MAT1320 LINEAR ALGEBRA EXERCISES IV-V

Name Surname:	**Group No:**
Student No:	**Duration:**
Department:	**Date:** 31 October 2022
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1. What is the determinant of the following matrix?

$$\begin{bmatrix} 2 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & 7 \\ -3 & 2 & 0 & -1 & -6 \\ 2 & -2 & -1 & 1 & 4 \\ 0 & 0 & 0 & 4 & 3 \end{bmatrix}$$

a) 112 b) -56 c) -112 d) -28 e) 28

2. If the following matrix A is invertible, which of the followings is all possible values of k?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$$

a) $\{1, 2\}$ b) $\mathbb{R} - \{1\}$ c) $\mathbb{R} - \{1, 2\}$
d) $\mathbb{R}$ e) There is no such k.

3. Let A and B be two real matrices of the size 3×3. If $\det(A) = \sqrt{3}$ and $\det(B) = \frac{1}{2}$, then what is $\det\left(2A^T B^{-3}\right)$?

a) $32\sqrt{3}$ b) $16\sqrt{3}$ c) 48 d) $4\sqrt{3}$ e) $64\sqrt{3}$

4. Let $A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{bmatrix}$ be an invertible matrix. What is $\text{Adj}\left(A^{-1}\right)$?

a) $\begin{bmatrix} 3/7 & 11/14 & -1/7 \\ 1/7 & -1/14 & 2/7 \\ 3/7 & 2/7 & -1/7 \end{bmatrix}$ b) $\begin{bmatrix} -1/14 & 1/14 & 3/14 \\ 1/7 & 0 & -1/7 \\ 1/14 & 3/14 & -1/7 \end{bmatrix}$

c) $\begin{bmatrix} 3/7 & 1/7 & 3/7 \\ 11/14 & -1/14 & 2/7 \\ -1/7 & 2/7 & -1/7 \end{bmatrix}$ d) $\begin{bmatrix} -1/14 & 1/7 & 1/14 \\ 1/14 & 0 & 3/14 \\ 3/14 & -1/7 & -1/7 \end{bmatrix}$

e) None of them

5. Which of the followings is true for the matrix $A = \begin{pmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{pmatrix}$?

I. A is invertible.

II. The reduced row echelon form of A is I_4.

III. $\text{Adj}(A)A = \begin{pmatrix} 60 & 0 & 0 & 0 \\ 0 & 60 & 0 & 0 \\ 0 & 0 & 60 & 0 \\ 0 & 0 & 0 & 60 \end{pmatrix}$

a) Onyl I b) I and II c) I and III
d) II and III e) I, II and III

6. If $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4$, then what is $\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$?

a) 8 b) 6 c) 4 d) 2 e) 1

7. Let $A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix}$. Recall that A_{ij} is the cofactor of the component a_{ij}. Then, what is $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$?

a) 0 b) -32 c) 32 d) 16 e) -16

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$A_{n\times n}$ matris $\Rightarrow |kA| = k^n |A|$

$|AB| = |A|\cdot|B|$

$|A^T| = |A|$

1.
$$\begin{bmatrix} 2 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & 7 \\ -3 & 2 & 0 & -1 & -6 \\ 2 & -2 & -1 & 1 & 4 \\ 0 & 0 & 0 & 4 & 3 \end{bmatrix}$$
matrisinin determinantı aşağıdakilerden hangisidir?

a) 112 b) -56 c) -112 d) -28 e) 28

$= 7\cdot(-1)^{2+5}\begin{vmatrix} 2 & 0 & 0 & -3 \\ -3 & 2 & 0 & -1 \\ 2 & -2 & -1 & 1 \\ 0 & 0 & 0 & 4 \end{vmatrix}$ (Laplace expansion for 2nd row)

$= -7\cdot 4\cdot(-1)^{4+4}\begin{vmatrix} 2 & 0 & 0 \\ -3 & 2 & 0 \\ 2 & -2 & -1 \end{vmatrix}$ (Laplace expansion for 4th row)

$= -28\cdot 2\,(-1)^{1+1}\begin{vmatrix} 2 & 0 \\ -2 & -1 \end{vmatrix} = -56\cdot -2 = 112$

Note: A^{-1} exist $\Leftrightarrow |A|\neq 0 \Leftrightarrow$ RREF of $A = I_n \Leftrightarrow \operatorname{Rank} A = n$.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{vmatrix} \xrightarrow[r_3\to r_3-r_1]{r_2\to r_2-r_1} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & k-1 \\ 1 & 3 & k^2-1 \end{vmatrix} = 1\cdot A_{11}$$

$= (-1)^{1+1}\begin{vmatrix} 1 & k-1 \\ 3 & k^2-1 \end{vmatrix} = k^2-1-3(k-1)$

$= k^2-3k+2 = (k-2)(k-1) = 0$

$\Rightarrow$ If $k\neq 1$, $k\neq 2$, $|A|\neq 0$ and then A^{-1} exist.

$\Rightarrow \mathbb{R}-\{1,2\}$

2. A terslenebilir bir matris ise k nın tüm değerleri aşağıdakilerden hangisidir?
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$$

a) $\{1,2\}$ b) $\mathbb{R}-\{1\}$ c) $\mathbb{R}-\{1,2\}$

d) $\mathbb{R}$ e) Böyle bir k sayısı yoktur.

3. A ve B, 3×3 boyutlu ve reel bileşenli iki matris olmak üzere $\det(A) = \sqrt{3}$ ve $\det(B) = \frac{1}{2}$ ise $\det\left(2A^TB^{-3}\right)$ değeri aşağıdakilerden hangisidir?

a) $32\sqrt{3}$ b) $16\sqrt{3}$ c) 48 d) $4\sqrt{3}$ e) $64\sqrt{3}$

$|2A^T\cdot B^{-3}| = 2^3\cdot|A^TB^{-3}|$

$= 8\cdot|A^T|\cdot|B^{-3}|$

$= 8\cdot|A|\cdot\left(\frac{1}{|B|}\right)^3$

$= 8\cdot\sqrt{3}\cdot\left(\frac{1}{\frac{1}{2}}\right)^3 = 64\sqrt{3}$

$|B^{-3}| = |B^{-1}B^{-1}B^{-1}| = |B^{-1}|^3 = \left(\frac{1}{|B|}\right)^3$

4) Recall that $A^{-1} = \frac{Adj(A)}{|A|}$

Replacing A by A^{-1}, we get

$(A^{-1})^{-1} = \frac{Adj(A^{-1})}{|A^{-1}|} \Rightarrow Adj(A^{-1}) = |A^{-1}|\cdot A = \frac{1}{|A|}\cdot A$

Then, we need $|A|$.

$$\begin{vmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 4 & 0 & -11 \end{vmatrix} = 1\cdot(-1)^{1+2}\begin{vmatrix} 2 & -2 \\ 4 & -11 \end{vmatrix}$$

$= -(-22+8) = 14 \Rightarrow Adj(A^{-1}) = \frac{1}{14}\cdot A$

4. $A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{bmatrix}$ terslenebilir bir matris olmak üzere Ek (A^{-1}) matrisi aşağıdakilerden hangisidir?

a) $\begin{bmatrix} 3/7 & 11/14 & -1/7 \\ 1/7 & -1/14 & 2/7 \\ 3/7 & 2/7 & -1/7 \end{bmatrix}$ b) $\begin{bmatrix} -1/14 & 1/14 & 3/14 \\ 1/7 & 0 & -1/7 \\ 1/14 & 3/14 & -1/7 \end{bmatrix}$

c) $\begin{bmatrix} 3/7 & 1/7 & 3/7 \\ 11/14 & -1/14 & 2/7 \\ -1/7 & 2/7 & -1/7 \end{bmatrix}$ d) $\begin{bmatrix} -1/14 & 1/7 & 1/14 \\ 1/14 & 0 & 3/14 \\ 3/14 & -1/7 & -1/7 \end{bmatrix}$

e) Hiçbiri

5. $A = \begin{pmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{pmatrix}$ ise aşağıdakilerden hangileri doğrudur?

I. A matrisinin tersi vardır.

II. A matrisinin satırca indirgenmiş eşelon formu I_4 birim matrisidir.

III. $\text{Adj}(A)A = \begin{pmatrix} 60 & 0 & 0 & 0 \\ 0 & 60 & 0 & 0 \\ 0 & 0 & 60 & 0 \\ 0 & 0 & 0 & 60 \end{pmatrix}$

a) Yalnız I b) I ve II c) I ve III
d) II ve III e) I, II ve III

$$|A| = \begin{vmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -3 & -1 & 1 \\ -2 & -6 & 0 & 7 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix}$$

$r_2 \to r_2 + 2r_1$

$$= (-1)\cdot(-1)^{1+3} \begin{vmatrix} -2 & -6 & 7 \\ 3 & -2 & 0 \\ 1 & -4 & 0 \end{vmatrix} = (-1)\cdot 7(-1)^{1+3} \begin{vmatrix} 3 & -2 \\ 1 & -4 \end{vmatrix}$$

$$= -7\cdot(-12+2) = 70$$

Since $|A| = 70 \neq 0$, A^{-1} exists.

Recall that

A^{-1} exists $\Leftrightarrow |A| \neq 0 \Leftrightarrow$ R.R.E.F. is $I_n \Leftrightarrow \text{Rank}\, A = n$

Recall that $A \cdot \text{adj}\, A = |A| \cdot I_n$

$$\Rightarrow A \cdot \text{adj}\, A = \begin{vmatrix} 70 & 0 & 0 & 0 \\ 0 & 70 & 0 & 0 \\ 0 & 0 & 70 & 0 \\ 0 & 0 & 0 & 70 \end{vmatrix}$$

6. $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4$ ise $\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$ değeri aşağıdakilerden hangisidir?

a) 8 b) 6 c) 4 d) 2 e) 1

$$6) \begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$$

$$= \frac{1}{2} \cdot \begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ c_1 & c_2 & 4c_3 - 2c_2 \end{vmatrix}$$

$c_3 \to c_3 + 2c_2$

$$= \frac{1}{2} \cdot \begin{vmatrix} a_1 & a_2 & 4a_3 \\ b_1 & b_2 & 4b_3 \\ c_1 & c_2 & 4c_3 \end{vmatrix} =$$

$$= \frac{4}{2} \cdot \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 2 \cdot 4 = 8$$

7. $A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix}$ matrisi verilsin. A_{ij}, a_{ij} bileşeninin kofaktörü olmak üzere $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$ değeri aşağıdakilerden hangisidir?

a) 0 b) -32 c) 32 d) 16 e) -16

a_{11}, a_{21}, a_{31}: the elements of the first column

A_{12}, A_{22}, A_{32}: the cofactors of the elements of the second column

Since they are different, this sum is equal to 0.