

Opdrachten lecture 2

1) Differentieer de volgende functies.

- $f(x) = 4x^3 - 3x^4 + x^{\frac{1}{2}} + 10x + 15$
- $f(x) = (-5x + 3x^2)(3 - x)$
- $f(x) = \frac{5x+2}{2+4x}$
- $f(x) = (6x^2 + 7)^3$
- $f(x) = 4e^{-x}$

Differentiëren


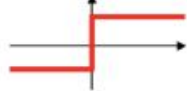
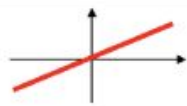


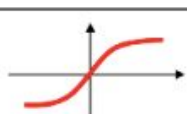

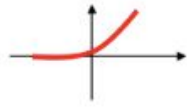
1) $f(x) = 4x^3 - 3x^4 + x^{\frac{1}{2}} + 10x + 15$
 $f'(x) = 12x^2 - 12x^3 + 0,5x^{-\frac{1}{2}} + 10$

2) $f(x) = (-5x + 3x^2)(3 - x)$
 $f'(x) = -5x - 5x^2 + 6x^2 + 3x^3$
 $= 3x^3 + x^2 - 5x$
 $= 3x^3 + x^2 - 5x$

4) $f(x) = (6x^2 + 7)^3$
 $u = 6x^2 + 7$
 $f'(x) = 3(6x^2 + 7)^2 \left(\frac{d}{dx}(6x^2 + 7) \right)$
 $= 3(6x^2 + 7)^2 (12x)$
 $f'(x) = 36x(6x^2 + 7)^2$

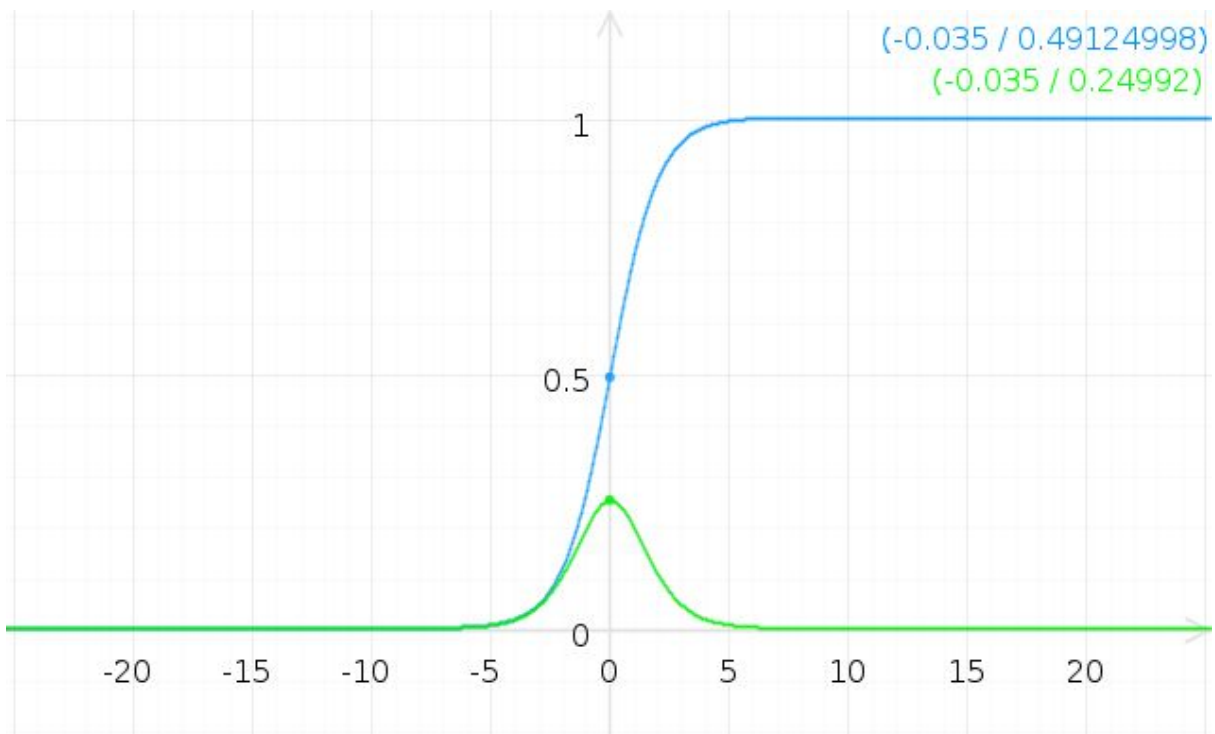
5) $f(x) = 4e^{-x}$
 $= (e^{\ln(4e)})^{-x}$
 $= (e^{\ln(4e)(-x)})$
 $= (e^{\ln(4e)(-x)}) \left(\frac{d}{dx}(\ln(4e))(-x) \right)$
 $= e^{\ln(4e)(-x)} (\ln(4e))(-1)$
 $= (e^{\ln(4e)})^{-x} (\ln(4e))(-1)$
 $f'(x) = (4e)^{-x} (\ln(4e))(-1)$

2) Differentieer een aantal activatiefuncties.

Activation function	Equation	Example	1D Graph
Unit step (Heaviside)	$\phi(z) = \begin{cases} 0, & z < 0, \\ 0.5, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Sign (Signum)	$\phi(z) = \begin{cases} -1, & z < 0, \\ 0, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Linear	$\phi(z) = z$	Adaline, linear regression	
Piece-wise linear	$\phi(z) = \begin{cases} 1, & z \geq \frac{1}{2}, \\ z + \frac{1}{2}, & -\frac{1}{2} < z < \frac{1}{2}, \\ 0, & z \leq -\frac{1}{2}, \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multi-layer NN	
Hyperbolic tangent	$\phi(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$	Multi-layer Neural Networks	
Rectifier, ReLU (Rectified Linear Unit)	$\phi(z) = \max(0, z)$	Multi-layer Neural Networks	
Rectifier, softplus	$\phi(z) = \ln(1 + e^z)$	Multi-layer Neural Networks	

Sigmoid

$$\phi(z) = \frac{1}{1+e^{-z}} \quad \begin{array}{l} p' = 0 \\ g' = -e^{-z} \end{array}$$
$$\phi'(z) = \frac{0 \cdot (1+e^{-z}) - 1 \cdot (e^{-z})}{(1+e^{-z})^2}$$
$$= \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1-1+e^{-z}}{(1+e^{-z})^2}$$
$$= \frac{1+e^{-z}}{(1+e^{-z})^2} - \frac{1}{(1+e^{-z})^2}$$
$$= \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2}$$
$$= \frac{1}{1+e^{-z}} \cdot \left(1 - \frac{1}{1+e^{-z}}\right)$$
$$\phi' = \phi(z)(1 - \phi(z))$$



Tanh

$$\begin{aligned}\phi(z) &= \frac{e^z - e^{-z}}{e^z + e^{-z}} \\ \phi'(z) &= \frac{(e^z + e^{-z})(e^z + e^{-z}) - (e^z - e^{-z})(e^z - e^{-z})}{(e^z + e^{-z})^2} \\ &= \frac{(e^z + e^{-z})^2}{(e^z + e^{-z})^2} - \frac{(e^z - e^{-z})^2}{(e^z + e^{-z})^2} \\ &= 1 - \left(\frac{e^z - e^{-z}}{e^z + e^{-z}} \right)^2 \\ &= 1 - (\phi(z))^2\end{aligned}$$

