

When $x(x) = \sin ax$ or $\cos ax$

Then $f(D)y = \sin ax$ or $\cos ax$

Where $f(D) = D^n + a_{n-1}D^{n-1} + \dots + a_1D + a_0$

The P.I. is $y_p = \frac{1}{f(D)} \sin ax$ or $\cos ax$

$$y_p = \frac{1}{f(-a^2)} \sin ax \text{ or } \cos ax$$

If $f(-a^2) = 0$ then

$$y_p = \frac{x}{f'(-a^2)} \sin ax \text{ or } \cos ax$$

If $f'(-a^2) = 0$ then

$$y_p = \frac{x^2}{f''(-a^2)} \sin ax \text{ or } \cos ax$$

Q. $(D^2 + 2D + 10)y + 37 \sin 3x = 0$

P.I. is $y_p = \frac{1}{D^2 + 2D + 10} (-37 \sin 3x)$

$$= \frac{1}{-9 + 2D + 10} (-37 \sin 3x)$$

$$= \frac{1}{2D + 1} (-37) \sin 3x$$

$$= -37 \frac{(2D - 1)}{4D^2 - 1} \sin 3x$$

$$= (2D - 1) \sin 3x$$

$$= 6 \cos 3x - \sin 3x$$