HW 5:Radiation from Moving Charges

Due: Oct. 1, 9:30AM on D2L

ASTR 589, Prof. Youdin

1. **Dipole antennas** radiate and receive the radio waves for cell phone use etc. Use the Larmor formula to estimate the current needed to power a 10^4 watt cell or FM tower.

   (a) What are the common frequencies and wavelengths, λ of cell communications? Of the community radio station 93.1 KXCI (broadcasting from Hotel Congress)?

   (b) On physical grounds, justify a simple expression for the electic dipole moment d in an antenna due to a current, I, that varies in time at the relevant radio frequencies $\nu = 2\pi\omega$, in an antenna of length, L. From this result, estimate $\ddot{d}$.

   (c) Derive the radiated power $P(I, L, \lambda)$. Assuming $L/\lambda \simeq 1$, what power is required, in Amps. Ignore uncertain order unity factors (2, π, etc.).

   (d) Justify the best orientation for a dipole antenna (horizontal, vertical, between?).

2. **Electron scattering opacity** This problem first calculates the electron scattering opacity, relevant in many stellar interiors. If you are not confident in your calculation, you can use $\kappa = 1$ cm^2/g (which is not quite right) to make the estimates that follow.

   (a) From the electron scattering cross section $\sigma_e = 0.665 \times 10^{-24}$ cm^2, calculate the scattering opacity, κ_e. Assume a fully ionized mixture of hydrogen, with mass fraction $X_H = X$, and helium, with mass fraction $X_{\rm He} = Y = 1 - X$. Approximate $m_p = m_n$ and $m_e = 0$ for protons, neutrons and electrons. Give your answer in terms of X.

   Hints for your derivation: The average mass per electron is ρ/n_e (mass density over number density of electrons). Consider n_e as the sum over the number density of electrons from each species, n_i, which depends on the atomic number, Z_i, atomic mass number, A_i and the mass density $\rho_i = X_i\rho$ of each species.

   (b) Estimate the total optical depth, τ, to the center of the Sun roughly, for electron scattering opacity and using the mean density throughout. (*This rough estimate is also a lower bound due to other sources of opacity*

   (c) In terms of $\tau, R_\odot, c$, what are N, the number of scatterings for a photon to escape, and $t_{\rm esc}$, the time for the photon to escape? Give the numerical values as well. Give some caveats of this estimate and why the timescale is relevant.

   (d) Make a crude estimate of the central temperature of the Sun, $T_c \sim T_{\rm diff}$ by assuming a constant radiative flux, which gives known solutions for $T(\tau)$. Then make a more robust, but still rough, virial approximation $T_c \sim T_{\rm vir}$, by equating mean thermal energies, kT and mean gravitational energies GMm_p/R (since the average particle mass is $m_p/2 \sim m_p$). Taking $T_{\rm vir}$ as typical, what is the ratio of radiation energy in the Sun to rest mass energy, $M_\odot c^2$?

3. **Classical Scattering** Start from the equation of motion of a bound electron with natural frequency ω_0, which is forced by an oscillating electric field with frequency ω

$$\ddot{x} = -\omega_0^2 x + \tau \dddot{x} + \frac{eE_0}{m} \exp(\imath\omega t)$$

(a) Explain the term with $\tau = 2r_0/(3c)$ in a few sentences. No detailed derivation needed.

(b) Reproduce the derivation of the frequency dependent cross section, $\sigma(\omega)$. Note that the derivation in Rybicki and Lightman make an unstated approximation involving switching ω and ω_0. Either explain this approximation or don't make it (it's not necessary).

(c) Evaluate $\sigma(\omega)$ in the 3 limiting cases $\omega \ll \omega_0$, $\omega \gg \omega_0$ and $\omega \simeq \omega_0$. Give brief descriptions and show that the scattering line shape matches the emission case.

4. **From classical to quantum** For a transition between states 1 and 2, with energy difference $E_2 - E_1 = h\nu_0$, the dimensionless (and often order unity) oscillator strength f_{12} gives a simple way to relate the results of classical oscillators to the more accurate results of quantum mechanics. In this exercise we start with the definition of frequency integrated cross section

$$\int \sigma_\nu d\nu = \frac{\pi e^2}{me^2} f_{12}$$

(a) Based on this definition calculate the Einstein coefficients B_{12}, B_{21} and A_{21}.

(b) Express A_{21} in terms of the classical line width (in frequency) $\Gamma = \omega_0^2 \tau$ with $\tau = 2r_0/(3c)$ the radiation reaction timescale.