Fast and Reliable Determination of Gravitational Wave Merger Sky Locations Using Fixed Masses and Variable Parameters

1. Introduction:

The detection of gravitational wave, ripples in fabric of spacetime, has opened a new window for observing the universe. Gravitational waves observations provided a unique opportunity to study the most energetic events in the universe, such as the mergers of black holes and neutrons stars. One of the key challenges in the field of gravitational wave astronomy is the rapid and accurate determination of the sky locations of the source of the gravitational waves.

The project "**Fast and Reliable Determination of Gravitational Wave Merger Sky Locations Using Fixed Masses and Variable Parameters**" aims to develop an efficient and reliable method for determining the sky locations of gravitational wave mergers. The project utilizes the PyCBC inference method, which involves fixing the masses of the merging objects and allowing other parameters to vary, to infer the sky location of the source of gravitational waves.

The project involves the analysis of the data from the LIGO **(Laser Interferometer Gravitational-Wave Observatory)** and Virgo gravitational wave observatories; LIGO – **H1**: LIGO Hanford Observatory, near Richland, Washington, **L1**: LIGO Livingston Observatory, in Livingston, Louisiana and **V1**: Virgo Interferometric gravitational wave detector, in Italy. The PyCBC inference method is used to analyze the data and determine the sky locations of the gravitational wave source using **SingleTemplate** model and two different samplers.

2. Background:

The study of gravitational waves has been revolutionized by the advent of advanced gravitational wave observatories such as the Laser Interferometer Gravitational-Wave Observatory (LIGO) and the Virgo detector. These observatories have enabled the detection of gravitational waves from a variety of sources, including black holes and neutron star mergers (GW170817).

One of the key challenges in gravitational wave astronomy is the accurate and reliable determination of the sky location of the source of the gravitational waves. Accurate sky localization is critical for follow-up observations with telescopes and other instruments to study the properties of the sources and their environments.

There have been several approaches for determining the sky locations of gravitational waves sources. These approaches can broadly be classified into two categories: template-based methods and Bayesian inference methods. Template-based methods rely on the comparison of the observed gravitational wave signal with a theoretical waveform to determine the sky location of the source. Bayesian inference methods, on the other hand, use a statistical approach to infer the sky location of the source.

The PyCBC inference is one such Bayesian inference method that has been developed for the analysis of gravitational wave data. This method involves fixing few parameters as static parameters (parameter's value do not change by the model and specified by the user) and variable parameters (parameter values are estimated by the model). The PyCBC inference method has been shown to be an effective tool for the analysis of gravitational wave data and has been used to study a variety of sources, including black hole and neutron star mergers.

In summary, the determination of the sky location of gravitational wave sources is a critical problem in the field of gravitational wave astronomy. The PyCBC inference method is a powerful tool for addressing this problem, and its effectiveness has been demonstrated in a variety of studies. However, there is still ongoing research to improve the method and address its limitations.

3. Result:

The weblink https://dcc.ligo.org/public/0146/P1700349/001/{}-{}1_LOSC_CLN_4_V1-1187007040-2048.gwf is used to access the raw gravitational data for the event GW170817 event observed by the LIGO and VIRGO observatories. By replacing the {}-{}1 part of the URL with the observatory name (H, L and V) data can be downloaded from each observatory.

Preconditioning the dataset of gravitational wave is an essential step in the analysis of gravitational waves. Raw data is collected from the observatories such as above [a] contains a lot of noise, which can make analysis harder to find the signal. Preconditioning involves processing the raw data to remove noise and enhance the signal-to-noise-ratio (SNR).

Firstly, a high pass filter is used to remove the low-frequency noise or baseline variations from the signal. Low frequency noise can cause problems as it masks the underlying gravitational signal. The high-pass filter allows high-frequency signals to pass through while attenuating or blocking the lower frequency signals. Thus, removing the low frequencies is an important step in processing raw gravitational data which helps to improve the quality of the signal.

After applying the high pass filter, it is common practice to resample the data to a fixed interval. It ensures that all the datapoints are evenly spaced, and it produces a consistent time series that can analysed using various signal processing techniques. Resampling also reduces the amount of data that needs to be processed, making it easier to manage and analyse.

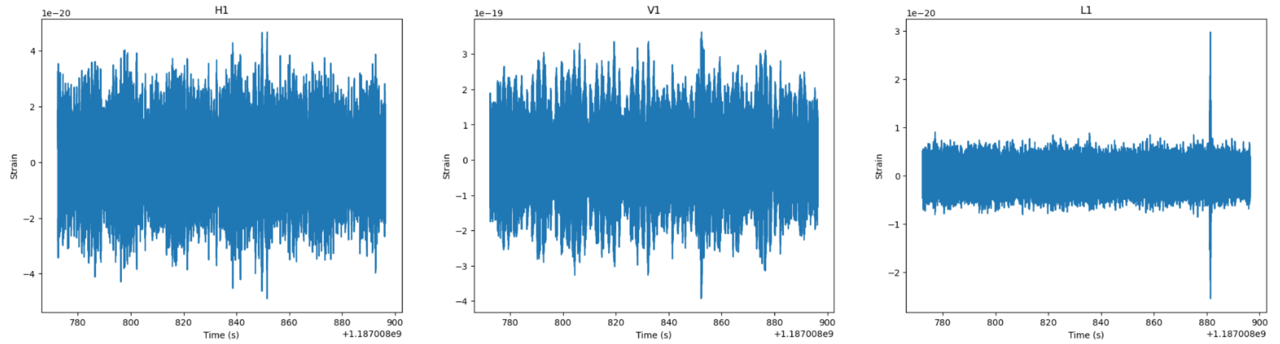

Fig 3.1: Strain data vs Time (s) for three observatories

Power spectral density (PSD) is an important tool in analysing the gravitational wave data. Gravitational waves are ripples in spacetime and they are passed through the detectors, it causes tiny fluctuations in the detector's position, which are recorded as a time series of data. To detect and analyze these tiny fluctuations, it is important to understand the noise characteristics of the detector. The PSD of the detector's noise is a measure of how much noise there is at different frequencies in the detector's output. The PSD is used to filter the data, by removing the noise at frequencies where it is dominant and amplifying the signals that are of interest.

Before finding the PSD, it is a good practice to use techniques such as interpolation and inverse spectrum truncation (IST). Interpolating is the process of estimating the value of a function at points between the known data points. It is used to estimate the PSD at intermediate frequencies, which are not directly sampled by the data, and to smooth the high-frequency noise in the PSD whereas IST is a technique used to remove the noise from the observed signal by dividing it by the inverse of noise's power spectral density in the frequency domain.

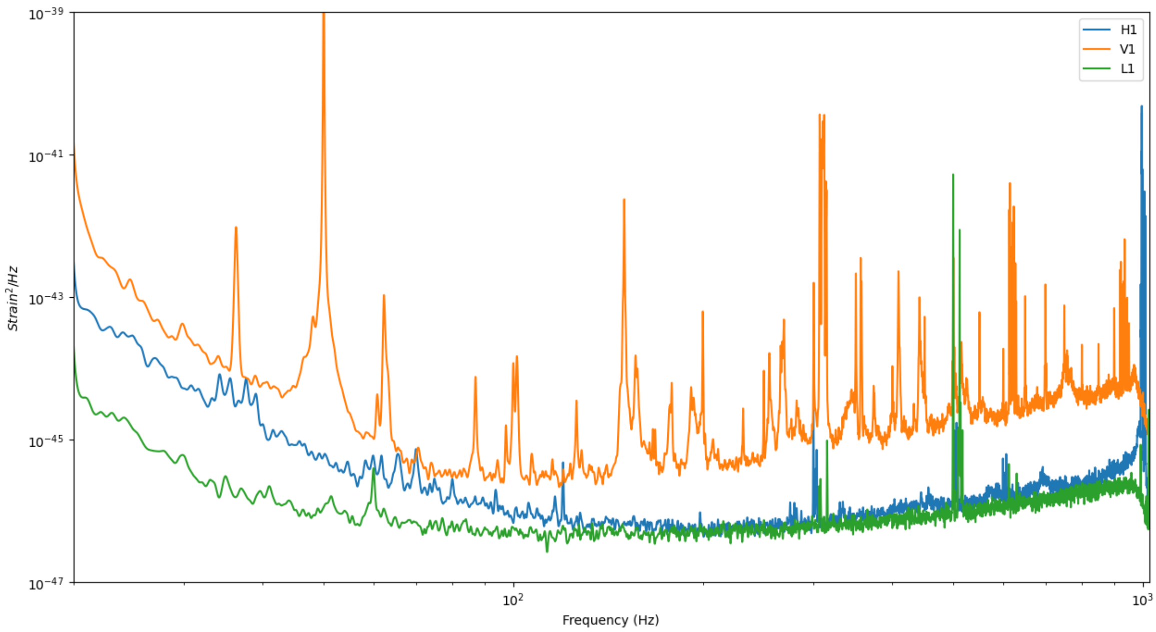

Fig 3.2: Power Spectral Density: shows how noise power varies over frequency in each detector.

Once the data has been pre-conditioned, we can apply the models and samplers to estimate the posterior distribution of the desired parameters. However, before utilizing the models and samplers in PyCBC, we need to provide the necessary input parameters such as static parameters, variable parameters, and a joint distribution of prior distributions for variable parameters.

Static parameters are parameters that are assumed to be constant for a given gravitational wave signal. These parameters are typically fixed during the parameter estimation process. Variable parameters are parameters that are allowed to vary during the parameter estimation process to find the best best-fit model for the observed data. The goal of the parameter estimation process is to estimate the posterior probability distribution of the variable parameters given the observed data. Samplers such as Markov Chain Monte Carlo (MCMC), Nested Sampling are used to explore parameter space.

We feed static parameters and variable parameters (as a joint distribution) to the model and then we use sampler to find the posterior distribution of the variable parameters.

Single Template Model is useful when we know all the intrinsic parameters of a source (i.e., component, masses, spins, etc) but we don't know the extrinsic parameters (i.e., sky location, distance, binary orientation). It is a search method that uses pre-defined waveform templates to match against our data **(Match Filtering)**. To use single template model, it needs to have specific set of data products such as frequency-domain data, power spectral density estimates and low frequency cut off for internal filtering in the model.

In PyCBC, samplers are algorithms that are used to explore the parameter space of a gravitational wave model and to estimate the posterior probability distribution of the model parameters given the observed data. Samplers are fundamental tools for gravitational wave parameter estimation, and are used in wide range of applications, including detection parameter estimation and model selection. In this project I have used two samplers namely: **Parallel Tempering (PT)** and **Nested Sampling (NS).**

Parallel Tempering is a Markov Chain Monte Carlo (MCMC) sampler that generates multiple chains with different temperatures, and exchange samples between chains at regular intervals to explore the parameter space more efficiently. **Nested Sampling (NS)** is a Bayesian Sampler that uses a set of "Live points" to explore the parameter space by iteratively replacing the least likely point with a new point that has higher likelihood.

Estimating sky locations of GW170817 for fixed masses:

Case – 1: Determining the sky locations using all parameters such as: Right Ascension (ra), Declination (dec), Inclination and Distance using PT Sampler:

In this particular analysis, I have considered certain parameters as static parameters, meaning that their values remain constant throughout the entire analysis. These parameters include the masses of both neutron stars (mass1 and mass2), low frequency cut-off, time of coalescence, polarization, and the approximant used for the waveform model. On the other hand, the parameters that are used to estimate the sky locations, such as right ascension, declination, distance, and inclination, are considered as variable parameters, and their respective prior distributions are used during the analysis. To perform the analysis, a Single Template model is defined with the gravitational wave data, static parameters, variable parameters, low frequency cut-off for each detector, sampling rate, prior distributions, and power spectral density, which is then fed to the sampler. The sampler used in this case is PT Sampler, and is run for 400 iterations.

Once the sampler has completed for 400 iterations, the posterior distribution for the right ascension (ra) and declination (dec) are plotted in the Fig 3.3. During the parameter estimation process, two clusters were formed, but they eventually merge into a single cluster located in the top-left corner of the plot. We can observe that the likelihood of the posterior distribution (from the right-side plot) is concentrated around the values of ra = 3.48 and dec = -0.42. The actual values for the ra and dec are ra = 3.44615914 and dec = -0.40808407 respectively.

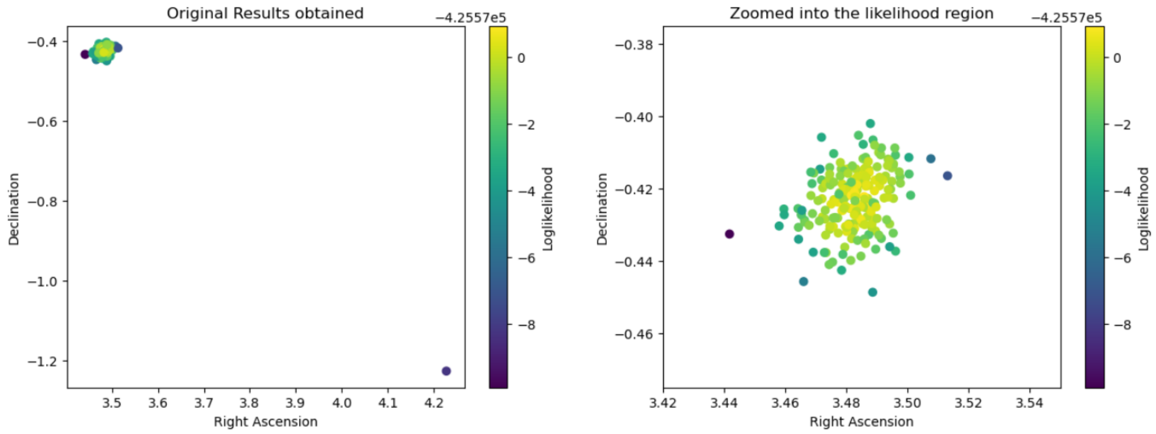

Fig 3.3 Posterior distribution of Right Ascension (ra) and Declination (dec)

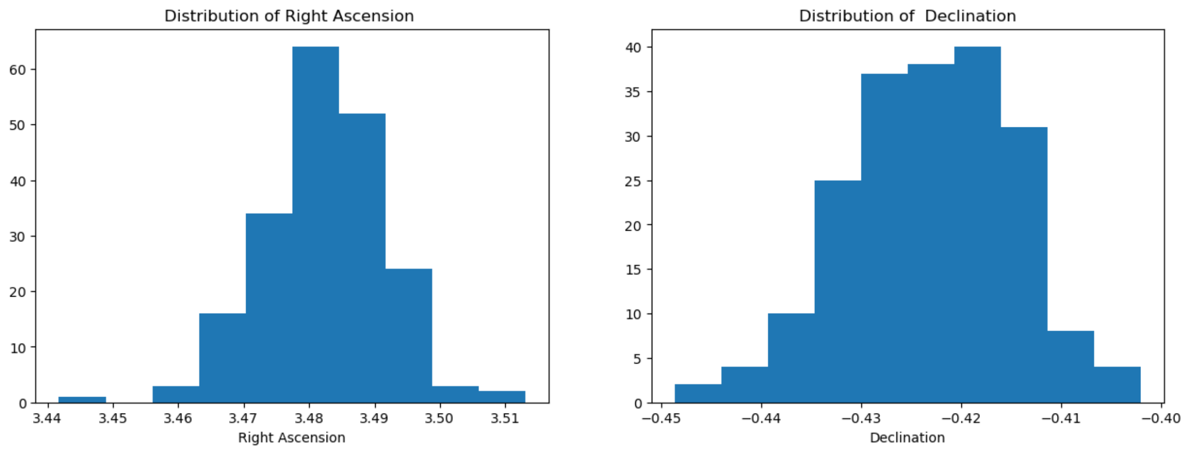

Fig 3.4 Distribution of Right Ascension (ra) and declination (dec)

The distributions of ra and dec are almost like normal distribution, it means that the values of these parameters are more likely to be clustered around the mean value of the distribution. The shape of the distribution implies that the values are closer to mean and are more likely to occur than the values that are further away from the mean.

Case – 2: Determining the sky locations by making inclination = 0 using PT Sampler:

Inclination is one of the key parameters in gravitational wave analysis that describes the angle between the orbital angular momentum of the binary system and the line of sight of observer. If we define inclination = 0, it means that the binary system is face-on with respect to the observer. In other words, the orbital angular momentum vector of the binary system is aligned with the line of sight of the observer.

In this particular analysis, during the parameter estimation process, we haven't got two clusters (as shown in the Fig 3.3). We can observe that the likelihood of the posterior distribution is concentrated around the values of ra = 3.515 and dec = -0.385. The actual values for the ra and dec are ra = 3.44615914 and dec = -0.40808407 respectively.\

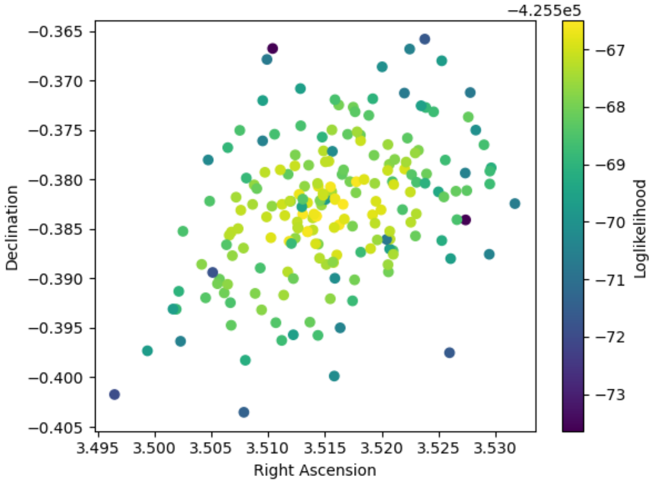

Fig 3.5: Shows the posterior distribution of Right Ascension (ra) and Declination (dec) when Inclination = 0

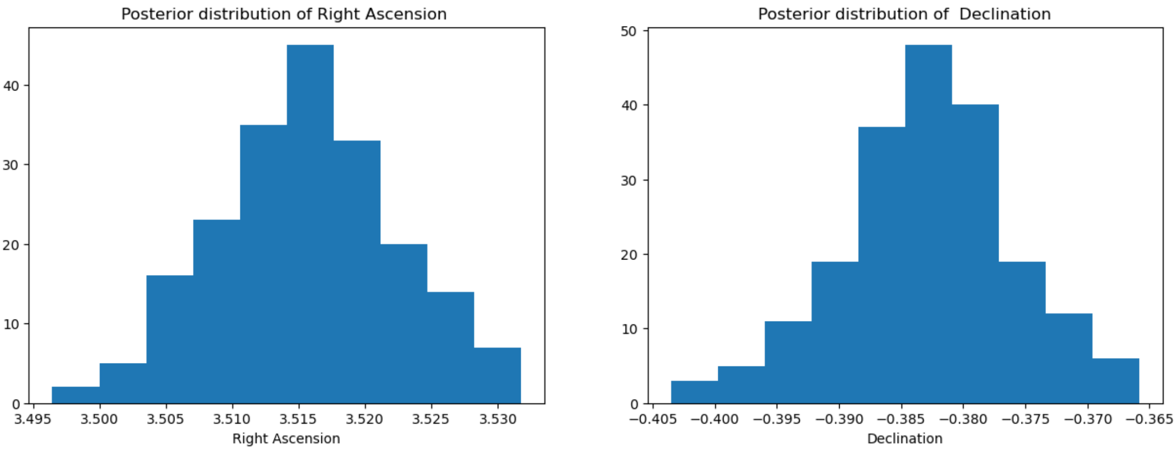

Fig 3.6: Distribution of Right Ascension (ra) and declination (dec), while inclination = 0

The distributions of ra and dec are almost like normal distribution, it means that the values of these parameters are more likely to be clustered around the mean value of the distribution. The shape of the distribution implies that the values are closer to mean and are more likely to occur than the values that are further away from the mean.

Case – 3: Determining the sky locations using all parameters such as: Right Ascension (ra), Declination (dec), Inclination and Distance with Nested Sampler using PyCBC Inference script:

The PyCBC inference script is a python script that uses the PyCBC inference package to perform Bayesian inference on gravitational wave data. The scripts typically include steps for pre-processing the data, defining the model parameters, specifying the prior distributions for the parameters, defining the likelihood function, and running the sampler to obtain the posterior distribution of the parameters.

The contour lines on the posterior plot in Fig 3.7 (1) tells us the regions where the probability density is higher, indicating the most probable values of the parameters. We got ra and dec values as 3.498 and -0.366 respectively. The sky map in Fig 3.7 (2) shows us the most probable location of the source in the sky as well as uncertainty associated with that location.

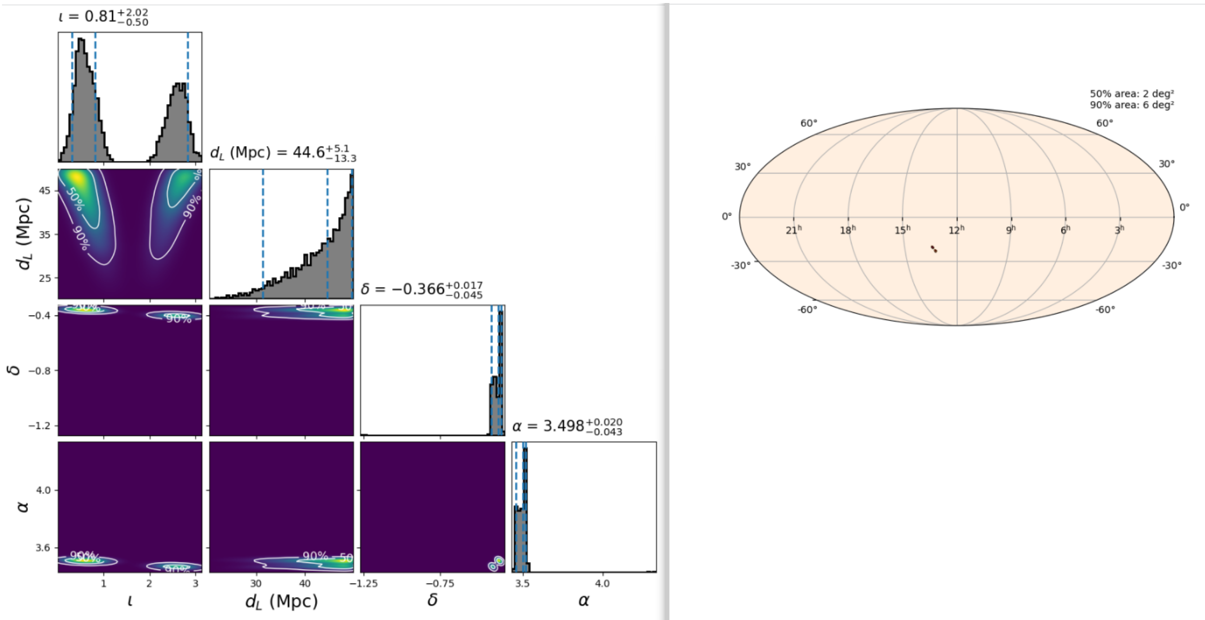

Fig 3.7 (1) Posterior distributions of inclination, distance, declination, and ra. Fig 3.7 (2) Sky map locations

Case – 4: Determining the sky locations by making inclination = 0 using Nested Sampler:

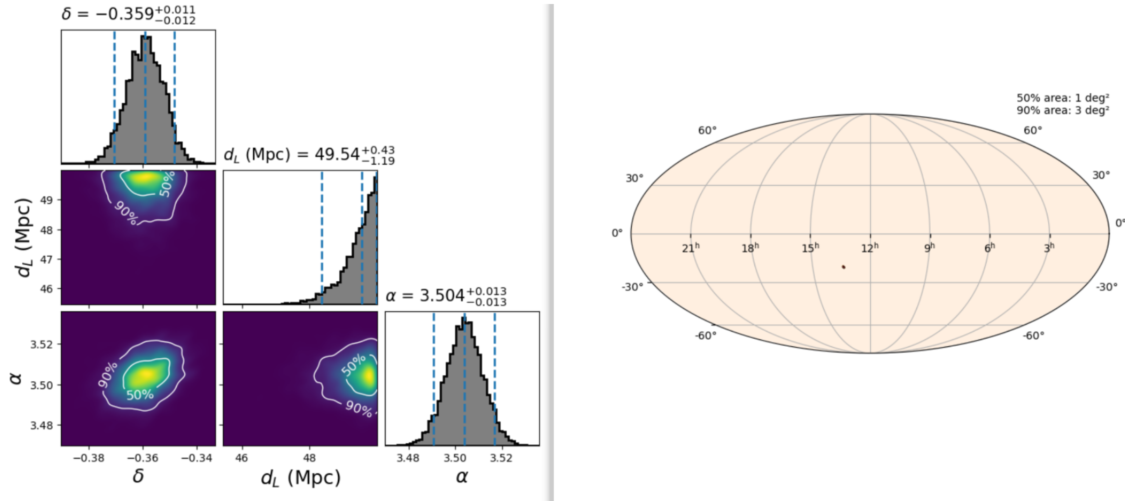

Fig 3.8 (1) Posterior distributions of dec, distance and ra. Fig 3.8 (2) Sky map locations

The contour lines on the posterior plot in Fig 3.8 (1) tells us the regions where the probability density is higher, indicating the most probable values of the parameters. We got ra and dec values as 3.5 and -0.359 respectively. The sky map in Fig 3.8 (2) shows us the most probable location of the source in the sky as well as uncertainty associated with that location.

4. Results:

#	Published Values	Value
1	Right Ascension (ra)	3.44615914
2	Declination (dec)	-0.40808407

Determining sky locations of the Merger GW170817

Sampler Name	Parameters	Time	Right Ascension	Declination
MCMC PT Sampler	All	346.883268	3.482089	-0.424477
MCMC PT Sampler	Partial	362.703101	3.514163	-0.383027
Nested Sampler	All	285.859406	3.498000	-0.366000
Nested Sampler	Partial	90.079029	3.504000	-0.359000

Fig 4.1: Time taken for different samplers with different number of sky location parameters.

Based on the results presented in Figure 4.1, it can be concluded that the Nested Sample method with inclination = 0 was the fastest way to estimate the Right Ascension (ra) and Declination (dec), with a computation time of 90.079 seconds. However, for more accurate results that were consistent with previously published results, the MCMC PT Sampler method with all the sky location parameters was more efficient, taking around 346.883268 seconds. These findings suggest that the MCMC PT Sampler may be more suitable for more detailed analyses requiring greater accuracy, while the Nested Sample method can be useful for quick and simple calculations. Further research could explore the strengths and limitations of these and other methods in more detail and investigate their applicability to other types of data and analysis scenarios.

References:

1. The basic physics of the binary black hole merger GW150914 LIGO Scientific and VIRGO Collaborations
2. Properties of the Binary Neutron Star Merger GW170817 B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration)
3. An introduction to Bayesian inference in gravitational-wave astronomy: parameter estimation, model selection, and hierarchical models Eric Thrane1 , 2 , ∗ and Colm Talbot1 , 2 , † 1 Centre for Astrophysics, School of Physics and Astronomy, Monash University, VIC 3800, Australia 2OzGrav: The ARC Centre of Excellence for Gravitational-Wave Discovery, Clayton, VIC 3800, Australia
4. https://www.gw-openscience.org/eventapi/html/GWTC-1-confident/GW170608/v3/H-H1_GWOSC_16KHZ_R1-1180920447-4096.gwf