Unit - 3

Set -

A set is any well defined ~~function~~ collection of distinct objects or entities of any kind.

for eg -

1. A is set of six letter alphabets a, u, v, c, y, z.
2. A is set of cities
3. A is set of capital cities of India.
4. (i) N = the set of natural numbers
   = 1, 2, 3, - - - - - - - - - - - -

   (ii) W = the set of whole numbers
   = 0, 1, 2, 3, - - - - - - - - -

   (iii) I or Z = the set of integers, includes 'zero' and all positive & negative numbers

   (iv) Q = the set of Rational numbers
   = P/q ($q \neq 0$)

   (v) R = the set of real numbers $\{-\infty$ to $\infty\}$

   (vi) C = the set of complex number $(x+iy)$

Date
Page

Representation of set -

(a) Rule method (Builder method)

(b) Roster method (Tabulation method)

(a) Rule method -

eg (1) If P = set of all prime no's then
$P = \{x : x \text{ is prime number}\}$

(2) if A = set of natural numbers b/w 10 and 100
$A = \{x : x \in N \text{ and } 10 < x < 100\}$

(b) Roster method -

eg (1) A = set of letters of word 'MATHEMATICS' then
$A = \{M, A, T, H, E, I, C, S\}$

★ **Null set or Empty set** = $\{\}$, ϕ

eg (1) $A = \{x : x^2+1 = 0 \text{ and } x \in Z\}$

$$x^2+1 = 0$$
$$x = \pm i \notin Z$$
$$A = \phi \text{ or } \{\}$$

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Date ______
Page ______

(2) The set of all even numbers between 2 & 4

(3) The set of all even prime number greater than 2

finite & infinite set - A set is said to be finite if it has finite number of elements otherwise it is said to be infinite.

for eg -

(1) The set of vowels in the english alphabet is a finite set.

(2) The set of natural number is an infinite set.

Cardinality of set - The number of elements in a finite set A is called its cardinal number of set & it is denoted by $n(A)$ or $|A|$.

Subset - A set A is a subset of the set B if & only if every element of A is also an element of B. it is denoted by $A \subseteq B$

Symbollically if $x \in A \Rightarrow x \in B$ then

$$A \subseteq B$$

Proper Subset – A set A is called proper subset of B if

(i) A is subset of B.
(ii) B is not subset of A.
& It is denoted by $\boxed{A \subset B}$

Super Set –

If A is a subset of B then B is called Super set of A.

eg

(i) if $A = \{0, 2, 9\}$

$B = \{0, 2, 7, 9, 11\}$

then $A \subset B$

(A is proper set of B)

(ii) if $A = \{1, 2, 4\}$

$B = \{2, 4, 6, 8\}$

A is proper subset of B &
B is super set of A.

Power Set -

The set of all subset of S is called power set of S. It is denoted by P(S) or 2^S

eg

$A = \{1, 2, 3\}$

$S = 3$

$P(A) = 2^3 = 8$

$B = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

Note- Every Empty Set ϕ is the subset of every set.

Note- Every set is also its own subset.

Q find the power set of the following

(i) $\{a, \{b\}\} = B$

$\Downarrow$

$2^2 = 4$

$\Downarrow$

$P(B) = \{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$

(ii) $\{1, \phi, \{\phi\}\} = A$

$\Downarrow$

$2^3 = 8$

$\Downarrow$

$P(A) = \{\phi, \{1\}, \{\phi\}, \{\{\phi\}\}, \{1, \phi\}, \{\phi, \{\phi\}\}, \{1, \{\phi\}\}, A\}$

Universal Set-

A set U which contains all the sets under consideration as subset is called a Universal Set.

Operations on Sets-

① Union of sets-

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Rules-

$$A \cup \phi = A$$
$$A \cup A = A$$
$$A \cup B = B \cup A$$
$$A \cup (B \cup C) = (A \cup B) \cup C$$

② Intersection of Sets-

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Rules-

$$A \cap \phi = \phi$$
$$A \cap A = A$$
$$A \cap B = B \cap A$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

(3) Difference of Sets-

If A & B be any two sets, the difference of B & A is written as A - B is set consisting of all elements of A which are not element of B.

$$A - B = \{x : x \in A \cap B, x \notin B\}$$

eg

$A = \{a, b, c\}$
$B = \{b, c, d, e, f, g\}$

$A - B = \{a\}$

Symmetric Difference - The symmetric difference of two sets A & B is defined as the smallest set containing elements that are either in A or in B. But not in both.

It is denoted by $\boxed{A \oplus B}$ or $\boxed{A \Delta B}$ & sometimes $\boxed{A + B}$

eg

$A = \{a, b, c, d, e\}$
$B = \{c, d, e, f, g\}$

$A - B = \{a, b\}$
$B - A = \{f, g\}$

$$A \oplus B = (A - B) \cup (B - A)$$
$$= \{a, b, f, g\}$$

Page

Rules on difference of Sets –

(i) $A^c = U - A$
(ii) $A - B = A \cap B^c$
(iii) $A - A = \phi$
(iv) $A - \phi = A$
(v) $A - B = B - A \Leftrightarrow A = B$
(vi) $A - B = A \Leftrightarrow A \cap B = \phi$
(vii) $A - B = \phi \Leftrightarrow A \subseteq B$

Rules on symmetric difference -

(i) $A \oplus \phi = A$
(ii) $A \oplus A = \phi$
(iii) $A \oplus B = B \oplus A$
(iv) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$
(v) $A \oplus B = (A - B) \cup (B - A)$

* Law of set theory –

① Independent law –
ⓐ $A \cup A = A$
ⓑ $A \cap A = A$

② Associative law -
ⓐ $(A \cup B) \cup C = (A) \cup (B \cup C)$
ⓑ $(A \cap B) \cap C = A \cap (B \cap C)$

③ Commutative law -

ⓐ $A \cup B = B \cup A$

ⓑ $A \cap B = B \cap A$

④ Distributive law -

ⓐ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ⓑ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

⑤ Identity law -

ⓐ $A \cap U = A$

ⓑ $A \cap \phi = \phi$

ⓒ $A \cup \phi = A$

ⓓ $A \cup U = U$ (U - universal set)

⑥ Involution law -

ⓐ $(A^c)^c = A$

ⓑ $A \cup A^c = U \rightarrow$ universal set

ⓒ $U^c = \phi$

⑦ Complement law -

ⓐ $A \cap A^c = \phi$

ⓑ $\phi^c = U$

⑧ Demorgan's law -

ⓐ $(A \cup B)^c = A^c \cap B^c$

ⓑ $(A \cap B)^c = A^c \cup B^c$

Relations

Cartesian Product of two set - Let A & B be two sets. The set of all ordered pair (a,b) where $a \in A$ & $b \in B$ is called cartesian Product of A & B.

for eg -

$A = \{1, 2, 3\}$ $B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

But $\Rightarrow$ $\boxed{A \times B \neq B \times A}$

Relation -

Let A & B be two non-empty sets then any subset of $A \times B$ is called "a relation from "A into B".

i.e

Let $a \in A$ & $b \in B$ then Pair (a,b) is called ordered pair of $(a,b) \in R$ then we write aRb which is read as "a" is related to "b".