DEFINITE INTEGRAL

1) Find the average values of the following functions for given intervals.

a) $f(x)=\sin x$, $[0,\pi]$

$$\bar{f}=\frac{1}{\pi-0}\int_0^\pi \sin x\,dx=\frac{1}{\pi}\left[-\cos x\right]_0^\pi=\frac{1}{\pi}\left[-\cos\pi+\cos 0\right]=\frac{1}{\pi}[1+1]=\frac{2}{\pi}$$

b) $f(x)=x^2$, $[1,4]$

$$\bar{f}=\frac{1}{4-1}\int_1^4 x^2\,dx=\frac{1}{3}\cdot\frac{x^3}{3}\Big|_1^4=\frac{1}{9}[64-1]=\frac{63}{9}=7$$

c) $f(x)=2x+\frac{2}{x}$, $[1,e]$ (Be careful : $0\notin[1,e]$)

$$\bar{f}=\frac{1}{e-1}\int_1^e\left(2x+\frac{2}{x}\right)dx=\frac{1}{e-1}\left[x^2+2\ln x\right]_1^e=\frac{1}{e-1}\left[e^2+2-1\right]=\frac{e^2+1}{e-1}$$

d) $f(x)=\sqrt{x}-\frac{1}{\sqrt{x}}$, $[1,4]$ (Be careful : $0\notin[1,4]$ and $x>0$)

$$\bar{f}=\frac{1}{4-1}\int_1^4\left(x^{1/2}-x^{-1/2}\right)dx=\frac{1}{3}\left[\frac{2}{3}\cdot x^{3/2}-2\cdot x^{1/2}\right]_1^4=\frac{1}{3}\left[\frac{16}{3}-4-\frac{2}{3}+2\right]=\frac{8}{9}$$

e) $f(x)=x(\sin x^2)$, $[0,10]$

$$\bar{f}=\frac{1}{10-0}\int_0^{10}x(\sin x^2)\,dx \qquad \begin{aligned} x^2&=t \\ 2x\,dx&=dt\end{aligned} \qquad \begin{aligned} x=0&\Rightarrow t=0 \\ x=10&\Rightarrow t=\sqrt{10}\end{aligned}$$

$$\bar{f}=\frac{1}{10}\int_0^{\sqrt{10}}\frac{\sin t}{2}\,dt=\frac{1}{20}(-\cos t)\Big|_0^{\sqrt{10}}=\frac{1}{20}\left(1-\cos\sqrt{10}\right)$$

f) $f(x) = -|x|, \quad [-2,1]$

$$\bar{f} = \frac{1}{1-(-2)}\int_{-2}^{1} -|x|\,dx = \frac{1}{3}\left[\int_{-2}^{0} x\,dx + \int_{0}^{1} -x\,dx\right] = \frac{1}{3}\left[\frac{x^2}{2}\Big|_{-2}^{0} - \frac{x^2}{2}\Big|_{0}^{1}\right]$$

$$= \frac{1}{3}\left[0 - \frac{4}{2} - \frac{1}{2} + 0\right] = \frac{1}{3}\cdot\frac{(-5)}{2} = -\frac{5}{6}$$

g) $f(x) = |x^2-1| + 2, \quad [-2,1]$

		-1		1	
x^2-1	$+$		$-$		$+$

$$\bar{f} = \frac{1}{1-(-2)}\int_{-2}^{1}(|x^2-1|+2)\,dx = \frac{1}{3}\left[\int_{-2}^{-1}[(x^2-1)+2]\,dx + \int_{-1}^{1}[(1-x^2)+2]\,dx\right]$$

$$= \frac{1}{3}\left[\int_{-2}^{-1}(x^2+1)\,dx + \int_{-1}^{1}(3-x^2)\,dx\right] = \frac{1}{3}\left[\left[\frac{x^3}{3}+x\right]_{-2}^{-1} + \left[3x - \frac{x^3}{3}\right]_{-1}^{1}\right]$$

$$= \frac{1}{3}\left[\left(\cancel{-\frac{1}{3}}\cancel{-1} + \frac{8}{3} + \cancel{2}\right) + \left(3 - \cancel{\frac{1}{3}} + 3 - \cancel{\frac{1}{3}}\right)\right] = \frac{1}{3}\left(\frac{8}{3}+6\right) = \frac{26}{9}$$

2) If $\int_{0}^{2} f(x)\,dx = 4$, what is the average value of $g(x) = 3f(x)$ on $[0,2]$?

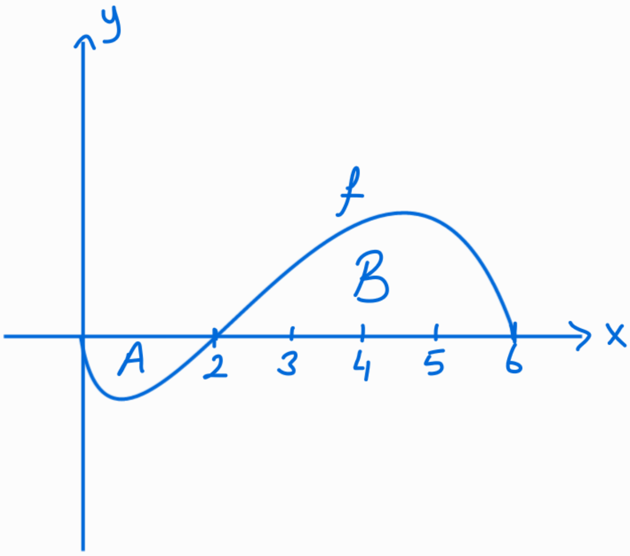
$$\bar{g} = \frac{1}{2-0}\int_{0}^{2} 3f(x)\,dx = \frac{1}{2}\cdot 3\int_{0}^{2} f(x)\,dx = \frac{3}{2}\cdot 4 = 6$$

3) Let $\int_{0}^{6} f(x)\,dx = 10$ and $\int_{0}^{4} f(x)\,dx = 7$. Evaluate $\int_{4}^{6} f(x)\,dx$.

We can write $\underbrace{\int_{0}^{6} f(x)\,dx}_{10} = \underbrace{\int_{0}^{4} f(x)\,dx}_{7} + \int_{4}^{6} f(x)\,dx$

$$\Rightarrow \int_{4}^{6} f(x)\,dx = 10 - 7 = 3$$

4) The graph of f is shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)dx = 3.5$. Use this information to answer the following.



a) $\int_0^2 f(x)dx$

b) $\int_2^6 f(x)dx$

c) $\int_0^6 |f(x)|dx$

d) $\int_0^2 -2f(x)dx$

e) $\int_0^6 [2+f(x)]dx$

f) The average value of f over the interval $[0,6]$.

$$A = \left| \int_0^2 f(x)dx \right| = -\int_0^2 f(x)dx \qquad B = \int_2^6 f(x)dx$$

$$\int_0^6 f(x)dx = \int_0^2 f(x)dx + \int_2^6 f(x)dx = -\underset{1.5}{\underset{\downarrow}{A}} + B = 3.5$$

$$\Rightarrow B = 5$$

a) $-A = -1.5$ b) $B = 5$ c) $A+B = 6.5$ d) $2A = 3$

e) $\int_0^6 2dx + \int_0^6 f(x)dx = 2x\Big|_0^6 + 3.5 = 12 + 3.5 = 15.5$

f) $\bar{f} = \frac{1}{6-0}\int_0^6 f(x)dx = \frac{1}{6}(3.5) = \frac{7}{12}$

5) Given $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, evaluate

a) $\int_0^6 f(x)dx$ b) $\int_6^3 f(x)dx$ c) $\int_3^6 -10 f(x)dx$

a) $\int_0^6 f(x)dx = \int_0^3 f(x)dx + \int_3^6 f(x)dx = 4 - 1 = 3$

b) $\int_6^3 f(x)dx = -\int_3^6 f(x)dx = -(-1) = 1$

c) $\int_3^6 -10f(x)dx = -10\int_3^6 f(x)dx = -10\cdot(-1) = 10$

6) Calculate the value of the definite integral of $I_1 = \int_3^5 \frac{3x}{x-2}dx$ if $I_2 = \int_3^5 \frac{dx}{x-2} = m$.

$$I_1 = \int_3^5 \frac{3x}{x-2}dx = \int_3^5 \frac{3x-6+6}{x-2}dx = \int_3^5 \frac{3\cancel{(x-2)}}{\cancel{x-2}}dx + 6\underbrace{\int_3^5 \frac{dx}{x-2}}_{m}$$

$$= \int_3^5 3dx + 6m = 3x\Big|_3^5 + 6m = 6 + 6m$$

7) The curve $f(x)$ is tangent to $y=x$ in the origin and its second derivative is $2x+1$. Evaluate the value of $f(1)$.

Curve is tangent to $y=x$ in the origin (at $x=0$, $y=0$)
$\Leftrightarrow f(0)=0$ and $f'(0)=1$ (slope of $y=x$ at $x=0$).

$f''(x) = 2x+1 \Rightarrow f'(x) = \int(2x+1)dx = x^2+x+C_1$ and $f'(0) = C_1 = 1$

$f(x) = \int(x^2+x+1)dx = \frac{x^3}{3} + \frac{x^2}{2} + x + C_2$ and $f(0) = C_2 = 0$

$\Rightarrow f(x) = \frac{x^3}{3} + \frac{x^2}{2} + x \Rightarrow f(1) = \frac{1}{3} + \frac{1}{2} + 1 = \frac{11}{6}$

8) Calculate the value of $f(x=e)$ if the derivative of $f(x^2)$ with respect to x is $\frac{6}{x}$ and $f(x=1)=0$.

$$\frac{d}{dx}f(x^2)=2x.f'(x^2)=\frac{6}{x} \Rightarrow f'(x^2)=\frac{3}{x^2}$$

We need $f(x)$, let $x^2=t \Rightarrow f'(t)=\frac{3}{t}$

Integrating both sides, we have $f(t)=3\ln|t|+c$

Since $f(1)=0 \Rightarrow 3\ln 1+c=0 \Rightarrow c=0 \Rightarrow f(e)=3\ln e=3$

9) Calculate the value of $f(x=-1)$ if $f'(\cos^2 x)=\cos 2x$ and $f(x=1)=1$.

$f'(\cos^2 x)=\cos 2x=2\cos^2 x-1$. Let $\cos^2 x=t$

$$f'(t)=2t-1 \xRightarrow{\text{Integrate}} \int f'(t)dt=\int(2t-1)dt \Rightarrow f(t)=t^2-t+c$$

$$f(1)=1-1+c=1 \Rightarrow c=1 \Rightarrow f(t)=t^2-t+1 \Rightarrow f(-1)=1+1+1=3$$

10) $\int_0^1 \frac{d}{dx}(x.e^x)dx=?$

$$x.e^x\Big|_0^1=1.e^1-0.e^0=e$$

11) $\frac{d}{dx}\int_{-3}^{3}(e^x+\sin x^2-\arctan x^3)dx=?$

Result: 0. Because, definite integral produces a number (constant) and its derivative is 0.

12) If $F(x)=\int_{\cos x}^{\sin x}\sqrt{1-t^2}\,dt$, $F'(x)=?$ $\left(0<x<\frac{\pi}{2}\right)$

$$F'(x)=\cos x.\underbrace{\sqrt{1-\sin^2 x}}_{|\cos x|=\cos x}-(-\sin x).\underbrace{\sqrt{1-\cos^2 x}}_{|\sin x|=\sin x}=\cos^2 x+\sin^2 x=1$$

(13) If $F(x)=\int_{e^{-x}}^{e^{x}} \ln t\,dt$, $F'(1)=?$

$$F'(x)=e^{x}\cdot\underbrace{\ln e^{x}}_{x}-(-e^{-x})\cdot\underbrace{\ln e^{-x}}_{-x}=x\cdot e^{x}-x\cdot e^{-x}\Rightarrow F'(1)=e-\frac{1}{e}.$$

(14) Let F be a continuous function which satisfies the equation $F(x)=\frac{1}{x}\int_{1}^{x^2}\left[e^{1-\sqrt{t}}-F'(\sqrt{t})\right]dt$ where $x\geq 1$. Evaluate the value of $F'(1)$.

$$F'(x)=-\frac{1}{x^2}\cdot\int_{1}^{x^2}\left[e^{1-\sqrt{t}}-F'(\sqrt{t})\right]dt+\frac{1}{\cancel{x}}\cdot 2\cancel{x}\left[e^{1-x}-F'(x)\right]$$

$$\Rightarrow F'(1)=-1\cdot\underbrace{\int_{1}^{1}\left[e^{1-\sqrt{t}}-F'(\sqrt{t})\right]dt}_{0}+2\cdot\left(e^{0}-F'(1)\right)\Rightarrow F'(1)=\frac{2}{3}.$$

(15) If $f(x)=e^{g(x)}$ and $g(x)=\int_{x^2}^{x^3}\frac{t^2\,dt}{1+t^2}$, find the value of $f'(1)$.

$$f'(x)=g'(x)\cdot e^{g(x)} \qquad g(1)=\int_{1}^{1}\frac{t^2\,dt}{1+t^2}=0 \text{ (1=1 (upper and lower))}$$

$$g'(x)=3x^2\cdot\frac{x^6}{1+x^6}-2x\cdot\frac{x^4}{1+x^4}\Rightarrow g'(1)=3\cdot\frac{1}{2}-2\cdot\frac{1}{2}=\frac{1}{2}$$

$$\Rightarrow f'(1)=\frac{1}{2}\cdot e^{0}=\frac{1}{2}.$$

(16) Find the value of $(f^{-1})'(0)$ where $f:[0,\infty)\to\mathbb{R}$, $f(x)=\int_{1}^{e^{x}}\frac{dt}{\sqrt{1+t^2}}$, considering that f^{-1} exists.

$$f^{-1}(a)=0\Rightarrow f(0)=\int_{1}^{e^{a}}\frac{dt}{\sqrt{1+t^2}}=0\Rightarrow e^{a}=1\Rightarrow a=0$$

$$f'(x)=e^{x}\frac{1}{\sqrt{1+e^{2x}}}\Rightarrow f'(0)=e^{0}\cdot\frac{1}{\sqrt{1+1}}=\frac{1}{\sqrt{2}} \qquad (f^{-1})'(0)=\frac{1}{f'(0)}=\frac{1}{\frac{1}{\sqrt{2}}}=\sqrt{2}.$$

17) Let $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$ and $g(x)=\int_0^{\cos x}[1+\sin t^2]dt$. $f'\left(\frac{\pi}{2}\right)=?$

$$f'(x)=g'(x)\cdot\frac{1}{\sqrt{1+g^3(x)}} \Rightarrow f'\left(\frac{\pi}{2}\right)=g'\left(\frac{\pi}{2}\right)\cdot\frac{1}{\sqrt{1+g^3\left(\frac{\pi}{2}\right)}}$$

$$g\left(\frac{\pi}{2}\right)=\int_0^{\cos 0=0}[1+\sin t^2]dt=0 \qquad g'(x)=-\sin x\cdot\left[1+\sin(\cos^2 x)\right]$$

$$g'\left(\frac{\pi}{2}\right)=-\sin\frac{\pi}{2}\cdot\left[1+\underbrace{\sin(\underbrace{\cos^2\frac{\pi}{2}}_{0})}_{0}\right]=-1$$

$$\Rightarrow f'\left(\frac{\pi}{2}\right)=\frac{-1}{\sqrt{1+0}}=-1$$

18) Find the function $f(x)$ that satisfies the equation $\int_0^x f(t)dt=x.e^{2x}+\int_0^x e^{-t}f(t)dt$, $\forall x\in\mathbb{R}$ where f is continuous.

Take the derivative of both sides.

$$f(x)=e^{2x}+2xe^{2x}+e^{-x}f(x) \Rightarrow f(x)-e^{-x}f(x)=e^{2x}(2x+1)$$

$$\Rightarrow f(x)=\frac{e^{2x}(2x+1)}{1-\frac{1}{e^x}}=\frac{e^{2x}(2x+1)}{\frac{e^x-1}{e^x}}=\frac{e^{3x}(2x+1)}{e^x-1}$$

19) Let $f(x)\neq 0$ be a differentiable function and $\int_0^x f(t)dt=[f(x)]^2$, $\forall x\in\mathbb{R}$. Find the function $f(x)$.

Take the derivative of both sides.

$$\cancel{f(x)}=2.\cancel{f(x)}.f'(x) \Rightarrow f'(x)=\frac{1}{2} \quad \text{Integrate } (0 \text{ to } x)$$

$$\int_0^x f'(t)dt=\int_0^x\frac{dt}{2} \Rightarrow f(x)-f(0)=\frac{x}{2} \Rightarrow f(x)=\frac{x}{2}+f(0)$$

Let $x=0$. $\int_0^0 f(t)dt=[f(0)]^2 \Rightarrow f(0)=0$.

$$\Rightarrow f(x)=\frac{x}{2}.$$

(20) Let $f(x)=\int_1^x g(t)dt$ and $g(t)=\int_1^{t^2} \frac{1+u^4}{u}du$. $f''(2)=?$

$$f'(x)=g(x) \text{ and } f''(x)=g'(x)=2x\cdot\frac{\sqrt{1+x^8}}{x^2}$$

$$f''(2)=\cancel{4}\cdot\frac{\sqrt{1+256}}{\cancel{4}}=\sqrt{257}$$

(21) $\frac{d^2}{dx^2}\int_0^x\left(\int_1^{\sin t}\sqrt{1+u^4}\,du\right)dt=?$

$$\frac{d^2}{dx^2}\int_0^x\left(\int_1^{\sin t}\sqrt{1+u^4}\,du\right)dt=\frac{d}{dx}\int_1^{\sin x}\sqrt{1+u^4}\,du=\cos x\sqrt{1+\sin^4 x}$$

(22) Let $f(x)=\int_{-1}^x (t^2+9)^{\sin t}dt$ $(x>-1)$. $f''(0)=?$

$$f'(x)=(x^2+9)^{\sin x} \Rightarrow \ln f'(x)=\sin x.\ln(x^2+9)$$

$$\frac{f''(x)}{f'(x)}=\cos x.\ln(x^2+9)+\sin x.\frac{2x}{x^2+9}$$

$$f''(x)=(x^2+9)^{\sin x}.\left[\cos x.\ln(x^2+9)+\sin x.\frac{2x}{x^2+9}\right]\Rightarrow f''(0)=\ln 9.$$

(23) Let $f(x)=\int_2^{x+1}(t-1)^t dt$ where $x>1$. Find the value of $f''(1)$.

$$f'(x)=1.(x+1-1)^{x+1}=x^{x+1} \Rightarrow \ln f'(x)=(x+1).\ln x$$

$$\frac{f''(x)}{f'(x)}=\ln x+\frac{x+1}{x} \Rightarrow f''(x)=x^{x+1}\left[\ln x+\frac{x+1}{x}\right]\Rightarrow f''(1)=2.$$

(24) Suppose that f is a function whose first and second derivatives exist and satisfies the equation

$$f(x)=(3x^2-1)+(2-2x)\int_0^x t^2 f(t)dt+2x\int_x^1(t-t^2)f(t)dt.$$

Find the value of $f''(0)$.

$$f'(x)=6x-2\int_0^x t^2 f(t)dt+\cancel{(2-2x)}.x^2 f(x)+2\int_x^1(t-t^2)f(t)dt-2x\cancel{(x-x^2)}f(x)$$

$$f''(x)=6-2\cancel{x^2 f(x)}dx-2xf(x)+2\cancel{x^2 f}(x)=6-2x.f(x)\Rightarrow f''(0)=6.$$

(25) A function f is defined as $f(x)=3+\int_0^x \frac{1+\sin t}{2+t^2}\,dt,\ \forall x\in\mathbb{R}$.

Find the polynomial $p(x)=ax^2+bx+c$ such that $f(0)=p(0)$, $f'(0)=p'(0)$, and $f''(0)=p''(0)$.

$$f(0)=3+\int_0^0 \frac{1+\sin t}{2+t^2}\,dt=3 \Rightarrow p(0)=a.0+b.0+c=c=3$$

$$f'(x)=\frac{1+\sin x}{2+x^2} \Rightarrow f'(0)=\frac{1+0}{2+0}=\frac{1}{2} \Rightarrow p'(x)=2ax+b \Rightarrow p'(0)=b=\frac{1}{2}$$

$$f''(x)=\frac{\cos x.(2+x^2)-(1+\sin x).2x}{(2+x^2)^2} \Rightarrow f''(0)=\frac{2-0}{4}=\frac{1}{2} \Rightarrow p''(x)=2a$$

$$\Rightarrow p''(0)=2a=\frac{1}{2} \Rightarrow a=\frac{1}{4} \Rightarrow p(x)=\frac{x^2}{4}+\frac{x^2}{2}+3.$$

(26) Find the intervals of which the function $F(x)=\int_0^x t.e^{-t}\,dt$ is increasing, decreasing, concave up, and concave down.

$F'(x)=x.e^{-x}=0 \Rightarrow x=0$: critical point

$F''(x)=e^{-x}-x.e^{-x}=e^{-x}(1-x)=0 \Rightarrow x=1$: inflection point

	$-\infty$	0		1	∞
F'	$-$	0	$+$		$+$
F''	$+$		$+$	0	$-$
F	$\searrow$ $\cup$	local min	$\nearrow$ $\cup$	inflection	$\nearrow$ $\cap$

$(-\infty,0)$: decreasing

$(0,\infty)$: increasing

$(-\infty,1)$: concave up

$(1,\infty)$: concave down

(27) $\lim_{x\to 0}\frac{1}{x}\int_2^{2+x}\sqrt{1+t^3}\,dt=?$ $(\infty.0)$

$$\lim_{x\to 0}\frac{\int_2^{2+x}\sqrt{1+t^3}\,dt}{x} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0}\frac{\sqrt{1+(2+x)^3}}{1}=\sqrt{9}=3.$$

(28) $\lim_{x\to 3}\frac{x}{x-3}\int_3^x\frac{\sin t}{t}\,dt=?$ $(\infty.0)$

$$\lim_{x\to 3}\frac{x.\int_3^x\frac{\sin t}{t}\,dt}{x-3} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 3}\frac{\int_3^x\frac{\sin t}{t}\,dt+\cancel{x}.\frac{\sin x}{\cancel{x}}}{1}=\sin 3.$$

29) $\lim\limits_{x\to 0} \frac{1}{x^4} \int_0^{x^2} \frac{\ln(1+t^2)}{t}\, dt = ?$ $(\infty . 0)$

$$\lim_{x\to 0} \frac{\int_0^{x^2} \frac{\ln(1+t^2)}{t}\, dt}{x^4} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0} \frac{2x . \frac{\ln(1+x^4)}{x^2}}{4x^3} = \lim_{x\to 0} \frac{\ln(1+x^4)}{2x^4}$$

$$\underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0} \frac{4\cancel{x^3}}{2.(1+x^4).4\cancel{x^3}} = \frac{1}{2}.$$

30) $\lim\limits_{x\to 0^+} \frac{1}{2x} \int_0^{x^2} \ln\sqrt{t}\, dt = ?$ $(\infty . 0)$

$$\lim_{x\to 0^+} \frac{\int_0^{x^2} \ln\sqrt{t}\, dt}{2x} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0^+} \frac{\cancel{2}x . \ln x}{\cancel{2}} \overset{(0.\infty)}{=} \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \underset{(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{x\to 0^+} \frac{\frac{1}{\cancel{x}}}{-\frac{1}{x^{\cancel{2}}}} = 0.$$

31) $\lim\limits_{x\to 1} \frac{2-2x-\int_1^{x^4} \sec^{2024}(t-1)dt}{x^{2024}-1} = ?$

$$\underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 1} \frac{-2-4x^3 \sec^{2024}(x^4-1)}{2024\, x^{2023}} = \frac{-2-4.1}{2024} = \frac{-6}{2024} = \frac{-3}{1012}.$$

32) $\lim\limits_{x\to 1} \frac{\int_1^{x^3} \tan(t-1)\, dt}{x^3-2x^2+x} = ?$

$$\underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 1} \frac{3x^2 . \tan(x^3-1)}{3x^2-4x+1} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 1} \frac{6x.\tan(x^3-1)+3x^2.3x^2.\sec^2(x^3-1)}{6x-4} = \frac{9}{2}.$$

33) If $f(x)=\sqrt{x}-\int_0^{\sqrt{x}} \cos(t^2)dt$, $\lim\limits_{x\to 0^+} f'(x) = ?$

$$f'(x) = \underbrace{\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}}\cos x}_{\frac{1}{2\sqrt{x}}(1-\cos x)} \Rightarrow \lim_{x\to 0^+} \frac{1-\cos x}{2\sqrt{x}} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0^+} \frac{\sin x}{\frac{1}{\sqrt{x}}} = 0.$$

34) $\lim_{x\to 0} \frac{1}{x} \cdot \frac{d}{dx} \int_0^x (1-\tan 2t)^{1/t}\,dt = ?$ $(\infty \cdot 0)$

$$\frac{d}{dx}\int_0^x (1-\tan 2t)^{1/t}\,dt = (1-\tan 2x)^{1/x}$$

$$\lim_{x\to 0} \underbrace{(1-\tan 2x)^{1/x}}_{A} = (1^\infty) \Rightarrow \ln A = \frac{1}{x}\ln(1-\tan 2x)$$

$$\lim_{x\to 0} \ln A = \lim_{x\to 0} \frac{\ln(1-\tan 2x)}{x} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0} \frac{1-2\sec^2(2x)}{1-\tan 2x} = \frac{-1}{1} = -1$$

$$\lim_{x\to 0} \ln A = -1 \Rightarrow \lim_{x\to 0} A = e^{-1} = \frac{1}{e}$$

$$\Rightarrow \lim_{x\to 0} \underbrace{\frac{1}{x}}_{\infty} \cdot \underbrace{\frac{d}{dx}\int_0^x (1-\tan 2t)^{1/t}\,dt}_{\frac{1}{e}} = \infty$$