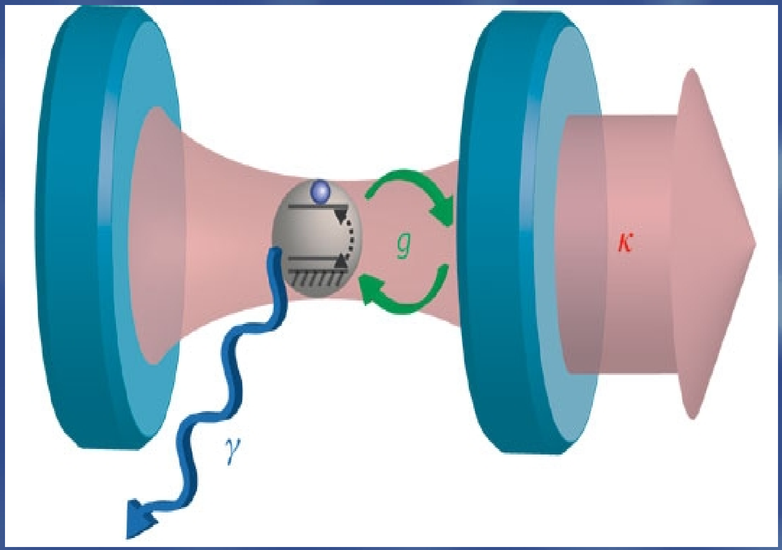
Force on an atom in a cavity and the Abraham-Minkowski paradox

June 25 2012 Committee Meeting



Nick Miladinovic
Duncan O'Dell

McMaster University
Inspiring Innovation and Discovery

Motivation

100 year old problem

1. $$\mathbf{p}_{\mathrm{Abr}} = \int dV \frac{\mathbf{E} \times \mathbf{H}}{c^2}$$ $$p_{Abr} = p_0/n$$

Abraham, Max (1909), "Zur Elektrodynamik bewegter Körper", Rendiconti del Circolo Matematico di Palermo 28: 1–28

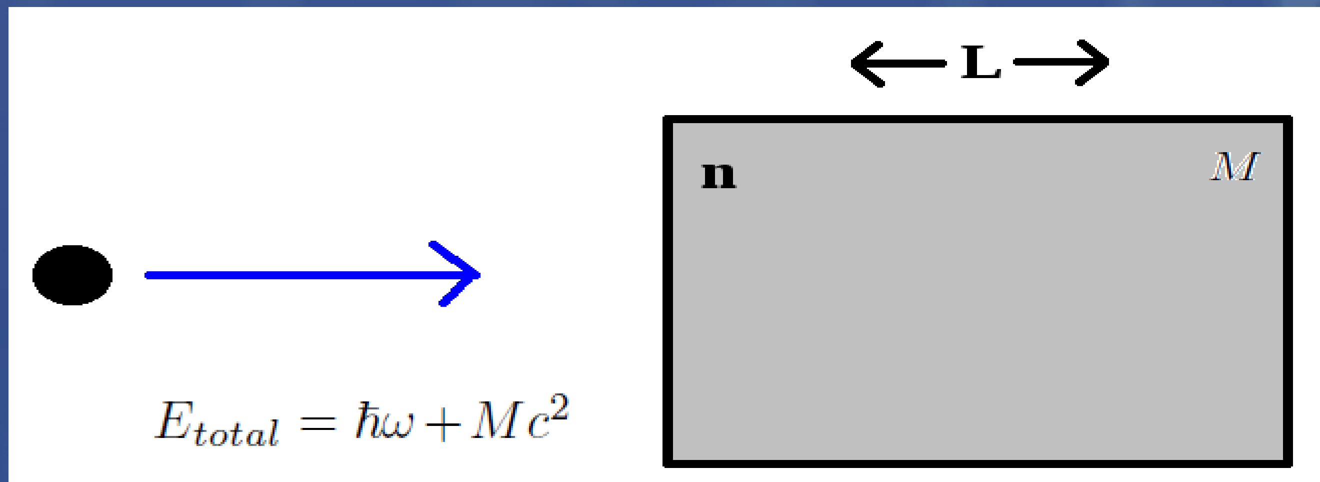
2. $$\mathbf{p}_{\mathrm{Min}} = \int dV \mathbf{D} \times \mathbf{B}$$ $$p_{Min} = p_0 n$$

Minkowski, Hermann (1908), "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern", Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse: 53–111

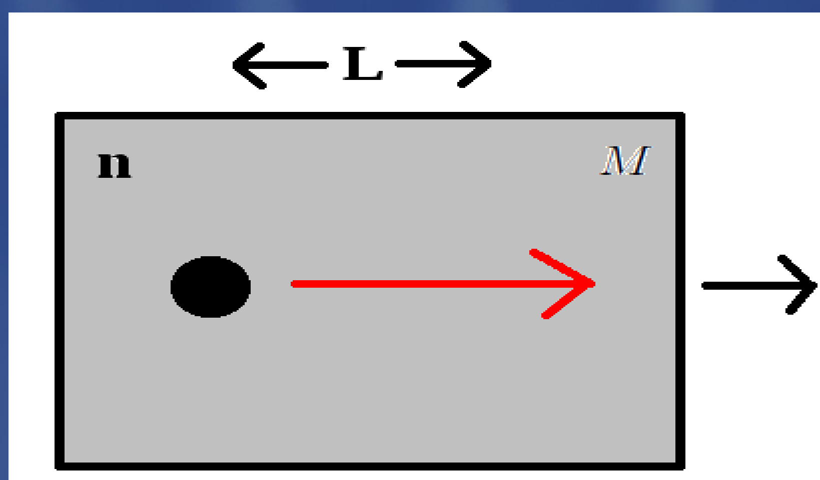
Outline

- Argument for Abraham
- Argument for Minkowski
- Delta-model
- Forces on an atom
- Comparing the two
- Next steps
- Conclusion

Abraham



$$E_{total} = \hbar\omega + Mc^2$$



$$L\,(n-1)\,\hbar\omega = \Delta z M c^2$$

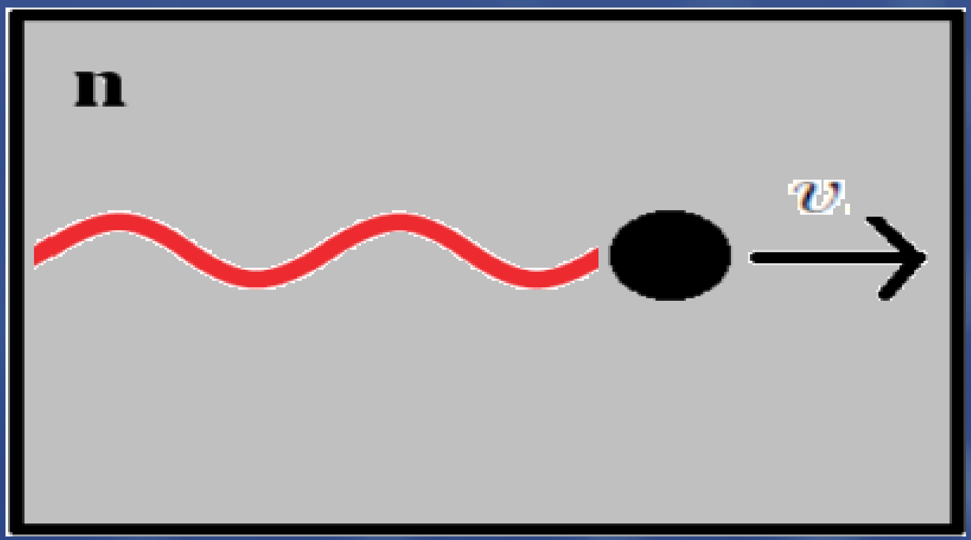
$$p_{photon} = \frac{\hbar\omega}{cn}$$

A. F. Gibson, M. F. Kimmitt, A. O. Koohian, D. E. Evans, G. F. D. Levy, Proc. R. Soc. Lond. A 370, 303 (1980).

R. Loudon, Fortschr. Phys. 52, 1134 (2004).

R. Loudon, S. M. Barnett, C. Baxter, Phys. Rev. A 71, 063802 (2005).

Minkowski



$$\omega = \omega_0 \left(1 - \frac{vn}{c}\right)$$

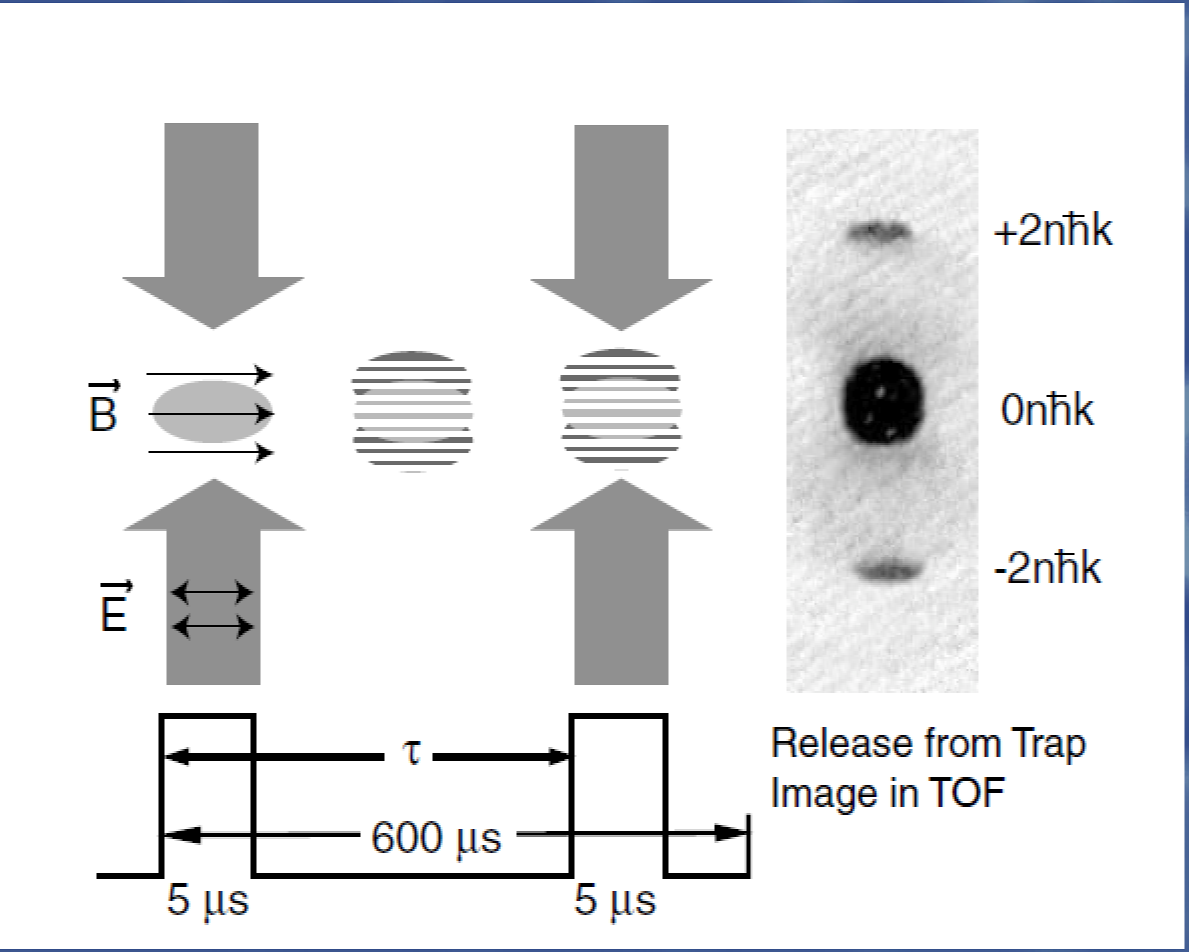
$$\frac{1}{2}mv_{final}^2 + \hbar\omega_0 = \frac{1}{2}mv_{initial}^2 + \hbar\omega$$

+

$$mv_{final} = mv_{initial} + p_{photon}$$

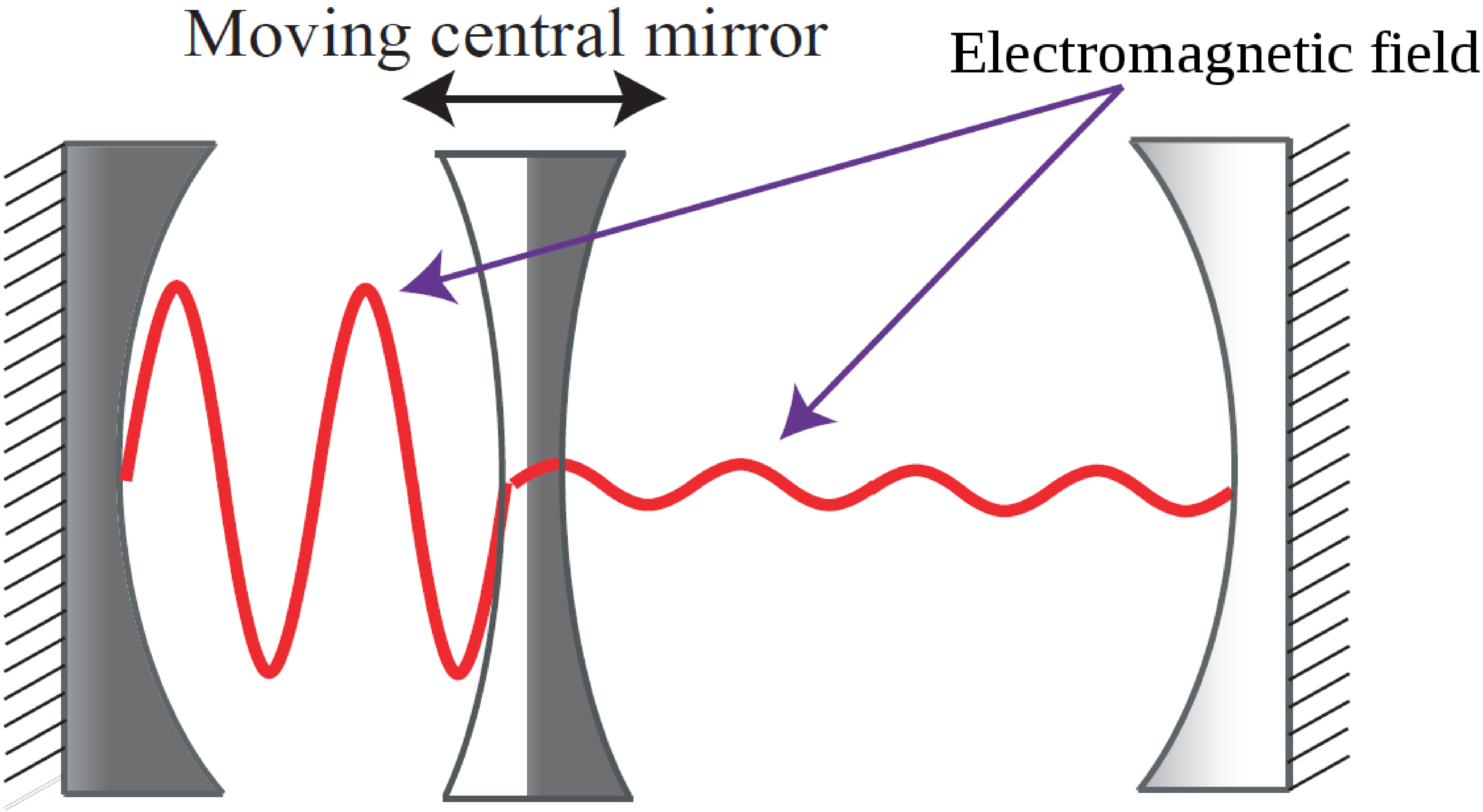
$$p_{photon} = \frac{\hbar\omega n}{c}\frac{2v_{initial}}{v_{initial} + v_{final}} \approx \frac{\hbar\omega n}{c}$$

A cold atom recoil experiment giving the Minkowski result



G. K. Campbell, A. E. Leanhardt, J. Mun, M. Boyd, E. W. Streed, W. Ketterle, and D. E. Pritchard, Phys. Rev. Lett. 94, 170403 (2005).

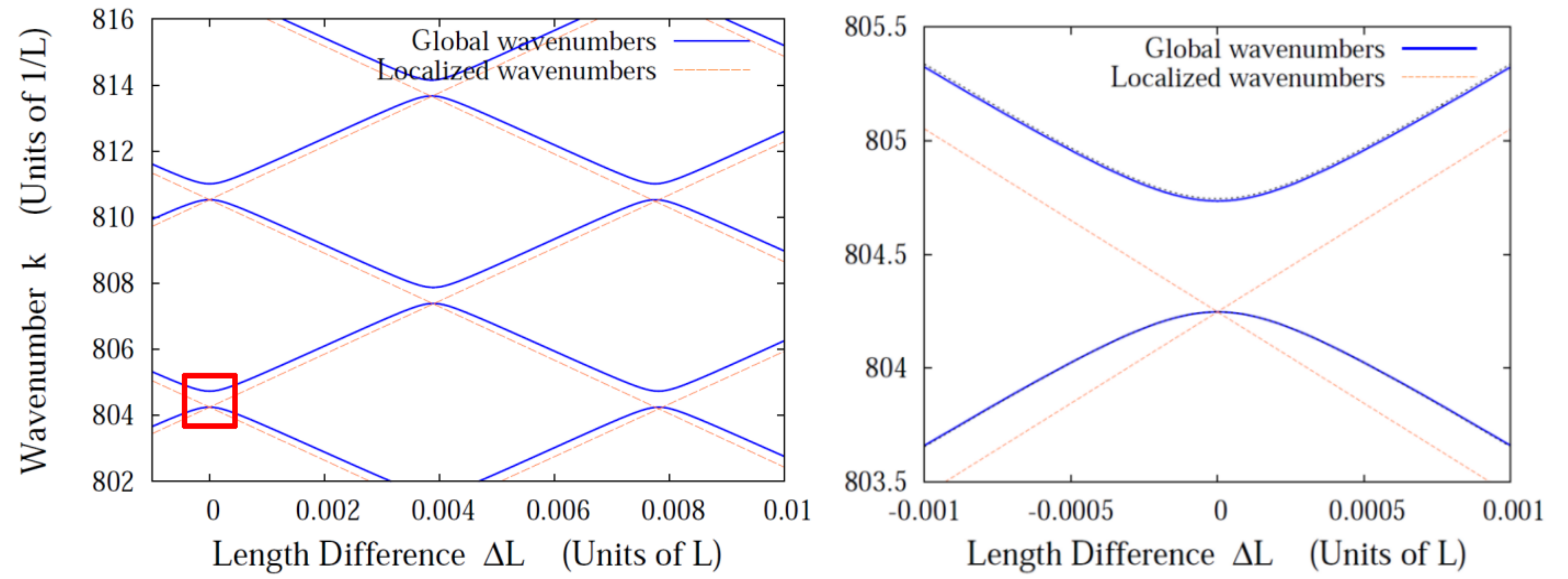
Our Setup



Maxwell's Equations

$$\frac{\partial^2 E(x,t)}{\partial x^2} - \mu_0 \varepsilon_0 (1 + \alpha\delta(x)) \frac{\partial^2 E(x,t)}{\partial t^2} = 0$$

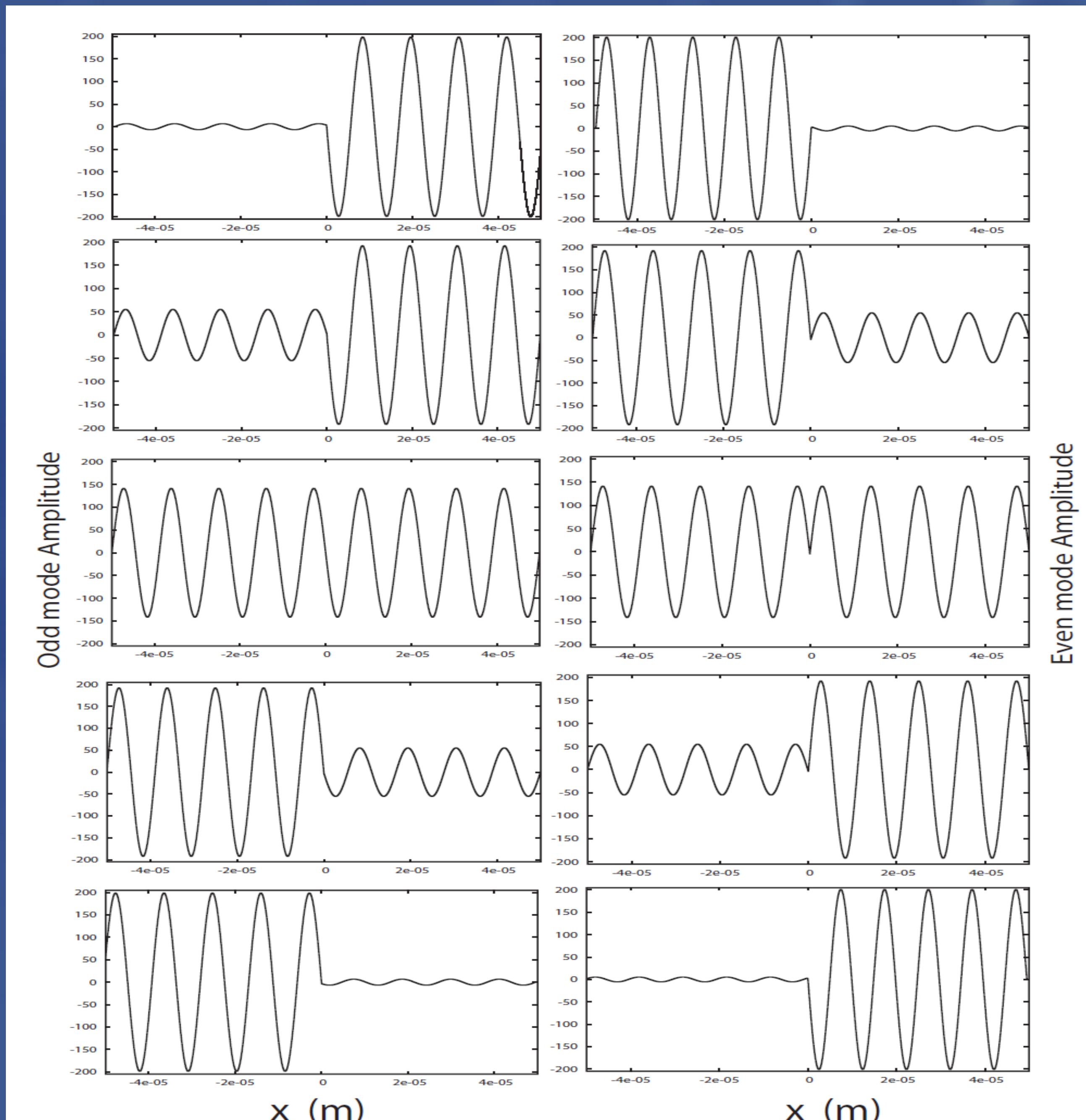
$$\tan(k_n L_2) = \frac{\tan(k_n L_1)}{\alpha k_n \tan(k_n L_1) - 1}$$



Mode Plots vs. Mirror Position

Amplitude

Central Mirror Position



Left Displaced Mirror

Slightly Left Displaced Mirror

Centered Mirror

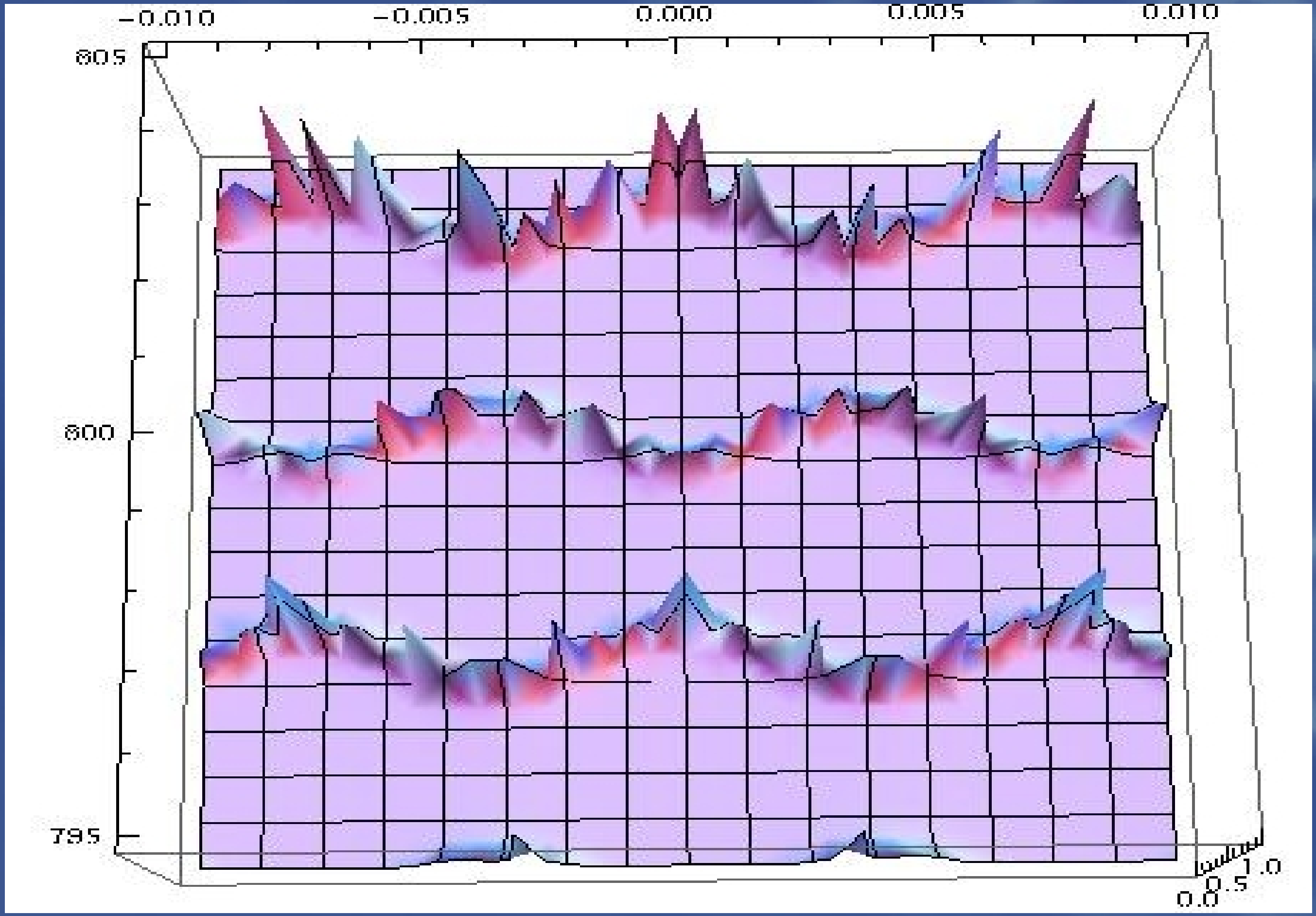
Slightly Right Displaced Mirror

Right Displaced Mirror

Open vs. closed cavity

Mirror displacement

Wavenumber



Microscopic expression for an atom and a mirror

Dipole force

$$F_{dipole} = -\frac{\hbar\delta}{4}\frac{\vec{\nabla}\Omega^2}{\frac{\Gamma^2}{4}+\delta^2+\frac{\Omega^2}{2}} = -\hbar u\vec{\nabla}\Omega$$

$$\Omega = -\frac{\mathbf{d}_{\mathrm{ba}}\cdot\mathbf{E}}{\hbar}$$

Reflection probability for a single atom

$$R = \frac{u\Omega\hbar/(2c)}{2\hbar k} = \frac{u\Omega}{4ck}$$

Reflection probability for delta model

$$R = \frac{1}{1+\frac{4}{k^2\alpha^2}}$$

Mapping onto our model

Solving for alpha

$$\alpha = \sqrt{\frac{u\Omega}{ck^3 - \frac{u\Omega k^2}{4}}} \approx \sqrt{\frac{u\Omega}{ck^3}}$$

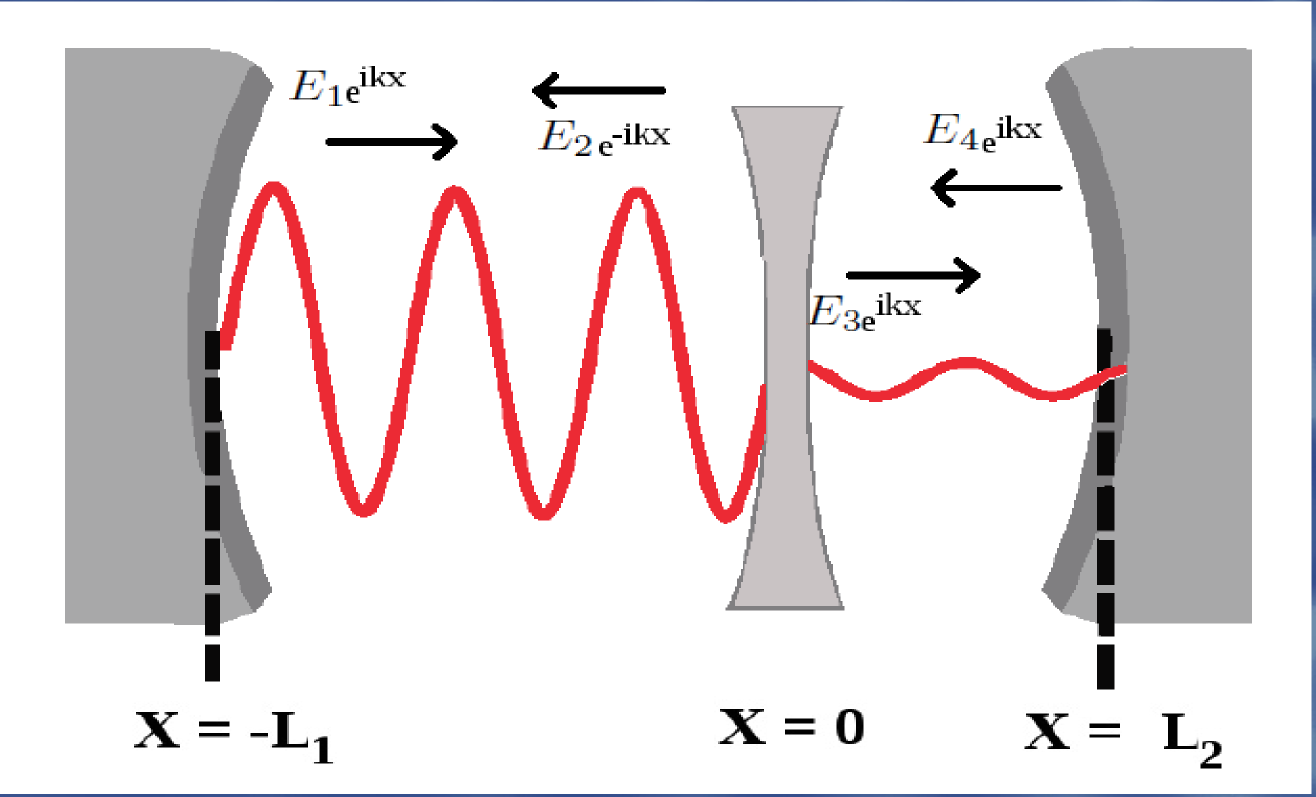
Wavenumber approximation

$$k_n = \frac{n\pi}{L}\left[\pm\frac{\alpha}{2L}\left(\cos(n\pi\frac{\Delta L}{L}) \mp 1\right) + 1\right]$$

Atom-cavity refractive index

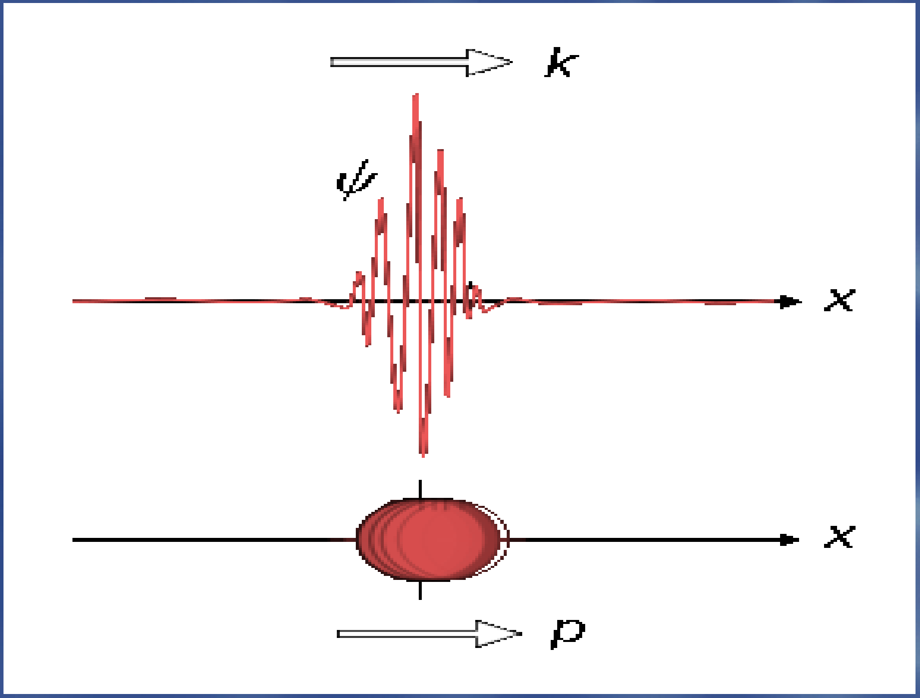
$$n_{\text{atom-cavity}} = 1 \pm \frac{1}{2L}\sqrt{\frac{u\Omega}{ck^3}}\left(\cos(n\pi\frac{\Delta L}{L}) \mp 1\right)$$

A second approach



$$F = \frac{\varepsilon_0}{2}\left(|E_1|^2 + |E_2|^2 - |E_3|^2 - |E_4|^2\right)$$

Future Work



Consider the atom and mirror with a wave function, and see whether we obtain Minkowski or Abraham

References

References

[1] Stephen M. Barnett, *Phys. Rev. Lett.* **104** 070401 (2010)

[2] J. Dalibard, and C. Cohen-Tannoudji, *Opt. Coc. Am. B*, **Vol 2** (1985)

[3] J.K. Asboth, H. Ritch, and P. Domokos, *Phys. Rev. A* **77** 063424 (2008)

[4] E.A Hinds, and M. Barnett, *Phys. Rev. Lett.* **102** 050403 (2009)

[5] N. Miladinovic et al., *Phys. Rev. A* **84** 043822 (2011)

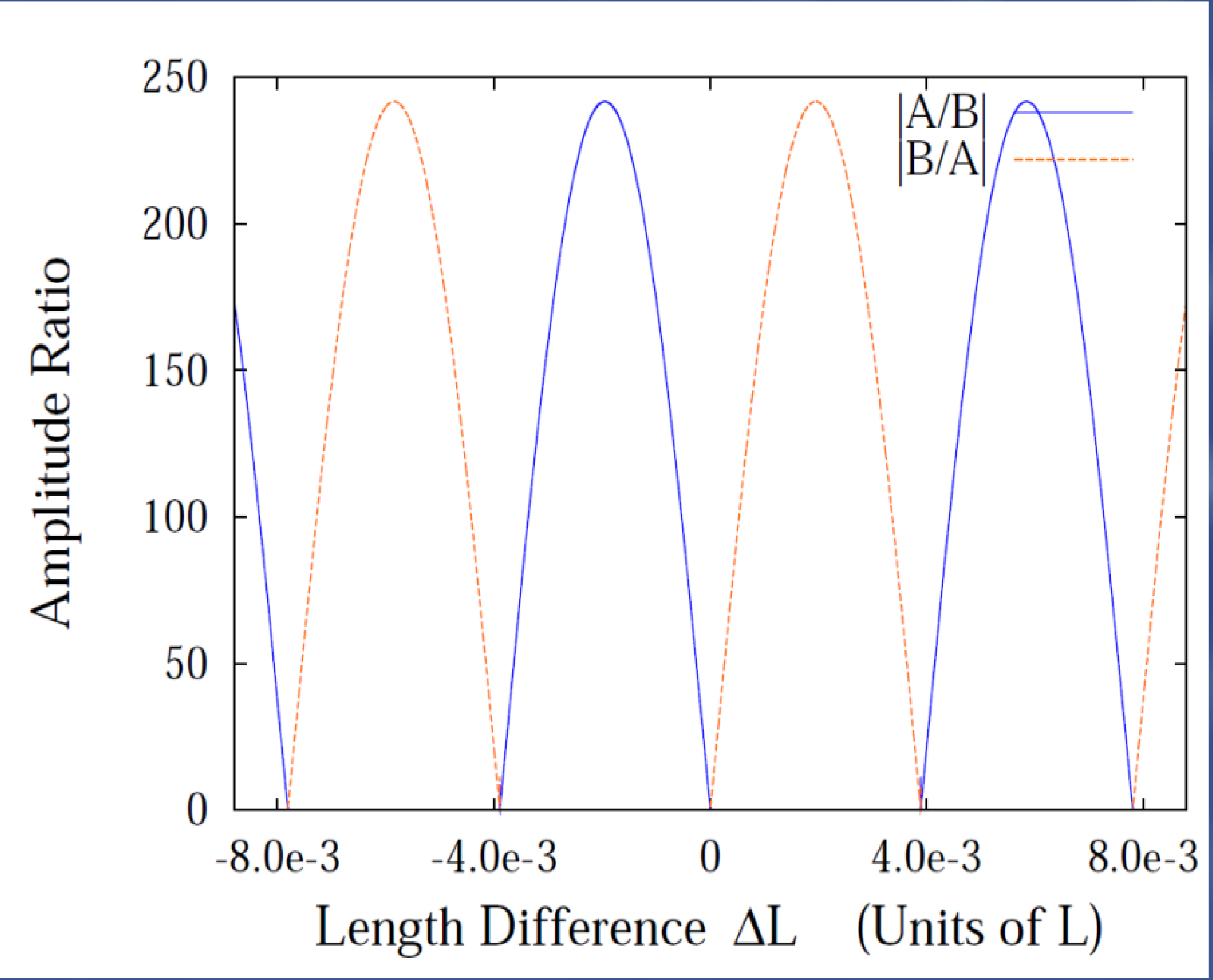
[6] R. J. Cook, *Phys. Rev.* Vol **20** (1979)

[7] Loudon, R., J. Mod. Opt. **49**: p. 821. (2002)

[8] G. Hechenlaikner, M. Gangl, P. Horak, and H. Ritsch, *Phys. Rev. A* **58** (1998)

Thanks!

Amplitude Transfer



$$U_m(x) = \begin{cases} A_m \sin(k_m(x + L_1)) & -L_1 \geq x \leq 0 \\ B_m \sin(k_m(x - L_2)) & 0 < x \leq L_2 \end{cases}$$

Using reasonable parameters we can easily obtain relative amplitude transfers in the range of 1:240

Cavity length L=100 microns
n = 124
alpha = 0.3L