Part 4

INTRODUCTION TO FUZZY LOGIC, CLASSICAL SETS AND FUZZY SETS

FUZZY LOGIC

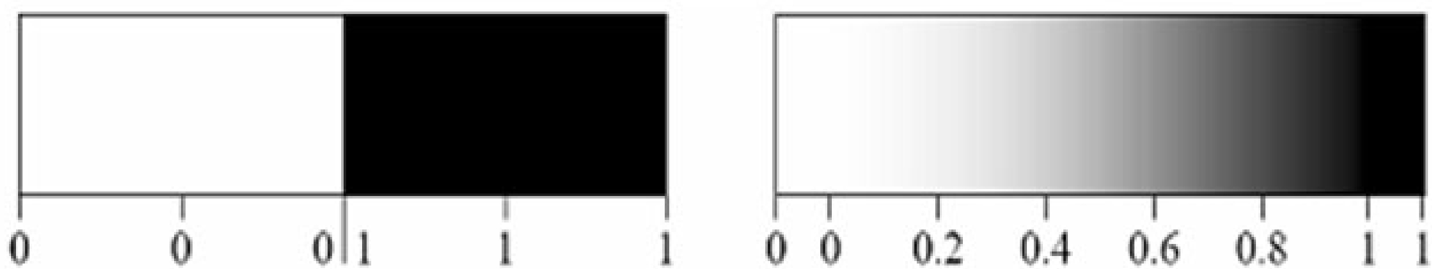
- Fuzzy logic is **the logic** underlying **approximate**, rather than exact, **modes of reasoning**.

- It is an extension of multivalued logic: **Everything**, including truth, **is a matter of degree**.

- It contains as special cases **not only** the classical two-value logic and multivalue logic systems, **but also** probabilistic logic.

- A proposition ***p*** has a **truth value**
  - 0 or 1 in two-value system,
  - element of a set T in multivalue system,
  - **Range over the fuzzy subsets of T** in fuzzy logic.

- Boolean logic uses sharp distinctions.
- Fuzzy logic reflects how people think.



(a) Boolean Logic. (b) Multi-valued Logic.

- Fuzzy logic is a set of mathematical principles for knowledge representation and reasoning based on degrees of membership.

NEED OF FUZZY LOGIC

- Based on intuition and judgment.
- No need for a mathematical model.
- Provides a smooth transition between members and nonmembers.
- Relatively simple, fast and adaptive.
- Less sensitive to system fluctuations.
- Can implement design objectives, difficult to express mathematically, in linguistic or descriptive rules.

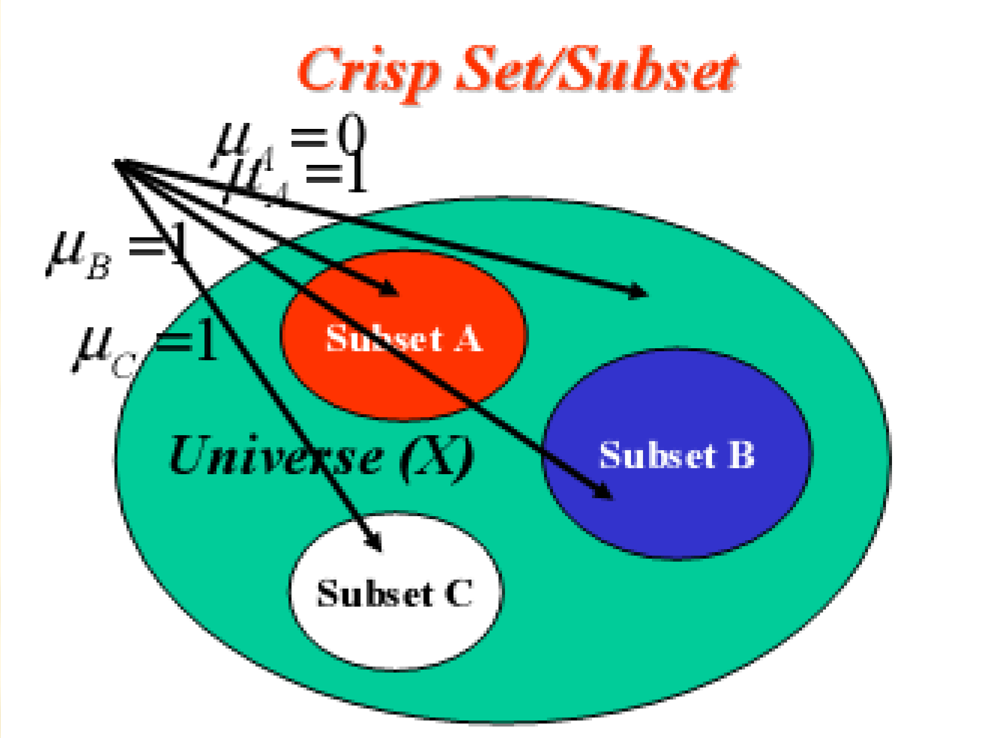
CLASSICAL SETS (CRISP SETS)

Conventional or crisp sets are Binary. An element either belongs to the set or does not.

{True, False}

{1, 0}

CRISP SETS



Crisp Set/Subset
$\mu_A = 0$
$\mu_A = 1$
$\mu_B = 1$
$\mu_C = 1$
Subset A
Subset B
Subset C
Universe (X)

OPERATIONS ON CRISP SETS

- UNION: $A \cup B = \{x | x \in A \text{ or } x \in B\}$
- INTERSECTION: $A \cap B = \{x | x \in A \text{ and } x \in B\}$
- COMPLEMENT: $\bar{A} = \{x | x \notin A, x \in X\}$
- DIFFERENCE:

$$\begin{aligned} A|B \text{ or } (A - B) &= \{x | x \in A \text{ and } x \notin B\} \\ &= A - (A \cap B) \end{aligned}$$

PROPERTIES OF CRISP SETS

The various properties of crisp sets are as follows:

1. Commutativity

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

2. Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

3. Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

4. Idempotency

$$A \cup A = A$$
$$A \cap A = A$$

5. Transitivity

$$\text{If } A \subseteq B \subseteq C, \text{ then } A \subseteq C$$

6. Identity

$$A \cup \phi = A, \quad A \cap \phi = \phi$$
$$A \cap X = A, \quad A \cap X = X$$

7. Involution (double negation)

$$\bar{\bar{A}} = A$$

8. Law of excluded middle

Two point crossover

$$A \cup \bar{A} = X$$

9. Law of contradiction

$$A \cap \bar{A} = \phi$$

10. DeMorgan's law

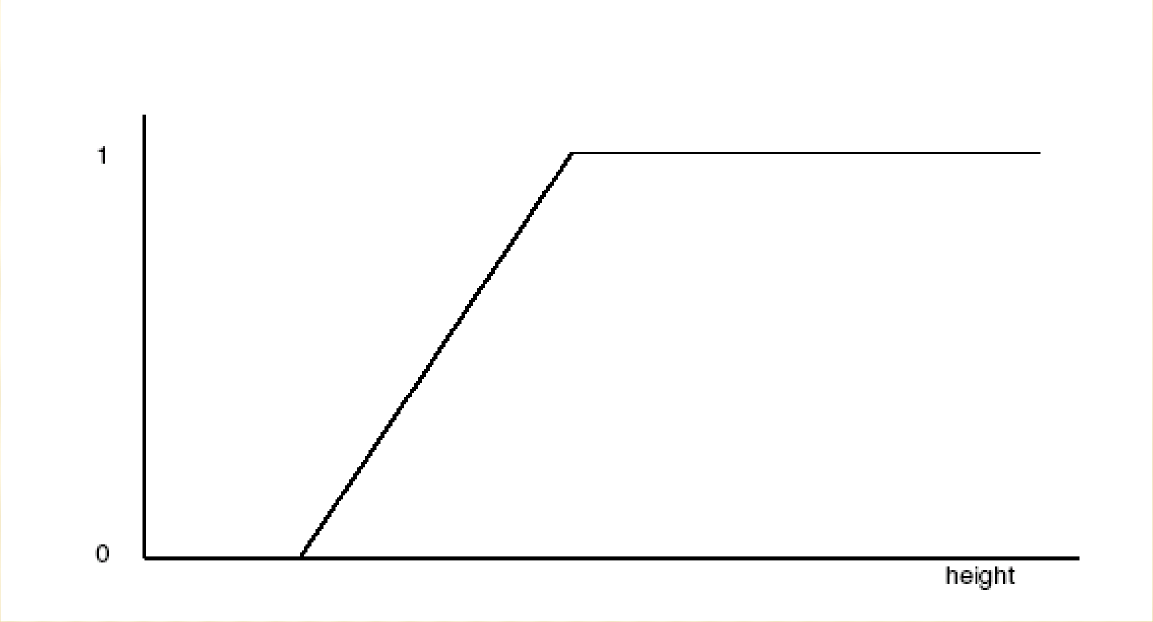
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

FUZZY LOGIC THEN . . .

- is particularly good at handling uncertainty, vagueness and imprecision.
- especially useful where a problem can be described linguistically (using words).
- Applications include:
  - robotics
  - washing machine control
  - nuclear reactors
  - focusing a camcorder
  - information retrieval
  - train scheduling

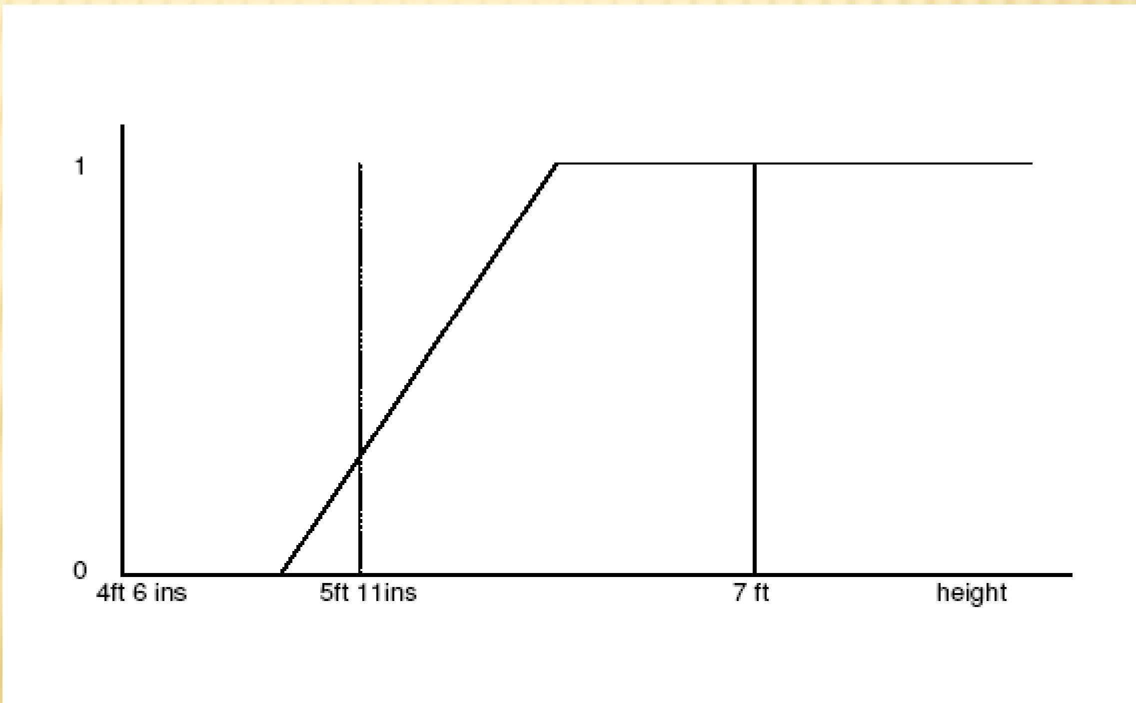
FUZZY SETS

- The shape you see is known as the membership function

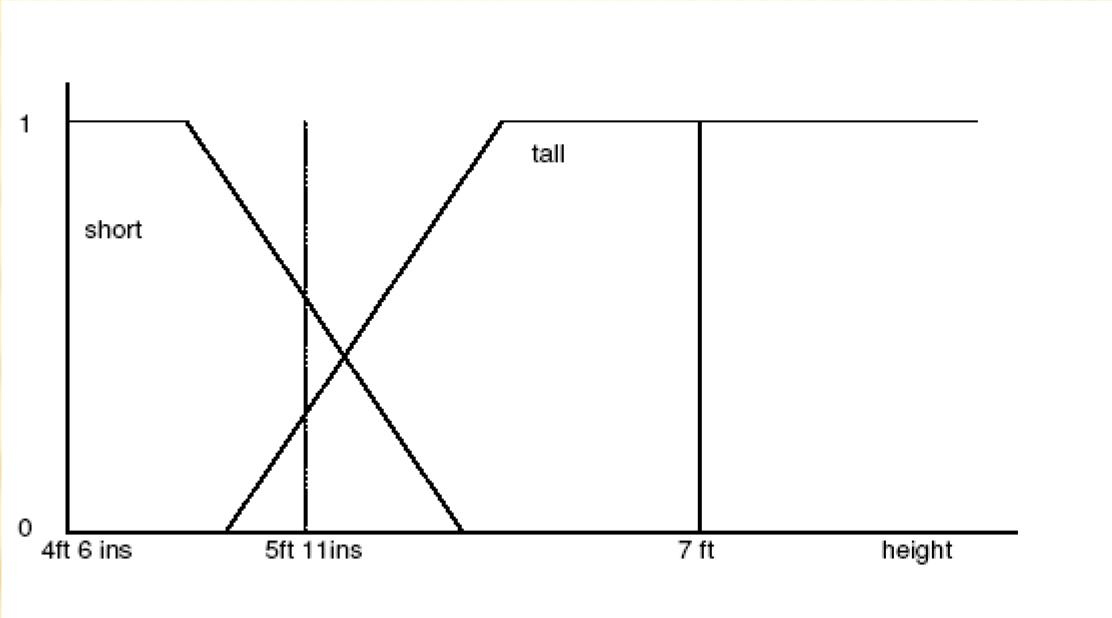


FUZZY SETS

- Now we have added some possible values on the height - axis



FUZZY SETS



Shows two membership functions: 'tall' and 'short'

NOTATION

- For any fuzzy set, A, the function μ_A represents the membership function for which $\mu_A(x)$ indicates the degree of membership of x (of the universal set X) in set A. It is usually expressed as a number between 0 and 1:

$$\mu_A(x) : X \to [0, 1]$$

NOTATION

For the member, x, of a discrete set with membership μ we use the notation μ/x . In other words, x is a member of the set to degree μ. Discrete sets are written as:

$$A = \mu_1/x_1 + \mu_2/x_2 + \dots\dots\dots + \mu_n/x_n$$

Or

$$A = \sum_{i=1,n} \mu_i/x_i$$

where x_1, $x_2 \ldots x_n$ are members of the set A and μ_1, μ_2, ..., μ_n are their degrees of membership. A continuous fuzzy set A is written as:

$$A = \int_X \mu(x)/x.$$

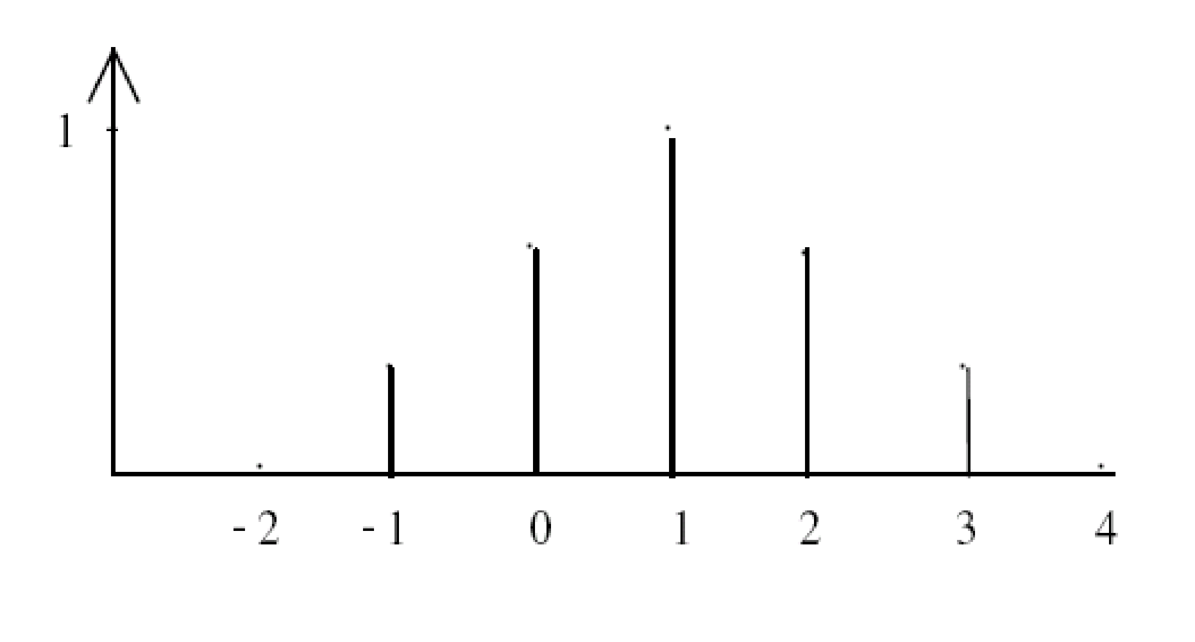
FUZZY SETS

- The members of a fuzzy set are members to some degree, known as a membership grade or degree of membership.
- The membership grade is the degree of belonging to the fuzzy set. The larger the number (in [0,1]) the more the degree of belonging. (N.B. This is <u>not</u> a probability)
- The translation from x to $\mu_A(x)$ is known as **fuzzification.**
- A fuzzy set is either continuous or discrete.
- Graphical representation of membership functions is very useful.

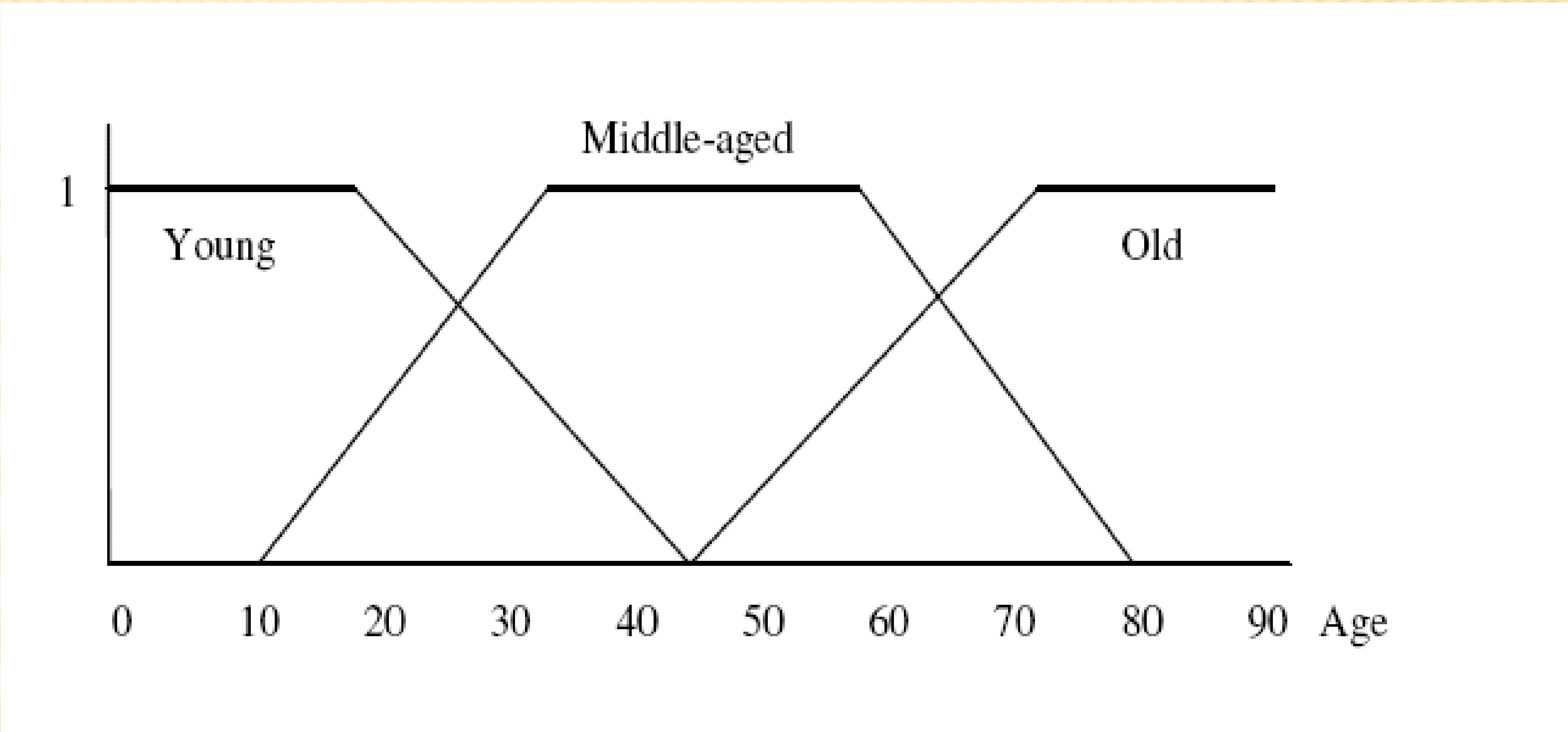
FUZZY SETS - EXAMPLE

"numbers close to 1"

$$A = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$$

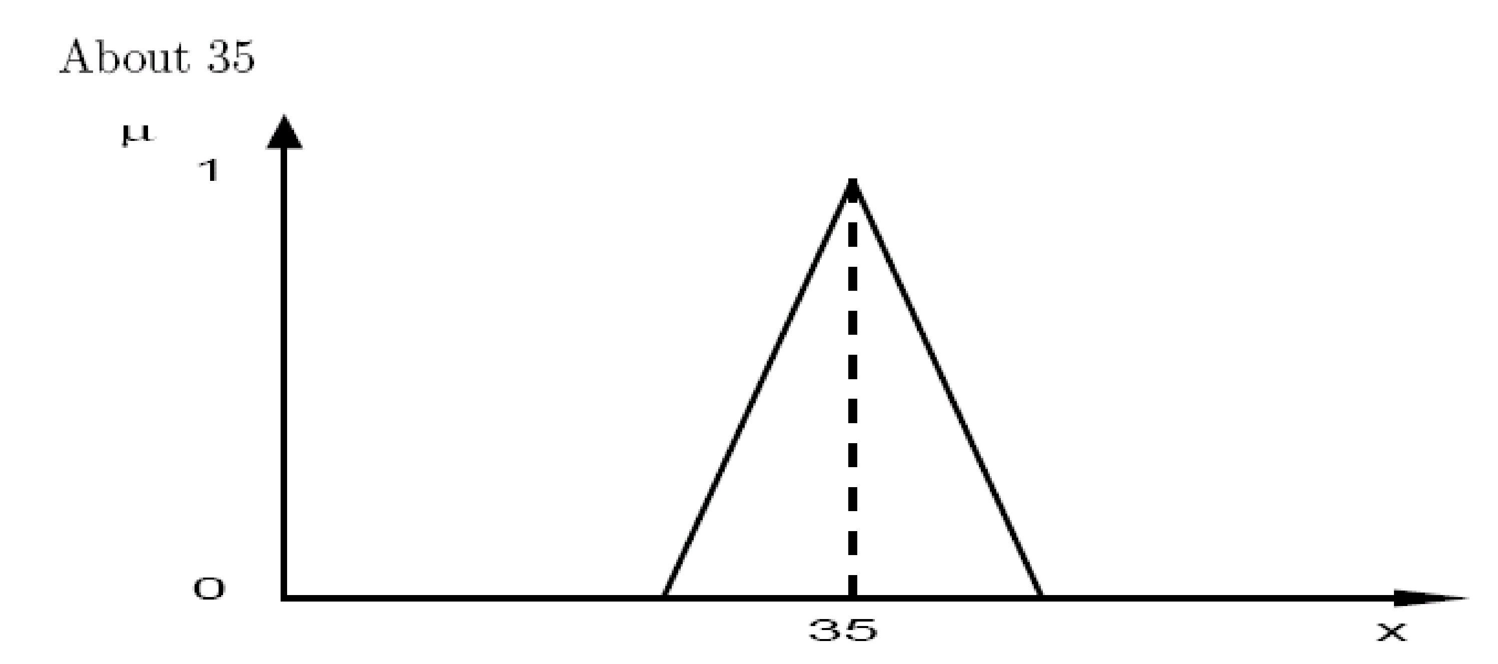


FUZZY SETS - EXAMPLE



Again, notice the overlapping of the sets reflecting the real world more accurately than if we were using a traditional approach.

IMPRECISION

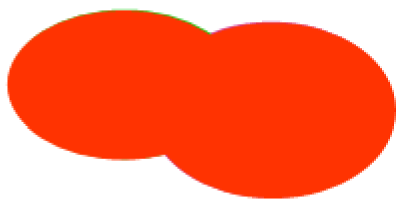


Words are used to capture imprecise notions, loose concepts or perceptions.

OPERATIONS ON FUZZY SETS

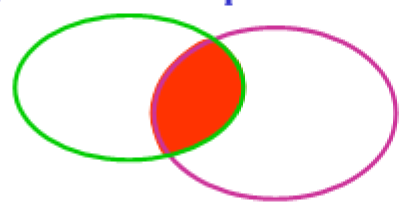
- Union: $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
- Intersection: $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$
- Complement: $\mu_{\neg A}(x) = 1 - \mu_A(x)$

Fuzzy union operation or fuzzy *OR*



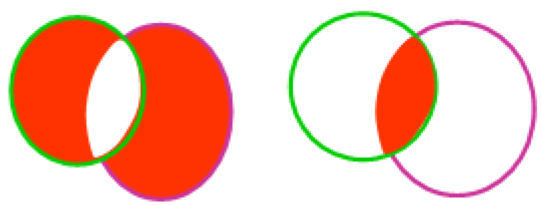
$$\mu_{A+B}(x) = \max[\mu_A(x), \mu_B(x)]$$

Fuzzy intersection operation or fuzzy *AND*



$$\mu_{A \cdot B}(x) = \min[\mu_A(x), \mu_B(x)]$$

Complement operation



$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

PROPERTIES OF FUZZY SETS

The same as for crisp sets

- Commutativity
- Associativity
- Distributivity
- Idempotency
- Identity
- De Morgan's Laws
- …

1. Commutativity

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \underset{\sim}{B} \cup \underset{\sim}{A}$$
$$\underset{\sim}{A} \cap \underset{\sim}{B} = \underset{\sim}{B} \cap \underset{\sim}{A}$$

2. Associativity

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cup \underset{\sim}{C}$$
$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cap \underset{\sim}{C}$$

3. Distributivity

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cap (\underset{\sim}{A} \cup \underset{\sim}{C})$$
$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cup (\underset{\sim}{A} \cap \underset{\sim}{C})$$

4. Idempotency

$$\underset{\sim}{A} \cup \underset{\sim}{A} = \underset{\sim}{A}$$
$$\underset{\sim}{A} \cap \underset{\sim}{A} = \underset{\sim}{A}$$

5. Identity

$$\underset{\sim}{A} \cup \phi = \underset{\sim}{A} \text{ and } \underset{\sim}{A} \cup U = U \text{(universal set)}$$
$$\underset{\sim}{A} \cap \phi = \phi \text{ and } \underset{\sim}{A} \cap U = \underset{\sim}{A}$$

6. Involution (double negation)

$$\overline{\overline{\underset{\sim}{A}}} = \underset{\sim}{A}$$

7. Transitivity

$$\text{If } \underset{\sim}{A} \subseteq \underset{\sim}{B} \subseteq \underset{\sim}{C}, \text{ then } \underset{\sim}{A} \subseteq \underset{\sim}{C}$$

8. Demorgan's law

$$\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}}$$

$$\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}$$

RELATIONS

- Relations represent mappings between sets and connectives in logic.
- A classical binary relation represents the presence or absence of a connection or interaction or association between the elements of two sets.
- Fuzzy binary relations are a generalization of crisp binary relations, and they allow various degrees of relationship (association) between elements.

CRISP CARTESIAN PRODUCT

Lets consider properties of crisp relations first and then extend the mechanism to fuzzy sets.

Definition of (crisp) Product set: Let A and B be two non-empty sets, the product set or Cartesian product $A \times B$ is defined as follows,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

(a set of ordered pairs a,b)

CRISP RELATIONS

Cartesian product of *n* sets

$$A_1 \times A_2 \times \ldots \times A_n = \prod_{i=1}^{n} A_i = \{(a_1, \ldots, a_n) \mid a_1 \in A_1, \ldots, a_n \in A_n\}$$

Definition of Binary Relation

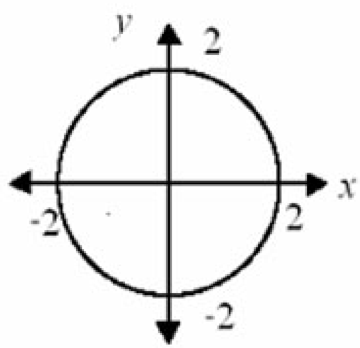
If A and B are two sets and there is a specific property between elements x of A and y of B, this property can be described using the ordered pair (x, y). A set of such (x, y) pairs, $x \in A$ and $y \in B$, is called a relation R.

$$R = \{ (x,y) \mid x \in A, y \in B \}$$

CRISP BINARY RELATIONS

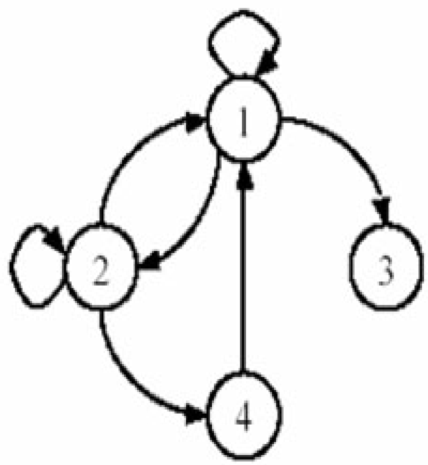
Examples of binary relations

Coordinate diagram

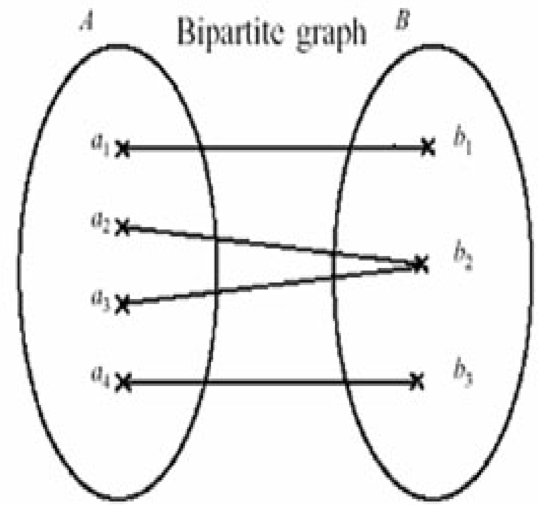


Relation of $x^2 + y^2 = 4$

Binary relation from A to B



Bipartite graph



Relation matrix

R	b_1	b_2	b_3
a_1	1	0	0
a_2	0	1	0
a_3	0	1	0
a_4	0	0	1

OPERATIONS ON CRISP RELATIONS

Function-theoretic operations for the two crisp relations (R, S) are defined as follows:

1. Union

$$R \cup S \to \chi_{R \cup S}(x, y) : \chi_{R \cup S}(x, y) = \max\left[\chi_R(x, y), \chi_S(x, y)\right]$$

2. Intersection

$$R \cap S \to \chi_{R \cap S}(x, y) : \chi_{R \cap S}(x, y) = \min\left[\chi_R(x, y), \chi_S(x, y)\right]$$

3. Complement

$$\overline{R} \to \chi_{\overline{R}}(x, y) : \chi_{\overline{R}}(x, y) = 1 - \chi_{\overline{R}}(x, y)$$

4. Containment

$$R \subset S \to \chi_R(x, y) : \chi_R(x, y) \leq \chi_S(x, y)$$

5. Identity

$$\phi \to \phi_R \text{ and } X \to E_R$$

PROPERTIES OF CRISP RELATIONS

The properties of crisp sets (given below) hold good for crisp relations as well.

- Commutativity,
- Associativity,
- Distributivity,
- Involution,
- Idempotency,
- DeMorgan's Law,
- Excluded Middle Laws.

COMPOSITION ON CRISP RELATIONS

The composition operations are of two types:

1. Max-min composition
2. Max-product composition.

The max-min composition is defined by the function theoretic expression as

$$T = R \circ S$$

$$\chi_T(x, z) = \bigvee_{y \in Y} [\chi_R(x, y) \wedge \chi_S(y, z)]$$

The max-product composition is defined by the function theoretic expression as

$$T = R \circ S$$

$$\chi_T(x, z) = \bigvee_{y \in Y} [\chi_R(x, y) \cdot \chi_S(y, z)]$$

FUZZY Relation

Let R be a fuzzy subset of M and S be a fuzzy subset of N. Then the Cartesian product $R \times S$ is a fuzzy subset of $N \times M$ such that

$$\forall \vec{x} \in M, \vec{y} \in N \; \mu_{R \times S}(\vec{x}, \vec{y}) = \min\left(\mu_R(\vec{x}), \mu_S(\vec{y})\right)$$

Example:

Let R be a fuzzy subset of {a, b, c} such that R = a/1 + b/0.8 + c/0.2 and S be a fuzzy subset of {1, 2, 3} such that S = 1/1 + 3/0.8 + 2/0.5. Then fuzzy relation R x S is given by

$$\begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0.5 & 0.9 \\ 0.8 & 0.5 & 0.8 \\ 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

FUZZY RELATION

A fuzzy relation R is a mapping from the Cartesian space X x Y to the interval [0,1], where the strength of the mapping is expressed by the membership function of the relation $\mu_R(x,y)$

$$\mu_R : A \times B \to [0, 1]$$

$$R = \{((x, y), \mu_R(x, y)) \mid \mu_R(x, y) \geq 0 \ , \ \ x \in A, \ \ y \in B\}$$

Crisp relation vs. Fuzzy relation

(a) Crisp relation (b) Fuzzy relation

Corresponding fuzzy relation matrix

A \ A	a	b	c	d
a	0.0	0.0	0.8	0.0
b	1.0	0.0	0.0	0.0
c	0.0	0.9	0.0	1.0
d	0.0	0.0	0.0	0.0

OPERATIONS ON FUZZY RELATION

The basic operation on fuzzy sets also apply on fuzzy relations.

1. Union:

$$\mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x, y) = \max\left[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)\right]$$

2. Intersection:

$$\mu_{\underset{\sim}{R} \cap \underset{\sim}{S}}(x, y) = \min\left[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)\right]$$

3. Complement:

$$\mu_{\overline{\underset{\sim}{R}}}(x, y) = 1 - \mu_{\underset{\sim}{R}}(x, y)$$

4. Containment:

$$\underset{\sim}{R} \subset \underset{\sim}{S} \Rightarrow \mu_{\underset{\sim}{R}}(x, y) \leq \mu_{\underset{\sim}{S}}(x, y)$$

5. Inverse:

The inverse of a fuzzy relation R on $X \times Y$ is denoted by R^{-1}. It is a relation on $Y \times X$ defined by $R^{-1}(y, x) = R(x, y)$ for all pairs $(y, x) \in Y \times X$.

6. Projection:

For a fuzzy relation $R(X, Y)$, let $[R \downarrow Y]$ denote the projection of R onto Y. Then $[R \downarrow Y]$ is a fuzzy relation in Y whose membership function is defined by

$$\mu_{[R\downarrow Y]}(x, y) = \max_{x} \mu_{\underset{\sim}{R}}(x, y)$$

The projection concept can be extended to an n-ary relation $R(x_1, x_2, \ldots, x_n)$.

PROPERTIES OF FUZZY RELATIONS

The properties of fuzzy sets (given below) hold good for fuzzy relations as well.

- Commutativity,
- Associativity,
- Distributivity,
- Involution,
- Idempotency,
- DeMorgan's Law,
- Excluded Middle Laws.

COMPOSITION OF FUZZY RELATIONS

Two fuzzy relations R and S are defined on sets A, B and C. That is, $R \subseteq A \times B$, $S \subseteq B \times C$. The composition $S \bullet R = SR$ of two relations R and S is expressed by the relation from A to C:

For $(x, y) \in A \times B$, $(y, z) \in B \times C$,

$$\mu_{S \bullet R}(x, z) = \max_{y} [\min(\mu_R(x, y), \mu_S(y, z))]$$

$$= \vee_{y} [\mu_R(x, y) \wedge \mu_S(y, z)]$$

$M_{S \bullet R} = M_R \bullet M_S$ (matrix notation)

(max-min composition)

Example:

$$X=\{x_1,x_2\},Y=\{y_1,y_2\},\quad \text{and} \quad Z=\{z_1,z_2,z_3\}$$

Consider the following fuzzy relations:

$$\tilde{R}=\begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix}\end{array} \quad \text{and} \quad \tilde{S}=\begin{array}{c} \\ y_1 \\ y_2 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.9 & 0.6 & 0.5 \\ 0.1 & 0.7 & 0.5 \end{bmatrix}\end{array}$$

Using max-min composition,

$$\left.\begin{aligned} \mu_{\tilde{T}}(x_1,z_1) &= \vee_{y\in Y}(\mu_{\tilde{R}}(x_1,y)\wedge\mu_{\tilde{S}}(y,z_1)) \\ &= \max[\min(0.7,0.9),\min(0.5,0.1)] \end{aligned}\right\} \quad \tilde{T}=\begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.7 & 0.6 & 0.5 \\ 0.8 & 0.6 & 0.4 \end{bmatrix}\end{array}$$

$$=0.7$$

Two fuzzy relations R and S are defined on sets A, B and C. That is, $R \subseteq A \times B$, $S \subseteq B \times C$. The composition $S \bullet R = SR$ of two relations R and S is expressed by the relation from A to C:

For $(x, y) \in A \times B$, $(y, z) \in B \times C$,

$$\mu_{S \bullet R}(x, z) = \max_{y} [\mu_R(x, y) \bullet \mu_S(y, z)]$$

$$= \vee_{y} [\mu_R(x, y) \bullet \mu_S(y, z)]$$

$M_{S \bullet R} = M_R \bullet M_S$ (matrix notation)

(max-product composition)

Max-product example:

$$X = \{x_1, x_2\},\quad Y = \{y_1, y_2\},\ \text{and}\ \ Z = \{z_1, z_2, z_3\}$$

Consider the following fuzzy relations:

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \end{array} \quad \text{and} \quad \tilde{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.9 & 0.6 & 0.5 \\ 0.1 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

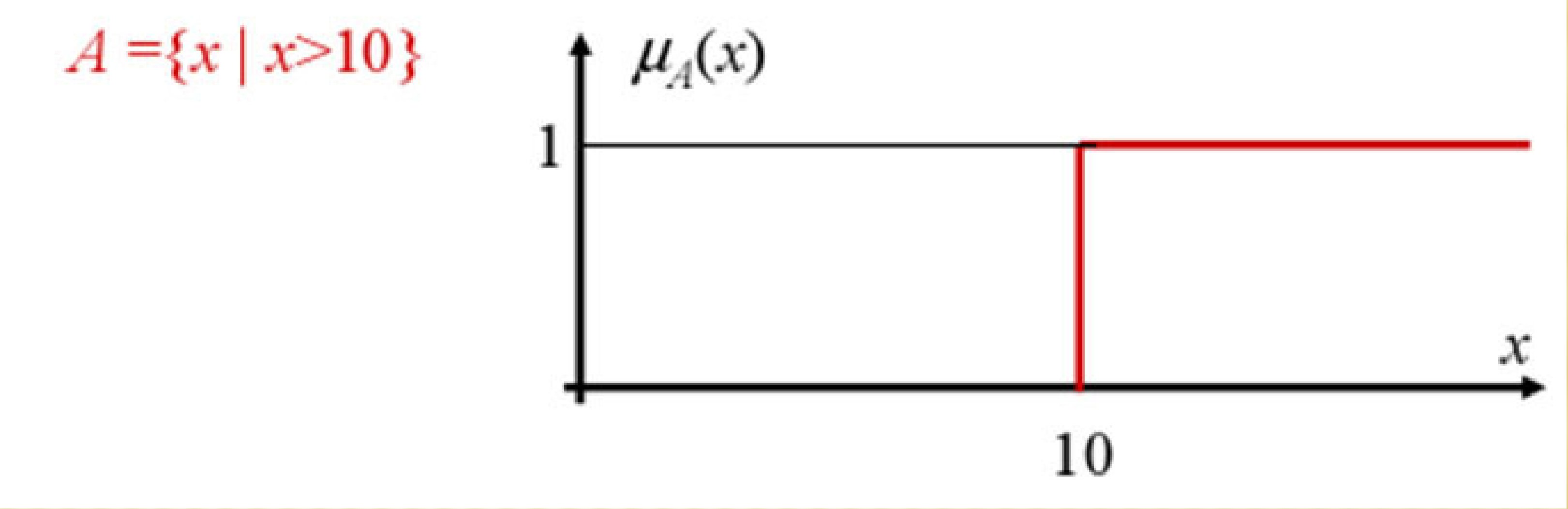
Using max-product composition,

$$\left.\begin{aligned} \mu_{\tilde{T}}(x_2, z_2) &= \bigvee_{y \in Y} (\mu_{\tilde{R}}(x_2, y) \bullet \mu_{\tilde{S}}(y, z_2)) \\ &= \max[(0.8, 0.6), (0.4, 0.7)] \\ &= 0.48 \end{aligned}\right\} \quad \tilde{T} = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} .63 & .42 & .25 \\ .72 & .48 & .20 \end{bmatrix} \end{array}$$

MEMBERSHIP FUNCTIONS

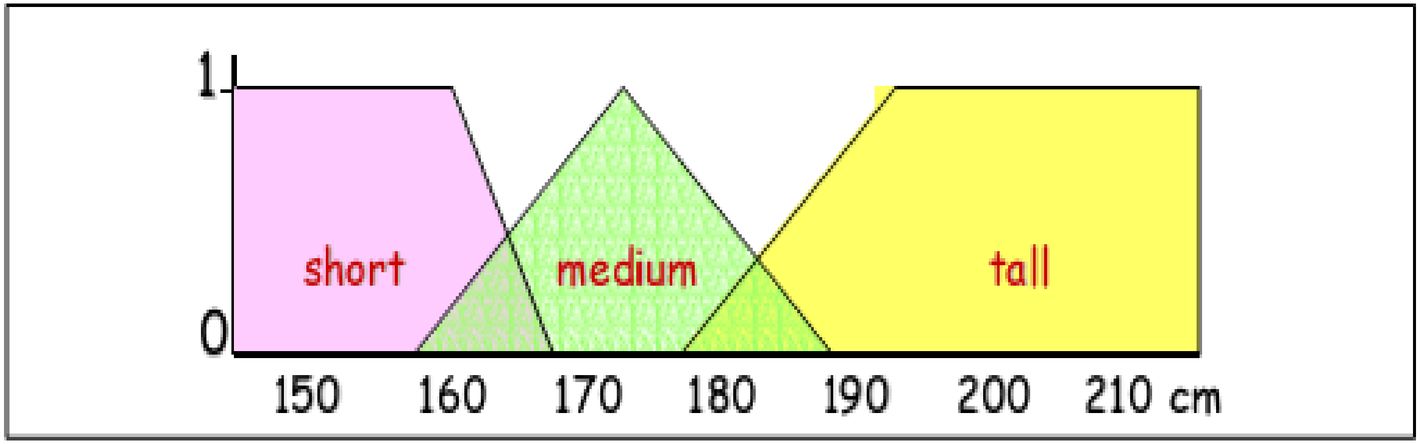
CRISP MEMBERSHIP FUCNTIONS

- Crisp membership functions (μ) are either one or zero.
- Consider the example: Numbers greater than 10. The membership curve for the set A is given by



REPRESENTING A DOMAIN IN FUZZY LOGIC

Fuzzy sets (men's height):



FUZZY MEMBERSHIP FUCNTIONS

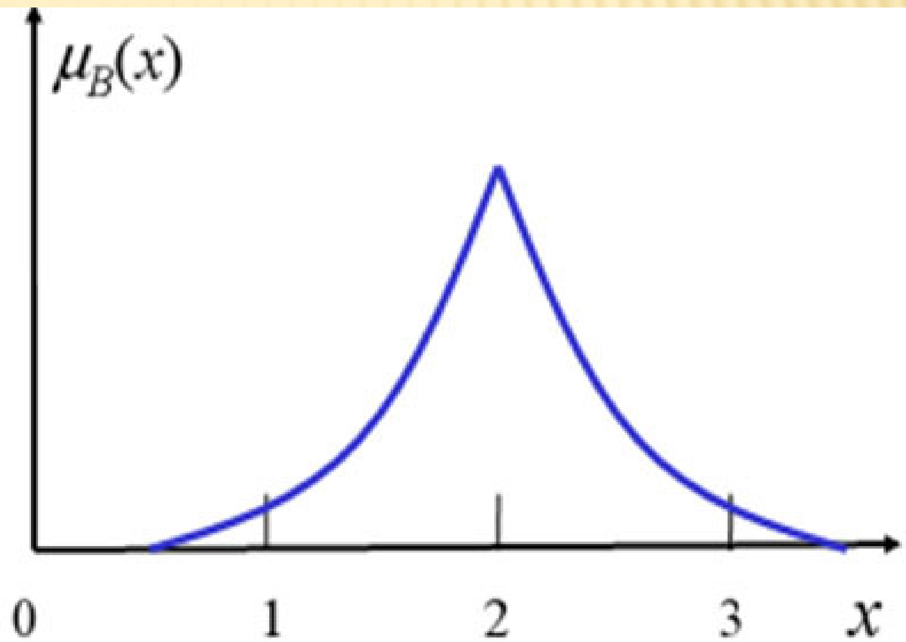
- Categorization of element x into a set A
  described through a membership function $\mu_A(x)$
- Formally, given a fuzzy set A of universe X
  $\mu_A(x): X \to [0,1]$, where
  $\mu_A(x) = 1$ if x is totally in A
  $\mu_A(x) = 0$ if x is totally not in A
  $0 < \mu_A(x) < 1$ if x is partially in A

  $\mu_{Tall}(200) = 1$
  $\mu_{Tall}(160) = 0$
  $0 < \mu_{Tall}(180) < 1$
- (Discrete) Fuzzy set A is represented as:
  $A = \{\mu_A(x_1)/x_1, \mu_A(x_2)/x_2, \ldots, \mu_A(x_n)/x_n\}$

  Tall = {0/160, 0.2/170, 0.8/180, 1/190}

The set B of numbers approaching 2 can be represented by the membership function

$$\mu_B(x) = e^{-|x-2|}$$

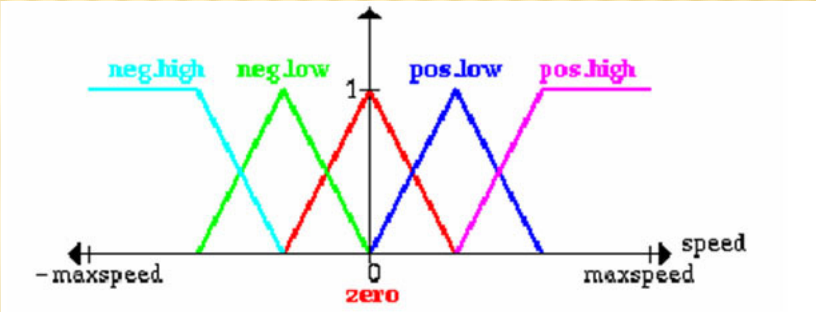


LINGUISTIC VARIABLE

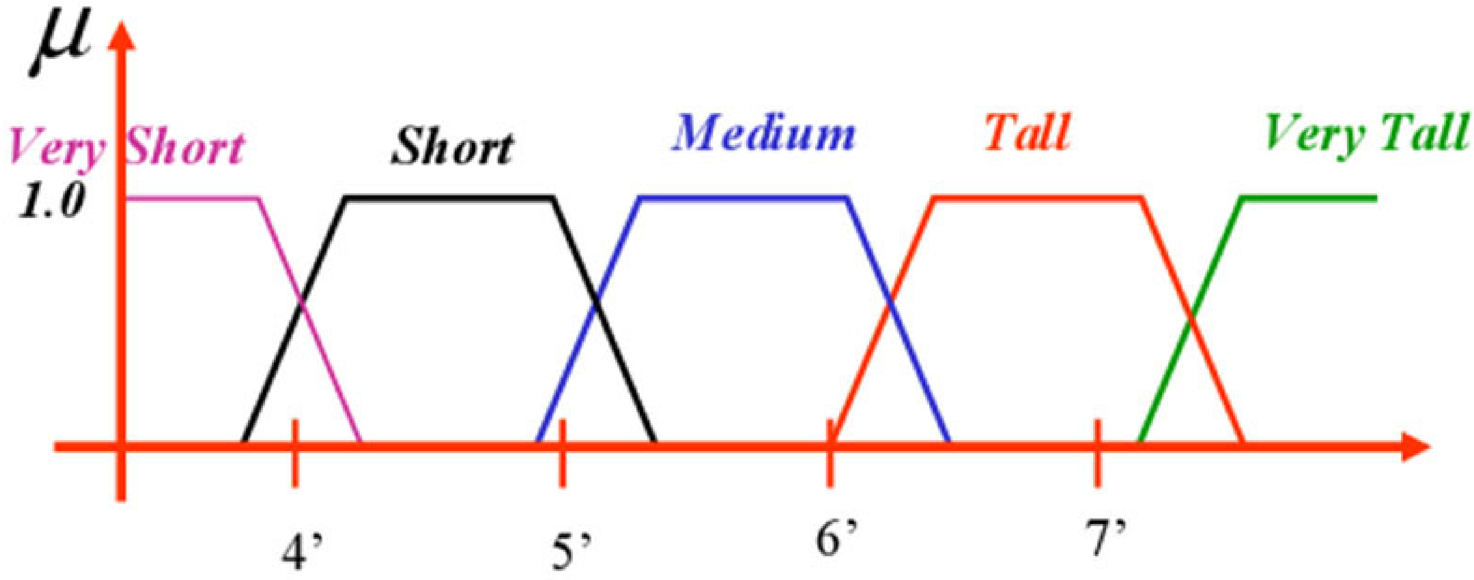
- Let x be a linguistic variable with the label "speed".
- Terms of x, which are fuzzy sets, could be "positive low", "negative high" from the term set T:

T = {PostiveHigh, PositiveLow, NegativeLow, NegativeHigh, Zero}

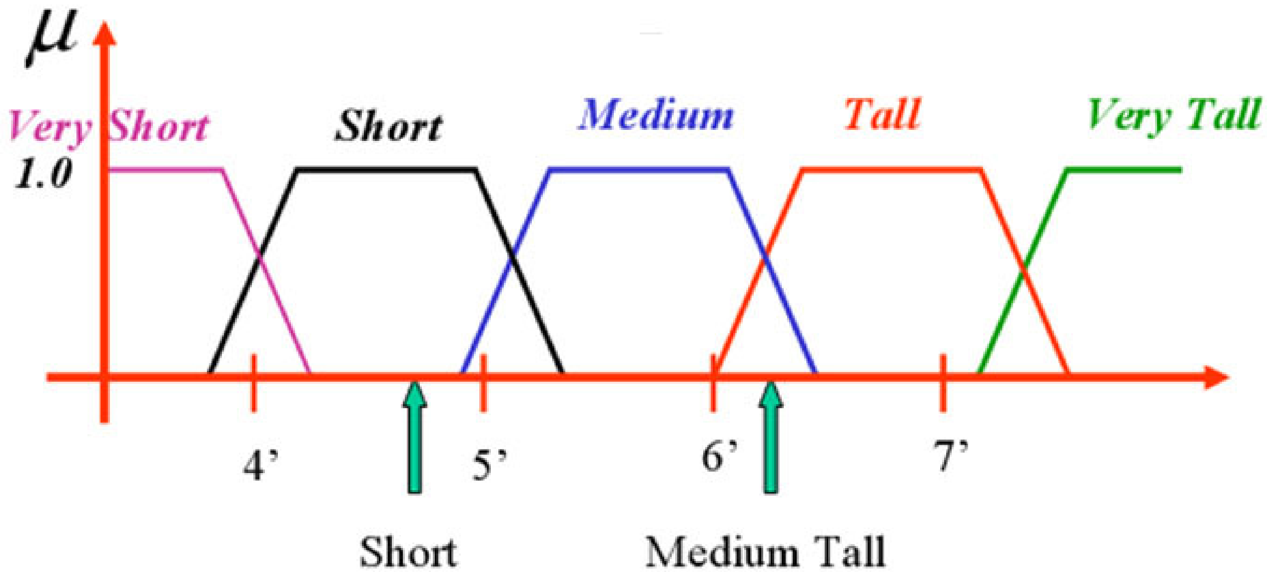
- Each term is a fuzzy variable defined on the base variable which might be the scale of all relevant velocities.



MEMBERSHIP FUCNTIONS



$$\mu = \left[\mu_{vs}, \mu_{s}, \mu_{m}, \mu_{t}, \mu_{vt}\right]$$



$$\mu = [0,\ 1,\ 0,\ 0,\ 0]$$

$$\mu = [0,\ 0,\ 0.5,\ 0.5,\ 0]$$

FEATURES OF MEMBERSHIP FUNCTIONS

- CORE:

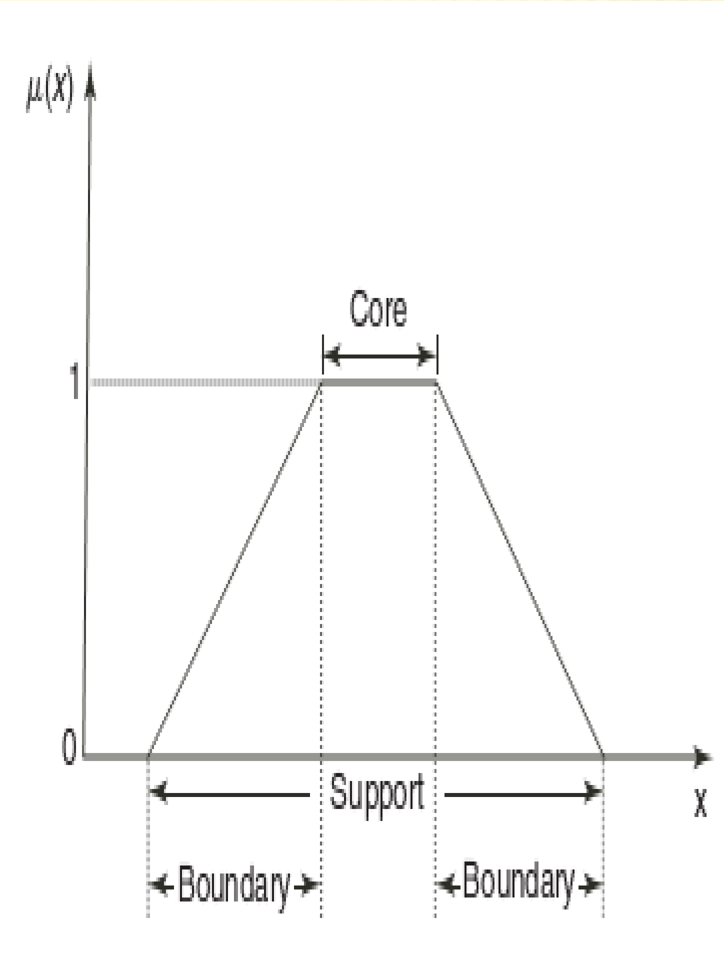
$$\mu_{\underset{\sim}{A}}(x) = 1$$

- SUPPORT:

- BOUNDARY:

$$\mu_{\underset{\sim}{A}}(x) > 0$$

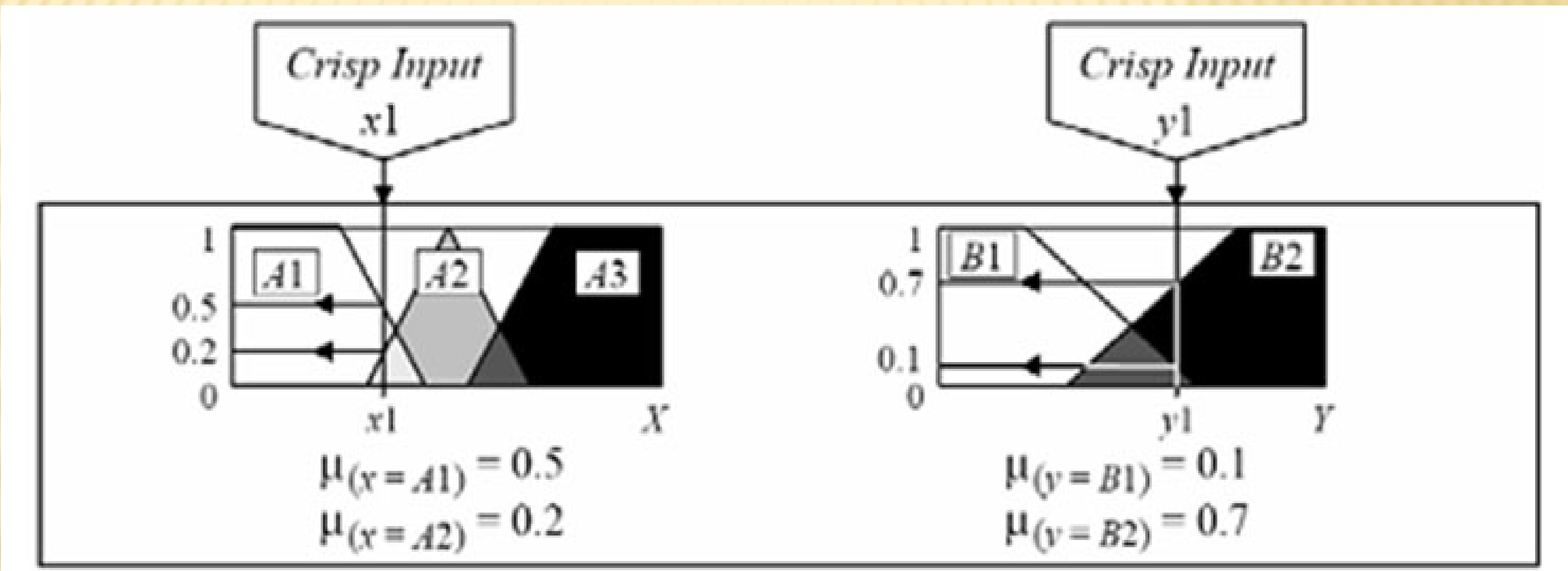
$$0 < \mu_{\underset{\sim}{A}}(x) < 1$$



FUZZIFICATION

- Fuzzifier converts a crisp input into a fuzzy variable.
- Definition of the membership functions must
  - reflects the designer's knowledge
  - provides smooth transition between member and nonmembers of a fuzzy set
  - simple to calculate
- Typical shapes of the membership function are Gaussian, trapezoidal and triangular.

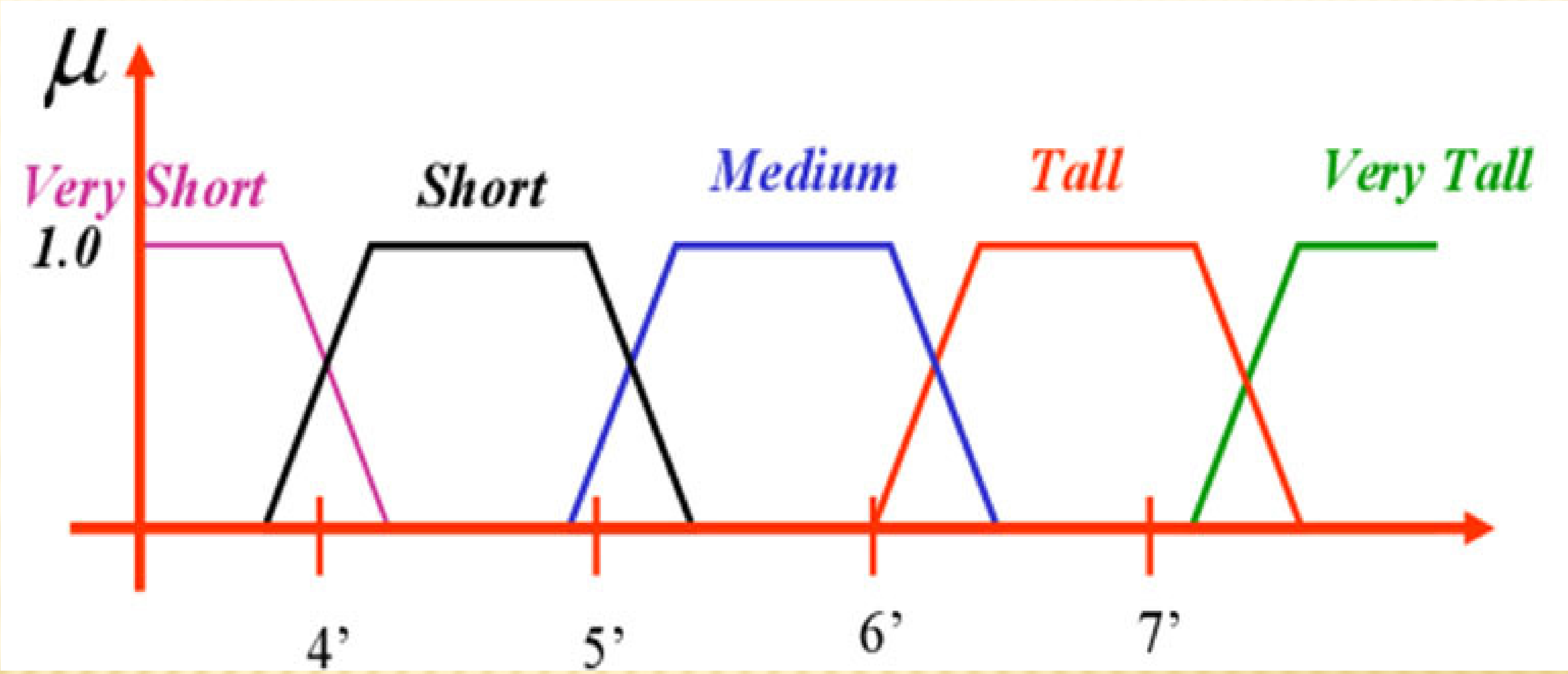
- Use crisp inputs from the user.
- Determine membership values for all the relevant classes (i.e., in right Universe of Discourse).



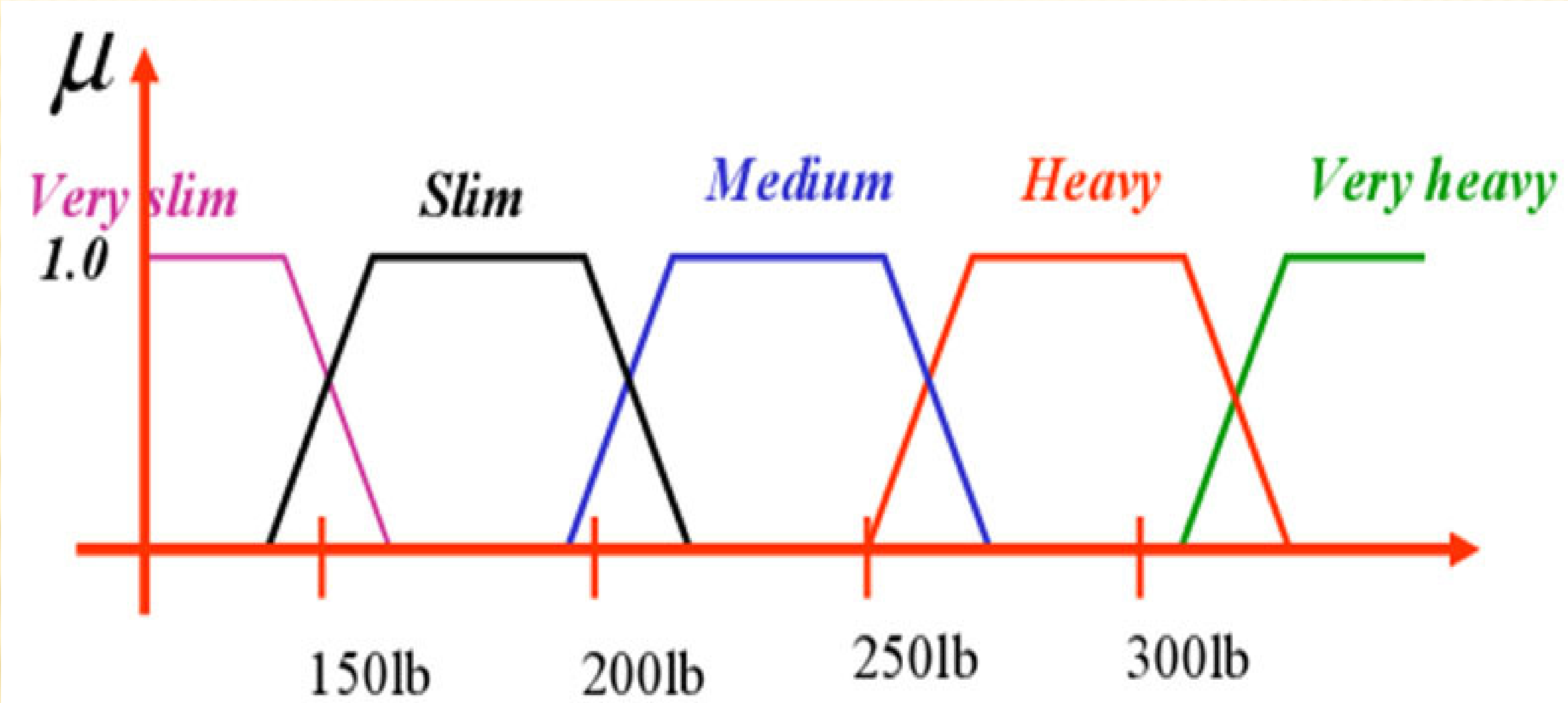
EXAMPLE - FUZZIFICATION

- Assume we want to evaluate the health of a person based on his height and weight.
- The input variables are the crisp numbers of the person's height and weight.
- Fuzzification is a process by which the numbers are changes into linguistic words

FUZZIFICATION OF HEIGHT



FUZZIFICATION OF WEIGHT



LAMBDA CUT FOR FUZZY SETS

Consider a fuzzy set $\underset{\sim}{A}$. The set $A_\lambda (0 < \lambda < 1)$ called the lambda (λ)-cut (or alpha [α]-cut) set is a crisp set of the fuzzy set and is defined as follows:

$$A_\lambda = \{x \mid \mu_{\underset{\sim}{A}}(x) \geq \lambda\}; \quad \lambda \in [0, 1]$$

The properties of λ-cut sets are as follows:

1. $(\underset{\sim}{A} \cup \underset{\sim}{B})_\lambda = A_\lambda \cup B_\lambda$
2. $(\underset{\sim}{A} \cap \underset{\sim}{B})_\lambda = A_\lambda \cap B_\lambda$
3. $(\bar{\underset{\sim}{A}})_\lambda \neq (\bar{A}_\lambda)$ except when $\lambda = 0.5$
4. For any $\lambda \leq \beta$, where $0 \leq \beta \leq 1$, it is true that $A_\beta \subseteq A_\lambda$, where $A_0 = X$.

LAMBDA CUT FOR FUZZY RELATIONS

Let $\underset{\sim}{R}$ be a fuzzy relation where each row of the relational matrix is considered a fuzzy set. The jth row in a fuzzy relation matrix $\underset{\sim}{R}$ denotes a discrete membership function for a fuzzy set $\underset{\sim}{R}_j$. A fuzzy relation can be converted into a crisp relation in the following manner:

$$R_\lambda = \{(x, y) | \mu_{\underset{\sim}{R}}(x, y) \geq \lambda\}$$

where R_λ is a λ-cut relation of the fuzzy relation $\underset{\sim}{R}$.

For two fuzzy relations $\underset{\sim}{R}$ and $\underset{\sim}{S}$ the following properties should hold:

1. $(\underset{\sim}{R} \cup \underset{\sim}{S})_\lambda = R_\lambda \cup S_\lambda$
2. $(\underset{\sim}{R} \cap \underset{\sim}{S})_\lambda = R_\lambda \cap S_\lambda$
3. $(\bar{\underset{\sim}{R}})_\lambda \neq (\bar{R}_\lambda)$ except when $\lambda = 0.5$
4. For any $\lambda \leq \beta$, where $0 \leq \beta \leq 1$, it is true that $R_\beta \subseteq R_\lambda$.

DEFUZZIFICATION

DEFUZZIFICATION

- Defuzzification is a mapping process from a space of fuzzy control actions defined over an output universe of discourse into a space of crisp (nonfuzzy) control actions.
- Defuzzification is a process of converting output fuzzy variable into a unique number.
- Defuzzification process has the capability to reduce a fuzzy set into a crisp single-valued quantity or into a crisp set; to convert a fuzzy matrix into a crisp matrix; or to convert a fuzzy number into a crisp number.

METHODS OF DEFUZZIFICATION

Defuzzification is the process of conversion of a fuzzy quantity into a precise quantity. Defuzzification methods include:

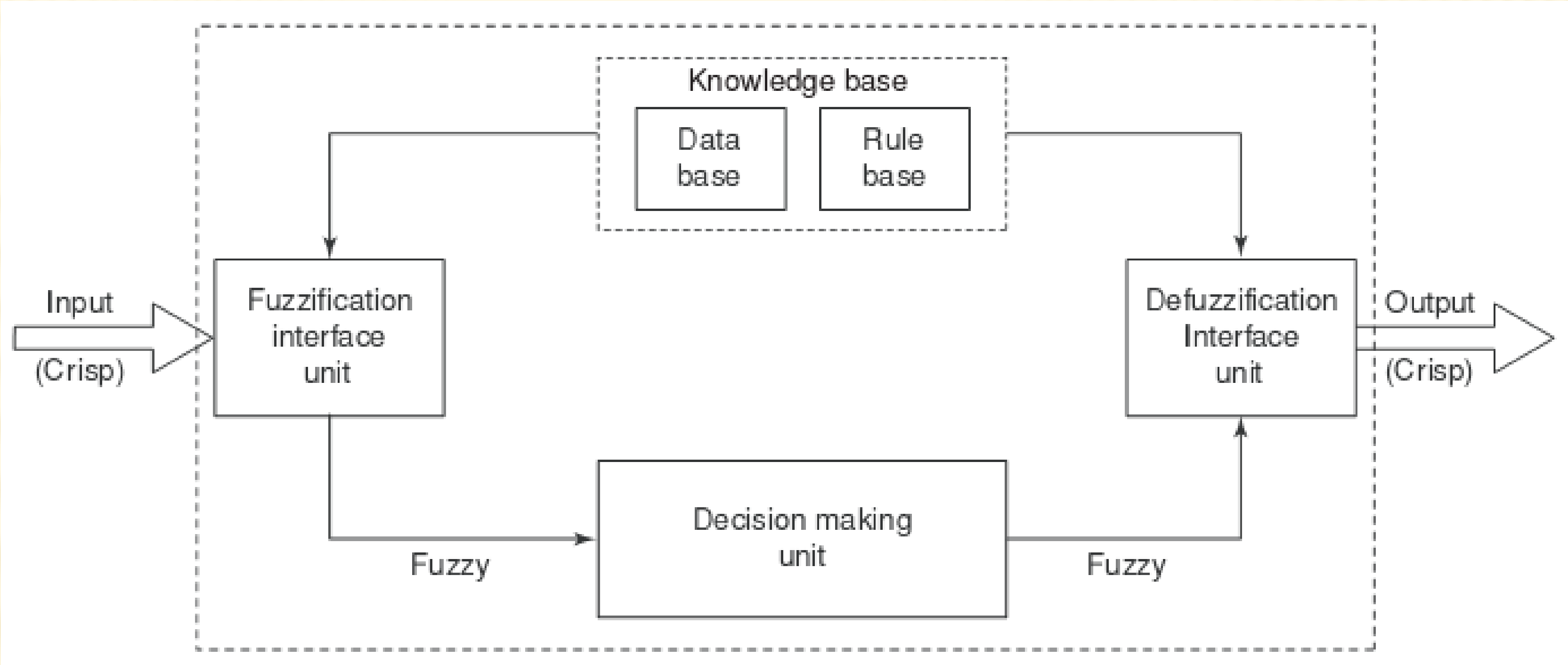
- Max-membership principle,
- Centroid method,
- Weighted average method,
- Mean-max membership,
- Center of sums,
- Center of largest area,
- First of maxima, last of maxima.

FUZZY INFERENCE SYSTEM

FUZZY INFERENCE SYSTEMS (FIS)

- Fuzzy rule based systems, fuzzy models, and fuzzy expert systems are also known as fuzzy inference systems.
- The key unit of a fuzzy logic system is FIS.
- The primary work of this system is decision-making.
- FIS uses “IF...THEN” rules along with connectors “OR” or “AND” for making necessary decision rules.
- The input to FIS may be fuzzy or crisp, but the output from FIS is always a fuzzy set.
- When FIS is used as a controller, it is necessary to have crisp output.
- Hence, there should be a defuzzification unit for converting fuzzy variables into crisp variables along FIS.

BLOCK DIAGRAM OF FIS



TYPES OF FIS

There are two types of Fuzzy Inference Systems:

- Mamdani FIS(1975)
- Sugeno FIS(1985)

MAMDANI FUZZY INFERENCE SYSTEMS (FIS)

- Fuzzify input variables:
  - Determine membership values.
- Evaluate rules:
  - Based on membership values of (composite) antecedents.
- Aggregate rule outputs:
  - Unify all membership values for the output from all rules.
- Defuzzify the output:
  - COG: Center of gravity (approx. by summation).

SUGENO FUZZY INFERENCE SYSTEMS (FIS)

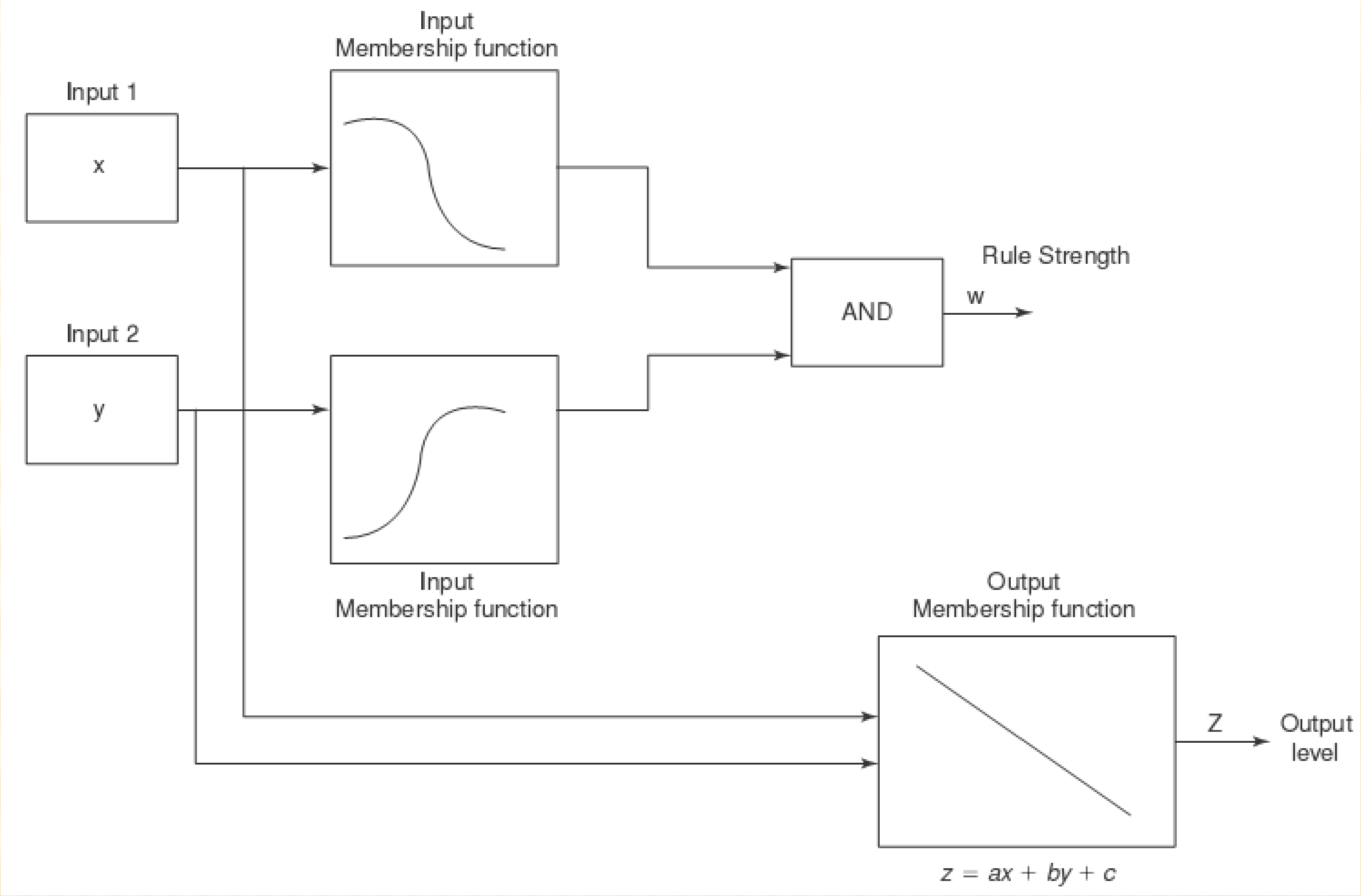
The main steps of the fuzzy inference process namely,

1. fuzzifying the inputs and

2. applying the fuzzy operator are exactly the same as in MAMDANI FIS.

The main difference between Mamdani's and Sugeno's methods is that Sugeno output membership functions are either linear or constant.

SUGENO FIS



Input
Membership function
Input 1
x
Input 2
y
AND
Rule Strength
w
Input
Membership function
Output
Membership function
Z
Output
level
z = ax + by + c

FUZZY EXPERT SYSTEMS

An expert system contains three major blocks:

- Knowledge base that contains the knowledge specific to the domain of application.
- Inference engine that uses the knowledge in the knowledge base for performing suitable reasoning for user's queries.
- User interface that provides a smooth communication between the user and the system.

References

- Principles of soft computing by S.N.Deepa and S.N. Shivanandan
- Multi objective optimization using evlotuniory algorithms by Kalyanmoy deo
- Genetic algorithms in search ,optimization and machine learning by Devid E.goldberg
- Neural Networks and Fuzzy Systems by Bart Kosko
- Neural Network ,Fuzzy Logic and Genetic Algorithm by Rahshekhran and Pai

- **Note :1. Students are advised to refer online resources and above mentioned books to get more information related to soft computing**

  2. Figures, formulae and many slides are taken as it is from the CD of book of soft computing techniques by shivanandan and Deepa