Size of Sets (Cardinality of Sets)

We know that the cardinality of a finite set is the number of elements in the set. The cardinalities of finite sets give idea about the size of sets wheather they are equal or not (which one bigger).
When we have infinite sets then we can't say anything about equallity or bigger/smaller size of sets directly.

The sets A and B have the same cardinality iff there is a one-to-one correspondence from A to B.

There is a one-to-one function from A to B, then
$|A| \le |B|$.

A set is called countable if it is either finite or has the same cardinality as the set of natural numbers.
A set that is not countable is called uncountable.
We denote the cardinality of Natural numbers by $\aleph_0$ (aleph nought)
$|\mathbb{N}| = \aleph_0$

An infinite set is countable iff it is possible to list the elements of the set in a sequence.

Ex: Show that the set of odd positive integers is a countable set.
To show that the set of odd positive integers is countable, we will exhibit a one-to-one correspondence between this set and the set of positive integers ($\mathbb{N}$).
Consider the function $f(n) = 2n-1$ from $\mathbb{N}$ to set of odd positive integers.
let $f(n) = f(m) \Rightarrow 2n-1 = 2m-1 \Rightarrow n = m \Rightarrow f$ is 1-1.
Suppose that t is an odd positive integer. Then t is

1 less than an even integer $2k$, where k is a natural number. So, $t = 2k - 1 = f(k)$. This shows f is onto. Hence, f is one-to-one correspondence.

1	2	3	4	5	6	7	8	9	10	11	12	...
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	
1	3	5	7	9	11	13	15	17	19	21	23	...

Ex Show that the set of all integers is countable.

0 1 -1 2 -2 ...

$$f(n) = \frac{n}{2}, \quad n \text{ even}$$

$$= -\frac{(n-1)}{2}, \quad n \text{ odd}$$

Ex. Show that the cardinality of natural numbers and integers is same.

Ex Show that the set of positive rational numbers is countable.

Every positive rational number is the quotient p/q of two positive integers. We can arrange the positive rational numbers by listing those with denominator $q = 1$ in the first row, those with denominator $q=2$ in the second row and so on.

The key to listing the rational numbers in a sequence is to first list the positive rational numbers p/q with $p+q=2$, followed by those with $p+q=3$ and so on. In this way, all the positive ~~integers~~ rational nos. can be written as a sequence. And by the definition of sequence, all positives integers is one-one correspondence with positive ~~int~~ rational numbers. Hence, +ve rationals are countable.