EXAMPLE 2.1 Testing Interconnections of Ideal Sources

Use the definitions of the ideal independent voltage and current sources to determine which interconnections in Fig. 2.3 are permitted and which violate the constraints imposed by the ideal sources.

Solution

Connection (a) is permitted. Each source supplies voltage across the same pair of terminals, marked a and b. This requires that each source supply the same voltage with the same polarity, which they do.

Connection (b) is permitted. Each source supplies current through the same pair of terminals, marked a and b. This requires that each source supply the same current in the same direction, which they do.

Connection (c) is not permitted. Each source supplies voltage across the same pair of terminals, marked a and b. This requires that each source supply the same voltage with the same polarity, which they do not.

Connection (d) is not permitted. Each source supplies current through the same pair of terminals, marked a and b. This requires that each source supply the same current in the same direction, which they do not.

Connection (e) is permitted. The voltage source supplies voltage across the pair of terminals marked a and b. The current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same voltage regardless of the current, and an ideal current source supplies the same current regardless of the voltage, this connection is permitted.

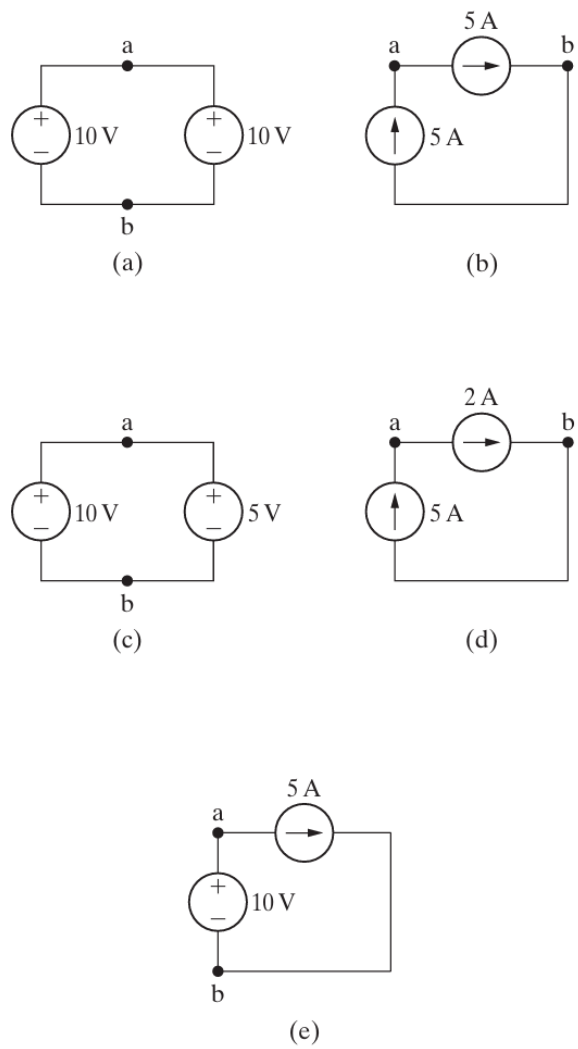


Figure 2.3 ▲ The circuits for Example 2.1.

EXAMPLE 2.2 Testing Interconnections of Ideal Independent and Dependent Sources

State which interconnections in Fig. 2.4 are permitted and which violate the constraints imposed by the ideal sources, using the definitions of the ideal independent and dependent sources.

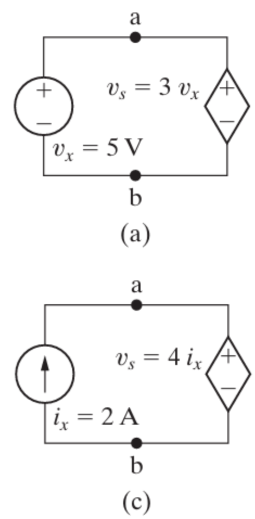


Figure 2.4 ▲ The circuits for Example 2.2.

Solution

Connection (a) is not permitted. Both the independent source and the dependent source supply voltage across the same pair of terminals, labeled a and b. This requires that each source supply the same voltage with the same polarity. The independent source supplies 5 V, but the dependent source supplies 15 V.

Connection (b) is permitted. The independent voltage source supplies voltage across the pair of terminals marked a and b. The dependent current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same voltage regardless of current, and an ideal current source supplies the same current regardless of voltage, this is a valid connection.

Connection (c) is permitted. The independent current source supplies current through the pair of terminals marked a and b. The dependent voltage source supplies voltage across the same pair of terminals. Because an ideal current source supplies the same current regardless of voltage, and an ideal voltage source supplies the same voltage regardless of current, this is a valid connection.

Connection (d) is not permitted. Both the independent source and the dependent source supply current through the same pair of terminals, labeled a and b. This requires that each source supply the same current in the same direction. The independent source supplies 2 A, but the dependent source supplies 6 A in the opposite direction.

EXAMPLE 2.3 Calculating Voltage, Current, and Power for a Simple Resistive Circuit

In each circuit in Fig. 2.8, either the value of or i is not known.

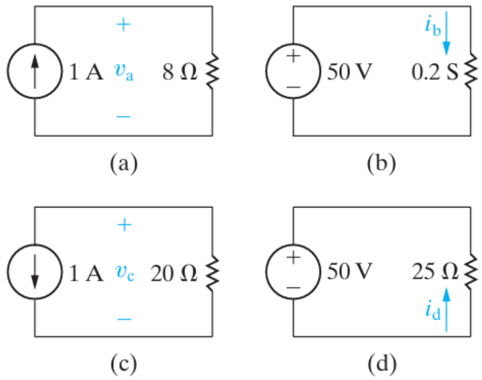


Figure 2.8 ▲ The circuits for Example 2.3.

a) Calculate the values of and i.

b) Determine the power dissipated in each resistor.

Solution

a) The voltage ¨$_a$ in Fig. 2.8(a) is a drop in the direction of the resistor current. The resistor voltage is the product of its current and its resistance, so,

$$v_a = (1)(8) = 8\text{ V}.$$

The current i_b in the resistor with a conductance of 0.2 S in Fig. 2.8(b) is in the direction of the voltage drop across the resistor. The resistor

current is the product of its voltage and its conductance, so

$$i_b = (50)(0.2) = 10\text{ A}.$$

The voltage $_c$ in Fig. 2.8(c) is a rise in the direction of the resistor current. The resistor voltage is the product of its current and its resistance, so

$$v_c = -(1)(20) = -20\text{ V}.$$

The current i_d in the 25 Ω resistor in Fig. 2.8(d) is in the direction of the voltage rise across the resistor. The resistor current is its voltage divided by its resistance, so

$$i_d = -\frac{50}{25} = -2\text{ A}.$$

b) The power dissipated in each of the four resistors is

$$p_{8\Omega} = \frac{(8)^2}{8} = (1)^2(8) = 8\text{ W}$$

(using Eq. 2.10 and Eq. 2.9);

$$p_{0.2S} = (50)^2(0.2) = 500\text{ W}$$

(using Eq. 2.12);

$$p_{20\Omega} = \frac{(-20)^2}{20} = (1)^2(20) = 20\text{ W}$$

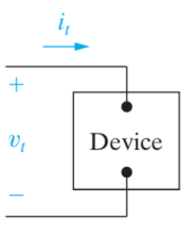
(using Eq. 2.10 and Eq. 2.9);

$$p_{25\Omega} = \frac{(50)^2}{25} = (-2)^2(25) = 100\text{ W}$$

(using Eq. 2.10 and Eq. 2.9).

EXAMPLE 2.5 Constructing a Circuit Model Based on Terminal Measurements

The voltage and current are measured at the terminals of the device illustrated in Fig. 2.13(a), and the values of $_t$ and i_t are tabulated in Fig. 2.13(b). Construct a circuit model of the device inside the box.



(a)

v_t (V)	i_t (A)
−40	−10
−20	−5
0	0
20	5
40	10

(b)

Figure 2.13 ▲ The (a) device and (b) data for Example 2.5.

Solution

Plotting the voltage as a function of the current yields the graph shown in Fig. 2.14(a). The equation of the line in this figure is $v_t = 4i_t$, so the terminal voltage is directly proportional to the terminal current. Using Ohm's law, the device inside the box behaves like a 4 Ω resistor. Therefore, the circuit model for the device inside the box is a 4 Ω resistor, as seen in Fig. 2.14(b).

We come back to this technique of using terminal characteristics to construct a circuit model after introducing Kirchhoff's laws and circuit analysis.

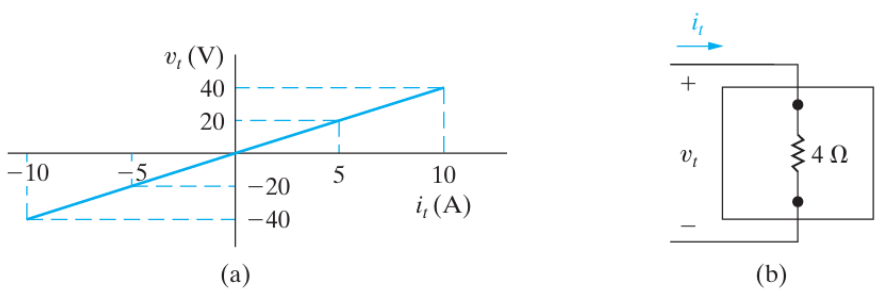


Figure 2.14 ▲ (a) The values of $_t$ versus i_t for the device in Fig. 2.13. (b) The circuit model for the device in Fig. 2.13.

EXAMPLE 2.6 Using Kirchhoff's Current Law

Sum the currents at each node in the circuit shown in Fig. 2.16. Note that there is no connection dot (•) in the center of the diagram, where the 4 Ω branch crosses the branch containing the ideal current source i_a.

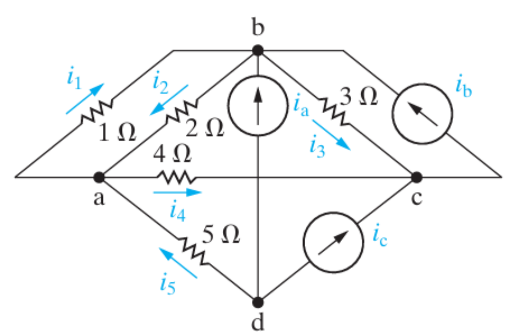


Figure 2.16 ▲ The circuit for Example 2.6.

Solution

In writing the equations, we use a positive sign for a current leaving a node. The four equations are

$$\text{node a} \qquad i_1 + i_4 - i_2 - i_5 = 0,$$

$$\text{node b} \qquad i_2 + i_3 - i_1 - i_b - i_a = 0,$$

$$\text{node c} \qquad i_b - i_3 - i_4 - i_c = 0,$$

$$\text{node d} \qquad i_5 + i_a + i_c = 0.$$

EXAMPLE 2.7 Using Kirchhoff's Voltage Law

Sum the voltages around each designated path in the circuit shown in Fig. 2.17.

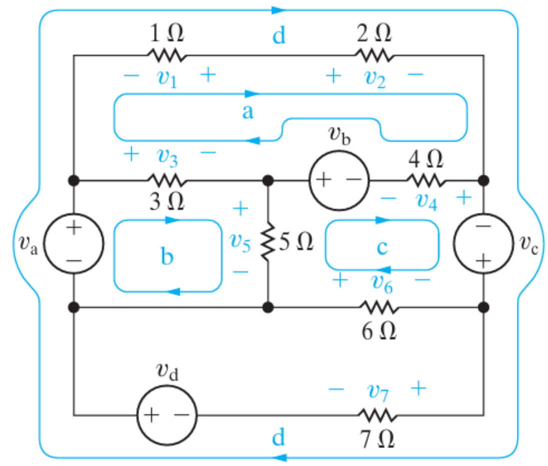


Figure 2.17 ▲ The circuit for Example 2.7.

Solution

In writing the equations, we use a positive sign for a voltage drop. The four equations are

$$\text{path a} \qquad -v_1 + v_2 + v_4 - v_b - v_3 = 0,$$

$$\text{path b} \qquad -v_a + v_3 + v_5 = 0,$$

$$\text{path c} \qquad v_b - v_4 - v_c - v_6 - v_5 = 0,$$

$$\text{path d} \qquad -v_a - v_1 + v_2 - v_c + v_7 - v_d = 0.$$

EXAMPLE 2.8 Applying Ohm's Law and Kirchhoff's Laws to Find an Unknown Current

a) Use Kirchhoff's laws and Ohm's law to find i_o in the circuit shown in Fig. 2.18.

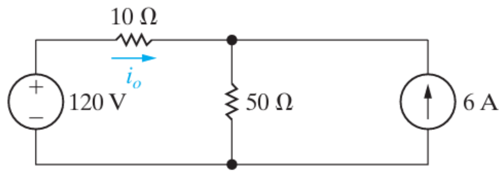


Figure 2.18 ▲ The circuit for Example 2.8.

b) Test the solution for i_o by verifying that the total power generated equals the total power dissipated.

Solution

a) We begin by redrawing the circuit and assigning an unknown current to the 50 Ω resistor and unknown voltages across the 10 Ω and 50 Ω resistors. Figure 2.19 shows the circuit. The nodes are labeled a, b, and c to aid the discussion.

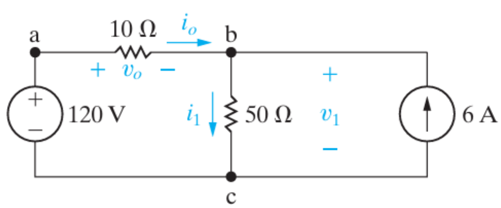


Figure 2.19 ▲ The circuit shown in Fig. 2.18, with the unknowns i_1, $_o$, and $_1$ defined.

Because i_o also is the current in the 120 V source, we have two unknown currents and therefore must derive two simultaneous equations involving i_o and i_1. One of the equations results from applying Kirchhoff's current law to either node b or c. Summing the currents at node b and assigning a positive sign to the currents leaving the node gives

$$i_1 - i_o - 6 = 0.$$

We obtain the second equation from Kirchhoff's voltage law in combination with Ohm's law. Noting from Ohm's law that $v_o = 10i_o$ and $v_1 = 50i_1$, we sum the voltages clockwise around the closed path c-a-b-c to obtain

$$-120 + 10i_o + 50i_1 = 0.$$

In writing this equation, we assigned a positive sign to voltage drops in the clockwise direction. Solving these two equations (see Appendix A) for i_o and i_1 yields

$$i_o = -3 \text{ A} \quad \text{and} \quad i_1 = 3 \text{ A}.$$

b) The power for the 50 Ω resistor is

$$p_{50\Omega} = (i_1)^2(50) = (3)^2(50) = 450 \text{ W}.$$

The power for the 10 Ω resistor is

$$p_{10\Omega} = (i_o)^2(10) = (-3)^2(10) = 90 \text{ W}.$$

The power for the 120 V source is

$$p_{120\text{V}} = -120i_o = -120(-3) = 360 \text{ W}.$$

The power for the 6 A source is

$$p_{6\text{A}} = -v_1(6), \text{ and } v_1 = 50i_1 = 50(3) = 150 \text{ V};$$

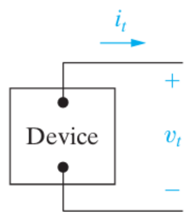
therefore

$$p_{6\text{A}} = -150(6) = -900 \text{ W}.$$

The 6 A source is delivering 900 W, and the 120 V source and the two resistors are absorbing power. The total power absorbed is $p_{6\text{A}} + p_{50\Omega} + p_{10\Omega} = 360 + 450 + 90 = 900$ W. Therefore, the solution verifies that the power delivered equals the power absorbed.

EXAMPLE 2.9 Constructing a Circuit Model Based on Terminal Measurements

We measured the terminal voltage and terminal current on the device shown in Fig. 2.20(a) and tabulated the values of $_t$ and i_t in Fig. 2.20(b).



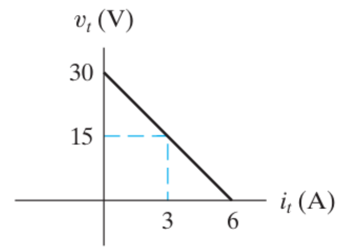
(a)

v_t (V)	i_t (A)
30	0
15	3
0	6

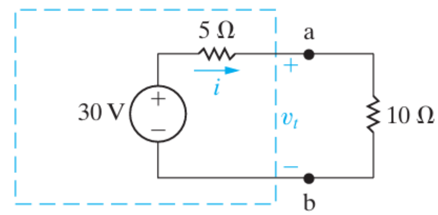
(b)

Figure 2.20 ▲ (a) Device and (b) data for Example 2.9.

a) Construct a circuit model of the device inside the box.
b) Using this circuit model, predict the power this device will deliver to a 10 Ω resistor.



(a)



(b)

Figure 2.21 ▲ (a) The graph of $_t$ versus i_t for the device in Fig. 2.20(a). (b) The resulting circuit model for the device in Fig. 2.20(a), connected to a 10 Ω resistor.

Solution

a) Plotting the voltage as a function of the current yields the graph shown in Fig. 2.21(a). The equation of the line plotted is

$$v_t = 30 - 5i_t.$$

What circuit model components produce this relationship between voltage and current? Kirchhoff's voltage law tells us that the voltage drops across two components in series add. From the equation, one of those components produces a 30 V drop regardless of the current, so this component's model is an ideal independent voltage source. The other component produces a positive voltage drop in the direction of the current i_t. Because the voltage drop is proportional to the current, Ohm's law tells us that this component's model is an ideal resistor with a value of 5 Ω. The resulting circuit model is depicted in the dashed box in Fig. 2.21(b).

b) Now we attach a 10 Ω resistor to the device in Fig. 2.21(b) to complete the circuit. Kirchhoff's current law tells us that the current in the 10 Ω resistor equals the current in the 5 Ω resistor. Using Kirchhoff's voltage law and Ohm's law, we can write the

equation for the voltage drops around the circuit, starting at the voltage source and proceeding clockwise:

$$-30 + 5i + 10i = 0.$$

Solving for i, we get

$$i = 2 \text{ A}.$$

This is the value of current flowing in the 10 Ω resistor, so compute the resistor's power using the equation $p = i^2R$:

$$p_{10\Omega} = (2)^2(10) = 40 \text{ W}.$$

EXAMPLE 2.10 Analyzing a Circuit with a Dependent Source

Find the voltage v_o for the circuit in Fig. 2.22.

Solution

The closed path consisting of the voltage source, the 5 Ω resistor, and the 20 Ω resistor contains the two unknown currents. Apply Kirchhoff's voltage law around this closed path, using Ohm's law to express the voltage across the resistors in terms of the currents in those resistors. Starting at node c and traversing the path clockwise gives:

$$-500 + 5i_\Delta + 20i_o = 0.$$

Now we need a second equation containing these two currents. We can't apply Kirchhoff's voltage law to the closed path formed by the 20 Ω resistor and the dependent current source because we don't know the value of the voltage across the dependent current source. For this same reason, we cannot apply Kirchhoff's voltage law to the closed path containing the voltage source, the 5 Ω resistor, and the dependent source.

We turn to Kirchhoff's current law to generate the second equation. Either node b or node c can be used to construct the second equation from Kirchhoff's current law, since we have already used node a to determine that the current in the voltage source and the 5 Ω resistor is the same. We select node b and produce the following equation, summing the currents leaving the node:

$$-i_\Delta + i_o - 5i_\Delta = 0.$$

Solve the KCL equation for i_o in terms of $i_\Delta (i_o = 6i_\Delta)$, and then substitute this expression for i_o into the KVL equation to give

$$500 = 5i_\Delta + 20(6i_\Delta) = 125i_\Delta.$$

Therefore,

$$i_\Delta = 500/125 = 4 \text{ A} \quad \text{and} \quad i_o = 6(4) = 24 \text{ A}.$$

Using i_o and Ohm's law for the 20 Ω resistor, we can solve for the voltage $_o$:

$$v_o = 20i_o = 480 \text{ V}.$$

EXAMPLE 2.11 Applying Ohm's Law and Kirchhoff's Laws to Find an Unknown Voltage

a) Use Kirchhoff's laws and Ohm's law to find the voltage $_o$ as shown in Fig. 2.23.
b) Show that your solution is consistent with the requirement that the total power developed in the circuit equals the total power dissipated.

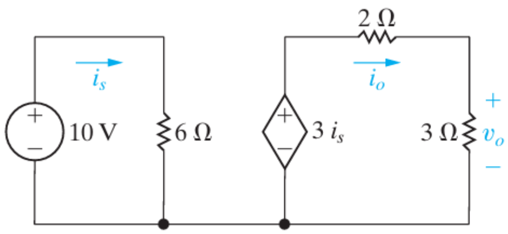


Figure 2.23 ▲ The circuit for Example 2.11.

Solution

a) A close look at the circuit in Fig. 2.23 reveals that:
- There are two closed paths, the one on the left with the current i_s and the one on the right with the current i_o.
- Once i_o is known, we can compute $_o$ using Ohm's law.

We need two equations for the two currents. Because there are two closed paths and both have voltage sources, we can apply Kirchhoff's voltage law to each, using Ohm's law to express the voltage across the resistors in terms of the current in those resistors. The resulting equations are:

$$-10 + 6i_s = 0 \quad \text{and} \quad -3i_s + 2i_o + 3i_o = 0.$$

Solving for the currents yields

$$i_s = 1.67 \text{ A} \quad \text{and} \quad i_o = 1 \text{ A}.$$

Applying Ohm's law to the 3 Ω resistor gives the desired voltage:

$$v_o = 3i_o = 3 \text{ V}.$$

b) To compute the power delivered to the voltage sources, we use the power equation, $p = vi$, together with the passive sign convention. The power for the independent voltage source is

$$p = -10i_s = -10(1.67) = -16.7 \text{ W}.$$

The power for the dependent voltage source is

$$p = -(3i_s)i_o = -(5)(1) = -5 \text{ W}.$$

Both sources are supplying power, and the total power supplied is 21.7 W.

To compute the power for the resistors, we use the power equation, $p = i^2R$. The power for the 6 Ω resistor is

$$p = (1.67)^2(6) = 16.7 \text{ W}.$$

The power for the 2 Ω resistor is

$$p = (1)^2(2) = 2 \text{ W}.$$

The power for the 3 Ω resistor is

$$p = (1)^2(3) = 3 \text{ W}.$$

The resistors all absorb power, and the total power absorbed is 21.7 W, equal to the total power supplied by the sources.

EXAMPLE 2.12 Applying Ohm's Law and Kirchhoff's Law in an Amplifier Circuit

The circuit in Fig. 2.24 represents a common configuration encountered in the analysis and design of transistor amplifiers. Assume that the values of all the circuit elements—R_1, R_2, R_C, R_E, V_{CC}, and V_0—are known.

a) Develop the equations needed to determine the current in each element of this circuit.
b) From these equations, devise a formula for computing i_B in terms of the circuit element values.

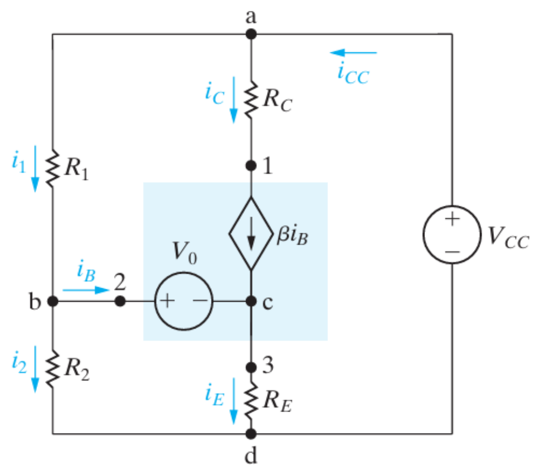


Figure 2.24 ▲ The circuit for Example 2.12.

Solution

Carefully examine the circuit to identify six unknown currents, designated i_1, i_2, i_B, i_C, i_E, and i_{CC}. In defining these six unknown currents, we observed that the resistor R_C is in series with the dependent current source βi_B, so these two components have the same current. We now must derive six independent equations involving these six unknowns.

a) We can derive three equations by applying Kirchhoff's current law to any three of the nodes a, b, c, and d. Let's use nodes a, b, and c and label the currents away from the nodes as positive:

$$(1)\quad i_1 + i_C - i_{CC} = 0,$$

$$(2)\quad i_B + i_2 - i_1 = 0,$$

$$(3)\quad i_E - i_B - i_C = 0.$$

A fourth equation results from imposing the constraint presented by the series connection of R_C and the dependent source:

$$(4)\quad i_C = \beta i_B.$$

We use Kirchhoff's voltage law to derive the remaining two equations. We must select two closed paths, one for each Kirchhoff's voltage law equation. The voltage across the dependent current source is unknown and cannot be determined from the source current βi_B, so select two closed paths that do not contain this dependent current source.

We choose the paths b-c-d-b and b-a-d-b, then use Ohm's law to express resistor voltage in terms of resistor current. Traverse the paths in the clockwise direction and specify voltage drops as positive to yield

$$(5)\quad V_0 + i_E R_E - i_2 R_2 = 0,$$

$$(6)\quad -i_1 R_1 + V_{CC} - i_2 R_2 = 0.$$

b) To get a single equation for i_B in terms of the known circuit variables, you can follow these steps:

- Solve Eq. (6) for i_1, and substitute this solution for i_1 into Eq. (2).
- Solve the transformed Eq. (2) for i_2, and substitute this solution for i_2 into Eq. (5).
- Solve the transformed Eq. (5) for i_E, and substitute this solution for i_E into Eq. (3). Use Eq. (4) to eliminate i_C in Eq. (3).
- Solve the transformed Eq. (3) for i_B, and rearrange the terms to yield

$$i_B = \frac{(V_{CC}R_2)/(R_1 + R_2) - V_0}{(R_1 R_2)/(R_1 + R_2) + (1 + \beta)R_E}. \tag{2.21}$$

Problem 2.38 asks you to verify these steps. Note that once we know i_B, we can easily obtain the remaining currents.