UNIT I

Number Systems & Boolean Algebra

Binary weighted & non-weighted codes & code conversion

CODES

Codes are the representation of information in a particular format. The information may include numbers, alphabets and symbols, which man and machine can recognize. All the computer systems understand only the machine code; the basic property of machine code is binary in nature. Different codes are used to store and transmit the data efficiently.

Classification of Codes

- Computer recognize binary data. Combination of 1 and 0 are referred as the code for information.
- When the data is transmitted over a long distance , it is transmitted in terms of **code-words** . During the transmission process , errors may be introduced . To detect and correct the errors, special codes are used in digital system communication.
- The commonly used binary codes are classified as:
- Weighted codes
- Non-weighted codes
- Self –complementary codes
- Unit distance codes
- Alphanumerical codes
- Error detecting and correcting codes

Weighted codes

- In weighted codes, the weight of a digit or a bit depends on its position . For example, in decimal code 589, the weight of 5 is 500, weight of 8 is 80 and weight of 9 is 9. similarly in binary code , the value of 1 depends on its position . Binary, BCD, 8-4-2-1 and 2-4-2-1 are the examples of weighted codes which will be discussed in the next section.

Self –complementary codes

- Excess-3 code is the example of self –complementary code. In this code, the 1's complement of the excess-3 code is the excess-3 code for the 9's complement of the corresponding decimal number. For example, the excess-3 code for 2 is 0101, the 1's complement of 0101 is 1010, it is the excees-3 code and 7 is the 9's complement of 2. next section will see.

Unit distance codes

- The name itself indicates that there is unit distance between two consecutive codes. i.e. the bit patterns for two consecutive numbers differ in only one bit position . **Gray code** is an example of unit distance code. Gray code will be discussed in next section.

Alphanumerical codes

- The binary codes of alphabets, numbers and special symbols are known as alphanumerical codes. ASCII (American Standard Code For Information Interchange) and EBCDIC(Extended Binary Coded Decimal Interchange Code) are commonly used alphanumerical codes.

Error detecting and correcting codes

- When information in digital form is transmitted to a long distance, errors may get introduced and 1 becomes 0 and 0 becomes 1. special codes are used are to detect and correct such errors. Parity and Hamming codes are commonly used for error detection and correction.

WEIGHTED CODES

1. Binary coded decimal Code (BCD CODE):

- It is a 4-bit binary coded decimal number. Each digit of the decimal number is represented by four –bits, the digit 0 in BCD is represented as 0000 and the digit 9 is represented as 1001. The weights of b3,b2,b1,b0 are 8,4,2 and 1 hence, BCD code is known as 8-4-2-1 .

Decimal Digit	BCD Code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

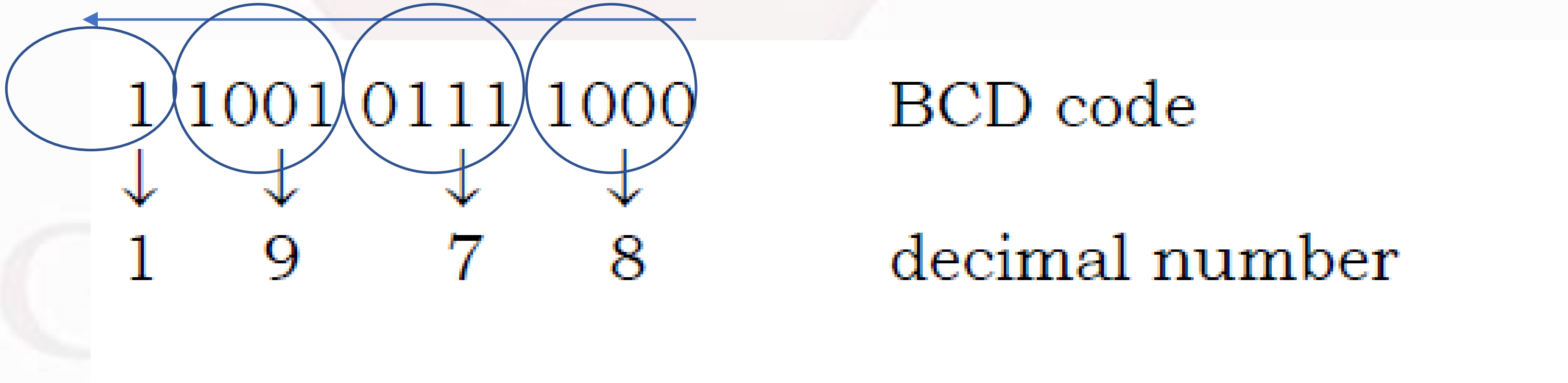
Decimal ➔ BCD Conversion

- Conversion between BCD and decimal is accomplished by replacing a 4-bit BCD for each decimal digit. For example, 874_{10} = 1000 0111 0100_{BCD}.

- BCD is not another number system like binary, octal, decimal and hexadecimal. It is in fact the decimal system with each digit encoded in its binary equivalent. A BCD code is not the same as a straight binary number. For example, the BCD code requires 12 bits, while the straight binary number requires only 10 bits to represent 873_{10}.

BCD ➔ Decimal Conversion

- A BCD code is converted into a decimal number by taking groups of 4 bits, starting from LSB, and replacing them with a BCD code. For example, 1 1001 0111 1000 $_{BCD}$ = 1978_{10}



An N digit decimal number is represented by 4*N bits in BCD code . The BCD codes of (15) is 00010101 and the binary code of (15) is eight bits and the binary 1111. The BCD code of 15 is eight bits and binary of 15 is four bits; BCD code is not efficient as compared to binary. The BCD code requires more space and time to transmit the information .

2. 2-4-2-1

This is a number code, where each digit of a decimal number is represented using four bits. It is a weighted code , the weight of binary symbol 1 depends on its position . The weights of b3,b2,b1,abd b0 are 2,4,2,1 respectively. Thus this code is referred to as 2-4-2-1 code.

NON-WEIGHTED CODES

Non-weighted codes are codes that are not positionally weighted. This means that each position within a binary number is not assigned a fixed value. Excess-3 codes and Gray codes are example of non-weighted codes.

1. Excess-3 Code

It is obtained by adding 3 to each digit of the decimal number and then . It is represented by a 4-bit binary code.

Example: Convert decimal number 9 into Excess-3 code:

1001 BCD code of 9

0011 Add 3

11 carry

1100 Excess -3 code of 9

Or the excess-3 code of 9=(9+3)=12=1100

Convert Excess-3 code 12 into decimal number

1100 BCD code of 12

0011 subtract 3

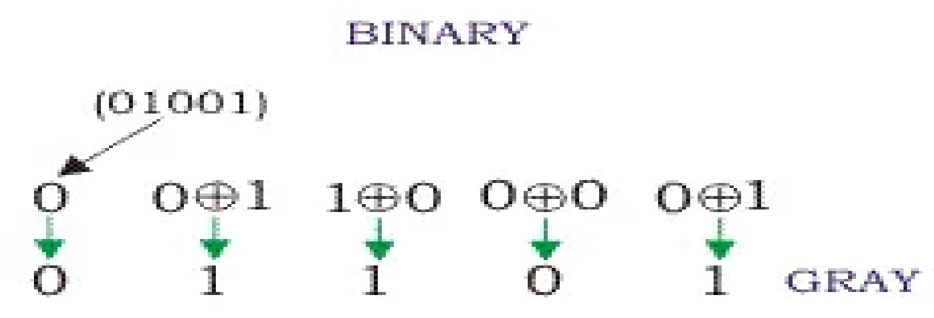
1001 Or decimal code of 12=(12-3)=9=1001

Gray Code:

- Gray code is non weighted code, it is not an arithmetic code.
- Decimal numbers, 0 to15, are represented by 4 bits binary codes.
- It has a very special feature that only one bit will change, each time the decimal number is incremented. This code is known **as unit distance code** because two consecutive codes differ in only one bit position.
- Gray code is not suitable for arithmetic operation, but it finds application in analog-to-digital converters.

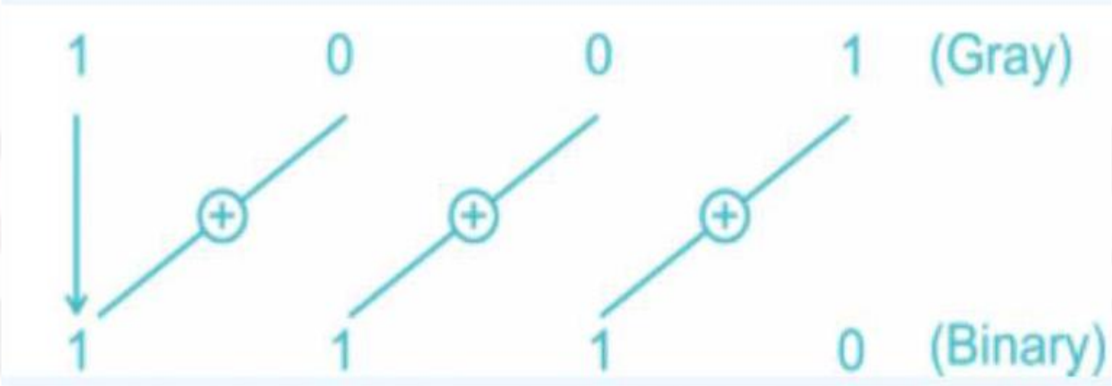
- **Binary to Gray Conversion:** A binary number can be converted to its Gray code when,

i. First bit (MSB) of Gray code is same as first bit of binary number;
ii. The second bit of Gray code equals to the exclusive-OR, of the first and second bits of binary number;
iii. The third Gray code bit equals to exclusive-OR of second and third bits of binary number, and so on.
iv. The binary number 01001 is converted into Gray Code 01101 as,



- **Gray to Binary Conversion:** Conversion of Gray code into its binary form involves following steps,

i. The first binary bit (MSB) is the same as that of the Gray code bit;
ii. If second Gray bit is 0, the second binary bit is the same as that of the first binary; if the second Gray bit is 1, the second binary bit is the inverse of its first binary bit.
iii. Step ii. is repeated for each successive bit.
iv. The Gray code 1001 is converted into binary number 1110 as,



Alphanumeric Codes:

The symbols are required to represent

- 26 alphabets with capital and small letters
- Number from 0 to 9
- Punctuation marks and other symbols.
- Alphanumeric codes are the codes that represent numbers and alphabetic characters(letters).
- The following three alphanumeric codes are very commonly used for the data representation.

  a) American Standard code for Information Interchange(ASCII)

  b) Extended Binary Coded-Decimal Interchange Code(EBCDIC)
- ASCII code is a 7 bit code whereas EBCDIC is an 8 bit code. ASCII code is more commonly used world wide while EBCDIC is used primarily in large IBM computers.

Error detecting and Correcting Codes:

- **Introduction**:

  When we talk about digital systems, be it a digital computer or a digital communication set-up, the issue of error detection and correction is of great practical significance. Errors creep into the bit stream owing to noise or other impairments during the course of its transmission from the transmitter to the receiver. While the addition of redundant bits helps in achieving the goal of making transmission of information from one place to another error free or reliable, it also makes it inefficient.

- **Some Common Error Detecting and Correcting Codes**

  Parity Code, Repetition Code, Cyclic Redundancy Check Code and Hamming Code

❖ Parity Code:

- A parity bit is an extra bit added to a string of data bits in order to detect any error that might have crept into it while it was being stored or processed and moved from one place to another in a digital system.
- This simple parity code suffers from two limitations. Firstly, it cannot detect the error if the number of bits having undergone a change is even.

❖ Repetition Code:

- The repetition code makes use of repetitive transmission of each data bit in the bit stream. In the case of threefold repetition, ‘1’ and ‘0’ would be transmitted as ‘111’ and ‘000’ respectively.
- The repetition code is highly inefficient and the information throughput drops rapidly as we increase the number of times each data bit needs to be repeated to build error detection and correction capability.

❖Cyclic Redundancy Check Code:

• Cyclic redundancy check (CRC) codes provide a reasonably high level of protection at low redundancy level.

• The probability of error detection depends upon the number of check bits, n used to construct the cyclic code. It is 100 % for single-bit and two bit errors. It is also 100 % when an odd number of bits are in error and the error bursts have a length less than $n + 1$.

• The probability of detection reduces to $1 - (1/2)^{n-1}$ for an error burst length equal to $n + 1$, and to $1 - (1/2)^{n}$ for an error burst length greater than $n + 1$.

❖ **Hamming Code:**

• An increase in the number of redundant bits added to message bits can enhance the capability of the code to detect and correct errors.

• If sufficient number of redundant bits arranged such that different error bits produce different error results, then it should be possible not only to detect the error bit but also to identify its location.

• In fact, the addition of redundant bits alters the 'distance' code parameter, which has come to be known as the Hamming distance.

Hamming Distance:

• The Hamming distance is nothing but the number of bit disagreements between two code words.

For example, the addition of single-bit parity results in a code with a Hamming distance of at least 2.

• The smallest Hamming distance in the case of a threefold repetition code would be 3.
• Hamming noticed that an increase in distance enhanced the code’s ability to detect and correct errors.
• Hamming’s code was therefore an attempt at increasing the Hamming distance and at the same time having as high an information throughput rate as possible.

The algorithm for writing the generalized Hamming code is as follows:

1. The generalized form of code is P1P2D1P3D2D3D4P4D5D6D7D8D9D10D11P5....... , where P and D respectively represent parity and data bits.
2. 2. We can see from the generalized form of the code that all bit positions that are powers of 2 (positions 1, 2, 4, 8, 16 ...) are used as parity bits.
3. 3. All other bit positions (positions 3, 5, 6, 7, 9, 10, 11 ...) are used to encode data.
4. 4. Each parity bit is allotted a group of bits from the data bits in the code word, and the value of the parity bit (0 or 1) is used to give it certain parity.

1. Groups are formed by first checking bits and then alternately skipping and checking bits following the parity bit. Here, is the position of the parity bit; 1 for P1 , 2 for P2 , 4 for P3 , 8 for P4 and so on.
2. 2. For example, for the generalized form of code given above, various groups of bits formed with different parity bits would be P1D1D2D4D5...., P2D1D3D4D6D7...., P3D2D3D4D8D9...., P4D5D6D7D8D9D10D11...., and so on. To illustrate the formation of groups further, let us examine the group corresponding to parity bit P3 .
3. 3. Now, the position of P3 is at number 4. In order to form the group, we check the first three bits N-1=3 and then follow it up by alternately skipping and checking four bits (N=4).

- The Hamming code is capable of correcting single-bit errors on messages of any length.
- Although the Hamming code can detect two-bit errors, it cannot give the error locations.
- The number of parity bits required to be transmitted along with the message, however, depends upon the message length.
- The number of parity bits n required to encode m message bits is the smallest integer that satisfies the condition . $(2^n - n) > m$
- The most commonly used Hamming code is the one that has a code word length of seven bits with four message bits and three parity bits.
- It is also referred to as the Hamming (7, 4) code
- The code word sequence for this code is written as P1P2D1P3D2D3D4 , with P1 , P2 and P3 being the parity bits and D1 , D2 , D3 and D4 being the data bits.

Thank You