ASSIGNMENT 15

Neeraj Pandey

Discrete Mathematics

Q1(a): There are k kinds of the postcards, but only in a limited number of each, there being a_i copies of the ith one. What is the number of possible ways of sending all of them to n friends? (We may send more than one copy of the same postcard to the same person. We many send different kinds of the postcard to the same person.)

Solution: a_i denote the ith kind of postcards.(given)
Let y_i denote the number of ways in which a_i postcards be distributed among n friends. Let x_j denote the number of postcards sent to the jth friend.There are a total of n friends.

Therefore total number of possible ways of sending all of the a_i postcards to n friends is the total number of solutions of the equation:

$$x_1 + x_2 + ... + x_n = a_i$$

So, total number of solutions will be:

$$\binom{\mathrm{a}_i + n - 1}{\text{n-1}}$$

let the total number of possible ways of postcards of k kinds will be Y:

$$\prod_{i=1}^{k} y_i$$

$$\prod_{i=1}^{k} \binom{\mathrm{a}_i + n - 1}{\text{n-1}}$$

Q2 : Let $f : \{1, 2, \ldots, m\} \implies \{1, 2, \ldots, n\}$. How many f's are possible which are monotonically (not strictly) increasing?

Solution: Let us construct a set D which contains the domain of the function
$f : \{1, 2, ..., m\} \implies \{1, 2, ..., n\}$. Similarly, construct a set C which contains the co-domain. $\therefore$ $|D| = m$ and $|C| = n$. So, all m elements of the domain map to m or less than m elements in the co-domain(because multiple elements from the domain can map to a single element in the co-domain.)

Therefore, the number of monotonically increasing functions, $f : \{1, 2, ..., m\} \implies \{1, 2, ..., n\}$, is equal to the number of ways we can choose m elements from n with repetition.

- Now, each m-combination of a set with n elements when repetition is allowed can be represented by a list of $n1$ bars and m crosses.
- The n 1 bars are used to mark off n different cells, with the ith cell containing a cross for each time the ith element of the set occurs in the combination.
- For instance, a 6-combination of a set with four elements is represented with three bars and six crosses.
- As we have seen, each different list containing $n1$ bars and m crosses corresponds to an m-combination of the set with n elements, when repetition is allowed. The number of such lists is $\binom{\text{m+n-1}}{\text{m}}$, because list corresponds to a choice of the m positions to place the m crosses from the $\binom{\text{m+n-1}}{\text{m}}$ positions that contain m crosses and $n1$ bars.
- . The number of such lists is also equal to $\binom{\text{m+n-1}}{\text{m}}$, because each list corresponds to a choice of the $n1$ positions to place the $n1$ bar.

  . Hence, there are $\binom{\text{m+n-1}}{\text{m}}$ f's possible which are monotonically increasing.