

$$y'' - 2y' + 5y = e^x \cos(2x)$$

$$r^2 - 2r + 5 = 0$$

$$\frac{2 \pm \sqrt{4 - 4(5)(1)}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2} \rightsquigarrow \begin{matrix} 1 + 2i \\ 1 - 2i \end{matrix}$$

$$y_{(H)} = C_1 e^{(1+2i)x} + C_2 e^{(1-2i)x}$$

$$C_1 e^x (\cos 2x + i \sin 2x) + C_2 e^x (\cos 2x - i \sin 2x)$$

$$C_1 e^x \cos 2x + C_1 i e^x \sin 2x + C_2 e^x \cos 2x - C_2 i e^x \sin 2x$$

$$e^x \cos 2x (C_1 + C_2) + e^x \sin 2x (C_1 - C_2)i$$

$$C_3 e^x \cos 2x + e^x \sin 2x \times C_4$$

$$y_H = e^x (C_3 \cos 2x + C_4 \sin 2x)$$

