L6: Function

Let A and B be nonempty sets. A function f from A to B is an relation in which every element of A is paired with only one element of B.

$A = \{1,2,3\}$, $B = \{4,5,6\}$,

$f = \{(1,4),(2,4),(3,4)\}$ — function | Notation: $f(1)=4$

$R_1 = \{(1,4),(1,5),(2,4),(3,6)\}$ it is not a funct

$R_2 = \{(1,5),(2,6)\}$ – not function

If f is a function from A to B, then A is the domain of f and B is the codomain of f. If $f(a)=b$, then b is the image of a and a is preimage of b. The set of all images of element of A under f is called range of f.

A function f is said to be one-to-one (injunction) iff $f(a)=f(b) \Rightarrow a=b$. i.e. when $a \neq b$ then $f(a) \neq f(b)$.

Ex: $f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = x^2$ not 1-1. because $1 \neq -1$ but $f(1)=1$, $f(-1)=1$

$f: \mathbb{Z}^+ \to \mathbb{Z}^+$

$f(x) = x^2$ is one-one

$f: \mathbb{R} \to \mathbb{R}$; $f(x) = x+1$ is one-one

A function f from A to B is called onto (surjection) iff for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

Ex. $f : \mathbb{Z} \to \mathbb{Z}$

$f(x) = x^2$ not onto $f(x) = -1$ no such x exist

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = x+1$ onto $y \in \mathbb{R}$, $f(y-1) = y-1+1$
$= y$

A function f is a one-to-one correspondence (bijection) if it is both one-to-one and onto.

e.g. $f : \{a, b, c, d\} \to \{1, 2, 3, 4\}$

$f(a) = 1$, $f(b) = 3$, $f(d) = 2$, $f(c) = 4$

When a function $f : A \to B$ is bijective then $|A| = |B|$.

Let f be a one-to-one correspondence from the set A to the set B. The inverse function of f is the function that assigns to an element $b \in B$ the unique element a in A such that $f(a) = b$.

Ex. $f : \{a, b, c\} \to \{1, 2, 3\}$: $f(a) = 2$, $f(b) = 3$, $f(c) = 1$
Here, f is one-one and onto. So, function is invertible
$f^{-1}(2) = a$, $f^{-1}(3) = b$, $f^{-1}(1) = c$.

Ex. $f : \mathbb{Z} \to \mathbb{Z}$ $f(x) = x+1$. This f is one-one & onto.
So, f is invertible. $f^{-1}(y) = y-1$.

› Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The composition of the functions f and g, denoted by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))\ \forall a \in A$.

Ex: $f : \mathbb{Z} \to \mathbb{Z}$, $g : \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2x+3$ $\qquad g(x) = 3x+2$

$$(f \circ g)(x) = f(g(x)) = f(3x+2) = 2(3x+2)+3$$
$$= 6x+7$$
$$(g \circ f)(x) = g(f(x)) = g(2x+3) = 3(2x+3)+2$$
$$= 6x+11$$

When $f \circ g = g \circ f = I$ then f and g are inverse functions of each other.

→ Let f_1 and f_2 be functions from A to B. Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to B defined by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x).$$
$$(f_1 f_2)(x) = f_1(x)\, f_2(x)$$