ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/348548255

A MATLAB based program for the Inversion of Time Lapse ERT Data

Conference Paper · February 2020

CITATIONS
0

READS
347

5 authors, including:

Neeraj Nainwal
Queen's University
4 PUBLICATIONS **0** CITATIONS

SEE PROFILE

Utsav Mishra
Indian Institute of Technology Kanpur
11 PUBLICATIONS **36** CITATIONS

SEE PROFILE

Animesh Mandal
Indian Institute of Technology Kanpur
58 PUBLICATIONS **649** CITATIONS

SEE PROFILE

All content following this page was uploaded by Neeraj Nainwal on 22 June 2022.

The user has requested enhancement of the downloaded file.

A MATLAB based program for the Inversion of Time Lapse ERT Data

Neeraj Nainwal[1], Anshul Dekate[2], Utsav Mishra[1], Animesh Mondal[1], Praveen Kumar[3]*
[1]Indian Institute of Technology, Kanpur, [2]Indian Institute of Technology, Roorkee
[3]National Geophysical Research Institute, Hyderabad

*Email: neerajn@iitk.ac.in**

Keywords
Damped Least-square method, 2-D Forward Modelling, Resistivity Inversion, Time Lapse ERT

Summary

In this work, we present the development of MATLAB based 2-D resistivity inversion algorithm using damped least square method and a comparison of a developed algorithm with the standard resistivity inversion software (i.e. RES2DINV) to check its compatibility. Further, a time-lapse Electrical Resistivity Tomography (ERT) data sets were acquired on the same profile during pre and post-monsoon at IIT Kanpur campus to identify the potential conduit for groundwater recharge

First, the forward problem is solved in the Fourier domain where finite difference equations are obtained using area discretization for Poisson's Equation. It includes an optimization method for the choice of a set of Ky for inverse Fourier transform. Singularity correction is done by separating primary and secondary potential to increase the accuracy near current electrodes. Inversion is carried out using a damped least square method (Marquardt –Levenberg algorithm). The best fit between observed and the calculated apparent resistivity data is attained by using a damping factor of 0.01. The code has been verified for analytical solutions and various benchmark models. And the robustness of the algorithm is showed by a synthetic model computation and then the field data inversion by comparison of results with widely acclaimed RES2DINV software. A satisfactory coherence is seen between them for delineation of the water-bearing formations and estimation of their depth and thickness.

Introduction

Groundwater exploration involves the delineation of a suitable aquifer in the subsurface. The physical property of the rock matrix gets affected due to the presence of water in the rock unit i.e., Resistivity. As a result, ERT is the most suitable geophysical approach in delineating the subsurface aquifer zones. Various authors have demonstrated the applicability of ERT to differentiate saturated and unsaturated zones based on the resistivity contrast (Loke and Barker, 1996). However, the resistivity values obtained from the ERT survey are apparent resistivity which depends upon the type of array chosen and heterogeneity in the subsurface. Now, in order to get the true subsurface resistivity variations, we need to invert the measured apparent resistivity using an appropriate numerical model as the analytical solution for the irregular bodies is complex. First, these theoretical aspects of the Finite Difference Method were used by Dey and Morrison (1979). It involves 3-D current source over the 2-D Earth model. Later Pseudo sections were improved by carrying out an algorithm (Loke and Barker 1995).

For solving optimization problems, one of the most popular techniques is least-square Inversion and its Extended work and reviews can be found here (e.g. Lowson and Hanson, 1974). A major problem that arises in the least square inversion is the ill-conditioning due to the fact that the matrix (JTJ) is singular. Lanczos (1960) treated non-singularity by neglecting the small eigenvalues. Marquardt (1963) purposed to introduce constraints to the optimization instead of rejecting the small eigenvalues. To overcome this, constrained were used using the Lagrange multiplier which is added to the diagonal elements of (J^TJ) matrix.

Marquardt (1963) demonstrated that if Langranian multiplier value is very small ($\mu \rightarrow 0$) or very large ($\mu \rightarrow \infty$) then damped least square equation becomes equivalent to the Gauss-Newton Technique or steepest gradient method). Constable et al. (1987) suggested a selection of LM values of μ=0.05 was optimum while for noisy data μ=0.5 was found satisfactory by testing several models.

Method/Theory

Inversion of Time Lapse ERT data

Ohm's Law governs the basic relation between current, electrical resistivity and electric potential. The principal aim of the resistivity survey is to determine the potential distribution in the ground due to a point source. Dey and Morrison (1979), formulated a mathematical expression which calculates the apparent resistivity value for a two-dimensional structure using finite-difference modelling approach. The generalized, poisons equation for 3-D space is:

$$-\nabla.[\sigma(x,y,z)\nabla\varphi(x,y,z)] = (\frac{I}{\Delta V})\delta(x-x_s)\delta(y-y_s)\delta(z-z_s) \quad (1)$$

Where $\sigma(x,y,z)$ represents the conductivity variations in space, I is the injected point current source, (x_s, y_s, z_s) are the location of point source and $\varphi(x,y,z)$ showing 3-D potential distribution.

In this approach, the sub-surface is discretized into the number of rectangular blocks and each block is assigned a known conductivity value which is the observed resistivity value, and thereafter, the potential is evaluated at each block. The discretization is done using discretization by area method which satisfies the relation,

$$\begin{aligned}\nabla^2\{\sigma(x,z)\tilde{\varphi}(x,K_y,z)\} + \sigma(x,z)\nabla^2\tilde{\varphi}(x,K_y,z) \\ - \tilde{\varphi}(x,K_y,z)\nabla^2\sigma(x,z) \\ - 2K_y^2\sigma(x,z)\tilde{\varphi}(x,K_y,z) \\ = -2Q\delta(x_s)\delta(z_s) \quad (2)\end{aligned}$$

Where σ is conductivity of the medium, $\tilde{\varphi}$ is the transforming potential (obtained by Fourier transform in the y-direction), Ky is the wavenumber,(Q =I/2ΔA). The above equation can be expressed in matrix form, which is written as,

$$C\tilde{\varphi} = S \quad (3)$$

Where C= capacitance matrix which takes the known conductivity value in each point, S is the source vector.

The transforming potential ($\tilde{\boldsymbol{\varphi}}$) is calculated using Gauss decomposition method for a sparse C matrix and then the potential for every node is determined by applying the Inverse Fourier transformation given by,

$$\varphi(x,y,z) = \frac{2}{\pi}\int_0^\infty \tilde{\varphi}\,(x,K_y,z)cos(K_y y)dK_y$$

(4)

This potential value is used to calculate the apparent resistivity value using the following equation

$$\rho_a = k\frac{\Delta\varphi}{I} \quad (5)$$

Where

$$k = 2\pi\,(\frac{1}{r_{C1P1}} - \frac{1}{r_{C1P2}} - \frac{1}{r_{C2P1}} + \frac{1}{r_{C2P2}})$$

is the geometric factor which depends upon the electrodes arrangement or array type and ρ_a is apparent resistivity in Ωm.

The apparent resistivity is then calculated which is plotted with apparent depth in the form of pseudo-section.

Geophysical inversion aims at estimating a model that predicts a response similar to an actual model with minimum relative error. It takes a model parameter (in this case resistivity) which is estimated from the observed data, i.e., apparent resistivity value. In order to achieve minimum error, optimization techniques are applied, which creates an initial model that is modified in iterations by reducing the difference between model response (generated by synthetic data) and observed data. Marquardt-Levenberg method treats the ill-conditioning by introducing constraints in optimization that reduce the unbound oscillation in the solutions. Langrage multipliers are used to solve this constraint multiplier. The mathematical representation of Marquardt-Levenberg Damped Least Square Inversion is represented as,

$$dm = (J^TJ + \mu I)^{-1}J^T d\rho$$

(6)

Where J is the Jacobian Matrix, μ is the Lagrangian Multiplier (widely known as the Marquardt's damping Factor), I is the Identity Matrix, dy is the difference in observed and modelled data and dx is the resistivity correction vector.

For this research work we have carried out damped least square methods (also known as Marquardt-Levenberg Method, or the Ridge–regression method) due to its fast convergence and stability and has been extensively used by (Pelton et al. 1978; Sasaki 1982; smith and vozoff, 1984; Rijo et al., 1997)

Case Study

In order to test our forward model and Inversion code on the real data sets, a time-lapse 2-D ERT data was acquired at inside the Hockey Ground, IIT Campus on pre-monsoon time (i.e. 30/06/2019) and post-monsoon time (i.e. 08/10/2019) on the same locations keeping the parameter same using instrument IRIS-Syscal R1 plus switch 72 channel (IRIS-France). The profile length was taken 240m based on the availability of space and the electrode spacing of 5 m is chosen using Wenner-Schlumberger array. IIT Kanpur is situated in Kanpur Nagar, Uttar Pradesh which forms a part of Indo-Gangetic plain comprises mainly of sand, gravel, silt, and clay (based on the bore log information obtained from the institute database). Further, the acquired data sets are processed in MATLAB using the Damped least square method. Also, the results are validated by processing the same data sets using the standard resistivity inversion software RES2DINV.

RESULTS AND DISCUSSION

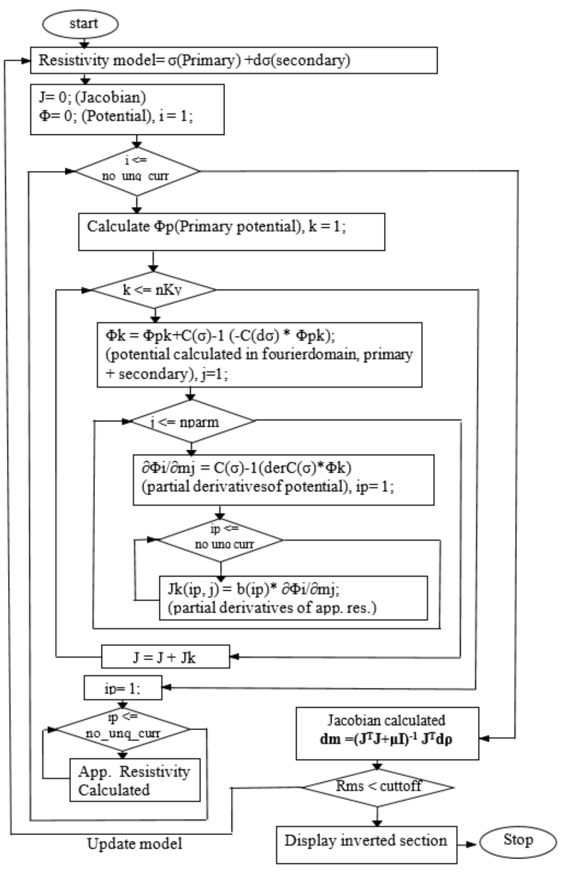

Figure1. flow chart of Algorithm

The initialization of the forward model used a homogeneous ground as a starting model with default 100 Ωm as a model parameter was given with a layer of low resistive of around 70 Ωm and thickness of about 7-25 m. The forward model (refer figure 2) is carried out with 5m spacing using the Finite difference method by selecting the optimum values of Ky wavenumber (Xu et al., 2000) to test the forward modelling algorithm. Solving Boundary problems for every source location shows a singularity which can lead to numerical errors at the electrode position. This singularity is removed by separating the primary and secondary potential as purposed by (Lowry et al., 1989).

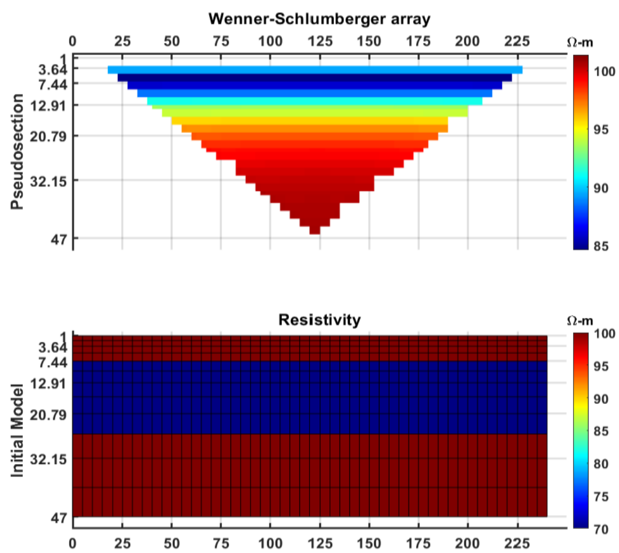

Figure 2. Calculated Initial model and Pseudosection using MATLAB.

2D Inversion of the wenner-schlumberger array was carried out using Damped least square method (refer to figure 1) with damping factor μ=0.01.

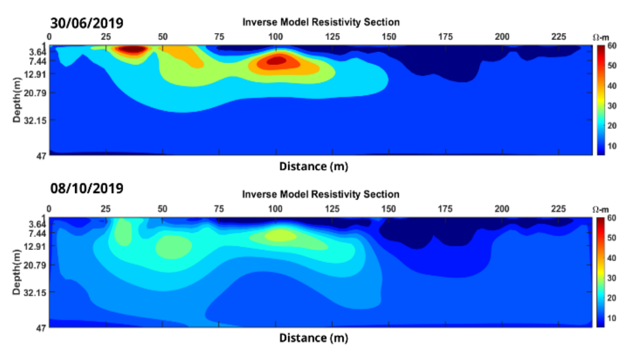

Figure 3. Inverse Model Resistivity Section of (a) Pre (b) Post Monsoon processed using MATLAB.

Inversion of Time Lapse ERT data

2-D inverted resistivity section of pre-monsoon data shown in Figure 3 (a), exhibits high resistive channel (25-60 Ωm), with a lateral variation of 0-140m, also, originating from the top of the profile and extending till depth more than 30 m. However, the resistivity of the same channel is showing a decreasing trend (25-35 Ωm) in the post-monsoon resistivity section (refer to Figure 3(b)). A major contrast in resistivity is due to the percolation of groundwater through the porous media to rock matrix, causing the water saturation of the channel.

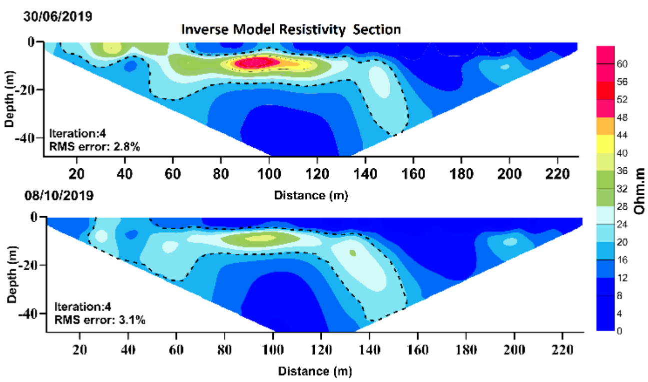

Figure 4. Inverse Model Resistivity Section (a) Pre (b) Post Monsoon processed using RES2DINV software.

To validate our results same field data is modelled in RES2DINV inverse modelling software based on the smoothness-constrained least-square scheme. A total of 4 numbers of iteration were carried out with a RMS error of 3.1% to achieve the near subsurface model. The demarcated zone shows the recharged ground water zone post monsoon same as interpreted through modelling.

Conclusions

The MATLAB algorithm developed in house and tested on the acquired time-lapse ERT data is showing close resemblance with the results obtained from the RES2DINV software. Also, We successfully demonstrated the approach of time-lapse monitoring of ERT data for identifying the potential conduit for recharging the subsurface porous strata.

References

Bharti, A.K., Pal, S.K., Saurabh, Singh, K.K.K., Singh, P.K., Prakash, A. and Tiwary, R.K. (2018b) Groundwater prospecting by inversion of cumulative data of Wenner-Schlumberger and dipole-dipole arrays: a case study at Turamdih, Jharkhand, India. Jour. Earth Syst. Sci. v.128(4):107.

Dey, A, and Morrison, H.F., Resistivity modelling for arbitrary shaped two-dimensional structures, 1979, *Geophysical Prospecting*, vol.27, pp. 106-136.

Lanczos, C. (1960). Linear differential operators. D. Van Nostrad Company Ltd.

Lawson, C.L. and Hanson, R.J. (1974) Solving Least Squares Problems. Prentice-Hall, New York

Constable, S. Parker, R., and Constable C. (1987). Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data. Geophysics, 52,289-300.

Loke, M.H. and Barker, R.D., Rapid least-squares inversion of apparent resistivity pseudo sections using a quasi-Newton method, 1996, *Geophysical Prospecting*, vol. 44, pp. 131-152.

Loke, M.H., Tutorial: 2-D and 3-D electrical imaging surveys, 2004.

Lowry, T., Allen, M.B. & Shive, P.N., 1989. Singularity removal: a refinement of resistivity modeling techniques, Geophysics, 54(6), 766–774.

Xu, Shi-zhe., Selection of the wavenumbers k using an optimization method for the inverse Fourier transform in 2.5D electrical modelling, 2000, *Geophysical Prospecting*, vol. 48, pp. 789-796.

View publication stats