Question Bank-4 (cat-2)
Discrete Maths (Important Question)

1.) Binary Operation:-
let G be a non-empty set and operation on G is a function * from $G \times G \rightarrow G$ is called a binary operation. we usally denote it by $a*b$ or ab.

2.) Internal Binary Operation

$$f : G \times G \rightarrow G$$

$(+, -, \times, \div) \leftarrow *$

$G = N, W, Z, Q, R, C$

3.) External Binary operation

$$f : F \times V \rightarrow V$$

or

$$f : V \times F \rightarrow V$$

i.e.

$$* : (\alpha, a) \rightarrow \alpha * a$$

4) ~~###~~ Group
let G be non-empty set together with binary operation * is called group if

(i) G is closed under *
(ii) * is an associative operation
(iii) $(G, *)$ has identity element
(iv) for each element a and G there is an element b in G such that

$$a*b = b*a = e$$

16.) The set of 2x2 matrix with real enteries is a semigroup but do not a group because it have not inverse when determinate of matrix become 0.

28.) (a.) $3*4$

$$a*b = (a+b) - ab$$
$$= (3+4) - 3\cdot 4$$
$$= 7 - 12 = -5$$

$$2*(-5) = -3 - (-10)$$
$$= -3 + 10 = 7$$

$$7 * 1/2 = \left(7 + \frac{1}{2}\right) - 7/2$$
$$= \frac{15}{2} - 7/2 = 8/2 = 4$$

(b.) For semigroup

(i) $G \times G \to G$

$Q \times Q \to Q$

$a*b = (a+b) - ab$

$a+b$ is ~~real~~ rational (sum of two ~~real~~ rational is ~~real~~ rational

ab is rational (product of two rational is rational)

then $(a+b) - ab$ is also rational

$\therefore$ G is undeer group $*: G \times G \to G$

(ii) ~~let~~ let (a,b) and.

let a, b and c are any arbitrary rational no
then we have to prove

$$a*(b*c) = (a*b)*c$$

LHS

$$a*(b*c) = a*\{(b+c)-bc\}$$
$$= a*\{(b+c)-bc\}$$
$$= a+\{(b+c)-bc\} - a\cdot\{(b+c)-bc\}$$
$$= a+(b+c)-bc - a(b+c)-abc$$
$$= a+b+c-bc-ab-ac-abc$$

RHS

$$(a*b)*c = \{(a+b)-ab\}*c$$
$$\{(a+b)-ab\}+b - \{(a+b)-ab\}\cdot c$$
$$= a+b+c-ab-ac-bc-abc$$

$\therefore$ RHS = LHS

So, it follows associative property

Hence $a*b$ is a semigroup.

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for commutative

$a*b = b*a$

LHS

$a*b = (a+b) - ab$

RHS

$b*a = (b+a) - ab$

yes it is commutative.

(c)

An identity relation is one in which every element of set is related to itself only

$a*e = e*a = a$

$a+e-ae = a$

$\cancel{e(1-a)}$

$e(1-a) = 0$

$e = 0$

2) let for all a, b in group G

By using the fact that

$a \cdot a = b \cdot b = (ab) \cdot (ab) = e$

Since $(ab)^2 = a^2 \cdot b^2 = e \cdot e = e$

Multiplying ba both side

$\Rightarrow ab \cdot ab \cdot ba = e \cdot ba$

$\Rightarrow ab \cdot a(b \cdot b) \cdot a = eba$

$\Rightarrow ab \cdot a \cdot b^2 \cdot a = eba$

$\Rightarrow ab \cdot a \cdot e \cdot a = eba$

(35)

$ab \cdot a \cdot a = ba$

$ab \cdot e = ba$

$ab = ba$

Hence, ~~$\mathcal{G}$~~ G is commutative

36.)

$G = \{1, \omega, \omega^2\}$

Composition Table

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	$\omega^3 = 1$
ω^2	ω^2	$\omega^3 = 1$	$\omega^4 = \omega$

① closure property

Since all enteries in the composition table are in A. Therefore, closure property is satisfied

② Associative law:-

$1 \times (\omega \times \omega^2) = (1 \times \omega) \times \omega^2$

$1 \times \omega^3 = \omega \times \omega^2$

$\omega^3 = \omega^3$

$1 = 1$

then ~~*~~ statisfied the associtive law

(3.) Identity element
Here Identity element 1 is in the set under multipication.

(4.) Inverse property

If all a belonging to G, there exis a, b such that $a*b=e$, then b is called the inverse of a

Clearly, Inverse of 1 is 1
Inverse of ω is ω^2
Inverse of ω^2 is ω

Since, all 4 properties of group are satisfied G is a group.

38.) Given that,
$G = \{0, 1, 2, 3, 4\}$

Binary operation = $+_5$ (Addition Modulo '5'

we have to prove $(G, +_5)$ is a ring

composition table

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(1) closure law

According composition table, we observe that all the elements of the table are also the elements of the set 'G'.

$\therefore$ Closure law is verified

(2.) Associative law

from the table, we identify, All the elements follow Associative law w.r.t $+_5$.

$\therefore$ Associative law is verified

(3.) Identity law:-

we know that,

'0' is the Additive Identity element.

$\boxed{e=0} \in G$

$\therefore$ Identity law is verified

(4.) Inverse law.

from the table.

$$a \cdot a^{-1} = a^{-1} \cdot a = e$$

$0^{-1} = 0$

$1^{-1} = 4$

$2^{-1} = 3$

$3^{-1} = 2$

$4^{-1} = 1$

Inverse of every element $\in G$

$\therefore$ Inverse law is verified

$\therefore$ G is in Group.

45.) (a.) $Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$a * b = ab/n = ab/7$

Compositional ~~Combinational~~ table

$*_7$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

(b) Semi-group

(i) Clowsor law

~~From~~ In compositional table, All the element are the element of $Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$\therefore$ clousor law is verified

(ii) Associative law

$$(a*b)*c = a*(b*c)$$

$$\left(\frac{ab}{7}\right)*c = a*\left(\frac{bc}{7}\right)$$

$$\frac{abc}{7}/7 = \frac{abc}{7}/7$$

$$\frac{abc}{49} = \frac{abc}{49}$$

LHS = RHS

$\therefore$ Associative law is verified

So, the given $(Z_n, *)$ is semi Group

(44.) $G = \{1, 2, 3, 4, 5, 6\}$

(a.) Table:-

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(b.)

from above table

$2^{-1} = 4$

$3^{-1} = 5$

$6^{-1} = 6$