Partial Order Relation

A relation R on a set A is called a partial order relation, if it is reflexive, anti-symmetric & transitive.

In this case, (A, R) is called a partially ordered set or poset.

partial order rel^n

e.g. $A = \{1, 2, 3, 4, 5\}$, $\leq$

(1.) Reflexive :- (aRa for all $a \in A$)

As $a \leq a$ for all $a \in A$.

$\therefore$ $\leq$ is reflexive.

(2) Anti-Symmetric :-

[if aRb and bRa then $a = b$ for all $a, b \in A$]

($R = \leq$)

Since $a \leq b$ and $b \leq a \Rightarrow a = b$ for all $a, b \in A$

$\Rightarrow \leq$ is anti-symmetric

(3.) Transitive:-

As $a \leq b$ and $b \leq c$
$\Rightarrow a \leq c$ for all $a, b, c \in A$

Thus, $\leq$ is transitive

Hence, $(A, \leq)$ is a poset

Ex-3:- $R = \geq$ defined on the set of ~~the~~ integers.

(1) Reflexive:- $a \geq a$ for all ~~the~~ integers a.

$\Rightarrow$ '$\geq$' is reflexive

(2) Anti-Symmetric:-
$a \geq b$ & $b \geq a$ for
$\Rightarrow a = b$ for all ~~the~~ integers a & b.

(3) Transitive:- $a \geq b$ & $b \geq c$
$\Rightarrow a \geq c$ for all ~~the~~ int.. a, b, c

$\Rightarrow \geq$ is a partial ordering on the set of integers

Hence, $(Z, \geq)$ is a poset.

Ex:- $\subseteq \rightarrow$ inclusion relation defined on the power set of a set S.

$R = \{(A, B) \mid A \subseteq B\}$

Sol:- ~~(A)~~ ~~R~~

(i) Reflexive:- [$(A, A) \in R$ or not for all $A \in P(S)$]

$\therefore A \subseteq A$ whenever $A \in P(S)$

$\Rightarrow \subseteq$ is reflexive

(2) Anti-Symmetric:-

If $(A, B) \in R$ & $(B, A) \in R$

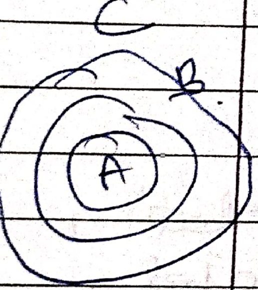
$\Rightarrow A \subseteq B$ & $B \subseteq A$

$\Rightarrow A = B$

Thus, $\subseteq$ is anti-symmetric

(3) Transitive:-

If $(A, B) \in R$ & $(B, C) \in R$



$\Rightarrow A \subseteq B$ & $B \subseteq C$

$\Rightarrow A \subseteq C$

$\Rightarrow \subseteq$ is transitive

Hence, $\subseteq$ is partial order rel^n & $(P(S), \subseteq)$ is a poset.