Q3 Doppler : $\Delta\nu_D$

Collisional : Γ

$\Delta\nu = \nu - \nu_0$ relative to line center

$\nu_0 = c/\lambda_0$

$$\phi(\Delta\nu) = \frac{1}{\sqrt{\pi}\,\Delta\nu_D} H(u,a)$$

$$H(u,a) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{a e^{-y^2}}{(u-y)^2 + a^2}\,dy$$

$$u = \frac{\Delta\nu}{\Delta\nu_D} \qquad a = \frac{\Gamma}{4\pi\Delta\nu_D}$$

a) As $a \to 0$, voigt reduces to doppler

Given : $\lim\limits_{a \to 0} \left(\frac{a}{x^2 + a^2}\right) \Rightarrow \pi\,\delta(x)$

Using this in $H(u,a)$

$$\lim_{a\to 0} H(v,a) = \lim_{a\to 0} \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{a e^{-y^2}}{(v-y)^2+a^2}\, dy$$

No we have $\displaystyle \lim_{a\to 0} \frac{a}{(v-y)^2+a^2}$

$$= \pi\,\delta(v-y)$$

Plugging it in,

$$\lim_{a\to 0} H(v,a) = \frac{1}{\pi}\int_{-\infty}^{\infty} \pi\delta(v-y)e^{-y^2}\, dy$$

Using $\displaystyle \int_{-\infty}^{\infty} \delta(v-y)e^{-y^2}\, dy = e^{-v^2}$

we get

$$\lim_{a\to 0} H(v,a) = \frac{1}{\pi}\cdot \pi e^{-v^2} = e^{-v^2}$$

Plugging this in $\phi(\Delta\nu)$ we get,

$$\phi(\Delta\nu) = \frac{1}{\sqrt{\pi}\Delta\nu_D} e^{-v^2}$$

Using $v = \frac{\Delta\nu}{\Delta\nu_D}$ we get

$$\phi(\Delta\nu) = \frac{1}{\sqrt{\pi}\,\Delta\nu_D} e^{-\frac{(\Delta\nu)^2}{(\Delta\nu_D)^2}}$$

$$\boxed{\phi(\Delta\nu) = \frac{1}{\sqrt{\pi}\,\Delta\nu_D} e^{-\frac{(\nu-\nu_0)^2}{\Delta\nu_D^2}}}$$

This is the doppler profile hence proved that as $a \to 0$, Voigt reduces to doppler