EXAMPLE 3.1 Applying Series-Parallel Simplification

a) Find the equivalent resistance seen by the current source in Fig. 3.9, using series and parallel simplifications.

b) Use your results in part (a) to find the power delivered by the current source.

Solution

a) Our goal is a circuit with the 50 mA current source and a single resistor. We start simplifying the circuit's right-hand side, moving left toward the current source. The 2 kΩ and 3 kΩ resistors are in series and can be replaced by a single resistor whose value is

$$2000 + 3000 = 5000 = 5\ \text{k}\Omega.$$

Figure 3.10(a) shows this simplified circuit where the 20 kΩ and the 5 kΩ resistors are now in parallel. We replace these parallel-connected resistors with a single equivalent resistor, calculating its value using the "product over the sum" equation (Eq. 3.3):

$$\frac{(20{,}000)(5000)}{20{,}000 + 5000} = 4000 = 4\ \text{k}\Omega.$$

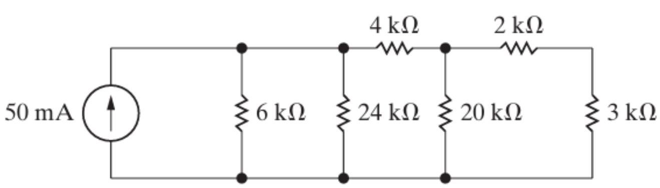


Figure 3.9 ▲ The circuit for Example 3.1.

Figure 3.10(b) shows this result, and now we see the two 4 kΩ resistors are in series. They can be replaced with a single resistor whose value is

$$4000 + 4000 = 8000 = 8\ \text{k}\Omega.$$

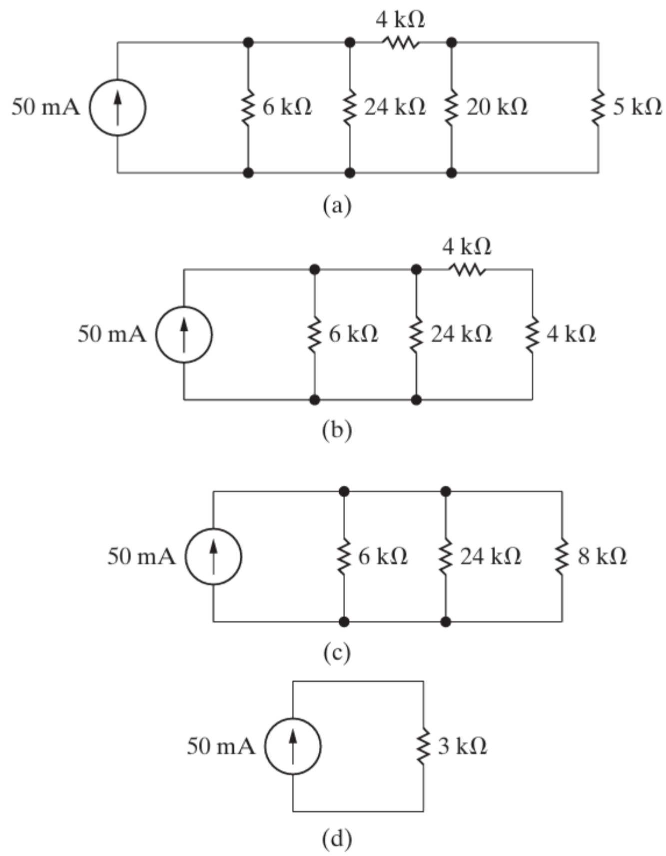


Figure 3.10 ▲ Simplifying the circuit in Fig. 3.9.

Figure 3.10(c) shows this simplification. Now the 24 kΩ, 6 kΩ, and 8 kΩ resistors are in parallel. To find their equivalent, we add their inverses and invert the result (Eq. 3.2):

$$\left(\frac{1}{24{,}000} + \frac{1}{6000} + \frac{1}{8000}\right)^{-1}$$

$$= \left(\frac{1}{24{,}000} + \frac{4}{24{,}000} + \frac{3}{24{,}000}\right)^{-1}$$

$$= \left(\frac{8}{24{,}000}\right)^{-1} = \frac{24{,}000}{8} = 3000 = 3\ \text{k}\Omega.$$

The equivalent resistance seen by the current source is 3 kΩ, as shown in Fig. 3.10(d).

b) The power of the source and the power of the equivalent 3 kΩ resistor must sum to zero. Using Fig. 3.10(d), we can easily calculate the resistor's power using its current and resistance to give

$$p = (0.05)^2(3000) = 7.5\ \text{W}.$$

The equivalent resistor is absorbing 7.5 W, so the current source must be delivering 7.5 W.

EXAMPLE 3.2 Solving a Circuit Using Series-Parallel Simplification

Find i_s, i_1, and i_2 in the circuit shown in Fig. 3.11.

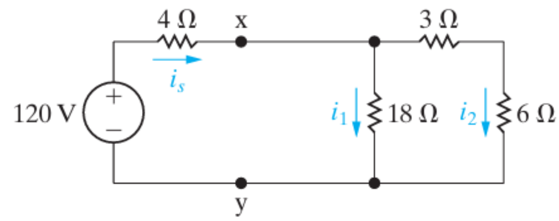


Figure 3.11 ▲ The circuit for Example 3.2.

Solution

Using series-parallel simplifications, we reduce the resistors to the right of the x-y terminals to a single equivalent resistor. On the circuit's right-hand side, the 3 Ω and 6 Ω resistors are in series. We replace this series combination with a 9 Ω resistor, reducing the circuit to the one shown in Fig. 3.12(a). Then we replace the parallel combination of the 9 Ω and 18 Ω resistors with a single equivalent resistance of $(18 \times 9)/(18 + 9)$, or 6 Ω. Figure 3.12(b) shows the resulting circuit. The nodes x and y marked on all diagrams should help you trace through the circuit simplification.

From Fig. 3.12(b) you can use Ohm's law to verify that

$$i_s = \frac{120}{(6 + 4)} = 12 \text{ A}.$$

Figure 3.13 shows this result and includes the voltage $_1$ to help clarify the subsequent discussion. Using Ohm's law, we compute the value of $_1$:

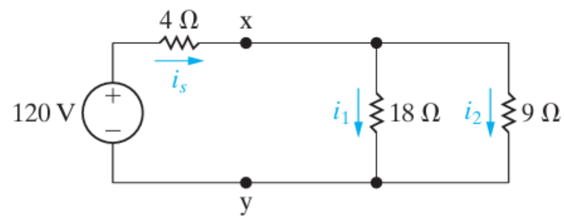
$$v_1 = (12)(6) = 72 \text{ V}.$$

Since v_1 is the voltage drop from node x to node y, we can return to the circuit shown in Fig. 3.12(a) and again use Ohm's law to calculate i_1 and i_2. Thus,

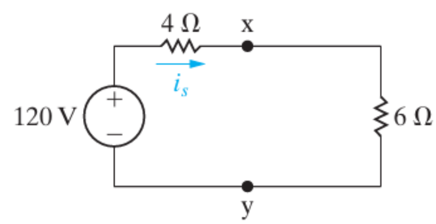
$$i_1 = \frac{v_1}{18} = \frac{72}{18} = 4 \text{ A},$$

$$i_2 = \frac{v_1}{9} = \frac{72}{9} = 8 \text{ A}.$$

We have found the three specified currents by using series-parallel reductions in combination with Ohm's law.



(a)



(b)

Figure 3.12 ▲ A simplification of the circuit shown in Fig. 3.11.

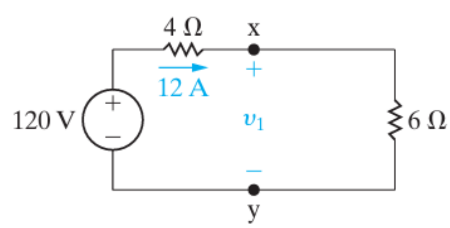


Figure 3.13 ▲ The circuit of Fig. 3.12(b) showing the numerical value of i_s.

EXAMPLE 3.3 Designing a Simple Voltage Divider

The voltage divider in Fig. 3.14 has a source voltage of 20 V. Determine the values of the resistors R_1 and R_2 to give $v_1 = 15$ V and $v_2 = 5$ V.

Solution

From Eqs. 3.4 and 3.5,

$$15 = \frac{R_1}{R_1 + R_2}(20) \qquad \text{and} \qquad 5 = \frac{R_2}{R_1 + R_2}(20).$$

Unfortunately, these two equations are not independent. If you solve each equation for R_1, you get $R_1 = 3R_2$. An infinite number of combinations of R_1 and R_2 yield the correct values for v_1 and v_2. For example, if you choose $R_2 = 10\,\text{k}\Omega$, then $R_1 = 30\,\text{k}\Omega$ gives the correct voltages, but if you choose $R_2 = 400\ \Omega$, then $R_1 = 1200\ \Omega$ gives the correct voltages.

EXAMPLE 3.4 Adding a Resistive Load to a Voltage Divider

a) For the voltage divider designed in Example 3.3, suppose resistors $R_2 = 10\,\text{k}\Omega$ and $R_1 = 30\,\text{k}\Omega$. Connect a resistor $R_L = 10\,\text{k}\Omega$ in parallel with R_2 and determine the voltage across R_L.

b) Repeat part (a) using resistors $R_2 = 400\,\Omega$ and $R_1 = 1200\,\Omega$, but the same value of R_L.

Solution

a) The voltage divider with the resistor R_L is shown in Fig. 3.15. The resistor R_L acts as a load on the voltage-divider circuit. A **load** on any circuit consists of one or more circuit elements that draw power from the circuit. The parallel combination of the two $10\,\text{k}\Omega$ resistors, one from the voltage divider and the other the load resistor R_L, gives an equivalent resistance of $5\,\text{k}\Omega$. Therefore, from Eq. 3.5,

$$v_o = \frac{5000}{30{,}000 + 5000}(20) = 2.86\ \text{V}.$$

This is certainly not the 5 V we were expecting the voltage divider to deliver to the load, because adding the load resistor changed the voltage-divider circuit.

b) The voltage divider with a different set of resistors and the same load resistor is shown in Fig. 3.16. Again, we expect the load resistor to change the voltage-divider circuit. The parallel combination of the $400\,\Omega$ and $10\,\text{k}\Omega$ resistors

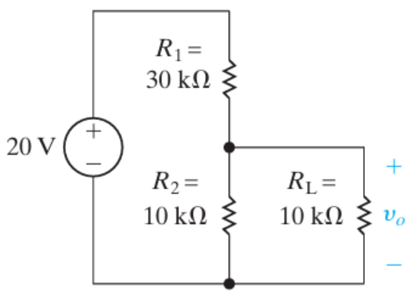


Figure 3.15 ▲ The voltage divider from Example 3.3 with a resistive load.

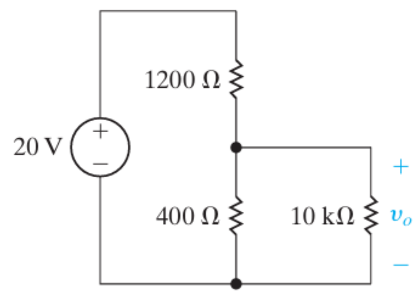


Figure 3.16 ▲ The voltage divider from Example 3.3 with a different choice of R_1 and R_2 resistors and a resistive load.

gives an equivalent resistance of $384.615\,\Omega$. Therefore, from Eq. 3.5,

$$v_o = \frac{384.615}{1200 + 384.615}(20) = 4.85\ \text{V}.$$

This is much closer to the 5 V we expected the voltage divider to deliver to the load. The effect of the load resistor is minimal because the load resistor value is much larger than the value of R_2 in the voltage divider.

EXAMPLE 3.5 The Effect of Resistor Tolerance on the Voltage-Divider Circuit

The resistors used in the voltage-divider circuit shown in Fig. 3.18 have a tolerance of $\pm 10\%$. Find the maximum and minimum value of $_o$.

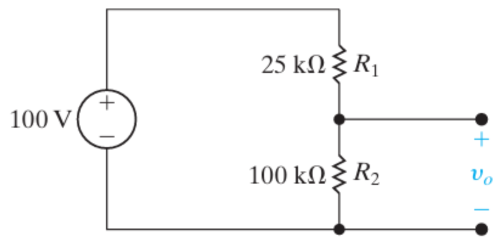


Figure 3.18 ▲ The circuit for Example 3.5.

Solution

From Eq. 3.5, the maximum value of $_o$ occurs when $R_2 = 110\,\text{k}\Omega$ (10% high) and $R_1 = 22.5\,\text{k}\Omega$ (10% low), and the minimum value of $_o$ occurs when $R_2 = 90\,\text{k}\Omega$ (10% low) and $R_1 = 27.5\,\text{k}\Omega$ (10% high). Therefore

$$v_o(\text{max}) = \frac{110\text{ k}}{110\text{ k} + 22.5\text{ k}}(100) = 83.02\text{ V}$$

and

$$v_o(\text{min}) = \frac{90\text{ k}}{90\text{ k} + 27.5\text{ k}}(100) = 76.60\text{ V}.$$

If we choose 10% resistors for this voltage divider, the no-load output voltage will lie between 76.60 and 83.02 V.

EXAMPLE 3.6 Designing a Current-Divider Circuit

Suppose the current source for the current divider shown in Fig. 3.19 is 100 mA. Assuming you have 0.25 W resistors available, what is the largest R_2 resistor you can use to get $i_2 = 50\text{ mA}$?

Solution

While you can use Eq. 3.8 to find the ratio of resistors, it should be clear that if the current in one resistor is 50 mA, the current in the other resistor must also be 50 mA, so both resistors must have the same value. Therefore, there are an infinite number of different resistor values which, when used for R_1 and R_2 will give $i_2 = 50\text{ mA}$. If the resistors have the same value and the same current, they absorb the same amount of power, which cannot exceed 0.25 W. From the power equation for resistors,

$$p = i^2R = (0.05)^2\,R = 0.25$$

so

$$R = \frac{0.25}{(0.05)^2} = 100\ \Omega.$$

The largest 0.25 W resistors that can be used to create a current $i_2 = 50\text{ mA}$ are 100 Ω resistors.

EXAMPLE 3.7 Using Voltage Division and Current Division to Solve a Circuit

Use current division to find the current i_o and use voltage division to find the voltage $_o$ for the circuit in Fig. 3.22.

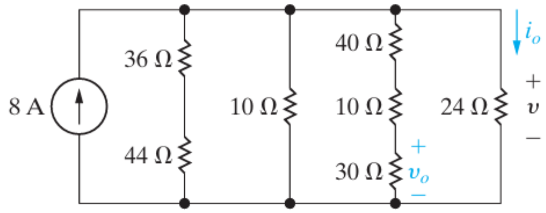


Figure 3.22 ▲ The circuit for Example 3.7.

Solution

We can use Eq. 3.10 if we can find the equivalent resistance of the four parallel branches containing resistors. Using "+" to represent series-connected resistors and "||" to represent parallel-connected resistors, the equivalent resistance is

$$R_{\text{eq}} = (36 + 44)||10||(40 + 10 + 30)||24$$

$$= 80||10||80||24 = \frac{1}{\frac{1}{80} + \frac{1}{10} + \frac{1}{80} + \frac{1}{24}} = 6\ \Omega.$$

Applying Eq. 3.10,

$$i_o = \frac{6}{24}(8\text{ A}) = 2\text{ A}.$$

We can use Ohm's law to find the voltage drop across the 24 Ω resistor:

$$v = (24)(2) = 48\text{ V}.$$

This is also the voltage drop across the branch containing the 40 Ω, the 10 Ω, and the 30 Ω resistors in series. Use voltage division to determine the voltage drop $_o$ across the 30 Ω resistor from the voltage drop across the series-connected resistors, using Eq. 3.9. The equivalent resistance of the series-connected resistors is $40 + 10 + 30 = 80\ \Omega$, so

$$v_o = \frac{30}{80}(48\text{ V}) = 18\text{ V}.$$

EXAMPLE 3.8 Using a d'Arsonval Ammeter

a) A 50 mV, 1 mA d'Arsonval movement is to be used in an ammeter with a full-scale reading of 10 mA. Determine R_A.

b) Repeat (a) for a full-scale reading of 1 A.

c) How much resistance is added to the circuit when the 10 mA ammeter is inserted to measure current?

d) Repeat (c) for the 1 A ammeter.

Solution

a) Look at the analog ammeter circuit in Fig. 3.26. The current in the ammeter must divide between the branch with the resistor R_A and the branch with the meter movement. From the problem statement we know that when the current in the ammeter is 10 mA, 1 mA is flowing through the meter coil, which means that 9 mA

must be diverted through R_A. We also know that when the movement carries 1 mA, the voltage across its terminals is 50 mV, which is also the voltage across R_A. Using Ohm's law,

$$9 \times 10^{-3} R_A = 50 \times 10^{-3},$$

or

$$R_A = 50/9 = 5.555\ \Omega.$$

b) When the full-scale deflection of the ammeter is 1 A, R_A must carry 999 mA when the movement carries 1 mA. In this case,

$$999 \times 10^{-3} R_A = 50 \times 10^{-3},$$

or

$$R_A = 50/999 \approx 50.05\ \text{m}\Omega.$$

c) Let R_m represent the equivalent resistance of the ammeter. When the ammeter current is 10 mA, its voltage drop is 50 mV, so from Ohm's law,

$$R_m = \frac{0.05}{0.01} = 5\ \Omega.$$

Alternatively, the resistance of the ammeter is the equivalent resistance of the meter movement in parallel with R_A. The resistance of the meter movement is the ratio of its voltage to its current, or $0.05/0.001 = 50\ \Omega$. Therefore,

$$R_m = 50 \| (50/9) = \frac{(50)(50/9)}{50 + (50/9)} = 5\ \Omega.$$

d) For the 1 A ammeter

$$R_m = \frac{0.05}{1} = 0.05\ \Omega,$$

or, alternatively,

$$R_m = 50 \| (50/999) = \frac{(50)(50/999)}{50 + (50/999)} = 0.05\ \Omega.$$

EXAMPLE 3.9 Using a d'Arsonval Voltmeter

a) A 50 mV, 1 mA d'Arsonval movement is to be used in a voltmeter in which the full-scale reading is 150 V. Determine R_v.

b) Repeat (a) for a full-scale reading of 5 V.

c) How much resistance does the 150 V meter insert into the circuit?

d) Repeat (c) for the 5 V meter.

Solution

a) Look at the analog voltmeter circuit in Fig. 3.27. The voltage across the voltmeter must divide between the resistor R_v and the meter movement. From the problem statement we know that when the voltage across the voltmeter is 150 V, the voltage across the meter coil must be 50 mV. The remaining 149.95 V must be the voltage across R_v. We also know that when the movement's voltage drop is 50 mV, its current is 1 mA, which is also the current in R_v. Using Ohm's law,

$$R_v = \frac{149.95}{0.001} = 149{,}950\ \Omega.$$

b) For a full-scale reading of 5 V, the voltage across R_v is 4.95 V and the current in R_v is still 1 mA, so from Ohm's law.

$$R_v = \frac{4.95}{0.001} = 4950\ \Omega.$$

c) Let R_m represent the equivalent resistance of the voltmeter. When the voltage across the voltmeter is 150 V, its current is 1 mA, so from Ohm's law,

$$R_m = \frac{150}{10^{-3}} = 150{,}000\ \Omega.$$

Alternatively, the resistance of the voltmeter is the equivalent resistance of R_v in series with the meter movement. The resistance of the meter movement is the ratio of its voltage to its current, or $50\,\text{m V}/1\,\text{mA} = 50\ \Omega$. Therefore,

$$R_m = 149{,}950 + 50 = 150{,}000\ \Omega.$$

d) For the 5 V voltmeter,

$$R_m = \frac{5}{10^{-3}} = 5000\ \Omega,$$

or, alternatively,

$$R_m = 4950 + 50 = 5000\ \Omega.$$

EXAMPLE 3.10 Using a Wheatstone Bridge to Measure Resistance

For the Wheatstone bridge in Fig. 3.30, R_3 can be varied from 10 Ω to 2 kΩ. What range of resistor values can this bridge measure?

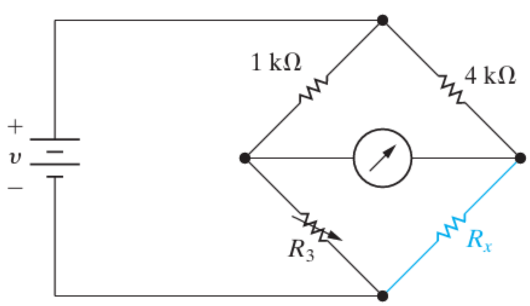


Figure 3.30 ▲ The circuit for Example 3.10.

Solution

When $R_3 = 10\ \Omega$, the bridge is balanced when

$$R_x = \frac{4000}{1000}(10) = 40\ \Omega.$$

When $R_3 = 2\,\text{k}\Omega$, the bridge is balanced when

$$R_x = \frac{4000}{1000}(2000) = 8\,\text{k}\Omega.$$

Therefore, the range of resistor values the bridge can measure is 40 Ω to 8 kΩ.

EXAMPLE 3.11 Applying a Delta-to-Wye Transform

Find the current and power supplied by the 40 V source in the circuit shown in Fig. 3.35.

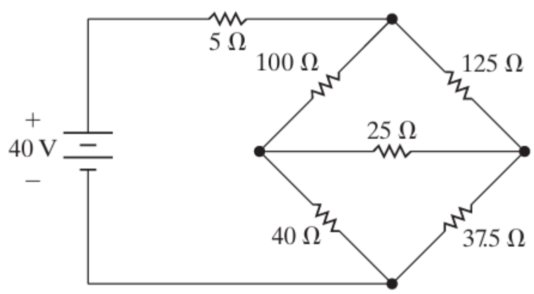


Figure 3.35 ▲ The circuit for Example 3.11.

Solution

This problem is easy to solve if we can find the equivalent resistance seen by the source. Begin this simplification by replacing either the upper Δ (100, 125, 25 Ω) or the lower Δ (40, 25, 37.5 Ω) with its equivalent Y. We choose to replace the upper Δ by computing the three Y resistances, defined in Fig. 3.36, using Eqs. 3.15 to 3.17. Thus,

$$R_1 = \frac{100 \times 125}{250} = 50\ \Omega,$$

$$R_2 = \frac{125 \times 25}{250} = 12.5\ \Omega,$$

$$R_3 = \frac{100 \times 25}{250} = 10\ \Omega.$$

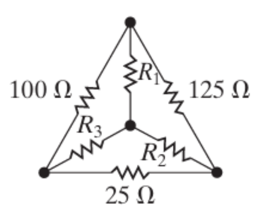


Figure 3.36 ▲ The equivalent Y resistors.

Substituting the Y resistors into the circuit shown in Fig. 3.35 produces the circuit shown in Fig. 3.37. From Fig. 3.37 we can easily calculate the resistance seen by the 40 V source using series-parallel simplifications:

$$R_{eq} = 5 + 50 + (10 + 40) \| (12.5 + 37.5)$$
$$= 55 + \frac{(50)(50)}{50 + 50} = 80\ \Omega.$$

The circuit simplifies to an 80 Ω resistor across a 40 V source, as shown in Fig. 3.38, so the 40 V source delivers current $i = 40/80 = 0.5$ A and power $p = 40(0.5) = 20$ W is delivered to the circuit.

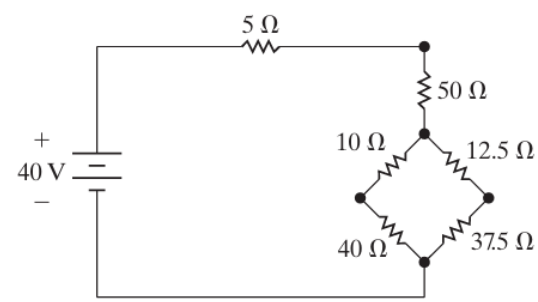


Figure 3.37 ▲ A transformed version of the circuit shown in Fig. 3.35.

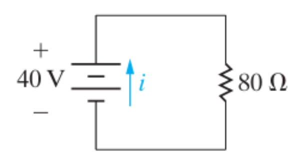


Figure 3.38 ▲ The final step in the simplification of the circuit shown in Fig. 3.35.