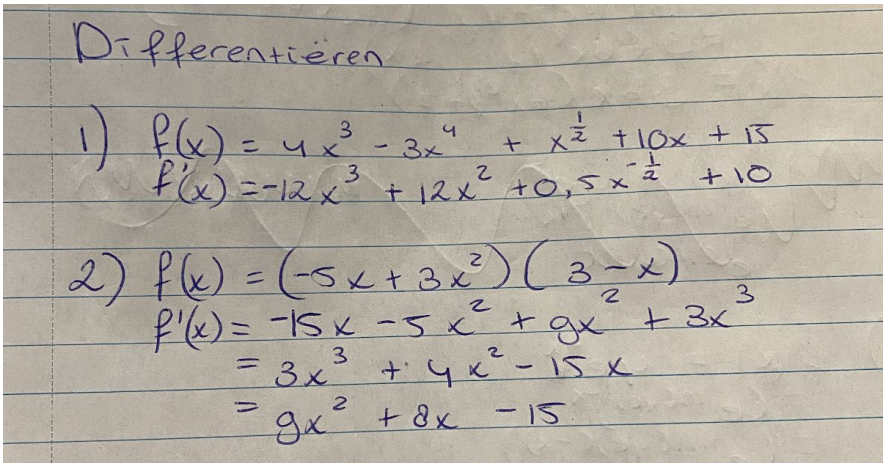
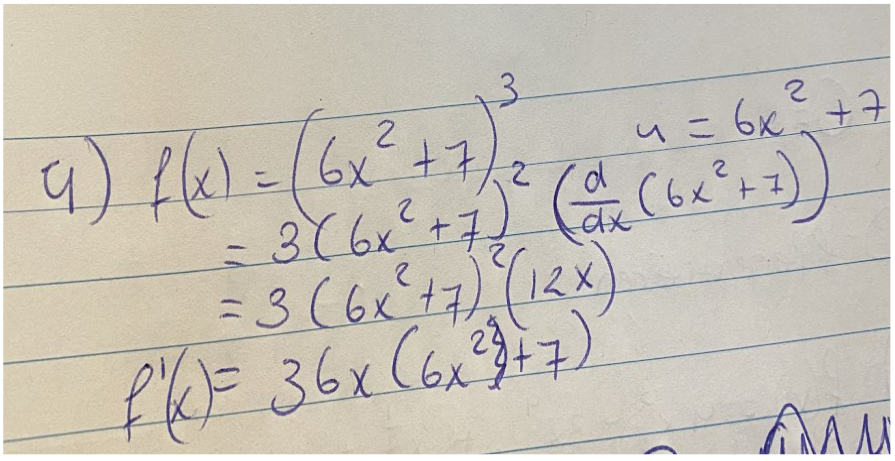
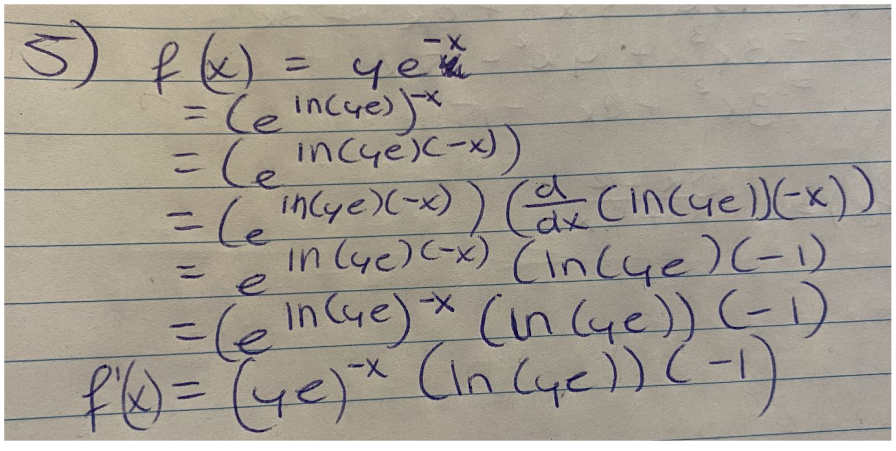
Opdrachten lecture 2

1) Differentieer de volgende functies.

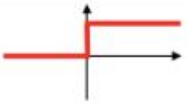
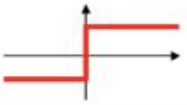
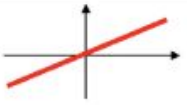
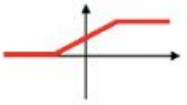
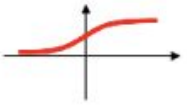
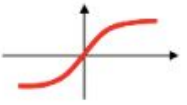
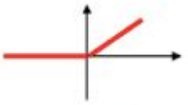
- $f(x) = 4x^3 - 3x^4 + x^{\frac{1}{2}} + 10x + 15$
- $f(x) = (-5x + 3x^2)(3 - x)$
- $f(x) = \frac{5x+2}{2+4x}$
- $f(x) = (6x^2 + 7)^3$
- $f(x) = 4e^{-x}$







2) Differentieer een aantal activatiefuncties.

Activation function	Equation	Example	1D Graph
Unit step (Heaviside)	$\phi(z) = \begin{cases} 0, & z < 0, \\ 0.5, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Sign (Signum)	$\phi(z) = \begin{cases} -1, & z < 0, \\ 0, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Linear	$\phi(z) = z$	Adaline, linear regression	
Piece-wise linear	$\phi(z) = \begin{cases} 1, & z \geq \frac{1}{2}, \\ z + \frac{1}{2}, & -\frac{1}{2} < z < \frac{1}{2}, \\ 0, & z \leq -\frac{1}{2}, \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multi-layer NN	
Hyperbolic tangent	$\phi(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$	Multi-layer Neural Networks	
Rectifier, ReLU (Rectified Linear Unit)	$\phi(z) = max(0, z)$	Multi-layer Neural Networks	
Rectifier, softplus	$\phi(z) = \ln(1 + e^{z})$	Multi-layer Neural Networks	

Sigmoid

$$\phi(z) = \frac{1}{1+e^{-z}} \qquad f' = 0 \qquad g' = -e^{-z}$$

$$\phi'(z) = \frac{0 \cdot (1+e^{-z}) - 1 \cdot (e^{-z})}{(1+e^{-z})^2}$$

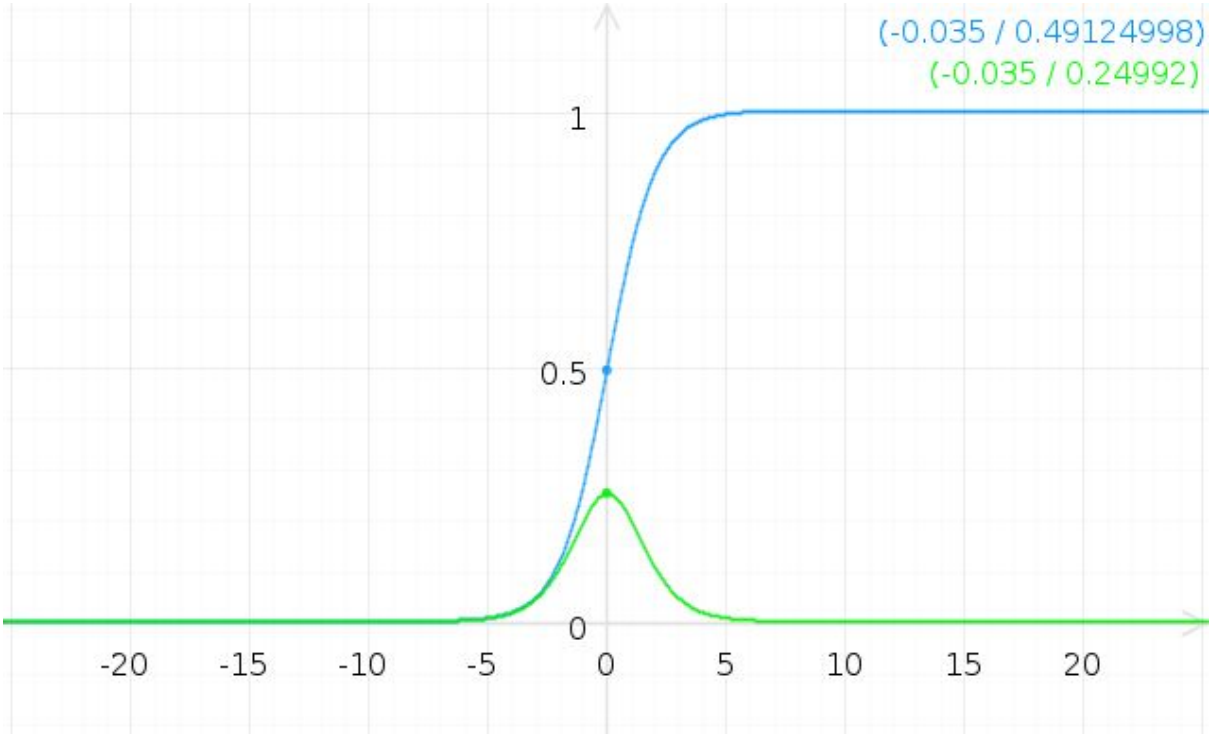
$$= \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1-1+e^{-z}}{(1+e^{-z})^2}$$

$$= \frac{1+e^{-z}}{(1+e^{-z})^2} - \frac{1}{(1+e^{-z})^2}$$

$$= \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2}$$

$$= \frac{1}{1+e^{-z}} \cdot \left(1 - \frac{1}{1+e^{-z}}\right)$$

$$\phi' = \phi(z)(1-\phi(z))$$



Tanh

$$\phi(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$\phi'(z) = \frac{(e^z + e^{-z})(e^z + e^{-z}) - (e^z - e^{-z})(e^z - e^{-z})}{(e^z + e^{-z})^2}$$

$$= \frac{(e^z + e^{-z})^2}{(e^z + e^{-z})^2} - \frac{(e^z - e^{-z})^2}{(e^z + e^{-z})^2}$$

$$= 1 - \left(\frac{e^z - e^{-z}}{e^z + e^{-z}}\right)^2$$

$$= 1 - (\phi(z))^2$$