Consider $f(D)y = xV(x)$

$$P.I. = \frac{1}{f(D)} xV(x) = x\frac{1}{f(D)}V(x) - \frac{f'(D)}{[f(D)]^2}V(x)$$

Ex: $(D^2+2D+1)y = 2x\sin x$

The A.E. is $m^2+2m+1=0$

$$(m+1)^2 = 0 \Rightarrow m = -1, -1$$

C.F. is $y_h = c_1 e^{-x} + c_2 x e^{-x}$

$$\text{P.I. is } y_p = \frac{1}{f(D)} 2x\sin x$$

$$= \frac{2}{(D^2+2D+1)} x\sin x$$

$$= x\frac{2}{D^2+2D+1}\sin x - \frac{2D+2}{(D^2+2D+1)^2}\sin x$$

$$= \frac{2x}{-1+2D+1}\sin x - \frac{2D+2}{(-1+2D+1)^2}\sin x$$

$$= \frac{2x}{2D}\sin x - \frac{2D+2}{4D^2}\sin x$$

$$= -x\cos x - \frac{2D+2}{-4}\sin x$$

$$= -x\cos x + \frac{1}{2}(D+1)\sin x$$

$$= -x\cos x + \frac{1}{2}D\sin x + \frac{1}{2}\sin x$$

$$= -x\cos x + \frac{1}{2}\cos x + \frac{1}{2}\sin x.$$

Ex. $(D^2-2D+1)y = xe^x \sin x$

The A.E. is $m^2-2m+1=0$

$\Rightarrow m = 1, 1$

C.F. $y_h = c_1 e^x + c_2 x e^x$

P.I. is $y_p = \dfrac{1}{(D^2-2D+1)} x e^x \sin x$

$= \dfrac{1}{(D-1)^2} e^x (x \sin x)$

$= e^x \dfrac{1}{(D+1-1)^2} x \sin x$

$= e^x \dfrac{1}{D^2} x \sin x$

$= e^x \left[x \dfrac{1}{D^2} \sin x - \dfrac{2D}{D^4} \sin x \right]$

$= e^x \left[x(-\sin x) - \dfrac{2}{D^3} \sin x \right]$

$= (-x \sin x - 2\cos x) e^x$

$\therefore$ The G.S. is $y = y_h + y_p$

$= c_1 e^x + c_2 x e^x + e^x (-x \sin x - 2\cos x)$