HW 3 : Radiation moments Neev

1. $$T^4 = \frac{3}{4} T_{eff}^4 \left(\tau + 2/3 \right)$$

$$\mathcal{E} = \frac{2\pi}{c} \left(I_+ + I_- \right)$$

$$F = \pi \left(I_+ - I_- \right)$$

$$P = \frac{2\pi}{3c} \left(I_+ + I_- \right)$$

$$F = \sigma T_{eff}^4 \qquad I_-(0) = 0 \qquad \cancel{E} = 0$$

$$\frac{dF}{d\tau^{(abs)}} = \frac{K}{K^{(abs)}} \frac{dF}{d\tau} = c\mathcal{E} - 4\pi \mathcal{S} = 0$$

$$\frac{dP}{d\tau} = \frac{F}{c}$$

$$E(0) = \frac{2\pi}{c}(I_+)$$

$$F(0) = \pi(I_+)$$

$$P(0) = \frac{2\pi}{3c}(I_+)$$

a) $$\boxed{\begin{aligned} E(0) &= \frac{2F(0)}{c} \\ P(0) &= \frac{2F(0)}{3c} \end{aligned}}$$

b) $F = \sigma T_{eff}^4$

$F(\tau) = F(0)$
$= \sigma T_{eff}^4$

$$\frac{dP}{d\tau} = \frac{F}{c}$$

$$\int_{P(0)}^{P(\tau)} dP = \int_0^\tau \frac{F}{c}\, d\tau$$

$$P(\tau) - \frac{2F(0)}{3c} = \frac{F\tau}{c}$$

$$P(\tau) = \frac{F}{c}\left(\tau + \frac{2}{3}\right)$$

$$P(\tau) = \frac{\sigma T_{eff}^4}{c}(\tau + 2/3)$$

$$P = \frac{E}{3}$$

$$\boxed{E(\tau) = \frac{3\sigma T_{eff}^4}{c}(\tau + 2/3)}$$

c) $F \Rightarrow$ const

$$cE = 4\pi B$$

$$B(\tau) = \frac{\cancel{c}}{4\pi} \cdot \frac{3\sigma T_{eff}^4}{\cancel{c}}(\tau + 2/3)$$

$$= \frac{3\sigma}{4\pi} T_{eff}^4 (\tau + 2/3)$$

$$B(\tau) = \frac{\sigma T^4}{\pi}$$

$$\therefore \boxed{T^4(\tau) = \frac{3}{4} T_{eff}^4 (\tau + 2/3)}$$

Eddington approximation

2. a) $P_{rad} = \frac{aT^4}{3}$

$\frac{F}{m} = \frac{1}{c} \kappa_F F$

$d\tau = -\rho\kappa dz$

$\frac{F}{V} = \frac{\rho\kappa_F F}{c}$

$\frac{dP \cancel{dxdy}}{\cancel{dxdy}dz} = -\frac{\rho\kappa_F F}{c}$

$\cancel{\frac{dP}{dz} = \frac{GM\rho}{R^2}}$

$\frac{dP_{rad}}{dz} = -\frac{\rho\kappa_F F}{c} = -\frac{\rho\kappa_F L}{c\, 4\pi R^2}$

$\frac{dP}{dz} = -\frac{GM(R)\rho}{R^2}$

$P \sim P_{rad}$

$\frac{\cancel{\rho}\kappa_F L_{edd}}{c 4\pi \cancel{R^2}} = \frac{GM(R)\cancel{\rho}}{\cancel{R^2}}$

$\boxed{L_{Edd} = \frac{4\pi GMc}{\kappa_F}}$

b) $\frac{dP_{gas}}{dz} = -\rho g$

$P_{gas}(\tau = 2/3)$?

$P_{gas}(0) = 0$

$d\tau = -\rho\kappa dz$

$\frac{dP_{gas}}{d\tau} = \frac{\cancel{\rho} g}{\cancel{\rho}\kappa}$

$P_{gas}(2/3) - P_{gas}(0) = \frac{2g}{3\kappa}$

$$\boxed{P_{gas}(2/3) = \frac{2g}{3K}}$$

c) $F = \pi B$

$P = U/3$

$U = \frac{4\pi}{c} B$

$$P = \frac{4\pi}{3c} B$$

$$P = \frac{4F}{3c}$$

$$P_{rad} = \frac{4}{3c} \frac{L}{4\pi R^2}$$

$$P_{rad} \geq P_{gas}$$

$$\frac{\cancel{4}}{\cancel{3}c} \frac{L}{\cancel{4}\pi R^2} \geq \frac{2g}{\cancel{3}K}$$

$$L \geq \frac{2g\pi R^2 c}{K} \qquad g \sim \frac{GM}{R^2}$$

$$L \geq 2\frac{GM}{\cancel{R^2}K}\pi \cancel{R^2} c$$

$$\boxed{L \geq 2\pi\frac{GMc}{K}}$$

earlier we got 4π instead of 2π

3. $T = T_{eff}\, q(\tau)$

$q(\tau) = [(3/4)(\tau + 2/3)]^{1/4}$

$x = \frac{h\nu}{kT_{eff}}$ $\quad \mathcal{F} = \frac{\nu F_\nu}{\sigma T_{eff}^4}$

a) $B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$ $\quad T = T_{eff}$

$F_\nu = \pi B_\nu$

$F_\nu = \frac{\pi\, 2h\nu^3/c^2}{e^{h\nu/kT} - 1}$

$\mathcal{F} = \frac{\nu F_\nu}{\sigma T_{eff}^4} = \frac{2\pi h\nu^4/c^2}{(e^{h\nu/kT_{eff}} - 1)\,\sigma T_{eff}^4}$

$x = \frac{h\nu}{kT_{eff}}$

$$\mathcal{F}_0(x) = \frac{\frac{2\pi h}{c^2\sigma \cancel{T_{eff}^4}} \frac{x^4 k^4 \cancel{T_{eff}^4}}{h^4}}{e^x - 1} = \frac{2\pi k^4}{c^2\sigma h^3} \frac{x^4}{e^x - 1}$$

$$\sigma = \frac{2\pi^5 k^4}{15 c^2 h^3}$$

$$\mathcal{F}_B(x) = \frac{\cancel{2}\pi \cancel{k^4}\ 15\cancel{c^2}\cancel{h^3}}{\cancel{c^2}\cancel{h^3}\ \cancel{2}\pi^5 \cancel{k^4}} \frac{x^4}{e^x - 1} = \boxed{\frac{15}{\pi^4}\ \frac{x^4}{e^x - 1}}$$

b) $F_\nu(0) = 2\pi \int_0^\infty E_2(\tau)\, B_\nu(T(\tau))\, d\tau$

$$\mathcal{F}_E = \frac{\nu F_\nu}{\sigma T_{eff}^4} = \frac{\nu\, 2\pi}{\sigma T_{eff}^4} \int_0^\infty E_2(\tau)\, B_\nu(T(\tau))\, d\tau$$

$$\frac{\nu 2\pi}{\sigma T_{eff}^4} \int_0^\infty E_2(\tau)\ \cancel{F_\nu (T(\tau)) d\tau} \quad \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

$$\Downarrow$$

$$\frac{2h}{c^2}\ \frac{x^3 k^3 T_{eff}^3}{h^3}$$

$$\mathcal{F}_E = \frac{x k \cancel{T_{eff}}\, 2\pi}{h \sigma \cancel{T_{eff}^4}} \int_0^\infty E_2(\tau)\ \frac{2k^3\ \cancel{T_{eff}^3}\ x^3}{c^2 h^2 \left(e^{\left(x \frac{T_{eff}}{T}\right)} - 1\right)}\, d\tau$$

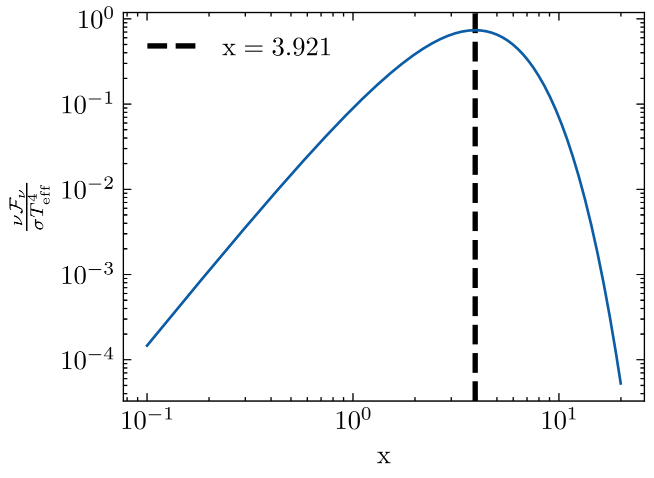
$$F_{B} = \frac{x^4 k^4 4\pi}{h^3 c^2 \sigma} \int_0^\infty \frac{E_2(\tau)\, d\tau}{e^{\left(x \frac{T_{ep}}{T}\right)} - 1}$$

$$F_E = \frac{x^4 \cancel{k^4} \cancel{4}^{2} \pi \; 15 \cancel{c^2} \cancel{h^3}}{\cancel{h^3} \cancel{c^2} \cancel{2} \pi^5 \cancel{k^4}} \int_0^\infty$$

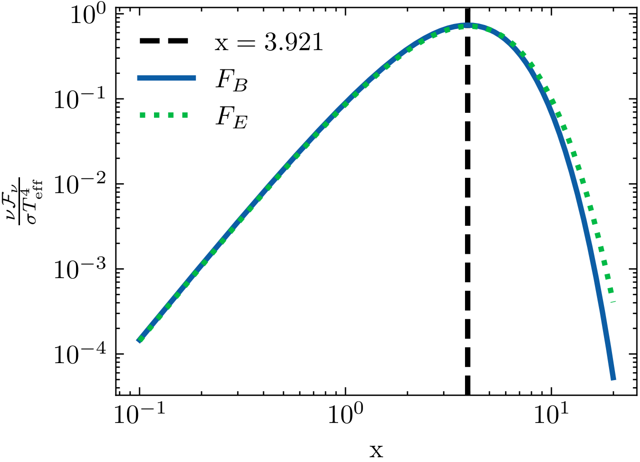
$$F_E = \frac{30 x^4}{\pi^4} \int_0^\infty \frac{E_2(\tau)\, d\tau}{e^{\left[\frac{x T_{ep}}{T(\tau)}\right]} - 1}$$

c) Gray atmosphere emits more than corresponding black body flux at higher frequencies while they are almost same at low frequencies.

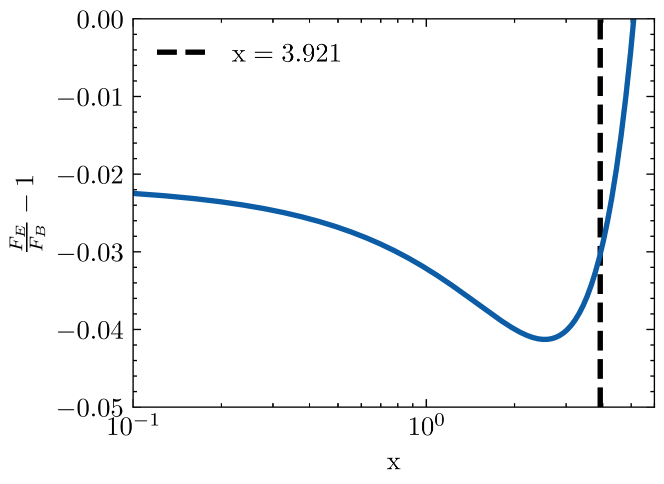
F_E is basically a superposition of planck functions for different layers parametrized by τ & therefore different temp $T(\tau)$. However this is weighted by $E_2(\tau)$ which decreases with increasing τ as one would expect as the most contribution to emergent flux would be from outer layers.
Due to the contribution (small but not 0) from inner layers (high τ), we see the flux F_E higher than F_B at high x and smaller (slightly) at low x.



x = 3.921
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\frac{\nu \mathcal{F}_\nu}{\sigma T_{\rm eff}^4}$
10^{-1}
10^0
10^1
x



x = 3.921
F_B
F_E
$\frac{\nu \mathcal{F}_\nu}{\sigma T_{\mathrm{eff}}^4}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
10^0
10^1
x



x = 3.921
$\frac{F_E}{F_B} - 1$
0.00
−0.01
−0.02
−0.03
−0.04
−0.05
10^{-1}
10^{0}
x