Recurrence Relation :

A recurrence relation for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, \ldots, a_{n-1}$, for all integers n with $n \geq n_0$, where n_0 is a nonnegative integer.

A sequence is called a solution of a recurrence relation if its terms satisfy the recurrence relation.

Ex: Determine whether the sequence $\{a_n\}$, where $a_n = 3n$ for every nonnegative integer n, is a solution of the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, \ldots$.
Answer the same question where $a_n = 2^n$ and where $a_n = 5$.

Soln:- Suppose that $a_n = 3n$ for every nonnegative integer n.

For $n \geq 2$, $2a_{n-1} - a_{n-2} = 2[3(n-1)] - 3(n-2)$

$= 6n - 6 - 3n + 6 = 3n = a_n$.

Therefore, $\{a_n\}$, where $a_n = 3n$, is a solution of the RR.

Suppose that $a_n = 2^n$ for $\forall n \in Z^+ \cup \{0\}$.

$2a_{n-1} - a_{n-2} = 2(2^{n-1}) - 2^{n-2} = 2^n - 2^{n-2} \neq 2^n$.

or $a_0 = 2^0 = 1$, $a_1 = 2$, $a_2 = 4$ and $a_2 = 2a_1 - a_0 \not\Rightarrow 4 = 2(2) - 1$

$\therefore \{2^n\}$ is not a solution of the RR.

Suppose that $a_n = 5$ for $\forall n \in Z^+ \cup \{0\}$. Then for $n \geq 2$

$a_n = 2a_{n-1} - a_{n-2} = 2(5) - 5 = 5 = a_n$.

$\therefore \{a_n\, 5\}$ is a solution of the RR.

inear Recurrence Relation :

A linear homogeneous recurrence relation of degree k with constant coefficients is a recurrence relation of the form

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$, where $c_1, c_2, \dots, c_k$ are real numbers, and $c_k \neq 0$.

Ex: $P_n = (1.11) P_{n-1}$ is a linear homogeneous RR of degree 1.

$f_n = f_{n-1} + f_{n-2}$ " " 2.

$a_n = a_{n-5}$ " " 5.

$a_n = a_{n-1} + a_{n-2}^2$ is not linear

$H_n = 2H_{n-1} + 1$ is not homogeneous

$B_n = nB_{n-1}$ does not have constant coefficients.