EXAMPLE 5.1 Analyzing an Op Amp Circuit

The op amp in the circuit shown in Fig. 5.7 is ideal.

a) Calculate v_o if $v_a = 1$ V and $v_b = 0$ V.

b) Repeat (a) for $v_a = 1$ V and $v_b = 2$ V.

c) If $v_a = 1.5$ V, specify the range of ${}_{b}$ that avoids amplifier saturation.

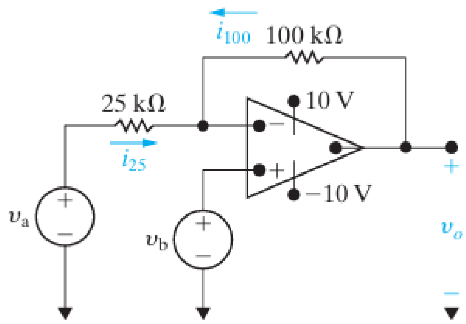

Figure 5.7 ▲ The circuit for Example 5.1.

Solution

a) **Step 1:** A negative feedback path exists from the op amp's output to its inverting input through the 100 kΩ resistor, so we assume the op amp is confined to its linear operating region.

Step 2: The voltage at the inverting input terminal is 0 because $v_p = v_b = 0$ from the connected voltage source, and ${}_{n} = {}_{p}$ from the voltage constraint in Eq. 5.2.

Step 3: Use KCL to sum the currents entering the node labeled v_n to get

$$i_{25} + i_{100} - i_n = 0.$$

Remember that i_n is the current entering the inverting op amp terminal. From Ohm's law,

$$i_{25} = \frac{v_a - v_n}{25{,}000} = \frac{1 - 0}{25{,}000} = \frac{1}{25{,}000};$$

$$i_{100} = \frac{v_o - v_n}{100{,}000} = \frac{v_o - 0}{100{,}000} = \frac{v_o}{100{,}000}.$$

The current constraint requires $i_n = 0$. Substituting the values for the three currents into the KCL equation, we obtain

$$\frac{1}{25} + \frac{v_o}{100} = 0.$$

Hence, v_o is -4 V.

Step 4: Because v_o lies between ± 10 V, our assumption that the op amp is in its linear region of operation is confirmed.

b) Using the same steps as in (a), we get

$$v_p = v_b = v_n = 2 \text{ V},$$

$$i_{25} = -i_{100}.$$

$$i_{25} = \frac{v_a - v_n}{25{,}000} = \frac{1 - 2}{25{,}000} = -\frac{1}{25{,}000};$$

$$i_{100} = \frac{v_o - v_n}{100{,}000} = \frac{v_o - 2}{100{,}000}.$$

Therefore, $v_o = 6$ V. Again, v_o lies within ± 10 V.

c) As before, ${}_{n} = {}_{p} = {}_{b}$, and $i_{25} = -i_{100}$. Because $v_a = 1.5$ V,

$$\frac{1.5 - v_b}{25{,}000} = \frac{v_o - v_b}{100{,}000}.$$

Solving for ${}_{b}$ as a function of ${}_{o}$ gives

$$v_b = \frac{1}{5}(6 + v_o).$$

Now, if the amplifier operates within its linear region, $-10 \text{ V} \le v_o \le 10$ V. Substituting these limits on ${}_{o}$ into the expression for ${}_{b}$, we find the range for v_b is

$$-0.8 \text{ V} \le v_b \le 3.2 \text{ V}.$$

EXAMPLE 5.2 Designing an Inverting Amplifier

a) Design an inverting amplifier (see Fig. 5.8) with a gain of 12. Use ± 15 V power supplies and an ideal op amp.

b) What range of input voltages, $_s$, allows the op amp in this design to remain in its linear operating region?

Solution

a) We need to find two resistors whose ratio is 12 from the realistic resistor values listed in Appendix H. There are lots of different possibilities, but let's choose $R_s = 1\,\text{k}\Omega$ and $R_f = 12\,\text{k}\Omega$. Use the inverting-amplifier equation (Eq. 5.4) to verify the design:

$$v_o = -\frac{R_f}{R_s} v_s = -\frac{12{,}000}{1000} v_s = -12 v_s.$$

Thus, we have an inverting amplifier with a gain of 12, as shown in Fig. 5.10.

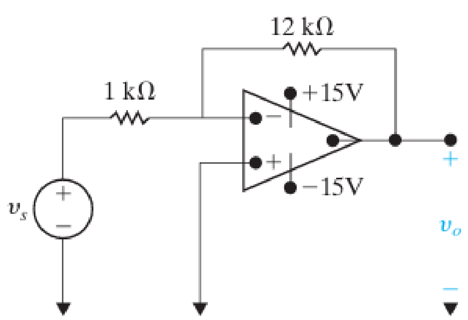

Figure 5.10 ▲ Inverting amplifier for Example 5.2.

b) Solve two different versions of the inverting-amplifier equation for $_s$, first using $v_o = +15$ V and then using $v_o = -15$ V:

$$15 = -12 v_s \text{ so } v_s = -1.25 \text{ V};$$

$$-15 = -12 v_s \text{ so } v_s = 1.25 \text{ V}.$$

Thus, if the input voltage is greater than or equal to -1.25 V and less than or equal to $+1.25$ V, the op amp in the inverting amplifier will remain in its linear operating region.

EXAMPLE 5.3 Designing a Summing Amplifier

a) Design a summing amplifier (see Fig. 5.11) whose output voltage is

$$v_o = -4v_a - v_b - 5v_c.$$

Use an ideal op amp with ± 12 V power supplies and a 20 kΩ feedback resistor.

b) Suppose $v_a = 2$ V and $v_c = -1$ V. What range of input voltages for v_b allows the op amp in this design to remain in its linear operating region?

c) Suppose $v_a = 2$ V, $v_b = 3$ V, and $v_c = -1$ V. Using the input resistor values found in part (a), how large can the feedback resistor be before the op amp saturates?

Solution

a) Use the summing-amplifier equation (Eq. 5.6) and the feedback resistor value to find the three input resistor values:

$$-\frac{R_f}{R_a} = -4 \quad \text{so} \quad R_a = \frac{20\text{ k}}{4} = 5\text{ k}\Omega;$$

$$-\frac{R_f}{R_b} = -1 \quad \text{so} \quad R_b = \frac{20\text{ k}}{1} = 20\text{ k}\Omega;$$

$$-\frac{R_f}{R_c} = -5 \quad \text{so} \quad R_c = \frac{20\text{ k}}{5} = 4\text{ k}\Omega.$$

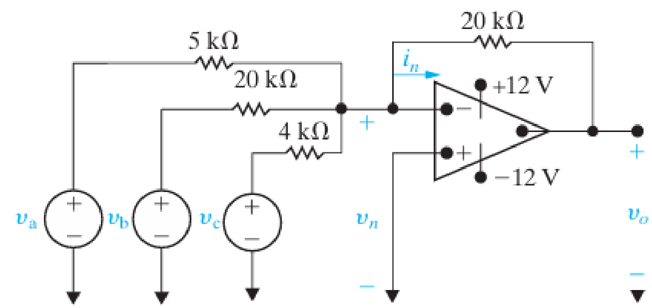

Figure 5.12 ▲ The summing amplifier for Example 3.3(a).

The resulting circuit is shown in Fig. 5.12.

b) Substitute the values for v_a and v_c into the equation for v_o given in the problem statement to get

$$v_o = -4(2) - v_b - 5(-1) = -3 - v_b.$$

Solving this equation for v_b in terms of v_o gives

$$v_b = -3 - v_o.$$

Now substitute the two power supply voltages for the output voltage to find the range of v_b values that keeps the op amp in its linear region:

$$-15\text{ V} \leq v_b \leq 9\text{ V}.$$

c) Starting with the summing-amplifier equation, Eq. 5.6, substitute the input resistor values found in part (a) and the specified input voltage values.

Remember that the feedback resistor is an unknown in this equation:

$$v_o = -\frac{R_f}{5000}(2) - \frac{R_f}{20{,}000}(3) - \frac{R_f}{4000}(-1) = -\frac{6R_f}{20{,}000}.$$

From this equation, it should be clear that if the op amp saturates, it will do so at its negative power supply value, −12 V. Using this voltage for v_o in the above equation and solving for the feedback resistance gives

$$R_f = 40\text{ k}\Omega.$$

Given the specified input voltages, this is the largest value of feedback resistance that keeps the op amp in its linear region.

EXAMPLE 5.4 Designing a Noninverting Amplifier

a) Design a noninverting amplifier (see Fig. 5.13) with a gain of 6. Assume the op amp is ideal.

b) Suppose we wish to amplify a voltage ${}_g$, where $-1.5\text{ V} \le v_g \le +1.5\text{ V}$. What are the smallest power supply voltages that could be used with the resistors selected in part (a) to ensure that the op amp remains in its linear region?

Solution

a) Using the noninverting-amplifier equation (Eq. 5.7),

$$v_o = \frac{R_s + R_f}{R_s} v_g = 6v_g \quad \text{so} \quad \frac{R_s + R_f}{R_s} = 6.$$

Therefore,

$$R_s + R_f = 6R_s, \quad \text{so} \quad R_f = 5R_s.$$

Look at the realistic resistor values listed in Appendix H. Let's choose $R_f = 10\text{ k}\Omega$, so $R_s = 2\text{ k}\Omega$. But there is not a $2\text{ k}\Omega$ resistor in Appendix H. We can create an equivalent $2\text{ k}\Omega$ resistor by combining two $1\text{ k}\Omega$ resistors in series. We can use a third $1\text{ k}\Omega$ resistor for R_g. The resulting circuit is shown in Fig. 5.14.

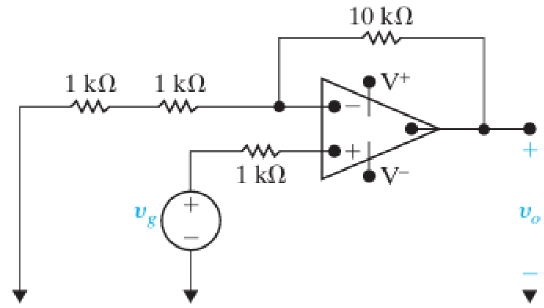

Figure 5.14 ▲ The noninverting amplifier design of Example 5.3.

b) Solve two different versions of the noninverting-amplifier equation for ${}_o$, first using $v_g = +1.5\text{ V}$ and then using $v_g = -1.5\text{ V}$:

$$v_o = 6(1.5) = 9\text{ V};$$

$$v_o = 6(-1.5) = -9\text{ V}.$$

Thus, if we use $\pm 9\text{ V}$ power supplies for the noninverting amplifier designed in part (a) and $-1.5\text{ V} \le v_g \le +1.5\text{ V}$, the op amp will remain in its linear operating region. The circuit resulting from the analysis in parts (a) and (b) is shown in Fig. 5.14, with $V^+ = 9\text{ V}$ and $V^- = -9\text{ V}$.

EXAMPLE 5.5 Designing a Difference Amplifier

a) Design a difference amplifier (see Fig. 5.15) that amplifies the difference between two input voltages by a gain of 8, using an ideal op amp and ± 8 V power supplies.

b) Suppose $v_a = 1$ V in the difference amplifier designed in part (a). What range of input voltages for v_b will allow the op amp to remain in its linear operating region?

Solution

a) Using the simplified difference-amplifier equation (Eq. 5.10),

$$v_o = \frac{R_b}{R_a}(v_b - v_a) = 8(v_b - v_a) \quad \text{so} \quad \frac{R_b}{R_a} = 8.$$

We want two resistors whose ratio is 8. Look at the realistic resistor values listed in Appendix H. Let's choose $R_b = 12\ \text{k}\Omega$, so $R_a = 1.5\ \text{k}\Omega$, although there are many other possibilities. Note that the simplified difference-amplifier equation requires that

$$\frac{R_a}{R_b} = \frac{R_c}{R_d}.$$

A simple choice for R_c and R_d is $R_c = R_a = 1.5\ \text{k}\Omega$ and $R_d = R_b = 12\ \text{k}\Omega$. The resulting circuit is shown in Fig. 5.16.

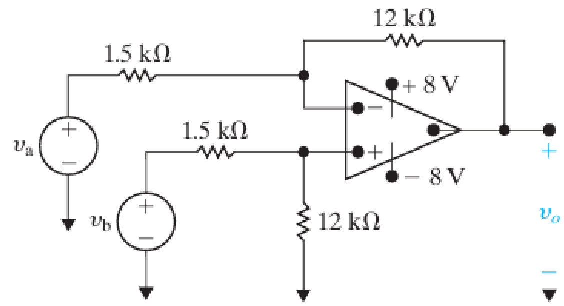

Figure 5.16 ▲ The difference amplifier designed in Example 5.4.

b) Using $v_a = 1$, solve two different versions of the simplified difference-amplifier equation (Eq. 5.10) for v_b in terms of v_o. Then substitute the two limiting values for the output voltage, $v_o = +8$ V and $v_o = -8$ V:

$$v_b = \frac{v_o}{8} + 1 = \frac{8}{8} + 1 = 2\ \text{V};$$

$$v_b = \frac{v_o}{8} + 1 = \frac{-8}{8} + 1 = 0\ \text{V}.$$

Thus, if $v_a = 1$ V in the difference amplifier from part (a), the op amp will remain in its linear region if $0\ \text{V} \leq v_b \leq 2\ \text{V}$.

EXAMPLE 5.6 Calculating the CMRR

a) Suppose the R_c resistor in the difference amplifier designed in Example 5.5, shown in Fig. 5.16, is 10% larger than its nominal value. All other resistor values are unchanged. Calculate the common mode gain, the difference mode gain, and the CMRR for the difference amplifier.

b) Repeat part (a) assuming the R_d resistor value is 10% larger than its nominal value and all other resistor values are unchanged.

Solution

a) Use the common mode gain equation in Eq. 5.15 with $R_c = 1500(1.1) = 1650\ \Omega$ to get

$$A_{cm} = \frac{(1500)(12{,}000) - (12{,}000)(1650)}{1500(1650 + 12{,}000)}$$
$$= -0.0879.$$

Then use the difference mode gain equation in Eq. 5.15 with $R_c = 1500(1.1) = 1650\ \Omega$ to get

$$A_{dm} = \frac{12{,}000(1500 + 12{,}000) + 12{,}000(1650 + 12{,}000)}{2(1500)(1650 + 12{,}000)}$$
$$= 7.956.$$

The CMRR (Eq. 5.20) is thus

$$\text{CMRR} = \left|\frac{7.956}{-0.0879}\right| = 90.5.$$

b) Use the common mode gain equation in Eq. 5.15 with $R_d = 12{,}000(1.1) = 13{,}200\ \Omega$ to get

$$A_{cm} = \frac{(1500)(13{,}200) - (12{,}000)(1500)}{1500(1500 + 13{,}200)}$$
$$= 0.08163.$$

Then use the difference mode gain equation in Eq. 5.15 with $R_d = 12{,}000(1.1) = 13{,}200\ \Omega$ to get

$$A_{dm} = \frac{13{,}200(1500 + 12{,}000) + 12{,}000(1500 + 13{,}200)}{2(1500)(1500 + 13{,}200)}$$
$$= 8.0408.$$

The CMRR (Eq. 5.20) is thus

$$\text{CMRR} = \left|\frac{8.0408}{0.08163}\right| = 98.5.$$

Example 5.7 analyzes a noninverting-amplifier circuit that employs the more realistic op amp model.

EXAMPLE 5.7 Analyzing a Noninverting-Amplifier Circuit using a Realistic Op Amp Model

Here we analyze the noninverting amplifier designed in Example 5.5 using the realistic op amp model in Fig. 5.18. Assume that the open-loop gain $A = 50{,}000$, the input resistance $R_i = 100\ \text{k}\Omega$, and the output resistance $R_o = 7.5\ \text{k}\Omega$. The circuit is shown in Fig. 5.21; note that there is no load resistance at the output.

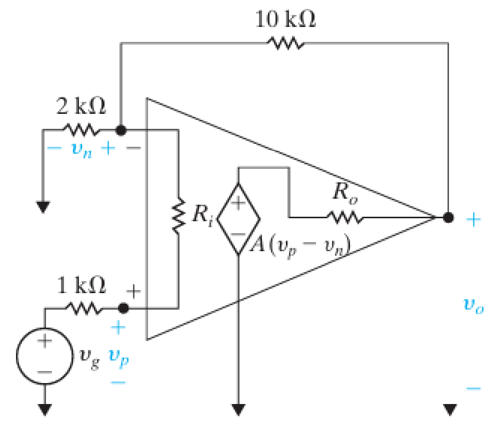

Figure 5.21 ▲ The difference amplifier from Example 5.5,using a realistic op amp model with $A = 50{,}000$, $R_i = 100\ \text{k}\Omega$, and $R_o = 7.5\ \text{k}\Omega$.

a) Calculate the ratio of the output voltage to the source voltage, v_o/v_g.

b) Find the voltages at the op amp input terminals op amp, v_n and v_p, with respect to the common node, when $v_g = 1$ V.

c) Find the voltage difference at the op amp input terminals, $(v_p - v_n)$, when $v_g = 1$ V.

d) Find the current in the signal source, i_g, when the voltage of the source $v_g = 1$ V.

Solution

) Using Eq. 5.27,

$$\frac{v_o}{v_g} = \frac{10\ \text{k} + 2\ \text{k} + \dfrac{(2\ \text{k})(7.5\ \text{k})}{(100\ \text{k})(50{,}000)}}{2\ \text{k} + \dfrac{7.5\ \text{k}}{50{,}000}\left(1 + \dfrac{2\ \text{k} + 1\ \text{k}}{100\ \text{k}}\right) + \dfrac{1}{50{,}000(100\ \text{k})}[(10\ \text{k})(2\ \text{k}) + (10\ \text{k} + 2\ \text{k})(100\ \text{k} + 1\ \text{k})]} = 5.9988.$$

Note how close this value is to the gain of 6 specified and achieved in Example 5.5 using the ideal op amp model.

b) From part (a), when $v_g = 1$ V, $v_o = 5.9988$ V. Now use Eq. 5.23 to solve for v_n in terms of v_o and v_g:

$$v_n\left(\frac{1}{2\ \text{k}} + \frac{1}{1\ \text{k} + 100\ \text{k}} + \frac{1}{10\ \text{k}}\right) = \frac{1}{100\ \text{k} + 1\ \text{k}} + \frac{5.9988}{10\ \text{k}};$$

$$v_n = 0.999803\ \text{V}.$$

Use Eq. 5.25 to solve for v_p:

$$v_p = \frac{R_g(v_n - v_g)}{R_i + R_g} + v_g = \frac{1\ \text{k}(0.999803 - 1)}{100\ \text{k} + 1\ \text{k}} + 1$$

$$= 0.999996\ \text{V}.$$

c) Using the results from part (b), we find that the voltage difference at the op amp input terminals is

$$v_p - v_n = 192.895\ \mu\text{V}.$$

While this voltage difference is very small, it is not zero, as we assume when using the ideal op amp model.

d) The current in the signal source is the current in the resistor R_g. Using Ohm's law,

$$i_g = \frac{{}_g - {}_p}{R_g} = \frac{1 - 0.999996}{1000} = 3.86\ \text{nA}.$$

This is also the current into the noninverting op amp terminal. It is very small but is not zero, as we assume when using the ideal op amp model.