HW 4

Example 4

$Y_1, \ldots Y_n \sim \text{Poisson}(\lambda)$

$\sum Y_i \rightarrow$ sufficient for λ

$$L(\lambda) = \prod_1^n \left(\frac{e^{-\lambda} \lambda^{y_i}}{y_i!} \right)$$

$$= \frac{e^{-n\lambda} \lambda^{\sum_1^n y_i}}{\prod_1^n y_i!} \rightarrow h(\{y_i\})$$

$$g(u, \theta) = e^{-n\lambda} \lambda^{\sum_1^n y_i}$$

$U = \sum_1^n y_i \Rightarrow$ sufficient statistic

Example 6

$$Y_i \sim f(y|\theta) = \frac{1}{\theta} e^{-y/\theta}$$

$$L(\theta) = \prod_1^n \frac{1}{\theta} e^{-y_i/\theta}$$

$$= \frac{e^{-\frac{\sum y_i}{\theta}}}{\theta^n}$$

$U = \sum y_i$ sufficient statistic for θ

$E(u) = E(\Sigma y_i) = n E(y_i)$

$= n\theta$

we want $E(\overset{h(u)}{\bullet}) = \theta^2 \rightarrow \text{var}$

$\text{var}(\bar{y}) = \theta^2/n$

$\cancel{E\left(\frac{(\Sigma y_i)^2}{\bullet}\right)}$

$= \cancel{n E(y_i^2)} +$

$E(\bar{y}^2) = \frac{\theta^2}{n} + \theta^2$

$E(\bar{y}^2) = \frac{n+1}{n}\theta^2$

$\cancel{E\left(\frac{\sum_i^n (y_i - \bar{y})^2}{n-1}\right)}$

$E\left(\frac{n}{n+1}\bar{y}^2\right) = \theta^2$

$= \cancel{E\left(\frac{\Sigma(y_i^2 + \bar{y}^2 - 2y_i\bar{y})}{n-1}\right)}$

$\boxed{MVUE = \frac{n}{n+1}\bar{y}^2}$

$= \cancel{\frac{1}{n-1}(\bullet\, n}$

$\cancel{E(\bar{y}^2)}$

Ex 9

$$Y_i \sim \text{Exp}(\theta)$$

$$f_Y(y|\theta) = \frac{1}{\theta} e^{-y/\theta}$$

$$L(\theta) = \frac{1}{\theta^n} e^{-\frac{\sum_{i}^{n} y_i}{\theta}}$$

$$I(\theta) = \log L(\theta) = -\frac{\sum_{i}^{n} y_i}{\theta} - n\log\theta$$

$$\frac{dI}{d\theta} = \frac{\sum_{i}^{n} y_i}{\theta^2} - \frac{n}{\theta} = 0$$

$$\theta = \frac{\sum_{i}^{n} y_i}{n} \longrightarrow \text{MLE}$$

$$\downarrow$$

$$\hat{\theta} = \bar{Y}$$

Lec 7

Ex 2,

$C_1 : p = 0.3$

$n = 15$

~~$H_0 : p = 0.5$~~ ~~$RR = \{y : y \le 2\}$~~

~~$\tilde{\beta} = P(\text{accept } H_0 \text{ when } H_a \text{ true})$~~

~~$= P(y > 7 \text{ when } p = 0.3)$~~

$\tilde{\beta} = P(\text{accept } H_0 \text{ when } H_a \text{ true})$

$= P(Y > 2 \text{ when } p = 0.3)$

$= 1 - P(Y \le 2 \text{ when } p = 0.3$

$1 - \left({}^{15}C_0 (0.3)^0 (0.7)^{15} + {}^{15}C_1 (0.3)^1 (0.7)^{14} + {}^{15}C_2 (0.3)^2 (0.7)^{13} \right)$

$1 - (0.7)^{13} \left((0.7)^2 + 15(0.3)(0.7) + \frac{15 \times 14}{2} (0.3)^2 \right)$

$\cancel{\tilde{B} \sim 1 - 0.2012}$

$\cancel{\tilde{B} \sim 0.7986}$

$\tilde{B} \sim 1 - (0.00474 + 0.0305 + 0.09156)$

$$\boxed{\tilde{B} \sim 0.8732}$$

Ex 3

$\tilde{\alpha} = P(\text{rejet } H_0 \text{ when } H_0 \text{ true})$

$= P(y \leq 5 \text{ when } p = 0.5)$

$= \sum_{i=0}^{5} {}^{15}C_i (0.5)^i (0.5)^{15-i}$

$$\boxed{\tilde{\alpha} \sim 0.15087}$$

$\tilde{B} = P(\text{accept } H_0 \text{ when } H_a \text{ true})$
$p = 0.3$

$= P(y > 5 \text{ when } p = 0.3)$

$= \sum_{i=6}^{15} {}^{15}C_i (0.3)^i (0.7)^{15-i}$

~~$\beta \sim 0.1300$~~ $\boxed{\beta \sim 0.27837}$

EX 7

$H_0 : \mu = 15$ $\mu_a = 16$

$\sigma^2 \sim 9$ $\alpha = \beta = 0.05$

$$k = \mu_a - Z_\beta \frac{\sigma}{\sqrt{n}}$$

$$n = \frac{(Z_\alpha + Z_\beta)^2 \sigma^2}{(\mu_a - \mu_0)^2}$$

$Z_{0.05} \sim 1.645$ (one tailed test)

$$n = \frac{(1.645 + 1.645)^2 9}{(16-15)^2} = 9 \times 10.8241$$

~ 97.4169

n should be at least 98

~~$n \sim 97 - 98$~~