ASSIGNMENT 2

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Discrete Mathematics

Q1. Show that among $n+1$ positive integers of which is greater than $2n$, there are two that are relatively prime. (Two numbers are said to be relatively prime if their greatest common divisor is 1).
Solution: Let us proof by using a Lemma,
Lemma: Every consecutive positive integer is prime. Proof by contradiction says that n and n+1 are positive integers such that prime number divides both of them.
Contradiction: k — 1, but k > 1 By lemma stated above, divide the $2n$ numbers into the following sets,

$$\{1,2\}$$

$$\{3,4\}$$

$$\{5,6\}$$

$$\{...\}$$

$$\{...\}$$

$$\{2n-1,2n\}$$

As there are n such sets and every number $\leq$ 2n, there are $n+1$ numbers and n sets. By Pigeonhole Principle there is atleast one set which has atleat 2 elements. As elements of this set are consecutive positive integers, therefore by Lemma they are relatively prime.

Q2. Suppose that we have n natural numbers none of which is greater than $2n$ such that the least common multiple of any two is greater than $2n$. Show that all n numbers are greater than $\frac{2n}{3}$.

Solution: Let us proof this by contradiction:

Let's say that there is a number a_1 among all the chosen numbers $\ni a_1 < 2_n$ and the rest of the numbers as $a_i > \frac{2n}{3}$.
We can see that following property from the above:

$$2a_1 < 3a_1 < 2_n$$

Take a set, S = $\{ 2a_1, 3a_1, a_2 .... a_n \}$
here, we will take a helping theorem (lemma)
Lemma 1: There are 2 positive integers among n + 1 which divides the other and is greater than $2n$.
So, we can have the following sets:

$$\{1, 2, 4, 8...\}$$

$$....$$

$$....$$

$$\{2n-1, 4n-2, 8n-4...\}$$

$\implies$ These are the following properties of the above sets:
1. There are 'n' sets in total.
2. There are 2 numbers from a set which divides one from the other.
3. Numbers smaller than 2n fall under these sets.
- From the lemma used above, there are 2 numbers that one divides from the other. As, $2a_1$ / $3a_1$ is not an integer, we can conclude that there are 2 numbers a_i and $a_k \ni a_k > a_i$ and a_i divides a_k.
Therfore, if $a_1 < 2n$, there there exist 2 numbers among $\{ a_1, a_2 .... a_n \}$ and their LCM is smaller than 3. Hence, proved by Contradiction.