Tautology:- A compound propn that is always true, no matter what the truth values of the propositions that occur in it, is called a tautology.

e.g. $(p \vee \neg p)$

| $p$ | $\neg p$ | $p \vee \neg p$ |
|---|---|---|
| T | F | T |
| F | T | T |

Contradiction:- A compound propn that is always false is called a contradiction.

e.g. $p \wedge \neg p$

| $p$ | $\neg p$ | $p \wedge \neg p$ |
|---|---|---|
| T | F | F |
| F | T | F |

Contingency:- is a compound propn that is neither a tautology nor a contradiction.

Logical Equivalence:- The compound propositions $p$ & $q$ are called logically equivalent if $p \leftrightarrow q$ is a tautology. (i.e. if they have the same truth table).

The notation $p \equiv q$ denotes that $p$ & $q$ are logically equivalent.

Q. Show that each of these conditional statements is a tautology by using truth tables

(1.) $(p \wedge q) \rightarrow p$

| $p$ | $q$ | $p \wedge q$ | $(p \wedge q) \rightarrow p$ |
|---|---|---|---|
| T | F | F | T |
| T | T | T | T |
| F | T | F | T |
| F | F | F | T |

(2.) $\neg p \rightarrow (p \rightarrow q)$

| $p$ | $q$ | $\neg p$ | $p \rightarrow q$ | $(\neg p) \rightarrow (p \rightarrow q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

(3.) $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

| $p$ | $q$ | $r$ | $p \rightarrow q$ | $q \rightarrow r$ | $(p \rightarrow q) \wedge (q \rightarrow r)$ | $p \rightarrow r$ | Ans |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| F | T | T | T | T | T | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| F | F | F | T | T | T | T | T |

Q. Show that $\neg(p \vee q)$ & $\neg p \wedge \neg q$ are logically equivalent.

Soln:-

| p | q | $p \vee q$ | $\neg(p \vee q)$ | $\neg p$ | $\neg q$ | $\neg p \wedge \neg q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

$\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$ is a tautology Hence, these compound propn are logically equivalent.

Q. Show that $p \vee (q \wedge r)$ & $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

Soln:-

| p | q | r | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | F | T | F | F | F | T | F |
| F | T | F | F | F | T | F | F |
| T | F | F | F | T | T | T | T |
| F | F | F | F | F | F | F | F |