Ex.: $\int \sin^2 x \cdot \underbrace{\cos x \, dx}$

$\sin x = t$

$\cos x \, dx = dt$

$$\int t^2 \, dt = \frac{t^3}{3} + C = \frac{\sin^3 x}{3} + C$$

Ex.: $\int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x \, dx}{e^{2x} + 1}$ $\quad e^x = t \Rightarrow e^x dx = dt$

$$\int \frac{dt}{t^2 + 1} = \arctan t + C = \arctan e^x + C$$

Ex.: $\int_0^{\pi/2} \sin^5 x \cdot \cos x \, dx$ $\quad \sin x = t \Rightarrow \cos x \, dx = dt$

$x = 0 \Rightarrow t = 0 \;/\; x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^1 t^5 \, dt = \frac{t^6}{6} \Big|_0^1 = \frac{1}{6}$$

$$\frac{t^6}{6} = \frac{\sin^6 x}{6} \Big|_{x=0}^{x=\pi/2}$$

Ex.: $\int_1^4 \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx$ $\quad \sqrt{x} = t$, $\frac{1}{2\sqrt{x}} dx = dt$ $\quad x = 1 \Rightarrow t = 1$, $x = 4 \Rightarrow t = 2$

$$\int_1^2 2\underbrace{\sqrt{1+t}}_{(1+t)^{\frac{1}{2}}} \, dt = 2 \cdot \frac{(1+t)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_1^2 = \underbrace{\frac{4}{3}\left[3\sqrt{3} - 2\sqrt{2}\right]}$$

$1 + \sqrt{x} = t$

$\underline{Ex.}$: If $\underbrace{\int_3^5 f(x-k)\,dx = 1}_{t}$, $x = 3-k$, compute $\underbrace{\int_{3-k}^{5-k} f(x)\,dx}_{\text{substituting}}$.

$3 \to 3-k$

$5 \to 5-k$

$x-k \to x$

$x = t+k$

$dx = dt$

$x=3 \Rightarrow t = 3-k$

$x=5 \to t = 5-k$

$$\int_{3-k}^{5-k} f(t)\,dt$$

$$\int_3^5 f(x-k)\,dx = 1 = F(x-k)\Big|_3^5 = F(5-k) - F(3-k) = 1$$

$$F'(x-k) = f(x-k) \implies F(x) = f'(x)$$

$$\int_{3-k}^{5-k} f(x)\,dx = F(x)\Big|_{3-k}^{5-k} = F(5-k) - F(3-k) = 1$$

$\underline{Ex.}$: $\int_0^{\pi} \sqrt{1-\sin^2 x}\,.\sin x\,dx$ $\qquad 1-\sin^2 x = \cos^2 x$

$$\int_0^{\pi} \sqrt{\cos^2 x}\,.\sin x\,dx = \int_0^{\pi} |\cos x|\,.\sin x\,dx$$

$x = \frac{\pi}{2}$ is root of $\underline{\cos x}$

$$\implies \int_0^{\pi/2} \cos x\,.\sin x\,dx + \int_{\pi/2}^{\pi} (-\cos x)\,.\sin x\,dx$$

$\cos x = t$ and $\sin x = t$ also work.

$$= \int_0^{\pi/2} \frac{\sin(2x)}{2}\,dx - \int_{\pi/2}^{\pi} \frac{\sin(2x)}{2}\,dx = -\frac{\cos(2x)}{4}\Big|_0^{\pi/2} + \frac{\cos(2x)}{4}\Big|_{\pi/2}^{\pi}$$

$$= \underbrace{\frac{-1}{4}\big(\underbrace{-1-1}_{-2}\big)}_{\frac{1}{2}} + \underbrace{\frac{1}{4}\big(\underbrace{1+1}_{2}\big)}_{\frac{1}{2}} = 1$$

$$\int \frac{\ln x}{x^2}\,dx \qquad (LIATE)$$

$\ln x = u \qquad \frac{dx}{x^2} = dv$

$\frac{1}{x}dx = du \qquad -\frac{1}{x} = v$

$$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \underbrace{\int \left(-\frac{1}{x}\right)\cdot \frac{1}{x}\,dx}_{+\int \frac{1}{x^2}dx \,=\, -\frac{1}{x}+C}$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$

<u>Ex.:</u> $\int \ln(x^2+1)\,dx$

$\ln(x^2+1) = u \qquad dx = dv$

$\frac{2x}{x^2+1}dx = du \qquad x = v$

$$\Rightarrow x.\ln(x^2+1) - \int \underbrace{x.\frac{2x}{x^2+1}}_{\frac{2x^2+2-2}{x^2+1} = \frac{2\cancel{(x^2+1)}}{\cancel{x^2+1}} - \frac{2}{x^2+1}}\,dx$$

$$\Rightarrow x.\ln(x^2+1) - \int\left(2 - \frac{2}{x^2+1}\right)dx = x.\ln(x^2+1) - 2x + 2.\arctan x + C$$

<u>Homework:</u> $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} x^3.\cos(x^2)\,dx \qquad \left(= -\frac{1}{2} - \frac{\pi}{4}\right)$

Ex.: $\int x\cdot e^{x^2}\cdot\cos x^2\,dx$

$x^2=t \Rightarrow 2x\,dx=dt$

$\int \frac{e^t\cdot\cos t}{2}\,dt$ $\qquad \cos t=u \qquad e^t\,dt=dv$

$\qquad -\sin t\,dt=du \qquad e^t=v$

$\Rightarrow \frac{1}{2}\left[e^t\cos t+\underbrace{\int e^t\cdot\sin t\,dt}_{e^t\sin t-\int e^t\cdot\cos t\,dt}\right]$ $\qquad \sin t=u \qquad e^t\,dt=dv$

$\qquad \cos t\,dt=du \qquad e^t=v$

$\Rightarrow \cancel{\frac{1}{2}}\underbrace{\int e^t\cdot\cos t\,dt}_{I}=\cancel{\frac{1}{2}}e^t\cos t+\cancel{\frac{1}{2}}e^t\cdot\sin t-\cancel{\frac{1}{2}}\underbrace{e^t\cos t}_{I}$

$2I=e^t(\cos t+\sin t)\Rightarrow I=\frac{e^t(\cos t+\sin t)}{2}=\frac{e^{x^2}\cdot(\cos x^2+\sin x^2)}{2}+C$

Check if it was $\frac{}{4}$!