1 Previous Work

Let us begin with Maxwell's wave equation in matter

$$\frac{\partial^2 E(x,t)}{\partial x^2} - \mu_0 \varepsilon(x) \frac{\partial^2 E(x,t)}{\partial t^2} = 0$$

Here we have assumed that the permeability is unchanging, and the permittivity is a function of position. This equation is only true for a static dielectric medium, however, it will hold approximately for low medium velocities. By considering the dielectric material to be linear and infinitely thin, we may write the permittivity function as

$$\varepsilon(x) = \varepsilon_0(1 + \chi_e \delta(x))$$

By writing the the electric field as

$$E_n(x,t) = U_n(x) e^{-i\omega_n t}$$

and inserting this into equation (1) we obtain

$$\frac{d^2 U_n(x)}{dx^2} + k_n^2(1 + \chi_e \delta(x)) U_n(x) = 0$$

This may be rewritten as (Asboth et. al)

$$\left[\partial_x^2 + k^2\right] U_n(x) = -\frac{\eta\alpha}{\varepsilon_0} k^2 \delta(x)$$

Here η is the surface density of particles per unit area, and α is the polarizability per particle. From here we may use the relation (Asboth et. al)

$$\alpha = -\frac{d_{eg}^2}{\hbar} \frac{1}{\Delta} \left(1 - i\frac{\Gamma}{2\Delta}\right)$$

where Δ is the detuning and d_{eg} is the dipole transition matrix element. The linewidth Γ is related to the atomic frequency ω_0 and d_{eg} by (Asboth et. al)

$$\Gamma = \frac{\omega_0^3}{3\pi\varepsilon_0\hbar c^3} d_{eg}^2$$

Comparing this with the wave equation used in our delta mirror model, we see that

$$\alpha_\delta = \frac{\eta\alpha}{\varepsilon_0} = \frac{d_{eg}^2 \eta}{\hbar\varepsilon_0\Delta} \left(1 - i\frac{\omega_0^3}{6\Delta\pi\varepsilon_0\hbar c^3} d_{eg}^2\right)$$

The dipole transition matrix element may be written in terms of the coupling strength

$$d_{eg} = g_0 \sqrt{\frac{\varepsilon_0 \hbar V}{\omega}}$$

where V is the cavity volume and ω the the photon frequency. Substituting this in yields

$$\alpha_\delta = \frac{\eta\alpha}{\varepsilon_0} = \frac{g_0^2\eta}{\hbar\varepsilon_0\Delta}\frac{\varepsilon_0\hbar V}{\omega}\left(1 - i\frac{\omega_0^3 g_0^2}{6\Delta\pi\varepsilon_0\hbar c^3}\frac{\varepsilon_0\hbar V}{\omega}\right) = \frac{g_0^2\eta V}{\omega\Delta}\left(1 - i\frac{\omega_0^3 g_0^2 V}{6\Delta\pi c^3\omega}\right)$$

The Rabi frequency is $\hbar\Omega(x) = -d_{eg}\,|E_n(x)|$. Esslinger claims a coupling strength of $g_0 = 2\pi \times 10.6$ (MHz).

With this in mind we can map this problem onto the delta mirror model from our paper. Let us consider the transcendental equation

$$\cos(k\triangle L) - \cos(kL) = \frac{2L}{\alpha}\frac{\sin kL}{kL}$$

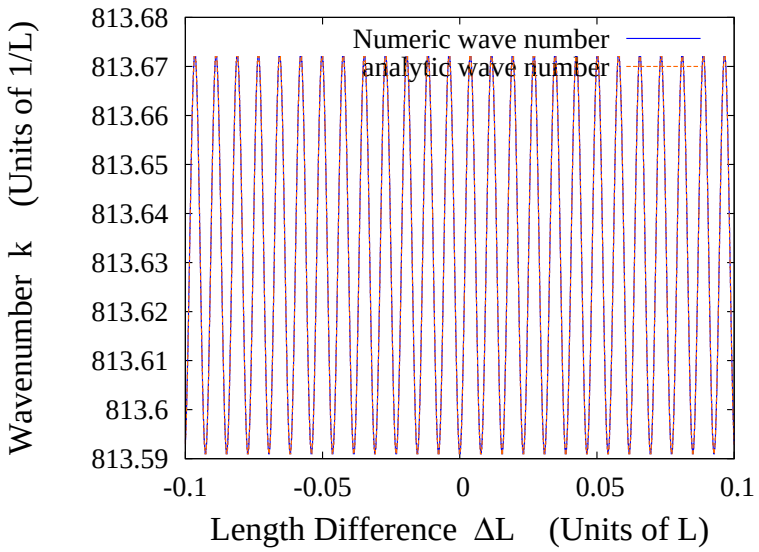
When α is very small, as we have found it indeed is, the right side must be of unitary order, thereforesin kL must be very close to zero. We therefore expand kL about $n\pi$ to first order

$$\cos(k\triangle L) \pm 1 = \pm\frac{2L}{\alpha}\left(\frac{1}{n\pi}(x - n\pi)\right)$$

Now $\cos(k\Delta L)$ has k in the argument, however as this doesn't deviate from $n\pi$ much, it is reasonable for small values of α to replace it with $n\pi$ as the cosine function is insensitive to such small perturbations

$$x = \pm\frac{\alpha n\pi}{2L}\left(\cos(n\pi\triangle L) \mp 1\right) + n\pi$$

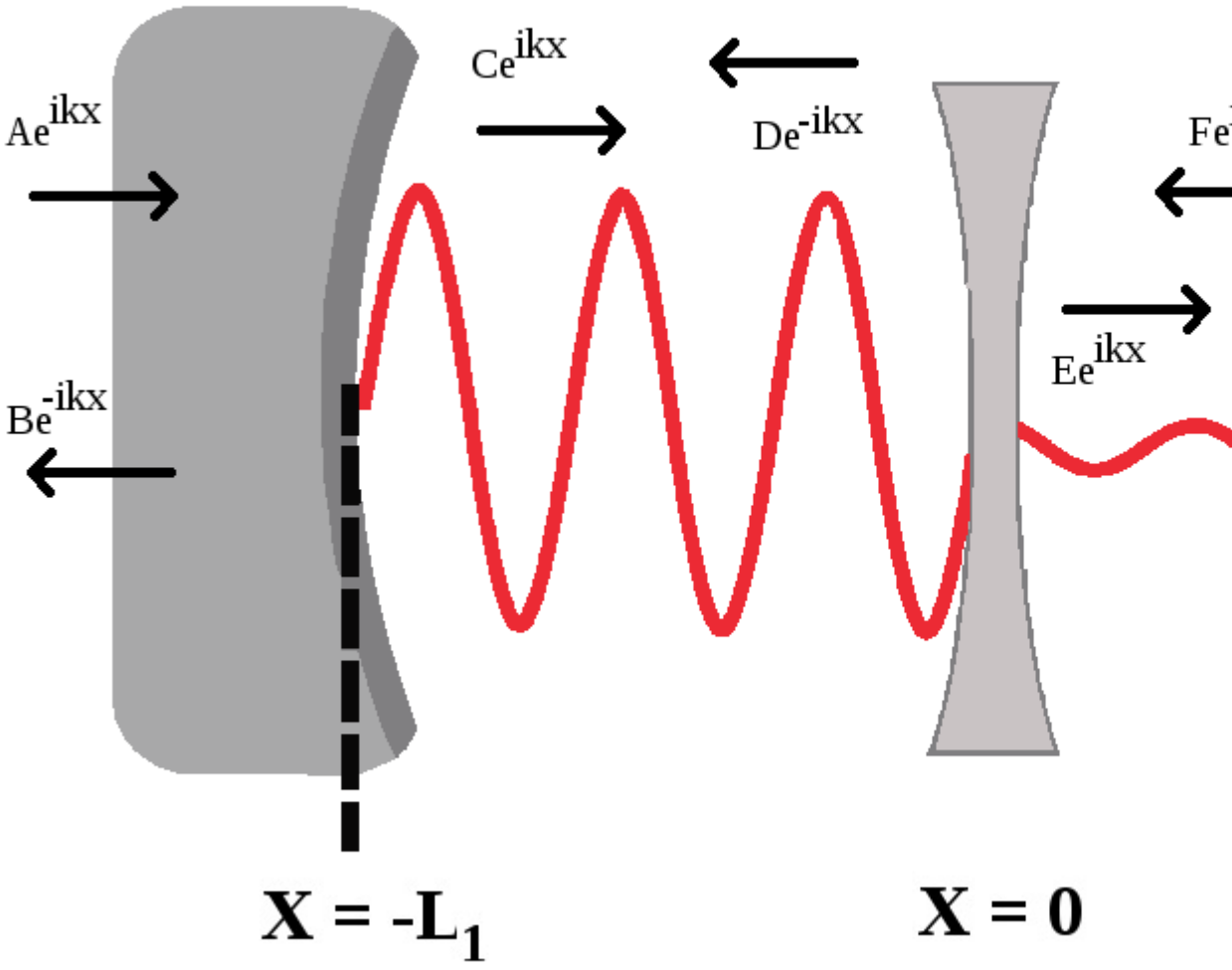
Here is a plot for $k \approx 8 \times 10^7$ and $\alpha = 10^{-9}$ as a function of $\triangle L$. Both numeric and analytic solutions are plotted.



We want to compare the size of these fluctuations relative to the linewidth due to cavity decay $\Gamma = \pi\kappa$, where κ is the cavity decay rate. Esslinger claims a decay rate of $\kappa = 2\pi \times 1.3$ (MHz). This corresponds to a line width of $\Gamma = 2.6$ (MHz).

2 March 1 2012

Consider a double cavity system in which the two end mirrors are not perfecty reflective. This scenario is fundamentally very different from the case in which we have perfectly reflecting end mirrors. In the latter case, the mirror position determines the allowed wavenumbers of the system. In the former case, the positioning does not change the wavenumber of the pumped laser, it only changes the amplitude. We wish to find the force on the central mirror as a function of the mirror reflectivities, wave number, and mirror position. To accomplish this, we solve Maxwell's equations in the four zones. We treat the mirrors as delta potentials similar to how we treated the central mirror in the thesis. Using the ansatz that in each region we have a plane wave propagating to the right and a plane wave propagating to the left, we may solve for the amplitude of the waves by matching boundary values.



Continuity of the electric field at the mirrors give the following relations:

$$Ae^{ikx_1} + Be^{-ikx_1} = Ce^{ikx_1} + De^{-ikx_1}$$

$$C + D = E + F$$

$$Ee^{ikx_3} + Fe^{-ikx_3} = Ge^{ikx_3}$$

Integrating over an infinitesimal region about each delta mirror gives the final 3 relations:

$$\frac{i}{k}\left(-Ae^{ikx_1} + Be^{-ikx_1} + Ce^{ikx_1} - De^{-ikx_1}\right) = -\alpha_{left}\left(Ae^{ikx_1} + Be^{-ikx_1}\right)$$

$$\frac{i}{k}\left(-C + D + E - F\right) = -\alpha_{central}\left(C + D\right)$$

$$\frac{i}{k}\left(-Ee^{ikx_3} + Fe^{-ikx_3} + Ge^{ikx_3}\right) = -\alpha_{right}\left(Ee^{ikx_3} + Fe^{-ikx_3}\right)$$

We assume here that $A = 1$. Solving this system of equations yields the following solution:

$$B = \frac{-e^{2ikx_1}k\left(e^{2ik(x_1+x_3)}\alpha_{central}\alpha_{left}\alpha_{right}k^2 - e^{2ikx_3}(-2i + k\alpha_{left})(-2i + k\alpha_{central})\alpha_{right} + (-2i + k\alpha_{left})\alpha\right.}{e^{2ik(x_1+x_3)}k^2(2i + k\alpha_{left})\alpha_{central}\alpha_{right} - e^{2ikx_3}k^2(-2i + k\alpha_{central})\alpha_{left}\alpha_{right} + k^2(2i + k\alpha_{right})\alpha_{cen}}$$

$$C = \frac{2e^{2ikx_1}\left(4 + \left(-1 + e^{2ikx_3}\right)k^2\alpha_{central}\alpha_{right} - 2ik\left(\alpha_{central} +\right.\right.}{e^{2ik(x_1+x_3)}k^2(2 - ik\alpha_{left})\alpha_{central}\alpha_{right} + e^{2ikx_3}k^2(2 + ik\alpha_{central})\alpha_{left}\alpha_{right} + k^2(2 - ik\alpha_{right})\alpha_{central}\alpha_{le}$$

$$D = \frac{2e^{2ikx_1}k\left(e^{2ikx_3}(2 + ik\alpha_{central})\alpha_{right} + \alpha_{central}\left(2 - ik\alpha\right.\right.}{-e^{2ik(x_1+x_3)}k^2(2i + k\alpha_{left})\alpha_{central}\alpha_{right} + e^{2ikx_3}k^2(-2i + k\alpha_{central})\alpha_{left}\alpha_{right} - k^2(2i + k\alpha_{right})\alpha_{centra}$$

$$E = \frac{4e^{2ikx_1}(2i + k\alpha_{right})}{e^{2ik(x_1+x_3)}k^2(2i + k\alpha_{left})\alpha_{central}\alpha_{right} - e^{2ikx_3}k^2(-2i + k\alpha_{central})\alpha_{left}\alpha_{right} + k^2(2i + k\alpha_{right})\alpha_{central}\alpha}$$

$$F = \frac{4e^{2ik(x_1+x_3)}k\alpha_{right}}{e^{2ik(x_1+x_3)}k^2(2i + k\alpha_{left})\alpha_{central}\alpha_{right} - e^{2ikx_3}k^2(-2i + k\alpha_{central})\alpha_{left}\alpha_{right} + k^2(2i + k\alpha_{right})\alpha_{central}\alpha}$$

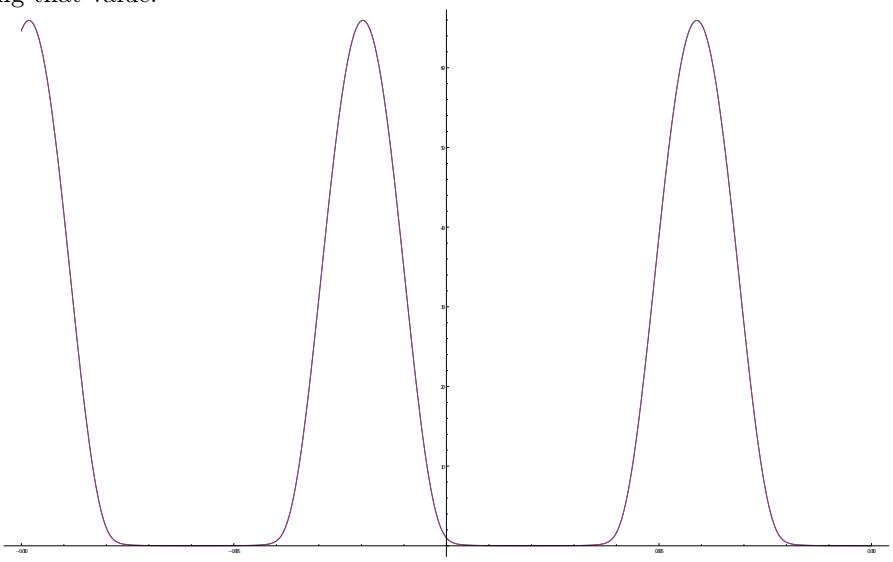
$$G = \frac{8e^{2ikx_1}}{e^{2ik(x_1+x_3)}k^2(2 - ik\alpha_{left})\alpha_{central}\alpha_{right} + e^{2ikx_3}k^2(2 + ik\alpha_{central})\alpha_{left}\alpha_{right} + k^2(2 - ik\alpha_{right})\alpha_{central}\alpha_{le}$$

3 March 12 2012

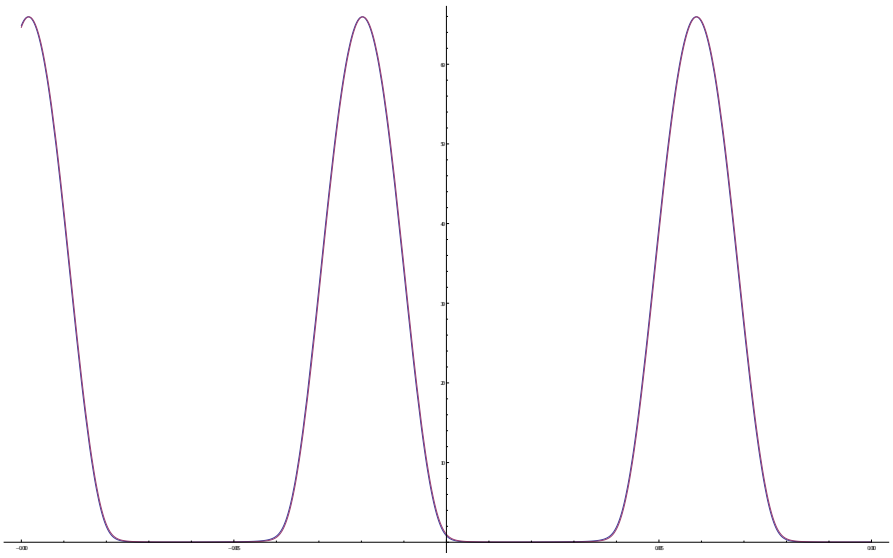
After much annoyance, I've constructed plots comparing the amplitude of the field intensity between mirrors 1 and 2 relative to the field intensity between mirrors 2 and 3. The first graph is the graph obtained by assuming that the end mirrors are infinitely reflective and then plotting the relative intensities. The equation used here is the one found in my thesis

$$\frac{A_n}{B_n} = -\frac{\sin(k_n L_2)}{\sin(k_n L_1)}$$

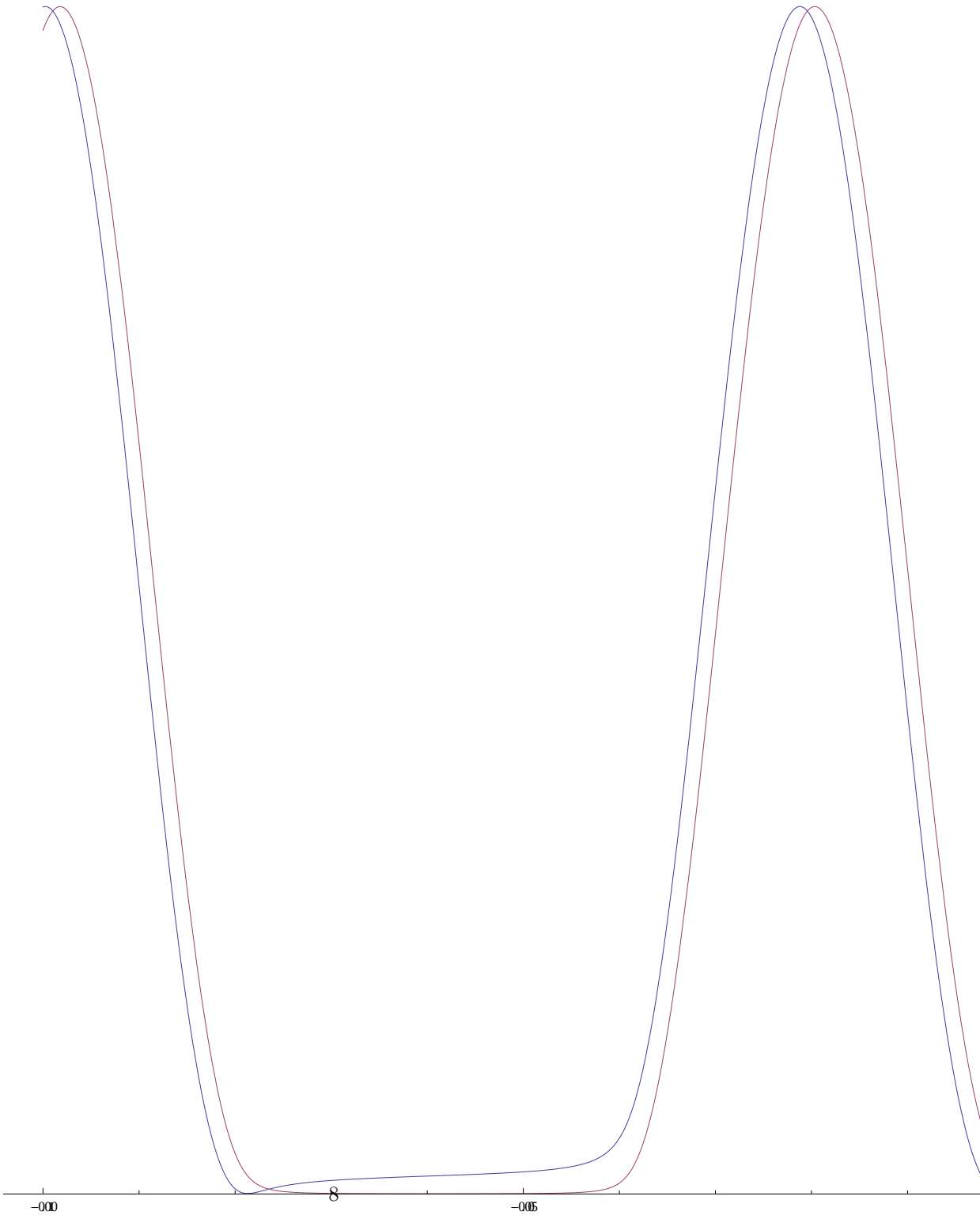
where A_n and B_n are the field amplitudes in the first and second cavity respectively. By squaring this quantity, I was able to create a plot for the relative intesities. The parameters used here are $k = 800000$, $L = 0.001$,$\alpha = 0.00001$. The first plot shows the scenario in which we let the two end mirrors are set to $\alpha_1 = \alpha_3 = 10^3\alpha_2$. I have also plotted the perfectly reflective case on top to show how well they match up. Note that in this graph, and all that follow, I have also changed the wavenumber to match the resonance wavenumber that would have been found in the perfectly reflecting case. By this I mean that for each change in$\triangle L$, I recalculate the wave number k using the transcendental equation for the perfect cavity case and then I solve the system of equations using that value.



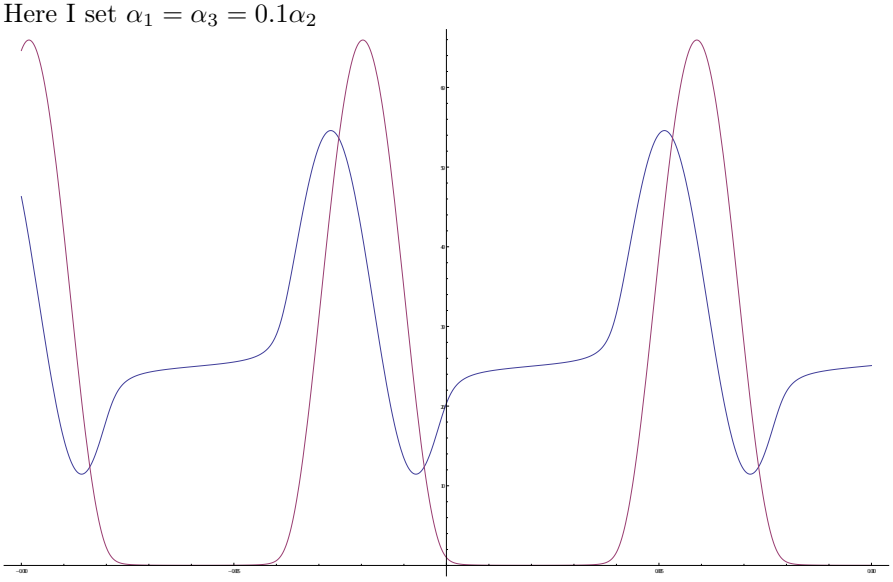
In the next graph, I have changed $\alpha_1 = \alpha_3 = 10\alpha_2$.



Here I set $\alpha_1 = \alpha_3 = \alpha_2$, which means all three mirrors are equal here.



Here I set $\alpha_1 = \alpha_3 = 0.1\alpha_2$



4 March 16 2012

Over the past few days, Faiyaz and I have been discussing Maxwell's equations in the cavity. He was able to show that the energy in the cavity is equally divided between the E-field and the B-field as is to be expected. The only weird thing that has come up is the fact that the B-field doesn't seem to be zero at the cavity wall. If you take the derivative of the E-field which should be a sine function, you'll end up with a cosine function which yields a unitary value at the boundary. I should look into this, as you can write a B-field as a $1/c$ E-field and get the proper result, but I do see based the solutions we have, what he means. I think it might have to do with the fact that we are not in a vacuum.

Duncan also mentioned some interesting things today on how to interpret what is actually having in these two cavity systems. The quasimode approximations assumes that in a cavity, an eigenmode is picked out. This allowed frequency is the only one which can live inside the cavity. It is given some width due to the fact that the end mirrors are not perfect and you have decay. This implies that the mode is coupled to other external modes and you can this fuzziness as you are in a superposition of many such global states. The flip side to this is to think that all these frequencies are allowed, it's just that the amplitudes for some are much greater than others as we have already found. Thus you get the same picture, but this time you are thinking of simply amplitude attenuation due to boudary conditions.

I have now started doing calculations based on the assumption that you have two laser sources pumping the cavity system from the right and to the left. This

corresponds to adding in an extra term in the system of equations.

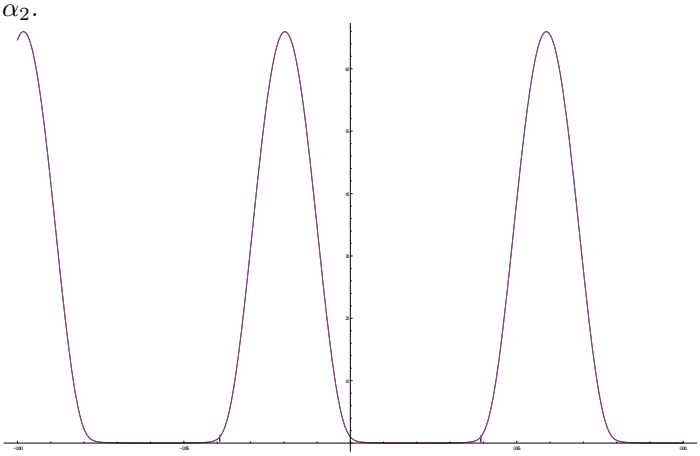
$$Ee^{ikx_3} + Fe^{-ikx_3} = Ge^{ikx_3} + 1e^{-ikx_3}$$

$$\frac{i}{k}\left(-Ee^{ikx_3} + Fe^{-ikx_3} + Ge^{ikx_3} - 1e^{-ikx_3}\right) = -\alpha_{right}\left(Ee^{ikx_3} + Fe^{-ikx_3}\right)$$

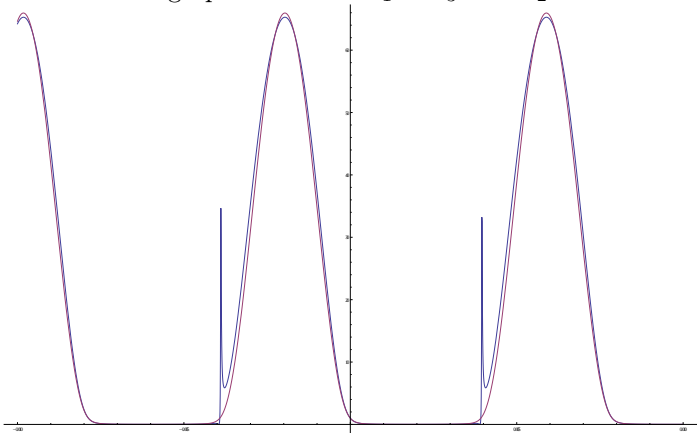
This is more in line with the type of experients people are doing, and eliminates any assymetries which pumping from only a single side may create.

5 March 23 2012

The following are the graphs done for the symmetric case. I have set the cavity length to $L = 10^{-3}m$, and $\alpha_2 = 10^{-5}$. In the first graph, I have set $\alpha_1 = \alpha_3 = 10^3\alpha_2$.



In the second graph I have set $\alpha_1 = \alpha_3 = 10\alpha_2$



6 April 2-15 2012

Review notes on optical forces

According to Ashkin (PRA 21:5,1979) the force on the atom can be viewed as the lorentz force acting on the induced dipole (classical), or as the momentum transfer due to absorbtion and reemission of light by the atom (quantumy). The total force has 3 basic components: absorbtion, spontaneous and stimulated emission. The first two are known as the scattering force. Due to the angular symmetry of the spontaneous emission, this force only contributes to fluctuations in the force. The stimulated emission is the cause of the dipole force. If we think of this classically, the dipole force is related to the potential energy of the induced dipole in the electric field. The dipole force is due to the real part (in phase) of the polarizability. The scattering force is due to the imaginary component.

One of my concerns is that in classical electrodynamics, light waves incident on a material induce small oscillations of polarization in the individual atoms (or oscillation of electrons, in metals), causing each particle to radiate a small secondary wave (in all directions, like a dipole antenna). All these waves add up to give specular reflection and refraction, according to the Huygens-Fresnel principle. This poses a problem for us when we are dealing with a single atom. The Huygens-Fresnel principle seems to only apply to a very large collection of atoms where you have specular reflection. For a single atom, you would get a very diffuse reflection. Therefore the argument that we can treat the single atom as a very thin dielectric slab and just tweak the parameters doesn't make sense to me. In the calculations done by other groups, they assume there is this specular reflection coefficient, which isn't the case for a single atom. Therefore doing stuff like solving Maxwell's equations for the double cavity system in which we have imposed boundary conditions for a mirror doesn't make sense to me. I think of Rayleigh scattering cross sections here.

A second concern is that in all the atomic force calculations I have seen, the driving field is taken to unaffected by the atom. I suppose in the strong field limit, the effect of the atom on the driving laser would be inconsequential. However, in the lower field limit where we don't have a very strong field, the atom would create a discontinuity in the field mode, and therefore taking the gradient doesn't make sense. What is the best way to deal with this? In classical electrodynamics when calculating the force on a material, we have to take the average of the force due to the field on either side. This is a way to eliminate the force cased by the material on itself. To see this, consider the total field E_{total}.

$$E_{total} = E_{patch} + E_{other}$$

By this I mean we can split the field acting on the patch of dielectric slab being considered as that due to the patch bound charge itself E_{patch}, plus that due to all other means E_{other}. Now of course the patch itself can't apply a force onto itself. What we can do to eliminate this is to so consider

$$E_{LeftOfSlab} = E_{other} + E_{patch}$$

$$E_{LeftOfSlab} = E_{other} - E_{patch}$$

Now the field due to a pure dipole is given by

$$E_{dipole}(r,\theta) = \frac{\rho}{4\pi\epsilon_0 r^3}\left(2\cos(\theta)\,\hat{r} + \sin(\theta)\,\hat{\theta}\right)$$

This shows that the field due to the patch on the left is equal but opposite in sign to the field due to the patch on the right. It is now clear why we use the average of the two fields

$$E_{other} = \frac{1}{2}\left(E_{LeftOfSlab} + E_{LeftOfSlab}\right) = E_{average}$$

Therefore we see that there is a field discontinuity whenever you have a dielectric material, but when you want to calculate the force on the material, you have to take the average of the two fields. Therefore I'm pretty sure we can deal with the gradient discontinuity we encounter in a similar manner. Why not consider a dielectric slab comprised of many atoms and solve for the fields on either side (as we have done in the paper). Doing so will give us the Electromagnetic field just to the left and just to the right of the slab. We may then take the average and calculate the force using this.

In our paper we found the modes to be

$$U_n(x) = \begin{cases} A_n \sin(k_n(x+L_1)) & -L_1 < x \leq 0 \\ B_n \sin(k_n(x-L_2)) & 0 < x < L_2 \end{cases}$$

It was also possible to find a good analytic solution for the wavenumber k_n. Consider the transcendental equation

$$\cos(k\triangle L) - \cos(kL) = \frac{2L}{\alpha}\frac{\sin kL}{kL}$$

When α is very small, as we have found it indeed is, the right side must be of unitary order, thereforesin kL must be very close to zero. We therefore expand kL about $n\pi$ to first order

$$\cos(k\triangle L) \pm 1 = \pm\frac{2L}{\alpha}\left(\frac{1}{n\pi}(x - n\pi)\right)$$

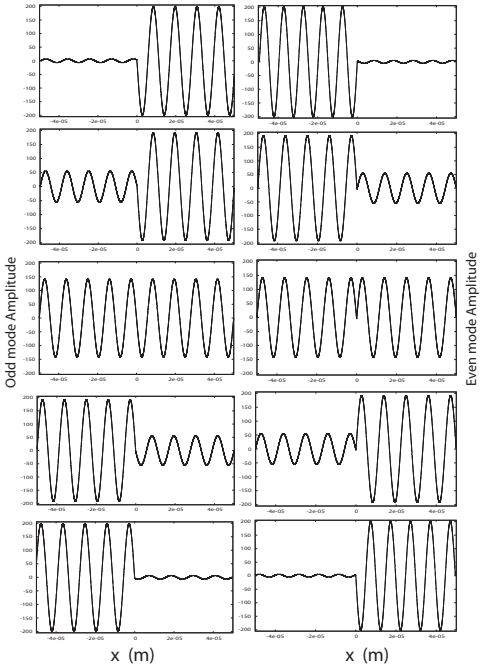
Now $\cos(k\Delta L)$ has k in the argument, however as this doesn't deviate from $n\pi$ much, it is reasonable for small values of α to replace it with $n\pi$ as the cosine function is insensitive to such small perturbations

$$k_n = \pm\frac{\alpha n\pi}{2L^2}\left(\cos(n\pi\triangle L) \mp 1\right) + \frac{n\pi}{L}$$

Ok good. So we have k_n as a function of $\triangle L$ (this isn't in the paper). Now all we need is the amplitudes A_n and B_n which isn't hard to find as

$$\frac{A_n}{B_n} = -\frac{\sin(k_n L_2)}{\sin(k_n L_1)}$$

If we consider the dielectric to be at the center, then $A_n = B_n$ and the only effect that the dielectric slab has is to alter the wavenumber. Now lets go back to the paper and look at the the wavemode graphs where we examined what the mode structure looks like as we move the central dielectric. In the third row we see the odd and even modes when $\triangle L = 0$.



Note that the odd mode is just the cavity standing wave. It is only in the even mode that we see a difference. However, if we believe the argument above for the force felt by the atom, then by symmetry we should get no force felt by the atom. This fact can be made more clear by considering the equation for the force on an atom applied by an electric field. Let the electric field be of the form:

$$\overrightarrow{E}(\overrightarrow{r}, t) = \overrightarrow{E_0}(\overrightarrow{r}) \cos\left[\omega t + \phi(\overrightarrow{r})\right]$$

Then the average force on the atom is given by

$$\left\langle \overrightarrow{F}(t) \right\rangle = \frac{\overrightarrow{\nabla} I(\overrightarrow{r})}{I_s} \frac{-\hbar\left(\frac{\delta}{2}\right)}{1 + \frac{I(\overrightarrow{r})}{I_s} + \left(\frac{2\delta}{\Gamma}\right)^2} - \hbar\Gamma\left(\overrightarrow{\nabla}\phi(\overrightarrow{r})\right) \frac{\Omega^2}{\Gamma^2 + 4\delta^2 + 2\Omega^2}$$

Here I_s is known as the saturation intensity and is given by $I_s = \frac{2\pi^2 \hbar c \Gamma}{3\lambda^3}$, Γ is the decay rate, Ω is the Rabi frequency, and δ is the detuning. The first term

in the force equation is known as the dipole force, and the second term is known as the scattering force. For a pure plane wave, the first term is zero because the gradiant of the intensity is zero as $E_0 = constant$. For a standing wave the second term is zero, as there is no phase term. Going back to our problem now, it is clear that the odd mode feels no scattering force, but feels a dipole force proportional to the gradient of the intensity. A classical picture would be something like a ball rolling down the slope of the hill. In the even mode however, we would still get zero scattering force due to the fact that we still have a standing wave, but we also should get no dipole force as the "atom" is parked in a potential ditch. Does this make sense? The problem to this classical argument is that in the other images (when we change $\triangle L$) the "atom" is still stuck in a ditch however, the average of $E_{LeftOfSlab}$ and $E_{RightOfSlab}$ will give a non-zero result and there will be a dipole force.

7 April 18 2012

The reason that we are in the dispersive regime is because we want to ignore the effects that the driving radiation has on the spontaneous emission rate. This is the case when the Rabi frequency is much smaller than the transition angular frequency of the atom.

It's good to have the block vectors written out too. The vectors are: (Cohen-Tannoudji)

$$u = \frac{1}{2}\left(\hat{\sigma}_{ab} + \hat{\sigma}_{ba}\right)$$

$$v = \frac{1}{2i}\left(\hat{\sigma}_{ab} - \hat{\sigma}_{ba}\right)$$

$$w = \frac{1}{2}\left(\hat{\sigma}_{bb} - \hat{\sigma}_{aa}\right)$$

These are time independent as we have $\hat{\sigma}_{ab} = \sigma_{ab}e^{i\omega_L t}$, $\hat{\sigma}_{ba} = \sigma_{ba}e^{-i\omega_L t}$, $\hat{\sigma}_{aa} = \sigma_{aa}$, $\hat{\sigma}_{bb} = \sigma_{bb}$. Using this we can write the average dipole as

$$\langle d \rangle = 2d_{ab}\left(u\cos\omega_L t - v\sin\omega_L t\right)$$

8 April 23 2012

From Cohen-Tannoudji (pg.369), between the time t and $t + dt$ the atomic electron moves from r to $r + dr$ and the driving field $E_0 \cos\omega_L t$ does work equal to

$$dW = qE_0\cos\omega_L t \cdot dr$$

We can therefore show that the average power absorbed by the atom is

$$\left\langle \frac{dW}{dt} \right\rangle = qE_0 \cos\omega_L t \left\langle \dot{d} \right\rangle$$

Using the formula for $\langle d \rangle$ yields

$$\left\langle \overline{\frac{dW}{dt}} \right\rangle = -2d_{ab} \cdot E_0 \omega_L \left(v\overline{\cos^2 \omega_L t} + u\overline{\sin\omega_L t \cos\omega_L t} \right) = \hbar\Omega\omega_L v$$

If we now divide this by the energy per photon $\hbar\omega_L$ we find the number of atoms absorbed per unit time is

$$\left\langle \frac{dN}{dt} \right\rangle = \Omega v$$

In the steady state, this is equal to the number of photons spontaneously emitted by the atom per unit time. It does not tell you about stimulated emission. We can show that the scattering force (or dissipative force) for a planewave is equal to

$$F_{scatt} = \left\langle \frac{dN}{dt} \right\rangle \hbar k_L$$

We can rewrite this as

$$F_{scatt} = \hbar k_L \frac{\Gamma}{2} \frac{\frac{\Omega^2}{2}}{\frac{\Gamma^2}{4} + \delta^2 + \frac{\Omega^2}{2}}$$

This force varies as a Lorentzian about the atomic frequency ω_0. At low intensities, it can be show that this is proportional to the intensity, while for high intensities it is proportional to the decay rate $F_{scatt} = \hbar k_L \frac{\Gamma}{2}$ and independent of intensity.

For dipole forces things are a bit different. Although there is no exchange in energy between the atom and the driving field, there is a redistribution of energy between the two plane waves forming the standing wave. This allows the momentum of the atom to vary as one can absorb a photon from one wave and redistribute in the other giving it a momentum kick of $2\hbar k_L$. Unlike the dissipative force, the dipole force increases without bound as the gradient of the intensity. It can be shown that the maximum value of F_{dip} is

$$|F_{dip}| = \hbar k_L \Omega$$

In the Cohen-Tannoudji paper, he states that at low intensities one finds that the average dipole force over a wavelength is just the sum of the radiation pressures caused by the two running waves. They claim that the dressed atom approach is the correct way to look at the dipole force.

9 April 24 2012

We are able to use the steady state values for the optical Bloch equations only if we assume the atom to be moving sufficiently slowly. It is also important to remember that because the dipole force is conservative, we can write it as a gradient of a potential.

The force on a standing wave of the form $E(x,t) = E_0(x)\cos\omega_L t$ can be written as

$$F = \frac{1}{2}\alpha\nabla E_0^2$$

where

$$\alpha = \frac{-2\delta d^2}{\hbar\left[\Gamma^2 + 4\delta^2 + 2\left(d\cdot E_0/\hbar\right)^2\right]}$$

is the atomic polarizability (R.J. Cook 1979). Also note that on resonance, the dipole force vanishes. The above form of the force is due to (Griffiths pg 165) $F = (p\cdot\nabla)E$. Interestingly, if there is both a standing and travelling wave, then a third force term must be considered, a sort of cross term. This is derived in the Cook paper. Note that the Griffiths force doesnt take into account the term $\dot{P}\times B$. There are higher order terms as well, but usually it's safe to drop them.

Let's look at the polarization term.

$$p = \alpha E$$

We also have

$$P = p/V$$

Combining this with

$$P = \epsilon_0\chi_e E$$

gives us

$$\alpha = \epsilon_0\chi_e V$$

This allows us to write the force per unit volume on a polarized material as (? only the electric force contribution?)

$$f = \frac{1}{2}\epsilon_0\chi_e\nabla E_0^2$$

Now in Griffiths, he derives the total electromagnetic force on the charges in a volume of material. He writes everything in terms of fields.

$$f = \nabla\cdot\overleftrightarrow{T} - \frac{1}{c^2}\frac{\partial S}{\partial t}$$

where

$$T_{ij} = \epsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$$

and

$$S = \frac{1}{\mu_0}(E \times B)$$

Using $\dot{P} = J$ and $\nabla \cdot P = -\rho$ in the Lorentz force equation, we get back the dipole force equation with the extra magnetic term.

Going back to the dipole force, we can find the dipole potential in the case where we are far off resonance by expanding the saturation parameter(atomoptics.uoregon.edu site that prassana sent) to be

$$V_{dip} = \frac{\hbar |\Omega(r)|^2}{4\delta}$$

10 April 26 2012

Going through Cohen-Tannoudji's paper, he derives an equation for the dipole force on an atom written in the dressed-atom basis.

$$f_{dip} = \frac{\hbar \nabla \Omega}{2}(\Pi_2 - \Pi_1) - \hbar \Omega \nabla \Theta (\rho_{12} + \rho_{21})$$

Where he defines

$$\Omega(r) = \left[\omega_1^2(r) + \delta^2\right]^{1/2}$$

as the generalized Rabi frequency. He also defines

$$\Pi_i(r) = \sum_n \langle i, n; r | \rho | i, n; r \rangle$$

$$\rho_{ij}(r) = \sum_n \langle i, n; r | \rho | j, n; r \rangle$$

the populations and coherences respectively. Finally we have

$$\nabla \Theta = \frac{-\delta \nabla \omega_1}{2(\delta^2 + \omega_1^2)}$$

Note that the Rabi frequency is defined in the paper as $\omega_1(r)e^{i\varphi(r)}$ so ω_1 is actually the real part and φ is the phase. It also is defined to include the photon number and the electromagnetic wave. This is different from other definitions where they only take the amplitude of the field. He next calculates the work dW that has to be provided to move the atom by dr. For an atom at rest he shows that you can write the steady state dipole force as

$$f_{dip}^{ss} = -\Pi_1 \nabla E_1 - \Pi_2 \nabla E_2$$

where

$$E_1(r) = \frac{1}{2}\hbar \Omega(r) = -E_2(r)$$

are the deviations of E_{1n} and E_{2n} from $(E_{1n} + E_{2n})/2$ which is independent of r. Also note that there is another term that must be considered if that atom

is moving, but it drops out for a static atom, and can be neglected for a slow moving atom. It must be stated that one must be careful in defining the energy of the system because we haven't taken into account the incoherent processes such as spontaneous emission. However, if we take a step back and consider the effects only on the atom. We can let our Hamiltonian take the form

$$H = H_{atom} + H_{dipole}$$

$$H \longrightarrow \frac{1}{2}\hbar \begin{pmatrix} -\delta & \Omega \\ \Omega & \delta \end{pmatrix}$$

in the excited and ground state basis. Then you can show (Thywessian's notes) that the energy shift on the ground state atom due to the applied laser field is

$$\Delta E = \frac{\hbar \Omega^2}{4\delta}$$

Now my question is, do we have to consider this as being lost by the field along with the force on the atom. The force gives the change in impulse on the center of mass of the atom, but doesn't describe the internal energy shifts of the atomic states.

11 April 27 2012

In the momentum exchange paper by Ed Hinds, he describes the Lorentz force on a slowly moving atom as

$$F_i = (d \cdot \nabla) E_i + \left(\dot{d} \times B\right)_i$$

which is the standard Lorentz force on a dielectric material. By using Maxwell's equations, this is usually rewritten as

$$F_i = F_{1i} + F_{2i} = d \cdot \frac{\partial}{\partial x_i} E + \frac{\partial}{\partial t} (d \times B)_i$$

Usually it is only the F_{1i} term that is used in the laser cooling literature. However, Hinds suggests that the first of the equations is more telling. Here the first term comes from the Coulomb force, and contributes nothing to the force. The second term is the magnetic Lorentz force, and the forces felt by the atom comes from this. Usually we write the averaged force due F_{1i} as (here we have assumed a wave travelling in the z-direction and therefore polarizations in x and y)

$$\left\langle \overline{F_{1z}} \right\rangle = D \left(u \nabla E_0 - vkE_0\right)$$

for a plane wave laser field of the form

$$E_0 (\omega t - kz) \cos (\omega t - kz)$$

The first term is of course just the dipole force and the second is the scattering. If now instead we had used the first equation instead of the second, we would have found that the first term (Coulomb) contributes nothing, and the second term gives us

$$\langle \overline{F_z} \rangle = D \left(\frac{\partial u}{\partial t} \frac{E_0}{c} - vkE_0 \right)$$

This gives the exact same momentum transfer of $DE_0 u/(2c)$ which is the same as that which would be found if considering the second equation. Note that here they had to take into account the second magnetic term.

12 April 30 2012

Reading the wikipedia entry on optical tweezers, they had some interesting insight into how to work with optical forces. The article had this to say:

"Proper explanation of optical trapping behavior depends upon the size of the trapped particle relative to the wavelength of light used to trap it. In cases where the dimensions of the particle are much greater than the wavelength, a simple ray optics treatment is sufficient. If the wavelength of light far exceeds the particle dimensions, the particles can be treated as electric dipoles in an electric field. For optical trapping of dielectric objects of dimensions within an order of magnitude of the trapping beam wavelength, the only accurate models involve the treatment of either time dependent or time harmonic Maxwell equations using appropriate boundary conditions."

The ray optics explanation is to simply use conservation of momentum and refration to explain why a macroscopic object tends to be drawn to the center of the beam. See the article for their image. For us, what is of interst is the dipole force explanation. The Lorentz force applied to one of the charges in the dipole is given by:

$$F_1 = q \left(E_1 + \frac{\partial x_1}{\partial t} \times B \right)$$

Since the two charges have opposite sign, the total force on the two charges is

$$F = q \left(-E_1 (x, y, z) + E_2 (x + \triangle x, y + \triangle y, z + \triangle y) + \frac{\partial (\triangle x)}{\partial t} \times B \right)$$

Expanding E_2 to first order around x yields

$$F = q \left(-E_1 + (\triangle x \cdot \nabla) E + E_1 + \frac{\partial (\triangle x)}{\partial t} \times B \right)$$

$$F = (p \cdot \nabla) E + \dot{p} \times B$$

Using the linearity of the dipole, we have

$$F = \alpha \left[(E \cdot \nabla) E + \dot{E} \times B \right]$$

Using Maxwell's equations, we obtain

$$F = \alpha \left[\frac{1}{2} \nabla E^2 + \frac{\partial}{\partial t} (E \times B) \right]$$

The first term is the gradiant of the intenstiy, while the second term is the rate of change of the Poynting vector, which in a standing wave is zero.

13 May 2 2012

We need to first investigatethe force on a dielectric median. How do you handle the problem when there are many atoms comrising the material? It is not just the sum of the individual forces.

A paper by Barnett and Loudon suggests the the force density induced on a dielectric median by an electromagnetic field is given by

$$f = - (\nabla \cdot P) E + \dot{P} \times B$$

if one treats the individual charges in the material. Or as

$$f = (P \cdot \nabla) E + \dot{P} \times B$$

if one treats the individual dielectric point particles. This latter expression is for the force at applied to the center of mass of the dielectric, while the former is for the force on each charge. They go on to show that integrating this over some volume gives the same total force, as is expected. E and B are the microscopic fields. It seems that they simply integrate this over a volume for a material. Griffiths does the same when he defines the dipole density function P as the dipole moment per unit volume. He states that outside the dielectric there is no real problem as the graininess is blurred by distance. He states that the true microscopic field would be impossible to calculate, and that whenever yoiu deal with fields inside dielectrics, you always use the average field. What you need to average over are regions much smaller than the dimension of the object itself, but still containing many thousands of atoms. This is why instead of integrating over all the dipoles, all you need is to find do is find the bound charges and then calculate the fields they produce. Griffiths justifies that what we actually calculate using these methods is the macroscopic field with the following argument:

Suppose you want to calculate the macroscopic field at some point r within the dielectric. Since you want to average the true microscopic field over an appropriate volume, he draws a small sphere about point r which has some

radius R. The macroscopic field at r then consisits of the field due the charges inside the sphere and those charges outside the sphere.

$$E = E_{outside} + E_{inside}$$

It can be shown that the average field over a sphere, produced by charges outside, is equal to the field they produce at the center. Therefore, $E_{outside}$ is the field at r due to the dipoles outside of the sphere. The argument then goes that these dipoles are far enough that you can safely use the continuous dipole density P. This shows that

$$V_{outside} = \frac{1}{4\pi\epsilon_0}\int_{outside}\frac{\widehat{r}\cdot P\left(r'\right)}{r^2}d\tau'$$

Now the average field inside the sphere can be shown to be

$$E_{inside} = -\frac{1}{3\epsilon_0}P$$

This follows from the fact that the average field inside a sphere of radius R due to all charges whitin the sphere can be written as

$$E = -\frac{1}{4\pi\epsilon_0}\frac{p}{R^3}$$

where p is the total dipole moment. (This is where the extra term in the Clausius-Mosotti equation comes from)The next step in the argument is that P does not vary significantly over the volume of the sphere, so that the term left out of the potential integral corresponds to the field at the center of a uniformly polarized sphere. We have just argued that this is exactly what E_{inside} puts back in. We are therefore justified in deriving the macroscopic field as an integral running over the entire volume of the dielectric using the polarization density. It correctly calculates the averaged macroscopic field inside a dielectric. This means that no matter how wild the actual microscopic charge configuration is, we may always replace it by a smooth distribution of perfect dipoles.

14 May 3rd 2012

The Clausius-Mosotti equations relate the dielectric constant to the polarization. Going through some of Feynmans notes (V2Ch32) now. The wave equation in a dielectric is given by:

$$\frac{\partial^2 E(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2} = -\frac{1}{\varepsilon_0}\nabla\left(\nabla\cdot P\right) + \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 P}{\partial t^2}$$

where P is the polarization.

Now he does something interesting here. He solves the wave equation above by assuming the wave of the form

$$E_0 e^{i(\omega t - kx)}$$

which he writes as

$$E_0 e^{-ik(x-vt)}$$

and since $v_{phase} = c/n$

$$E_0 e^{i\omega(t-xc/n)}$$

therefore you can find n by finding out what value of k solves the wave equation and using $n = kc/\omega$. The first term on the right disappears because the polarization doesn't vary in the polarization direction of the field (let's say z). Finally, assuming that the polarization varies as $e^{i\omega t}$ we get

$$-k^2 E_z + \frac{\omega^2}{c^2} E_z = -\frac{\omega^2}{\varepsilon_0 c^2} P_z$$

We can write

$$P_z = \varepsilon_0 \alpha N E_z$$

Therefore we arrive at

$$k^2 = \frac{\omega^2}{c^2}(1 + \alpha N)$$

and so

$$n^2 = (1 + \alpha N)$$

This derivation is fine for gasses, but not for dense materials as it doesn't take into account the field produced by the dipoles themeselves. In this case, we must use:

$$E_{local} = E_z + \frac{P}{3\varepsilon_0}$$

Thus we should rewrite

$$P_z = \varepsilon_0 \alpha N E_{local}$$

and then we find for an isotropic solid (also require the spacing between atoms to be far less than the wavelength of light) that

$$n^2 = 1 + \frac{\alpha N}{1 - \frac{N\alpha}{3}}$$

This is known as the Clausius-Mosotti equation.

15 May 7 2012

These are the notes from Cohen-Tannoudji's optics review. One thing he says is that you can interpret the dipole force as a redistribution of photons between two modes of the laser. For a standing wave comprised of two plane waves, it's simply the absorbtion and stimulated emission from one plane wave mode, into the other. He also states that another way to look at it is in the dressed atom approach. In this case, the eigen-energies E_1 and E_2 for the eigenmodes$|1\rangle$ and $|2\rangle$ of the dressed manifold are used. The atom feels a force proportional to the gradiant of these forces multiplied by the steady state populations for each.

$$F = \Pi_1 F_1 + \Pi_2 F_2$$

Now on to the good stuff. It is easy enough to connect the radiation pressure force felt by an atom in a planewave to that felt by a macro object irradiated by an electromagnetic field. The atom absorbs and then spontaneously emits a photon which gives it a kick. Now in a standing wave this picture doesn't work. In general you can't just use the spontaneous force from one wave and add the spontaneous force produced by the counter propagating wave. The reason for this is that thre are cross terms that you need to consider. For an atom which is moving, the spatial average value of these cross terms is zero, but not for a moving atom. This arises due to the fact that the dipole for wave (1) goes as $e^{i(\omega t - kx)}$ while the field of (2) varies as $e^{i(\omega t + kx)}$. Multiplying these two together yields a spatially modulated term with period $\lambda/2$. It is only true that you can ignore these terms if the field intensity is small enough.

$$\left\langle \overline{F} \right\rangle = \langle d_{ss} \rangle^{(1)} \nabla E^{(1)} + \langle d_{ss} \rangle^{(2)} \nabla E^{(2)} + \langle d_{ss} \rangle^{(2)} \nabla E^{(1)} + \langle d_{ss} \rangle^{(1)} \nabla E^{(2)}$$

Mathematically, in the weak intensity regime, the saturation parameter s is taken to be $\ll 1$.

16 May 9 2012

Here we go over the calculation for the index of refraction of an atom. Now we know that the reactive (dipole) force on an atom is given by:

$$F_{dipole} = -\frac{\hbar\delta}{4} \frac{\overrightarrow{\nabla}\Omega^2}{\frac{\Gamma^2}{4} + \delta^2 + \frac{\Omega^2}{2}}$$

Then comparing this with the form of a classical pure dipole force

$$F = \frac{1}{2}\alpha \nabla E_0^2$$

One is able to obtain

$$\alpha = \frac{-2\delta d^2}{\hbar\left[\Gamma^2 + 4\delta^2 + 2\left(d \cdot E_0/\hbar\right)^2\right]}$$

This is the result obtained by both R.J. Cook (1979) and Cohen-Tannoudji (Atom-photon interactions). Cohen-Tannoudji then goes on to use this to obtain the susceptibility using the formula for molecular polarizability $p = \alpha E_{local}$. This introduces a complication as the local field may differ from the overall applied field. As $P = Np$, and $P = \varepsilon_0 \chi_e E$, then we get:

$$\chi_e = \frac{N\alpha}{\varepsilon_0}\frac{E_{local}}{E}$$

Therefore we only get the Cohen-Tannoudji result of

$$Re\left[\chi_{atom}\right] = \frac{-2\delta d^2 N}{\hbar\varepsilon_0\left[\Gamma^2 + 4\delta^2 + 2\left(d \cdot E_0/\hbar\right)^2\right]}$$

if the local field is equal to the ambient field. This is fine for gases and single atoms, but is not true for solids. In this case, one must use the Clausius-Mosotti equation:

$$n^2 = 1 + \frac{\alpha N}{1 - \frac{N\alpha}{3}}$$

which takes into account the effects of the dipoles on the field acting on it. Ignoring this, we can write $n \approx 1 + \frac{1}{2}Re\left[\chi_{atom}\right]$ and arrive at

$$n_{atom} = 1 - \frac{\delta d^2 N}{\hbar\varepsilon_0\left[\Gamma^2 + 4\delta^2 + 2\left(d \cdot E_0/\hbar\right)^2\right]} = 1 - \frac{d^2 N u_{ss}}{2\Omega\hbar\varepsilon_0}$$

Where I have used the steady state Bloch vector u_{ss}

$$u_{ss} = \frac{\Omega}{2}\frac{\delta}{\frac{\Gamma^2}{4} + \delta^2 + \frac{\Omega^2}{2}}$$

The reflected intensity R of a wave incident on a dielectric interface at normal incidence is given by

$$R = \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

For us, $n_1 = 1$ and $n_2 = n_{atom}$. Plugging this in we obtain

$$R = \left(\frac{\frac{\delta d^2 N}{\hbar\varepsilon_0\left[\Gamma^2 + 4\delta^2 + 2(d \cdot E_0/\hbar)^2\right]}}{2 - \frac{\delta d^2 N}{\hbar\varepsilon_0\left[\Gamma^2 + 4\delta^2 + 2(d \cdot E_0/\hbar)^2\right]}}\right)^2 \approx \left(\frac{\delta d^2 N}{2\hbar\varepsilon_0\left[\Gamma^2 + 4\delta^2 + 2\left(d \cdot E_0/\hbar\right)^2\right]}\right)^2$$

Now the difference in intensities going in vs. coming back is $I_I - I_R = I_I(1-R) = \frac{c\varepsilon_0}{2}|E|^2\left(1-\left(\frac{\delta d^2 N}{2\hbar\varepsilon_0\left[\Gamma^2+4\delta^2+2(d\cdot E_0/\hbar)^2\right]}\right)^2\right)$

This equation can be rewritten as

$$\frac{c\varepsilon_0}{2}|E|^2\left(1-\left(\frac{d^2 N u_{ss}}{2\hbar\varepsilon_0\Omega}\right)^2\right)$$

Then the momentum $\langle P\rangle = \langle I\rangle / c^2$ can be written as

$$P \approx \frac{\varepsilon_0}{2c}|E|^2\left(\frac{1}{1+\left(\frac{d^2 N u_{ss}}{2\hbar\varepsilon_0\Omega}\right)^2}\right)$$

This we can write as

$$P = P_0$$