A vector space that has no finite basis is called infinite-dimensional.

Thm: Let W be a subspace of FDVS V. Then:
ⓐ W is also FDVS and $\dim V \le \dim V$
ⓑ $\dim W = \dim V$ iff $W = V$.

Table

$\dim V$	V
3	$\mathbb{R}^3$
2	Plane through the origin, $\mathbb{R}^2$
1	Line through the origin, $\mathbb{R}$
0	$\{0\}$
$n+1$	P_n {polynomials of degree $\le n$}
mn	$M_{m\times n}$ (Matrices of order $m\times n$)
infinite	$\mathbb{R}(\mathbb{Q})$
$\sum n$	Symmetric matrices Vector space of order n

Thm: Let V be a FDVS with $\dim V = n$. Then:

ⓐ Any LI set in V contains at most n vectors.
ⓑ Any spanning set for V contains at least n vectors.
ⓒ Any LI set of exactly n vectors in V is a basis for V.
ⓓ Any spanning set for V consisting of exactly n vectors is a basis for V.
ⓔ Any LI set in V can be extended to a basis for V.
ⓕ Any spanning set for V can be reduced to a basis for V.

Q(31) Find the dimension of subspace $\{(x_1, x_2, x_3, x_4, x_5) : 3x_1 - x_2 + x_3 = 0\}$ of $\mathbb{R}^5$.

To know dimension, need to find basis:

$(x_1, x_2, x_3, x_4, x_5) = (x_1, 3x_1 + x_3, x_3, x_4, x_5) = x_1(1,3,0,0,0) + x_3(0,1,1,0,0) + x_4(0,0,0,1,0) + x_5(0,0,0,0,1)$. $\boxed{\therefore \dim = 4}$