Review notes on optical forces

According to Ashkin (PRA 21:5,1979) the force on the atom can be viewed as the lorentz force acting on the induced dipole (classical), or as the momentum transfer due to absorbtion and reemission of light by the atom (quantumy). The total force has 3 basic components: absorbtion, spontaneous and stimulated emission. The first two are known as the scattering force. Due to the angular symmetry of the spontaneous emission, this force only contributes to fluctuations in the force. The stimulated emission is the cause of the dipole force. If we think of this classically, the dipole force is related to the potential energy of the induced dipole in the electric field. The dipole force is due to the real part (in phase) of the polarizability. The scattering force is due to the imaginary component.

One of my concerns is that in classical electrodynamics, light waves incident on a material induce small oscillations of polarization in the individual atoms (or oscillation of electrons, in metals), causing each particle to radiate a small secondary wave (in all directions, like a dipole antenna). All these waves add up to give specular reflection and refraction, according to the Huygens-Fresnel principle. This poses a problem for us when we are dealing with a single atom. The Huygens-Fresnel principle seems to only apply to a very large collection of atoms where you have specular reflection. For a single atom, you would get a very diffuse reflection. Therefore the argument that we can treat the single atom as a very thin dielectric slab and just tweak the parameters doesn't make sense to me. In the calculations done by other groups, they assume there is this specular reflection coefficient, which isn't the case for a single atom. Therefore doing stuff like solving Maxwell's equations for the double cavity system in which we have imposed boundary conditions for a mirror doesn't make sense to me. I think of Rayleigh scattering cross sections here.

A second concern is that in all the atomic force calculations I have seen, the driving field is taken to unaffected by the atom. I suppose in the strong field limit, the effect of the atom on the driving laser would be inconsequential. However, in the lower field limit where we don't have a very strong field, the atom would create a discontinuity in the field mode, and therefore taking the gradient doesn't make sense. What is the best way to deal with this? In classical electrodynamics when calculating the force on a material, we have to take the average of the force due to the field on either side. This is a way to eliminate the force cased by the material on itself. To see this, consider the total field E_{total}.

$$E_{total} = E_{patch} + E_{other}$$

By this I mean we can split the field acting on the patch of dielectric slab being considered as that due to the patch bound charge itself E_{patch}, plus that due to all other means E_{other}. Now of course the patch itself can't apply a force onto itself. What we can do to eliminate this is to so consider

$$E_{LeftOfSlab} = E_{other} + E_{patch}$$

$$E_{LeftOfSlab} = E_{other} - E_{patch}$$

Now the field due to a pure dipole is given by

$$E_{dipole}(r,\theta) = \frac{\rho}{4\pi\epsilon_0 r^3}\left(2\cos(\theta)\,\hat{r} + \sin(\theta)\,\hat{\theta}\right)$$

This shows that the field due to the patch on the left is equal but opposite in sign to the field due to the patch on the right. It is now clear why we use the average of the two fields

$$E_{other} = \frac{1}{2}\left(E_{LeftOfSlab} + E_{LeftOfSlab}\right) = E_{average}$$

Therefore we see that there is a field discontinuity whenever you have a dielectric material, but when you want to calculate the force on the material, you have to take the average of the two fields. Therefore I'm pretty sure we can deal with the gradient discontinuity we encounter in a similar manner. Why not consider a dielectric slab comprised of many atoms and solve for the fields on either side (as we have done in the paper). Doing so will give us the Electromagnetic field just to the left and just to the right of the slab. We may then take the average and calculate the force using this.

In our paper we found the modes to be

$$U_n(x) = \begin{cases} A_n\sin(k_n(x+L_1)) & -L_1 < x \leq 0 \\ B_n\sin(k_n(x-L_2)) & 0 < x < L_2 \end{cases}$$

It was also possible to find a good analytic solution for the wavenumber k_n. Consider the transcendental equation

$$\cos(k\triangle L) - \cos(kL) = \frac{2L}{\alpha}\frac{\sin kL}{kL}$$

When α is very small, as we have found it indeed is, the right side must be of unitary order, thereforesin kL must be very close to zero. We therefore expand kL about $n\pi$ to first order

$$\cos(k\triangle L) \pm 1 = \pm\frac{2L}{\alpha}\left(\frac{1}{n\pi}(x - n\pi)\right)$$

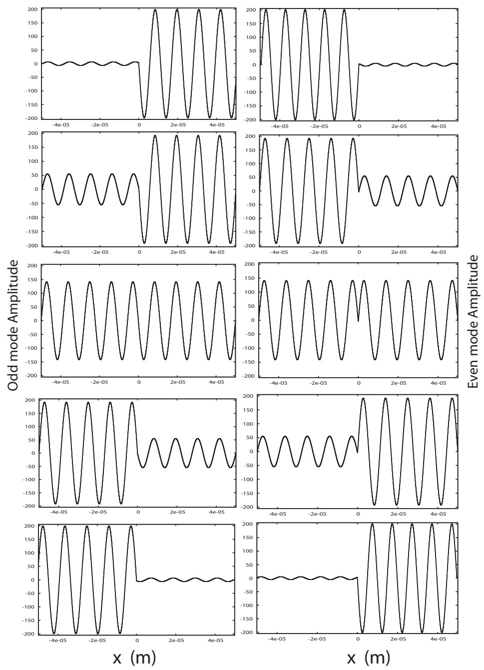
Now $\cos(k\Delta L)$ has k in the argument, however as this doesn't deviate from $n\pi$ much, it is reasonable for small values of α to replace it with $n\pi$ as the cosine function is insensitive to such small perturbations

$$k_n = \pm\frac{\alpha n\pi}{2L^2}\left(\cos(n\pi\triangle L) \mp 1\right) + \frac{n\pi}{L}$$

Ok good. So we have k_n as a function of $\triangle L$ (this isn't in the paper). Now all we need is the amplitudes A_n and B_n which isn't hard to find as

$$\frac{A_n}{B_n} = -\frac{\sin(k_nL_2)}{\sin(k_nL_1)}$$

If we consider the dielectric to be at the center, then $A_n = B_n$ and the only effect that the dielectric slab has is to alter the wavenumber. Now lets go back to the paper and look at the the wavemode graphs where we examined what the mode structure looks like as we move the central dielectric. In the third row we see the odd and even modes when $\triangle L = 0$.



Note that the odd mode is just the cavity standing wave. It is only in the even mode that we see a difference. However, if we believe the argument above for the force felt by the atom, then by symmetry we should get no force felt by the atom. This fact can be made more clear by considering the equation for the force on an atom applied by an electric field. Let the electric field be of the form:

$$\overrightarrow{E}(\overrightarrow{r}, t) = \overrightarrow{E_0}(\overrightarrow{r}) \cos\left[\omega t + \phi(\overrightarrow{r})\right]$$

Then the average force on the atom is given by

$$\left\langle \overrightarrow{F}(t) \right\rangle = \frac{\overrightarrow{\nabla} I(\overrightarrow{r})}{I_s} \frac{-\hbar\left(\frac{\delta}{2}\right)}{1 + \frac{I(\overrightarrow{r})}{I_s} + \left(\frac{2\delta}{\Gamma}\right)^2} - \hbar\Gamma\left(\overrightarrow{\nabla}\phi(\overrightarrow{r})\right) \frac{\Omega^2}{\Gamma^2 + 4\delta^2 + 2\Omega^2}$$

Here I_s is known as the saturation intensity and is given by $I_s = \frac{2\pi^2\hbar c\Gamma}{3\lambda^3}$, Γ is the decay rate, and δ is the detuning. The first term in the force equation is known as the dipole force, and the second term is known as the scattering force. For a pure plane wave, the first term is zero because the gradiant of the intensity is zero as $E_0 = constant$. For a standing wave the second term is zero, as there is no phase term. Going back to our problem now, it is clear that the odd mode feels no scattering force, but feels a dipole force proportional to the gradient of the intensity. A classical picture would be something like a ball rolling down the slope of the hill. In the even mode however, we would still get zero scattering force due to the fact that we still have a standing wave, but we also should get no dipole force as the "atom" is parked in a potential ditch. Does this make sense? The problem to this classical argument is that in the other images (when we change $\triangle L$) the "atom" is still stuck in a ditch however, the average of $E_{LeftOfSlab}$ and $E_{RightOfSlab}$ will give a non-zero result and there will be a dipole force.