A **A** **A**

Name Surname: **Student No:** **Duration: 100 minutes** **22.12.2022**

A particle of mass *m*, while traveling at a constant velocity of $\vec{\upsilon} = \upsilon\hat{i}$, collides with another particle of mass 4*m*, which is at rest. After the collision, the particle with mass *m* deviates 60° from its initial direction of motion (*x*-axis) and moves with $\upsilon/2$ speed. Based on this information, answer the following three questions (1-3). (sin30°=cos60°=1/2, cos30°= sin60°=$\sqrt{3}/2$, sin45°=cos45°=$\sqrt{2}/2$, sin0°=cos 90°=0, cos0°= sin90°=1)

1) By how many degrees will the 4*m* particle deviate from the *x*-axis after the collision?

a) 0° b) 30 ° c) 45° d) 60° e) 90°

2) What is the speed of the particle having 4*m* mass after the collision in (m/s) unit?

a) $\frac{\upsilon}{\sqrt{2}}$ b) $\frac{\upsilon\sqrt{3}}{2}$ c) $\frac{\upsilon\sqrt{2}}{3}$ d) $\frac{\upsilon\sqrt{3}}{8}$ e) $\frac{\upsilon\sqrt{3}}{4}$

3) What is the absolute value of the ratio of kinetic energy loss to pre-collision kinetic energy in this collision?

a) $\frac{9}{16}$ b) $\frac{7}{16}$ c) $\frac{5}{16}$ d) $\frac{4}{16}$ e) $\frac{3}{16}$

A 125 (kg) astronaut (including space suit) acquires a speed of 2 (m/s) by pushing off with her legs from a 2000 (kg) space capsule. Based on this information, answer the following three questions (4-6).

4) What is the change in speed of the space capsule in (m/s) unit?

a) $-\frac{1}{3}$ b) $-\frac{1}{4}$ c) $-\frac{1}{5}$ d) $-\frac{1}{6}$ e) $-\frac{1}{8}$

5) If the push lasts 0.5 (s), what is the magnitude of the average force exerted by the astronaut and space capsule on each other in (N) unit? (As the reference frame, use the position of the capsule before the push.)

a) 125 b) 250 c) 500 d) 600 e) 750

6) What is the kinetic energy of each after the push in (J) unit? (K_a is the kinetic energy of the astronaut and K_{sc} is the kinetic energy of the space capsule.)

a) $K_a = 250$, $K_{sc} = \frac{125}{8}$ b) $K_a = 125$, $K_{sc} = \frac{125}{4}$ c) $K_a = 250$, $K_{sc} = \frac{125}{4}$ d) $K_a = 125$, $K_{sc} = \frac{125}{2}$ e) $K_a = 125$, $K_{sc} = \frac{250}{8}$

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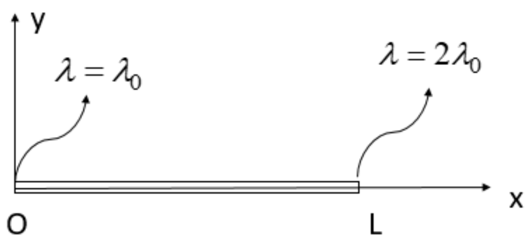
7) The linear mass density of a thin rod of mass *m* of length *L* is $\lambda = \lambda_0$ at its left end and $\lambda = 2\lambda_0$ at its right end. The linear mass density varies <u>uniformly</u> from its left end to the right end. How far is the center of mass of the rod from its left end? (Guidance: Derive an *x* dependent function for λ.)



a) $\frac{L}{4}$ b) $\frac{3L}{8}$ c) $\frac{5L}{6}$ d) $\frac{3L}{4}$ e) $\frac{5L}{9}$

8) A person sits stationary at the back end of a long trolley moving uniformly with speed on a frictionless horizontal surface. If the person gets up and runs forward on the trolley, which of the comments given in the options is true for the speed of the system's center of mass? (The system constitutes the person and trolley.)

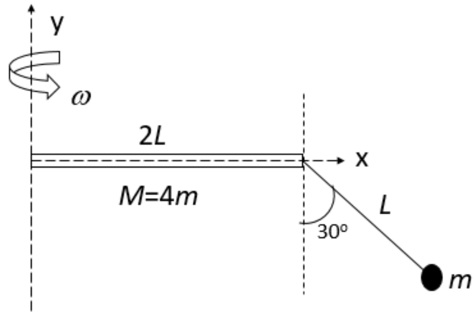
a) The speed of the center of mass of the system increases.
b) The speed of the center of mass of the system decreases.
c) The speed of the center of mass of the system remains the same.
d) The speed of the center of mass of the system increases and then decreases.
e) Not enough information has been given to make a judgment.

9) When a rigid object rotates about a fixed axis, which of the following statements is true about all points in the object?

A. They all have the same tangential speed.
B. They all have the same tangential acceleration.
C. They all have the same angular speed.
D. They all have the same angular acceleration.
E. They all have the same radial acceleration.
F. They all have the same total acceleration.

a) A, B, C, and F b) C, D, and F c) A, D, E, and F d) C and D e) B, C, and D

In the system shown in the figure, the object with mass m is suspended from a massless rope of length L to a rod with mass M=4m and length 2L with homogeneous mass distribution. The system is rotated with constant angular speed ω around an axis passing through the other end of the rod and perpendicular to the rod. In this case, the rope makes an angle of 30° with the vertical. Based on this information, answer the following four questions (10-13).



(The moment of inertia of a homogeneous rod of mass m and length L about an axis perpendicular to the rod and passing through the center of mass is defined by $\frac{1}{12}mL^2$.) (sin30°=cos60°=1/2, cos30°= sin60°=$\sqrt{3}/2$)

10) What is the distance between the center of mass of the system and the y-axis?

a) $\frac{33}{50}L$ b) $\frac{65}{50}L$ c) $\frac{11}{50}L$ d) $\frac{17}{50}L$ e) $\frac{34}{50}L$

11) What is the moment of inertia of the system about the y-axis in terms of the physical quantities given in the question?

a) $\frac{99}{12}mL^2$ b) $\frac{106}{12}mL^2$ c) $\frac{121}{12}mL^2$ d) $\frac{117}{12}mL^2$ e) $\frac{139}{12}mL^2$

12) What is the rotational kinetic energy of the system in terms of the physical quantities given in the question?

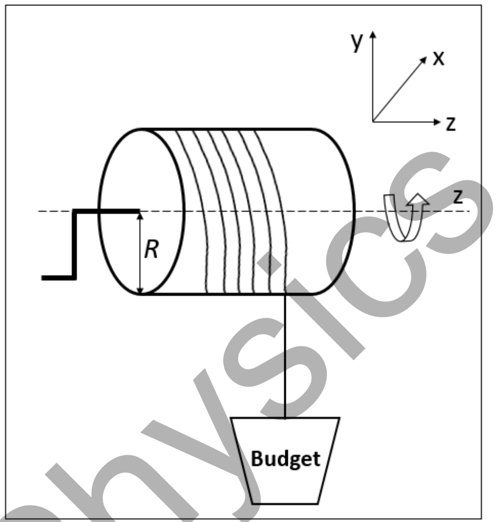
a) $\frac{99}{24}mL^2\omega^2$ b) $\frac{106}{24}mL^2\omega^2$ c) $\frac{121}{24}mL^2\omega^2$ d) $\frac{117}{24}mL^2\omega^2$ e) $\frac{139}{24}mL^2\omega^2$

13) What is the linear speed of the object with mass m?

a) $2\omega L$ b) $\frac{9}{4}\omega L$ c) $\frac{5}{2}\omega L$ d) $3\omega L$ e) $\frac{7}{2}\omega L$

The bucket attached to a rope wound on a spinning wheel of radius R=0.5 (m) is released in the vertical direction as shown in the figure. The mass of the spinning wheel is m=25 (kg). The spinning wheel is a uniform solid cylinder; and its moment of inertia about the axis of rotation is $\frac{1}{2}mR^2$. All frictions and rope masses in the system are neglected. The rope breaks when the tensile force exceeds 100 (N).

Based on this information, answer the following four questions (14-17). (In the figure, the yz is the page plane; the x-axis is oriented into the page plane.)



14) What should be the maximum linear acceleration of the bucket in (m/s^2) unit so that the rope does not break when the bucket is released?

a) 8 b) 4 c) 12 d) 6 e) 10

15) If the force exerted by the rope on the spinning wheel is 100 (N), how much work is done in (J) unit by this force if this force rotates the spinning wheel one complete cycle from the rest?

a) 125 b) 150 c) 250 d) 300 e) 330

16) If the force exerted by the rope on the spinning wheel is 100 (N), what will be the angular speed in (rad/s) unit of the spinning wheel if this force rotates the spinning wheel one complete cycle from the rest?

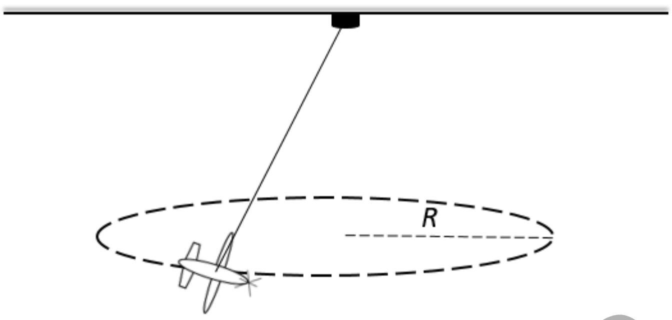
a) $6\sqrt{3}$ b) $8\sqrt{3}$ c) $9\sqrt{3}$ d) $2\sqrt{3}$ e) $3\sqrt{3}$

17) For the situation in question 15, when the spinning wheel is rotated one complete cycle by the rope, what is the power in (W) unit delivered to the spinning wheel by the rope?

a) $150\sqrt{3}$ b) $200\sqrt{3}$ c) $400\sqrt{3}$ d) $300\sqrt{3}$ e) $120\sqrt{3}$

A model airplane with a mass of 0.6 (kg) is flying on a horizontal circular path of radius R=30 (m) as it is attached to the ceiling by a wire as shown in the figure. The airplane engine provides a net thrust of 0.9 N <u>perpendicular</u> to the tethering wire.

Based on this information, answer the following four questions (18-20).



18) What is the magnitude of the torque produced by the net thrust about the center of the circle in (N.m) unit?

a) 27 b) 18 c) 36 d) 54 e) 45

19) What is the angular acceleration of the model airplane in (rad/s^2) unit? (Guidance: Consider the model air plane as a point like object.)

a) 0.0125 b) 0.1250 c) 0.2500 d) 0.0500 e) 0.5000

20) What is the translational acceleration of the model airplane that is tangent to its flight path in (m/s2) unit?

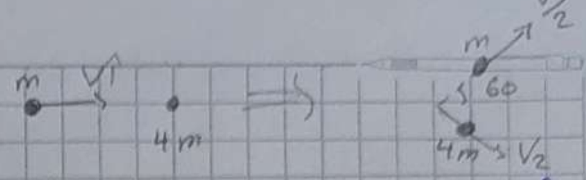
a) 1.125 b) 1.500 c) 1.250 d)1.000 e) 2.500

No		No	
1.	B	**11.**	E
2.	D	**12.**	E
3.	A	**13.**	C
4.	E	**14.**	A
5.	C	**15.**	D
6	A	**16.**	B
7	E	**17.**	C
8.	C	**18.**	A
9.	D	**19.**	D
10.	B	**20.**	B

$m = m$ $\quad$ $m = 4m$ $\quad$ $\vec{V}_1' = \frac{V}{2}$

$\vec{V}_1 = V \uparrow$ $\quad$ $\vec{V}_2 = 0$ $\quad$ $\Theta = 60$

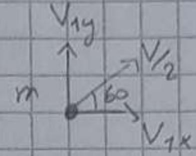
before collusion



1) by how many degrees will the 4m particle deviate from the x axis.

Consirvation of Momentum.

$$P_i = P_f \Rightarrow P_{ix} = P_{fx} \text{ and } P_{iy} = P_{fy}$$



$$\cancel{m} V = \cancel{m} \cdot V_{1x} + 4\cancel{m} V_{2x}$$

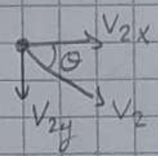
$$V_{1x} = \frac{V}{2} \cdot \cos 60$$

$$V_{1x} = \frac{V}{4}$$

$$V = \frac{V}{4} + 4 V_{2x}$$

$$V_{1y} = \frac{V}{2} \cdot \sin 60 = \frac{\sqrt{3} V}{4}$$

$$\Rightarrow V_{2x} = \frac{3V}{16}$$



$$P_{iy} = P_{fy}$$

$$\tan\Theta = \frac{V_{2x}}{V_{2y}}$$

$$0 = \cancel{m} V_{1y} + 4\cancel{m} V_{2y}$$

$$\Theta = \arctan\left(\frac{V_{2y}}{V_{2x}}\right)$$

$$-\frac{\sqrt{3} V}{4} = 4 V_{2y} \Rightarrow V_{2y} = -\frac{\sqrt{3} V}{16}$$

$$\Theta = \arctan\left(\frac{V_{2y}}{V_{2x}}\right) = \arctan\left(\frac{\frac{\sqrt{3}\cancel{V}}{\cancel{16}}}{\frac{3\cancel{V}}{\cancel{16}}}\right) = \arctan\left(\frac{\sqrt{3}}{3}\right) = 30^\circ$$

b

2) what's the speed of ~~V~~ 4m ? $= |V_2|$

$$= \sqrt{(V_{2x})^2 + (V_{2y})^2} = \sqrt{\frac{9V^2}{16^2} + \frac{3V^2}{16^2}} = \sqrt{\frac{12 V^2}{16^2}} = \frac{\sqrt{12}\, V}{16} = \frac{2\sqrt{3}\, V}{16}$$

$$= \frac{\sqrt{3}\, V}{8}$$

d

3) $\left|\frac{\Delta K}{K_i}\right| = ?$

$$\Delta K = K_f - K_i = \frac{m 7 V^2}{32} - \frac{m V^2}{2} = m V^2 \left(-\frac{9}{32}\right)$$

$$K_i = \frac{1}{2} m \cdot V^2$$

$$K_f = \frac{1}{2} m \frac{V^2}{4} + \frac{1}{2} \cancel{4} m \cdot \frac{3V^2}{\underset{16}{\cancel{64}}} = \frac{mV^2}{8} + \frac{m3V^2}{32} = \frac{m 7 V^2}{32}$$

$$\frac{\Delta K}{K_i} = \frac{\cancel{mV^2} \frac{9}{32}}{\cancel{mV^2} \frac{1}{2}} = \frac{9}{16}$$

a

$m = 125$ $V_1 = 2$ $M = 2000$

4) what's the final speed of the capsule

$P_i = P_f$

$0 = mV_1 + MV_2 \Rightarrow mV_1 = -MV_2$

no movement

$\Rightarrow V = 0$

$125 \cdot 2 = -2000 V_2 \Rightarrow V_2 = \frac{-1}{8}$ (d)

5) if $t = 0{,}5 \Rightarrow \bar{F} = ?$

$J = \Delta P = \bar{F} \cdot t$

impulse   momentum difference of the human

$\Delta P = P_f - \cancel{P_i}^{\,0}$

$= mV_1 = 125 \cdot 2 = 250$

$\Delta P = \bar{F} \cdot t$

$250 = \frac{\bar{F}}{2} \Rightarrow \bar{F} = 500$ (e)

6) $K_f = ??$

$K_a = \frac{1}{2} m v^2 = \frac{1}{2} \cdot 125 \cdot 4 = 250$ (a)

$K_{sc} = \frac{1}{2} 2000 \cdot \frac{1}{64} = \frac{125}{8}$

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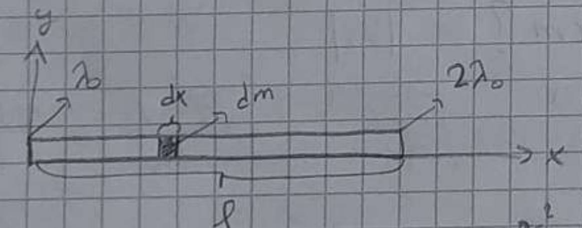
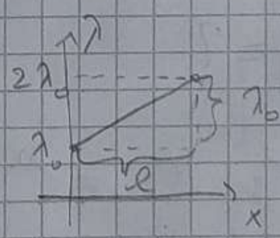
7)



$X_{cm} = ?$

$\lambda = \frac{dm}{dx} \Rightarrow dm = \lambda\, dx$



$X_{cm} = \frac{1}{M} \cdot \int x\, dm$

$= \dfrac{\int_0^\ell \left(\frac{\lambda_0}{\ell} x^2 + \lambda_0 x\right) dm}{\int_0^\ell \left(\frac{\lambda_0}{\ell} x + \lambda_0\right) dm}$

$= \dfrac{\left(\frac{\lambda_0 x^3}{3\ell} + \frac{\lambda_0 x^2}{2}\right|_0^\ell}{\left(\frac{\lambda_0 x^2}{2\ell} + \lambda_0 x\right|_0^\ell}$

$= \dfrac{\frac{\lambda_0 \ell^3}{3\ell} + \frac{\lambda_0 \ell^2}{2}}{\frac{\lambda_0 \ell^2}{2\ell} + \lambda_0 \ell}$

$= \dfrac{\frac{2\ell^2 + 3\ell^2}{6}}{\frac{3\ell}{2}}$

$= \dfrac{2\ell^2 + 3\ell^2}{9\ell}$

$= \frac{5\ell}{9}$ (c)

$\lambda - \lambda_0 = \frac{\lambda_0}{\ell}(x - 0)$

$\lambda = \frac{\lambda_0}{\ell} x + \lambda_0$

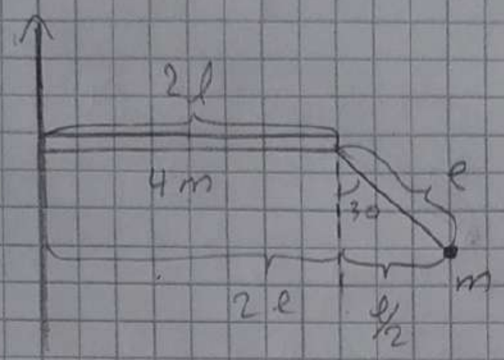
$\Rightarrow dm = \left(\frac{\lambda_0 x}{\ell} + \lambda_0\right) dx$

and $M = \int dm$

8) for Isolated systems, speed of CM is zero. c

9) C and D ~~tangential~~ means it has the radius in it, so position affect it.

d



10) $X_{cm} = ?$

CM is at the middle of the rod

$$= \frac{\cancel{m} \cdot \frac{5l}{2} + 4\cancel{m}\, l}{5\cancel{m}}$$

$$= \frac{13l}{10} \cdot \frac{5}{5} = \frac{65l}{50}$$

b

11) $I_y^{system} = ? = I_y^{rod} + I_y^{m}$

$I_y^{m} = m r^2 = m \left(\frac{5l}{2}\right)^2 = m l^2 \frac{25}{4}$

$I_y^{rod} = I_{cm} + M d^2 = \frac{1}{12} 4m\, 4l^2 + 4m \cdot l^2 = 4ml^2 \cdot \left(\frac{4}{3}\right) = ml^2 \left(\frac{16}{3}\right)$

parallel axis theorem

$I^{sys} = ml^2 \left(\frac{25}{4} + \frac{16}{3}\right) = ml^2 \frac{139}{12}$

e

12) $K_R = ? = \cancel{\frac{1}{2}} = \frac{1}{2} I \omega^2 = \frac{139}{24} ml^2 \omega^2$ e

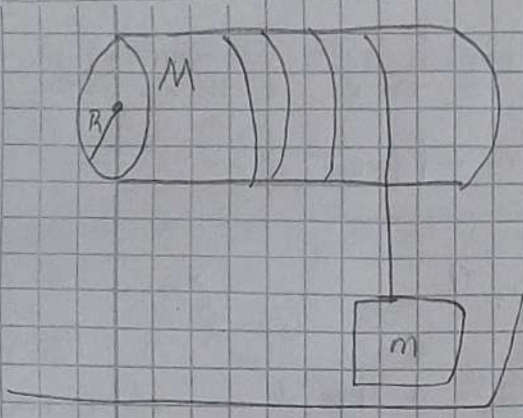
13) what's V of m ?

$V = R \cdot \omega = \frac{5}{2} \cdot \omega$

c

$R = 0{,}5 \qquad M = 25$

max force on the rope = 100



$I_{CM} = \frac{1}{2} M R^2$

14) what's the max acc?

m)

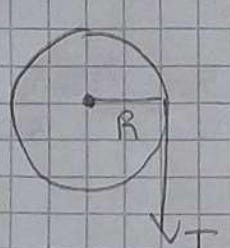
$\sum F = ma$

$mg - T = ma$

$10m - 100 = m \cdot 8$

$m = 50$

cylen)



$\sum \tau = I \cdot \alpha$

$T \cdot \cancel{R} = \frac{1}{2} M R^{\cancel{2}} \alpha$

$\alpha = \frac{a}{R}$

$T = \frac{1}{2} M a$

max T = 100

$100 = \frac{1}{2} \cdot 25 \cdot a$

$a = 8$ // (a)

15) $W_T = ?$ for one cycle

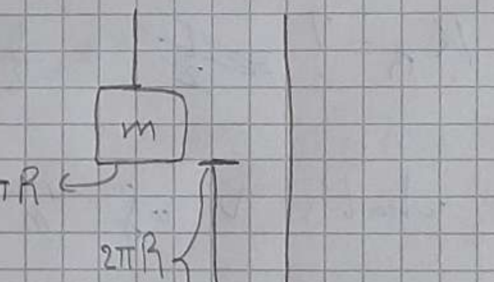
$W = \tau \cdot \Delta\theta = R \cdot T \cdot 2\pi = \frac{1}{2} \cdot 100 \cdot 2 \cdot 3 = 300$ // (d)

16) $\omega_f = ?$ for one cycle.

$E_i = E_f$

$\cancel{K_i}^{\,0} + U_i = K_f + \cancel{U_f}^{\,0}$

$mg\,2\pi R = \underbrace{\frac{1}{2} I \omega_f^2}_{\text{energy in cylender}} + \underbrace{\frac{1}{2} m V_f^2}_{\text{energy in m}}$



$50 \cdot 10 \cdot 3 = \frac{1}{2} \frac{25}{8} \cdot \omega_f^2 + \frac{1}{2} \cdot 50\, \omega_f^2 \frac{1}{4}$

$V_f = \omega_f R$

$V_f^2 = \omega_f^2 R^2$

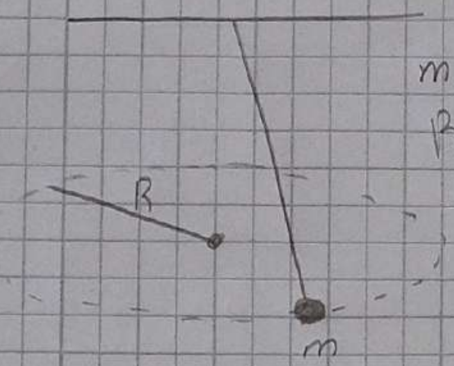
$I_{cy} = \frac{1}{2} M R^2 = \frac{25}{8}$

$1500 = \omega_f^2 \left(\frac{125}{16}\right)$

$\omega_f^2 = \frac{16 \cdot 1500}{125}$

$\omega_f^2 = 16 \cdot 12 \Rightarrow \omega_f = 8\sqrt{3}$ // (b)

17) $P = ? = \tau \omega = 50 \cdot 8\sqrt{3} = 400\sqrt{3}$



$m = 0{,}6$

$R = 30$

$F = 0{,}9$

18) $\tau = ? = \vec{R} \times \vec{F} = R F \sin 90$

$= 30 \cdot 0{,}9 = 27$ a

19) $\alpha = ?$

$\Sigma \tau = I \alpha$

$I = mR^2$

$= 0{,}6 \cdot 30^2$

$= \frac{6}{10} \cdot 900$

$= 540$

$27 = 540 \cdot \alpha$

$\alpha = \frac{27}{540} = \frac{1}{20} = 0{,}05$ d

20) $a_t = ?$

$= R\alpha = 30 \cdot 0{,}05 = 1{,}5$ b

سبحان الله وبحمده ٥

سبحان الله العظيم