

# Digital logic

- properties of different gates

	Gates	Idempotence	Commutative	Associative
1	NOT -	X	X	X
2	AND ·	✓	✓	✓
3	OR +	✓	✓	✓
4	NAND ↑	X	✓	X
5	NOR ↓	X	✓	X
6	XOR ⊕	X	✓	✓
7	XNOR ◎	X	✓	✓

Idempotence :  $A \cdot A = A$

Commutative :  $A \cdot B = B \cdot A$

Associative :  $(A + B) + C = A + (B + C)$

Imp: K map

MUX

Latches

Number system: Fixed , Floating

Binary Single precision

Conversion Double precision

Code

Hexa

Decimal

Octa

### ① Properties

i) Property of 0:  $p+0=p$   
 $p \cdot 0=0$

ii) Biconditional:  
 $(p \Leftrightarrow q) = (p \Rightarrow q) \cdot (q \Rightarrow p)$   
 $= (\bar{p} + q) \cdot (p + \bar{q})$

iii) Property of 1:  $p+1=1$   
 $p \cdot 1=p$

### iii) Distribution

$$\begin{aligned} X + \bar{X}Y &= X + \bar{Y} \\ X + YZ &= (X+Y)(X+Z) \\ X(Y+Z) &= XY + XZ \end{aligned}$$

iv) Complementarity:  $p+\bar{p}=1$   
 $p \cdot \bar{p}=0$

v) Commutativity:  $p+q=q+p$   
 $p \cdot q=q \cdot p$

vi) Idempotence:  $p+p=p$   
 $p \cdot p=p$

vii) Involution:  $\bar{\bar{p}}=p$

viii) DeMorgan:  $\overline{p+q}=\bar{p} \cdot \bar{q}$   
 $\overline{pq}=\bar{p}+\bar{q}$

ix) Associativity:  $p+(q+r)=(p+q)+r$   
 $p(qr)= (pq)r$

x) Absorption:  $p+qp=p$   
 $p \cdot (pq)=p$

xi) Conditional:  $p \Rightarrow q = \bar{p} + q$

### Logic Gates

#### ② Base Gates

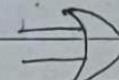
AND



all 1 = 1

$A \cdot B$

OR



any 1 = 1

$A+B$

NOT

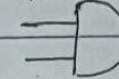


$1 \rightarrow 0, 0 \rightarrow 1$

$\bar{A}=A$

#### ③ Universal Gate

NAND



all 1 = 0

$\bar{AB}=1$

NOR

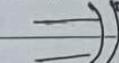


opposite of OR

$\bar{A}+\bar{B}=1$

#### ④ Arithmetic Gate (used in half, full adder, subtractor)

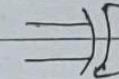
XOR



odd 1 = 1 = diff

$[A \oplus B = A\bar{B} + \bar{A}B]$

XNOR



same input = 1 = same

$AB + \bar{A}\bar{B}$

#### ★ Making different gates using NOR and NAND

NOT



1

1

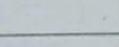
AND



2

3

OR



3

2

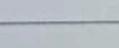
XOR



4

5

XNOR



5

4

NAND  $\leftrightarrow$  NOR

\* Buffer: agar zero pass to zero, agar 1 pass to 1

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Eg: Using XOR as buffer ( $A\bar{B} + \bar{A}\bar{B}$ )

- Invertor i)  $A \oplus \bar{A} = 1 \rightarrow A \oplus 1 = \bar{A}$   
Buffer ii)  $A \oplus 0 = A \rightarrow A \oplus A = 0$   
iii)  $A \oplus A \oplus A \oplus \dots = [ \text{even} = 0  
[ \text{odd} = A \text{ (buffer)} ] ]$

Eg: Using XNOR ( $\bar{A}\bar{B} + A\bar{B}$ )

- i)  $A \odot A = 1$   
ii)  $A \odot \bar{A} = 0$   
Invertor iii)  $A \odot 0 = \bar{A}$   
Buffer iv)  $A \odot 1 = A$   
v)  $A \odot A \odot A \odot \dots = [ \text{even} = 1  
[ \text{odd} = A \text{ (buffer)} ] ]$

### ⑤ Canonical Sum of Product

SOP (Sum of Product) can have all the literals but  
Canonical form of SOP must have all product term containing  
all literals either in complementary or uncomplementary form

$$f(x, y, z) = \begin{array}{l} xyz + yz \\ xyz + \bar{x}\bar{y}\bar{z} \\ xyz + \bar{x}\bar{y} \\ \bar{x}\bar{y}z + \bar{x}yz \end{array} \begin{array}{l} \text{SOP} \\ \text{CSOP} \\ \text{SOP} \\ \text{CSOP} \end{array}$$

x	y	z	f	SOP
0	0	0	0	
0	0	1	1	✓
0	1	0	0	
0	1	1	1	✓
1	0	0	0	
1	0	1	1	✓
1	1	0	0	
1	1	1	1	✓

$$f(x, y, z) = \bar{x}\bar{y}z + \bar{x}yz + \bar{x}\bar{y}\bar{z} + xyz$$

Sum of all minterms of 'f' which 'f' assumes 1 is the  
canonical SOP or disjunctive normal form:

$$f(x, y, z) = \sum m_1 + m_3 + m_5 + m_7$$
$$\sum m(1, 3, 5, 7)$$

⑥ Duality Theorem: Variables are not changed but

$$\cdot \leftrightarrow +$$

$$OR \leftrightarrow AND$$

$$NOT \leftrightarrow NOT$$

$$XOR \leftrightarrow XNOR$$

$$NAND \leftrightarrow NOR$$

$$0 \leftrightarrow 1$$

Eg.  $\bar{A}B + A\bar{B}$  apply duality theorem  
 $(\bar{A}+B) \cdot (\bar{A}+\bar{B})$

$$ABC + \bar{A}BC + A\bar{B}C$$

$$(A+B+C) \cdot (\bar{A}+B+C) \cdot (A+\bar{B}+C)$$

→ Self dual fng: A fng is said to be self dual if and only if its dual is equivalent to given fng

$$\begin{aligned} \text{Eg: } f(x,y,z) &= (XY + YZ + ZX) = (x+y)(y+z)(z+x) \\ &= (xy + xz + yz) (z^n) \\ &= yz + xy + xz + xz \\ &= yz + xy + xz \end{aligned}$$

• How many self dual fng possible with 1 variable

$$\rightarrow \text{Find total no. of combinations} = \bar{A}, A \quad (2) \cdot (2^n)$$

$$\rightarrow \text{Find total no. of Boolean fng (logically)} = 0, 1, A, \bar{A} \quad (2) \cdot (2^n)$$

$$\rightarrow \text{Total no. of self dual fng: } (2) = 2^{2^{n-1}}$$

$$\rightarrow n = \text{no. of variables}$$

• How many self dual fng with 2 variables

$$\rightarrow \text{Total no. of combinations} = 4 = 2^n$$

$$\rightarrow \text{Total no. of Boolean fng} = 2^{2^n} = 16$$

$$\rightarrow \text{Total no. of self dual fng} = 4$$

A	B	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	$f_6$	$f_7$	$f_8$	$f_9$	$f_{10}$	$f_{11}$	$f_{12}$	$f_{13}$	$f_{14}$	$f_{15}$	$f_{16}$
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	1	1	0	1	0	1	0	0	1	0	0	0
1	0	0	0	0	1	0	1	0	1	1	1	0	0	0	1	0	0
1	1	0	1	0	0	0	0	1	1	0	0	1	1	0	0	0	0

$$\begin{aligned} (f_1 - f_{16}) &= \cancel{x}, \cancel{X}, A, \bar{A}, B, \bar{B}, \cancel{AB}, \cancel{A}\bar{B}, \cancel{AB}, \\ &\quad \cancel{A}\bar{B}, \cancel{AXB}, \cancel{\bar{A}XB}, \cancel{\bar{A}X\bar{B}}, \cancel{AB} + \bar{A}\bar{B}, \bar{A}\bar{B} \cancel{+ AB} \\ &= A, \bar{A}, B, \bar{B} \end{aligned}$$

1 variable      2 variables

$$\rightarrow \text{Total combinations}$$

$$2^n$$

$$\rightarrow \text{Total Boolean fng}$$

$$2^{2^n}$$

$$\rightarrow \text{Total self dual fng}$$

$$2^{2^{n-1}}$$

$$2^n$$

⑦ K Map: Minimization using K map: graphical way to reduce a boolean expression.

Given fng:  $f(A, B) = \sum m(2, 3)$

A	B	$\bar{f}$	$\bar{B}$	B
0	0	0	0	1
1	0	1	0	1
2	1	0	1	1
3	1	1	1	1

$$= \bar{A} \bar{B} + A B$$

$$\therefore A(\bar{B} + B) = A$$

→ Karnaugh Map:

→ Boolean expression having  $n$  variables, no. of cells required in K Map =  $2^n$  cells

→ It is based on Gray code (Unit distance code)

→ Based on 3 types of input values (0, 1, don't care)

Eg: If map of 3 variables:

$f(A, B, C) =$

	$\overline{BC}$	$\overline{BC}$	$\overline{BC}$	$\overline{BC}$	$\overline{BC}$
A	00	01	11	10	
A	0	1	3	2	
A	1	5	7	6	

Pg: K map of 4 variables

$\{A, B, C, D\}$

$\bar{A}\bar{B}$	$CD$	$\bar{C}D$	$\bar{C}\bar{D}$	$CD$	$\bar{C}\bar{D}$
$AB$	00	01	11	11	10
$\bar{A}B$	00	01	11	11	10
$A\bar{B}$	00	01	11	11	10
$\bar{A}\bar{B}$	00	01	11	11	10
$\bar{A}B$	01	11	11	11	10
$A\bar{B}$	01	11	11	11	10
$AB$	11	11	11	11	10
$\bar{A}\bar{B}$	10	11	11	11	10
$\bar{A}B$	11	11	11	11	10
$A\bar{B}$	11	11	11	11	10
$AB$	11	11	11	11	10

$y f(A, B, C, D)$

if (C,D,A,B)

Eg: K map of 2 variables

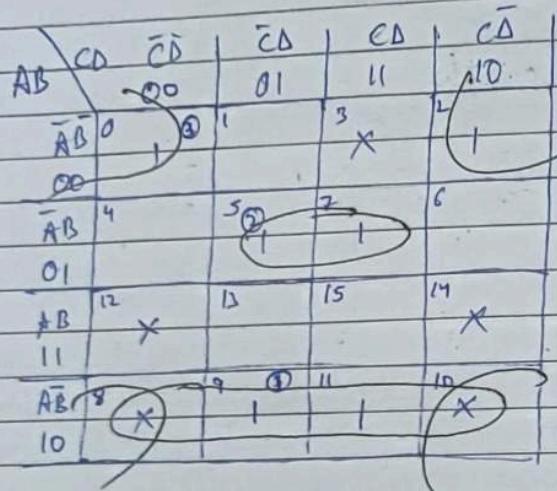
$(A, B)$

		B	B
		O	1
A	O	0	1
A	1	2	3

Eg: use of don't care (d)

$$F(A, B, C, D) = \sum m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$$

don't care are drawn on the K-map but only used when needed to have bigger groups, can be ignored



as its SOP (minterms) i.e value of 1's

$$f(A, B, C, D) = A\bar{B} + \bar{A}BC + \bar{C}\bar{D}$$

①  $A\bar{B}$

②  $\bar{A}BC$

③  $\bar{C}\bar{D}$

④ Redundant Group/Essential prime: atleast one '1' should be uncommon in intersecting group same for '0' for POS and SOP

⑤ Half adder: (addition of 2 bits)

	X	Y	Carry	Sum
	0	0	0	0
	0	1	0	1
	1	0	0	1
	1	1	1	0

2 I  $\rightarrow$  2 O

$$S(\text{minterm}) = \bar{x}y + xy = x \oplus y$$

$$C(\text{minterm}) = xy$$

⑥ Full adder: (addition of 3 bits)

	x	y	z	carry	sum
	0	0	0	0	0
	0	0	1	0	1
	0	1	0	0	1
	0	1	1	1	0
	1	0	0	0	1
	1	0	1	1	0
	1	1	0	1	0
	1	1	1	1	1

3 I  $\rightarrow$  2 O

$$S(\text{minterm}) = x \oplus y \oplus z$$

$$C(\text{minterm}) = \bar{x}y + yz + zx$$

$$= (\bar{x} \oplus y)z + xy$$

## ① Half Subtractor: (Subtraction of 2 bits)

$2I \rightarrow 2O$

x	y	barrow sum
0	0	0 0
0	1	1 1
1	0	0 1
1	1	0 0

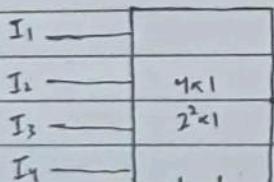
$$S(\text{minutum}) = x \oplus y$$

$$B(\text{minutum}) = \bar{x}y$$

## ② Multiplexer:

- It is a combination of circuit that has  $2^n$  input lines and a single output lines
- A multiplexer is an electronic switch that connects one input to output
- It is functionally complete i.e all boolean function can be realized using one multiplexer without any other gate

### → $4 \times 1$ Multiplexer:

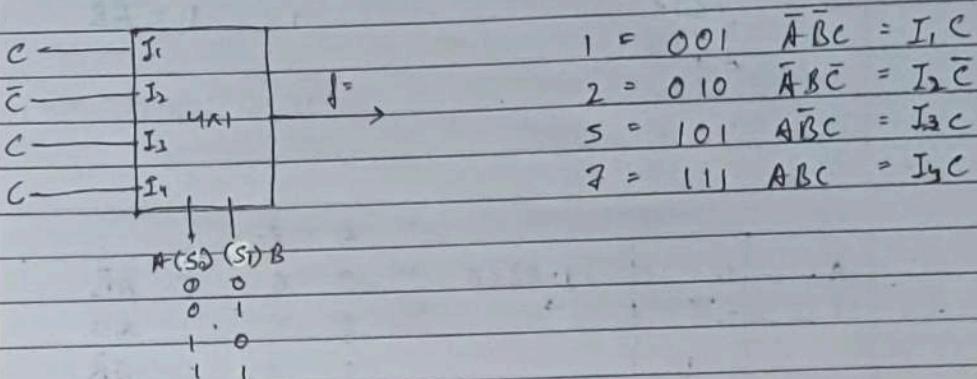


$4 = 2^2$  jittu parvar utni  
select lines

S<sub>0</sub> S<sub>1</sub>  
0 0  
0 1  
1 0  
1 1

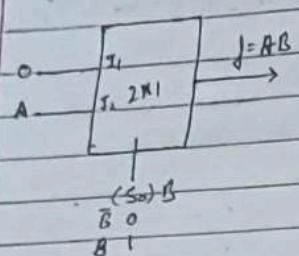
$$f = (\bar{S}_0 \bar{S}_1) I_1 + (\bar{S}_0 S_1) I_2 + (S_0 \bar{S}_1) I_3 + (S_0 S_1) I_4$$

$$\text{Pg: } f(A, B, C) = E(1, 2, 5, 7)$$



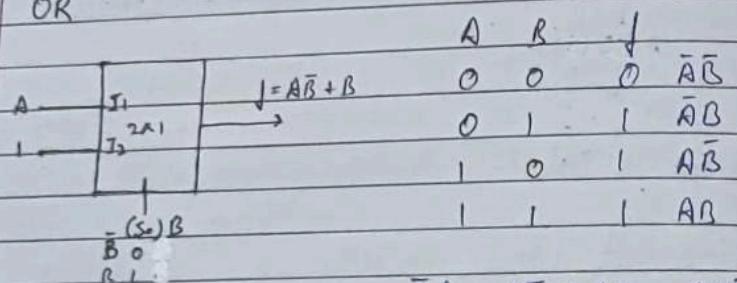
$$\begin{aligned} 1 &= 001 \quad \bar{A}\bar{B}C = I_1 C \\ 2 &= 010 \quad \bar{A}B\bar{C} = I_2 \bar{C} \\ 5 &= 101 \quad A\bar{B}C = I_3 C \\ 7 &= 111 \quad ABC = I_4 C \end{aligned}$$

→ AND



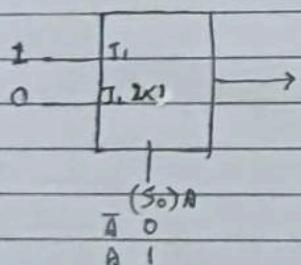
A	B	J
0	0	0
0	1	0
1	0	0
1	1	1 = AB

→ OR



$$\bar{A}B + A\bar{B} + AB = B(\bar{A} + A) + A\bar{B} \\ = B + A\bar{B}$$

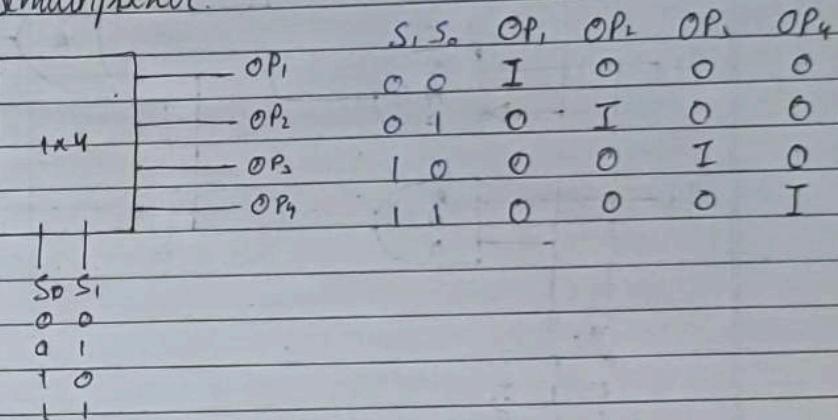
→ NOT



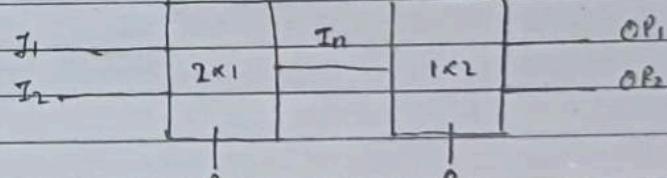
⑬ Demultiplexer:

- It is a combination of circuits that has  $2^n$  output lines and a single input line.
- Input bus ek current hai jisko direct karna hai yun hai koi ek output ko at a single time.

→ 1x4 Demultiplexer

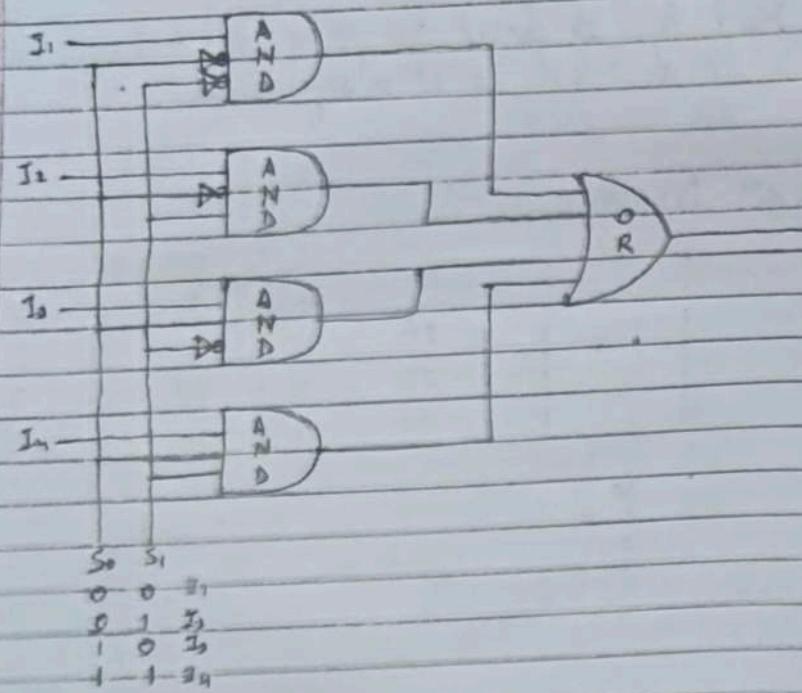


→ Convector (Multiplexer + Demultiplexer)



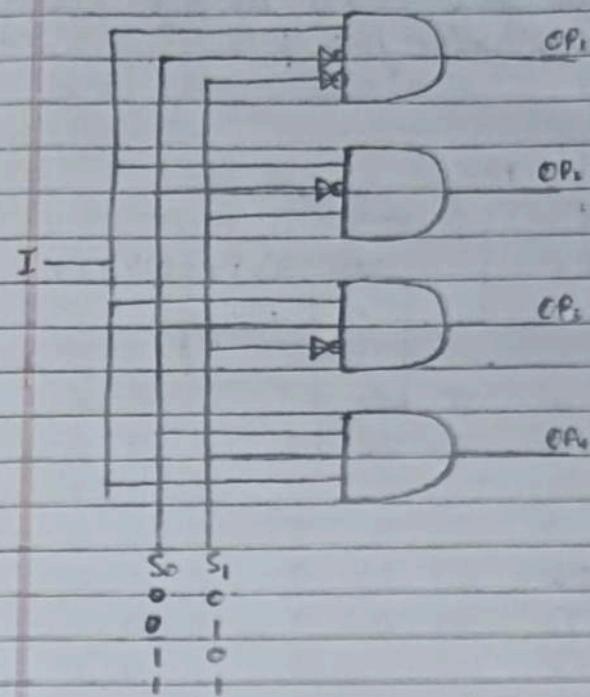
A	B	I <sub>1</sub>	I <sub>2</sub>	OP <sub>1</sub>	OP <sub>2</sub>
0	0	I <sub>1</sub>	I <sub>2</sub>	OP <sub>1</sub>	
0	1	I <sub>2</sub>	I <sub>1</sub>	OP <sub>1</sub>	
1	0	I <sub>1</sub>	I <sub>2</sub>	OP <sub>1</sub>	
1	1	I <sub>2</sub>	I <sub>1</sub>	OP <sub>2</sub>	

→ Working of Multiplexer ( $4 \times 1$ )



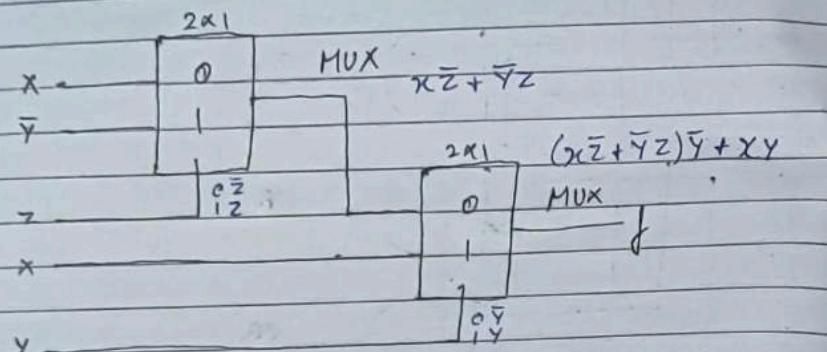
$$Z = \bar{S}_0 \bar{S}_1 I_0 + \bar{S}_0 S_1 I_3 + S_0 \bar{S}_1 I_2 + S_0 S_1 I_4$$

→ Working of Demultiplexer ( $1 \times 4$ )



### (14) Cascaded multiplexer

Q: Consider the circuit below. Which one of the following option correctly represents  $f(x, y, z)$ ?



cascading i.e passing

$$(X\bar{Z} + \bar{Y}Z)\bar{Y} + XY$$

$$X\bar{Y}\bar{Z} + \bar{Y}\bar{Y}Z + XY$$

$$X\bar{Y}\bar{Z} + \bar{Y}Z + XY$$

$$X(\bar{Y}\bar{Z} + Y) + \bar{Y}Z$$

$$X(Y + \bar{Y})(Y + \bar{Z}) + \bar{Y}Z$$

$$XY + X\bar{Z} + \bar{Y}Z$$

### (15) Decoders

- It is multiple input and multiple output device
- Decoder is a combinational circuit that converts n lines input into  $2^n$  lines output ( $n: 2^n$ )
- Application : converting binary code to

i) Octal  $(3 \times 8)$

$$8 = 2^3 \quad n \times 2^n$$

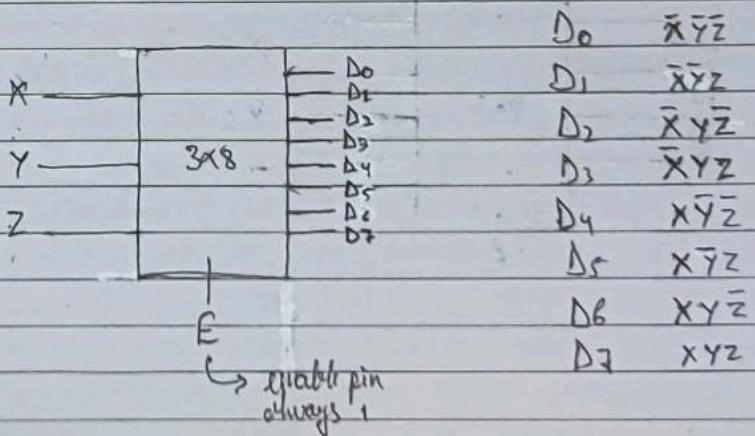
ii) Hexadecimal  $(4 \times 16)$

$$16 = 2^4$$

iii) Decimal  $(4 \times 10)$

$2^n$  is max less  
than it is allowed

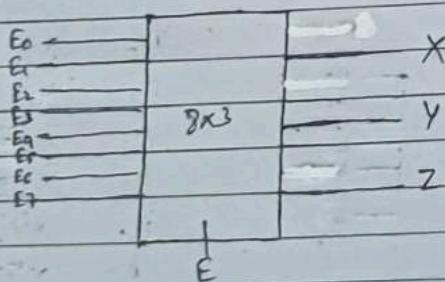
X	Y	Z	D <sub>7</sub>	D <sub>6</sub>	D <sub>5</sub>	D <sub>4</sub>	D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	E
0	0	0	0	0	0	0	0	0	0	1	1
0	0	1	0	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	0	0	1
0	1	1	0	0	0	0	1	0	0	0	1
1	0	0	0	0	0	1	0	0	0	0	1
1	0	1	0	0	1	0	0	0	0	0	1
1	1	0	0	1	0	0	0	0	0	0	1
1	1	1	1	0	0	0	0	0	0	0	1



## (16) Encoder

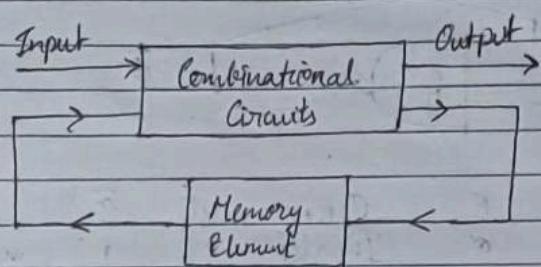
- It is also a multiple input and multiple output device
- Encoder is a combinational circuit that converts  $2^n$  input lines into  $n$  output lines. ( $2^n : n$ )
- Application i) Octal to binary ( $8 : 3$ )  
ii) Hexadecimal to binary ( $16 : 4$ )  
iii) Decimal to binary ( $10 : 4$ )

$E_7$	$E_6$	$E_5$	$E_4$	$E_3$	$E_2$	$E_1$	$E_0$	$X$	$Y$	$Z$	$E$
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1	1
0	0	0	0	0	1	0	0	0	1	0	1
0	0	0	0	1	0	0	0	0	1	1	1
0	0	0	0	1	0	0	0	0	1	0	1
0	0	0	1	0	0	0	0	1	0	0	1
0	0	1	0	0	0	0	0	1	0	0	1
0	0	1	0	0	0	0	0	1	0	1	1
0	1	0	0	0	0	0	0	1	0	1	1
0	1	0	0	0	0	0	0	1	1	0	1
1	0	0	0	0	0	0	0	1	1	1	1



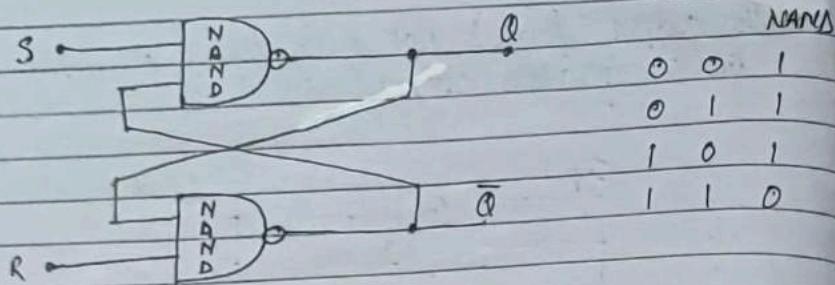
## (17) Sequential Circuit

- The output (final) not only depend on input but also on the previous output.



$\text{Input} + Q_n \rightarrow \text{Final Output } (Q_{n+1})$

(B) SR latch (NAND)

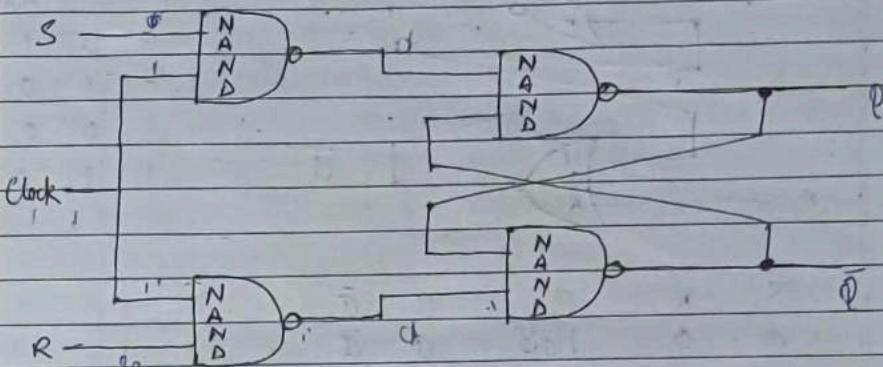


S	R	$Q_{n+1}$	$Q$	$\bar{Q}$	
0	0	Invalid	1	1	not possible
0	1	1	1	0	
1	0	0	0	1	
1	1	Hold	$Q$	$\bar{Q}$	

$$\boxed{\overline{I \cdot \bar{Q}} = \overline{I} + \overline{\bar{Q}} = \overline{0} + \overline{Q} = \overline{Q}}$$

$$\boxed{\overline{I \cdot Q} = \overline{I} + \overline{Q} = \overline{0} + \overline{0} = \overline{0}}$$

(19) SR flip flop (NAND)



clock	S	R	$Q_{n+1}$	$Q$	$\bar{Q}$
x	x	x	$x(Q_n)$	x	x
1	0	0	Hold	$Q$	$\bar{Q}$
1	0	1	0	0	1
1	1	0	1	1	0
1	1	1	Invalid	1	1

\* when using NOR Q is above S  
when using NAND Q is above R

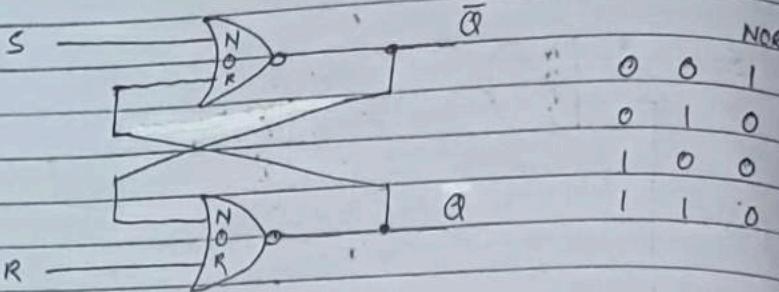
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## (20) SR latch (NOR)

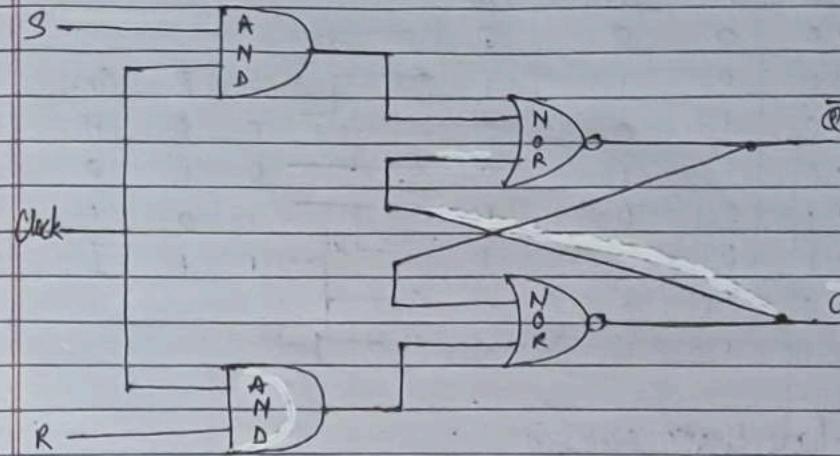


S	R	$Q_{n+1}$	Q	$\bar{Q}$
0	0	Hold	Q	$\bar{Q}$
0	1	0	0	1
1	0	1	1	0
1	1	Invalid	0	0

$$\overline{Q + \bar{Q}} = 1 \cdot \bar{Q} = \bar{Q}$$

$$\overline{Q + \bar{Q}} = 1 \cdot \bar{Q} = Q$$

## (21) SR flip flop (NOR)



clock	S	R	$Q_{n+1}$	Q	$\bar{Q}$
x	x	x	x	x	x
1	0	0	Hold	Q	$\bar{Q}$
1	0	1	0	0	1
1	1	0	1	1	0
1	1	1	Invalid	1	1

Ex: 3 inputs SR latch (flip flop) → characteristic Table

S	R	$Q_n$	$Q_{n+1}$	
→ 0	0	0	0	hold
↑↑ 0	0	1	1	
→ 0	1	0	0	Reset
→ 0	1	1	0	
↑↑ 1	0	0	1	
↑↑ 1	0	1	1	Set
1	1	0	X	
1	1	1	X	Invalid

→ characteristic equation:

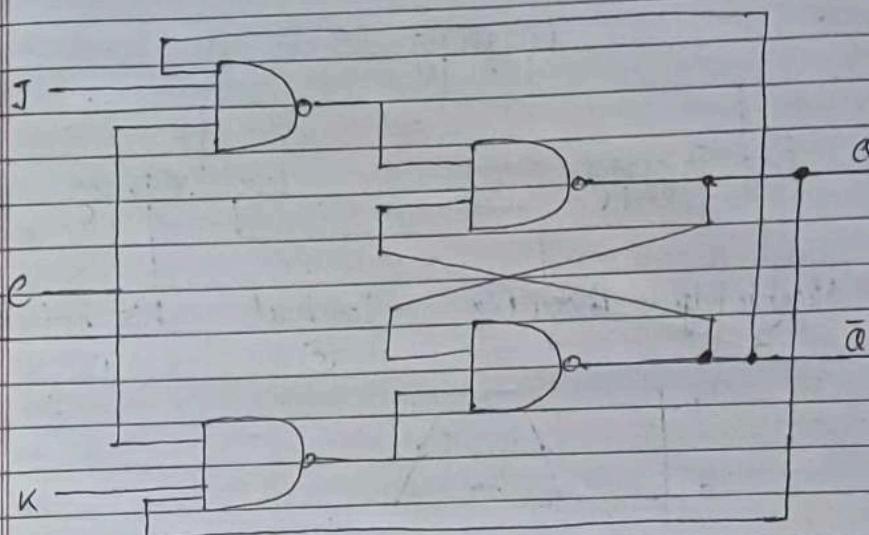
S	$R\bar{Q}$	$\bar{R}\bar{Q}$	$\bar{R}Q$	$RQ$	$R\bar{Q}$
$\bar{S}$	0	1	1	3	2
S	1	1	X	X	

$$Q_{n+1} = S + \bar{R}Q_n$$

→ excitation table

$Q_n$	$Q_{n+1}$	S	R
0	0 → 0	X (unpredictable)	
0	1 → 1	0	
1	0 ↑ 0	1	
1	1 ↑ X	0	

(22) JK flip flop



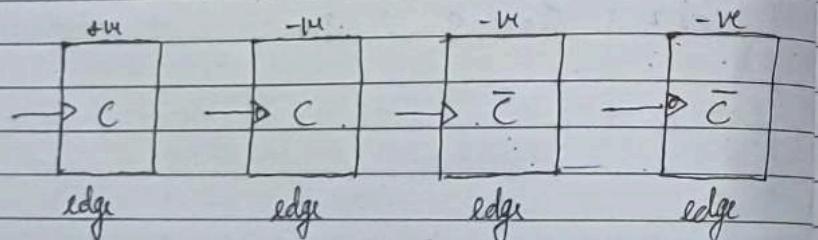
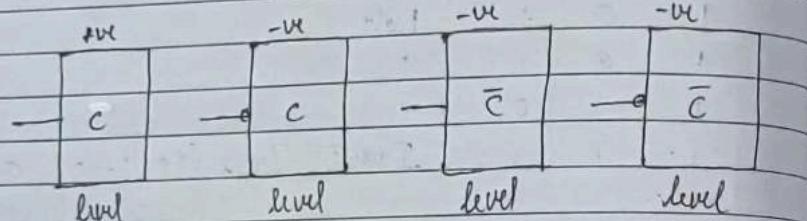
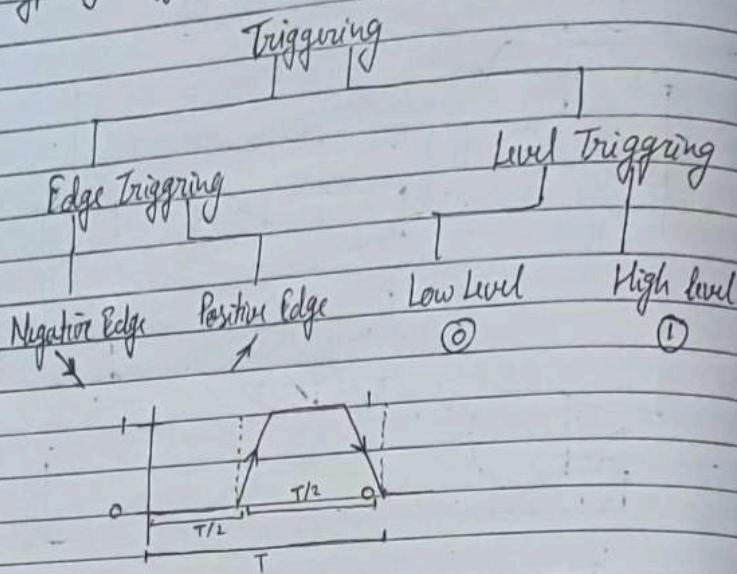
clock	J	K	$Q_{n+1}$
1	0	0	Hold
1	0	1	0
1	1	0	-1

Invalid Toggle (i.e. 1 → 0, 0 → 1)

case 1 :  $Q_n = 1 \rightarrow Q_{n+1} = 0$

case 2 :  $Q_n = 0 \rightarrow Q_{n+1} = 1$

② Types of triggering:



Eg 3 input JK latch (flip flop)

→ characteristic table

J	K	$Q_n$	$Q_{n+1}$	
→ 0	0	0	0	Hold
↔ 0	0	1	1	
→ 0	1	0	0	Reset
↔ 0	1	1	0	
→ 1	0	0	1	Set
↔ 1	0	1	1	
→ 1	1	0	1	Toggle
↔ 1	1	1	0	

→ characteristic equations

$\bar{J} K_n$	$\bar{K} \bar{Q}_n$	$\bar{K} Q_n$	$K \bar{Q}_n$	$K Q_n$
0	1	1	3	2
1	4	5	7	6
1	10	11	12	13
1	14	15	16	17

$$Q_{n+1} = \bar{K} Q_n + J \bar{Q}_n$$

→ excitation table

$Q_n$	$Q_{n+1}$	J	K
0	0	→ 0	X
0	1	↔ 1	X
1	0	↔ X	1
1	1	↔ X	0

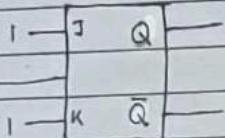
## (24) Race around condition

Condition 1: level triggered JK flip flop

Condition 2: When  $J=K=1$  (Toggle mode)

Condition 3:  $T_w \gg T_d$

- \* all 3 conditions must be fulfilled
- \* only level trigger clock



$T_w$  = clock time

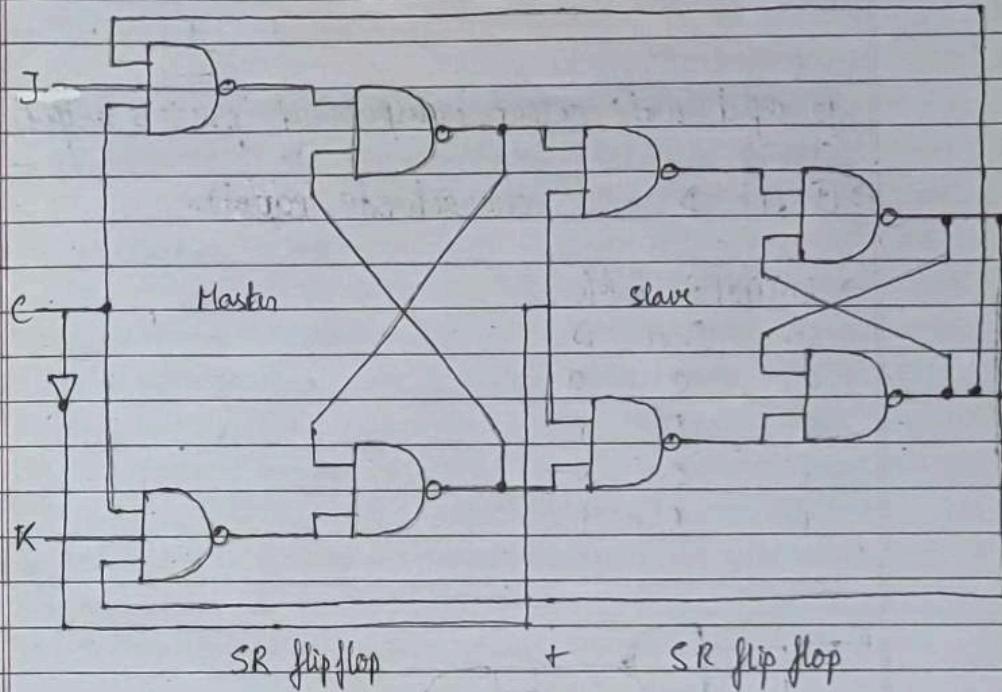
$T_d$  = processing time

jab processing time kam hai or clock time zyada toh  
ek clock time mai do bar valai (output) toggle hogayi  
i.e.  $1 \rightarrow 0 \rightarrow 1$  toh humko pata nahi chala  
ki kuch change hua bhi hai kinaki  
ek hi clock pulse mai do bar toggle hogayi toh  
hum decide ki nahi kar pa rahi ki actual output  
kya hai

## (25) Master slave JK flip flop

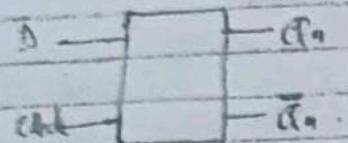
This has been developed to tackle the JK flip flops race around condition problem.

\* edge trigger can solve this problem but what can be solution using level trigger is solved by master slave



## ⑥ D flip flop (Transparent flip flop)

Block diagram: (Storage device)



characteristic table

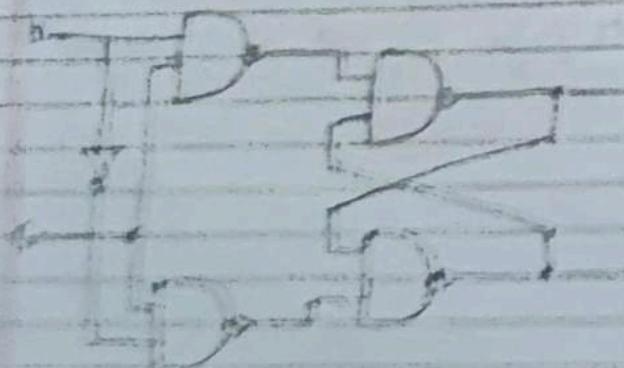
D	Q <sub>n</sub>	Q <sub>n+1</sub>
0	0	0
0	1	0
1	0	1
1	1	1

ie input with output i.e. input of previous output

$$\rightarrow Q_{n+1} = D \quad \text{characteristic equation}$$

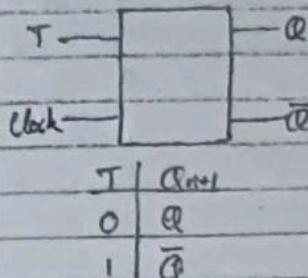
→ excitation table

Q <sub>n</sub>	Q <sub>n+1</sub>	D
0	0	0
0	1	1
1	0	1
1	1	0



## ⑦ T flip flop (Toggle flip flop)

Block diagram:



characteristic table

T	Q <sub>n</sub>	Q <sub>n+1</sub>
0	0	0
0	1	1
1	0	1
1	1	0

0 pulse, 1 pc toggle

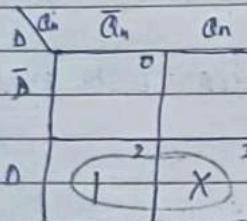
$$\rightarrow Q_{n+1} = T \oplus Q_n \quad \text{XOR : characteristic equation}$$

→ excitation table

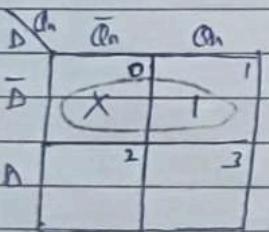
Q <sub>n</sub>	Q <sub>n+1</sub>	T
0	0	0
0	1	1
1	0	1
1	1	0

Q. Convert SR flip flop to D flip flop  
given required

D	$\bar{Q}_{n+1}$	S	R	On State SR
0	0	0	0	X
0	1	0	0	1
1	0	1	1	0
1	1	1	X	0



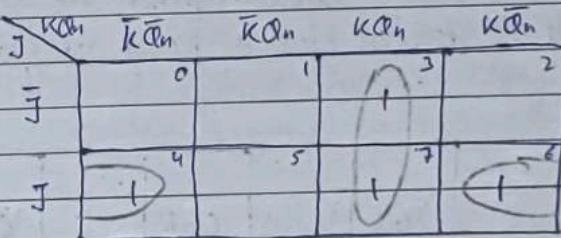
$$S = D$$



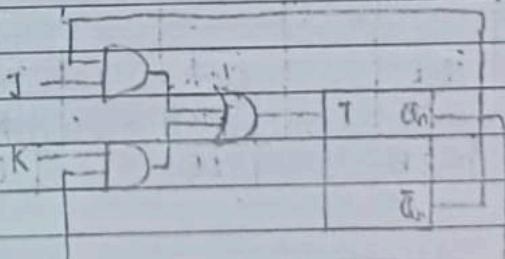
$$R = \bar{D}$$

Q. Convert T flip flop to JK flip flop  
given required

J	K	$\bar{Q}_n$	$\bar{Q}_{n+1}$	T	T excitation
0	0	0	0	0	0 0 0
0	0	1	1	0	0 0 0
0	1	0	0	0	1 0 1
0	1	1	0	1	1 0 1
1	0	0	1	1	1 1 0
1	0	1	1	0	1 1 0
1	1	0	1	1	1 1 0

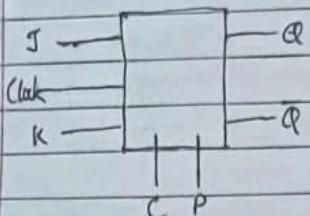


$$T = K\bar{Q}_n + J\bar{Q}_n : \text{Characteristic equation}$$



(28) Preset and clear, input in flip flop

pins used when making counters



clear to active : value = 0

preset to active : value = 1

High enable to active clear or preset i.e giving 1  
Low enable to deactivate clear or preset i.e giving 0

→ clear

Clock	Clock	Clock	Clock
EN	EN	EN	EN
HE	9 LE	LE	P LE

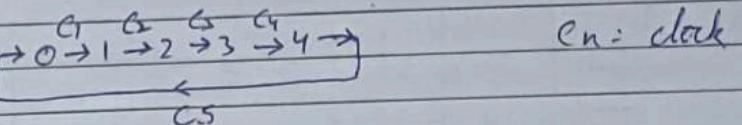
→ preset

Clock	Clock	Clock	Clock
EN	EN	EN	EN
HE	Y LE	LE	P LE

(29) Counters :

- It is a device that is used to store the number of times a particular event or process has occurred, often in relationship to a clock.
- A counter circuit is usually constructed of a number of flip flops connected in cascade.
- provide clock pulse to trigger
- random, ascending, descending order of counting
- now using edge trigger

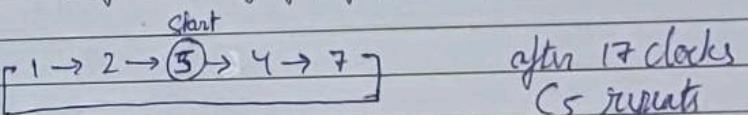
Q: How to find the mod value of a counter?



\* mod = no. of different states + largest value in sequence  
mod = 5 (0, 1, 2, 3, 4)

ii) after 21 clocks what will be output = ?

Q: How to find no. of flip flops required to design mod-n counter



after 17 = 7 ; so Mod 5 clocks required

$$2^n \geq M$$

n = no. of ff required

$$\text{or } 2^n \geq 5 \quad n=3 \text{ i.e. 3 ff required}$$

$$\text{ceil}(\log_2 M)$$

$$\text{for } M=272 : \text{ceil}(\log_2 272) = 8 \dots \approx 9 \text{ i.e. 9 ff required}$$

ii)  $[0 \rightarrow 4 \rightarrow 8 \rightarrow 1 \rightarrow 5]$

$$\text{Mod } 5 \Rightarrow \log_2 5 = 3 \quad X$$

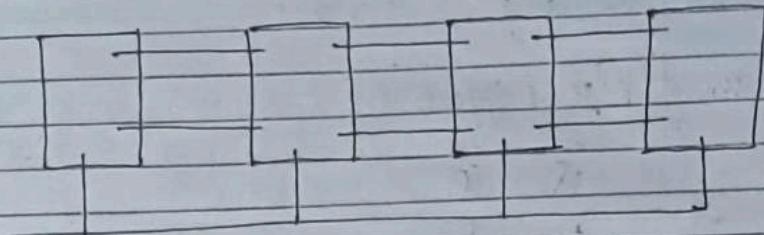
bit of 8 is 1000  
 $3ff$  i.e.  $2^3 = 8$  as 1  $ff$  stores 1 bit.  
 num combination is 7 i.e. 111

for 8: 4  $ff$  required = 4 ✓

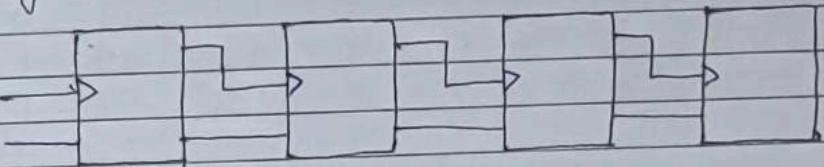
### ③ Synchronous and Asynchronous

→ Synchronous

$\curvearrowright 0-4-2-6-3$



- sabko individually clock signal mil raha magar eksath
- ring, T-R (Johnson Counter)
- Asynchronous
- Jumping sequence



- ek ka signal piche wali pe dependent hai
  - little slower than synchronous counter
  - low cost
  - easy design
  - ripple counters
- (follows only the sequential order)
- Only up or only down

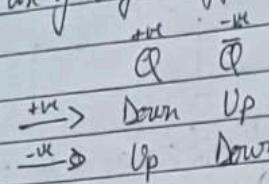
$\curvearrowright 0-1-2-3-4$

$\curvearrowright 4-3-2-1-0$

② Up/Down counter

up: ascending      1 2 3 4 5  
 down: descending    5 4 3 2 1

use of edge trigger



same: down  
 diff: up

Design of synchronous counter

$[0 \rightarrow 1 \rightarrow 3 \rightarrow 2]$

2 flip flops (D type) required  
 2 states : i) present  
 ii) next

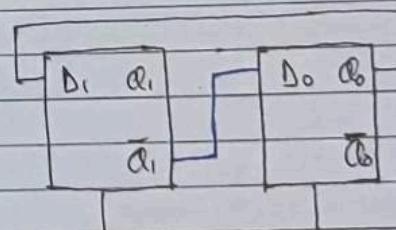
D	0 0	0
0 1	0 1	1
1 0	1 0	0
1 1	1 1	1

Present	Next	D flip flops
$Q_1\ Q_0$	$Q_1^+\ Q_0^+$	$D_1\ D_0$
0 0	0 1	0 1
0 1	1 1	1 1
1 0	0 0	0 0
1 1	1 0	1 0

$D_1$	$D_0$				
$Q_1$	$Q_0$	$Q_1$	$Q_0$	$Q_1$	$Q_0$
0	1	0	1	1	1
1	2	1	2	2	3
2	3	0	0	0	0

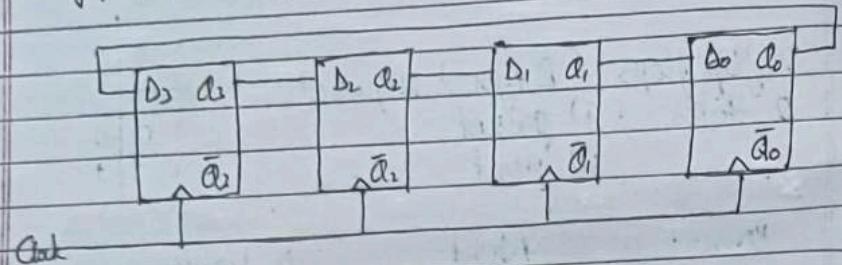
$$D_1 = Q_0$$

$$D_0 = \bar{Q}_1$$



### (32) Ring counter

4 flip flop ( $D$  type)



$$Q_3^{(n+1)} = D_3 = Q_0(n)$$

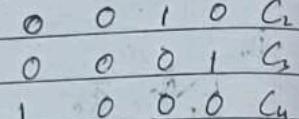
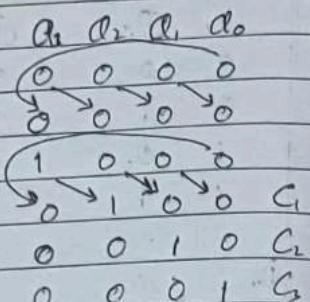
$$Q_2^{(n+1)} = D_2 = Q_3(n)$$

$$Q_1^{(n+1)} = D_1 = Q_2(n)$$

$$Q_0^{(n+1)} = D_0 = Q_1(n)$$

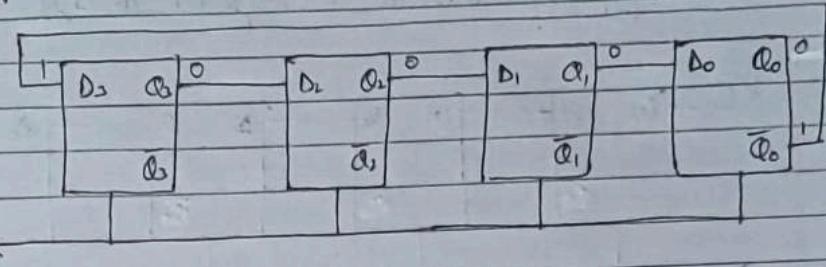
$$2^n \rightarrow n$$

$$16 \rightarrow 4$$



### (33) Johnson counter (Twisted ring counter)

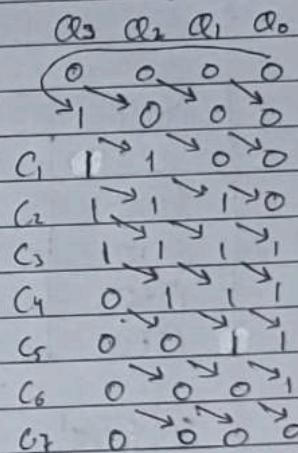
4 flip flops ( $D$  type)



$$Q_3^{(n+1)} = D_3 = \bar{Q}_0(n)$$

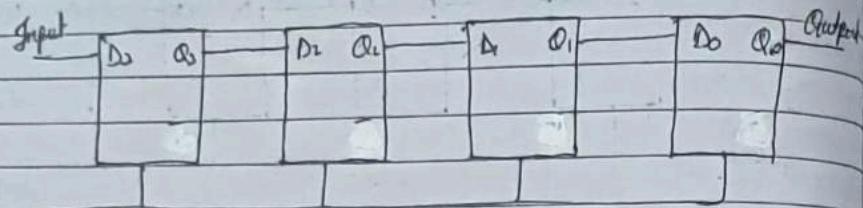
$$Q_2^{(n+1)} = D_2 = \bar{Q}_3(n)$$

$$2^n = 16 - 8 = 8$$



## (34) Shift Registers

- They are used to implement arithmetic operations
- e.g. left shift, right shift
- D type flip flop used in the register in DFF



Mode	Clocks needed for n-bit shift register		
	Loading	Reading	Total
SISO	n	n-1	2n-1
SIPD	n	0	n
PISO	1	n-1	n
PIPO	1	0	1

S: serial

I: input

P: parallel

O: output

Choti se badi = multiply  
Badi se choti = divide

## (35) Binary Number System

- 1) Conversions
- binary to decimal

a) 101<sub>2</sub>

$$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 4 + 0 + 1 \\ = 5_{10}$$

b) 1101.11<sub>2</sub>

$$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} \\ = 8 + 4 + 0 + 1 + 0.5 + 0.25 \\ = 13.75_{10}$$

c) 101101.101<sub>2</sub>

$$= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 + 1 \times 2^{-2} \\ = 32 + 0 + 8 + 4 + 0 + 1 + 0.5 + 0.125 \\ = 45.625$$

→ Decimal to Binary

a)  $25_{10} \rightarrow \underline{\hspace{2cm}}_2$

2	25	1
2	12	0
2	6	0
2	3	1
1		

$11001_2$

b)  $25.025_{10} \rightarrow \underline{\hspace{2cm}}_2$

$0.025 \times 2 = 0.05$

0

$0.05 \times 2 = 0.1$

0

$0.1 \times 2 = 0.2$

0

$0.2 \times 2 = 0.4$

0

$0.4 \times 2 = 0.8$

0

$0.8 \times 2 = 1.6$

1

$1.6 \times 2 = 3.2$

1

now 0.2 will repeat so we stop

$25.025_{10} = 11001.000011_2$

c)  $45.125_{10}$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
1		

$0.125 \times 2 = 0.25 = 0$

$0.25 \times 2 = 0.50 = 0$

$0.50 \times 2 = 1.0 = 1$

$101101.001_2$

d)  $67.125$

2	67	1
2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
1		

$0.0125 \times 2 = 0.025$

$0.025 \times 2 = 0.050$

$0.050 \times 2 = 0.10$

$0.10 \times 2 = 0.20$

$0.20 \times 2 = 0.40$

$0.40 \times 2 = 0.80$

$0.80 \times 2 = 1.60$

$1.60 \times 2 = 3.2$

$0.2 \times 2 = \text{Dropped}$

0.2 repeats

$67.125_{10} = 1000011.00000011$

→ Decimal to Octal

a)  $356_{10} \rightarrow 8$

$$\begin{array}{r} 356 \\ 8 | 44 \quad 4 \\ 8 | 5 \quad 4 \\ 0 \quad 5 \end{array}$$

$544_8$

b)  $512 \cdot 375_{10}$

$$\begin{array}{r} 412 \\ 8 | 51 \quad 4 \\ 8 | 6 \quad 3 \\ 0 \quad 6 \end{array}$$

$$0.375 \times 8 = 3$$

$634.3_8$

c)  $513 \cdot 5625_{10}$

$$\begin{array}{r} 513 \\ 8 | 64 \\ 8 | 3 \quad 0 \\ 8 | 1 \quad 0 \\ 0 \quad 1 \end{array}$$

$$0.5625 \times 8 = 44$$

$1601.44_8$

→ Octal to decimal

d)  $142_8$

$$= 1 \times 8^2 + 4 \times 8^1 + 2 \times 8^0$$

$$= 64 + 32 + 2$$

$$= 98_{10}$$

e)  $512 \cdot 125_8$

$$= 5 \times 8^2 + 1 \times 8^1 + 2 \times 8^0 + 1 \times 8^{-1} + 2 \times 8^{-2} + 5 \times 8^{-3}$$

$$= 320 + 8 + 2 + 1/8 + 2/64 + 5/512$$

$$= 330.15805_{10}$$

f)  $173 \cdot 062_8$

$$1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 0 + 6/64 + 2/512$$

$$123.09765_{10}$$

→ Octal to Binary

a)  $4513_8 \rightarrow 2$

4 5 1 3

100 101 001 011

$100101001011_2$

b)  $1004.241_8$

1 0 0 4 . 2 4 1

001 000 000 100 010 100 001 11

$100000100.010100001_2$

c)  $516.23_8$

5 1 6 - 2 3

101 001 110 010 011

$101001110.010011_2$

→ Binary to Octal

a)  $101101111_2 \rightarrow 8$

101 101 111  
5 5 7

$557_8$

b)  $10110111$

0+10 110 111  
2 6 7

$267_8$

c)  $1110101.11_2$

0011 110 101 - 11+0  
1 6 5 6

$165.6_8$

→ Decimal to Hexadecimal

a)

$$\begin{array}{r} 16 \mid 422 \\ 16 \quad 26 \quad 6 \\ 16 \quad \underline{1} \quad A \\ \quad \quad \quad 1 \end{array}$$

 $1A6_{16}$ 

b)

$422 \cdot 03125_{10}$

$0.03125 \times 16 = 0.5$

$0.5 \times 16 = 8.0$

 $1A6 \cdot 08_{16}$ 

c)

$$\begin{array}{r} 16 \mid 916 \\ 16 \quad 13 \quad 8 \\ \quad \quad \quad \uparrow \\ \quad \quad \quad D \end{array}$$

 $D8_{16}$ 

d)

$512 \cdot 0125_{10}$

$16 \mid 512$

$16 \mid 32 \quad 0$

$16 \mid 2 \quad 0$

2

 $200 \cdot 05_{16}$ 

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

3	3
4	4
5	5
6	6
7	7
8	8
9	9

10	A
11	B
12	C
13	D
14	E
15	F

16	10
17	11
18	12
19	13
20	14
21	15
22	16
23	17
24	18
25	19
26	1A
27	1B
28	1C
29	1D
30	1E
31	1F
32	20
33	21
34	22
35	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

11	11
10	10
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
1A	1A
1B	1B
1C	1C
1D	1D
1E	1E
1F	1F
20	20
21	21
22	22
23	23

→ Hexadecimal to Decimal

a)

$216_{16}$

$2 \times 16^2 + 1 \times 16^1 + 6 \times 16^0$

$512 + 16 + 6$

$534_{10}$

b)

$13A \cdot 2B16$

$13 \cdot 16 \cdot 2 \cdot 11$

$= 1 \times 16^2 + 3 \times 16^1 + 10 \times 16^0 + 2 / 16^1 + 11 / 16^2$

$= 256 + 48 + 160 + 0.125 + 0.6$

$= 464.725$

$= 464.812_{10}$

→ Hexadecimal to Binary

a)  $2A1_6$

$$\begin{array}{r} 2 \quad A \quad 1 \\ 2 \quad A \quad 1 \\ 0010 \quad 1010 \quad 0001 \\ \hline 1010100001_2 \end{array}$$

b)  $B_2 F_2 C_2 D_2$

$$\begin{array}{ccccccccc} B & 2 & F & 2 & C & 2 & D & 2 & 8 \\ 1100 & 0010 & 1111 & 0111 & 1101 & 1101 & 1101 & 1101 & \\ 11000010111011111011110_2 \end{array}$$

c)  $BEE_2 \cdot FACE_6$

$$\begin{array}{ccccccccc} B & E & -E & 2 & \cdot & F & A & -C & E \\ 1100 & 1110 & 1110 & 0010 & \cdot & 1111 & 1010 & 1101 & 1110 \\ 1100111011100010 \cdot 1111101011011110_2 \end{array}$$

→ Binary to Hexadecimal

a)  $101101011_2$

$$\begin{array}{r} 0001|0110|1011 \\ 1 \quad 6 \quad B \end{array}$$

$16B_{16}$

b)  $110101011-11$

$$\begin{array}{r} 0001|0101|1011 \cdot 11001 \\ 1 \quad A \quad B \cdot C \end{array}$$

$1AB-C_{10}$

c)  $1010110000 \cdot 01_2$

$$\begin{array}{r} 10010|1011|0000 \cdot 01+00 \\ 2 \quad C \cdot O \cdot 4 \end{array}$$

$2CO \cdot 4_{16}$

d)  $1100010101 \cdot 001_2$

$$\begin{array}{r} 10011|0001|0101 \cdot 0010 \\ 3 \quad 1 \quad 5 \cdot 2 \end{array}$$

$315 \cdot 2_{16}$

→ Octal to Hexadecimal

a)  $6704.053_8$

$6 \quad 7 \quad 0 \quad 4 \cdot 0 \quad 5 \quad 11 \quad 3$   
 $110 \quad 111 \quad 000 \quad 100 \cdot 000 \quad 101 \quad 011$

$110 | 1100 | 0100 \cdot 000 | 0101 | +000$   
D C 4 · 1 5 1

$DC4.151_8$

b)  $513.258$

$5 \quad 1 \quad 3 \cdot 2 \quad 5 \quad 1 \quad 1 \quad 1$

$101 \quad 001 \quad 011 \cdot 010 \quad 101$

~~$010 | 000 | 0011 \cdot 0010 |$~~

$000 | 0100 | 011 \cdot 010 | 01 + 00$   
1 4 B - 5 4

$14B.54_8$

Page \_\_\_\_\_

Date \_\_\_\_\_

Page \_\_\_\_\_

Date \_\_\_\_\_

→ Hexadecimal to Octal

a)  $A \cdot 2 C 316 \rightarrow \underline{\hspace{2cm}}_8$

$A \cdot 2 \quad C \quad 3 \quad 1 \quad 1 \quad 3$   
 $1010 \quad 0010 \quad 1100 \quad 0011$

$00 | 010 \cdot 001 | 011 | 000 | 001 | 1$   
1 2 - 1 3 0 3 · 1  
 $12 \cdot 1303_8$

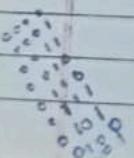
b)  $F \cdot D 0 \cdot E 716$

$F \cdot 1 \quad D \quad 0 \cdot E \quad 7$   
 $1111 \quad 0001 \quad 1101 \quad 0000 \cdot 1110 \quad 0111$

$00 | 111 | 000 | 110 | 0000 \cdot 111 | 001 | 11 + 0$   
1 7 0 7 2 0 - 7 1 6

$170720.716_8$

c)  $E D \cdot A \cdot A B E 16$



② Unsigned binary integers:

$$\begin{array}{r}
 & \text{+} & 7 & & 0 & 0 & 1 & 1 & 1 \\
 & \text{+} & 3 & & 0 & 0 & 0 & 1 & 1 \\
 & \text{+} & 1 & 0 & & 0 & 1 & 0 & 0
 \end{array}$$

$$\begin{array}{r} b) \quad +4 \\ \underline{+6} \\ +10 \end{array} \quad \begin{array}{r} 00100 \\ 00110 \\ 01010 \end{array}$$

$$\begin{array}{r} \text{c) } +7 & 00111 \\ +8 & \underline{01000} \\ +15 & 01111 \end{array}$$

man is 15 after that  
more decimal are used

8)

- True form = positive form without any -ve sign
  - 1's complement : -ve form having inverted binary  
i.e  $0 \rightarrow 1$  and  $1 \rightarrow 0$
  - 2's complement : by adding 1 to LSB of the 1's form  
of the number

$$\begin{array}{r}
 a) \quad \begin{array}{r} +7 \\ -3 \\ +4 \end{array} & \begin{array}{r} 00111 \\ 11100 \\ 00011 \end{array} & \begin{array}{l} \text{True form} \\ 1's \\ \dots \\ +1. \\ \hline 00100 \end{array} & \begin{array}{l} \text{True form} \end{array}
 \end{array}$$

\* in i's form when there's a carry, then it's get added to the LSR

$$\begin{array}{r} b) \quad -7 & 11000 & 18 \\ +3 & \underline{00011} & \text{True form} \\ -4 & 11011 & 18 \end{array}$$

$$\begin{array}{r}
 c) \quad -7 & 11000 & 18 \\
 -3 & \underline{11100} & 18 \\
 -10 & 10100 & \\
 & \hline & 1 \\
 & 10101 &
 \end{array}$$

Pg: Addition using 1's complement

a)  $\begin{array}{r} +4 \\ +2 \\ +6 \end{array} \quad \begin{array}{r} 100 \\ 010 \\ \hline 110 \end{array}$

b)  $\begin{array}{r} -4 \\ +2 \\ +2 \end{array} \quad \begin{array}{r} 011 \\ 010 \\ \hline 101 \end{array} \quad \begin{array}{l} 1's \\ \text{True} \end{array}$

c)  $\begin{array}{r} +4 \\ -2 \\ +2 \end{array} \quad \begin{array}{r} 100 \\ 101 \\ \hline 001 \end{array} \quad \begin{array}{l} \text{True} \\ 1's \\ 1 \\ \hline 010 \quad \text{True} \end{array}$

Page \_\_\_\_\_  
Date \_\_\_\_\_

Page \_\_\_\_\_  
Date \_\_\_\_\_

(37)

Rules for subtraction

add M to r's complement of N  
 $r_1 = \text{base}$   
 $M + (r^n - N)$

For subtraction we need to convert 3 bit into 4 bits

case 1: positive ( $M > N$ )

$$M - N \quad (M + r's \text{ comp of } N)$$

Ex (5-3)

$$\begin{array}{r} +5 \\ -3 \\ +2 \end{array} \quad \begin{array}{r} 0101 \\ 1100 \\ \hline 0001 \\ \hline 0010 \end{array} \quad \begin{array}{l} \text{True} \\ 1's \\ 1 \\ \hline 0010 \end{array}$$

case 2: negative

Here the sum doesn't produce any carry  
To obtain the result take there r's complement to the sum and place the negative sign in front of the result

ii)  $M < N$

Ex:  $\begin{array}{r} 3 - 5 \\ +3 \quad 0011 \\ -5 \quad 1010 \\ -2 \quad 1101 \end{array}$

Ex: 65 - 63 using 1's complement

2	65	1	2	63	1
2	32	0	2	31	1
2	16	0	2	15	1
2	8	0	2	7	1
2	4	0	2	3	1
2	2	0			1
	1				

65: 1000001 True

63: 0111111 True

SB

+65 0100001 True

-63 11000000 1s

00000001

+2 00000010 True

Ex: 65 - 63 using 2's complement

$$\begin{array}{r} +65 \quad 01000001 \quad \text{True} \\ -63 \quad 11000000 \quad 1s \\ \hline +1 \quad 1 \quad 1 \\ \hline 11000001 \quad 2's \end{array}$$

$$\begin{array}{r} 01000001 \\ 11000001 \\ \hline 00000010 \end{array}$$

\* in 2's complement the extra 1 after addition (carry) is dropped as 2's complement is created by adding 1 to 1's complement

Ex:  $X = 1010100, Y = 1000011$  ( $X - Y$ ) using 1's

$$0111100 \xrightarrow{1s}$$

$$1010100 \quad 0111101 \xrightarrow{2's}$$

0111100

0010000

1

0010001  $(17)_{10}$

28

1010100

0111101

0010001  $(17)_{10}$

drop

+9 : 00001001 True  
-9 : 10001001 mag  
11110110 1's  
11110111 2's

### (3) Signed and Unsigned bit

→ Signed bit

+9 : 00001001

-9 : 10001001

→ Unsigned bit

Unsigned numbers ( $0 \rightarrow (2^n - 1)$ )

$n=8$        $0 - (256 - 1)$   
 $0 - 255$       Binary  
 $00 - FF$       Hexadecimal

Pg -6      11111010      2's  
+13      00001101      True  
+7      00000111

+6      00000110      True

-6      11111001      1's

-6      +1      2's  
11111010

-6      11111010      1's  
+13      00001101      True  
+7      00000111

Page \_\_\_\_\_  
Date \_\_\_\_\_

Page \_\_\_\_\_  
Date \_\_\_\_\_

Eg: -6      11111001      1's  
+13      00001101      True  
+7      00000110  
+1  
00000111      True

Eg: Sign Magnitude representation:

-3 : 101111  
-7 : 111111

Eg: 1's Complement Representation

-3 : 1100  
-7 : 1000

Eg: 2's Complement Representation

-3 : 1101  
-7 : 1001

Eg: -6      -6  
+13      0110 (6)      1's Comp  
+7      1110 (-6)      1001      2's Comp  
+1  
1010  
10000110      11111001      11111001  
+1  
11111010

Rg:

-6	10000110
-13	10001101

$$M > N$$

~~10000110~~

18:1111001

28: 1111001

11111010

-G 11111010 28  
-D 11110011 28  
① 11101101 28

-13 11110011 28  
④ 11121121 28

① 11101101 28

七

19 : 00010011 True

-19 : 10010011 sign

11101100 13

11101101 28

Page \_\_\_\_\_

Page \_\_\_\_\_  
Date \_\_\_\_\_

Eg Addition using 1's complement  
True 13

True

13

$$\begin{array}{r}
 a) \\
 \begin{array}{r}
 \begin{array}{r} +4 \\ +2 \\ +6 \end{array} & \begin{array}{r} 0100 \\ 0010 \\ \hline 1000 \end{array} & \begin{array}{r} 111 \\ \hline 1101 \end{array} \\
 \hline & & 1
 \end{array}
 \end{array}$$

1001 is 6

$$\begin{array}{r} \text{b) } -4 & 1100 \text{ (d4)} & 1011 & \text{in d4} \\ +2 & 0010 & \underline{0010} & \\ \hline =2 & & 1101 & \end{array}$$

$$\begin{array}{r} \text{c) } +4 \\ \quad -2 \\ \hline +2 \end{array} \qquad \begin{array}{r} 0100 \\ 1010 \\ \hline 1101 \end{array} \qquad \begin{array}{r} 0100 \\ 0001 \\ \hline 0010 \end{array}$$

	-4	-2	
d)	-4	1100	1010
	-2	1011	1101
	-6		

1000	1000
100	100
100	100
100	100
100	100

Eg)

-4

-2

using 2's complement

$$\begin{array}{r}
 -4 \\
 -2 \\
 \hline
 \text{True (Sign)} & 1100 & 1010 \\
 1's & 1011 & 1101 \\
 2's & 1100 & 1110
 \end{array}$$

$$\begin{array}{r}
 -4 \\
 -2 \\
 -6 \\
 \hline
 1100 \\
 1110 \\
 \text{drop}
 \end{array}$$

Eg:

+4

-2

+2

$$\begin{array}{r}
 +4 \\
 -2 \\
 \hline
 \text{True (Sign)} & 0100 & 1010 \\
 1's & 1101 \\
 2's & 1110
 \end{array}$$

$$\begin{array}{r}
 +4 \\
 -2 \\
 +2 \\
 \hline
 0100 \\
 1110 \\
 \text{drop}
 \end{array}$$

Page \_\_\_\_\_

Date \_\_\_\_\_

Page \_\_\_\_\_

Date \_\_\_\_\_

Eg

-4

+2

-2

-4

+2

True(sign)

1's

2's

1100

0010

1011

1100

-4

+2

0010

1110

2's of (-2)

Eg:

+4

+2

+6

+4

+2

0100

0010

+4

+2

+6

0100

0010

0110

Eg: Subtraction using 2's complement

$$\begin{array}{r}
 \text{a) } +4 \\
 -2 \\
 +2 \\
 \hline
 \text{True} & 0100 \\
 1's & 1101 \\
 2's & \underline{1110}
 \end{array}$$

$$\begin{array}{r}
 +4 \\
 -2 \\
 +2 \\
 \hline
 \text{True} & 0100 \\
 1's & 1101 \\
 2's & \underline{1110}
 \end{array}$$

Overflow

$$\begin{array}{r}
 \text{b) } +4 \\
 -2 \\
 +2 \\
 \hline
 \text{True} & 0100 \\
 1's & 1101 \\
 2's & \underline{1110}
 \end{array}$$

$$\begin{array}{r}
 +4 \\
 -2 \\
 \hline
 0010
 \end{array}$$

The reason BCD is full as often i.e 10, 11, 12... gets two digits binary

③

### BCD (Binary Coded Decimal)

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Highest 4bit :  $(1111)_2 = (15)_{10}$

$$\begin{array}{r}
 \text{a) } 4+5=9 \\
 0100 \\
 +0101 \\
 \hline
 1001
 \end{array}$$

$$\begin{array}{r}
 \text{b) } 4+8=12 \\
 0100 \\
 +1000 \\
 \hline
 1100
 \end{array}$$

i.e  $12 > 9$  ✓  
 (12) This no. is outside  
 BCD limit so we need to  
 add the binary equivalent  
 of 6 to the resultant

$$\begin{array}{r}
 1100 \\
 +0110 \\
 \hline
 10010
 \end{array}$$

1      2

0001 0010 BCD representation of 12

c)  $8+9 = 1000$

$$\begin{array}{r} 1001 \\ + 1001 \\ \hline 10001 \end{array}$$

$$\begin{array}{r} 10001 \\ + 00110 \\ \hline 10111 \end{array} \quad \text{adding 6: } 00010111 \quad \text{BCD representation for } 17$$

$0001\ 0001 = 1\ 7 = 17$

d) BCD coded addition:

$$\begin{array}{r} 184 \\ + 576 \\ \hline \end{array}$$

$$\begin{array}{r} 1\ 0001 \\ 5\ 0101 \\ \hline 0110 \\ 0111 \end{array} \quad \begin{array}{r} 8\ 1000 \\ 7\ 0111 \\ \hline 1111 \\ 0110 \\ 0110 \end{array} \quad \begin{array}{r} 4\ 0100 \\ 6\ 0110 \\ \hline 1010 \\ + 0110 \\ \hline 0000 \end{array}$$

$(0111|0110|0000)$  BCD

$1+5=6 < 9$  so no need to add (+6) to it

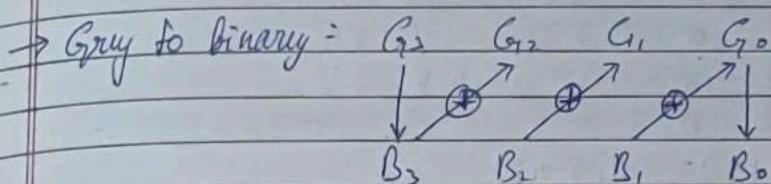
$8+7=15 > 9$

$4+6=10 > 9$

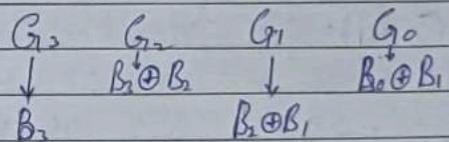
\* Hexadecimal =  $2^4=16$  so 4 bits ( $A=10$ ) 1010  
 Binary :  $2^1=2$  so 1 bit ( $0,1$ ) 0,1  
 Octal :  $2^3=8$  so 3 bits ( $7:$ ) 111

#### ⑩ Gray Code

Gray	Decimal	Binary
0000	0	
0001	1	
0011	2	
0010	3	
0110	4	



→ Binary to Gray:  $B_2 \oplus B_1 \oplus B_0$



Eg: Convert GC: 1010 to BC

$$B_3 = 1$$

$$B_2 = 1 \oplus 0 = 1$$

$$B_1 = 1 \oplus 1 = 0$$

$$B_0 = 0 \oplus 0 = 0 \quad (1100)_2$$

## ① Boolean algebra

$$n+0 = n$$

$$n+\bar{n} = 1$$

$$n+n = n$$

$$n+1 = 1$$

$$\bar{n} = n$$

$$n \cdot 1 = n$$

$$n \cdot \bar{n} = 0$$

$$n \cdot n = n$$

$$n \cdot 0 = 0$$

$$n(y+z) = ny + nz$$

$$(n+y) = \bar{n}\bar{y}$$

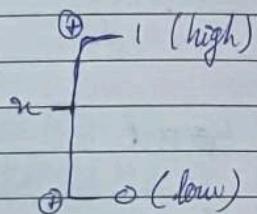
$$n+ny = n$$

$$n+yz = (n+y) \cdot (n+z)$$

$$(\bar{n}y) = \bar{n}y$$

$$n(n+y) = n$$

- \* add gives higher result
- \* multiply gives lower result



## ② Canonical & Standard form

Canonical form: when the terms of the expression has each term of the given function.

$$F(ABC) = \underline{1} + \underline{3} + \underline{5} \quad \text{ie } ABC$$

$$F(ABCD) = \underline{4} + \underline{4} + \underline{4} \quad \text{ie } ABCD$$

$$\begin{aligned} a) \quad F &= (\bar{A}+B) \cdot (A+\bar{B}) \\ &= \bar{A}A + \bar{A}\bar{B} + AB + B\bar{B} \\ &\Rightarrow \bar{A}\bar{B} + AB \end{aligned}$$

Eg: Represent the Boolean fcn as SOP (sum of minterms)

$$\begin{aligned} F &= A + \bar{B}C \\ &\Rightarrow A(B+\bar{C}) + (A+\bar{A})BC \\ &\Rightarrow (AB + A\bar{B})(C + \bar{C}) + ABC + \bar{ABC} \\ &\Rightarrow ABC + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{ABC} \\ &\Rightarrow ABC + \bar{A}\bar{B}C + ABC + \bar{ABC} + A\bar{B}\bar{C} + \bar{A}\bar{B}C \end{aligned}$$

7 4 6 5 4 1

$m_7 \quad m_6 \quad m_5 \quad m_4 \quad m_1$

$$F(ABC) = \sum (1, 4, 5, 6, 7) : \text{SOP}$$

Eg

$$F(XYZ) = XY + \bar{X}Z$$

$$x = ny, y = \bar{n}, z = z$$

$$n+yz = (n+y)(n+z)$$

distribution property

(POS)

$$F = ny + \bar{n}z$$

$$= (ny + \bar{n})(ny + z)$$

$$= (\bar{n} + n) \cdot (\bar{n} + y) \cdot (n + z) (z + y)$$

$$= (\bar{n} + n) \cdot (\bar{n} + y) \cdot (\bar{n} + z) (z + y)$$

$$= (\bar{n} + y + z \cdot \bar{z}) \cdot (\bar{n} + z + y \cdot \bar{y}) \cdot (\bar{n}\bar{n} + y + z)$$

$$= (\bar{n} + y) + z\bar{z} \cdot ((n + z) + y\bar{y}) \cdot (\bar{n}\bar{n} + (y + z))$$

$$= (\bar{n} + y + z) \cdot (\bar{n} + y + \bar{z}) (n + y + z) (n + \bar{y} + z)$$

$$(n + y + z) \cdot (\bar{n} + y + z)$$

$$= M_0 M_2 M_4 M_5$$

a)

(SOP)

$$\text{Eg: } F(nyz) = (ny + z)(y + nz)$$

$$= ny + nz + \bar{y}z + nz$$

$$= ny + z\bar{n} + \bar{y}z$$

$$= (nyz + ny\bar{z} + ny\bar{z} + \bar{y}z + nyz + \bar{y}z)$$

$$= nyz + ny\bar{z} + \bar{y}z + \bar{y}z$$

$$= nyz + ny\bar{z} + \bar{y}z + \bar{y}z$$

$$= M_2 M_6 M_5 M_3$$

Page \_\_\_\_\_

Date \_\_\_\_\_

Page \_\_\_\_\_

Date \_\_\_\_\_

$$\begin{aligned} \text{Pg: } F(nyz) &= (ny + z)(y + z)(y + \bar{z})(y + z) \\ &= (\bar{y} + y)(y + z)(y + \bar{z})(y + z)(y + \bar{z}) \\ &= (\bar{y} + n + y)(\bar{y} + z + y\bar{y})(y + z + \bar{z})(y + z + \bar{z}) \\ &= (\bar{y} + n + z)(\bar{y} + z + y\bar{y})(y + z + \bar{z})(y + z + \bar{z}) \end{aligned}$$

$$\begin{aligned} &= (\bar{n} + y + z)(\bar{n} + \bar{y} + z)(\bar{n} + y + z)(\bar{n} + y + \bar{z}) \\ &= (0 \ 0 \ 0)(0 \ 1 \ 0)(1 \ 0 \ 0)(0 \ 0 \ 1) \\ &\quad \begin{matrix} 0 & 2 & 4 & 1 \\ M_0 & M_2 & M_4 & M_1 \end{matrix} \end{aligned}$$

$$\begin{aligned} \text{Pg: } F &= \bar{A}B(\bar{\Delta} + \bar{CD}) + B(A + \bar{ACD}) \\ &= \bar{A}B(\bar{\Delta} + \bar{C}\bar{D}) + B(A + \bar{ACD}) \\ &= B(\bar{A}\bar{\Delta} + \bar{A}\bar{C}\bar{D} + A + \bar{ACD}) \\ &= B(\bar{A}(\bar{\Delta} + \bar{C}\bar{D} + CD) + A) \\ &= B(\bar{A}(\bar{\Delta} + \bar{A}(\bar{C} + C) + A) + A) \\ &= B(\bar{A}(\bar{A}(\bar{\Delta} + D) + A) + A) \\ &= B(\bar{A} + A) \\ &= B \end{aligned}$$

or

$$\begin{aligned} &= \bar{A}B\bar{\Delta} + \bar{A}B\bar{C}\bar{D} + AB + \bar{ABC}\bar{D} \\ &= \bar{ABC}(1) + AB + \bar{ABC}\bar{D} \\ &= \bar{AB} + AB \\ &= B(\bar{A} + A) \\ &= B \end{aligned}$$

$$F = ny + (\bar{n} + \bar{y}) wz$$

$$F = ny + (\bar{n}y) * wz$$

$$= A + \bar{A}B$$

$$= (A + B)(\bar{A} + \bar{B})$$

$$= (ny + wz) \cancel{(1)}$$

$$= ny + wz$$

Eg:  $F(A, B) = \sum(0, 2, 3)$

$$= 00, 10, 11$$

$$= \bar{A}\bar{B} + A\bar{B} + AB$$

$$= \bar{A}\bar{B} + A$$

$$= (A + \bar{A})(A + \bar{B})$$

$$= (A + \bar{B})$$

Eg:  $F = (A + B)(A + \bar{B})(\bar{A} + B)(\bar{A} + \bar{B})$

$$= (A \cdot A + A\bar{B} + AB + B\bar{B})(\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}B + B\bar{B})$$

$$= (A + AB)(\bar{A} + BB)$$

$$= (B)(0)$$

$$= 0$$

A	B	C	SOP
0	0	0	$\bar{A}\bar{B}\bar{C}$
1	0	1	$\bar{A}\bar{B}C$
1	0	0	$\bar{A}B\bar{C}$
0	1	1	$\bar{A}BC$
1	0	0	$A\bar{B}\bar{C}$
1	0	1	$A\bar{B}C$
1	1	0	$AB\bar{C}$
1	1	1	$ABC$

POS
$A + B + C$
$A + B + \bar{C}$
$A + \bar{B} + C$
$A + \bar{B} + \bar{C}$
$\bar{A} + B + C$
$\bar{A} + B + \bar{C}$
$\bar{A} + \bar{B} + C$
$\bar{A} + \bar{B} + \bar{C}$

### (43) Logic gates

1) AND

2) OR

3) NOT

4) NOR

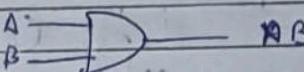
5) NOR

6) NAND

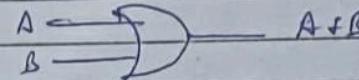
7) XNOR

] universal gate

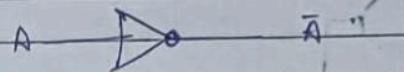
1) AND



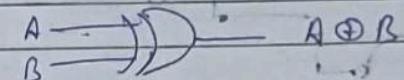
2) OR



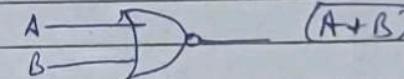
3) NOT



4) NOR



5) NOR



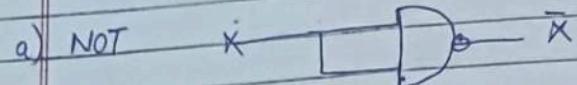
6) NAND



\* Jaise change hota hai vo hake ruse all should be taken.

Page \_\_\_\_\_  
Date \_\_\_\_\_

### (44) NAND as universal gate



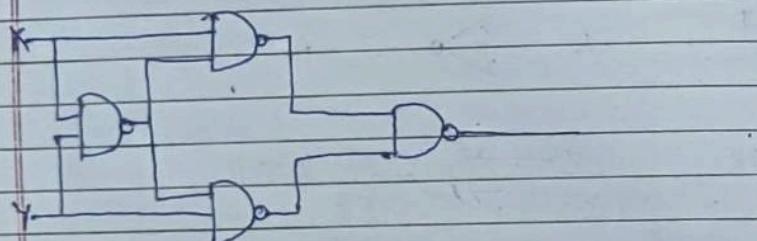
b) AND

$$F(n, y, z) = \sum(2, 3, 4, 5)$$

c) OR

$$(x + y) = (\bar{x} \bar{y})$$

d) XOR



(45) K-map

a)  $F(n, y, z) = \sum(2, 3, 4, 5)$

$\bar{n} \bar{y} \bar{z}$	00	01	11	10
$\bar{n} \bar{y} z$	0	1	1	1
$n \bar{y} \bar{z}$	1	1	0	1
$n y \bar{z}$	1	0	1	0

$n\bar{y} + \bar{n}y$

b)  $F(n, y, z) = \sum(3, 4, 6, 7)$

$\bar{n} \bar{y} \bar{z}$	00	01	11	10
$\bar{n} \bar{y} z$	0	0	1	1
$n \bar{y} \bar{z}$	1	1	0	1
$n y \bar{z}$	1	0	1	0

$n\bar{y} + \bar{y}\bar{z} + y\bar{z}$   
 $n\bar{y} + \bar{z}(y + \bar{y})$

c)  $F(w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

$w \bar{x}$	$y \bar{z}$	$\bar{y} \bar{z}$	$\bar{y} z$	$y \bar{z}$	$y z$
00	00	01	11	10	11
$\bar{w} x$	0	1	1	1	1
01	1	1	1	0	1
$\bar{w} x$	1	1	0	1	1
11	1	0	1	1	0
$w x$	0	1	1	1	0
10	1	1	0	1	1
$\bar{w} x$	1	0	1	1	1

$\bar{y} + \bar{w}y\bar{z} + w\bar{x}\bar{z}$

$$F(A \bar{B} C \bar{D} E) = (0, 2, 4, 7, 8, 10, 12, 16, 18, 20, 23, 24, 28, 26, 27, 29)$$

Page \_\_\_\_\_  
Date \_\_\_\_\_

		BC		DE		A=0			
		00	01	10	11	00	01	10	11
		1	4	5	7	1	4	5	7
		12	13	15	14	12	13	15	14
		8	9	11	10	8	9	11	10

		BC		DE		A=1			
		00	01	10	11	00	01	10	11
		16	17	19	18	16	17	19	18
		20	21	23	22	20	21	23	22
		25	27	29	31	25	27	29	31
		24	26	28	30	24	26	28	30

$$\bar{D}\bar{E} + \bar{C}\bar{E} + A\bar{B}\bar{C} + \bar{B}C\bar{D}$$

$$e) F(A \bar{B} C \bar{D} E) = (0, 1, 2, 4, 5, 6, 19, 13, 14, 18, 21, 22, 24, 26, 28, 29, 20)$$

		BC		DE		A=0			
		00	01	10	11	00	01	10	11
		1	4	5	7	1	4	5	7
		12	13	15	14	12	13	15	14
		8	9	11	10	8	9	11	10

		BC		DE		A=1			
		00	01	10	11	00	01	10	11
		16	17	19	18	16	17	19	18
		20	21	23	22	20	21	23	22
		25	27	29	31	25	27	29	31
		24	26	28	30	24	26	28	30

$$( \bar{A} \bar{B} \bar{C} \bar{D} \bar{E} + \bar{A} \bar{B} \bar{C} \bar{D} ) + ( \bar{A} \bar{B} \bar{C} \bar{E} + \bar{A} \bar{B} \bar{C} \bar{D} ) + ( \bar{A} \bar{B} \bar{C} \bar{D} \bar{E} + \bar{A} \bar{B} \bar{C} \bar{D} ) + ( \bar{A} \bar{B} \bar{C} \bar{D} \bar{E} + \bar{A} \bar{B} \bar{C} \bar{D} )$$

$$\begin{aligned} F(AB) &= \bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB \\ &= \bar{A} + A \\ &= 1 \end{aligned}$$

Page \_\_\_\_\_  
Date \_\_\_\_\_

Page \_\_\_\_\_  
Date \_\_\_\_\_

\* Eg: Complement function for SOP:

$$f(ABC) = ABC + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C$$

What is the POS expression for this function?

$$f(ABC) = \sum(7, 0, 2, 3)$$

0	1		
1	0	✓	
2	1		
3	1		
4	0	✓	$\bar{A} + B + C$
5	0	✓	
6	0	✓	
7	1		

$$f(ABC) = \prod(1, 4, 5, 6)$$

$$F_1 = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + A\bar{B}\bar{C} = (0, 2, 3, 5, 6)$$

$$\begin{aligned} F_1 &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + A\bar{B}\bar{C} \\ &= (A+B+C) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+\bar{B}+C) \\ &= 0 \quad 2 \quad 3 \quad 5 \quad 6 \end{aligned}$$

POS we consider 0 for counting

SOP we consider 1 for counting

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + A\bar{B}\bar{C}$$

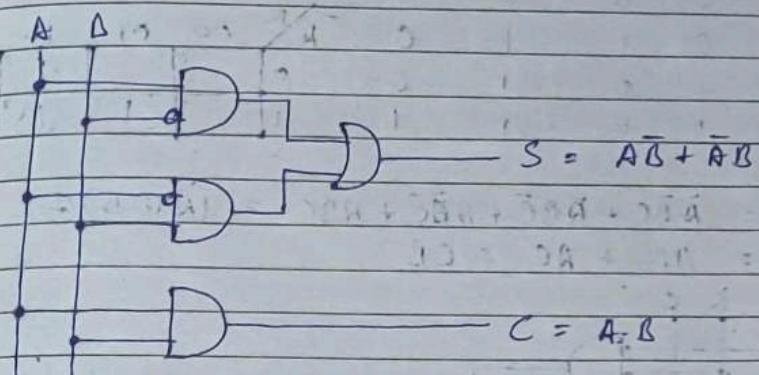
$$(000) + (010) + (011) + (101) + (110)$$

$$\begin{aligned} \bar{F} &= (A+B+C) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+\bar{B}+C) \\ &= (111) \cdot (101) \cdot (100) \cdot (101) \cdot (001) \end{aligned}$$

(46) Half adder:

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$\begin{aligned} S &= A \oplus B \\ C &= A \cdot B \end{aligned}$$



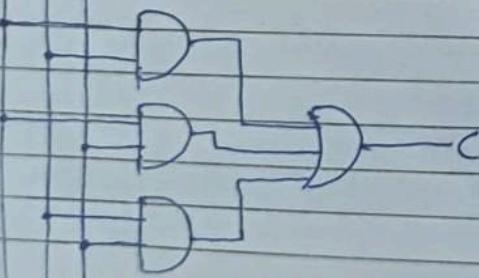
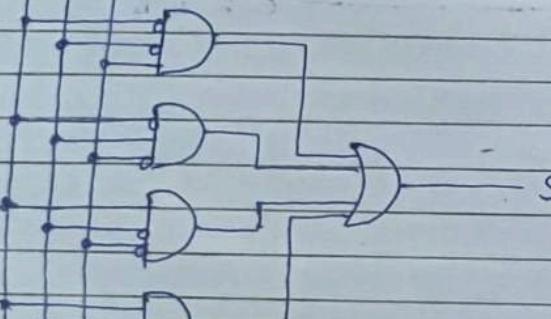
## (47) Full Adder:

A	B	C	$C_i$	S
0	0	0	0	0
1	0	0	0	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

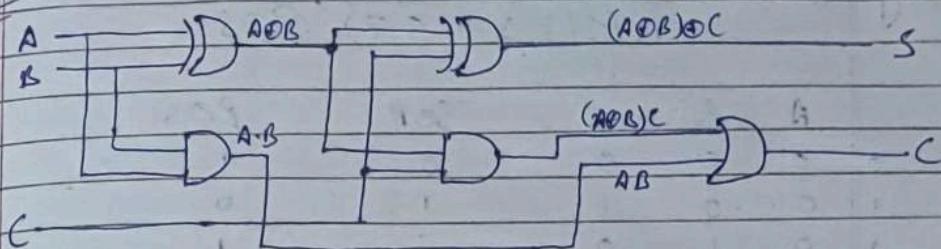
$$S = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC = (A \oplus B) \oplus C$$

$$C = AB + AC + CB$$

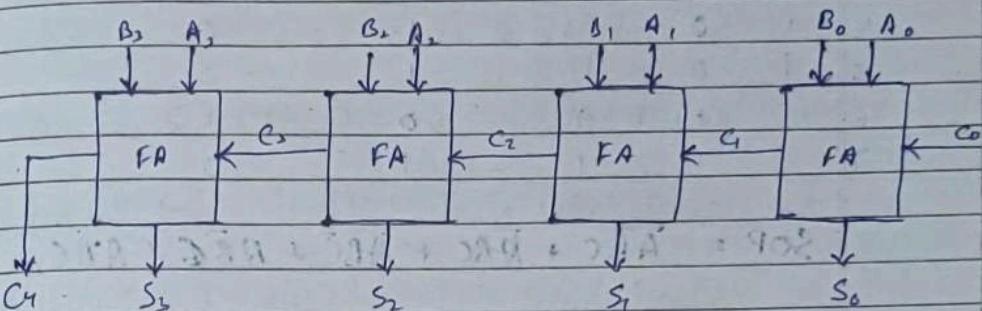
A B C



3 bit



4 bit



for 1st FA:  $A_0, B_0, C_0$  are 2 bits to be added then  
the output of 1st FA is used as the 3 bit  
input of 2nd FA as  $C_1$

$$\text{Ex: } F(ABC) = \sum m(1, 3, 4, 5, 7), \text{ SOP} \\ \text{ATM}(1, 3, 4, 5, 7). \text{ POS}$$

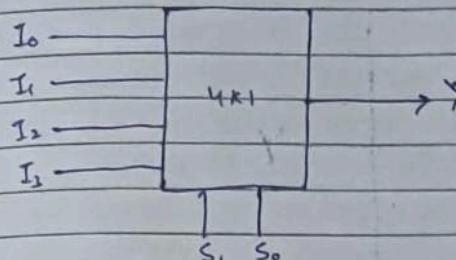
	A	B	C	SOP	POS
0	0	0	0	0	1
1	0	0	1	1	0
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	1	0
5	1	0	1	1	0
6	1	1	0	0	1
7	1	1	1	1	0

$$SOP = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC.$$

$$POS = (A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

Eg: i) E(0, 2, 3, 5, 6)      ii) F(0, 1, 2, 4)      iii) E(1, 3, 4, 6)      ] SOP & POS

(43) MUX



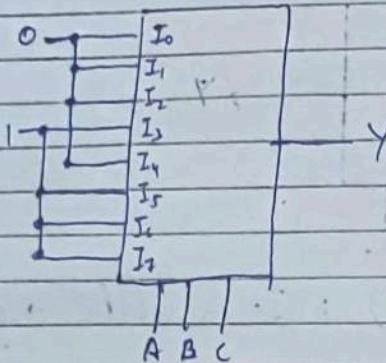
when  $(S_1 S_0) = (0, 0)$  :  $I_0$  is selected only

when  $(S_1, S_2) = (0, 1)$  :  $I_1$  is selected only

when  $(S, S_0) = (1, 0)$  :  $I_2$  is selected only

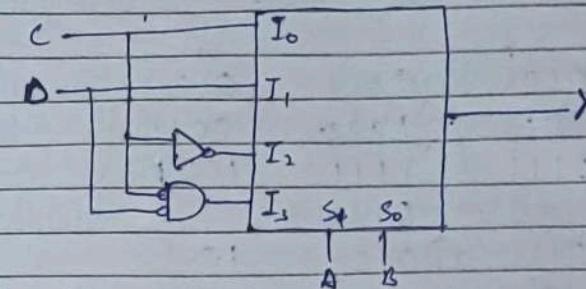
when  $(S, S_0) : (1, 1) = I_1$  is selected only

Eg:  $f(ADC) = \sum(3, 5, 6, 7)$  using 8x1



	A	B	C	$\bar{A}\bar{B}C$
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	✓ 1 $\bar{A}BC$
4	1	0	0	0
5	1	0	1	✓ 1 $A\bar{B}C$
6	1	1	0	✓ 1 $A\bar{B}\bar{C}$
7	1	1	1	✓ 1 ABC

Eg:



S <sub>1</sub>	S <sub>0</sub>	A	B	C	D	
0	0	0	0	0	0	0
1	0	0	0	0	1	0
2	0	0	1	0	1	2
3	0	0	1	1	1	3
4	0	1	0	0	0	4
5	0	1	0	1	1	5
6	0	1	1	0	0	6
7	0	1	1	1	1	7
8	1	0	0	0	1	8
9	1	0	I <sub>1</sub>	0	1	9
10	1	0	I <sub>2</sub>	0	0	10
11	1	0	I <sub>3</sub>	1	1	11
12	1	1	0	0	1	12
13	1	1	I <sub>2</sub>	0	1	13
14	1	1	I <sub>3</sub>	1	0	14
15	1	1	I <sub>4</sub>	1	1	15

$$F(ABCD) = \sum_m(2, 3, 5, 7, 8, 9, 12)$$

\* Ek digit ko n bits se represent karna hota ha

Page \_\_\_\_\_  
Date \_\_\_\_\_

Eg: add 183 to 576 using BCD

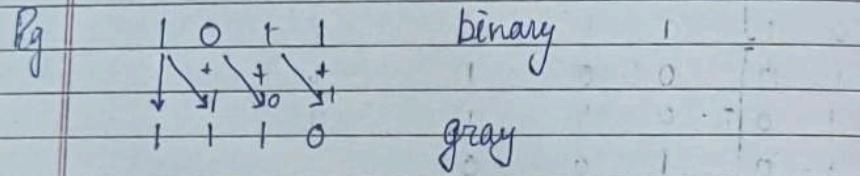
$$\begin{array}{r}
 576 \quad 0101 \\
 183 \quad 0001 \\
 \hline
 759 \quad 0111
 \end{array}
 \quad
 \begin{array}{r}
 0111 \\
 1000 \\
 \hline
 0110
 \end{array}
 \quad
 \begin{array}{r}
 0110 \\
 0011 \\
 \hline
 1001
 \end{array}$$

↓      ↓      ↓

7      5      9

Eg: Binary to Gray code

Record MSB as it is  
add MSB to next bit record sum neglect carry  
repeat process

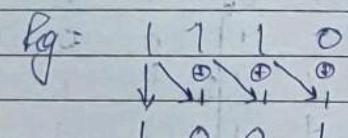


$$g_3 = b_3$$

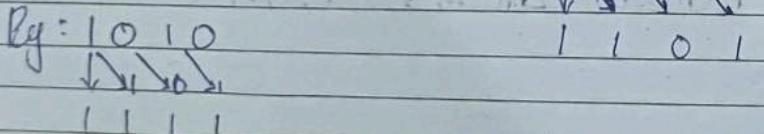
$$g_2 = b_2 \oplus b_3$$

$$g_1 = b_1 \oplus b_2$$

$$g_0 = b_0 \oplus b_1$$



Eg: 1001



\* from G to B & B to G MSB doesn't change

Page \_\_\_\_\_  
Date \_\_\_\_\_

Gray code =

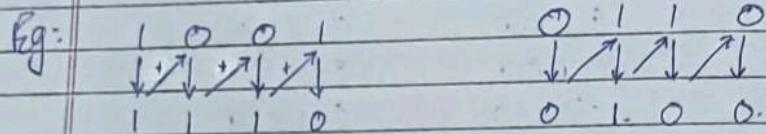
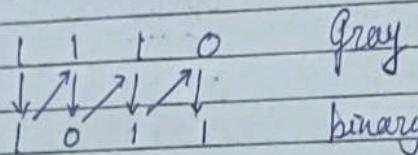
Minimum error code, two successive value differ in only 1 bit, binary is converted to gray to reduce switching.

Decimal	Binary	Gray
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000
16	10000	0000

ek koi aise bit flip karne jisse next term ke obtain ho na li pichla wala.

### Fig: Gray to binary

Record MSB as it is  
Add MSB to the next bit of Gray record from right to left.  
Repeat process



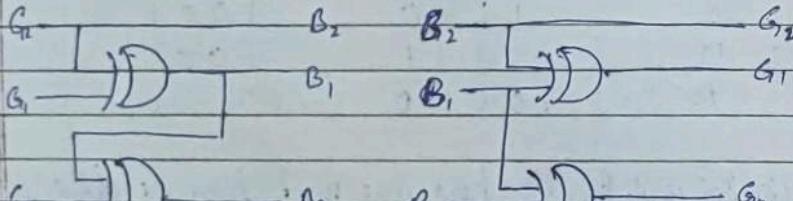
$$b_1 = g_2$$

$$b_2 = b_3 \oplus g_1$$

$$b_3 = b_4 \oplus g_1$$

$$b_4 = b_1 \oplus g_0$$

G to B



B to G



### Q9) Full subtractor

A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

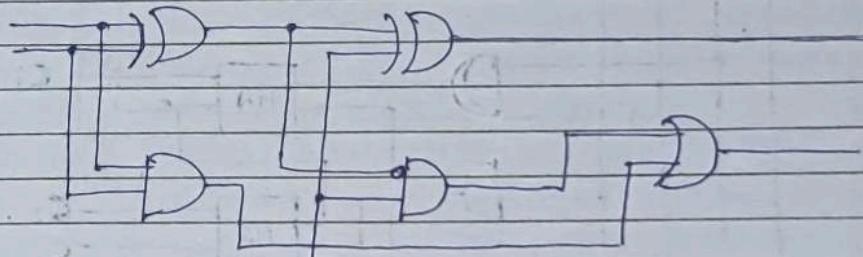
A	$\bar{B}$	$\bar{C}$	$\bar{B}C$	$B\bar{C}$	$\bar{B}\bar{C}$
0	0	1	0	1	
1	0	1	1	0	

Difference

A	$\bar{B}$	$\bar{C}$	$\bar{B}C$	$B\bar{C}$	$\bar{B}\bar{C}$
0	0	1	0	1	
0	0	0	0	0	0

Bout

$$\bar{A}C + BC + \bar{A}\bar{B}$$



Pg:  $C = K + Z_8Z_4 + Z_8Z_2$

$$= Z_8Z_1 + Z_8Z_2, Z_1 + Z_8Z_4 + Z_8Z_4Z_1 + Z_8Z_4Z_2 + Z_8Z_4Z_2Z_1 \\ + K + KZ_1 + KZ_2 + KZ_1$$

$$\Rightarrow Z_8Z_2 + Z_8Z_4 + Z_8Z_4Z_2 + \dots + KZ_2$$

$$= Z_8Z_2 + Z_8Z_4 + K$$

### (50) Binary Multiplier

$B_1 \quad B_0$

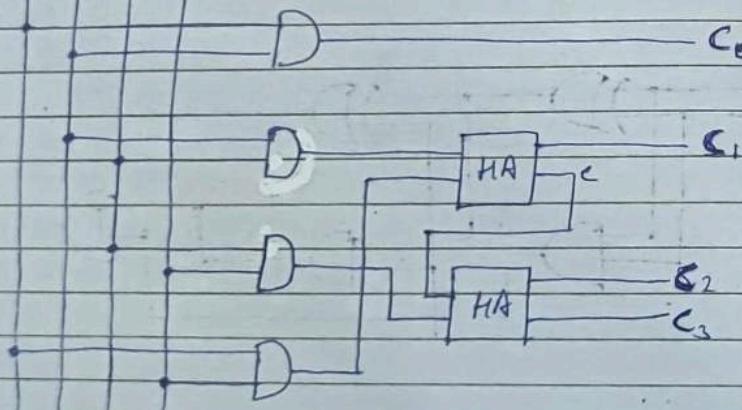
$A_1 \quad A_0$

$A_0B_1 \quad A_0B_0$

$$\begin{array}{l} A_1B_1 \quad A_0B_0 \\ \hline A_1B_1 \quad A_0B_1 + A_1B_0 \quad A_0B_0 \end{array}$$

$$C_1 \quad C_0 \quad C_0$$

$A_0 \quad B_0 \quad A_1 \quad B_1$

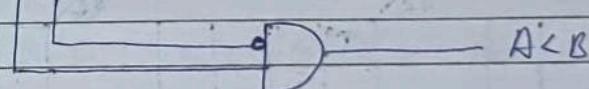
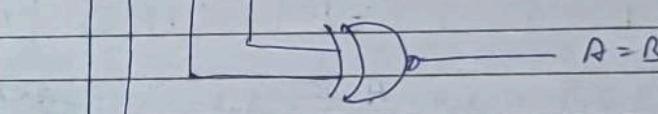
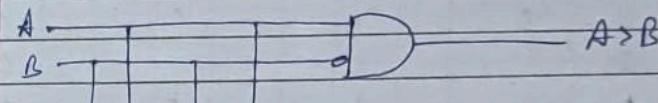


⑤ Comparators:  $>, <, =$

a) 1 Bit comparator

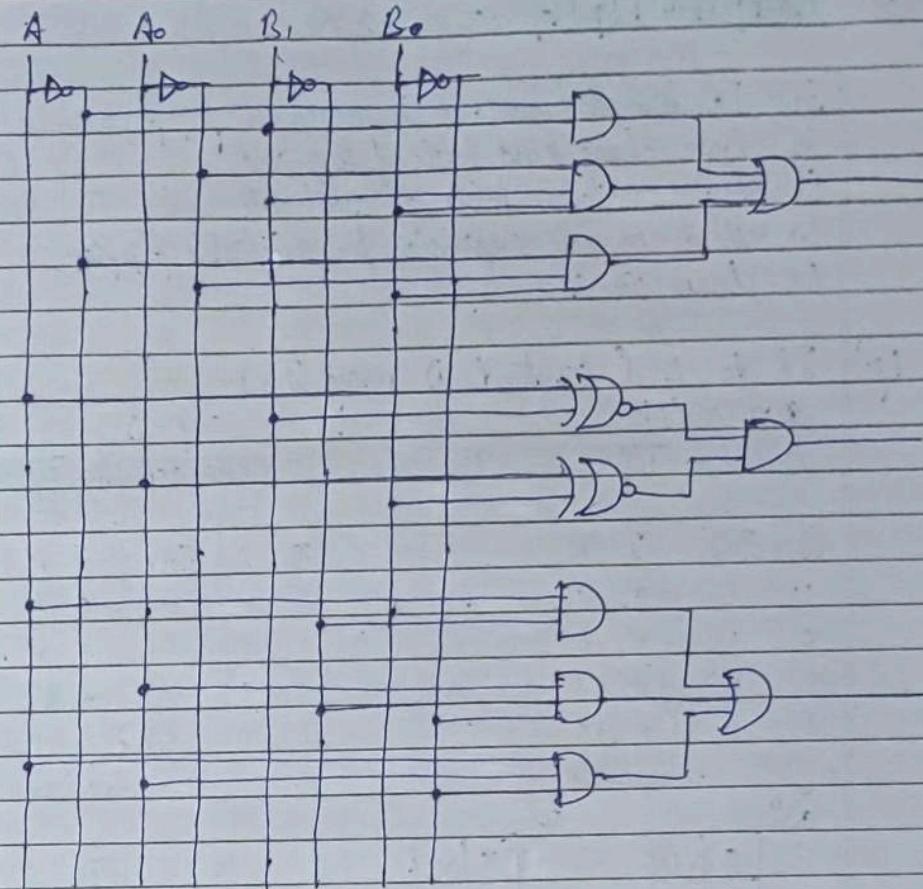
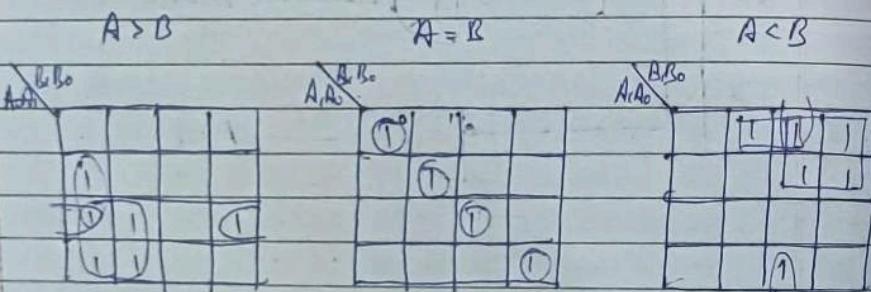
B	A	$A > B$	$A = B$	$A < B$
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

$A > B$	$A = B$	$A < B$
$\begin{array}{ c c } \hline 0 & 0 \\ \hline 1 & 0 \\ \hline \end{array}$	$\begin{array}{ c c } \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$	$\begin{array}{ c c } \hline 0 & 1 \\ \hline 0 & 0 \\ \hline \end{array}$



b) 2 bit comparator

A	B	$A > B$	$A = B$	$A < B$
0 0	0 0	0	1	0
0 0	0 1	0	0	1
0 0	1 0	0	0	1
0 0	1 1	0	0	1
0 1	0 0	1	0	0
0 1	0 1	0	1	0
0 1	1 0	0	0	1
0 1	1 1	0	0	1
1 0	0 0	1	0	0
1 0	0 1	1	0	0
1 0	1 0	0	1	0
1 0	1 1	0	0	1
1 1	0 0	1	0	0
1 1	0 1	1	0	0
1 1	1 0	1	0	0
1 1	1 1	0	1	0



$$A > B : A_0 B_0 + A_0 A_1 + B_0 B_1 = \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_1 B_1 + \bar{A}_0 B_1 B_0$$

$$A = B : (\bar{A}_0 \bar{B}_0 + A_0 B_0)(\bar{A}_1 \bar{B}_1 + A_1 B_1)$$

$$A < B : A_0 \bar{B}_0 + A_1 \bar{B}_1 + A_1 A_0 \bar{B}_0$$

### (5) Multiplanner Flip Flop conversion:

S<sub>1</sub>: Identify the available & required flip flops.

S<sub>2</sub>: Make characteristic table of required

S<sub>3</sub>: Make excitation table of available

S<sub>4</sub>: Write Boolean expression for available flip flop

S<sub>5</sub>: Draw circuit

### (Fig) JK to D (construct D using JK)

ava = JK

excitation TT

req = D  
characteristic TT

Ch. Anti JK

0 0 0 K

0 1 1 X

1 0 X 1

1 1 X 0

Ch. D Ch. Anti JK

0 0 0 . 0 X

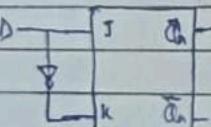
0 1 1 . 1 X

1 0 0 X 1

1 1 1 X 0

$D$	$J$	$K$	$Q_n$
0 1	X X	X X	
X X	0 0	0 0	

$J = D$        $K = \bar{D}$



### (Fig) T to D (construct D using T)

ava = T

excitation TT

req = D

characteristic TT

Ch. T Ch. Anti

0 0 0

0 1 1

1 0 1

1 1 0

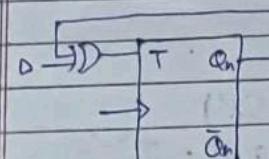
Ch. D Ch. Anti T

0 0 0 0 0

0 1 1 1 1

1 0 0 0 1

1 1 1 1 0



$$T = D \oplus Q_n$$

### (Fig) SR to JK (construct JK using SR)

ava = SR

excitation TT

req = JK

characteristic TT

Ch. Anti SR

0 0 0 X

0 1 1 0

1 0 0 1

1 1 X 0

Ch. J K Ch. Anti S R

0 0 0 0 0 X

0 0 1 0 0 X

0 1 0 1 1 0

1 0 1 1 1 0

1 0 0 1 X 0

1 0 1 0 0 1

1 1 0 1 X 0

1 1 1 0 0 1

1 0 1

1 1 1 Junchel

J K Ch. Anti

0 0 Hold

0 1 0

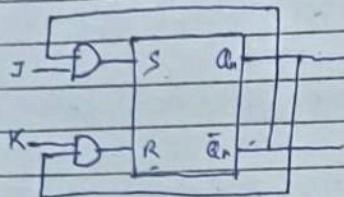
1 0 1

1 1 1 Junchel

$S$	$\bar{Q}_n$	$J$	$K$	$R$	$\bar{Q}_n$
0	0	1	1	X	X
X	0	0	X	0	0

$$S = \bar{Q}_n J$$

$$R = \bar{Q}_n K$$



#### (Pg3) SR to T (construct T using SR)

$$\text{qua} = SR$$

excitation

$$Q_n \text{ Out } SR$$

$$0 \ 0 \ X \ O \ X$$

$$0 \ 1 \ I \ O$$

$$I \ 0 \ O \ I$$

$$I \ I \ X \ O$$

$$\text{req} = T$$

characteristic

$$Q_n, T \text{ Out } S \ R$$

$$0 \ 0 \ . \ 0 \ O \ X$$

$$0 \ 1 \ . \ 1 \ I \ O$$

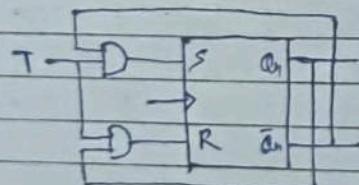
$$I \ 0 \ . \ 1 \ X \ O$$

$$I \ I \ . \ 0 \ O \ I$$

$T$	$\bar{Q}_n$	$J$	$K$	$R$	$\bar{Q}_n$
0	1	X	0	X	0
X	0	0	I	0	I

$$S = \bar{Q}_n T$$

$$R = \bar{Q}_n T$$



(Pg3)

D to JK (construct JK using D)

$$\text{qua} = D$$

$$\text{req} : JK$$

excitation

$$Q_n \text{ Out } D$$

$$0 \ 0 \ . \ 0$$

$$0 \ 1 \ . \ 1$$

$$I \ 0 \ . \ 0$$

$$I \ 1 \ . \ 1$$

$$1 \ 0 \ . \ 1$$

$$1 \ 1 \ . \ 1$$

$$1 \ 1 \ . \ 0$$

$$1 \ 1 \ . \ 0$$

$$1 \ 1 \ . \ 0$$

$$1 \ 1 \ . \ 0$$

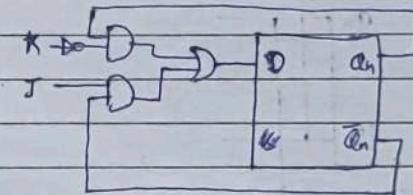
$$1 \ 1 \ . \ 0$$

$$1 \ 1 \ . \ 0$$

$$1 \ 1 \ . \ 0$$

$$D = J\bar{Q}_n + KQ_n$$

$D$	$\bar{Q}_n$	$J$	$K$	$R$	$\bar{Q}_n$
1	1	1	1	1	1



(Pq6)

T to JK (construct JK using T)

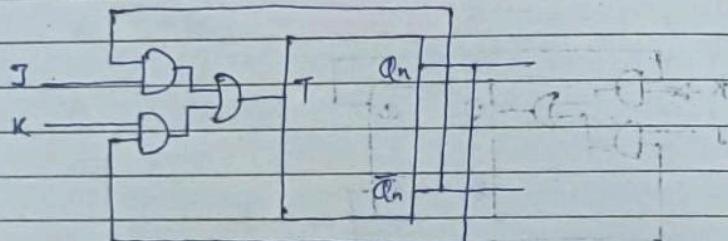
$$ava = T$$

excitation

$J_{eq} = JK$   
characteristic

$Q_n$	$Q_{n+1}$	T	J	K	$Q_n$	$Q_{n+1}$	T
0	0	0	0	0	0	0	0
0	1	1	0	0	1	1	0
1	0	1	0	1	0	0	0
1	1	0	0	1	1	0	1
		T	J	K			
			1	0	1	1	0
			1	1	1	0	1

$$T = KQ_n + \bar{J}Q_n$$



PYQ

(Pq1) 2's complement of 43

64	32	16	8	4	2	1
0	1	0	1	0	1	1

001010111  $\star$  2's complement of +ve no. is the no.  
itself

range of 2's complement  $= -2^{n-1} \text{ to } 2^{n-1} - 1$

(Pq2)

2's complement of -539<sub>10</sub> in hexadecimal

001000011011	True	2   539 1
11011100100	is	2   268 1
+1	2	2   134 0
11011110101	2's	2   67 1
D E 5		2   33 1
		2   16 0
		2   8 0
		2   4 0
		2   2 0

(Pq3) Decimal expression:  $16^3 \times 9 + 16^2 \times 7 + 16^1 \times 5 + 16^0 \times 3$   
no. of 1's in the unsigned binary are

9 7 5 3

1001011101010011

= 9 times 1

binary digits  
turn to decimal

$9753_{16} \rightarrow$

Pg4)  $A = a_3, a_2, a_1, a_0$   
 $B = b_3, b_2, b_1, b_0$

multiply digits  $c_i$  of the product C

c)  $c_1 a_0 b_1 + a_1 b_0$

Pg5) The value of  $n$   $\sqrt[3]{(224)_n} = (13)_n$

$$224_n = (13)_n^3$$

$$2 \times n^2 + 2 \times n^1 + 4 \times n^0 = (1 \times n^2 + 3 \times n^1)^3$$

$$2n^2 + 2n + 4 = n^6 + 9n^5 + 27n^4 + 54n^3 + 81n^2 + 54n + 27$$

$$n^2 - 4n - 5 = 0$$

$$(n-5) + (n+1) = 0$$

$$n = 5, -1$$

$$n = 5$$

Pg6) In 2's complement addition overflow

b) cannot occur when a +ve value is added to -ve value

Pg7) decimal value of 0.25

$$0.25 \times 2 = 0.0 \quad 0 \downarrow$$

$$0.50 \times 2 = 0.01 \quad 1 \downarrow$$

$$0.01$$

7003

Pg8) both in 2's form division of 11111011 by

- a) 11100111      00011001 (-25) 2's
- b) 11100100
- c) 11010111
- d) 11011011

$$\begin{array}{r} 11111011 \\ \text{Phase } (-s) \\ 00000100 \\ \hline \end{array}$$

$$\begin{array}{r} 00000101 \\ +1 \\ \hline 00000101 \end{array} \quad 2s$$

7002

Pg9) 2's represent of -15

$$00001111$$

$$11110000$$

$$\begin{array}{r} 11110001 \\ +1 \\ \hline 11110001 \end{array}$$

d) 11110001

7004

Pg10)  $73_n$  is equal to  $54_y$  the possible values of  $n & y$  in based no. system, y based number system.

$$73_n = 54_y$$

$$7n^1 + 3n^0 = 5y^1 + 4y^0$$

$$7n + 3 = 5y + 4$$

$$7n - 5y = +1$$

d) 8, 11

Q Pg 14)  $A = 11111010$  ] in 2's form find the product of A & B  
 $B = 00001010$  product also in 2's complement

$$A \quad 00000101 \Rightarrow -6(11111010)$$

$$\begin{array}{r} +1 \\ \hline 00000110 = 6 \end{array}$$

$$B \quad 00001010 = 10$$

$A \times B = 60$  = 2's form = True form for +ve

64 32 16 8 4 2

$$a) \quad 11000100 = -60 \quad 60 = 0.01 + 1100$$

$$\begin{array}{r} 11000011 \\ +1 \\ \hline 11000100 \end{array}$$

Pg 12) 123456<sub>8</sub> equals to

$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 2 & 1 & 3 & 0 & 2 & 3 & 2 \end{array}$

22130232

001010|01110010|110

$$a) \quad A \quad 7 \quad 2 \quad R_{16} \quad A72R$$

Pg 13) 16 bits 2's complement of decimal (-28)

$$28 = 00011100$$

$$11100011$$

+1

$$c) \quad 111111111100100$$

(Pg 14)

1217<sub>8</sub> equivalent to

- a) 1217<sub>16</sub>
- b) 028F<sub>16</sub>
- c) 2297<sub>10</sub>
- d) 0B17<sub>16</sub>

1217  
001010001111  
2 8 F

$$(Pg 15) (123)_r = (x8)_y$$

$$\begin{aligned} 1x5^2 + 2x5^1 + 3x5^0 &= ny^1 + 8 \\ 25 + 10 + 3 &= ny + 8 \\ 38 - 8 &= ny \\ 30 &= ny \end{aligned}$$

jab base 5 ka tak digit, yaada jab y ko lea hua  
 to digits de hogaye i.e. base bada hogega he and x ishota  
 hogya y se, n unknown hai to ek hi golu hai biggest ka  
 8 and y > 8 hogya hi tabhi y ka mai represent ho sakta hai  
 $x < y$ ,  $y > 8$

$$n = 1, y = 30$$

$$n = 2, y = 15$$

$$n = 3, y = 10$$

$$n = 4, y = 7.5 \quad \text{decimal not allowed}$$

$$n = 5, y = 6 \quad y > 8 : 6 \text{ not allowed}$$

3 possible solutions.

Q16)  $(43)_n = (y3)_8$

$$4n + 3 = 8y + 3$$

$$4n - 8y = 0$$

$$n = 2y$$

$$y < 8$$

$$n \leq 8$$

$$n \geq 4$$

$$n = 6, y = 3$$

$$n = 8, y = 4$$

$$n = 10, y = 5$$

$$n = 12, y = 6$$

$$n = 14, y = 7$$

5 possible solutions.

Q17) 1111 1111 1111 0101 : 16 bits 2's  
find decimal representation:

$$\begin{array}{r} 11110101 \\ 00001010 \\ +1 \\ \hline 00001011 \end{array}$$

$$\begin{array}{r} 00001011 \\ +1 \\ \hline -11 \end{array}$$

$$\begin{array}{r} 00001011 \\ -11 \\ \hline 11110100 \end{array}$$

$$-11$$

Q18)

$312 = 13 \cdot 1$  the base of the number system so that  
the above equation hold is.

$$\begin{array}{l} (312)_n = (13 \cdot 1)_n \\ (20)_n \end{array}$$

$$\begin{array}{l} 3n^2 + n + 2 = n^2 + 3 + 1/n \\ 2n + 0 \end{array}$$

$$\begin{array}{l} 3n^2 + n + 2 = 2n^2 + (5n) + 2 \\ 0 = 2n^2 - 3n^2 + 5n \\ n^2 - 5n = 0 \\ n = 0, n = 5 \end{array}$$

Q18) P is a k-bit signed integer P is  $(F87B)_{16}$  of 2's complement  
 $8 \times P = ?$  (in 2's complement) in hexadecimal

<del>F 8 7 B</del>	
<del>1111000010110101</del>	<del>= 3979<sub>10</sub></del>
<del>2'2'</del>	<del><math>\times 8</math></del>
<del>31832<sub>10</sub></del>	<del>2   15916 0</del>
<del>16   31832</del>	<del>2   7958 0</del>
<del>011110001010000</del>	<del>2   3979 1</del>
<del>FF FF</del>	<del>2   1989 1</del>
<del>83 A 8</del>	<del>2   994 0</del>
<del>1000101110101000</del>	<del>2   497 1</del>
<del>8 3 A 8</del>	<del>2   248 0</del>
<del>1000101110101000</del>	<del>2   124 0</del>
<del>8 3 A 8</del>	<del>2   62 0</del>
<del>1000101110101000</del>	<del>2   31 1</del>
<del>8 3 A 8</del>	<del>2   15 1</del>
<del>1000101110101000</del>	<del>2   3 1</del>
<del>8 3 A 8</del>	<del>2   1 1</del>

$$n = \text{no. of shifts} = 2^n$$

$$3 = 2^3 = 3 \text{ no. of shifts}$$

$$4 = 2^2 = 2 \text{ no. of shifts}$$

$$2 = 2^1 = 1 \text{ no. of shift}$$

P19 P is a 16 bit signed integer. P is  $(F87B)_{16}$  of  $2^8$  complement  
 $8 \times P = ?$  ( $2^8$  complement) and hexadecimal

- a) C3D8
- b) 187B
- c) F878
- d) 987B

M1 F87B

$$\begin{array}{r} 1111100001111011 \\ \times 8 \\ \hline 008888 \end{array}$$

$$\begin{array}{r} 0111110000111011 \\ \times 8 \\ \hline 0100000000000000 \end{array}$$

M2 agar 4 hai 0100 = 0010 i.e. divided by 2

left shift multiply by 2 if only once shift  
 no. of shifts =  $2^m$

$$\begin{array}{r} 1111100001111011 + 000 \\ \times 8 \\ \hline 0000000000000000 \end{array}$$

P20 Consider  $Z = X - Y$  when  $X, Y, Z$  are all signed.  $X$  &  $Y$  each represent  $n$  bits. To avoid overflow the representation of  $Z$  would require a min of

y bits

$$X = 16$$

$$Y = -5$$

$n$  bits  $\times$  not possible data + sign bit  $\Rightarrow$  requires 5 bits

$$X - Y = 11$$

① ② ③  
 sign more 3 bits

- a) n
- b) n-1
- c) n+1
- d) n+2

P21 For  $W, X, Y, Z$  boolean variables which is incorrect

$$\begin{array}{l} a) WX + (X+Y)W + X(X+Y) = X + WY \\ b) WX + WY + WY + X \\ \quad WY + WY + X \\ \quad X + WY \end{array}$$

$$\begin{array}{l} b) (W\bar{X}(Y+\bar{Z})) + \bar{W}X = \bar{W} + X + \bar{Y}Z \\ \quad (\bar{W}\bar{X} + \bar{Y}Z) + \bar{W}X \\ \quad (\bar{W} + X) + \bar{Y}Z + \bar{W}X \\ \quad \bar{W} + X + \bar{W}X + \bar{Y}Z \\ \quad \bar{W} + X + \bar{Y}Z \end{array}$$

$$\begin{array}{l} c) (W\bar{X}(Y+X\bar{Z}) + \bar{W}\bar{X})Y = X\bar{Y} \\ \quad (W\bar{X}Y + W\bar{X}\bar{Z} + \bar{W}\bar{X})Y \\ \quad W\bar{X}Y + \bar{W}\bar{X}Y \\ \quad \bar{X}Y \end{array}$$

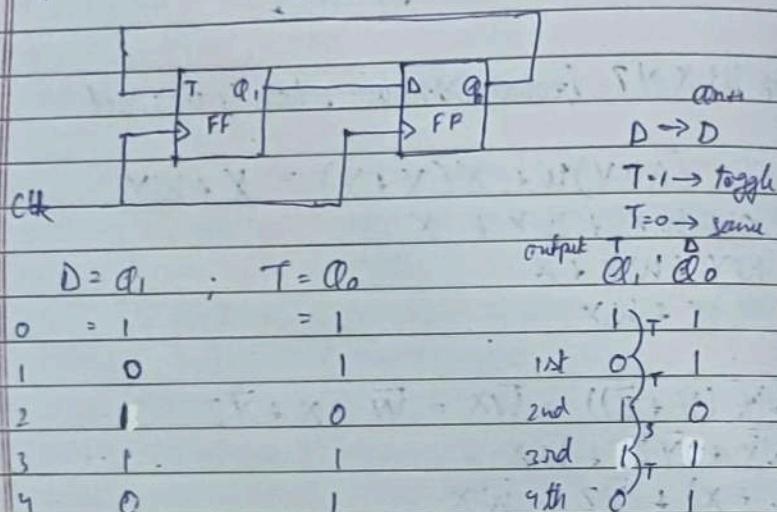
Q22.  $f(WXYZ) = \sum m(0, 1, 2, 3, 7, 8, 10) + \sum d(5, 6, 11, 15)$   
Which of following is a min POS form of  $f(WXYZ)$

$$\bar{f} = \bar{X}\bar{Z} + W\bar{Z}$$

$$f = (\bar{W} + \bar{Z})(\bar{X} + Z)$$

W	X	Y	Z
1	1	1	1
0	X	1	X
1	0	X	0

Q23. Computation of TDA ff, initially  $D_0 = Q_1 = 1$  before 1st division  
for 3rd & 4th cycle



Q24. An equivalent 2's complement representation of 2's complement number 1101 is :

① 101 i.e. no is -ve  
MSB

1101

110100

001101

110111

111101

as many signed bits added of same may

Q25. 2's complement of 16 bit number (1 sign bit & 15 mag bits) is FFFF  
Its magnitude in decimal representation is

① 1111111111111111 i.e. -ve no.

0000000000000000

+1

0000000000000001

= 1

Q26. Two 2's complement no. having sign bits are added and the sign bit of the result is 2. Then the occurrence of overflow is indicated by breakaway function.

Let MSB (sign bit MSB)

A  $n$  ... b<sub>0</sub>

+ B  $y$  ... G<sub>0</sub>

S  $z$  ... S<sub>0</sub>

range  
 $(-2^{n-1})$  to  $(2^{n-1}-1)$

$n=4$

$-8$  to  $+7$

$b_0 = 0$

breakaway: sum of  $2^k a_k g_{k+1} (-v_k)$   
sum of  $2^k a_k g_{k+1} (+v_k)$   
MSB = 1

Sum of 2 to give a (-ve) number

$$\begin{array}{ll} A = +v \\ B = +v \\ S = -v \end{array} \quad \begin{array}{ll} n = 0 \\ y = 0 \\ z = 1 \end{array} \quad \bar{y}\bar{z} = 1$$

Sum of 2 to give a (+ve) number

$$\begin{array}{ll} A = -v \\ B = -v \\ S = +v \end{array} \quad \begin{array}{ll} n = 1 \\ y = 1 \\ z = 0 \end{array} \quad ny\bar{z}$$

$$\bar{y}z + ny\bar{z}$$

<sup>no</sup> Pg 23 4 bit 2's complement representation of a decimal no. is 1000. Thus,

1000 gives a negative no.

0111

+1

$$1000 = 8$$

(-8) = -ve decimal

$$A \xrightarrow{-2^b} -A \xrightarrow{2^b} A$$

<sup>Ques</sup> Pg 28) The range of signed decimal numbers can be represented by 6 bit is complement form is:

$$\begin{aligned} \text{range} &= (-2^{n-1}) \text{ to } (2^{n-1} - 1) \\ &= 2^{6-1} - 1 \text{ to } 2^{6-1} - 1 \\ &= -31 \text{ to } +31 \end{aligned}$$

d)

<sup>Ques</sup> Pg 29) 11001, 1001, 111001 corresponds to 2's complement representation of which one of the following sets of numbers

- a) 25, 9, 57
- b) -6, -6, -6
- c) -7, -7, -7
- d) -25, 9, -57

MSB represent

111001

MSB = -ve as its 1

11001

1001

(-7)

1001

0110

+1

$$0111 = (7)$$

4 - 2005  
Pg 30) 43<sub>10</sub> into binary and BCD

00101011  
2 B

~~0010 1011~~  
~~0110~~  
~~0011~~  
 $43 = 0100\ 0011$

2006 Pg 31) A new binary coded binary (BCP) is proposed in which every digit of base 5 number is represented by its corresponding 3 bit binary code. Eg: 5 base number 24 will be represented by its BCP code 010100. In this number system, the BCP code 100010011001 corresponds to the following number in base 5 system.

$n=5$  : 0 to 4  
0 000  
1 001  
2 010  
3 011  
4 100

$24 = 2 \cdot 4^2 + 4$

given: 010 100  
100010011001  
4 2 3 1

4231<sub>5</sub>

2007

Pg 32) X = 01110, Y = 11001 are two 5 bit binary numbers represented by 2's complement form. The sum of X & Y represents 2's complement form using 6 bits is.

My  
(+w)X = 01110      (-w)Y = 11001  
not had  $\begin{array}{r} 11001 \\ 00110 \\ \hline 00111 \end{array}$   
for w  $\begin{array}{r} 11001 \\ +x \\ 10010 \\ \hline 00111 \end{array}$   
 $X = +14$        $Y = -7$

11001  
 $\begin{array}{r} 01110 \\ 11001 \\ \hline 10011 \end{array}$   
↓ drop  
 $X + Y = -14 - 7 = +7$

2008 Pg 33) The no. of bytes required to represent decimal no. 1856357 in BCD

1 byte 1 byte 1 byte 1 byte  
0 1 8 5 6 3 5 7

00000001 10000101, 01100011, 01010111,

i.e total of 4 bytes required

Pg 34)  $P = 11101101$ ,  $Q = 11100110$  (both 2's form)  
 $Q$  is subtracted from  $P$  the value obtained in signed 2's

MSB of  $P \& Q = 1$  i.e. both -ve

$$P = 11101101$$

$$\underline{00010010}$$

$$\begin{array}{r} +1 \\ \hline 100111 \end{array} \quad (19)$$

$$P = (-u) = -19$$

$$Q = 11100110$$

$$\underline{00011001}$$

$$+1$$

$$\underline{00011010}$$

$$Q = (-u) = -26$$

$$P - Q = -19 - (-26)$$

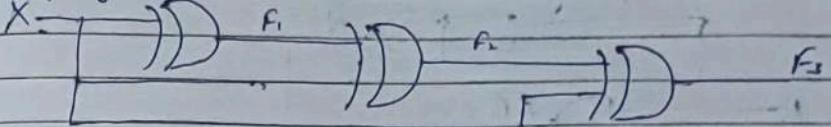
$$= 26 - 19$$

$$-19 \quad 00011010$$

$$-26 \quad \underline{11101101}$$

b)  $\begin{array}{r} 00000111 \\ +7 \\ \hline \text{drop} \end{array}$

Pg 35) Output of NORs



for similar inputs

$$Y = A \oplus B$$

$$= A\bar{B} + \bar{A}B$$

$$F_1 = 0$$

$$F_2 = 0\bar{A} + \bar{O}A$$

$$= A$$

$$Y = A \oplus A$$

$$= A\bar{A} + \bar{A}A$$

$$F_3 = 0$$

a)

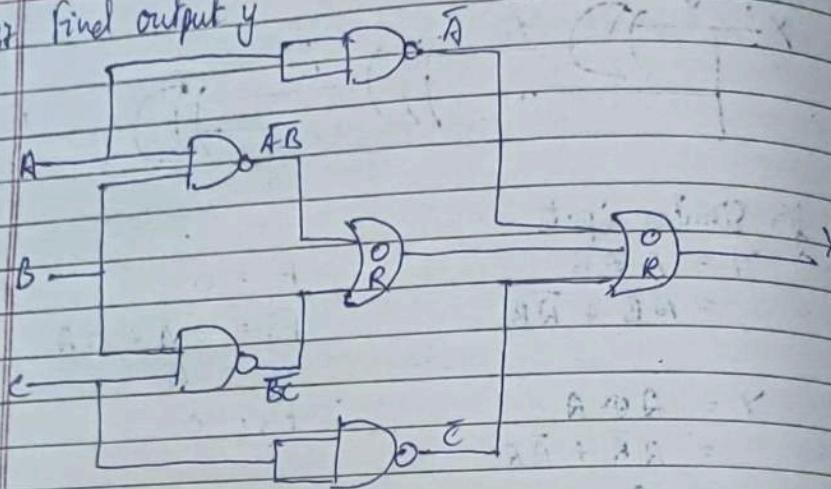
Pg 36) Min no. of 2 inputs NAND gate required to implement the fng  
 $F = (\bar{X} + \bar{Y})(Z + W)$

$$F = (W + Z)(\bar{X}Y)$$

$$= \bar{X}Y.W + \bar{X}Y.Z$$

b)  $y = ?$

1993 Pg 37) Find output  $y$



$$Y = \bar{A} + \bar{A} \cdot \bar{B} + \bar{B} \cdot \bar{C} + \bar{C}$$

$$c) = \bar{A} \bar{B} + \bar{B} \bar{C} + \bar{A} + \bar{C}$$

1993 Pg 38) Output from XNOR input A & B



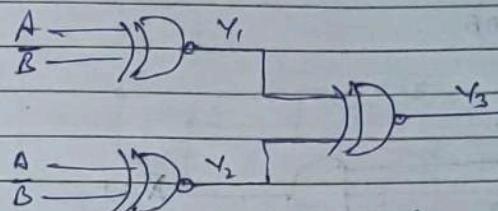
$$Y = \overline{A \oplus B} = XNOR$$

$$= \overline{AB} + \overline{A} \overline{B}$$

$$= (\bar{A} + B) \cdot (A + \bar{B})$$

c)

1993 Pg 39) The output:



$$Y_1 = A \cdot \bar{B} = Y_2$$

$$Y_3 = Y_1 \odot Y_2$$

$$= \overline{YY} + YY$$

$$= \overline{Y} + Y$$

$$= 1$$

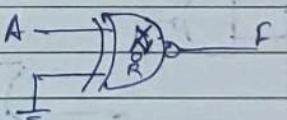
b)

1993 Pg 40) Min NAND for  $A + A\bar{B} + A\bar{B}C$

$$\begin{array}{c} A + A\bar{B} \\ \hline A \end{array}$$

a) ①

1993 Pg 41) Output of logic gate



d)

$$\bar{A}$$

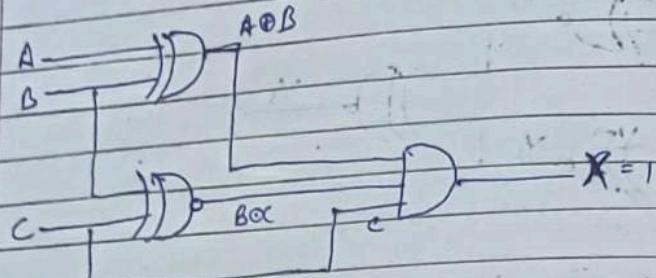
$$F = A \odot O$$

$$= \overline{AO} + AO$$

$$= \overline{A} I$$

$$= \overline{A}$$

2000 Pg 41 What input to get  $X=1$  (as output)



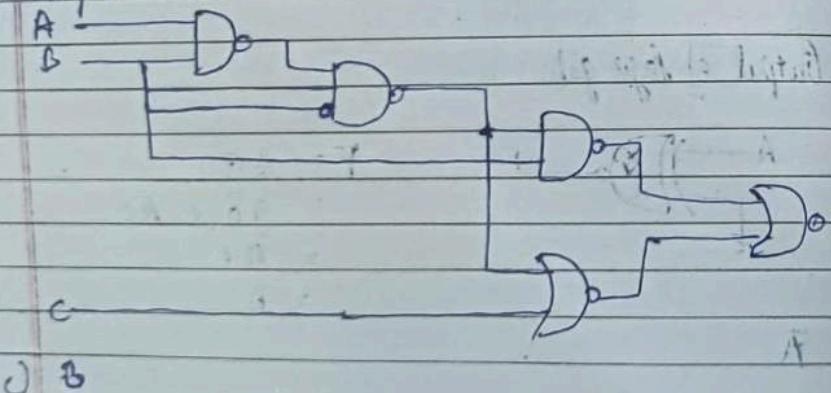
$$(A \oplus B) \cdot (B \oplus C) \cdot C$$

A	B	$A \oplus B$	$B \cdot C$	$B \oplus C$	$C = 1$
0	0	0	0	1	
0	1	1	0	0	
1	0	1	0	0	
1	1	0	1	1	

$A = 0 \text{ OR } C$   
 $B = 1 \text{ OR } C$

d)  $A \oplus B \oplus C = 101$

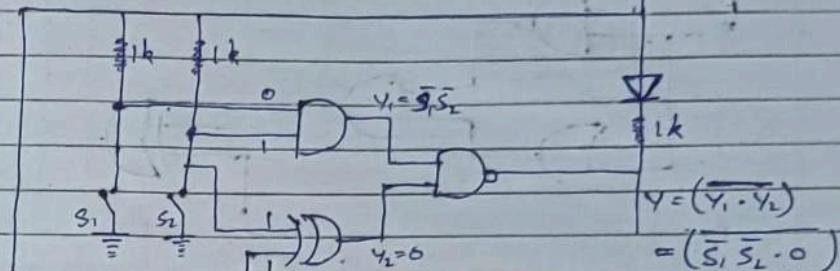
2000 Pg 42 Output



2001 Pg 42

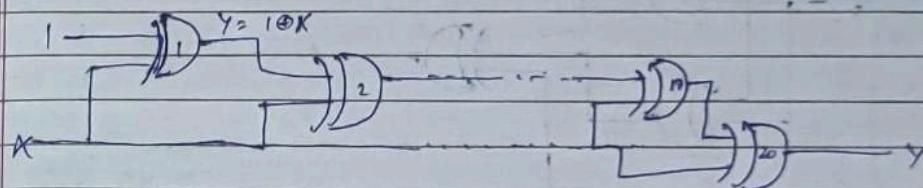
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$$V_{CC} = 5V$$



d) does not depend on  $S_1$  &  $S_2$

2002 Pg 43) Cascading 20 XOR find output

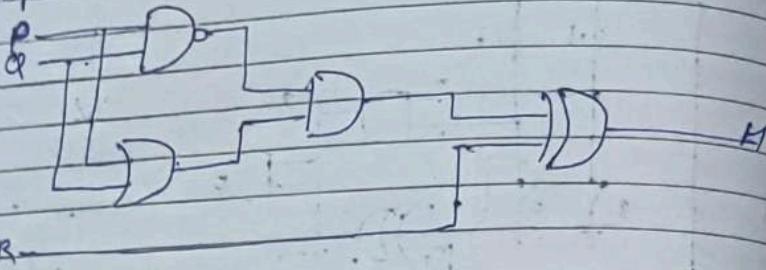


$$Y_1 = \bar{x}x + x\bar{x} = \bar{x}$$

$$Y_2 = \bar{\bar{x}}x + \bar{x}\bar{x} = 1$$

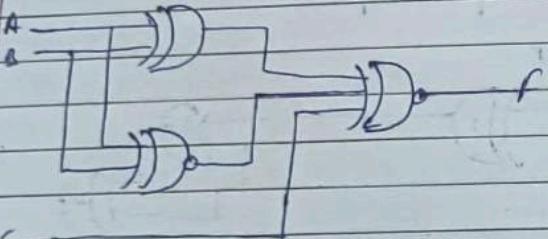
b) 1

9003  
Pg 45) Output



$$M = P \oplus Q \oplus R$$

1010  
Pg 46)  $f = 1$



$$F = (A \oplus B) \odot (A \oplus B) \odot C = 0 + 1 + 0 + 1 = 1$$
$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} = 1$$

a)  $A = 1 \quad B = 1 \quad C = 0$

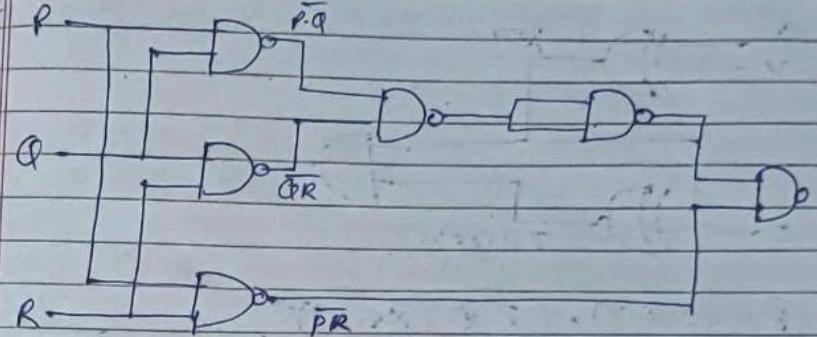
b)  $A = 1 \quad B = 0 \quad C = 0$

c)  $A = 0 \quad B = 1 \quad C = 0$

d)  $A = 0 \quad B = 0 \quad C = 1$

2011

Pg 47



$$Y = P\bar{Q} + \bar{Q}R + PR$$

(d) 2 or more are 1  $\Rightarrow Y = 1$

$$P = 1$$

$$Q = 1 \quad 1 + 1 + 1 = 1$$

$$R = 1$$

odd are 0

$$P = 0$$

$$Q = 1 \quad 0 + 1 + 0$$

$$R = 1 \quad 1 = 1$$

$$P = 0$$

$$Q = 0$$

$$R = 0$$

$$0 + 0 + 0$$

$$= 0$$

$$P = 0$$

$$Q = 0 \quad 0 + 0 + 0$$

$$R = 0 \quad = 0$$

odd are 1

$$P = 1$$

$$Q = 0$$

$$R = 0$$

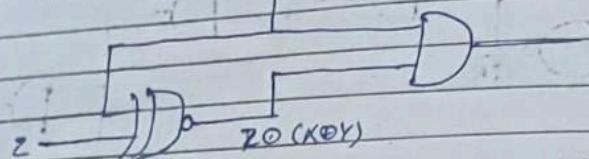
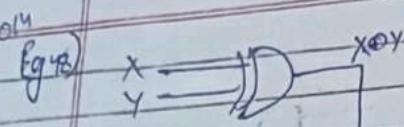
$$P = 1$$

$$Q = 1$$

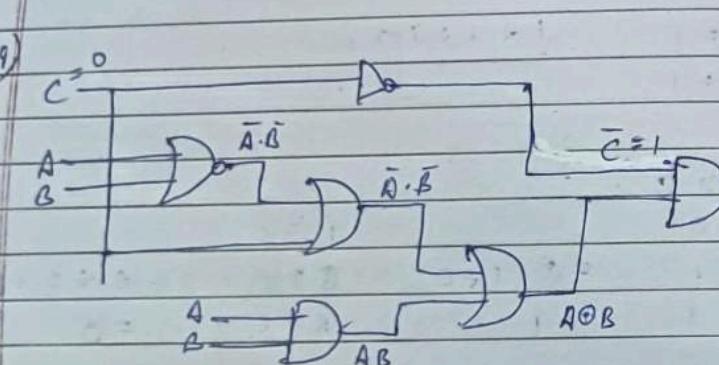
$$R = 1$$

$$1 + 1 + 1$$

$$= 1$$



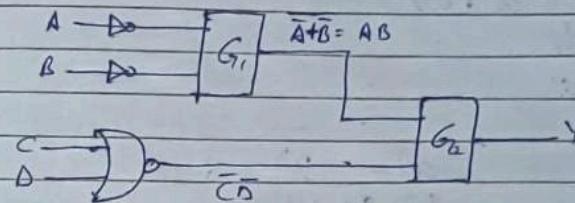
$$\begin{aligned}
 F &= (X \oplus Y) \cdot (Z \oplus (X \oplus Y)) \\
 &= (X \oplus Y) \cdot (X \oplus Y) \cdot Z + (X \oplus Y) \cdot (X \oplus Y) \cdot \bar{Z} \\
 &= X \oplus Y \cdot Z + 0 \\
 &= (X\bar{Y} + \bar{X}Y) Z \\
 &= X\bar{Y}Z + \bar{X}YZ
 \end{aligned}$$



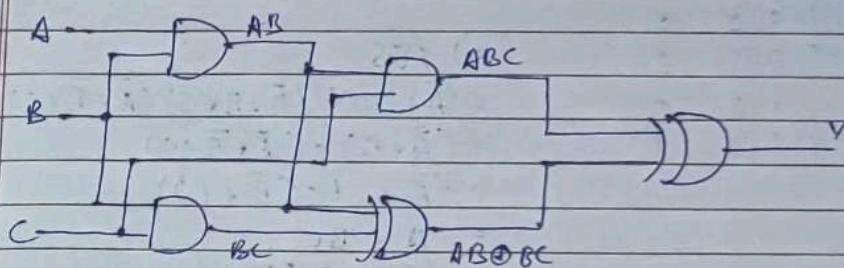
$$Y = A\bar{B} + \bar{A}B$$

a)  $Y = A \oplus B$

2015 Pg 50 identify gate  $G_1$  &  $G_2$ .  $Y = AB + \bar{C}\bar{D}$



a)  $G_1 = \text{NOR}$ ,  $G_2 = \text{OR}$



$$Y = AB \oplus BC \oplus ABC = \bar{ABC} + ABC + ABC$$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$\begin{aligned}
 &= \bar{ABC} + ABC + ABC \\
 &= BC(\bar{A}C + A) \\
 &= B((\bar{A} + A) \rightarrow (C + A)) \\
 &= B(C + A)
 \end{aligned}$$

- 1992  
Q52) For  $A, B, C$ :  $f = \text{true}$  when
- $A$  is false &  $B$  is true
  - $A$  is false &  $C$  is true
  - $A, B, C$  are false
  - $A, B, C$  are true

$A$	$B$	$C$	$F$
0	0	0	1
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

a)  $= 2^3 = 8$

b) SOP ( $C$ )

$$P = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C + A\bar{B}C$$

$$= \bar{A} + ABC$$

$$= \bar{A} + BC$$

c) POS :

$$= (\bar{A} + B + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

$$= (\bar{A} + B) \cdot (\bar{A} + \bar{B} + C)$$

$$= (\bar{A} + B \cdot (\bar{B} + C))$$

$$= \bar{A} + BC$$

$$= (\bar{A} + B)(\bar{A} + C)$$

1993  
Q53) logic circuit using NOR for  $A, B, C$  and output  $y$

LSB A -	0	1	0	1	0	1	
B	0	0	1	1	0	0	1
MSB C	0	0	0	0	1	1	1
Y	1	1	1	0	1	0	0

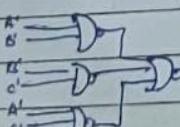
$\bar{A}$        $\bar{B}$        $\bar{C}$

$$f = (A+B+C) \cdot (A+B+C) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C})$$

$$\bar{f} = ((\bar{B}+\bar{A}) (\bar{C}+\bar{B}) (\bar{C}+\bar{A}))''$$

$$\bar{f} = (\bar{B}+\bar{A}) + (\bar{C}+\bar{B}) + (\bar{C}+\bar{A})$$

Ans



1998

Q54) dual form  $AB + \bar{A}C + BC = \bar{A}B + AC$

-  $\leftrightarrow$  +

+  $\leftrightarrow$  +  $(A+B) \cdot (\bar{A}+C) \cdot (B+C) = (\bar{A}+B) \cdot (\bar{A}+C)$

+  $\leftrightarrow$  -

0  $\leftrightarrow$  1

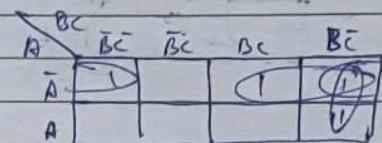
variable remain same

1999

Q55) reduced form of  $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}$

Ans Boolean laws

Ans K map



$$\bar{B}\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C}$$

2003

Q56) no. of distinct boolean expression for  $n$  variables

$$2^{2^n} = 2^{2^n} = 2^{16} = 65536$$

2007

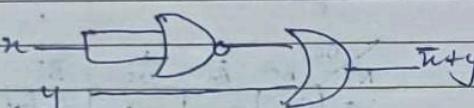
Eg57 Boolean function  $f$  of 2 variables  $x$  &  $y$  is defined as follows.

$$\begin{cases} f(x, y) = \\ f(0, 0) = f(0, 1) = f(1, 1) = 1, \quad f(1, 0) = 0 \end{cases}$$

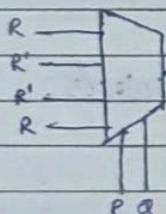
Assuming complements of  $x$  &  $y$  are not available, a min cost solution for realizing  $f$  using only 2 inputs NOR gates and 2 input OR gate (each having unit cost) would have total cost of

$x$	$y$	
0	0	
0	1	1
1	0	0
1	1	1

$$\begin{aligned} F(x, y) &= \bar{x}\bar{y} + \bar{x}y + xy \\ &= \bar{x}y + xy \\ &= \bar{x}y \end{aligned}$$



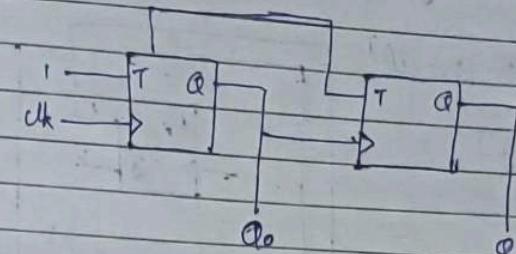
d) 2 units

2010 Eg58 Boolean expression  $f$ 

$$\begin{aligned} f &= \bar{P}\bar{Q}\bar{R} + \bar{P}\bar{Q}\bar{R}' + \bar{P}\bar{Q}'\bar{R} + P\bar{Q}R \\ &= \bar{P}(\bar{Q}\oplus R) + P(\bar{Q}\otimes R) \\ &= 001 + 010 + 100 + 111 \\ &= P \oplus Q \otimes R \end{aligned}$$

A Asynchronous : ak ka output durak hi clock  
Synchronous : ak ka clock pure mai

2010

Eg59 For sequential circuit below if initial value of  $Q_1, Q_0$  is 0 what are the next  $q$  values $Q_0, Q_1$ 0 0  $\rightarrow$  Q1 change due to  $Q_0 = 1$ 1 1  $\rightarrow$  no change due to  $Q_0 = 0$ 0 1  $\rightarrow$  Q1 change due to  $Q_0 = 1$ 1 0  $\rightarrow$  Q1 no change due to  $Q_0 = 0$ 0 0  $\rightarrow$  Q1 no change due to  $Q_0 = 0$  $\rightarrow$  +ve edge trigger T (FF) $\rightarrow$  Asynchronous $\rightarrow$  flip flop only change with rising edge

2011

Eg60 no. of D(FF) to design nwd 258 counter (min no.)

$$256 = 2^8 \text{ (8 bits)}$$

$$258 = 2^8 + 2 \text{ (9 bits)}$$

a) 9 bits

Eg61 Simplified SOP for expression  $(P + \bar{Q} + \bar{R}) \cdot (P + \bar{Q} + R) \cdot (P + Q + \bar{R})$ 

$$(0 \ 1 \ 1) \cdot (0 \ 1 \ 0) \ (0 \ 0 \ 1)$$

$$\text{fmin} = \pi M(1, 2, 3) \Rightarrow \text{minimizing SOP} = \Sigma(0, 4, 5, 6, 7)$$

b)

1	0	0	0
0	1	1	1

$$P + \bar{Q} \bar{R}$$

2011  
Eg61

If all FF were reset at 0 at power on what is the total no. of distinct output states generated by PQR generated by the counter is

→ Synchronous (as all have same clk).

$$\rightarrow D_p = R \quad (\text{1st})$$

$$P_{t+1} = D_p$$

$$D_q = \overline{(P+R)} \quad (\text{2nd})$$

$$Q_{t+1} = D_q$$

$$D_R = \overline{R}Q$$

$$R_{t+1} = D_R$$

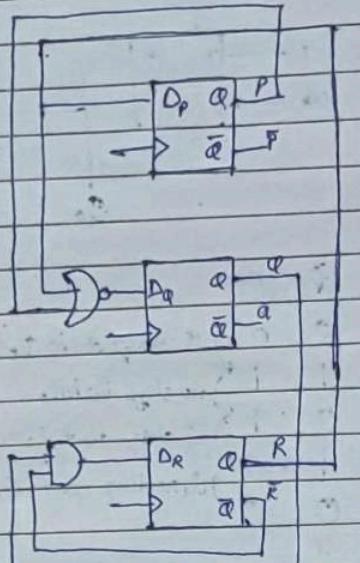
→ PQR

0 0 0	$R \quad (\overline{P+R})$	$\overline{R} \cdot 0$
1 0 1	$(0 \quad (\overline{0+0})$	$\overline{0} \cdot 0$
1 0 1	$(0 \quad (\overline{0+0})$	$\overline{0} \cdot 1$
1 1 0	$(1 \quad (\overline{1+0})$	$\overline{0} \cdot 0$
0 0 0	$(0 \quad (\overline{1+0})$	$\overline{0} \cdot 0$

b) 4

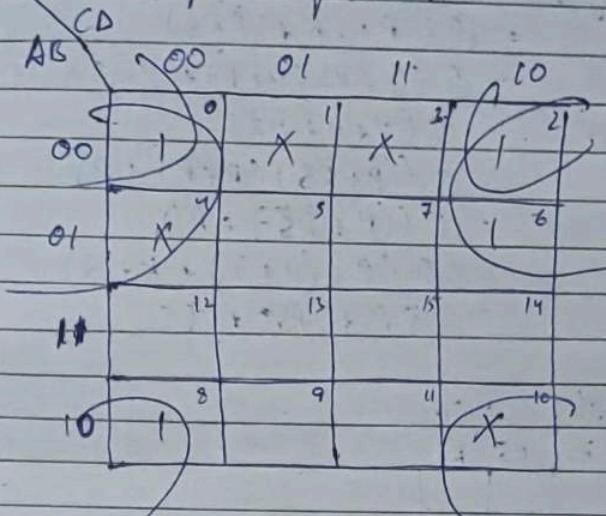
2011  
Eg62 No. of minterms for n variables

$$2^{n-1}$$



2012

Eg62 Reduced expression for SOP



~~ABCD~~  $B\bar{D} + \bar{A}\bar{D}$

Eg For TT, V=1 if & only if the input is valid

Input				Output		
A <sub>0</sub>	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	X <sub>0</sub>	X <sub>1</sub>	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

- a) priority encoder (3:8). valid output only in encoder
- b) Decoder (8:3) input 2-nd bit output 1st
- c) Multiplexer (4:1)
- d) Demultiplexer (1:4)

2014

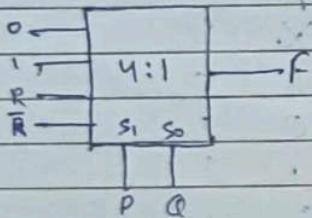
Eg63 Minimal SOP of  $F = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$

$$\begin{aligned}
 &= Q(P + \bar{P}R + \bar{P}\bar{R}S) \\
 &= Q((P+R)(P+\bar{P}) + \bar{P}\bar{R}S) \\
 &= Q(P+R+\bar{P}\bar{R}S) \\
 &= Q((P+\bar{R}S)(P+\bar{P}) + R) \\
 &= Q((P+\bar{R}S) + R) \\
 &= Q(P + (R+\bar{R}) + R + S) \\
 &= QP + RQ + QS
 \end{aligned}$$

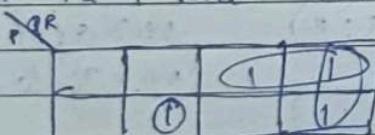
Ans

4 Variable K-map

$P\bar{R}$	$\bar{R}S$	$\bar{R}\bar{S}$	$R\bar{S}$	$RS$
$\bar{P}\bar{R}$	1	1	1	1
$\bar{P}R$	1	1	1	1
$P\bar{R}$	1	1	1	1

2014 Eg64 for 4:1 multiplex with  $S_0, S_1$  selection lines. SOP for F

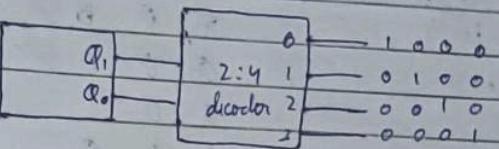
$$\begin{array}{cccc}
 000 & 101 & 110 & 111 \\
 0 + \bar{P}Q + R\bar{P}\bar{Q} + \bar{R}PQ
 \end{array}$$



$$\bar{P}Q + \bar{O}R + P\bar{O}R$$

2014

Eg65 Let  $k = 2^n$ . A circuit is built by giving the output of an n-bit binary counter as input to an n-to  $2^n$  bit decoder. This circuit is equivalent to a

 $A_1 \quad A_0$ 

0 = 0	0	1000	0 bit line lit
1 = 0	1	0100	1 bit line lit
2 = 1	0	0010	2 bit line lit
3 = 1	1	0001	3 bit line lit

- k bit binary up counter
- k bit binary down counter
- k bit ring counter
- k bit johnson counter

ring johnson

1000 0000

0100 1000

0010 1100

0001 1110

Pg 66

Function of block having variables  $n, y, a, b$ .

~~Inputs~~  $a, b$  are inputs  $y$  as output  
which digital logic block is implemented (combinational)

$$f(n, y, a, b) \{$$

$$y(n) =$$

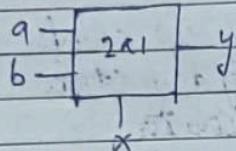
$$\begin{cases} y = a \\ y = b \end{cases}$$

	$n$	$a$	$b$	$y_a$	$y_b$	$y_n$
	0	0	0	0	0	0
	0	0	1	0	1	1

	0	1	0	1	0	0
	0	1	1	1	1	1
	1	0	0	0	0	0

	0	1	0	1	0	0
	0	1	1	1	1	1
	1	0	0	0	0	0

- a) Full adder  $\times$  multiple outputs
  - b) Priority encoder  $\times$
  - c) Multiplexer
  - d) Flip Flop (sequential)
- |   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

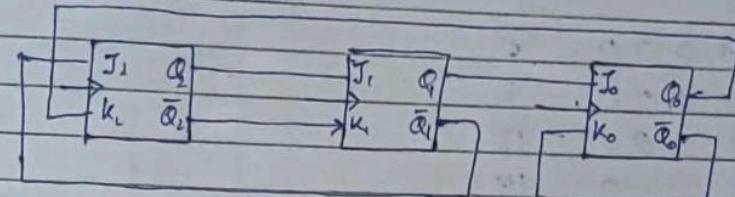


2014 Pg 67

$$\begin{aligned}
 F(P, Q) &= ((P \oplus Q) \oplus (P \otimes Q)) \oplus ((P \otimes Q) \oplus (Q \otimes P)) \\
 &= (\overline{P} \oplus P \oplus Q) \oplus (P \oplus Q \oplus \overline{Q}) \\
 &\quad (1 \oplus Q) \oplus (P \oplus 0) \\
 &= \overline{Q} \oplus P \\
 &= P \otimes \overline{Q} \\
 &= \frac{P \oplus Q}{P \otimes Q} \quad ] XNOR \\
 &= P \oplus \overline{Q}
 \end{aligned}$$

$$\begin{aligned}
 1 \oplus P &= \overline{P} \\
 0 \oplus P &= P \\
 P \oplus \overline{P} &= 1 \\
 P \otimes P &= 0
 \end{aligned}$$

The sequential circuit is built using JK flip flop which is initialized by  $Q_2 Q_1 Q_0 = 000$ . The output for next 3 cycles



$$J_1 = \overline{Q}_1$$

$$K_1 = Q_1$$

$$J_2 = Q_2$$

$$K_2 = \overline{Q}_2$$

$$J_3 = Q_3$$

$$K_3 = \overline{Q}_3$$

$$Q_1(t+1) = J_1 \overline{Q}_1 + \bar{K}_1 Q_1$$

$$Q_2(t+1) = J_2 \overline{Q}_2 + \bar{K}_2 Q_2$$

$$Q_3(t+1) = J_3 \overline{Q}_3 + \bar{K}_3 Q_3$$

$Q_2 \quad Q_1 \quad Q_0$

$$0 \quad 0 \quad 0$$

$$1 \quad 0 \quad 0$$

$$1 \quad 1 \quad 0$$

$$1 \quad 1 \quad 1$$

$$1 \quad 0 \quad 0$$

$$(1 \cdot 1 + 1 \cdot 0) \quad (0 \cdot 1 + 0 \cdot 0) \quad (0 \cdot 1 + 0 \cdot 1)$$

$$(1 \cdot 1) \cdot (1 \cdot 0) \cdot (0 \cdot 1)$$

c) 100, 110, 111

\* functionally complete if it satisfy NOT and any of AND or OR

Page \_\_\_\_\_  
Date \_\_\_\_\_

Ques Pg 61  
Ques Pg 62  
Consider a 4 bit Johnson counter with initial value 0000  
The counting sequence of this counter is

d)  
0 0 0 0 0  
1 0 0 0 8  
1 1 0 0 12  
1 1 1 0 14  
1 1 1 1 15  
0 1 1 1 7  
0 0 1 1 3  
0 0 0 1 1  
0 0 0 0 0

Ques Pg 63  
Which is functionally complete

$$f(x, y, z) = \bar{x}yz + x\bar{y} + \bar{y}z$$

$$g(\bar{x}, y, z) = \bar{x}yz + \bar{x}y\bar{z} + xy$$

Ans  
 $\Rightarrow f(x, 1, 1) = \bar{x}11 + x \cdot 0 + 0 \cdot 0$   
 $= \bar{x}$

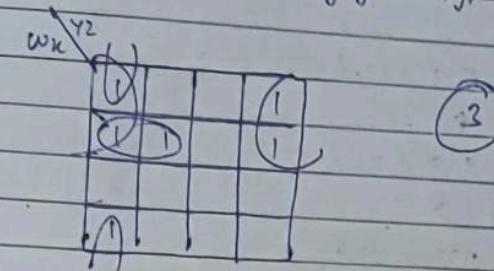
for

$$f(x, 0, f(1, 1)) = x + z$$

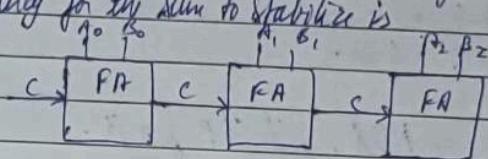
Ans  
 $\Rightarrow g(\bar{x}, y, z) = y (\bar{x}z + \bar{x}\bar{z} + x)$   
 $= y (x + \bar{z} + x)$   
 $= (x+1)y$   
 $= y$

only f is functionally complete

Ques Pg 71  
Total no. of prime implicants of  $f(w, x, y, z) = \sum (0, 2, 4, 5, 6, 10)$  is



Ques Pg 72  
Consider a 8 bit ripple carry adder for computing the sum A & B where A & B are integers represented in 2's complement form. If the decimal value of A is one, the decimal value of B that leads to longest latency for the sum to stabilize is



longest latency is the waiting period for A FA to get the carry which is to be added highest when 111 is added lowest when no carry is generated.

$$\begin{array}{r} 000 \quad 001 \quad 111 \quad 000 \\ 001 \quad 111 \quad 111 \quad 000 \\ \hline 001 \quad 1000 \quad 1110 \quad 000 \end{array}$$

highest: Lowest

in 8 bit 2's is  $(1)111111 \rightarrow 00000000$

$$\begin{array}{r} +1 \\ \hline 100000000 \end{array}$$

$+1$   
 $-1$

2016

Pg73 Min no. of JK (FF) used to create a synchronous counter to count  
0-1-0-2-0-3

00 0000 - 1st

01 0001

00 0100 2nd

10 0010

00 1100 3rd

11 0011

1 for sequence

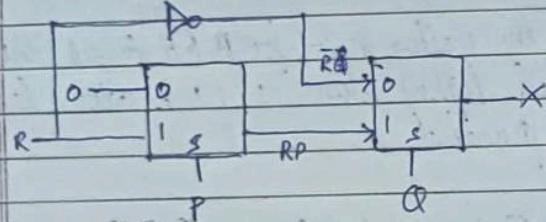
2 for zero

3 for 1st & 2nd zeros

4 for 2nd & 3rd zeros

2016

Pg74 2 cascading 2:1 MUX. find SOP for X



$$PQR + \bar{R}\bar{Q}$$

2016

Pg75  $(f_1 \text{ AND } f_2) \oplus f_3$

$$f_1 = \sum (0, 2, 5, 8, 14)$$

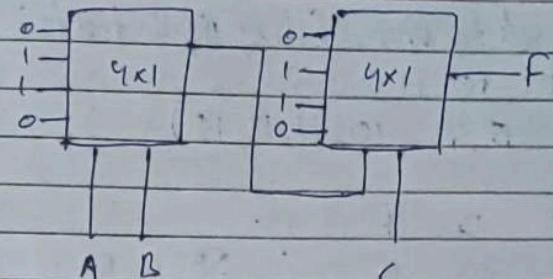
$$f_2 = \sum (2, 3, 6, 8, 14, 15)$$

$$f_3 = \sum (2, 7, 11, 14)$$

$$(f_1 \oplus f_2) \oplus f_3 = (2, 8, 14) \oplus (2, 7, 11, 14)$$

$$= (7, 14, 8)$$

Pg76



case 0 is 1st then C

A	B	C	$f_1$	F
0	0	0	0	0
0	0	1	0	$\bar{A}\bar{B}C$
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	0
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	1	0
1	1	0	0	0
1	1	1	0	$ABC$

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

Fig 72 Let AB be selection lines. C,D be input to obtain F

$$AB = 00, 01, 10, 11$$

$$F = \sum C(1, 2, 5, 7, 8, 10, 11, 13, 15)$$

A	B	C	D	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	0	1	0
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	1
9	1	0	0	0
10	1	0	1	1
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	1
15	1	1	1	1

$$D = F_2 = F_4, \quad F_1 = C \oplus A, \quad F_3 = \bar{D} + C, \quad F$$

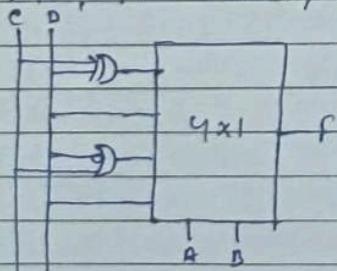


Fig 78 Input  $A_3, A_2, A_1, A_0$ , output  $B_3, B_2, B_1, B_0$   
where  $B = A$  for  $A < 10$   
 $B = A - 4$  for  $A > 10$

$A_3$	$A_2$	$A_1$	$A_0$	$B_3$	$B_2$	$B_1$	$B_0$
0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0
2	0	0	1	0	0	0	1
3	0	0	1	1	0	0	1
4	0	1	0	0	0	1	0
5	0	1	0	1	0	1	0
6	0	1	1	0	0	1	1
7	0	1	1	1	0	1	1
8	1	0	0	0	1	0	0
9	1	0	0	1	1	0	0
10	1	0	1	0	0	1	0
11	1	0	1	1	0	1	1
12	1	1	0	0	1	0	0
13	1	1	0	1	1	0	0
14	1	1	1	0	1	0	1
15	1	1	1	1	0	1	1

$$B_0 = A_0$$

$$B_1 = A_1$$

$$B_2 = \sum (4, 5, 6, 7, 10, 11)$$

$$B_3 = \sum (8, 9, 12, 13, 14, 15)$$

$B_2$	$A_3 A_2$	$B_3$	$A_2 A_1$	$Q, R_0$
1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1	1	1	

$$B_2 = \bar{A}_3 A_2 + A_1 A_3 \bar{A}_2$$

$$B_3 = A_2 A_2 + A_2 \bar{A}_1$$

Eg) When  $< 3 = 1, \geq 3 = 0$  for 3 variables

A B C F

0 0 0 1

0 0 1 1

0 1 0 1

0 1 1 0

1 0 0 0

1 0 1 0

1 1 0 0

1 1 1 0

$\bar{A} \bar{B} C$	$\bar{A} B C$	$A \bar{B} C$	$A B \bar{C}$
0 0 0	0 0 1	0 1 0	0 1 1
0 0 1	0 1 0	1 0 0	1 0 1
0 1 0	1 0 0	1 1 0	1 1 1
0 1 1	1 1 0	1 1 1	1 1 1

$$F = \bar{A}\bar{B} + \bar{A}\bar{C}$$

Eg: Design a max for  $\sum m(0, 1, 5, 6, 8, 10, 12, 15)$

$D_0$	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$
0	0	1	2	3	4	5	7
1	3	9	10	11	12	13	14

1  $\bar{A}$  A 0 A  $\bar{A}$ : A  $\cdot$   $\bar{A}$

Karnaugh  
table

1	$D_0$
0	$D_1$
1	$D_2$
0	$D_3$
1	$D_4$
0	$D_5$
1	$D_6$
0	$D_7$

8X1

Q3) Counters:

• 2 bit synchronous counter by JK FF

S1: Identify no. of bits and flip flops

S2: Write Excitation table of JK FF

S3: draw State table & diagram

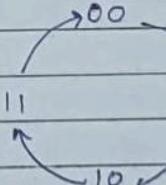
S4: Solve from K map by getting boolean expression

S5: Make the circuit from equation obtained.

S1: 2 bits + JK FF

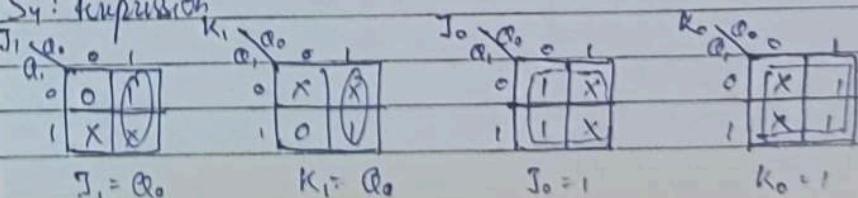
$Q_1$	$Q_0$	$Q_1^{*}$	$Q_0^{*}$	J	K
0	0	0	X		
0	1	1	X		
1	0	X	1		
1	1	X	0		

S2: State diagram + table



$Q_1$	$Q_0$	$Q_1^{*}$	$Q_0^{*}$	J <sub>1</sub> , K <sub>1</sub>	J <sub>0</sub> , K <sub>0</sub>
0	0	0	1	0, X	1, X
0	1	1	0	1, X	X, 1
1	0	1	1	X, 0	1, X
1	1	0	0	X, 1	X, 1

S4: Expression

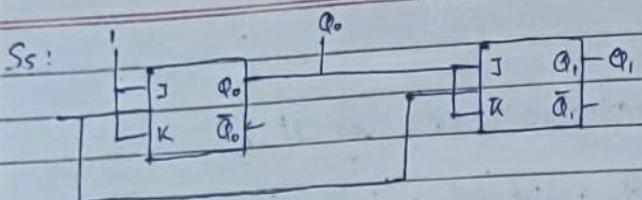


$$J_1 = Q_0$$

$$K_1 = Q_0$$

$$J_0 = 1$$

$$K_0 = 1$$



• 3 bit synchronous counted by T FF

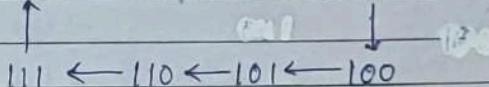
S<sub>1</sub>: 3 bits + T FF

S<sub>2</sub>: excitation table of T, FF

$Q_n$	$D_{n+1}$	T
0	0	0
0	1	1
1	0	1
1	1	0

S<sub>3</sub>: State diagram + table

$$000 \rightarrow 001 \rightarrow 010 \rightarrow 011$$



$Q_2$	$Q_1$	$Q_0$	$Q_2^*$	$Q_1^*$	$Q_0^*$	$T_2$	$T_1$	$T_0$
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

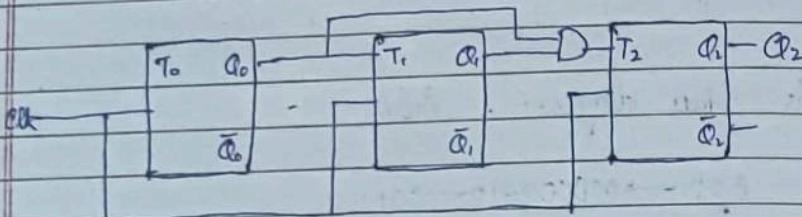
S4: Boolean expression

	$T_0$	$T_1$	$T_2$	$\bar{T}_0$	$\bar{T}_1$	$\bar{T}_2$	$Q_0$	$Q_1$	$Q_2$	$\bar{Q}_0$	$\bar{Q}_1$	$\bar{Q}_2$	$T$
	0 0 0	0 1 1	1 0 0	1 0 0	0 1 1	0 0 0	0 0 0	1 1 1	1 1 1	1 1 1	0 0 0	0 0 0	0 0 0
	1 0 0	1 1 0	0 0 1	0 0 1	1 1 0	1 0 0	1 0 0	0 1 1	0 1 1	0 1 1	1 1 0	1 1 0	1 1 0

$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = \bar{Q}_0 Q_1$$



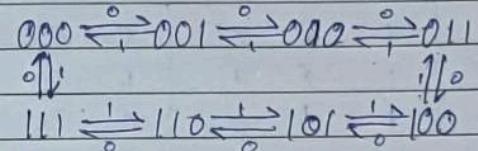
• 3 bit synchronous up / down counter

S1: 3 bit ;  $T \text{ FF}$

S2: excitation table

$Q_{n-1}$	$Q_{n-2}$	$T$	
0	0	0	$m=0$ up
0	1	1	$m=1$ down
1	0	1	
1	1	0	

S3: state diagram & table



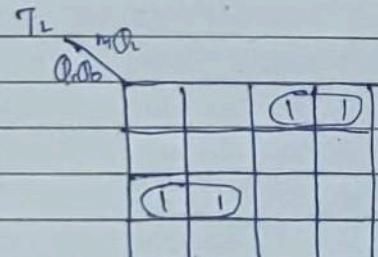
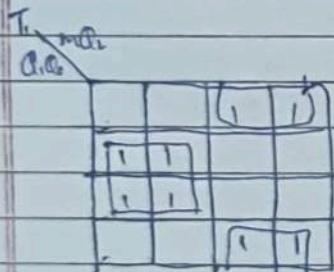
S5: Circuit diagram

84!

Page \_\_\_\_\_  
Date \_\_\_\_\_

m	$Q_1$	$Q_2$	$Q_0$	$Q_1'$	$Q_2'$	$Q_0'$	$T_1$	$T_2$	$T_0$
0	0	0	0	0	0	1	0	0	1
0	0	0	1	0	1	0	0	1	1
0	0	1	0	0	1	1	0	0	1
up	0	0	1	1	0	0	1	1	1
0	1	0	0	1	0	1	0	0	1
0	1	0	1	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0	1
0	1	1	1	0	0	0	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1	1
down	1	0	1	0	1	0	0	0	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	1	0	0	0	0	1
1	1	1	0	1	0	1	0	1	1
1	1	1	1	1	0	0	0	0	1

$$T_0 = 1$$

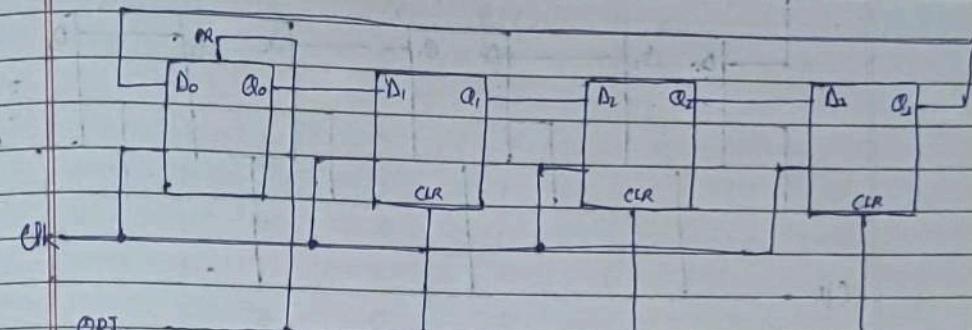


$$T_1 = Q_0 \bar{m} + \bar{Q}_0 m$$

$$T_2 = Q_1 Q_0 \bar{m} + \bar{Q}_1 \bar{Q}_0 m$$

## Ring counter

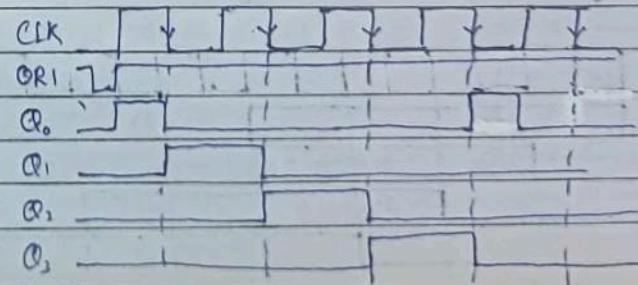
It is application of shift register  
no. of states = no. of bits  
no. of flip flop



ORI  
overriding input

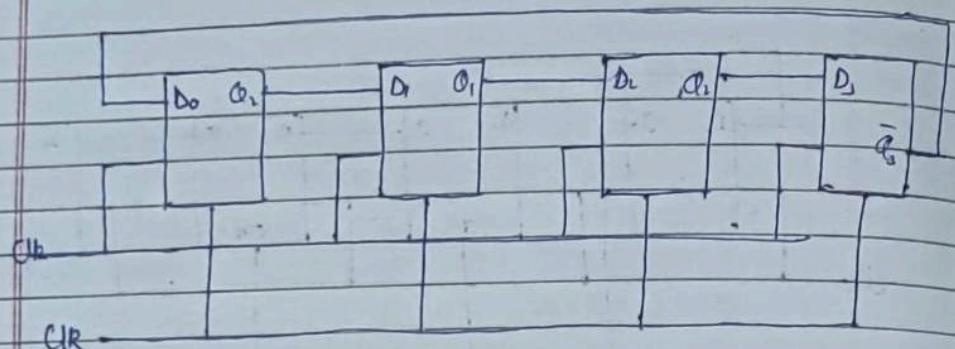
If CLR = 0, Q = 0

ORI	CLK	$Q_0$	$Q_1$	$Q_2$	$Q_3$
X		1	0	0	0
1	↓	0	1	0	0
1	↓	0	0	1	0
1	↓	1	0	0	1
1	↓	1	0	0	0



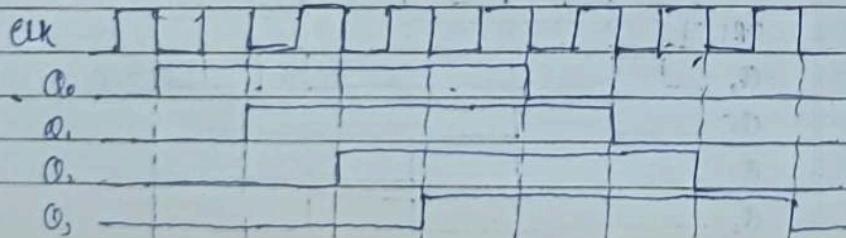
### Johnson Counter / Twisted / Switch Tail

It's the application of shift register  
 No. of states =  $2 \times$  no. of flip flops  
 $= 2^n$  no. of bits.



CLR CLK Q0 Q1 Q2 Q3

	X	0	0	0	0
1	↓	1	0	0	0
1	↓	-	1	0	0
1	↓	-	-	1	0
1	↓	-	-	-	1
1	↓	0	1	1	1
1	↓	0	0	1	1
1	↓	0	0	0	1
1	↓	0	0	0	0



### (54) State Tables

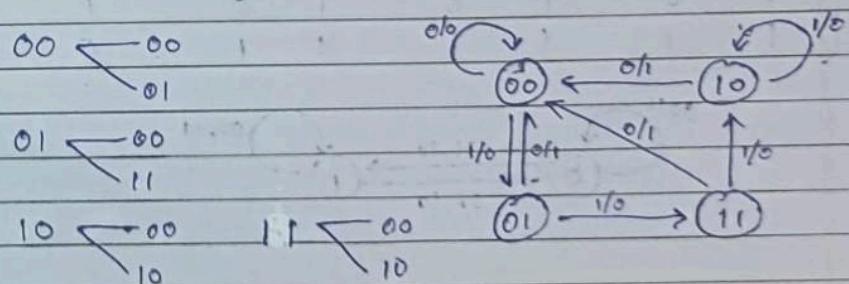
(q i)  $A(t+1) = A(t)z(t) + B(t)\bar{z}(t)$   
 $B(t+1) = \bar{A}(t)z(t)$   
 $y(t) = [A(t) + B(t)]\bar{z}(t)$

A	B	$z$	$A(t+1)$	$B(t+1)$	$y$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

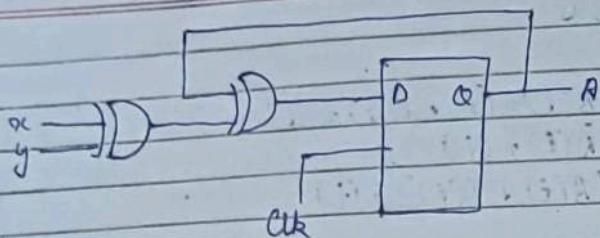
draw state diagram  
using this

→ Next state

		$n=0$	$n=1$	$n=2$	$n=3$
A	B	$A$	$B$	$A$	$B$
0	0	0	0	0	1
0	1	0	0	1	1
1	0	0	0	1	0
1	1	0	1	0	1



Pg ii)



Present state	Inputs	Next state
A	x y	A
0	0 0	0
0	0 1	1
0	1 0	1
0	1 1	0
1	0 0	1
1	0 1	0
1	1 0	0
1	1 1	1

$$A(t+1) = (x \oplus y) \oplus A$$

$$= A \oplus x \oplus y$$

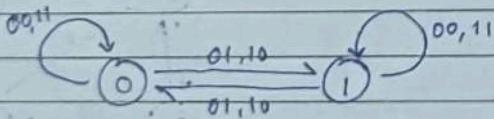
$A=0$

$A=1$

$A=0$

$A=1$

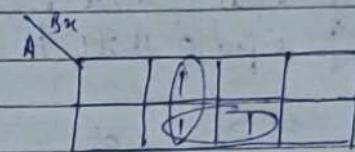
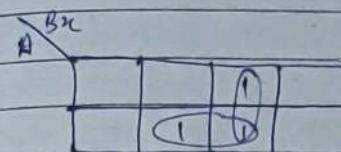
x	y	x	y	x	y	$A(t+1)$	$A(t+1)$
0	0	0	0	0	0	0	1
0	1	0	1	0	1	1	0
1	0	1	0	1	0	1	0
1	1	1	1	1	1	0	1



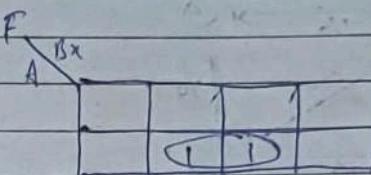
Pg iii)

$D_A = 3, 5, 7$

$D_B = 1, 5, 7$



$F = 5, 7$



$$D_A = Ax + Bx$$

$$D_B = Ax + \bar{B}x$$

$$F = Ax$$

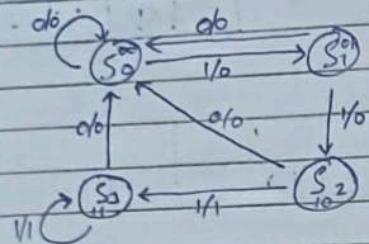
\* If D mentioned used that else default is D FF

Page \_\_\_\_\_  
Date \_\_\_\_\_

Eg iii) If consecutive 3(1s) then move to S<sub>3</sub>

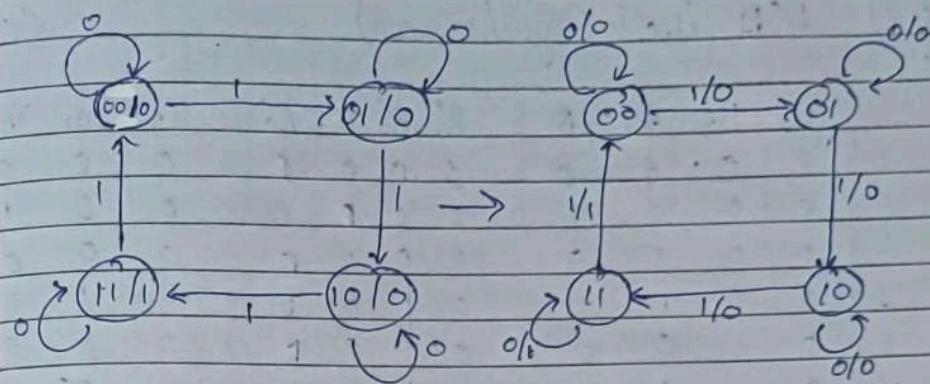
If input 1 move forward

If input 0 move to S<sub>0</sub>



Present A B	Input n	A(t+1)		B(t+1)		y
		0	1	0	1	
0 0	0	0	0	0	0	0
0 0	1	0	1	0	0	0
0 1	0	0	0	0	0	0
0 1	1	1	0	0	0	0
1 0	0	0	0	0	0	0
1 0	1	1	1	1	1	1
1 1	0	0	0	0	0	0
1 1	1	1	1	1	1	1

Eg iii)



andar 1 hai to wki se jarr wali sabhe root ke y = 1 hoga

A	B	n	A'	B'	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	a	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1

Pg v) BCD decade (synchronous)

	$B_3$	$B_2$	$B_1$	$B_0$	$B_3'$	$B_2'$	$B_1'$	$B_0'$	$T_{B_3}$	$T_{B_2}$	$T_{B_1}$	$T_{B_0}$
0	0	0	0	0	0	0	0	1	0	0	0	1
1	0	0	0	1	0	0	1	0	0	0	1	1
2	0	0	1	0	0	0	1	1	0	0	0	1
3	0	0	1	1	0	1	0	0	0	1	1	1
4	0	1	0	0	0	1	0	1	0	0	0	1
5	0	1	0	1	0	1	1	0	0	0	1	1
6	0	1	1	0	0	1	1	1	0	0	0	1
7	0	1	1	1	1	0	0	0	1	1	1	1
8	1	0	0	0	1	0	0	1	0	0	0	1
9	1	0	0	1	0	0	0	0	1	0	1	1
10	1	0	1	0	X	X	K	K	X	X	K	K
11	1	0	1	1	X	K	K	X	X	K	X	K
12	1	1	0	0	X	K	K	K	X	X	K	K
13	1	1	0	1	X	X	X	X	X	X	K	K
14	1	1	1	0	X	X	X	X	X	K	X	K
15	1	1	1	1	X	X	X	X	K	K	X	X

$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4$   
 $\uparrow$                     $\downarrow$   
 $9 \leftarrow 8 \leftarrow 7 \leftarrow 6 \leftarrow 5$

$T_{B_3}$	$B_3 B_2$	$B_3 B_2$

$T_{B_3}$	$B_3 B_2$	$B_3 B_2$

$T_{B_3}$	$B_3 B_2$	$B_3 B_2$	$T_{B_0}$	$B_1 B_0$	$B_1 B_0$
1	1	1	1	1	1
1	1	1	1	1	1
X	X	X	X	X	X
X	X	X	X	X	X

$$T_{B_3} = B_2 B_0 + B_2 B_1 B_0$$

$$T_{B_2} = B_1 B_0$$

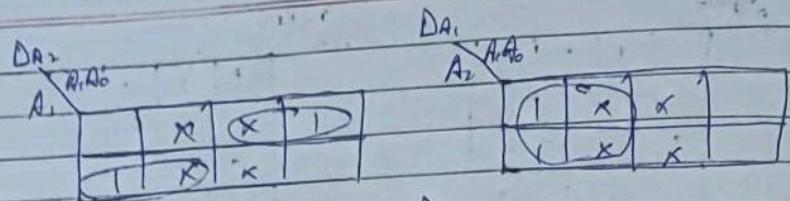
$$T_{B_1} = B_0 \bar{B}_2$$

$$T_{B_0} = 1$$

Pg vi)  $0 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 0$

$000 \rightarrow 010$   
 $\uparrow$                     $\downarrow$   
 $110 \leftarrow 100$

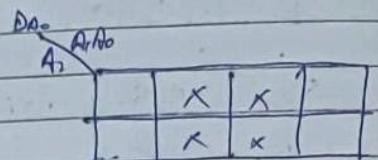
Present	Next			
$A_3 A_2 A_1 A_0$	$A_3' A_2' A_1' A_0'$	$D_3$	$D_2$	$D_1$
0 0 0 0	0 1 0 0	0	1	0
0 0 1 0	X X X	X	X	X
0 1 0 0	1 0 0 0	1	0	0
0 1 1 0	X X X	X	X	X
1 0 0 0	1 1 0 0	1	1	0
1 0 1 0	X X X	X	X	X
1 1 0 0	0 0 0 0	0	0	0
1 1 1 0	X X X	X	X	X



$$D_{A_2} = A_2 \bar{A}_1 + \bar{A}_2 A_1 = A_2 \oplus A_1$$

$$D_{A_1} = \bar{A}_1$$

$$D_{A_0} = 1$$



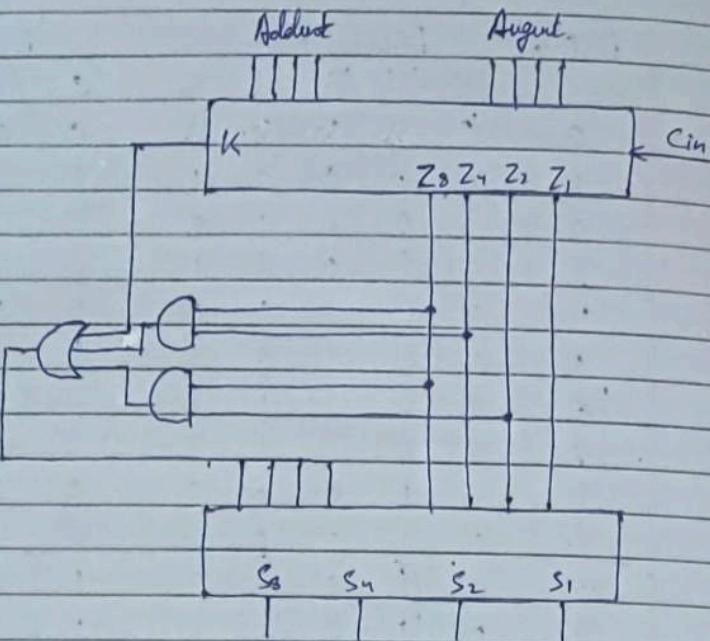
(55)

### 4 bit BCD adder

Decimal	Binary Sum	BCD sum
	C' S <sub>3</sub> S <sub>2</sub> S <sub>1</sub> S <sub>0</sub>	C S <sub>3</sub> S <sub>2</sub> S <sub>1</sub> S <sub>0</sub>
0	0 0 0 0 0	0 0 0 0 0
1	0 0 0 0 1	0 0 0 0 1
2	0 0 0 1 0	0 0 0 1 0
3	0 0 0 1 1	0 0 0 1 1
4	0 0 1 0 0	0 0 1 0 0
5	0 0 1 0 1	0 0 1 0 1
6	0 0 1 1 0	0 0 1 1 0
7	0 0 1 1 1	0 0 1 1 1
8	0 1 0 0 0	0 1 0 0 0
9	0 1 0 0 1	0 1 0 0 1
10	0 1 0 1 0	0 0 0 0 0
11	0 1 0 1 1	0 0 0 0 1
12	0 1 1 0 0	0 0 0 1 0
13	0 1 1 0 1	0 0 0 1 1
14	0 1 1 1 0	0 0 1 0 0
15	0 1 1 1 1	0 0 1 0 1
16	1 0 0 0 0	0 1 1 0 0
17	1 0 0 0 1	0 1 1 0 1
18	1 0 0 1 0	0 1 1 0 0
19	1 0 0 1 1	0 1 1 0 1

$$C = K + Z_3 Z_4 + Z_8 Z_2$$

equation for C :  $C = C' + S'_3 S'_4 + S'_8 S'_2$



5 0101

+ 7 0111

----- 1100

$$\begin{array}{r} 0001 | 0010 \\ \hline 110 \end{array} \quad (10010)_{BCD}$$