MAT1320-LINEAR ALGEBRA FINAL EXAM QUESTIONS **A**

Name Surname:	**Grup No:**
Student ID:	**Duration:** 90 mins.
Department:	**Date:** 23 August 2022
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Attention: *The 9th article of Student Disciplinary Regulations of Higher Education Council (YÖK) Law No. 2547 states that people who are cheating or helping to cheat or attempt to cheat in exams" will be punished by suspension of one or two semesters.*

1. Which of the following is the coefficient matrix and the augmented matrix, respectively, of the system $\left.\begin{array}{r} -x+z-3=y \\ y+z=2+x \\ z+x+y=0 \end{array}\right\}$?

a) $A=\begin{pmatrix} -1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $[A\mid b]=\left(\begin{array}{ccc|c} -1 & -1 & 1 & 3 \\ -1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right)$

b) $A=\begin{pmatrix} -1 & -1 & -3 \\ -1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$, $[A\mid b]=\left(\begin{array}{ccc|c} -1 & -1 & -3 & 1 \\ -1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right)$

c) $A=\begin{pmatrix} -1 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$, $[A\mid b]=\left(\begin{array}{ccc|c} -1 & -1 & 1 & -3 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right)$

d) $A=\begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $[A\mid b]=\left(\begin{array}{ccc|c} -1 & 1 & 1 & -3 \\ -1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right)$

e) $A=\begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$, $[A\mid b]=\left(\begin{array}{ccc|c} -1 & 1 & -1 & 3 \\ 1 & 1 & -1 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right)$

2. Let $B=\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$ and $C=\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ be two matrices such that $(AB)^{-1}=(CB)^T$. Then which of the following is the inverse of A ?

a) $\begin{pmatrix} 4 & 0 \\ 13 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 0 & -1 \\ \frac{1}{4} & \frac{13}{4} \end{pmatrix}$ c) $\begin{pmatrix} -1 & 13 \\ 0 & 4 \end{pmatrix}$

d) $\begin{pmatrix} 4 & 13 \\ 0 & -1 \end{pmatrix}$ e) $\begin{pmatrix} 13 & 4 \\ -1 & 0 \end{pmatrix}$

3. Let A and B be two matrices of size 3×3 such that $\det(A)=108$ and $\det(B)=4$. Then which of the following is the value of $\det\left(\frac{1}{3}AB^{-1}\right)$?

a) 18 b) 3 c) 1

d) 9 e) None of them.

4. For which value or values of k does the matrix $A=\begin{pmatrix} 3 & 2 & 1 \\ 5 & 3 & 2 \\ 2 & k & -3 \end{pmatrix}$ has an inverse?

a) $k=\mathbb{R}-\{1\}$ b) $k=\mathbb{R}-\{-5\}$ c) $k=5$

d) $k=\mathbb{R}-\{5\}$ e) $k=\mathbb{R}-\{-1\}$

5. Which of the following is a basis for the subspace $\mathcal{W}=\left\{\begin{pmatrix} x & y & z \\ 0 & x & 0 \end{pmatrix}\middle| x=y+z,\ x,y,z\in\mathbb{R}\right\}\subset \mathcal{M}_{2\times 3}(\mathbb{R})$?

a) $\mathcal{B}=\left\{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix},\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\right\}$

b) $\mathcal{B}=\left\{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix},\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\right\}$

c) $\mathcal{B}=\left\{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix},\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}\right\}$

d) $\mathcal{B}=\left\{\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}\right\}$

e) $\mathcal{B}=\left\{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix},\begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\right\}$

6. Let the trace and determinant of a 2×2 square matrix A be $Tr\,(A)=10$ and $\det(A)=16$, respectively. Which of the following are the eigenvalues of the matrix A ?

a) $\lambda_1=-2,\ \lambda_2=-8$ b) $\lambda_1=-2,\ \lambda_2=8$
c) $\lambda_1=2,\ \lambda_2=-8$ d) $\lambda_1=2,\ \lambda_2=8$
e) $\lambda_1=-2,\ \lambda_2=4$

7. For which value of t the vectors $\vec{\mathbf{v}}_1=(2,0,-2,0)$, $\vec{\mathbf{v}}_2=(0,t,2,1)$, $\vec{\mathbf{v}}_3=(1,1,1,2)$ and $\vec{\mathbf{v}}_4=(2,1,0,3)\in\mathbb{R}^4$ are linearly dependent?

a) 2 b) 1 c) -1 d) -2 e) 0

8. Which of the following is **always** true for the square matrix with characteristic polynomial $P(x)=x^3-2x$?

I. $A^3=2A$

II. $A^4=2A^2$

III. $A=2A^{-1}$

a) Only I b) Only II
c) Only I and II d) All of them.
e) None of them.

9. Let $\mathcal{P}_2$ be the vector space of all polynomials with real coefficients of degree at most two.Which of the following is a basis for $\mathcal{P}_2$?

I. $\{1,x,x^2\}$

II. $\{1+x,1+x^2\}$

III. $\{1,1+x,1+x^2\}$

a) Only I b) Only II c) Only III
d) II and III e) I and III

10. Let $\mathcal{B}_1=\{\vec{\mathbf{u}}_1,\vec{\mathbf{u}}_2\}$ and $\mathcal{B}_2=\{\vec{\mathbf{v}}_1,\vec{\mathbf{v}}_2\}$ be two ordered bases of the vector space V such that $\vec{\mathbf{v}}_1=2\vec{\mathbf{u}}_1-\vec{\mathbf{u}}_2$ and $\vec{\mathbf{v}}_2=\vec{\mathbf{u}}_1+\vec{\mathbf{u}}_2$. Which of the following is the base transition matrix $[M]_{\mathcal{B}_1}^{\mathcal{B}_2}$ from the basis $\mathcal{B}_1$ to the basis $\mathcal{B}_2$?

a) $\begin{pmatrix}2&-1\\1&1\end{pmatrix}$ b) $\begin{pmatrix}2&1\\-1&1\end{pmatrix}$ c) $\begin{pmatrix}\frac{1}{3}&\frac{-1}{3}\\\frac{1}{3}&\frac{2}{3}\end{pmatrix}$
d) $\begin{pmatrix}\frac{1}{3}&\frac{1}{3}\\\frac{-1}{3}&\frac{2}{3}\end{pmatrix}$ e) $\begin{pmatrix}2&1\\1&-1\end{pmatrix}$

11. For which values of b the system of linear equations $\begin{cases} x+2y+3z=0 \\ (b+1)\,x+4y+6z=0 \\ -x+by-3z=0 \end{cases}$ can have nonzero solution or solutions?

a) $b \neq 1$ and $b \neq -2$ b) $b=-1$ or $b=2$
c) $b=2$ or $b=0$ d) $b=1$ or $b=-2$
e) $b=-1$ or $b=0$

12. Let $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ be one of the eigenvector of the matrix $A = \begin{pmatrix} 4 & x \\ 1 & 1 \end{pmatrix}$. Then which of the following is the value of the multiplication of the eigenvalues of A?

a) -3 b) 6 c) 0 d) -6 e) 3

13. If the eigenvalues of the matrix $\begin{pmatrix} 2 & 3 \\ b & 0 \end{pmatrix}$ are -1 and 3, then which of the following is the value of b?

a) -3 b) -1 c) 0 d) 1 e) 3

14. If one of the eigenvalue of the matrix $\begin{pmatrix} 3 & a \\ b & -5 \end{pmatrix}$ is 1, then which of the following is the other?

a) 2 b) 3 c) -1 d) -2 e) -3

15. Let $\overrightarrow{P_1P_2}$ be the vector formed by the points $P_1(2,1,5)$ and $P_2(4,3,6)$. Which of the following is the value of the inner (dot) product of the vector $\vec{\mathbf{v}} = -6i+9j-3k$ and the unit vector in the direction of the vector $\overrightarrow{P_1P_2}$?

a) -1 b) 1 c) 2 d) -2 e) $\sqrt{3}$

16. Which of the following is the area of the triangle with vertices $P_1(1,0,1)$, $P_2(2,1,2)$ and $P_3(1,1,2)$?

a) $\frac{\sqrt{6}}{2}$ b) $\frac{\sqrt{5}}{2}$ c) 2 d) $\frac{\sqrt{2}}{2}$ e) $\frac{\sqrt{3}}{2}$

17. Let $\begin{vmatrix} 1 & -2 & 2 & 3 \\ 1 & b & 0 & 4 \\ -2 & 0 & b & 1 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 0$. Which of the following is the sum of all possible values of b?

a) -6 b) -4 c) 0 d) 4 e) 6

18. Which of the following subset is the subspace of the specified vector space?

I. $\mathcal{T} = \{(x,0,z,1) \mid x,z \in \mathbb{Q}\} \subset \mathbb{R}^4$

II. $\mathcal{W} = \{(x,x+2,0) \mid x \in \mathbb{R}\} \subset \mathbb{R}^3$

III. $\mathcal{H} = \{(x,0,0,y) \mid x,y \in \mathbb{R}\} \subset \mathbb{R}^4$

a) Only I b) Only III
c) Only II and III d) Only I and III
e) Only II

19. Let $V = \{Z\}$ be the set consisting of single element Z such that $Z + Z = Z$ and $\forall r \in \mathbb{R}$, $rZ = Z$. Then which of the following is/are **always** correct?

I. V is a real vector space.

II. V is a complex vector space.

III. V is a subspace of the set of real numbers $\mathbb{R}$.

IV. V is a subspace of the set of complex numbers $\mathbb{C}$.

a) Only I b) Only II
c) Only I and III d) Only II and IV
e) Only I, II and IV

20. Which of the following is the dimension of the subspace $\mathcal{W} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| y = x + z \text{ and } x,y,z \in \mathbb{R} \right\} \subset \mathbb{R}^3$?

a) 0 b) 1 c) 2 d) 3 e) 4