Matrix Contd

Normal form of a matrix

Every non-zero matrix of rank 'r' can be reduced using sequence of elementary transformations (row and column both) to the form

$$\left[\begin{array}{c:c} I_r & O \\ \hdashline O & O \end{array}\right]$$ called the NORMAL FORM.

Thus, rank of a matrix can be obtained by reducing it to the normal form.

Ex. Reduce the matrix A to its normal form and hence find its rank.

Sol. (Please note the process carefully)

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ by $R_1 \leftrightarrow R_2$ to make $a_{11} = 1$.

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$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix} \text{ by } R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1,\ R_4 \to R_4 - 6R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix} \text{ by } C_2 \to C_2 + C_1,\ C_3 \to C_3 + 2C_1,\ C_4 \to C_4 + 4C_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix} \text{ by } R_2 \to R_2 - R_3 \text{ to make } a_{22} = 1.$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix} \text{ by } R_3 \to R_3 - 4R_2,\ R_4 \to R_4 - 9R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix} \text{ by } C_3 \to C_3 + 6C_2,\ C_4 \to C_4 + 3C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{bmatrix} \text{ by } C_3 \to \frac{1}{33}C_3 \ \& \ C_4 \to \frac{1}{22}C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \text{ by } C_4 \to C_4 - C_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \text{ by } R_4 \to R_4 - 2R_3$$

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$$= \left[\begin{array}{c|c} I_3 & O \\ \hline O & O \end{array}\right]$$

This is the required normal form of A showing that $\rho(A) = 3$.

<u>Exercises</u> Reduce the following matrices to normal form and hence find their rank.

i) $\begin{bmatrix} 0 & 1 & 2 & 1 & -6 \\ 0 & 2 & 2 & 1 & -5 \\ 1 & -2 & -3 & -2 & 1 \\ 3 & -2 & 0 & -1 & -7 \end{bmatrix}$ <u>Ans. 4</u>

ii) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$ <u>Ans. 3</u>

iii) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ <u>Ans. 2</u>

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