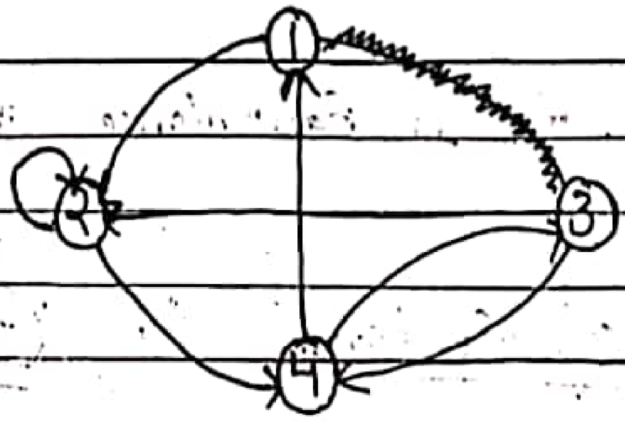
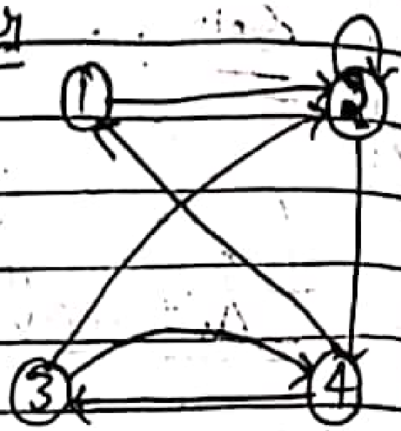
Q. Find the directed graph from the following set.

$A = \{1, 2, 3, 4\}$

$R = \{(1,2); (2,2), (2,4); (3,2), (3,4), (4,1), (4,3)\}$



or

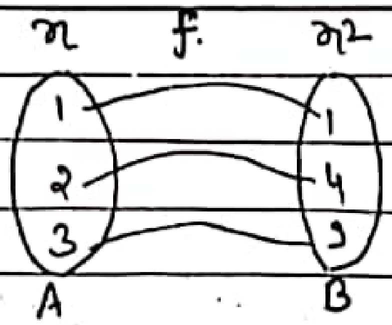


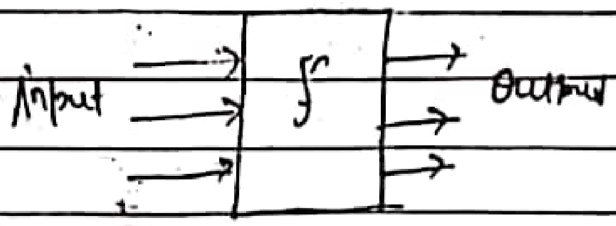
Function :⇒ function is a mapping from set A to B under certain Rules. In another word the each element of set A each is assigned with unic element of set B, the collection of such assignments is called function.

i.e. $f : A \rightarrow B$

which is Read as function f is from set A into B.

$f(x) = x^2$





Types of function

① **One - one function (Surjective) :⇒**

Or

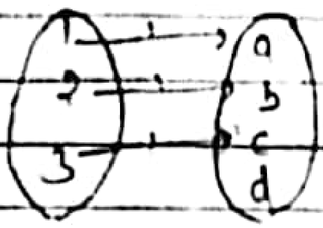
one - to - one function (1-1) function :⇒

A function $f : A \to B$ is said to be one-to-one function if different elements in the domain A have distinct different images in set B. in another words. f is one-one function

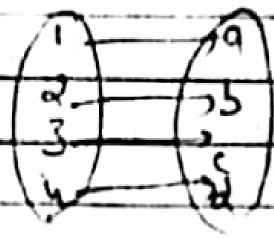
$f(a) = f(a')$ implies $a = a'$ Codomain f Range

for Ex :⇒

(i) A B

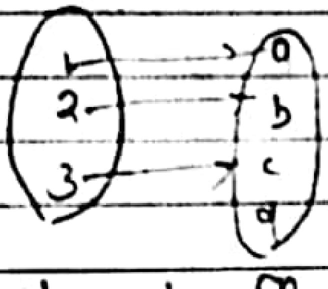


(ii) A B



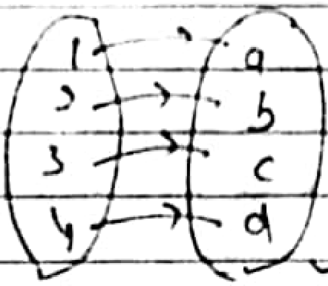
② **on to function :⇒** A function $f : A \to B$ is said to be one to f^n if each element of B is the image of some elements of A. in other words f^n f is said to be onto f^n if the image of f is the entire Codomain.

(i)



not onto f^n

(ii)



one - one function

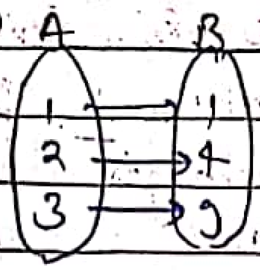
onto function

having range = Codomain.

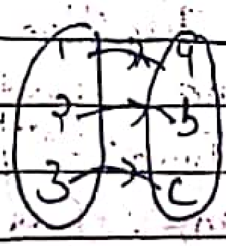
(3) **One-one onto function (Bijective fn) :-**

A function $f: A \to B$ is said to be one-one onto f^n (Bijective) if function is one-one & onto.

For ex :→ (1)



(2)



(4) (one-one onto होना चाहिए) **Inverse function or Invertiable f^n :-**

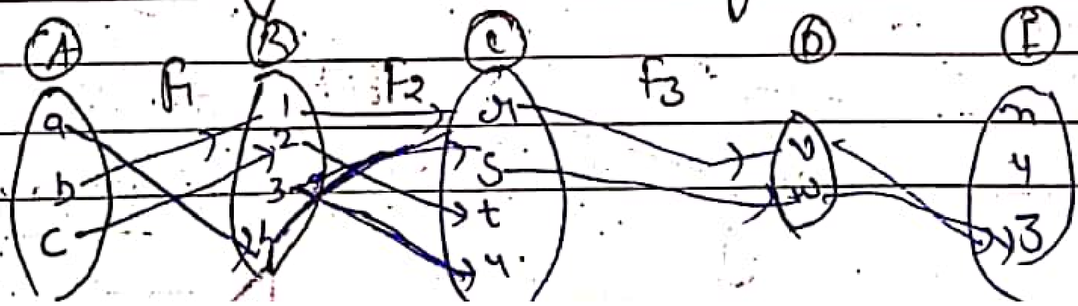
A function $f: A \to B$ is said to be invertiable if its inverse relations f^{-1} is a function from B to A.

(*) In general the Inverse Relation f^{-1} may not be a function.

Theorem :-1 A function $f: A \to B$ is said to be invertiable if and only if is both one to one and onto.

Classify the following function from following diagram / figure :-

(1)



$f_1 \to$ one - one

$f_2 \to$ one - one ~~onto~~ (invertible)

$f_3 \to$ Not one - one & not onto.

$f_4 \to$ Niether one - one Nor onto.

Composition of function :⇒

Consider the function $f: A \to B$ and $g: B \to C$ that is where the co-domain of f is the domain of g. the we may define a new function from A to C called the composition of f and g. which is written as $g o f$. as follows $(gof)(a) = g(f(a))$.

For Ex :⇒ let f, g, h are function on $X = \{1, 2, 3\}$ or

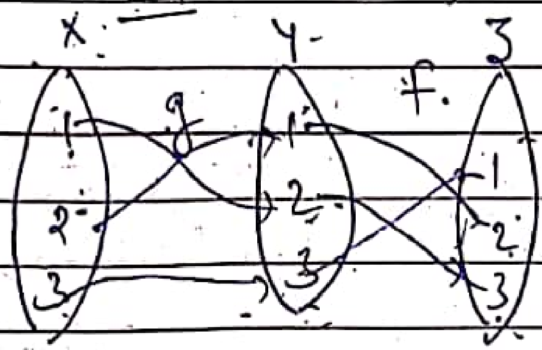
(a) $f = \{(1,2), (2,3), (3,1)\}$

$g = \{(1,2), (2,1), (3,3)\}$

$h = \{(1,1), (2,2), (3,1)\}$

Comute fog, gof, $fogoh$ and $fohog$

Soln :⇒ $fog = f(g(x))$



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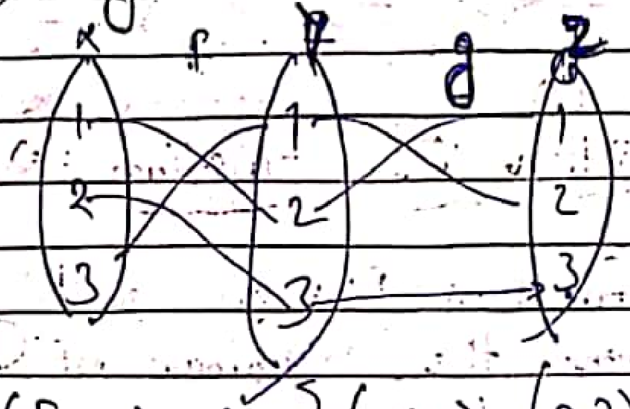
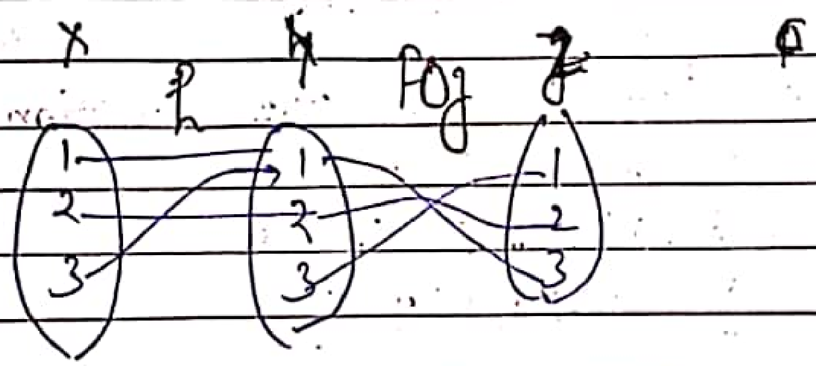
① fog = $f(g(x))$
$= \{(1,3), (2,2), (3,1)\}$

② gof :=



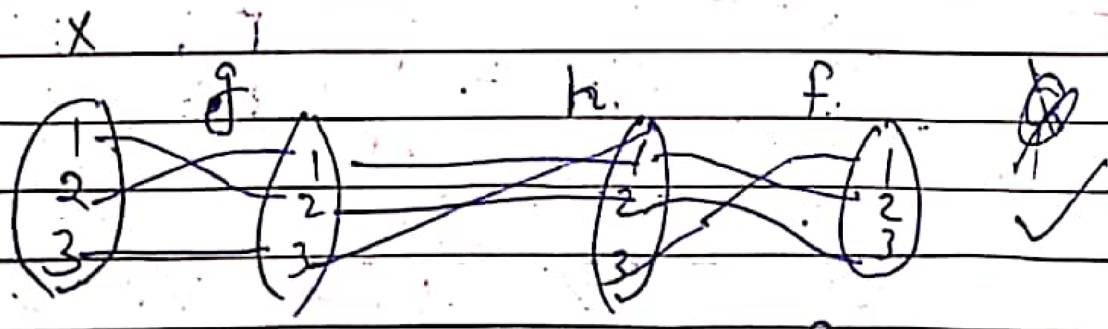
$g(f(x) \Rightarrow \{((1,1), (2,3)\ (3,2)\}$

③ fogoh



Fogoh = $(fog)(h(x)) = \{(1,3), (2,2), (3,3)\}$

④ fohog



Fohog = $(Foh)(g(x)) \Rightarrow \{(1,3), (2,2), (3,2)\}$

Que :-)

Let f, g, h are function from N to N where N is set of natural no. so that

$f(n) = n+1$ $\quad$ $g(n) = 2n$

$h(n) = 0$ $\quad$ if n is even

$h(n) = 1$ $\quad$ if n is odd

Determine (a) $f o f$ (b) $f o g$ (c) $g o f$ (d) $g o h$ (e) $(f o g) h$.

Sol n.-) (1) $f o f = f(f(n))$

$= f(n+1)$

$(n+1)+1$

$f o f = n+2$

(2) $f o g$

$f(g(n))$

$f(2n)$

$2(n+1)$

$= \cancel{2n}+2$

$2n+1$

(3) $g o f = g(f(n))$

$g(n+1)$

$2(n+1) = 2n+2$

(4) $g o h = g(h(n))$

$g(0)$

$= 0$ if even

(e) $g(h(n))$

$g(1)$

$2 \times 1 = 2$ if odd.

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(c) $(f \circ g) h = \cancel{(f \circ g)(h(n))}$

$\cancel{(f \circ g)(0)}$

$\cancel{f \circ (g(0))}$

$(f \circ g)(h(n)) = (f \circ g)(1)$

$f \circ (g(1))$

$f \circ (2n)$

$2n+1$ if ~~even~~ odd

$(f \circ g)(h(n)) = (f \circ g)(0)$

$f \circ (g(0))$

$f \circ (0)$

$\Rightarrow 1$ if even

Mathematical function-

Floor & Ceiling function-

Let x be any real number

$\lfloor x \rfloor$, called the floor of x, denotes the greatest integer that does not exceed x.

$\lceil x \rceil$, called ceiling of x, denotes the least integer that is not less than x.

* If x is itself an integer, then

$\lfloor x \rfloor = \lceil x \rceil$ other wise $\lfloor x \rfloor + 1 = \lceil x \rceil$

for eg -

$\lfloor 3.14 \rfloor = 3$ $\quad$ $\lfloor \sqrt{5} \rfloor = 2$

$\lfloor -8.5 \rfloor = -9$ $\quad$ $\lceil 3.14 \rceil = 4$

$\lceil \sqrt{5} \rceil = 3$ $\quad$ $\lceil -8.5 \rceil = -8$

$\lfloor 7 \rfloor = 7$ $\quad$ $\lfloor -4 \rfloor = -4$

$\lceil 7 \rceil = 7$ $\quad$ $\lceil -4 \rceil = -4$

Recursively Defined function-

A function is said to be recursively defined if the function definition refers to itself. The function definition must have the following two property-

(1) There must be certain arguements, called base values, for which the function does not refer to itself.

(2) Each time the function does refers to itself, the arguement of the function must be closer to a base value.

eg factorial function

(a) if $n=0$ then $n! = 1$ or $0! = 1$

(b) if $n>0$ then $n! = n(n-1)!$

fibonacci Series

(a) if $n=0$ or $n=1$ then $F_n = n$

(b) if $n>1$ then $F_n = F_{n-2} + F_{n-1}$