

# hw3\_ipynb

September 16, 2024

```
[ ]: import numpy as np
import matplotlib.pyplot as plt
import scienceplots
import astropy.units as u
import scipy
import astropy.constants as const

plt.style.use('science')
```

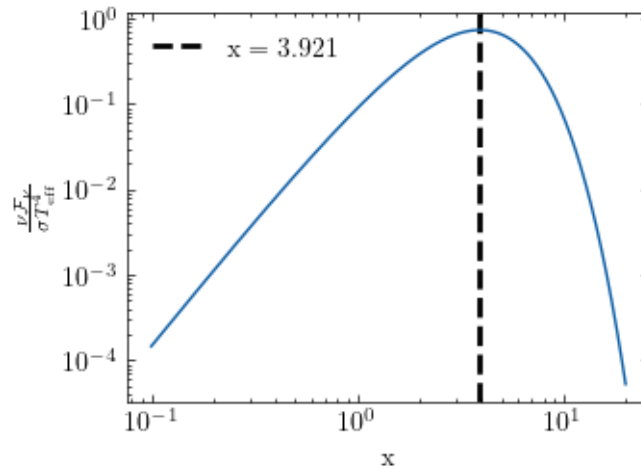
```
[ ]: h = const.h
c = const.c
k = const.k_B
sigma = const.sigma_sb
pi = np.pi
```

```
[ ]: (2 * pi * k**4 / (c**2 * sigma * h**3)).decompose().value, 15 / pi**4
```

```
[ ]: (0.15398973382026504, 0.15398973382026507)
```

```
[ ]: def F_B(x):
    prefactor = 15 / pi**4 #2 * pi * k**4 / (c**2 * sigma * h**3)
    return prefactor * (x**4 / (np.exp(x) - 1))
```

```
[ ]: x = np.logspace(-1,1.3,1000)
plt.axvline(3.921, color = 'k', linestyle = 'dashed', lw = 2, label = r'$\rm x_{\rm eff} = 3.921$')
plt.plot(x,F_B(x))
plt.xscale('log')
plt.yscale('log')
plt.legend()
plt.xlabel('x')
plt.ylabel(r'$\frac{\nu}{\sigma} \mathcal{F}_{\nu} / \sigma T_{\rm eff}^4$')
plt.savefig('nu_f_nu_B.pdf', dpi = 300)
```



```
[ ]: def Teff_T(tau):
      return ((4/3) / (tau + 2/3))**0.25
```

```
[ ]: def integrand(tau,x):
      return scipy.special.expn(2,tau) / (np.exp(x * Teff_T(tau)) - 1)
```

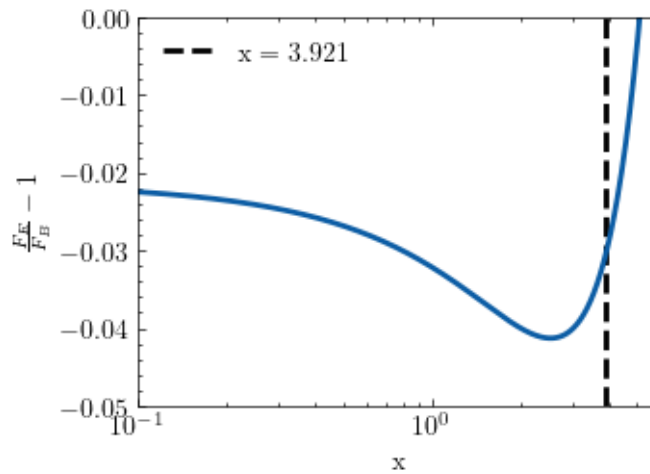
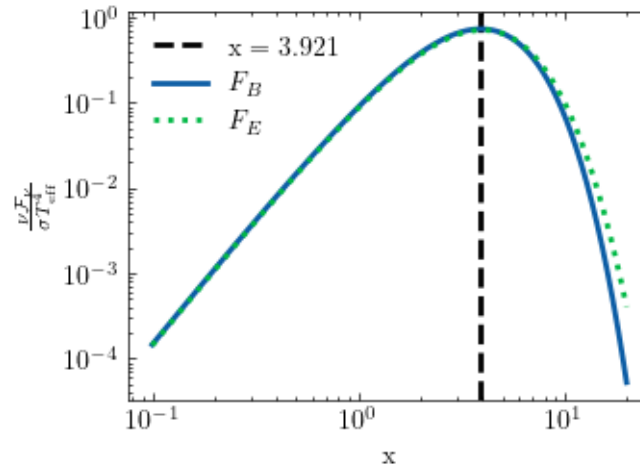
```
[ ]: x = np.logspace(-1,1.3,1000)

integral = [scipy.integrate.quad(integrand, a = 0, b = 100, args = (i))[0] for
            i in x]
F_E = (30 * (x/pi)**4) * integral
```

```
[ ]: FB = F_B(x)
plt.axvline(3.921, color = 'k', linestyle = 'dashed', lw = 2, label = r'$x_{\rm eff} = 3.921$')
plt.plot(x,F_B(x), label = r'$F_B$', lw = 2)
plt.plot(x,F_E, linestyle = 'dotted', label = r'$F_E$', lw = 2)
plt.xscale('log')
plt.yscale('log')
plt.legend(loc = 'upper left')
plt.xlabel('x')
plt.ylabel(r'$\frac{\nu \mathcal{F}_{\nu}}{\sigma T_{\rm eff}^4}$')
plt.savefig('F_E.pdf', dpi = 300)
plt.show()

plt.axvline(3.921, color = 'k', linestyle = 'dashed', lw = 2, label = r'$x_{\rm eff} = 3.921$')
plt.plot(x,(F_E/FB) - 1, lw = 2)
plt.xscale('log')
#plt.yscale('log')
```

```
plt.legend(loc = 'upper left')
plt.xlabel('x')
plt.ylabel(r'$\frac{F_E}{F_B} - 1$')
plt.xlim(1e-1,6)
plt.ylim(-0.05,0)
plt.savefig('dF_E_F_B.pdf', dpi = 300)
```



```
[ ]: F_E/FB - 1
```

```
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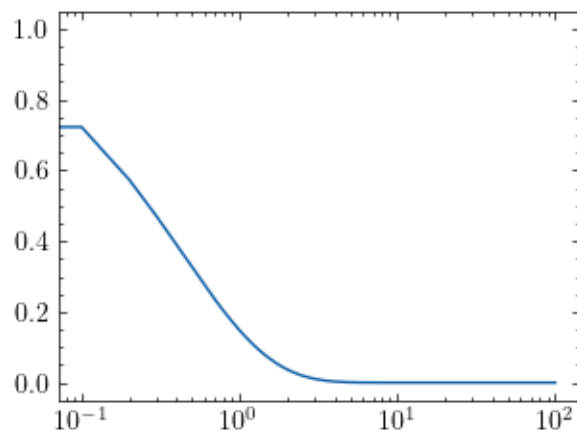
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[ ]: tau = np.linspace(0,100,1000)
     e2tau = scipy.special.expn(2,tau)

     plt.plot(tau,e2tau)
     plt.xscale('log')
     # plt.yscale('log')

```



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[ ]:

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