Rules of Inference

An **argument** in propositional logic is a sequence of propositions in which all but the final proposition are called premises and the final proposition is called the conclusion.

An argument is **valid** if the truth of all its premises implies that the conclusion is true.

Rules of Inference	Tautology	Name
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens (Law of detachment)
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens (Law of Contraposition)
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical Syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive Syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Ex. Show that the premises "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny," "If we do not go swimming then we will take a canoe trip", and "If we take a canoe trip then we will be home by sunset" lead to the conclusion "We will be home by sunset".

Soln: Let p be the proposition "It is sunny this afternoon"

q: It is colder than yesterday

r: We will go swimming

s: We will take a canoe trip

t: We will be home by sunset

Premises are $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$ and $s \rightarrow t$ and conclusion t.

Step	Reason
1. $\neg p \wedge q$	Premise
2. $\neg p$	Simplification using 1
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus tollens using 2 & 3
5. $\neg r \rightarrow s$	Premise
6. s	Modus ponens using 4 & 5
7. $s \rightarrow t$	Premise
8. t	Modus ponens using 6 & 7

Ex. Show that the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ imply the conclusion $p \vee s$.

Soln:-