HW ASTR513 Neel

1. $x_1 \ldots x_n \rightarrow N(\mu, \sigma^2)$

a) $U_1 = \frac{(x_1 + x_2 + \ldots + x_n)}{n}$

$= N(a, b^2)$

$a = \frac{1}{n} n\mu$

$b^2 = \frac{n\sigma^2}{n^2} \quad b^2 = \frac{\sigma^2}{n}$

$N(\mu, \sigma^2/n)$

b) $U_2 = x_1 + x_2 + \ldots x_n$

$= N(a, b^2)$

$a = \sum_1^n \mu = n\mu$

$b^2 = n\sigma^2$

$N(n\mu, n\sigma^2)$

c) $U_3 = X_1 - X_2$

$\sim N(a, b^2)$

$a = 1.(\mu) - 1(\mu) = 0$

$b^2 = 1^2\sigma^2 + (-1)^2\sigma^2$

$= 2\sigma^2$

$N(0, 2\sigma^2)$

d) $U_4 = \frac{X_1}{\sigma\sqrt{2}} - \frac{X_2}{\sigma\sqrt{2}}$

$\sim N(a, b^2)$

$a = \frac{1}{\sigma\sqrt{2}}\mu - \frac{1}{\sigma\sqrt{2}}\mu = 0$

$b^2 = \frac{1}{2\sigma^2}\sigma^2 + \frac{1}{2\sigma^2}\sigma^2 = 1$

$N(0, 1)$

e) $U_5 = X_1 + X_2 + X_3 - 3X_4$

$\sim N(a, b^2)$

$a = 1(\mu) + 1(\mu) + 1(\mu) - 3(\mu)$

$= 0$

$b^2 = 1(\sigma^2) + 1(\sigma^2) + 1(\sigma^2) + 9(\sigma^2)$

$= 12\sigma^2$

$= N(0, 12\sigma^2)$

$\chi^2_\nu \sim \text{Gamma}(\nu/2, 2)$

$Z_i \sim N(0, 1)$

$\sum Z_i^2 \sim \chi^2_\nu$

Sample variance S

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$\frac{\sum (x_i - \bar{x})^2}{\sigma^2} \sim \chi^2_{n-1}$

HW3 ASTR513 Neev

$y_1, y_2, y_3 \sim N(0,1)$

1. $U_1 = \bar{y}$

$$\mu_U = \frac{0+0+0}{3} = 0$$

$$\sigma_U^2 = \frac{\sigma^2}{3}$$

$$N(\mu_U, \sigma_U^2) \simeq N(0, \sigma^2/3)$$

2. $U_2 = y_1^2 + y_2^2 + y_3^2$

$= \chi_3^2$ (can be derived using mgf)

↓

Chisquare

3 dof

3. $U_3 = \dfrac{(Y_1+Y_2)/\sqrt{2}}{\sqrt{Y_3^2}}$

If $Z \sim N(0,1)$ $W \sim \chi_v^2$ & Z, W indep

$$T = T_v = \frac{Z}{\sqrt{W/v}}$$

$$\frac{Y_1+Y_2}{\sqrt{2}} \sim N(0,1)$$

$Y_3^2 \sim \chi_1^2$ → chi sq dof 1

$\therefore U_3 = T_1$

students t with 1 dof

4. $U_4 = \dfrac{Y_1}{\sqrt{0.5(Y_2^2 + Y_3^2)}}$

$Y_1 \sim N(0,1)$

$Y_2^2 + Y_3^2 \sim \chi^2_2$ → chi square 2 dof

$\therefore U_4 = T_2$ students T with 2 dof

5. $U_5 = \dfrac{2Y_1^2}{Y_2^2 + Y_3^2}$

$F = \dfrac{W_1/v_1}{W_2/v_2}$ W_1, W_2 indep χ^2 RV with v_1, v_2 dof

F distribution $\sim F_{v_1, v_2}$ with v_1 & v_2 df

$\sum_1^j y_i^2 \rightarrow \chi^2_j$

$U_5 = \dfrac{Y_1^2/1}{(Y_2^2 + Y_3^2)/2}$

$U_5 \sim \dfrac{\chi^2_1}{\chi^2_2} \sim F_{1,2}$