Rules of Inference for Quantified Statements

Rules of Inference	Name
$\dfrac{\forall x\, P(x)}{\therefore P(c)}$	Universal Instantiation
$\dfrac{P(c) \text{ for any arbitrary } c}{\therefore \forall x\, P(x)}$	Universal generalization
$\dfrac{\exists x\, P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore \exists x\, P(x)}$	Existential generalization

Ex. Show that the premises, "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."

Soln: Let $C(x)$: x is in this class, $B(x)$: x has read the book and $P(x)$: x passed the first exam.

The premises are $\exists x\,(C(x) \wedge \neg B(x))$ and $\forall x\,(C(x) \rightarrow P(x))$.

The conclusion is $\exists x\,(P(x) \wedge \neg B(x))$.

	Step	Reason
1.	$\exists x\,(C(x) \wedge \neg B(x))$	Premise
2.	$C(a) \wedge \neg B(a)$	Existential instantiation from (1)
3.	$C(a)$	Simplification from (2)
4.	$\forall x\,(C(x) \rightarrow P(x))$	Premise
5.	$C(a) \rightarrow P(a)$	Universal instantiation from (4)
6.	$P(a)$	Modus Ponens from (3) and (5)
7.	$\neg B(a)$	Simplification from (2)
8.	$P(a) \wedge \neg B(a)$	Conjunction from (6) and (7)
9.	$\exists x\,(P(x) \wedge \neg B(x))$	Existential generalisation from (8)