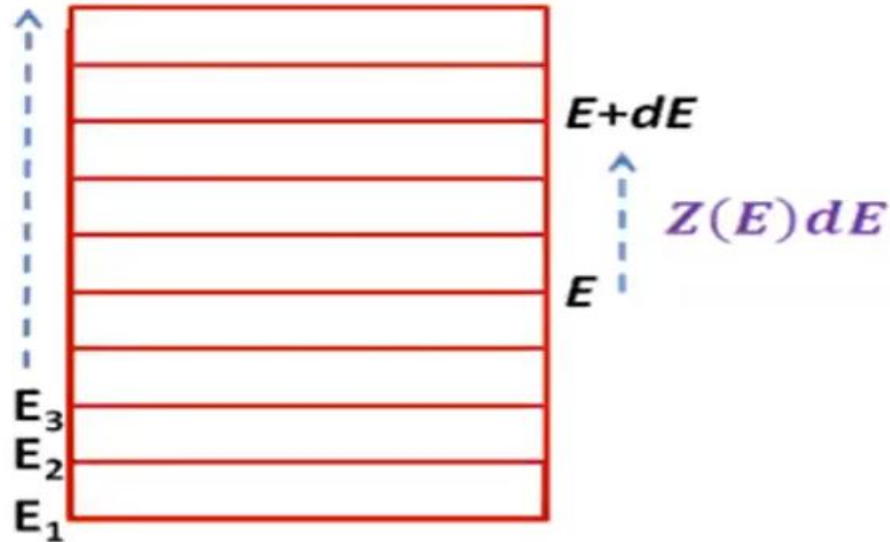


Fig. 1 Density of energy states calculation

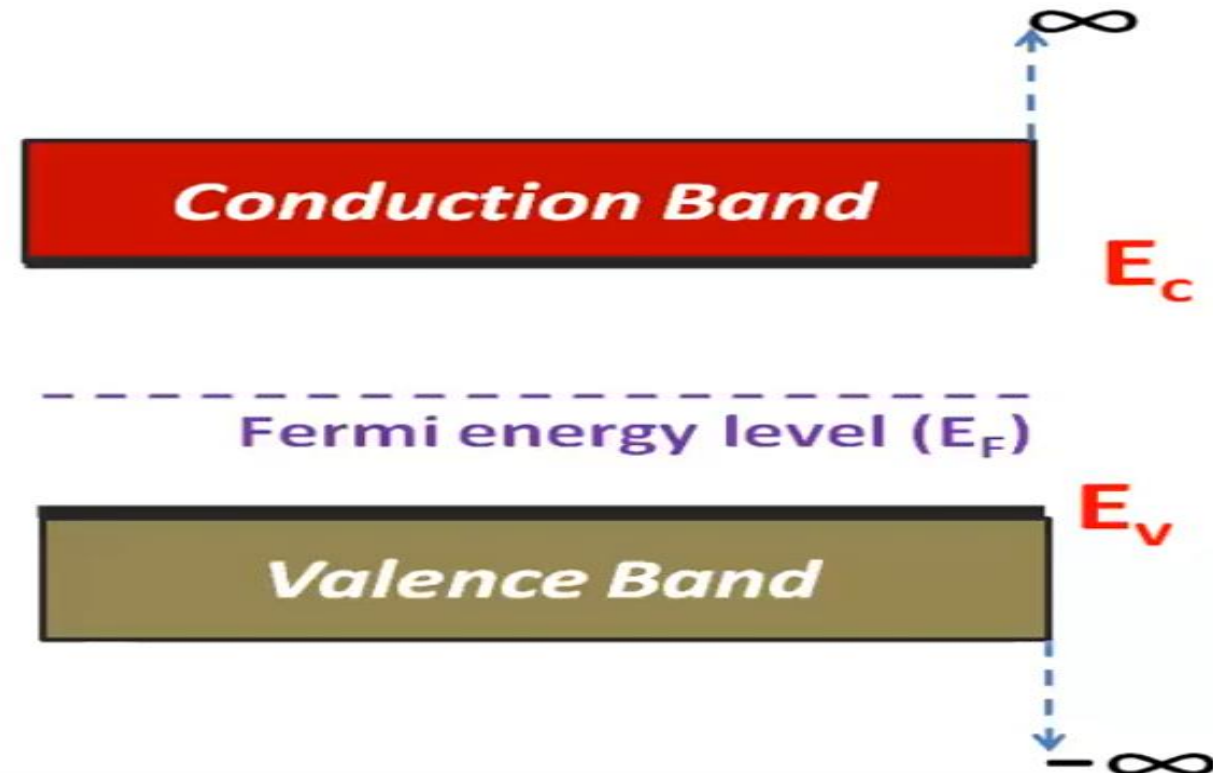
To understand the method of calculating the density of electrons (electron concentration)

$$\therefore \text{Density of electrons } (n) = \int F(E) \times Z(E) dE$$

$$n = \int \left(\frac{1}{\exp\left(\frac{E-E_f}{KT}\right) + 1} \right) \times \left(\frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E^{\frac{1}{2}} dE \right)$$

1) Density of electrons in conduction band of intrinsic semiconductor

$$n = \int_{E_c}^{\infty} \left(\frac{1}{\exp\left(\frac{E-E_f}{KT}\right) + 1} \right) \times \left(\frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E^{\frac{1}{2}} dE \right)$$



Density of electrons in conduction band of an intrinsic semiconductor

Let, dn be the number of electrons available between the energy interval E and $E+dE$ in the conduction band. Then, dn is defined as,

$$dn = (\text{Probability of occupancy}) \times (\text{Density of states})$$

$$dn = F(E) \times Z(E) dE$$

The total number of electrons in the conduction band is given by

$$n = \int_{E_c}^{\infty} dn = \int_{E_c}^{\infty} F(E)Z(E)dE \text{ -----1}$$



$$Z(E) dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E)^{1/2} dE$$

Where E starts from the bottom of the conduction band (E_c), hence E is replaced with $(E - E_c)$.

Therefore,

$$Z(E) dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_c)^{1/2} dE \text{ -----2}$$

The probability of an electron occupying the energy state E is given by,

$$F(E) = \frac{1}{\exp\left(\frac{E-E_f}{KT}\right) + 1}$$

For, all possible temperatures, $(E - E_f) \gg KT$

$$\underline{F(E)} = \exp^{-\left(\frac{E-E_f}{KT}\right)} = \exp\left(\frac{E_f-E}{KT}\right) \text{ -----3}$$

Substitute 2 and 3 in 1

$$n = \int_{E_c}^{\infty} dn = \int_{E_c}^{\infty} \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_c)^{1/2} \exp\left(\frac{E_f - E}{KT}\right) dE$$

$$n = \int_{E_c}^{\infty} dn = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left(\frac{E_f - E}{KT}\right) dE$$

$$n = \int_{E_c}^{\infty} dn = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{E_f}{KT}\right) \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left(\frac{-E}{KT}\right) dE$$

$$\text{Let, } (E - E_c)/KT = x$$

$$\text{Then, } E \rightarrow E_c \Rightarrow x \rightarrow 0$$

$$\& \quad E \rightarrow \infty \Rightarrow x \rightarrow \infty$$

$$dE = KT dx$$

Density of electron in conduction band of an intrinsic semiconductor

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{E_f}{KT}\right) \int_0^\infty (KTx)^{1/2} \exp\left(\frac{-E_c}{KT} - x\right) KT dx$$

$$n = \int_{E_c}^\infty dn = \frac{4\pi}{h^3} (2m_e^* KT)^{3/2} \exp\left(\frac{E_f - E_c}{KT}\right) \int_0^\infty x^{1/2} \exp(-x) dx$$

From Gamma function, it can be written as, $\int_0^\infty x^{1/2} \exp(-x) dx = \frac{\sqrt{\pi}}{2}$

$$n = \int_{E_c}^\infty dn = \frac{4\pi}{h^3} (2m_e^* KT)^{3/2} \exp\left(\frac{E_f - E_c}{KT}\right) \frac{\sqrt{\pi}}{2}$$

$$n = \int_{E_c}^\infty dn = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \exp\left(\frac{E_f - E_c}{KT}\right)$$

Density of electrons in conduction band and holes in valance band of an intrinsic semiconductor

$$n = \int_{E_c}^{\infty} dn = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \exp\left(\frac{E_f - E_c}{KT}\right)$$

$$p = \int_{-\infty}^{E_v} dp = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \exp\left(\frac{E_v - E_f}{KT}\right)$$

For Intrinsic carrier concentration:

Therefore, from case-1 and case-2,

The intrinsic carrier concentration is given by, $n = p = n_i$ and

$$n_i^2 = np = 4 \left(\frac{2\pi kT}{h^2} \right)^3 (m_e^* m_h^*)^{\frac{3}{2}} \exp\left(\frac{E_v - E_c}{KT}\right) = 4 \left(\frac{2\pi kT}{h^2} \right)^3 (m_e^* m_h^*)^{\frac{3}{2}} \exp\left(\frac{-E_g}{KT}\right)$$

$$n_i = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* m_h^*)^{\frac{3}{4}} \exp\left(\frac{-E_g}{2KT}\right)$$

For an intrinsic semiconductor $n = p = n_i$

Equating (1) and (2), we get $2 \left[\frac{2 \pi m_e^* kT}{h^2} \right]^{3/2} e^{\frac{(E_F - E_C)}{kT}} = 2 \left[\frac{2 \pi m_h^* kT}{h^2} \right]^{3/2} e^{\frac{(E_V - E_F)}{kT}}$

$$\frac{e^{\frac{(E_F - E_C)}{kT}}}{e^{\frac{(E_V - E_F)}{kT}}} = \left[\frac{m_h^*}{m_e^*} \right]^{3/2} ; \quad e^{\frac{2E_F - E_C - E_V}{kT}} = \left[\frac{m_h^*}{m_e^*} \right]^{3/2}$$

Taking log on both the sides, we get

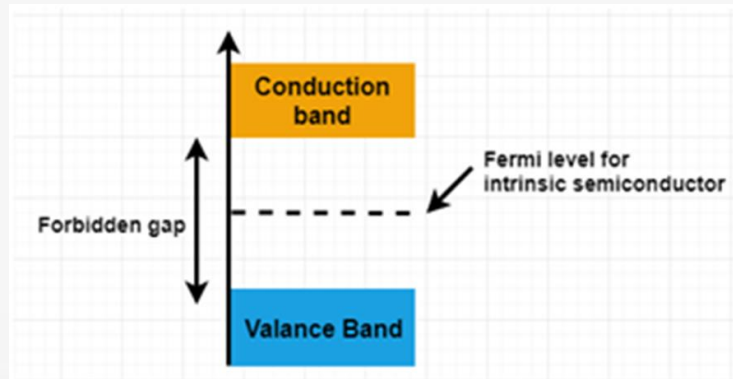
$$\frac{2E_F - E_C - E_V}{kT} = \frac{3}{2} \log_e \frac{m_h^*}{m_e^*} ; \quad E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \log_e \left(\frac{m_h^*}{m_e^*} \right)$$

Fermi level in Intrinsic Semiconductor

Strictly speaking the effective mass of electron m_e^* is different than that of a hole m_h^* but to a good approximation m_e^* and m_h^* may be assumed to be equal and so eqn. reduces to

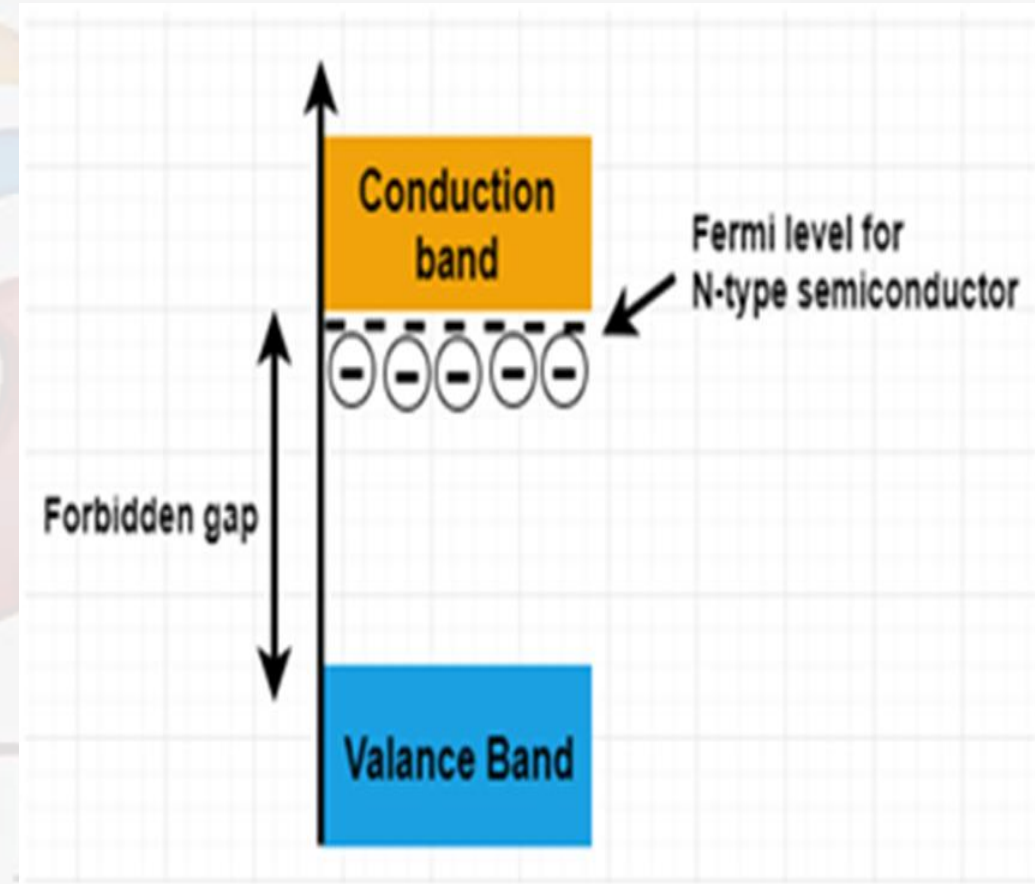
$$E_F = \frac{E_C + E_V}{2}$$

This shows that in an intrinsic semi-conductor, the Fermi-level essentially lies at the centre of the forbidden energy gap E_g .



n-type semiconductor:

At 0 K all allowed energy levels in the valence band are filled by electrons. All donor levels are filled by unbound electrons. The conduction band is free. So charge carriers do not exist, and the semiconductor behaves as an insulator. At 0 K the Fermi level is between the donor levels and the bottom of the conduction band.



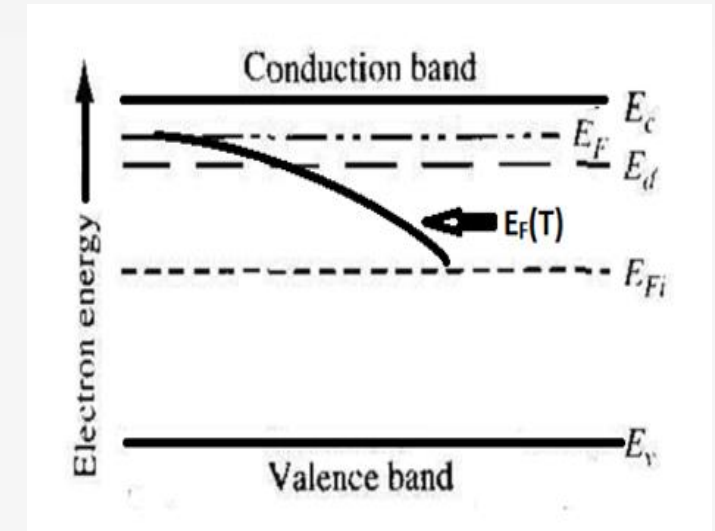
Fermi level in extrinsic Semiconductor (Temp. effect)

When the temperature is raised, by absorption of suitable energies, some donor atoms are ionised, so that their electrons are elevated to the conduction band. If the density of ionised donor atom is N_d and the density of electron-hole pairs in the intrinsic semi-conductors is n_i at room temperature T , then $N_d \gg n_i$.

The Fermi energy is :

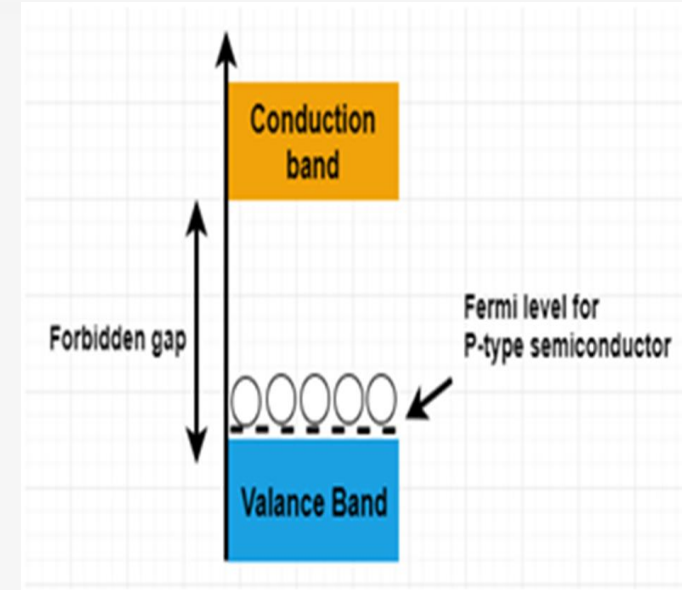
$$E_F = E_C - kT \log_e \left(\frac{N_C}{N_d} \right) \text{ where } N_C = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2}$$

This shows that in an N-type semiconductor the Fermi-level lies below the bottom of the conduction band. As temperature rises, the Fermi level goes on falling below E_C . When the temperature is sufficiently raised, the electrons and holes generated due to thermal agitation increase significantly and at a stage become fully dominant over the extrinsic carriers. Then the Fermi level approaches the middle of forbidden energy gap.



P-type Semiconductor:

The energy band diagram of a P-type semiconductor is shown in the Figure. The acceptor level is shown by E_a near the top of the valence band, the Fermi level by E_F . At absolute zero, all the holes are in acceptor levels, but as the temperature rises, the electrons from valence band jump into acceptor level on the absorption of energy ($E_A - E_V$) by each electron. As a result, these electrons are trapped in the acceptor levels and an equal number of holes are created in the valence band. These holes provide conduction currents. At the room temperature, almost all acceptor atoms trap electrons and thus the number of holes available in the valence band is almost equal to the number of impurity atoms added.

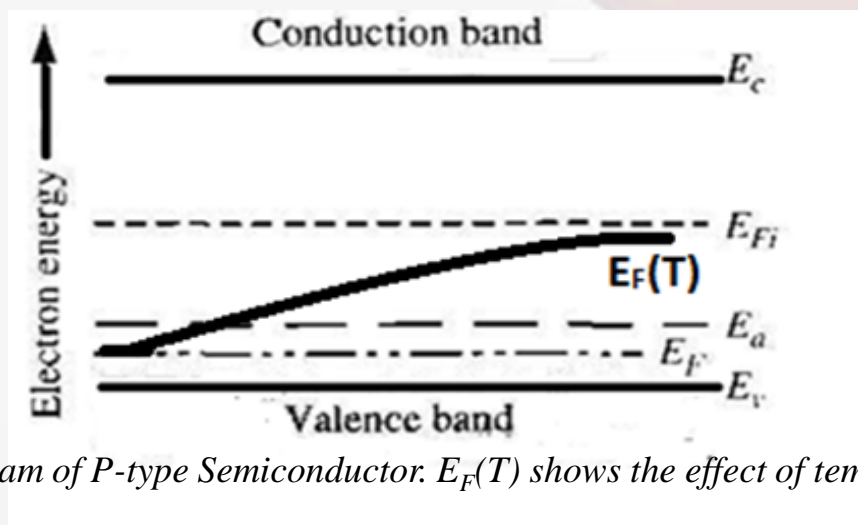


The variation of Fermi energy with temperature is given by:

$$E_F = E_V + kT \log_e \left(\frac{N_V}{N_a} \right) \text{ where } N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

Fermi level in extrinsic Semiconductor (Temp. Effect)

This shows that Fermi level lies above the top of the valence band. The position of Fermi level depends upon the temperature and the number of impurity atoms. When the number of impurity atoms increases, the number of holes in the valence band will increase and the Fermi level will shift towards the valence band. When the concentration of acceptor impurity atoms is sufficiently high, the number of holes will be far greater than the conduction electrons and the Fermi level may even lie in the valence band. As the temperature is sufficiently increased, electrons from the valence band are excited to the conduction band and finally the P-type crystal will start behaving like an intrinsic semi-conductor when the number of electrons in the conduction band will be nearly equal to the valence holes. Thus at extremely high temperatures the Fermi level shifts towards the middle of forbidden energy gap.



Energy diagram of P-type Semiconductor. $E_F(T)$ shows the effect of temperature on Fermi energy.

Remarks:

- (i) The doped semi-conductor (N-type or P-type) as a whole remains electrically neutral.
- (ii) In N-type semi-conductor the free electrons are majority charge carriers and holes are minority charge carriers. Therefore current flow in N-type semi-conductor is mainly due to free electrons
- (iii) In P-type semi-conductor the holes are majority charge carriers and free electrons are minority charge carriers. Therefore current flow in P-type semi-conductor is mainly due to holes.
- (iv) A doped semi-conductor acts like resistor. The resistance of doped semi-conductor is called bulk resistance. Greater is doping less is bulk resistance.

The logo of Galgotias University is a circular emblem with three concentric, curved bands in shades of pink, yellow, and light blue, creating a stylized 'G' or a spiral effect.

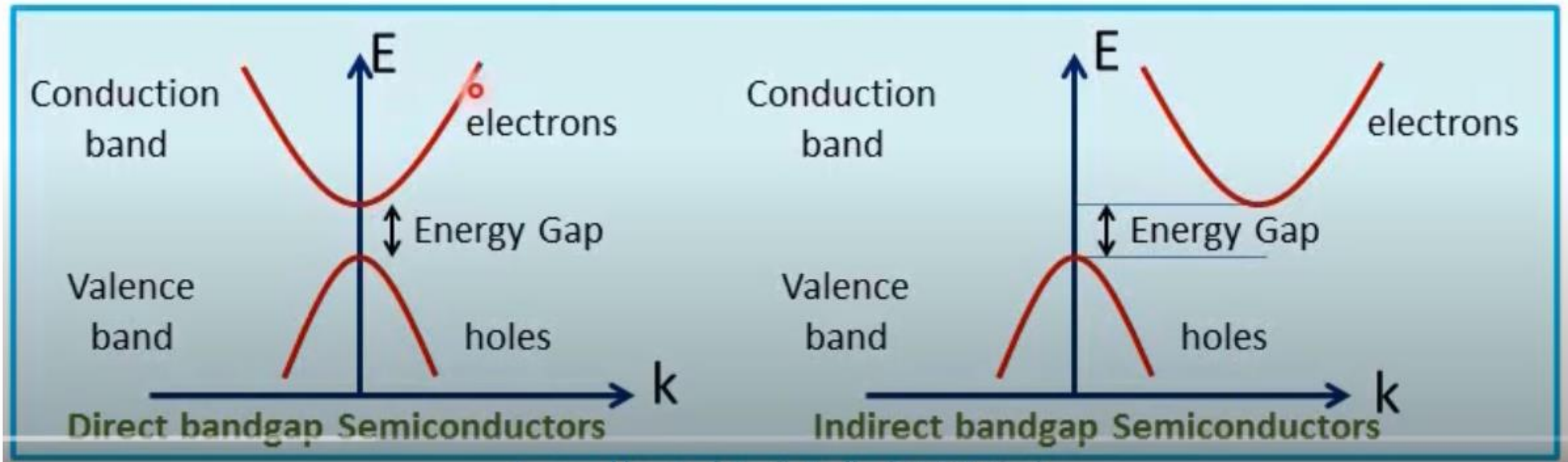
Direct and Indirect band gap Semiconductors

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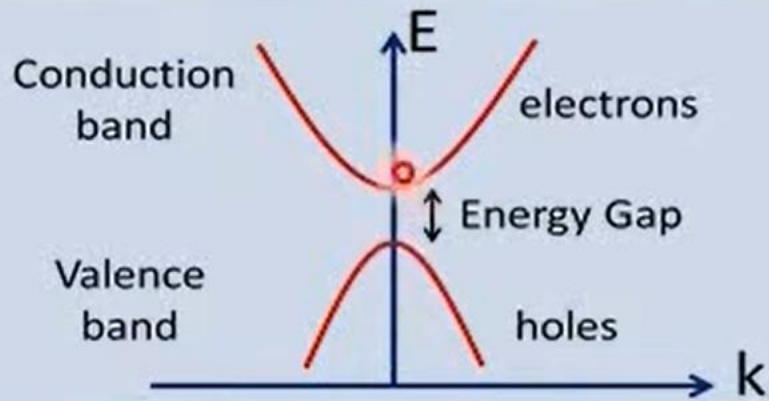
Direct Gap and Indirect Gap Semiconductors

The energy of an electron is given by $E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$ where p is momentum, m is mass of an electron, \hbar is Planck's constant and k is propagation constant

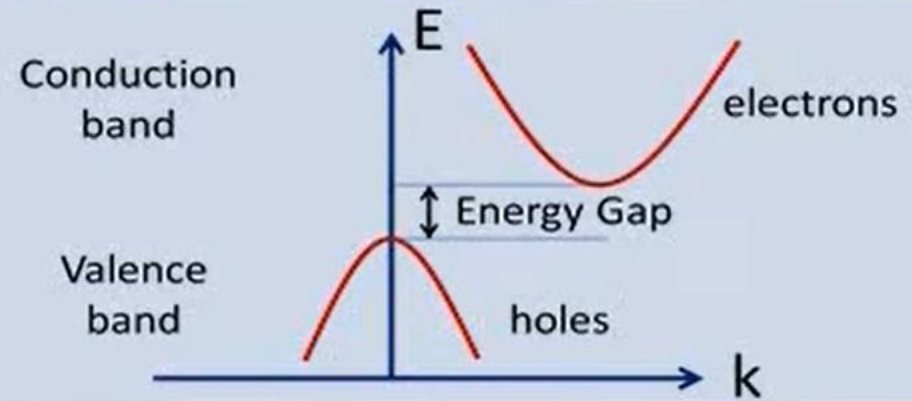
Thus $E \propto k^2$ which is an equation of **parabola**. The graph of E vs k is shown below -



Direct bandgap Semiconductors



Indirect bandgap Semiconductors



Maximum of valence band and minimum of conduction band occur at same momentum values

Electron making a transition from valence band to conduction band need not undergo any change in its momentum.

The compound semiconductors such as GaAs, are direct gap semiconductors

These direct gap semiconductors are used in LED and Semiconductor Lasers.

Maximum of valence band and minimum of conduction band occur at two different momentum values.

In order to make a transition from maximum point in valence band to minimum point in conduction band, the electron requires energy for the change in momentum in addition to the energy gap E_g

All elemental semiconductors such as Si, Ge, are indirect gap semiconductors

Not useful for LEDs and Semiconductor Lasers

1. Define Fermi Energy and its expression.
1. Discuss Fermi Dirac distribution function.
2. Illustrate density of energy states.
3. Explain the Fermi level for intrinsic and extrinsic semiconductors with suitable diagrams..
4. Explain with suitable diagram direct and indirect band gap semiconductors and their examples.

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