# • Summary for 10th Week Topics

• **Vector field** : $\vec{F}(x,y,z) = M(x,y,z)\vec{i} + N(x,y,z)\vec{j} + P(x,y,z)\vec{k}$

$= \langle M(x,y,z), N(x,y,z), P(x,y,z) \rangle$

• **Gradient vector field** : $\vec{\nabla} f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$

$= \langle f_x, f_y, f_z \rangle$

for a function $f(x,y,z)$.

## • Line Integrals

C : curve

① **Using Arc Length** : • There are a function $f(x,y,z)$ and a curve C.

• Find a parametrization for C.

$$\left.\begin{aligned} x &= g(t) \\ y &= h(t) \\ z &= k(t) \end{aligned}\right\} a \le t \le b$$

• $$\int_C f(x,y,z)\,ds = \int_a^b f(g(t),h(t),k(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

**Note :** If the curve $C = C_1 \cup C_2 \cup \dots \cup C_n$,
then $\int_C f\,ds = \int_{C_1} f\,ds + \int_{C_2} f\,ds + \dots + \int_{C_n} f\,ds$

## ② Using Vector Fields :

- There are a vector field $\vec{F}(x, y, z)$ and a parametrization $\vec{r}(t) = \langle g(t), h(t), k(t) \rangle$, $a \le t \le b$.
- 1st step : Find $\vec{F}(\vec{r}(t)) = \vec{F}(g(t), h(t), k(t))$
- 2nd step : Find $\frac{d\vec{r}}{dt} = r'(t)$
- 3rd Step : Find $\vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt}$

It is dot product!

Then,

$$\boxed{\int_C \vec{F} \cdot d\vec{r} = \int_a^b \left( \vec{F}(\vec{r}(t)) \cdot \frac{dr}{dt} \right) dt}$$

dot Product!

## ③ Using the xyz-coordinates :

- There are $\int_C M(x,y,z)\,dx + N(x,y,z)\,dy + P(x,y,z)\,dz$ and $\vec{r}(t) = g(t)\vec{i} + h(t)\vec{j} + k(t)\vec{k}$, $a \le t \le b$.

- $\int_C M(x,y,z)\,dx + N(x,y,z)\,dy + P(x,y,z)\,dz =$

$$\int_a^b \left[ M(r(t)) \cdot \frac{dg}{dt} + N(r(t)) \cdot \frac{dh}{dt} + P(r(t)) \cdot \frac{dk}{dt} \right] dt$$