ASSIGNMENT 8

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Discrete Mathematics

Q1: Prove that $\sqrt{p}$ is an irrational number for any prime p.

Solution: Let us consider $\sqrt{p}$ as rational.
Every rational number is of the form $\frac{a}{b}$
Squaring both the sides:

$$p = a^2 + b^2$$
$$\implies a^2 = p \cdot b^2$$
$$\implies b|a^2$$

Using the fundamental theorem of arithmetic, there exists a prime number k which divides b.
If $x|b$ then$x|a^2$ and if $x|a^2$ then $x|a$.

$x|gcd(a, b) = 1$ is a contradiction unless b = 1 and a = 1.

Now, if b=1, there is no integer for which $a^2 = p$ and same for a = 1. (where p is prime)

Therefore, $\sqrt{p}$ is irrational.

Q1: Prove that $2^n \nmid n!$ for all $n \geq 1$.

Solution: According to Fundamental Theorem of Arithmetic:

$$n! = 2^{a_1} \cdot k_2^{k_2} ....k_a^{a_a}$$

where $k_1, k_2....k_k$ are prime divisors,
a_1 should be $\geq n$:

$$a_1 = [\frac{n}{2}] + [\frac{n}{2^2}] + ...$$
$$[\frac{n}{2}] + [\frac{n}{4}] + ...$$
$$< \frac{n}{2} \cdot \frac{1}{1 - \frac{1}{2}}$$
$$\implies < n$$

Hence proved.