Ex.: $\int \sin^2 x \cdot \cos^1 x \, dx$ $\qquad \sin x = t \Rightarrow \cos x \, dx = dt$

$$\int t^2 \, dt = \frac{t^3}{3} + c = \frac{\sin^3 x}{3} + c$$

Ex.: $\int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x \, dx}{e^{2x} + 1}$ $\qquad e^x = t \Rightarrow e^x \, dx = dt$

$$\int \frac{dt}{t^2 + 1} = \arctan t + c = \arctan e^x + c$$

Ex.: $\int_{x=0}^{x=\pi/2} \sin^5 x \cdot \cos x \, dx$ $\qquad \sin x = t \Rightarrow \cos x \, dx = dt$

$$\boxed{x = 0 \Rightarrow t = 0 \ / \ x = \frac{\pi}{2} \Rightarrow t = 1}$$

$$\int_{t=0}^{t=1} t^5 \cdot dt = \frac{t^6}{6} \Big|_0^1 = \frac{1}{6}$$

$$\int t^5 \, dt = \frac{t^6}{6} = \frac{\sin^6 x}{6} \Big|_{x=0}^{x=\frac{\pi}{2}} = \frac{1}{6}$$

Ex.: $\int_1^4 \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx$ $\qquad 1 + \sqrt{x} = t^2 \Rightarrow \frac{1}{2\sqrt{x}} dx = 2t \, dt$

$x = 1 \Rightarrow t = \sqrt{2} \ / \ x = 4 \Rightarrow t = \sqrt{3}$

$$\int_{\sqrt{2}}^{\sqrt{3}} 4t^2 \, dt = \frac{4}{3} t^3 \Big|_{\sqrt{2}}^{\sqrt{3}} = \frac{4}{3}\left[3\sqrt{3} - 2\sqrt{2}\right]$$

Ex.: If $\int_3^5 f(\underset{\substack{\downarrow \\ t}}{x} - k) \, d\underset{\substack{\downarrow \\ t}}{x} = 1$, compute $\int_{3-k}^{5-k} f(x) \, dx$

$x = u - k$ $\qquad x = 3 - k \Rightarrow u = 3$

$dx = du$ $\qquad x = 5 - k \Rightarrow u = 5$

$$\int_{3-k}^{5-k} f(x) \, dx = \int_3^5 f(u - k) \, du = \int_3^5 f(\underset{\substack{\downarrow \\ t}}{x} - k) \, d\underset{\substack{\downarrow \\ t}}{x} = 1$$

Ex.: $\int_0^{\pi} \underbrace{\sqrt{1-\sin^2 x}}_{\sqrt{\cos^2 x}} \cdot \sin x \, dx$ $\qquad 1-\sin^2 x = \cos^2 x$

$\downarrow$

$|\cos x|$ at $x=\frac{\pi}{2}$ $\cos x$ changes sign

$\int_0^{\pi/2} \cos x \cdot \sin x \, dx + \int_{\pi/2}^{\pi} (-\cos x) \cdot \sin x \, dx$ $\qquad \cos x = t$ or $\sin x = t$

$\int_0^{\pi/2} \frac{\sin(2x)}{2} dx - \int_{\pi/2}^{\pi} \frac{\sin(2x)}{2} dx = \frac{-\cos(2x)}{4} \Big|_0^{\pi/2} + \frac{\cos(2x)}{4} \Big|_{\pi/2}^{\pi} = \underbrace{-\frac{1}{4}(-1-1)}_{\frac{1}{2}} + \underbrace{\frac{1}{4}(1+1)}_{\frac{1}{2}}$ $\qquad \frac{1}{2} + \frac{1}{2} = 1$

Ex.: $\int \frac{\ln x}{x^2} dx$ $\qquad$ (LIATE)

$\ln x = u \qquad \frac{dx}{x^2} = dv$

$\frac{dx}{x} = du \qquad -\frac{1}{x} = v$

$\Rightarrow \frac{-\ln x}{x} + \int \underbrace{\frac{1}{x} \cdot \frac{1}{x}}_{\frac{1}{x^2}} dx = -\frac{\ln x}{x} - \frac{1}{x} + C$

Ex.: $\int \ln(x^2+1) dx = I$ $\qquad \Rightarrow x \cdot \ln(x^2+1) - \int \underbrace{x \cdot \frac{2x}{x^2+1}}_{\frac{2x^2+2-2}{x^2+1}} dx$

$\ln(x^2+1) = u \qquad dx = dv$

$\frac{2x}{x^2+1} dx = du \qquad x = v$

$\int \left(\frac{2\cancel{(x^2+1)}}{\cancel{x^2+1}} - \frac{2}{x^2+1} \right) dx$

$I = x \cdot \ln(x^2+1) - 2x + 2 \cdot \arctan x + C$

Ex.: $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} x^3 \cdot \cos(x^2) dx$ $\qquad x^2 = t \Rightarrow 2x\,dx = dt$

$x^2 \cdot x$

$x = \sqrt{\pi/2} \Rightarrow t = \pi/2 \ / \ x = \sqrt{\pi} \Rightarrow t = \pi$

$\int_{\pi/2}^{\pi} \frac{t \cdot \cos t}{2} dt$

1 $t=u \quad \cos t\,dt = dv$

$dt = du \quad \sin t = v$

$$\Rightarrow \frac{1}{2}\left[t.\sin t - \int \sin t\,dt\right]_{\pi/2}^{\pi} = \frac{1}{2}\left[t.\sin t + \cos t\right]_{\pi/2}^{\pi} = \frac{1}{2}\left[-1-\frac{\pi}{2}\right]$$

2 $\cos t = u \quad t\,dt = dv$

$-\sin t\,dt = du \quad \frac{t^2}{2} = v$

$$\Rightarrow \frac{1}{2}\left[\frac{t^2}{2}.\cos t + \int \frac{t^2}{2}.\sin t\,dt\right] \Rightarrow \underline{\text{DONT}}$$

Ex.: $\int x.e^{x^2}.\cos x^2\,dx = I \quad x^2 = t \Rightarrow 2x\,dx = dt$

$\frac{1}{2}\int e^t.\cos t\,dt \quad \cos t = u \quad e^t dt = dv$

$-\sin t\,dt = du \quad e^t = v$

$$\frac{1}{2}\int e^t.\cos t\,dt = \frac{1}{2}\left[e^t.\cos t + \int e^t.\sin t\,dt\right] \quad \sin t = u \quad e^t dt = dv$$

$\cos t\,dt = du \quad e^t = v$

$$\underbrace{\frac{1}{2}\int e^t.\cos t\,dt}_{I} = \frac{1}{2}\left[e^t.\cos t + e^t.\sin t - \underbrace{\int e^t.\cos t\,dt}\right]$$

$$2I = \frac{1}{2}\left[e^t.\cos t + e^t.\sin t\right] + c$$

$$\Rightarrow I = \frac{1}{4}\left[e^{x^2}.\cos x^2 + e^{x^2}.\sin x^2\right] + c$$