ASSIGNMENT 6

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Discrete Mathematics

Q1: Prove that $4^n + n^4$ is not a prime for $n > 1$.

Solution:

- If n is even, then the equation is even:

  $$4^n + n^4 = 4^2x + (2x)^4 \in 2x = n$$

  Therefore, it is not even

- if n is odd, $n = 2x + 1$,

  $$= 4^n + n^4$$
  $$= 4^{(2x+1)} + n^4$$
  $$= 4^{(2x)} \cdot 4 + n^4$$
  $$= 4(2^x)^4 + n^4$$
  $$= (2(2^x)^2)^2 + 4n^2(2^x)^2 - 4n^2(2^x)^2$$
  $$= (n^2 + 2 \cdot (2^x))^2 - (2n \cdot 2^x)^2$$
  $$= (n^2 + 2 \cdot (2^x) + (2n \cdot 2^x)) \cdot (n^2 + 2 \cdot (2^x) - (2n \cdot 2^x))$$
  $$= ((n + 2^x)^2 + 4^x) \cdot ((n - 2^x)^2 + 4^k)$$

  When n is odd, the equation $4^n + n^4$ is not prime because it can be expressed as a product of more than two factors and 2 or more than 2 are greater than 1.
  Hence, proved.

Q2: Let a, b, c, d are integers such that $a \neq c$. Suppose $a - c|ab + cd$. Is it true that $ac|ad + bc$? Justify your answer.

Solution: From the given details,

$$(a - c)x = ab + cd \;\; for \;\; x \in Z$$
$$= (ab + cd) - (ad + bc)$$
$$= ab + cd - ad - bc$$
$$= a(b - d) + c(b - d)$$
$$= (a - c) \cdot (b - d)$$
$$\implies (ab + cd) - (ad + bc) = (a - c) \cdot (b - d)$$
$$(ad + bc) = (ab + cd) - ((a - c) \cdot (b - d))$$
$$(ad + bc) = (a - c)x - ((a - c) \cdot (b - d))$$
$$\implies (ad + bc) = (a - c) \cdot (x - b + d)$$

By Closure property, $a - c|ab + cd$. hence, proved.