Introduction to Artificial Intelligence

DA 221

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IIT Guwahati

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Lecture 16 (21-Feb-22)

Probability

- A way of summarizing the uncertainty that comes from our laziness or ignorance

- It is too much work to list the complete set of rules or use all rules
- Sometimes we do not have complete theory to know all the rules
- Even if above two don't hold, often our observations are partial.

Uncertainty and rational decisions

Utility theory: every state has a degree of usefulness, or utility, to an agent and that the agent will prefer states with higher utility

Decision theory = probability theory + utility theory

Maximum expected utility: an agent is rational iff it chooses the action that yields the highest expected utility, averaged over all possible outcomes of the action

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- Whereas logical assertions say which possible worlds are strictly ruled out (all those in which the assertion is false), probabilistic assertions talk about how probable the various worlds are.
- In probability SAMPLE SPACE theory, the set of all possible worlds is called the sample space.
  - For example, if we are about to roll two (distinguishable) dice, there are 36 possible worlds to consider: (1,1), (1,2), . . ., (6,6).
- The Greek letter Ω (uppercase omega) is used to refer to the sample space, and ω (lowercase omega) refers to elements of the space, that is, particular possible worlds.

Probability

- In probability SAMPLE SPACE theory, the set of all possible worlds is called the sample space.

A fully specified probability model associates a numerical probability P(ω) with each possible world.

$$\boxed{0 \leq P(\omega) \leq 1 \text{ for every } \omega \text{ and } \sum_{\omega \in \Omega} P(\omega) = 1}$$

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- Conditional Probability

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$$P(a \wedge b) = P(a \mid b)P(b)$$

$$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$$

Probability

$$\text{For any proposition } \phi, \; P(\phi) = \sum_{\omega \in \phi} P(\omega)$$

- Conditional Probability
- Marginal Probability
- Conditioning

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the $Toothache$, $Cavity$, $Catch$ world.

$$P(cavity \lor toothache) = 0.108 + 0.012 + 0.072 + 0.008 + 0.016 + 0.064 = 0.28$$

$$P(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$$

$$\mathbf{P}(Cavity) = \sum_{\mathbf{z} \in \{Catch, Toothache\}} \mathbf{P}(Cavity, \mathbf{z})$$

- Marginalization

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

$$\begin{aligned} P(cavity \mid toothache) &= \frac{P(cavity \land toothache)}{P(toothache)} \\ &= \frac{0.108 + 0.012}{0.108 + 0.012 + 0.016 + 0.064} = 0.6 \end{aligned}$$

Probability

- Conditional Probability
- Marginal Probability
- Conditioning
- Independence
- Bayes rule

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