L17

Method to find PI when $r(x) = x^m$, m being a +ve integer

Consider $f(D)y = x^m$ so that

$$P.I. = y_p = \frac{1}{f(D)} x^m$$

$$= \frac{1}{(D+a)} x^m$$

$$= \frac{1}{a\left[1+\frac{D}{a}\right]} x^m$$

$$= \frac{1}{a}\left[1+\frac{D}{a}\right]^{-1} x^m$$

$$= \frac{1}{a}\left[1-\frac{D}{a}+\frac{D^2}{a^2}-\cdots+(-1)^m\frac{D^m}{a^m}\right] x^m$$

$$(1+x)^{-1} = 1-x+x^2-x^3+\cdots+(-1)^m x^m+\cdots$$

$$(1-x)^{-1} = 1+x+x^2+x^3+\cdots+x^m+\cdots$$

Ex. $y''+3y'+2y = 12x^2$

The A.E. is $m^2+3m+2=0$

$$\Rightarrow m = -1, -2$$

So, the C.F. is $y_h(x) = c_1 e^{-x} + c_2 e^{-2x}$

$$P.I. = y_p(x) = \frac{1}{f(D)} 12x^2 = \frac{12}{D^2+3D+2} x^2$$

$$= \frac{12}{2}\,\frac{1}{1+\left(\frac{3}{2}D+\frac{1}{2}D^2\right)} x^2$$

$$= 6\left[1+\left(\frac{3}{2}D+\frac{1}{2}D^2\right)\right]^{-1} x^2$$

$$= 6\left[1-\left(\frac{3}{2}D+\frac{1}{2}D^2\right)+\left(\frac{3}{2}D+\frac{1}{2}D^2\right)^2+\cdots\right] x^2$$

$$= \left[1 - \frac{3}{2}D + \frac{1}{2}D^2 + \frac{9}{4}D^2 + \frac{3}{2}D^3 + \frac{1}{4}D^4 + \cdots\right]x^2$$

$$= \left[1 - \frac{3}{2}D + \frac{11}{4}D^2 + \cdots\right]x^2$$

$$= x^2 - \frac{3}{2}Dx^2 + \frac{11}{4}D^2x^2$$

$$= x^2 - 3x + \frac{11}{2}$$

$\therefore$ The G.S. is $y = y_h(x) + y_p(x)$

$$= c_1e^{-x} + c_2e^{-2x} + x^2 - 3x + \frac{11}{2}.$$

Consider $f(D)y = e^{\alpha x}V(x)$

$$P.I. = \frac{1}{f(D)}e^{\alpha x}V(x) = e^{\alpha x}\frac{1}{f(D+\alpha)}V(x)$$

Ex. $y'' + 4y' + 4y = e^{-x}\cos x$

The A.E. is $m^2 + 4m + 4 = 0$

$$(m+2)^2 = 0 \Rightarrow m = -2, -2$$

The C.F. is $= c_1e^{-2x} + c_2xe^{-2x}$

The P.I. is $\frac{1}{f(D)}e^{-x}\cos x$

$$= \frac{1}{(D+2)^2}e^{-x}\cos x$$

$$= e^{-x}\frac{1}{(D+2-1)^2}\cos x$$

$$= e^{-x}\frac{1}{D^2+2D+1}\cos x$$

$$= \frac{e^{-x}}{-1+2D+1}\cos x$$

$$= \frac{e^{-x}}{2D}\cos x$$

$$= \frac{e^{-x}D}{2D^2}\cos x$$

$$= -\frac{e^{-x}}{2}D\cos x$$

$$= \frac{1}{2}e^{-x}\sin x$$