Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Ans) B

let time taken for service transmission= T

T is normally distributed with μ = 45 minutes and standard deviation σ = 8 minutes.

Time delay= 10 minutes

Time available to finish the work= 60-10=50 minutes.

Therefore from the equation $Z=(T-\mu)/\sigma$

$P(T\leq 50)=p(Z\leq(50-45)/8)=p(Z\leq 0.625)= 0.7324$(using z table)

Therefore $p(T>50)=1-p(\leq 50)= 1-0.7324= 0.2676$

(Or)

Using R-function : [1-pnorm(50,45,8)]=

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> 1-pnorm(50,45,8)
[1] 0.2659855
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2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.

   A. More employees at the processing center are older than 44 than between 38 and 44.

Ans) False.

• 68% of the data falls within one standard deviation of the mean ($\mu+\sigma$).

Here μ=38, σ=6

Then, $\mu+\sigma$= 38+6=44

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans) True

$Z=(X-\mu)/\sigma$

$P(X\leq 30)=p(Z\leq(30-38)/6)=p(Z\leq -1.33)= 0.0918$(using z table)

Expected count=0.0918*400= 36.72

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans) 2 X_1 is simply a larger scale version of the random variable X_1. If X_1 is normally distributed then 2X1 is also normally distributed.

X_1 and X_2 are normal distributed, the associated sums and random samples are exactly (and not just approximately) normal, with the appropriate parameters.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Ans) D

Here we need range of 99% data which lies between 3rd standard deviation of the mean.

Here $\mu=100$, $\sigma=20$

From empirical rule, $\mu \pm 3\sigma = 100 \pm 3*20 => (100-60, 100+60) => (40,160)$.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5th percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans) let, X is the sum of two random variables having normal distribution.

E[X]= E[45*(profit 1+profit 2)]= 45*(5+7)=540 million rupees

SD[X]= SD[profit 1 +profit 2]=> $45*(\sqrt{var(profit1) + var(profit2)})$

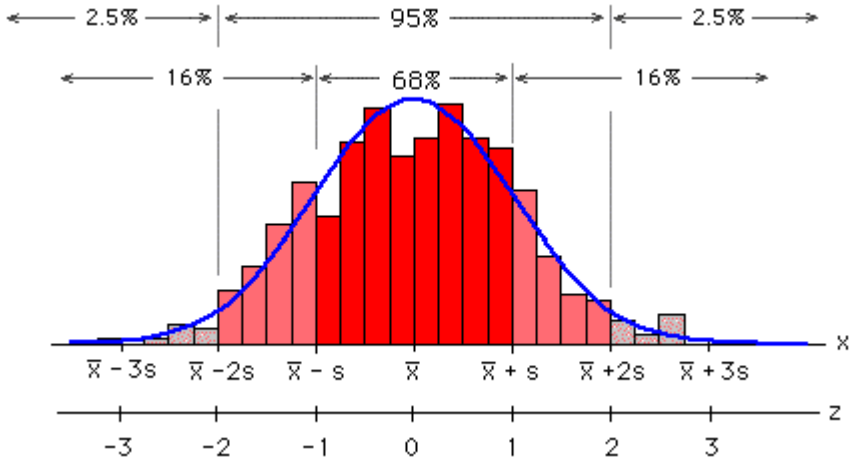
$= 45*\sqrt{9 + 16} = 225$ million rupees.

Therefore, X~ $N(540, 225^2)$

A) From the empirical rule, Approximately 95% of the data falls within two standard deviation of the mean.

$\mu \pm 2\sigma = 540 \pm 2*225 => (540-450, 540+450) =>$ **(90,990)**

B)



From the above normal distribution we can say that to find 5th percentile from the left side we can use the formula,

$\mu - 1.5\sigma$ => 540-(1.5*225) =>202.5 million rupees.

c) this question concerns the original profit distributions.

For division1= Z score for a profit of zero: $Z=(X-\mu)/\sigma$ => (0-5)/3 => -1.66=0.0485

(or)

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> pnorm(0,5,3)
[1] 0.04779035
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For division2= Z score for a profit of zero: $Z=(X-\mu)/\sigma$ =(0-7)/4 => -1.75= .0401

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> pnorm(0,7,4)
[1] 0.04005916
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Division2 has a higher probability of making a loss.