INDETERMINATE FORMS AND L'HOPITAL'S RULE

① $$\lim_{x\to 0}\frac{3^{5x}-1}{\tan 3x} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0}\frac{5.3^{5x}.\ln 3}{3.\sec^2 3x} = \frac{5.\ln 3}{3}$$

② $$\lim_{x\to 0}\frac{e^x-1}{x.\sin x} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0}\frac{e^x}{\sin x + x.\cos x} = \infty$$

③ $$\lim_{x\to 0^+}\frac{e^x-(1+x)}{x^3} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0^+}\frac{e^x-1}{3x^2} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0^+}\frac{e^x}{6x} = \infty$$

④ $$\lim_{x\to \infty} x.(2\arctan x-\pi) \underset{(\infty.0)}{=} \lim_{x\to\infty}\frac{2\arctan x-\pi}{\frac{1}{x}} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to\infty}\frac{\frac{2}{1+x^2}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty}\frac{-2x^2}{1+x^2} = -2$$

⑤ $$\lim_{x\to \infty}\frac{x^3}{e^{x^2}} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to\infty}\frac{3x^{\cancel{2}}}{2\cancel{x}.e^{x^2}} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to\infty}\frac{3}{4x.e^{x^2}} = 0$$

⑥ $$\lim_{x\to 1^+}\left(\underset{(x-1)}{\frac{3}{\ln x}} - \underset{(\ln x)}{\frac{2}{x-1}}\right) = \lim_{x\to 1^+}\frac{3x-3-2\ln x}{\ln x.(x-1)} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 1^+}\frac{3-\frac{2}{x}}{\frac{x-1}{x}+\ln x} = \infty$$

⑦ $$\lim_{x\to 0}\frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}}-\frac{1}{2}}{2x} = \lim_{x\to 0}\frac{1-\sqrt{1+x}}{4x\sqrt{1+x}}$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0}\frac{-\frac{1}{2\sqrt{1+x}}}{4.\left(\sqrt{1+x}+\frac{x}{2\sqrt{1+x}}\right)} = \frac{-\frac{1}{2}}{4.\left(1+\frac{0}{2.1}\right)} = -\frac{1}{8}$$

⑧ $$\lim_{x\to 0}\frac{e^x-e^{-x}}{\sin x} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0}\frac{e^x+e^{-x}}{\cos x} = \frac{1+1}{1} = 2$$

(9) $$\lim_{x\to0^+}\frac{\ln(\sin(2x))}{\ln(\sin(3x))} \underset{(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{x\to0^+}\frac{\frac{2\cos(2x)}{\sin(2x)}}{\frac{3\cos(3x)}{\sin(2x)}} = \lim_{x\to0^+}\frac{2\cot(2x)}{3\cot(3x)} \quad \left(\cot mx=\frac{1}{\tan mx}\right)$$

$$=\lim_{x\to0^+}\frac{2\tan(3x)}{3\tan(2x)}=\frac{2}{3}\cdot\frac{3}{2}=1 \quad \left(\lim_{x\to0}\frac{\tan mx}{\tan nx}=\frac{m}{n}\right)$$

(10) $$\lim_{x\to\frac{\pi}{2}^-}\frac{\tan x}{\sec x}=\left(\frac{\infty}{\infty}\right)\Rightarrow \lim_{x\to\frac{\pi}{2}^-}\frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\cancel{\cos x}}}=1$$ (Do try to solve this by L'Hopital's)

(11) $$\lim_{x\to0}\frac{e^{x-\arcsin x}-1}{\sqrt{1-x^2}-1} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to0}\frac{\left(1-\frac{1}{\sqrt{1-x^2}}\right)\cdot e^{x-\arcsin x}}{\frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}}$$

$$=\lim_{x\to0}\frac{(\sqrt{1-x^2}-1)\cdot \overbrace{e^{x-\arcsin x}}^{1}}{-x}=\lim_{x\to0}\frac{\sqrt{1-x^2}-1}{-x} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to0}\frac{-\cancel{2}x}{-\cancel{2}\sqrt{1-x^2}}=0$$

(12) $$\lim_{x\to\infty}x^2\cdot e^{\sqrt{x}-x^2} \underset{(\infty\cdot0)}{=} \lim_{x\to\infty}\frac{x^2}{e^{x^2-\sqrt{x}}} \underset{*(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{x\to\infty}\frac{2x}{\left(2x-\frac{1}{2\sqrt{x}}\right)\cdot e^{x^2-\sqrt{x}}}$$

$$=\lim_{x\to\infty}\frac{2\cancel{x}}{\cancel{x}\left(2-\underbrace{\frac{1}{2x\sqrt{x}}}_{0}\right)\cdot \underbrace{e^{x^2-\sqrt{x}}}_{\infty}}=0$$

$$*\ \lim_{x\to\infty}e^{x^2-\sqrt{x}}=e^{\lim_{x\to\infty}(x^2-\sqrt{x})}=e^{\lim_{x\to\infty}x^2\left(1-\frac{1}{x\sqrt{x}}\right)}=e^{\infty}=\infty$$

(13) $$\lim_{x\to\infty}\frac{e^{\arctan x}-x}{\ln(1+x^2)+x} \underset{*(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{x\to\infty}\frac{\frac{1}{1+x^2}\cdot e^{\arctan x}-1}{\frac{2x}{1+x^2}+1}=\frac{0\cdot e^{\frac{\pi}{2}}-1}{0+1}=-1$$

$$*\ \lim_{x\to\infty}e^{\arctan x}-x=e^{\frac{\pi}{2}}-\infty=-\infty \quad \left(\arctan\infty=\frac{\pi}{2}\right)$$

$$\text{(14)}\ \lim_{x\to\infty}\left[x\cdot\sin\left(\frac{x}{x^2+\ln x}\right)\right] \underset{(\infty\cdot 0)}{=} \lim_{x\to\infty} \frac{x\cdot\frac{x}{x^2+\ln x}\cdot\sin\left(\frac{x}{x^2+\ln x}\right)}{\frac{x}{x^2+\ln x}} \quad 1$$

$$= \lim_{x\to\infty}\frac{x^2}{x^2+\ln x} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to\infty}\frac{2x}{2x+\underbrace{\frac{1}{x}}_{0}} = 1$$

$$*\lim_{x\to\infty}\frac{x}{x^2+\ln x} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to\infty}\frac{1}{2x+\underbrace{\frac{1}{x}}_{0}} = 0$$

$$\text{(15)}\ \lim_{x\to\frac{\pi}{2}^+}\left[\ln\left(x-\frac{\pi}{2}\right)\cdot\cos x\right] \qquad x-\frac{\pi}{2}=t \qquad x\to\frac{\pi}{2}^+ \Rightarrow t\to 0^+$$

$$x = t+\frac{\pi}{2}$$

$$\lim_{t\to 0^+}\left[\ln t\cdot\underbrace{\cos\left(t+\frac{\pi}{2}\right)}_{-\sin t}\right] \underset{(\infty\cdot 0)}{=} \lim_{t\to 0^+}\frac{\ln t}{-\csc t} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{t\to 0^+}\frac{\frac{1}{t}}{\csc t\cdot\cot t}$$

$$= \lim_{t\to 0^+}\frac{\sin^2 t}{t\cdot\cos t} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{t\to 0^+}\frac{2\cdot\sin t\cdot\cos t}{\cos t - t\cdot\sin t} = \lim_{t\to 0^+}\frac{\sin 2t}{\cos t - t\sin t} = \frac{0}{1-0} = 0.$$

$$\text{(16)}\ \lim_{x\to 0}\left(\frac{3+2x}{3-2x}\right)^{1/x} = (1^\infty) \qquad \lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{\ln\left(\frac{3+2x}{3-2x}\right)}{x} \left(=\frac{4}{3}\right)$$

$$= \lim_{x\to 0}\frac{\ln(3+2x)-\ln(3-2x)}{x} \underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0}\frac{\frac{2}{3+2x}-\frac{-2}{3-2x}}{1} = \frac{2}{3}+\frac{2}{3} = \frac{4}{3}$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{3+2x}{3-2x}\right)^{1/x} = e^{4/3}$$

$$\text{(17)}\ \lim_{x\to 0}\left(\cos(2x)\right)^{1/x^2} = (1^\infty) \qquad \lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{\ln(\cos(2x))}{x^2} \ (=-2)$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0}\frac{\frac{-\cancel{2}\cdot\sin(2x)}{\cos(2x)}}{\cancel{2}x} = \lim_{x\to 0}\frac{-\tan(2x)}{x} = -2$$

$$\Rightarrow \lim_{x\to 0}\left(\cos(2x)\right)^{1/x^2} = e^{-2} = \frac{1}{e^2}$$

18) $\lim_{x\to \frac{\pi}{2}^-} (\tan x)^{\cos x} = (\infty^0)$ $\quad \lim_{x\to \frac{\pi}{2}^-} \ln y = \lim_{x\to \frac{\pi}{2}^-} \frac{\ln(\tan x)}{\sec x}$ $(=0)$

$$\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{\sec^{\cancel{2}}x}{\tan x}}{\cancel{\sec x}\cdot \tan x} = \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{1}{\cancel{\cos x}}}{\frac{\sin^2 x}{\cos^{\cancel{2}} x}} = \lim_{x\to \frac{\pi}{2}^-} \frac{\cos x}{\sin^2 x} = 0$$

$$\Rightarrow \lim_{x\to \frac{\pi}{2}^-} (\tan x)^{\cos x} = e^0 = 1$$

19) $\lim_{x\to 0^+} (\cot x)^{1/\ln x} = (\infty^0)$ $\quad \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(\cot x)}{\ln x}$ $(=-1)$

$$\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to 0^+} \frac{\frac{-\csc^2 x}{\cot x}}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{-\tan x \cdot x}{\sin^2 x} = \lim_{x\to 0^+} \frac{-\overset{1}{x\cdot \cancel{\sin x}}}{\cos x\cdot \sin x\cdot \cancel{\sin x}} = -1$$

$$\Rightarrow \lim_{x\to 0^+} (\cot x)^{1/\ln x} = e^{-1} = \frac{1}{e}$$

20) $\lim_{x\to 0^+} (e^x + x)^{1/x} = (1^\infty)$ $\quad \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(e^x + x)}{x}$ $(=2)$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}} \lim_{x\to 0^+} \frac{e^x + 1}{e^x + x} = \frac{1+1}{1+0} = 2 \quad \Rightarrow \lim_{x\to 0^+} (e^x + x)^{1/x} = e^2$$

21) $\lim_{x\to 0^+} x^{\sqrt{x}} = (0^0)$ $\quad \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln x}{x^{-1/2}}$ $(=0)$

$$\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}} = \lim_{x\to 0^+} \frac{-2\cdot \cancel{x}\sqrt{x}}{\cancel{x}} = 0 \quad \Rightarrow \lim_{x\to 0^+} x^{\sqrt{x}} = e^0 = 1$$

22) $\lim_{x\to \infty} x^{\ln 2/(1+\ln x)} = (\infty^0)$ $\quad \lim_{x\to \infty} \ln y = \lim_{x\to \infty} \frac{\ln 2 \cdot \ln x}{1 + \ln x}$ $(=\ln 2)$

$$\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}} \lim_{x\to \infty} \frac{\ln 2\cdot \cancel{\frac{1}{x}}}{\cancel{\frac{1}{x}}} = \ln 2 \quad \Rightarrow \lim_{x\to \infty} x^{\ln 2/(1+\ln x)} = e^{\ln 2} = 2$$

(23) $\lim_{x\to 0^+}\left(1+\frac{1-e^x}{1+\sin x}\right)^{1/x}=(1^\infty)$ $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln\left(\frac{1+\sin x+1-e^x}{1+\sin x}\right)}{x}$ $(=-1)$

$$\lim_{x\to 0^+}\frac{\ln(2+\sin x-e^x)-\ln(1+\sin x)}{x}\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\left(\underbrace{\frac{\cos x-e^x}{2+\sin x-e^x}}_{0}-\underbrace{\frac{\cos x}{1+\sin x}}_{1}\right)=-1$$

$$\Rightarrow \lim_{x\to 0^+}\left(1+\frac{1-e^x}{1+\sin x}\right)^{1/x}=e^{-1}=\frac{1}{e}$$

(24) $\lim_{x\to 0^+}\left(1-\sqrt{1-x}\right)^x=(0^0)$ $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln(1-\sqrt{1-x})}{\frac{1}{x}}$ $(=0)$

$$\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\frac{\frac{+1}{2\sqrt{1-x}}\cdot\frac{1}{1-\sqrt{1-x}}}{-\frac{1}{x^2}}=\frac{1}{2}\cdot\lim_{x\to 0^+}\frac{-x^2}{1-(1-x)^{1/2}}\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\frac{1}{\cancel{2}}\cdot\lim_{x\to 0^+}\frac{-2x}{+\frac{1}{\cancel{2}}(1-x)^{-1/2}}$$

$$=\lim_{x\to 0^+}-2x\sqrt{1-x}=0\Rightarrow \lim_{x\to 0^+}\left(1-\sqrt{1-x}\right)^x=e^0=1.$$

(25) $\lim_{x\to 0^+}(1+\sin(2x))^{1/x}=(1^\infty)$ $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln(1+\sin(2x))}{x}$ $(=2)$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\frac{2\cos(2x)}{1+\sin(2x)}=2\Rightarrow \lim_{x\to 0^+}(1+\sin(2x))^{1/x}=e^2$$

(26) $a,b\in\mathbb{R}^+\Rightarrow \lim_{x\to 0^+}\left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}}=(1^\infty)$ $\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{2\ln\left(\frac{a^x+b^x}{2}\right)}{x}$

$$=\lim_{x\to 0^+}\frac{2[\ln(a^x+b^x)-\ln 2]}{x}\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\frac{2(a^x\cdot\ln a+b^x\cdot\ln b)}{a^x+b^x}$$

$$=\frac{\cancel{2}\cdot(\ln a+\ln b)}{\cancel{2}}=\ln(ab)\Rightarrow \lim_{x\to 0^+}\left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}}=e^{\ln(ab)}=ab$$

(27) $\lim_{x\to 0^+} \left(2-e^{\sqrt{x}}\right)^{\frac{1}{x}} = (1^\infty)$ $\quad \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(2-e^{\sqrt{x}})}{x}$ $(-\infty)$

$$\underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0^+} \frac{-\frac{1}{2\sqrt{x}}\cdot e^{\sqrt{x}}}{2-e^{\sqrt{x}}} = \frac{-\infty}{1} = -\infty \Rightarrow \lim_{x\to 0^+}\left(2-e^{\sqrt{x}}\right)^{\frac{1}{x}} = e^{-\infty} = 0$$

(28) $\lim_{x\to 0^+} x^{\frac{1}{\ln(e^x-1)}} = (0^0)$ $\quad \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln x}{\ln(e^x-1)}$ $(=1)$

$$\underset{(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{e^x}{e^x-1}} = \lim_{x\to 0^+} \frac{e^x-1}{x\cdot e^x} \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0^+} \frac{\cancel{e^x}}{\cancel{e^x}+x\cdot \cancel{e^x}} = \frac{1}{1+0} = 1$$

$$\Rightarrow \lim_{x\to 0^+} x^{\frac{1}{\ln(e^x-1)}} = e^1 = e$$

(29) $\lim_{x\to 1^+} \left(\frac{4}{\pi}\arctan x\right)^{\frac{3}{x^2+2x-3}} = (1^\infty)$ $\quad \lim_{x\to 1^+} \ln y = \lim_{x\to 1^+} \frac{3\ln\left(\frac{4}{\pi}\arctan x\right)}{x^2+2x-3}$ $\left(=\frac{3}{2\pi}\right)$

$$\underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 1^+} \frac{3\cdot\cancel{\frac{4}{\pi}}\cdot\frac{1}{1+x^2}}{\cancel{\frac{4}{\pi}}\arctan x\cdot(2x+2)} = \frac{3\cdot\frac{1}{2}}{\frac{\pi}{\cancel{4}}\cdot\cancel{4}} = \frac{3}{2\pi} \Rightarrow \lim_{x\to 1^+}\left(\frac{4}{\pi}\arctan x\right)^{\frac{3}{x^2+2x-3}} = e^{\frac{3}{2\pi}}$$

(30) $\lim_{x\to 0} \left(1+2\sin^2 x\right)^{1/x^2} = (1^\infty)$ $\quad \lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln(1+2\sin^2 x)}{x^2}$ $(=2)$

$$\underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0} \frac{\cancel{2}\cdot 2\cdot \sin x\cdot\cos x}{\underbrace{(1+2\sin^2 x)}_{1}\cdot \cancel{2}x} \quad 1 = \lim_{x\to 0} 2\cos x = 2 \Rightarrow \lim_{x\to 0}\left(1+2\sin^2 x\right)^{1/x^2} = e^2$$

(31) $\lim_{x\to 0} (1+\arctan x)^{\frac{1}{x^2+2x}} = (1^\infty)$ $\quad \lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln(1+\arctan x)}{x^2+2x}$ $\left(=\frac{1}{2}\right)$

$$\underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0} \frac{\frac{1}{1+x^2}}{(1+\arctan x)\cdot(2x+2)} = \frac{1}{1\cdot 2} = \frac{1}{2} \Rightarrow \lim_{x\to 0}(1+\arctan x)^{\frac{1}{x^2+2x}} = e^{\frac{1}{2}} = \sqrt{e}$$

(32) $\lim\limits_{x\to 0^+}\left[\frac{1}{x}-\frac{2x}{1-\cos(2x)}\right]^{\frac{1}{x}}=(\infty-\infty)^{\infty}$ $\quad A=\frac{1}{x}-\frac{2x}{1-\cos(2x)}$

$\Rightarrow \lim\limits_{x\to 0^+} A=?=0$

$$\lim_{x\to 0^+} A=\lim_{x\to 0^+}\left[\frac{1}{x}-\frac{\cancel{2}x}{\cancel{2}\sin^2 x}\right]=\lim_{x\to 0^+}\left[\frac{1}{x}-\frac{x}{\underbrace{\sin x}_{1}\cdot\sin x}\right]=\lim_{x\to 0^+}\frac{\sin x - x}{x\cdot\sin x}$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\frac{\cos x-1}{\sin x+x\cos x}\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\frac{-\sin x}{\underbrace{\cos x+\cos x}_{2}-x\sin x}=\frac{-0}{2-0}=0$$

$$\lim_{x\to 0^+}\left[\frac{1}{x}-\frac{2x}{1-\cos(2x)}\right]^{\frac{1}{x}}=0^{\infty}=0$$ (0^{∞} is not an indeterminate form!)

(33) $\lim\limits_{x\to\infty}\left[1+2^x+3^x\right]^{\frac{1}{x}}=(\infty^0)$ $\quad \lim\limits_{x\to\infty}\ln y=\lim\limits_{x\to\infty}\frac{\ln(1+2^x+3^x)}{x}$ $(=\ln 3)$

$$\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}}\lim_{x\to\infty}\frac{2^x\cdot\ln 2+3^x\cdot\ln 3}{1+2^x+3^x}=\lim_{x\to\infty}\frac{2^x\cdot\ln 2}{1+2^x+3^x}+\lim_{x\to\infty}\frac{3^x\cdot\ln 3}{1+2^x+3^x}$$

$$=\underbrace{\lim_{x\to\infty}\frac{\cancel{2^x}\cdot\ln 2}{\cancel{2^x}\left(\underbrace{2^{-x}}_{0}+1+\underbrace{\left(\frac{3}{2}\right)^x}_{\infty}\right)}}_{0}+\underbrace{\lim_{x\to\infty}\frac{\cancel{3^x}\cdot\ln 3}{\cancel{3^x}\cdot\left(\underbrace{3^{-x}}_{0}+\underbrace{\left(\frac{2}{3}\right)^x}_{0}+1\right)}}_{\ln 3}=0+\ln 3=\ln 3$$

$$\Rightarrow \lim_{x\to\infty}\left[1+2^x+3^x\right]^{\frac{1}{x}}=e^{\ln 3}=3$$ $\left(\text{For } \lim\limits_{x\to\infty}\left(\frac{a}{b}\right)^x=\begin{cases}0 & \text{if } a<b\\ \infty & \text{if } a>b\end{cases}\right)$

(34) $\lim\limits_{x\to 0^+} x^{x\ln x}=(0^{0\cdot\infty})$ $\quad \lim\limits_{x\to 0^+}\ln y=\lim\limits_{x\to 0^+} x\cdot\ln x\cdot\ln x=\lim\limits_{x\to 0^+}\frac{\ln^2 x}{\frac{1}{x}}$ $(=0)$

$$\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\frac{2\cdot\ln x\cdot\frac{1}{\cancel{x}}}{-\frac{1}{x^{\cancel{2}}}}\underset{\left(\frac{\infty}{\infty}\right)}{\overset{L}{=}}\lim_{x\to 0^+}\frac{\frac{2}{\cancel{x}}}{+\frac{1}{x^{\cancel{2}}}}=\lim_{x\to 0^+}2x=0$$

$$\Rightarrow \lim_{x\to 0^+} x^{x\ln x}=e^0=1$$ $(\ln^2 x\neq \ln x^2)$

35) Find the value of $a+b+c$ such that
$$\lim_{x\to 0}\frac{\ln(a-x)+bx+cx^2}{x^3}=-\frac{1}{3}$$

$\lim\limits_{x\to 0}\frac{A}{x^3}=\infty$, but we find finite limit. There must be $\left(\frac{0}{0}\right)$.

$$\lim_{x\to 0}\ln(a-x)+b\underset{\downarrow}{x}\underset{0}{}+c\underset{\downarrow}{x^2}\underset{0}{}=0 \Rightarrow \ln(a-\underset{\downarrow}{x}\underset{0}{})=0 \Rightarrow a-0=1 \Rightarrow \underline{a=1}$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{x\to 0}\frac{\frac{-1}{1-x}+b+2cx}{3x^2}=-\frac{1}{3}\left(\frac{0}{0}\right)\Rightarrow \underbrace{\frac{1}{1-x}}_{-1}+b+\underbrace{2cx}_{0}=0 \Rightarrow \underline{b=1}$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{x\to 0}\frac{\frac{-1}{(1-x)^2}+2c}{6x}=-\frac{1}{3}\left(\frac{0}{0}\right)\Rightarrow \underbrace{\frac{-1}{(1-x)^2}}_{-1}+2c=0 \Rightarrow c=\frac{1}{2}$$

$$\Rightarrow a+b+c=1+1+\frac{1}{2}=\frac{5}{2}$$

36) Find the value of $a+b$ if $\lim\limits_{x\to\infty}\left(a+\frac{b}{x}+\frac{1}{x^2}\right)^x=e^3$.

$\lim\limits_{x\to\infty}(A)^x=\infty$, but we find finite limit. There must be (1^∞)

$$\lim_{x\to\infty}a+\frac{b}{x}+\frac{1}{x^2}=1 \Rightarrow a+0+0=1 \Rightarrow \underline{a=1}$$

$$\lim_{x\to\infty}\ln y=\ln e^3=3 \Rightarrow \lim_{x\to\infty}x\cdot\ln\left(1+\frac{b}{x}+\frac{1}{x^2}\right)=3$$

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{b}{x}+\frac{1}{x^2}\right)}{\frac{1}{x}}=3\quad\left(\frac{1}{x}=t\quad \begin{matrix}x\to\infty\\ t\to 0\end{matrix}\right)\quad \lim_{t\to 0}\frac{\ln(1+bt+t^2)}{t}=3$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L}{=}}\lim_{t\to 0}\frac{b+2t}{1+bt+t^2}=3$$ (Divisor $\neq 0$, no more L'Hopital's)

$$\Rightarrow \frac{b}{1}=3 \Rightarrow \underline{b=3} \Rightarrow a+b=1+3=4$$