Board Question Paper: March 2013
Mathematics and Statistics

Time: 3 Hours **Total Marks: 80**

Note:

i. All questions are compulsory
ii. Figures to the right indicate full marks.
iii. Solution of L.P.P. should be written on graph paper only.
iv. Answers to both the sections should be written in the same answer book.
v. Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following: **(6)[12]**

i. The principal solution of the equation $\cot x = -\sqrt{3}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $-\frac{5\pi}{6}$

ii. If the vectors $-3\hat{i}+4\hat{j}-2\hat{k}, \hat{i}+2\hat{k}, \hat{i}-p\hat{j}$ are coplanar, then the value of p is

(A) – 2 (B) 1
(C) –1 (D) 2

*iii. If the line $y = x + k$ touches the hyperbola $9x^2 - 16y^2 = 144$, then k = ______

(A) 7 (B) – 7
(C) $\pm\sqrt{7}$ (D) $\pm\sqrt{19}$

(B) Attempt any THREE of the following: **(6)**

i. Write down the following statements in symbolic form:
 a. A triangle is equilateral if and only if it is equiangular.
 b. Price increases and demand falls.

ii. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, then find A^{-1} by adjoint method.

iii. Find the separate equations of the lines represented by the equation $3x^2 - 10xy - 8y^2 = 0$.

*iv. Find the equation of the director circle of a circle $x^2 + y^2 = 100$.

v. Find the general solution of the equation $4\cos^2 x = 1$.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. Without using truth table show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

ii. If θ is the measure of acute angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$, $a + b \neq 0$.

*iii. Show that the line $x + 2y + 8 = 0$ is tangent to the parabola $y^2 = 8x$. Hence, find the point of contact.

(B) Attempt any TWO of the following: **(8)**

i. The sum of three numbers is 9. If we multiply third number by 3 and add to the second number, we get 16. By adding the first and the third number and then subtracting twice the second number from this sum, we get 6. Use this information and find the system of linear equations. Hence, find the three numbers using matrices.

ii. Find the general solution of $\cos x + \sin x = 1$.

iii. If $\overline{a}$ and $\overline{b}$ are any two non-zero and non-collinear vectors, then prove that any vector $\overline{r}$ coplanar with $\overline{a}$ and $\overline{b}$ can be uniquely expressed as $\overline{r} = t_1\overline{a} + t_2\overline{b}$, where t_1 and t_2 are scalars.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Using truth table examine whether the following statement pattern is tautology, contradiction or contingency.

$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

ii. Find k, if the length of the tangent segment from (8, – 3) to the circle $x^2 + y^2 - 2x + ky - 23 = 0$ is $\sqrt{10}$ units.

iii. Show that the lines given by

$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1}$ and $\frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$ intersect.

Also find the co-ordinates of the point of intersection.

(B) Attempt any TWO of the following: **(8)**

*i. Find the equation of the locus of the point of intersection of two tangents drawn to the hyperbola $\frac{x^2}{7} - \frac{y^2}{5} = 1$ such that the sum of the cubes of their slopes is 8.

ii. Solve the following L.P.P. graphically:

Maximize : $Z = 10x + 25y$

Subject to: $x \leq 3, y \leq 3, x + y \leq 5, x \geq 0, y \geq 0$

iii. Find the equations of the planes parallel to the plane $x + 2y + 2z + 8 = 0$ which are at the distance of 2 units from the point (1, 1, 2).

Section – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following: **(6)[12]**

i. Function $f(x) = x^2 - 3x + 4$ has minimum value at $x =$ ______

(A) 0 (B) $-\frac{3}{2}$

(C) 1 (D) $\frac{3}{2}$

ii. $\int \frac{1}{x} \cdot \log x \, dx =$ _____

(A) $\log(\log x) + c$ (B) $\frac{1}{2}(\log x)^2 + c$

(C) $2 \log x + c$ (D) $\log x + c$

iii. Order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\frac{d^2y}{dx^2}$$ are respectively –

(A) 2, 3 (B) 3, 2
(C) 7, 2 (D) 3, 7

(B) Attempt any THREE of the following: **(6)**

i. If $x = at^2$, $y = 2at$, then find $\frac{dy}{dx}$.

ii. Find the approximate value of $\sqrt{8.95}$.

iii. Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 3$.

*iv. For the bivariate data $r = 0.3$, $\text{cov}(X, Y) = 18$, $\sigma_x = 3$, find σ_y.

*v. A triangle bounded by the lines $y = 0$, $y = x$ and $x = 4$ is revolved about the X-axis. Find the volume of the solid of revolution.

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. A function $f(x)$ is defined as

$$\begin{aligned} f(x) &= x + a \quad , \quad x < 0 \\ &= x \quad , \quad 0 \le x < 1 \\ &= b - x \quad , \quad x \ge 1 \end{aligned}$$

is continuous in its domain. Find a + b.

ii. If $x = a\left(t - \frac{1}{t}\right)$, $y = a\left(t + \frac{1}{t}\right)$, then show that $\frac{dy}{dx} = \frac{x}{y}$.

iii. Evaluate : $\int \frac{1}{3 + 5\cos x}\,dx$

(B) Attempt any TWO of the following: **(8)**

i. An insurance agent insures lives of 5 men, all of the same age and in good health. The probability that a man of this age will survive the next 30 years is known to be $\frac{2}{3}$. Find the probability that in the next 30 years at most 3 men will survive.

ii. The surface area of a spherical balloon is increasing at the rate of 2 cm^2/sec. At what rate is the volume of the balloon is increasing when the radius of the balloon is 6 cm?

iii. The slope of the tangent to the curve at any point is equal to $y + 2x$. Find the equation of the curve passing through the origin.

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. If u and v are two functions of x, then prove that

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

ii. The time (in minutes) for a lab assistant to prepare the equipment for a certain experiment is a random variable X taking values between 25 and 35 minutes with p. d. f.

$f(x) = \frac{1}{10}$, $25 \le x \le 35$ = 0, otherwise.

What is the probability that preparation time exceeds 33 minutes? Also find the c. d. f. of X.

iii. The probability that a certain kind of component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 tested components survive.

(B) Attempt any TWO of the following: **(8)**

i. If $ax^2 + 2hxy + by^2 = 0$, show that $\frac{d^2y}{dx^2} = 0$.

ii. Find the area of the region common to the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$.

*iii. For 10 pairs of observations on two variables X and Y, the following data are available:

$\sum(x-2) = 30$, $\sum(y-5) = 40$, $\sum(x-2)^2 = 900$,

$\sum(y-5)^2 = 800$, $\sum(x-2)(y-5) = 480$.

Find the correlation coefficient between X and Y.

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SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following: **(6)[12]**

i. Which of the following represents direction cosines of the line?

(A) $0, \frac{1}{\sqrt{2}}, \frac{1}{2}$ (B) $0, \frac{-\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

(C) $0, \frac{\sqrt{3}}{2}, \frac{1}{2}$ (D) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

ii. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and A (adj A) = KI, then the value of 'K' is _______.

(A) 2 (B) – 2
(C) 10 (D) – 10

iii. The general solution of the trigonometric equation $\tan^2\theta = 1$ is _______.

(A) $\theta = n\pi \pm \frac{\pi}{3}, n \in Z$ (B) $\theta = n\pi \pm \frac{\pi}{6}, n \in Z$

(C) $\theta = n\pi \pm \frac{\pi}{4}, n \in Z$ (D) $\theta = n\pi, n \in Z$

(B) Attempt any THREE of the following: **(6)**

i. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are the position vectors of the points A, B, C respectively and $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment AB.

ii. The cartesian equation of a line is $\frac{x-6}{2} = \frac{y+4}{7} = \frac{z-5}{3}$, find its vector equation.

iii. Equation of a plane is $\bar{r} \cdot \left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) = 8$. Find the length of the perpendicular from the origin to the plane.

iv. Find the acute angle between the lines whose direction ratios are 5, 12, – 13 and 3, – 4, 5.

v. Write the dual of the following statements:
a. $(p \vee q) \wedge T$
b. Madhuri has curly hair and brown eyes.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k.

ii. Prove that three vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ are coplanar, if and only if, there exists a non-zero linear combination $x\bar{a} + y\bar{b} + z\bar{c} = \bar{0}$.

iii. Using truth table, prove that
$\sim p \wedge q \equiv (p \vee q) \wedge \sim p$.

(B) Attempt any TWO of the following: **(8)**

i. In any ΔABC, with usual notations, prove that
$b^2 = c^2 + a^2 - 2ca \cos B$.

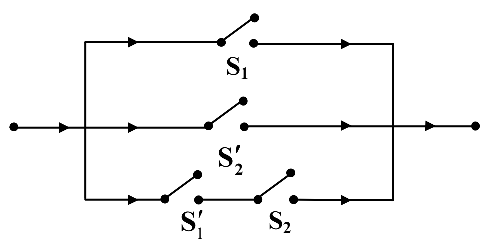
ii. Show that the equation $x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$ represents a pair of lines. Find the acute angle between them. Also find the point of intersection of the lines.

iii. Express the following equations in the matrix form and solve them by the method of reduction:
$2x - y + z = 1$, $x + 2y + 3z = 8$, $3x + y - 4z = 1$.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Prove that a homogeneous equation of degree two in x and y (i.e., $ax^2 + 2hxy + by^2 = 0$), represents a pair of lines through the origin if $h^2 - ab \geq 0$.

ii. Find the symbolic form of the following switching circuit, construct its switching table and interpret it.



iii. If A, B, C, D are (1, 1, 1), (2, 1, 3), (3, 2, 2), (3, 3, 4) respectively, then find the volume of the parallelopiped with AB, AC and AD as the concurrent edges.

(B) Attempt any TWO of the following: **(8)**

i. Find the equation of the plane passing through the line of intersection of the planes
$2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z-3}{5}$.

ii. Minimize: $Z = 6x + 4y$
Subject to: $3x + 2y \geq 12$,
$x + y \geq 5$,
$0 \leq x \leq 4$,
$0 \leq y \leq 4$.

iii. Show that: $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following: (6)[12]

i. If $y = 1 - \cos\theta$, $x = 1 - \sin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(A) -1 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

ii. The integrating factor of linear differential equation

$\frac{dy}{dx} + y \sec x = \tan x$ is

(A) $\sec x - \tan x$ (B) $\sec x \cdot \tan x$

(C) $\sec x + \tan x$ (D) $\sec x \cdot \cot x$

iii. The equation of tangent to the curve $y = 3x^2 - x + 1$ at the point (1, 3) is

(A) $y = 5x + 2$ (B) $y = 5x - 2$

(C) $y = \frac{1}{5}x + 2$ (D) $y = \frac{1}{5}x - 2$

(B) Attempt any THREE of the following: (6)

i. Examine the continuity of the function

$$f(x) = \sin x - \cos x, \quad \text{for } x \neq 0$$
$$= -1 \quad , \quad \text{for } x = 0$$

at the point $x = 0$.

ii. Verify Rolle's theorem for the function

$f(x) = x^2 - 5x + 9$ on [1, 4].

iii. Evaluate: $\int \sec^n x \cdot \tan x \, dx$

iv. The probability mass function (p.m.f.) of X is given below:

$X = x$	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Find $E(X^2)$.

v. Given that X ~ B (n = 10, p). If E (X) = 8, find the value of p.

Q.5. (A) Attempt any TWO of the following: (6)[14]

i. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then prove that $y = f[g(x)]$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

ii. Obtain the differential equation by eliminating the arbitrary constants A, B from the equation:

$y = A \cos(\log x) + B \sin(\log x)$

iii. Evaluate: $\int \frac{x^2}{(x^2+2)(2x^2+1)} dx$

(B) Attempt any TWO of the following: **(8)**

i. An open box is to be made out of a piece of a square cardboard of sides 18 cms by cutting off equal squares from the corners and turning up the sides. Find the maximum volume of the box.

ii. Prove that: $\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$

iii. If the function f(x) is continuous in the interval [–2, 2], find the values of a and b, where

$$f(x) = \frac{\sin ax}{x} - 2, \qquad \text{for } -2 \le x < 0$$
$$= 2x + 1, \qquad \text{for } 0 \le x \le 1$$
$$= 2b\sqrt{x^2+3} - 1, \qquad \text{for } 1 < x \le 2$$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Solve the differential equation: $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$.

ii. A fair coin is tossed 8 times. Find the probability that it shows heads at least once.

iii. If $x^p y^q = (x + y)^{p+q}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

(B) Attempt any TWO of the following: **(8)**

i. Find the area of the sector of a circle bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant.

ii. Prove that:

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$$

iii. A random variable X has the following probability distribution:

X = x	0	1	2	3	4	5	6
P[X = x]	k	3k	5k	7k	9k	11k	13k

a. Find k.

b. Find P(0 < X < 4).

c. Obtain cumulative distribution function (c.d.f.) of X.

BOARD QUESTION PAPER : MARCH 2015
MATHEMATICS AND STATISTICS

Time: 3 Hours **Total Marks: 80**

Note:

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vi. Use of logarithmic table is allowed.

SECTION – I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^6 =$ _______.

(A) 6A (B) 12A (C) 16A (D) 32A

ii. The principal solution of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{2}$

iii. If an equation $hxy + gx + fy + c = 0$ represents a pair of lines, then
(A) fg = ch (B) gh = cf (C) fh = cg (D) hf = – cg

(B) Attempt any THREE of the following: **(6)**

i. Write the converse and contrapositive of the statement-
"If two triangles are congruent then their areas are equal."

ii. Find 'k', if the sum of slopes of lines represented by equation $x^2 + kxy - 3y^2 = 0$ is twice their product.

iii. Find the angle between the planes $\bar{r}\cdot\left(2\hat{i}+\hat{j}-\hat{k}\right) = 3$ and $\bar{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right) = 1$.

iv. The cartesian equations of line are $3x - 1 = 6y + 2 = 1 - z$. Find the vector equation of line.

v. If $\bar{a} = \hat{i} + 2\hat{j}$, $\bar{b} = -2\hat{i} + \hat{j}$, $\bar{c} = 4\hat{i} + 3\hat{j}$, find x and y such that $\bar{c} = x\bar{a} + y\bar{b}$.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. If A, B, C, D are (1, 1, 1), (2, 1, 3), (3, 2, 2), (3, 3, 4) respectively, then find the volume of the parallelopiped with AB, AC and AD as the concurrent edges.

ii. Discuss the statement pattern, using truth table: $\sim(\sim p \wedge \sim q) \vee q$

iii. If point $C(\bar{c})$ divides the segment joining the points $A(\bar{a})$ and $B(\bar{b})$ internally in the ratio m : n, then prove that $\bar{c} = \frac{m\bar{b} + n\bar{a}}{m + n}$.

(B) Attempt any TWO of the following: **(8)**

i. Find the direction cosines of the line perpendicular to the lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$.

ii. In any ΔABC, if a^2, b^2, c^2 are in arithmetic progression, then prove that cot A, cot B, cot C are in arithmetic progression.

iii. The sum of three numbers is 6. When second number is subtracted from thrice the sum of first and third number, we get number 10. Four times the third number is subtracted from five times the sum of first and second number, the result is 3. Using above information, find these three numbers by matrix method.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. If θ is the acute angle between the lines represented by equation $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a + b}\right|$, $a + b \neq 0$.

ii. If the lines $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$ and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1}$ intersect each other, then find the value of 'k'.

iii. Construct the switching circuit for the following statement:
$[p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$

(B) Attempt any TWO of the following: **(8)**

i. Find the general solution of $\cos x - \sin x = 1$.

ii. Find the equations of the planes parallel to the plane $x - 2y + 2z - 4 = 0$, which are at a unit distance from the point (1, 2, 3).

iii. A diet of a sick person must contain at least 48 units of vitamin A and 64 units of vitamin B. Two foods F_1 and F_2 are available. Food F_1 costs ` 6 per unit and food $F_2$ costs ` 10 per unit. One unit of food F_1 contains 6 units of vitamin A and 7 units of vitamin B. One unit of food F_2 contains 8 units of vitamin A and 12 units of vitamin B. Find the minimum cost for the diet that consists of mixture of these two foods and also meeting the minimal nutritional requirements.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. A random variable X has the following probability distribution:

X = x	–2	–1	0	1	2	3
P(x)	0.1	0.1	0.2	0.2	0.3	0.1

Then E(x) =

(A) 0.8 (B) 0.9
(C) 0.7 (D) 1.1

ii. If $\int_0^{\alpha} 3x^2\,dx = 8$, then the value of α is

(A) 0 (B) –2
(C) 2 (D) ±2

iii. The differential equation of $y = \frac{c}{x} + c^2$ is

(A) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = y$ (B) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(C) $x^3\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = y$ (D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

(B) Attempt any THREE of the following: **(6)**

i. Evaluate: $\int e^x\left[\frac{\sqrt{1-x^2}\cdot\sin^{-1}x + 1}{\sqrt{1-x^2}}\right]dx$

ii. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$, then show that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

iii. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\,dx$

iv. If $y = e^{ax}$, show that $x\frac{dy}{dx} = y \log y$.

v. A fair coin is tossed five times. Find the probability that it shows exactly three times head.

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. Integrate: $\sec^3 x$ w.r.t. x.

ii. If $y = (\tan^{-1}x)^2$, show that

$(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} - 2 = 0$

iii. If $f(x) = \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}}$, for $x \neq 0$

$= k$, for $x = 0$

is continuous at $x = 0$, find k.

(B) Attempt any TWO of the following: **(8)**

i. Find the co-ordinates of the points on the curve $y = x - \frac{4}{x}$, where the tangents are parallel to the line $y = 2x$.

ii. Prove that: $\int\sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

iii. Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\sin x}\,dx$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Find a and b, so that the function $f(x)$ defined by

$f(x) = -2\sin x$, for $-\pi \le x \le -\frac{\pi}{2}$

$= a\sin x + b$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$= \cos x$, for $\frac{\pi}{2} \le x \le \pi$ is continuous on $[-\pi, \pi]$.

ii. If $\log_{10}\left(\dfrac{x^3 - y^3}{x^3 + y^3}\right) = 2$, then show that $\dfrac{dy}{dx} = -\dfrac{99x^2}{101y^2}$.

iii. Let the p.m.f. (probability mass function) of random variable x be

$P(x) = \binom{4}{x}\left(\dfrac{5}{9}\right)^x\left(\dfrac{4}{9}\right)^{4-x}$, $x = 0, 1, 2, 3, 4$.

$= 0$, otherwise

Find $E(x)$ and Var (x).

(B) Attempt any TWO of the following: (8)

i. Examine the maxima and minima of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$. Also, find the maximum and minimum values of $f(x)$.

ii. Solve the differential equation $(x^2 + y^2)dx - 2xydy = 0$.

iii. Given the p.d.f. (probability density function) of a continuous random variable x as:

$f(x) = \dfrac{x^2}{3}$, $-1 < x < 2$

$= 0$, otherwise

Determine the c.d.f. (cumulative distribution function) of x and hence find $P(x < 1)$, $P(x \le -2)$, $P(x > 0)$, $P(1 < x < 2)$.

BOARD QUESTION PAPER : MARCH 2016
MATHEMATICS AND STATISTICS

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SECTION – I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6) [12]**

i. The negation of $p \wedge (q \rightarrow r)$ is

(A) $p \vee (\sim q \vee r)$ (B) $\sim p \wedge (q \rightarrow r)$
(C) $\sim p \wedge (\sim q \rightarrow \sim r)$ (D) $\sim p \vee (q \wedge \sim r)$

ii. If $\sin^{-1}(1 - x) - 2 \sin^{-1}x = \frac{\pi}{2}$ then x is

(A) $-\frac{1}{2}$ (B) 1
(C) 0 (D) $\frac{1}{2}$

iii. The joint equation of the pair of lines passing through (2, 3) and parallel to the coordinate axes is

(A) $xy - 3x - 2y + 6 = 0$ (B) $xy + 3x + 2y + 6 = 0$
(C) $xy = 0$ (D) $xy - 3x - 2y - 6 = 0$

(B) Attempt any THREE of the following: **(6)**

i. Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -2 \end{bmatrix}$

ii. Find the vector equation of the plane passing through a point having position vector $3\hat{i} - 2\hat{j} + \hat{k}$ and perpendicular to the vector $4\hat{i} + 3\hat{j} + 2\hat{k}$.

iii. If $\bar{p} = \hat{i} - 2\hat{j} + \hat{k}$ and $\bar{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are position vector (P.V.) of points P and Q, find the position vector of the point R which divides segment PQ internally in the ratio 2:1.

iv. Find k, if one of the lines given by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$.

v. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are at right angle then find the value of k.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. Examine whether the following logical statement pattern is tautology, contradiction or contingency.
$[(p \rightarrow q) \wedge q] \rightarrow p$

ii. By vector method prove that the medians of a triangle are concurrent.

iii. Find the shortest distance between the lines $\bar{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$ where λ and μ are parameters.

(B) Attempt any TWO of the following: **(8)**

i. In ΔABC with the usual notations prove that

$$(a - b)^2 \cos^2\left(\frac{C}{2}\right) + (a + b)^2 \sin^2\left(\frac{C}{2}\right) = c^2.$$

ii. Minimize $z = 4x + 5y$ subject to $2x + y \geq 7$, $2x + 3y \leq 15$, $x \leq 3$, $x \geq 0$, $y \geq 0$. Solve using graphical method.

iii. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ` 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ` 90 whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ` 70. Find the cost of each item per dozen by using matrices.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Find the volume of tetrahedron whose coterminus edges are $7\hat{i} + \hat{k}$, $2\hat{i} + 5\hat{j} - 3\hat{k}$ and $4\hat{i} + 3\hat{j} + \hat{k}$.

ii. Without using truth table show that

$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

iii. Show that every homogeneous equation of degree two in x and y, i.e., $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through origin if $h^2 - ab \geq 0$.

(B) Attempt any TWO of the following: **(8)**

i. If a line drawn from the point A(1, 2, 1) is perpendicular to the line joining P(1, 4, 6) and Q(5, 4, 4) then find the co-ordinates of the foot of the perpendicular.

ii. Find the vector equation of the plane passing through the points $\hat{i} + \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$. Hence find the cartesian equation of the plane.

iii. Find the general solution of $\sin x + \sin 3x + \sin 5x = 0$.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If the function

$f(x) = k + x$, for $x < 1$

$\quad = 4x + 3$, for $x \geq 1$

is continuous at $x = 1$ then k =

(A) 7 (B) 8 (C) 6 (D) –6

ii. The equation of tangent to the curve $y = x^2 + 4x + 1$ at $(-1, -2)$ is

(A) $2x - y = 0$ (B) $2x + y - 5 = 0$

(C) $2x - y - 1 = 0$ (D) $x + y - 1 = 0$

iii. Given that $X \sim B(n = 10, p)$. If $E(X) = 8$ then the value of p is

(A) 0.6 (B) 0.7 (C) 0.8 (D) 0.4

(B) Attempt any THREE of the following: **(6)**

i. If $y = x^x$, find $\frac{dy}{dx}$.

ii. The displacement 's' of a moving particle at time 't' is given by $s = 5 + 20t - 2t^2$. Find its acceleration when the velocity is zero.

iii. Find the area bounded by the curve $y^2 = 4ax$, X-axis and the lines $x = 0$ and $x = a$.

iv. The probability distribution of a discrete random variable X is:

$X = x$	1	2	3	4	5
$P(X = x)$	k	2k	3k	4k	5k

Find $P(X \leq 4)$.

v. Evaluate: $\int \frac{\sin x}{\sqrt{36-\cos^2 x}}\,dx$

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x then prove that $y = f(g(x))$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

ii. The probability that a person who undergoes kidney operation will recover is 0.5. Find the probability that of the six patients who undergo similar operations.

a. None will recover.

b. Half of them will recover.

iii. Evaluate: $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}\,dx$

(B) Attempt any TWO of the following: **(8)**

i. Discuss the continuity of the following functions. If the function have a removable discontinuity, redefine the function so as to remove the discontinuity.

$$\left.\begin{aligned} f(x) &= \frac{4^x - e^x}{6^x - 1}, \text{ for } x \neq 0 \\ &= \log\left(\frac{2}{3}\right), \text{ for } x = 0 \end{aligned}\right\} \text{at } x = 0$$

ii. Prove that:

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

iii. A body is heated at 110°C and placed in air at 10°C. After 1 hour its temperature is 60°C. How much additional time is required for it to cool to 35°C?

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Prove that: $\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$

ii. Evaluate: $\int \frac{1+\log x}{x(2+\log x)(3+\log x)}\,dx$

iii. If $y = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$, find $\frac{dy}{dx}$

(B) Attempt any TWO of the following: **(8)**

i. Solve the differential equation $\cos(x + y)dy = dx$

Hence find the particular solution for $x = 0$ and $y = 0$.

ii. A wire of length l is cut into two parts. One part is bent into a circle and other into a square. Show that the sum of areas of the circle and square is the least, if the radius of circle is half the side of the square.

iii. The following is the p.d.f. (Probability Density Function) of a continuous random variable X:

$$\begin{aligned} f(x) &= \frac{x}{32}, \quad 0 < x < 8 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

a. Find the expression for c.d.f. (Cumulative Distribution Function) of X.

b. Also find its value at $x = 0.5$ and 9.

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Note:
i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Answer to every new question must be written on a new page.
v. Answers to both sections should be written in the same answer book.
vi. Use of logarithmic table is allowed.

SECTION – I

Q.1. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions: **(6) [12]**

(i) If the points A(2, 1, 1), B(0, –1, 4) and C(k, 3, –2) are collinear, then k = _______.
(A) 0 (B) 1
(C) 4 (D) –4

(ii) The inverse of the matrix $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$ is _______.

(A) $\frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$ (B) $\frac{1}{13}\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

(C) $\frac{1}{13}\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ (D) $\frac{1}{13}\begin{bmatrix} 1 & 5 \\ 3 & -2 \end{bmatrix}$

(iii) In Δ ABC, if a = 13, b = 14 and c = 15, then $\sin\left(\frac{A}{2}\right)$ = _______.

(A) $\frac{1}{5}$ (B) $\sqrt{\frac{1}{5}}$

(C) $\frac{4}{5}$ (D) $\frac{2}{5}$

(B) Attempt any THREE of the following: **(6)**

i. Find the volume of the parallelopiped whose coterminus edges are given by vectors $2\hat{i}+3\hat{j}-4\hat{k}$, $5\hat{i}+7\hat{j}+5\hat{k}$ and $4\hat{i}+5\hat{j}-2\hat{k}$.

ii. In Δ ABC, prove that, a (b cos C – c cos B) = $b^2 - c^2$.

iii. If from a point Q (a, b, c) perpendiculars QA and QB are drawn to the YZ and ZX planes respectively, then find the vector equation of the plane QAB.

iv. Find the cartesian equation of the line passing throught the points A(3, 4, –7) and B(6, –1, 1).

v. Write the following statement in symbolic form and find its truth value:
$\forall\, n \in N$, $n^2 + n$ is an even number and $n^2 - n$ is an odd number.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. Using truth tables, examine whether the statement pattern $(p \wedge q) \vee (p \wedge r)$ is a tautology, contradiction or contingency.

ii. Find the shortest distance between the lines
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

iii. Find the general solution of the equation
$\sin 2x + \sin 4x + \sin 6x = 0$

(B) Attempt any TWO of the following: **(8)**

i. Solve the following equations by method of reduction:
$x - y + z = 4$, $2x + y - 3z = 0$, $x + y + z = 2$

ii. If θ is the measure of the acute angle between the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$ where $a + b \neq 0$ and $b \neq 0$.

Find the condition for coincident lines.

iii. Using vector method, find incentre of the triangle whoose vertices are P(0, 4, 0), Q(0, 0, 3) and R(0, 4, 3).

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Construct the switching circuit for the statement
$(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$.

ii. Find the joint equation of the pair of lines passing through the origin which are perpendicular respectively to the lines represented by $5x^2 + 2xy - 3y^2 = 0$.

iii. Show that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

(B) Attempt any TWO of the following: **(8)**

i. If l, m, n are the direction cosines of a line, then prove that $l^2 + m^2 + n^2 = 1$. Hence find the direction angle of the line with the X axis which makes direction angles of 135° and 45° with Y and Z axes respectively.

ii. Find the vector and cartesian equations of the plane passing through the points A(1, 1, −2), B(1, 2, 1) and C(2, −1, 1).

iii. Solve the following L.P.P. by graphical method:
Maximise : $Z = 6x + 4y$
subject to $x \leq 2$, $x + y \leq 3$, $-2x + y \leq 1$, $x \geq 0$, $y \geq 0$

SECTION – II

Q.4. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. Derivatives of $\tan^3\theta$ with respect to $\sec^3\theta$ at $\theta = \frac{\pi}{3}$ is _______.

(A) $\frac{3}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $-\frac{\sqrt{3}}{2}$

ii. The equation of tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3) is _______.

(A) $5x - y = 2$ (B) $x + 5y = 16$

(C) $5x - y + 2 = 0$ (D) $5x = y$

iii. The expected value of the number of heads obtained when three fair coins are tossed simultaneously is _______.

(A) 1 (B) 1.5

(C) 0 (D) –1

(B) Attempt any THREE of the following: **(6)**

i. Find $\frac{dy}{dx}$ if $x \sin y + y \sin x = 0$.

ii. Test whether the function, $f(x) = x - \frac{1}{x}, x \in R, x \neq 0$, is increasing or decreasing.

iii. Evaluate: $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

iv. Form the differential equation by eliminating arbitrary constants from the relation $y = Ae^{5x} + Be^{-5x}$

v. The probability that a bomb will hit a target is 0.8. Find the probability that out of 10 bombs dropped, exactly 4 will hit the target.

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. Solve: $\frac{dy}{dx} = \cos(x + y)$

ii. If u and v are two functions of x, then prove that:

$$\int u\,v\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

iii. If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$, for $x \neq 0$, is continuous at $x = 0$, find f(0).

(B) Attempt any TWO of the following: **(8)**

i. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ where $\frac{dy}{dx} \neq 0$.

Hence find $\frac{d}{dx}\left(\tan^{-1} x\right)$.

ii. A telephone company in a town has 5000 subscribers on its list and collects fixed rent charges of ₹ 3,000 per year from each subscriber. The company proposes to increase annual rent and it is believed that for every increase of one rupee in the rent, one subscriber will be discontinued. Find what increased annual rent will bring the maximum annual income to the company.

iii. Evaluate: $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Discuss the continuity of the following function, at $x = 0$.

$$\begin{aligned} f(x) &= \frac{x}{|x|}, \text{ for } x \neq 0 \\ &= 1, \text{ for } x = 0 \end{aligned}$$

ii. If the population of a country doubles in 60 years, in how many years will it be triple under the assumption that the rate of increase in proportional to the number of inhabitants?
[Given : log 2 = 0.6912 and log 3 = 1.0986.]

iii. A fair coin is tossed 8 times. Find the probability that it shows heads
a. exactly 5 times
b. at least once.

(B) Attempt any TWO of the following: **(8)**

i. Evaluate: $\int \frac{d\theta}{\sin\theta + \sin 2\theta}$

ii. Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

iii. Given the probability density function (p.d.f.) of a continuous random variable X as,

$$\begin{aligned} f(x) &= \frac{x^2}{3}, -1 < x < 2 \\ &= 0, \text{ otherwise} \end{aligned}$$

Determine the cumulative distribution function (c.d.f.) of X and hence find P (X < 1), P (X > 0), P (1 < X < 2).