Controller Design of Two-wheeled Differential Drive with a passive castor wheel using Feedback Linearization

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Abstract—***In this paper motion control problem of two-wheeled differential drive with a passive castor wheel is studied where the environment is assumed to be obstacle-free. A differential drive robot is considered to be the simplest of wheeled mobile robots as it do not require additional steering motion. Dynamic feedback linearization is an efficient design tool for trajectory tracking when the robot is subjected to follow a circular path in addition to a straight line path. The implementation of this approach is done by designing a simulator on MATLAB.***

Keywords — Wheeled Mobile Robot (WMR), castor wheel, feedback linearization

I. INTRODUCTION

Wheeled Mobile Robots (WMR) have wide range applications in the automation of industrial processes and particularly in areas like agriculture, defense and security where autonomous motion capabilities are needed on smooth surfaces and ground[1]. They consume less energy and move at a faster pace than other locomotion mechanisms (e.g., legged robots or tracked vehicles). Beyond the obvious relevance in applications, the problem of motion planning and control of WMRs has attracted the interest of researchers in its theoretical challenges. In fact, these systems are a typical example of non-holonomic mechanisms due to the perfect rolling constraints i.e.no longitudinal or lateral slipping of the wheel. As WMR constitute a class of mechanical systems characterized by kinematic constraint which are non-integerable, so these constraints cannot be eliminated from the equations of the model. Thus, the standard planning and control algorithm developed for robotic manipulator is no more applicable [2].

Different control methods are, therefore, developed in literature for planning the motion of wheeled robots. These methods are mainly developed for unicycle and car-like mobile robots which corresponds to types (2, 0) and (1, 1) respectively as shown in the Table I [3]. In addition, several mobility configurations like number and types of wheels, their location etc. can be found in [3]. Five generic types of WMR based on degree of mobility (δ_m) and degree of steeribility (δ_s) are also explained in [3] which can be seen in tabulated form in Table I. There, δ_m represents the number of WMR velocities that can be assigned instantaneously and δ_s denotes the number of orientable wheels that can be steered independently[3]. Furthermore, WMR comes in a number of different kinematic structures. Most common among the types is a differential-drive type of mobile robots.They have two-coaxial wheels which are active and a passive castor wheel for maintaining the static stability. These robots are non-holonomically constrained which require high control effort and demonstrate two degrees of freedom. They are, therefore, classified as type (2,0) in the classification of WMR which means that their degree of mobility δ_m is 2 and degree of steeribility δ_s is 0 [3].

Although navigation and planning of mobile robots have beenextensively investigated in the literature but the work on dynamic control with non-holonomic constraints of mobile robots is much more recent [3]. A non-holonomic constraint is the limitation on velocity i.e. the directions where the robot cannot move. When properly defined, the orientation of WMR in motion lies along the tangential direction of the motion trajectory. So, a robot needs to be able to determine a driving wheel torque in accordance to the desired moving speed. Furthermore, kinematic model for

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implementation requires the WMR to achieve path tracking in its movement and to estimate its own position, direction, speed and acceleration.

In the absence of any obstacle, the basic motion tasks assigned to a WMR are framed as i) following a given trajectory and ii) moving between two robot postures. In this paper, we have selected the trajectory following WMR i.e. to make the robot follow the circular path in addition to straight line path. The design of our control system employs unicycle type kinematics of WMR. We present a control algorithm for the two-wheeled differential drive robot which is subjected to non-holonomic constraints. Moreover, key objective here is to solve trajectory tracking problem using feedback linearization framework. The rest of the paper is organized as follows: In Section II, system description and control tasks are discussed. Modeling of our system is described in Section III. The controller design and simulation results are discussed in Section IV. Conclusion is given in Section V.

II. System Description And Task Specification

The differential drive robot, which is an example of WMR, is dynamically an unstable system requiring a controller to maintain the balance[4]. When it is subjected to an obstacle-free environment, it can be considered to perform any of the two essential motion tasks i.e. i) Trajectory following ii) Point-to-point path planning. In trajectory following a reference point on the robot must follow a desired path in the Cartesian space from a given initial configuration whereas in point-to-point a desired goal configuration must be reached with a desired velocity from a given initial configuration.

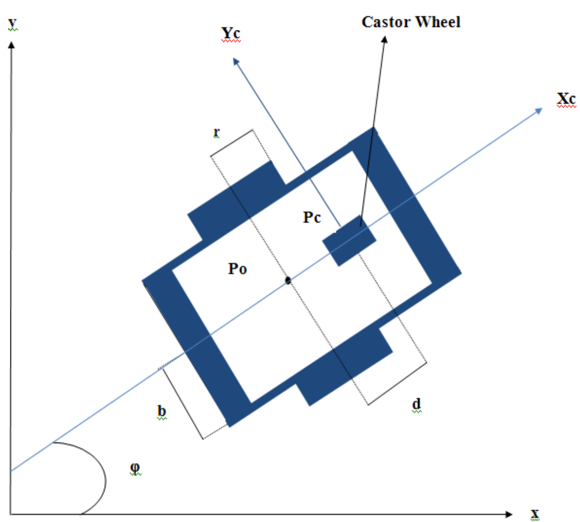

Figure 1 Two-wheeled differential drive with a passive castor wheel

The robot under consideration is assumed to be made up of a rigid cart which is equipped with two non-deformable wheels along with a passive castor wheel. We have assumed that robot is moving on a horizontal plane. The position of the robot can be explained by a vector $\xi=[x,y,\varphi]^t$ in an arbitrary reference frame as shown in Figure 1. Here x and y are the coordinates of the robot and φ is the orientation with respect to the reference frame which results in two degrees of freedom for the robot. The robot has two drive wheels that are assumed to be driven by two different motors which are controlled independently. However, for producing the forward motion, both wheels should be driven at the same rate. Similarly, for turning left or right, wheel must be rotated at a higher rate than the left wheel and vice versa for turning right. Hence, the accuracy of the robot motion is largely dependent upon the synchronization of the two wheel velocities.

Type (δ_m,δ_s)	**Type 1 (3,0)**	**Type 2 (2,0)**	**Type 3 (2,1)**	**Type 4 (1,1)**	**Type 5 (1,2)**
δ_m	3	2	2	1	1
δ_s	0	0	1	1	2

TABLE I . Five Types of WMR

In order to cope with the non-holonomic velocity constraint, wheel orientation should be actively steered in the desired velocity direction. This implies that the wheel orientation should be aligned before the movement of the robot. Additionally, the non-slip and pure rolling conditions must also be fulfilled i.e. the velocity of center of the wheel should be parallel with the wheel plane and proportional to the velocity of wheel rotation [3].

Now for a non-holonomically constrained aforementioned system, the equation of motion using Lagrange multiplier rule is described as follows:

$$M(z)\ddot{z} + C(z,\dot{z}) + G(z) = F(z)u + D^T(z)\lambda_n \qquad (1)$$

where M(z) is the [n x n] dimensional positive definite inertia matrix, $C(z,\dot{z})$ is the n-dimensional velocity-dependent force vector of Coriolis and Centrifugal forces, G (z) is the gravitational force vector, u is the r-dimensional vector of actuator force/torque, F(z) is the [n x r] dimensional matrix mapping the actuator space into the generalized coordinate space and λ is an m-dimensional vector of Lagrangian Multipliers. D(z) is an [m x n] dimensional matrix of constraints of a mechanical system that is subjected to m velocity levels such that:

$$D(z)\dot{z} = 0 \qquad (2)$$

where $z=(x_c,y_c,\varphi,\Theta_r,\Theta_l)^t$ is the n-dimensional matrix of generalized coordinates. As the constraints are independent so we can deduce that D (z) has rank m.

III. MODELING

For accurate description of a system, it is necessary to create a proper dynamic model. The development of an efficient control system for the drive robot necessitates the dynamics of the system to be described by a mathematical model. This model takes into consideration various properties like inertia, mass, torque, frictional forces etc.

A. Constraint Equation

Here the constraint equation for the two-wheeled differential drive is calculated considering two driving wheels and a single passive castor wheel. The passive castor wheel does not provide an additional velocity constraint on the robot's motion as it is needed only for stability of the robot.

Notation	Description
m_c	Mass of the robot body without driving wheels
m_w	Mass of each driving wheels
b	Distance between the driving wheels and axis of symmetry
r	Radius of each driving wheel
I_w	Moment of inertia of each wheel about the wheel axis
I_m	Moment of inertia of each wheel about wheel diameter
P_o	Intersection of driving wheel axis with the axis of symmetry
P_c	Center of mass of robot body
d	Distance from P_o to P_c
I_c	Moment of inertia of robot body without wheels about a vertical axis through P_o

TABLE II. NOMENCLATURE OF NOTATIONS

Let z be the n-vectors of generalized coordinates for the differential drive robot. As these systems are non-holonomic so they are well characterized by n-m non-integrable linear constraints on their generalized velocities. These constraints arise from the rolling without slipping condition for the wheels. Therefore, the robot is subjected to three constraint equations which are given below:

$$\dot{y}_c\cos\varphi - \dot{x}_c\sin\varphi - d\dot{\varphi} = 0 \tag{3}$$

$$\dot{x}_c\cos\varphi + \dot{y}_c\sin\varphi + b\dot{\varphi} = r\dot{\theta}_r \tag{4}$$

$$\dot{x}_c\cos\varphi + \dot{y}_c\sin\varphi - b\dot{\varphi} = r\dot{\theta}_l \tag{5}$$

where (x_c, y_c) are the coordinates of the center of mass P_c and φ is the heading angle measured from the x-axis of world coordinate system. Θ is the driving angles of both wheels of the robots. Equation (3) implies that the robot must move in the direction of axis of symmetry whereas equation (4, 5) suggests that the driving wheels should not slip. The above three constraints can be written in the form of a 3x5 dimensional matrix D(z) as:

$$D(z) = \begin{bmatrix} -\sin\varphi & \cos\varphi & -d & 0 & 0 \\ -\cos\varphi & -\sin\varphi & -b & r & 0 \\ -\cos\varphi & -\sin\varphi & b & 0 & r \end{bmatrix} \tag{6}$$

The notations used for the derivation of constraint equations and dynamic equation are shown in Table II.

B. Dynamic Equation

Mobile robot dynamics is needed for planning robot trajectory. Lagrangian approach is used to generate the ordinary differential equations (ODEs) of the system. So existing energies of the system is observed in order to develop the Lagrangian equation[5]. Lagrangian (£) in Newtonian mechanics is chosen to be

$$£=T-U \tag{7}$$

where T is the total kinetic energy and U is the total potential energy[6]. Here as we are neglecting the gravity factor so U=0 and also G (z) =0 in equation (1). The total kinetic energies of robot body and wheels are calculated separately. Non-linear equations of system are developed by substituting the existing energies in equation (1) which are given below:

$$m\ddot{x}_c - m_c d(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) - \lambda_1\sin\varphi - (\lambda_2 + \lambda_3)\cos\varphi = 0 \tag{8}$$

$$m\ddot{y}_c + m_c d(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi) + \lambda_1\cos\varphi - (\lambda_2 + \lambda_3)\sin\varphi = 0 \tag{9}$$

$$-m_c d(\ddot{x}_c\sin\varphi - \ddot{y}_c\cos\varphi) + I\ddot{\varphi} - d\lambda_1 + b(\lambda_3 - \lambda_2) = 0 \tag{10}$$

$$I_w\ddot{\theta}_r + \lambda_2 r = \tau_r \tag{11}$$

$$I_w\ddot{\theta}_l + \lambda_3 r = \tau_l \tag{12}$$

where

$$m = m_c + 2m_w$$

$$I = I_c + 2m_w(d^2 + b^2) + 2I_m$$

and τ_l and τ_r are the left and right torques acting on the wheel axis respectively. The above five motion equations can be written in the vector form as

$$M(z)\ddot{z} + C(z,\dot{z}) = F(z)u - D^T(z)\lambda_n \tag{13}$$

The matrix D^T (z) has been in equation (6) and the matrices M(z), $C(z,\dot{z})$ and F (z) are given by

$$
M(z) = \begin{bmatrix} m & 0 & -m_c d\sin\varphi & 0 & 0 \\ 0 & m & m_c d\cos\varphi & 0 & 0 \\ -m_c d\sin\varphi & m_c d\cos\varphi & I & 0 & 0 \\ 0 & 0 & 0 & I_w & 0 \\ 0 & 0 & 0 & 0 & I_w \end{bmatrix}
$$

$$
C(z,\dot{z}) = \begin{bmatrix} -m_c d\dot{\varphi}^2\cos\varphi \\ -m_c d\dot{\varphi}^2\sin\varphi \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
F(z) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

Now the equation of motion (13) and the constraint equation (2) is represented in state space by a 5x2 dimensional matrix $N(z)$ which describes the generalized velocities of the system such that $D(z)N(z) = 0$.

$$
N(z) = \begin{bmatrix} n_1(z) & n_2(z) \end{bmatrix} = \begin{bmatrix} c(b\cos\varphi - d\sin\varphi) & c(b\cos\varphi + d\sin\varphi) \\ c(b\sin\varphi + d\cos\varphi) & c(b\sin\varphi - d\cos\varphi) \\ c & -c \\ 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

Here the constant $c = \frac{r}{2b}$. From the constraint equation (2), $\dot{Z}$ is in the null space of $D(z)$. As the two columns of $N(z)$ are in the null space of $D(z)$ and are linearly independent, so we can express $\dot{Z}$ as a linear combination of the two columns of $N(z)$ i.e.

$$
\dot{z} = N(z)\upsilon \tag{14}
$$

where

$$
\upsilon = \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix}
$$

Differentiating equation (14) we get

$$
\ddot{z} = \dot{N}\upsilon(t) + N\dot{\upsilon}(t) \tag{15}
$$

Substituting the expression into equation (13) and pre-multiplying by N^T the equation becomes

$$
N^T(MN\dot{\upsilon}(t) + M\dot{N}\upsilon(t) + C) = \tau \tag{16}
$$

Let x be a state space vector such that

$$
x = \begin{bmatrix} z^T & \upsilon^T \end{bmatrix}^T
$$

Then

$$
\dot{x} = \begin{bmatrix} N\upsilon \\ f_2 \end{bmatrix} + \begin{bmatrix} 0 \\ (N^T M N)^{-1} \end{bmatrix} \tau \tag{17}
$$

where

$$
f_2 = (N^T M N)^{-1}(-N^T M N \upsilon - N^T C)
$$

By applying the following nonlinear feedback

$$
\tau = N^T M N (u - f_2) \tag{18}
$$

the state equation in (17) is further simplified to

$$
\dot{x} = \begin{bmatrix} N\upsilon \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{19}
$$

where u is the new input that could linearize equation (17) and then integrating equation (19) we get the vector x which gives every state variable in the system.

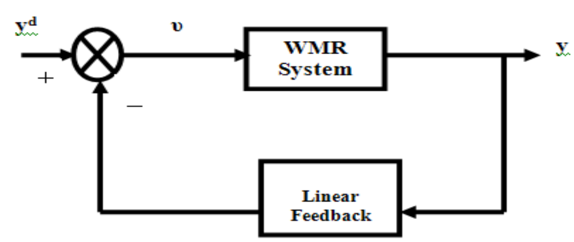

Figure 2 Feedback Linearization

IV. CONTROL SYSTEM DESIGN AND SIMULATION RESULTS

A non-linear kinematics model can be converted into a controllable system by using feedback linearization. Any number of system equations can be transformed into simple integrators (input–output linearization and decoupling). The system in equation (17) is linearized by designing a feedback controller where u is used as an input to the system. A non-linear feedback is used to achieve input-output linearization as in the block diagram of the feedback system shown in Figure 2.The output y is given by

$$y = h(z) = \begin{bmatrix} x_r & y_r \end{bmatrix}^T \quad (20)$$

where x_r and y_r are x and y coordinate of reference point on the desired trajectory. Differentiate (20) to get

$$\dot{y} = (\partial h(z) / \partial z)\dot{z} = J_h(N\upsilon) = (J_h N)\upsilon = \Phi\,\upsilon \quad (21)$$

where Φ is known as decoupling matrix. Again differentiating equation (21) and using Proportional-Derivative (PD) control we get

$$\ddot{y} = v = \ddot{y}^d + K_d(\dot{y}^d - \dot{y}) + K_p(y^d - y) \quad (22)$$

where y^d is the desired path, K_p is the proportional gain and K_d is the derivative gain of the PD controller. The feedback error is given by

$$e = y^d - y \quad (23)$$

Since the controller gains are sensitive to the performance of the system so a careful selection from a very large feasible set of PD gains is required. Thus, the trial and error method is employed for tuning the controller until the error is minimized. Here, a simulator on MATLAB is developed for simulating a two-wheeled differential drive. To assess the performance of the feedback linearization controller, we present the simulation result in Figure 4. The desired trajectory of the slanting line and a circle is given as input which can be seen in blue whereas the path followed by the robot is visible in red. The tracking of the reference trajectory is very accurate and the residual error is mainly because of the robot that takes a short time period to achieve the needed heading angle. But once it is directed it precisely follow the trajectory. We noted that the actual trajectory coincides with the desired trajectory closely.

The differential drive robot has a singularity that it cannot travel in the direction along the axis of rotation. Moreover, robot trajectory is also affected by small errors in the relative velocities of the two wheels. These robots are prone to slight change in the ground plane too. So the two wheels of the robot are driven with exactly the same angular displacement which can be seen in Figure 5. The linear velocity profile of the system can be seen in Figure 3 where after achieving a desired overshoot the profile become stable. Hence, the results prove to be quite satisfactory and here successful implementation of trajectory tracking problem using feedback linearization technique is accomplished.

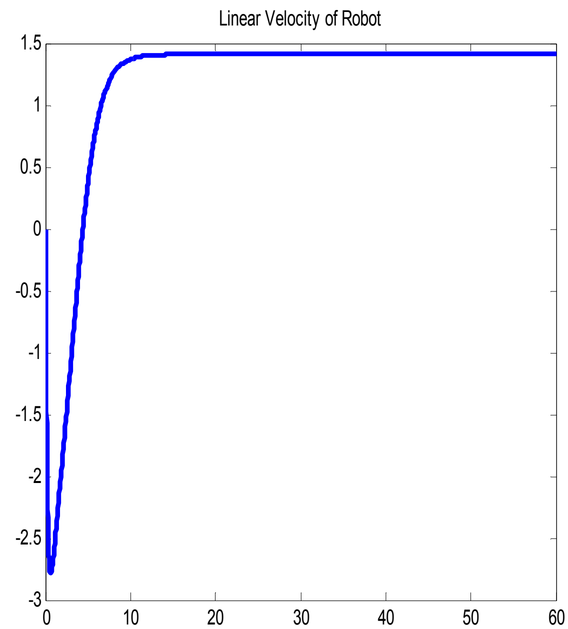

Figure 3. Linear Velocity of Robot

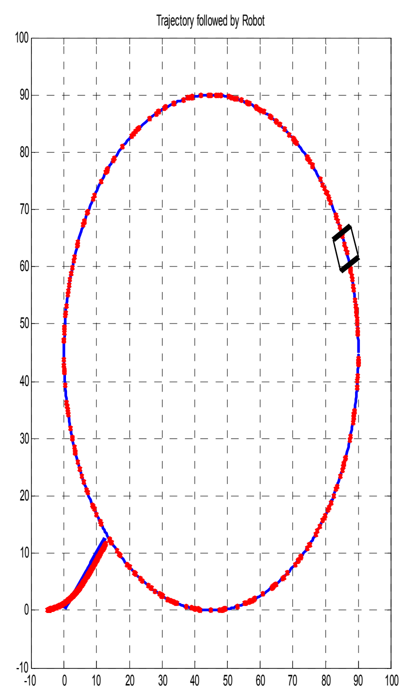

Figure 4. Simulation Results

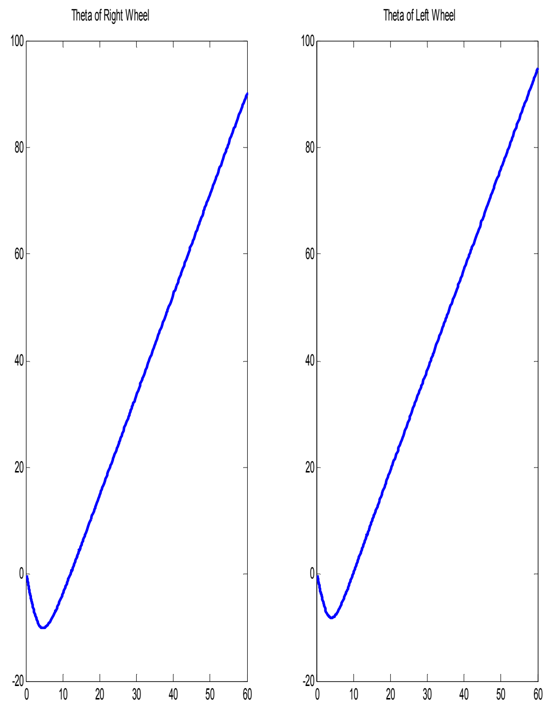

Figure 5 Angular Displacement of two wheels

V. CONCLUSION

In this paper, we have summarized our observations in the form of a useful guideline for implementation of the control strategy on other WMRs. From the point of view of control parameters tuning, especially for more complex WMRs, the feedback linearization technique appears to be simpler as it makes the choice of stabilizing gains for integrator quite easier. It has a simple mechanical structure and kinematic model. A control algorithm is designed using feedback linearization. As the robot is subjected to non-holonomic constraints, the dynamic system governing the motion of the robot is not input-state linearizable. Systematic errors are easy to calibrate. However, it allows only bidirectional movement so the robot has to be initially oriented to the reference. For future work, a system could be designed that can travel in an unknown environment between any initial and goal configuration of the robot.

REFERENCES

1. L.Gracia, J.T., *Kinematic Control of Wheeled Mobile Robots.* Latin American Applied Research, 2008.
2. Guy Campion, G.B. *Structural Properties and Classification of Kinematic and Dynamic Model of Wheeled Mobile Robots*. in *IEEE Transaction on Robotics and Automation*. 1996.
3. Bruno Siciliano, O.K., *Springer Handbook of Robotics,*. 2008: Springer.
4. Daniel R. Jones, K.A.S. (2010) *Modelling and Stability Control of Two-Wheeled Robots in Low-Traction Environments*. acra.
5. Hamid Reza, J.-Y.C., *Design and Parametric control of co-axes driven two-wheeled balancing robot*, in *Microsyst Technol*. 2011, Springer.
6. Blair (2003) *Lecture Notes*. Massachusetts Institute of Technology, Aerospace Dynamics
7. Roland Siegwart, I.R.N., *Intro to Autonomous Mobile Robots*. 2004: MIT Press.
8. J.Craig, J., *Introduction to Robotics Mechanics and Control*. Third ed. 2007: Pearson Education.
9. Chitsaz, H., et al. *Minimum wheel-rotation paths for differential-drive mobile robots*. in *Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on*. 2006.
10. T.Mathavaraj Ravikumar, R.S., *Modeling and Optimization of Odometry Error in a Two Wheeled Differential Drive Robot.* International Journal of Scientific and Research Publications, 2013 Volume 3(Issue 12).
11. Changbae Jung, C.-b.M., *Design of test track for accurate calibration of two wheel differential mobile robots.* International Journal of Precision Engineering and Manufacturing, 2014 Volume 15(Issue 1).