EE200 Practical Report

Frequency Mixer and De-Mixer

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Q1. Frequency Mixer – Beauty and the Blur

Objective

The aim of this exercise is to understand how different spatial frequencies in an image contribute to human perception. Specifically, we:

- Compute and visualize the 2D Discrete Fourier Transform (DFT) of an image.
- Shift the spectrum to center low-frequency components.
- Analyze the effect of rotating the image on its frequency spectrum.
- Creatively fuse two images—one carrying low-frequency (structure) and the other high-frequency (details)—to form a hybrid image.

2D Discrete Fourier Transform (DFT)

For a discrete image $I(x, y)$ of size $M \times N$, the 2D DFT is given by:

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I(x, y) \cdot e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

The inverse transform is:

$$I(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \cdot e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

Fourier Spectrum Visualization

We used NumPy's `fft2()` and `fftshift()` functions to compute and visualize the magnitude spectrum of a grayscale image.

Observation: The original spectrum is centered at the corners. After applying `fftshift()`, low-frequency components are relocated to the center, making interpretation easier.

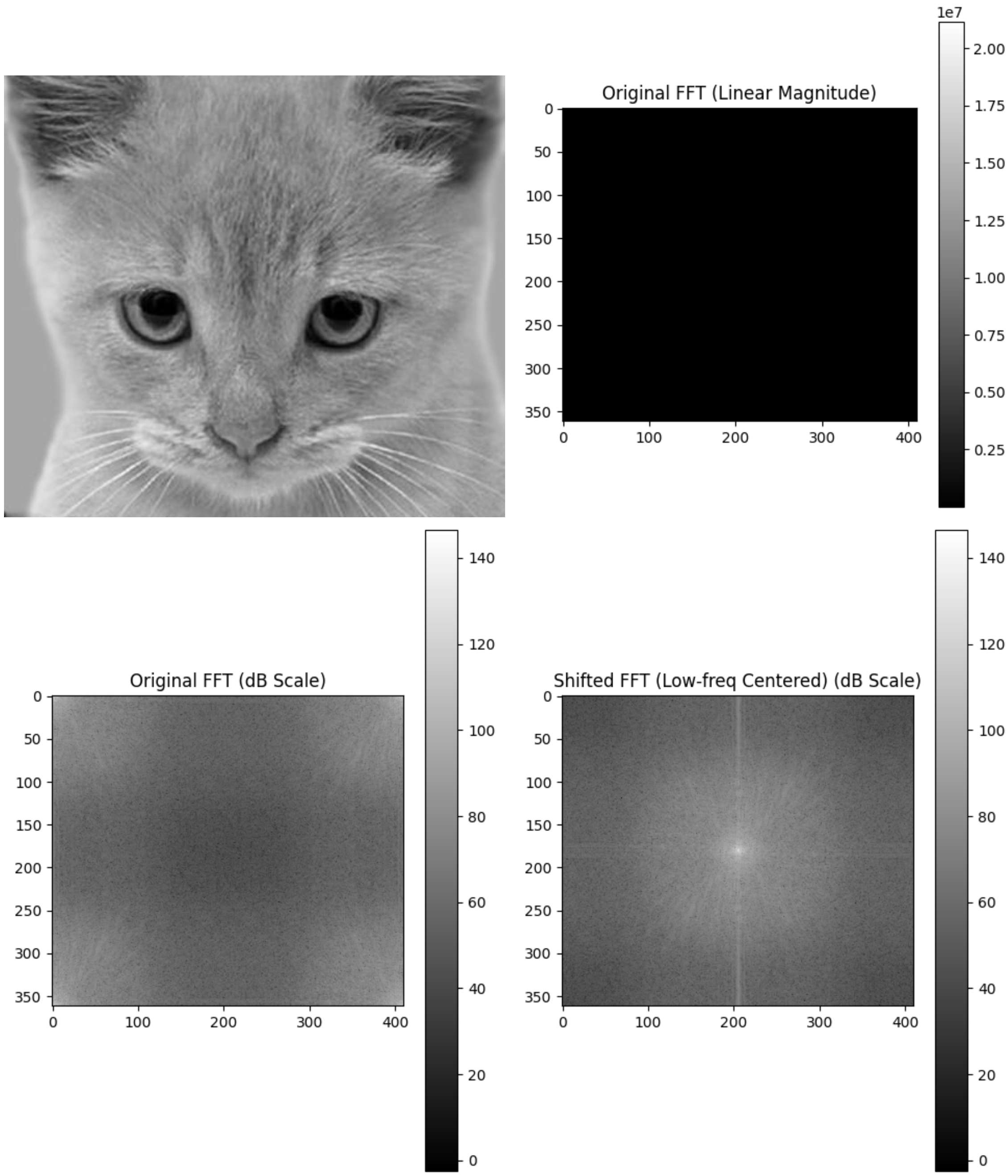

Figure 1: Top left: Original image; Top right: FFT magnitude; Bottom left: FFT in dB scale; Bottom right: Shifted FFT with low frequencies at center.

Rotation and Spectrum Comparison

We rotated the image by 90 degrees counter-clockwise and computed its FFT. As expected, the spectrum is also rotated by 90 degrees.

Observation: The frequency orientation is preserved: rotating the image causes a similar rotation in the frequency domain.

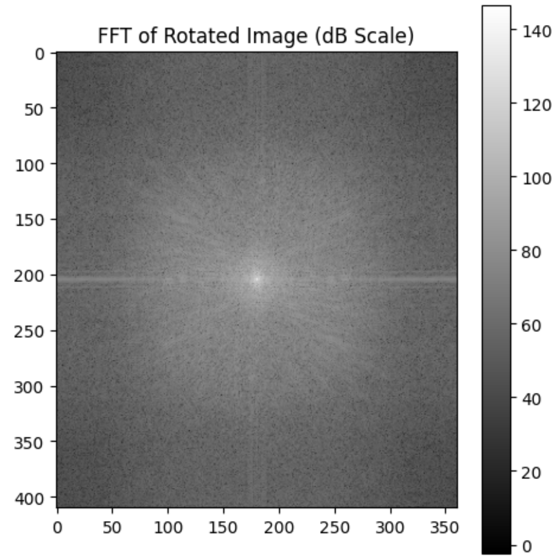

Figure 2: Left: Rotated image; Right: Corresponding rotated FFT magnitude spectrum.

Hybrid Image Construction (Frequency Mixer)

To create a hybrid image:

- We applied a Gaussian blur to one image to extract low-frequency components.
- We subtracted a blurred version from another image to extract high-frequency details.
- The final hybrid image is the sum of these two filtered components.

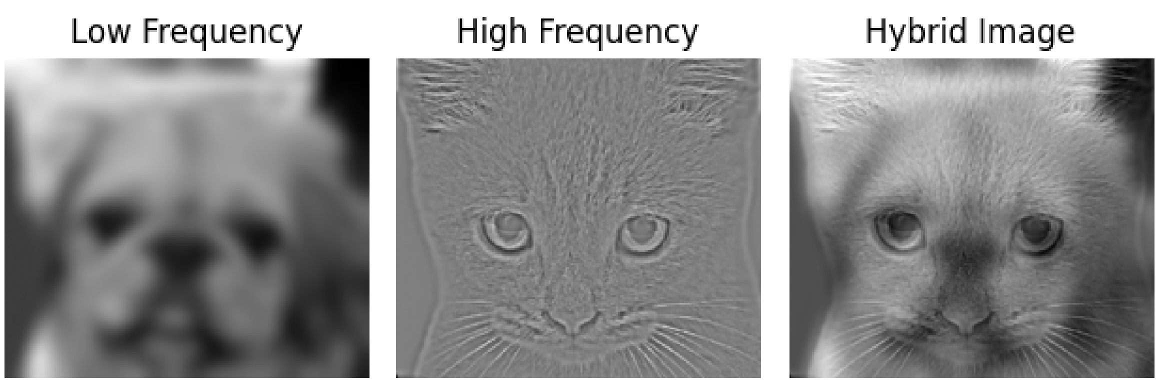

Figure 3: Frequency mixer output using low and high frequency fusion.

Interpretation: When viewed closely, high-frequency details dominate (e.g., edges), but from a distance, low-frequency structure becomes prominent. This mimics human visual perception.

Conclusion

We successfully demonstrated how Fourier analysis and frequency-domain filtering can be used to:

- Analyze and visualize spatial frequency content in images.
- Rotate images and observe corresponding spectral transformations.
- Fuse images using low and high frequency components.

This system effectively mimics biological visual mechanisms and is referred to as a **frequency mixer**.

Q2. Frequency de-mixer: 'Unwanted Solo'

Objective

This task aims to clean a corrupted audio signal (with piccolo instrument noise) using signal processing tools. We:

- Visualize waveform and spectrogram of the original signal.
- Identify noisy frequency bands using the Power Spectral Density (PSD).
- Design and apply a band-stop filter to suppress the unwanted frequency band.
- Compare filtered vs original signal using waveform and spectrograms.

Waveform and Spectrogram

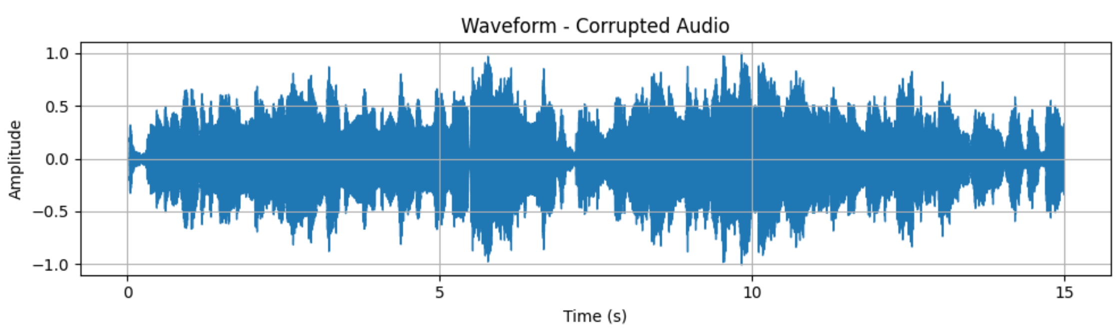

Figure 4: Waveform of the corrupted audio signal.

Power Spectral Density (PSD)

We applied Welch's method to estimate the power distribution over frequencies.

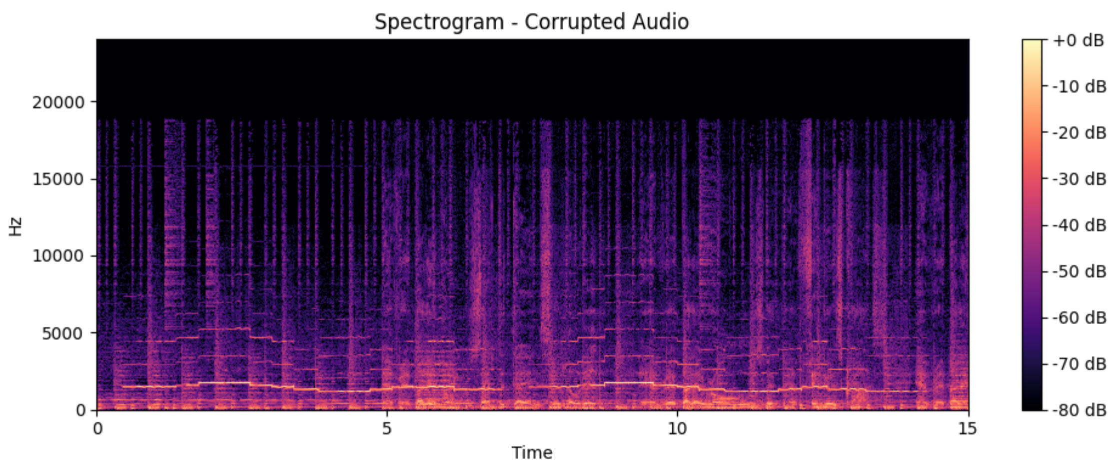

Figure 5: Spectrogram of the corrupted audio signal.

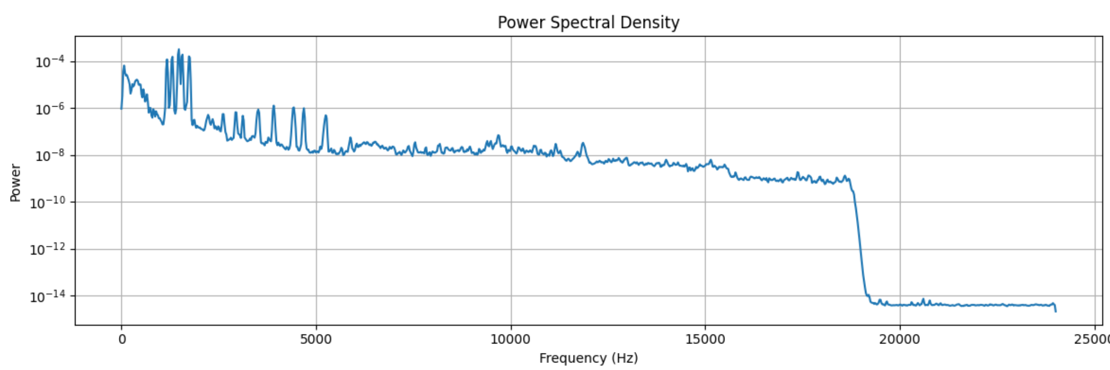

Figure 6: Power Spectral Density showing prominent peaks (likely piccolo tones).

Filter Design and Response

We designed a band-stop Butterworth filter with cutoff range between 600 Hz and 20,000 Hz to eliminate the piccolo content.

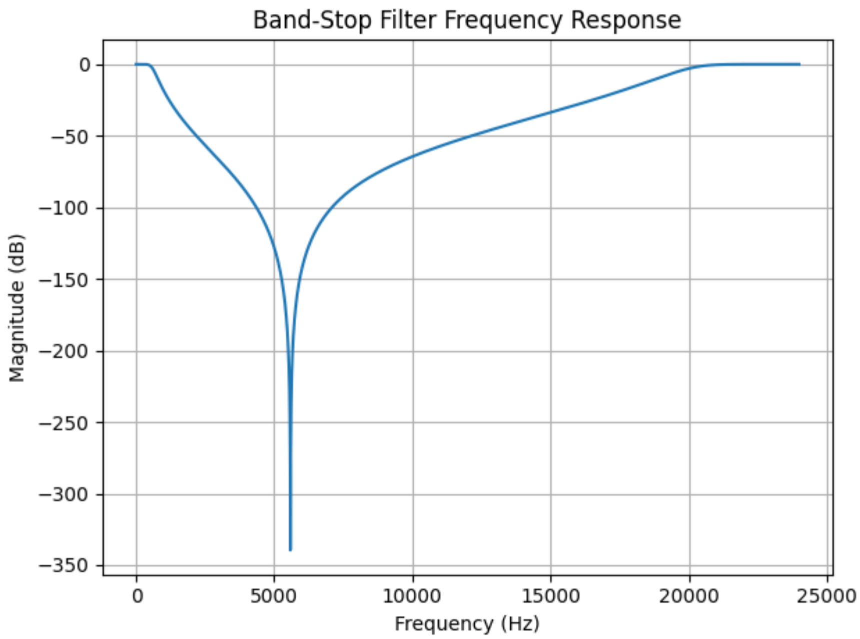

Figure 7: Frequency response of the band-stop Butterworth filter.

Waveform Comparison (Before and After Filtering)

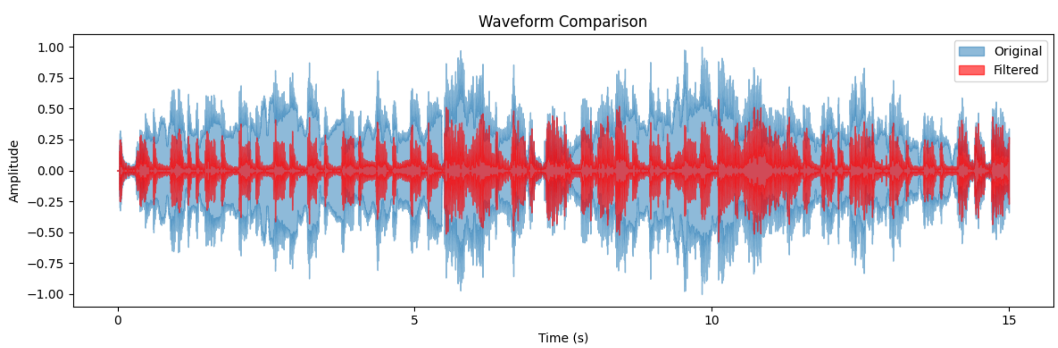

Figure 8: Comparison of original vs filtered waveform.

Filtered Spectrogram

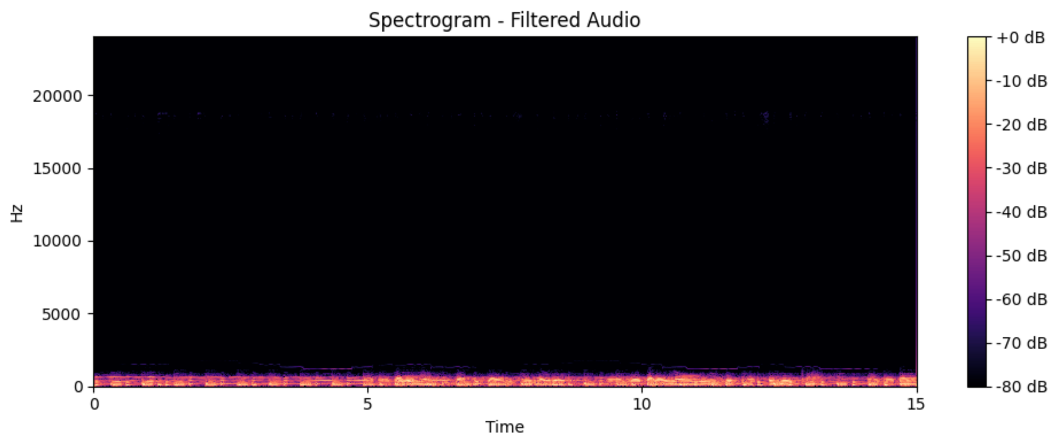

Figure 9: Spectrogram of the filtered audio. Noise bands are successfully suppressed.

Conclusion

We successfully removed high-pitched piccolo sounds from a corrupted audio file using signal processing techniques. Specifically:

- The Power Spectral Density (PSD) and spectrogram revealed strong unwanted frequency components starting around **600 Hz**, extending well into the upper frequency range beyond **20,000 Hz**.
- Based on this observation, we designed a band-stop Butterworth filter with cutoff frequencies at **600 Hz and 20,000 Hz**.
- The filtered audio signal demonstrated a clear suppression of the intrusive piccolo tones.
- Visual comparisons of waveform and spectrogram before and after filtering confirmed the effectiveness of the filter.

The final output audio, `restored_audio.wav`, was perceptually much cleaner, validating our filter design and overall signal processing pipeline.