# Principle of Mathematical Induction-

Let P be proposition defined on the integer $n \geq 1$ such ~~tha~~ that -
(i) P(1) is true
(ii) P(n+1) is true whenever P(n) is true.

Then P is true for every integer $n \geq 1$

Q Let P be the proposition that the sum of the first n odd number is $n^2$.

i.e;

$$1 + 3 + 5 + 7 + ----+(2n-1) = n^2$$

by mathematical induction-

$$P(1) = 1^2 = 1$$

Suppose P(n) is true Adding eqn (1) (2n+1) to both side of P(n), we get -

$$1 + 3 + 5 + ------- + (2n-1) + (2n+1) = n^2 + (2n+1)$$

$$\text{So } P(n+1) = (n+1)^2$$

We have shown that P(n+1) is true whenever P(n) is true. By PMI P is true for all n.

Q Proof the proposition that sum of the first n (+)ve integer is $\frac{n(n+1)}{2}$.

$$1+2+3+ \cdots + n = \frac{n(n+1)}{2}$$

$$P(1) = 1 \times \frac{2}{2} = 1$$

$$1+2+3+ \cdots + n + (n+1) = \frac{n(n+1)}{2} + (n+1)$$

$$P(n+1) = \frac{(n+1)(n+2)}{2}$$

We have shown that $P(n+1)$ is true whenever $p(n)$ is true.

Q Proof the proposition P that sum of the square of first n (+)ve integers is $\frac{n(n+1)(2n+1)}{6}$

$$P(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$P(1) = 1 \times \frac{6}{6} = 1$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$P(n+1)$$

$$P(n+1) = (n+1)\left[\frac{2n^2+n+6n+6}{6}\right]$$

$$P(n+1) = (n+1)\left(\frac{2n^2+7n+6}{6}\right)$$

$$= (n+1)\left(\frac{2n^2+4n+3n+6}{6}\right)$$

$$P(n+1) = \frac{(n+1)(n+2)(2n+3)}{6}$$

We have shown that $P(n+1)$ is true whenever $P(n)$ is true.

**Second form of MI** (Mathematical Induction)

Let $P$ be a proposition which is defined on the integer $n \geq 1$ such that -

(1) $P(1)$ is true.
(2) $P(n)$ is true whenever $P(K)$ is true for all $1 \leq K \leq n$

Then $P$ is true for every integer $n \geq 1$

Q Suppose $a \neq 1$ Let $P$ is the proposition on $n \geq 1$ defined by

$$P(n) = 1 + a + a^2 + \text{-----} + a^n = \frac{a^{n+1}-1}{a-1}$$

P(1) is true since

$$1+a = \frac{a^2-1}{a-1}$$

$$1+a+a^2+ \cdots +a^n+a^{n+1} = \frac{a^{n+1}-1}{a-1}+a^{n+1}$$

$$P(n+1) = \frac{a^{n+2}-1}{a-1}$$

Which is true for P(n+1)

## Well Ordering Principle-

**Theorem-** Let S be a non-empty set of positive integer
Then S ~~contin~~ contains a least element that is, S contain an element a such that $a \leq s$ for every s in S.

**In other words-**

An ordered set S is said to be well ordered if every subset of S contains a first element.