EXAMPLE 4.1 Identifying Node, Branch, Mesh, and Loop in a Circuit

For the circuit in Fig. 4.3, identify

a) all nodes.

b) all essential nodes.

c) all branches.

d) all essential branches.

e) all meshes.

f) two paths that are not loops or essential branches.

g) two loops that are not meshes.

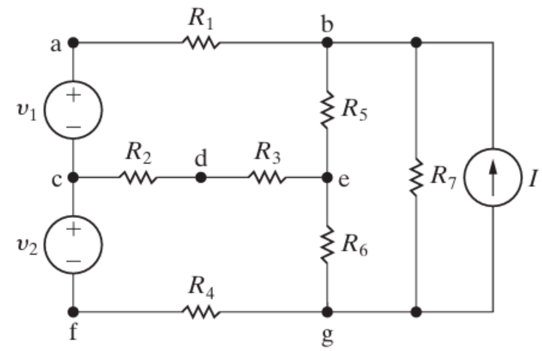


Figure 4.3 ▲ A circuit illustrating nodes, branches, meshes, paths, and loops.

Solution

a) The nodes are a, b, c, d, e, f, and g.

b) The essential nodes are b, c, e, and g.

c) The branches are $_1$, $_2$, R_1, R_2, R_3, R_4, R_5, R_6, R_7, and I.

d) The essential branches are $v_1 - R_1$, $R_2 - R_3$, $v_2 - R_4$, R_5, R_6, R_7, and I.

e) The meshes are $v_1 - R_1 - R_5 - R_3 - R_2$, $v_2 - R_2 - R_3 - R_6 - R_4$, $R_5 - R_7 - R_6$, and $R_7 - I$.

f) $R_1 - R_5 - R_6$ is a path, but it is not a loop (because it does not have the same starting and ending nodes), nor is it an essential branch (because it does not connect two essential nodes). $v_2 - R_2$ is also a path but is neither a loop nor an essential branch, for the same reasons.

g) $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ is a loop but is not a mesh because there are two loops within it. $I - R_5 - R_6$ is also a loop but not a mesh.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problem 4.1.

EXAMPLE 4.2 Using Essential Nodes and Essential Branches to Write Simultaneous Equations

The circuit in Fig. 4.4 has six essential branches, denoted $i_1 - i_6$, where the current is unknown. Use the systematic approach to write the six equations needed to solve for the six unknown currents.

Solution

The essential nodes in the circuit are labeled b, c, e, and g, so $n_e = 4$. From the problem statement we know that the number of essential branches where

[1]Applying KCL to the last unused node (the nth node) does *not* generate an independent equation. See Problem 4.4.

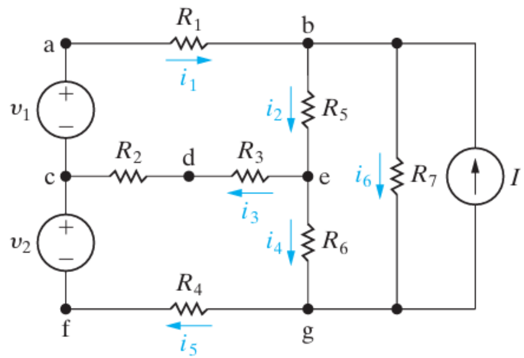


Figure 4.4 ▲ The circuit shown in Fig. 4.3 with six unknown branch currents defined.

the current is unknown is $b_e = 6$. Note that there are seven essential branches in the circuit, but the current in the essential branch containing the current source is known. We need to write six independent equations because there are six unknown currents.

We derive three of the six independent equations by applying Kirchhoff's current law to any three of the four essential nodes. We use the nodes b, c, and e to get

$$-i_1 + i_2 + i_6 - I = 0,$$
$$i_1 - i_3 - i_5 = 0,$$
$$i_3 + i_4 - i_2 = 0.$$

We derive the remaining three equations by applying Kirchhoff's voltage law around three meshes. Remember that the voltage across every component in each mesh must be known or must be expressed as the product of the component's resistance and its current using Ohm's law. Because the circuit has four meshes, we need to dismiss one mesh. We eliminate the $R_7 - I$ mesh because we don't know the voltage across I.[2] Using the other three meshes gives

$$R_1 i_1 + R_5 i_2 + i_3(R_2 + R_3) - v_1 = 0,$$
$$-i_3(R_2 + R_3) + i_4 R_6 + i_5 R_4 - v_2 = 0,$$
$$-i_2 R_5 + i_6 R_7 - i_4 R_6 = 0.$$

Rearranging the six equations to facilitate their solution yields the set

$$-i_1 + i_2 + 0i_3 + 0i_4 + 0i_5 + i_6 = I,$$
$$i_1 + 0i_2 - i_3 + 0i_4 - i_5 + 0i_6 = 0,$$
$$0i_1 - i_2 + i_3 + i_4 + 0i_5 + 0i_6 = 0,$$
$$R_1 i_1 + R_5 i_2 + (R_2 + R_3) i_3 + 0i_4 + 0i_5 + 0i_6 = v_1,$$
$$0i_1 + 0i_2 - (R_2 + R_3) i_3 + R_6 i_4 + R_4 i_5 + 0i_6 = v_2,$$
$$0i_1 - R_5 i_2 + 0i_3 - R_6 i_4 + 0i_5 + R_7 i_6 = 0.$$

EXAMPLE 4.3 Using the Node-Voltage Method

a) Use the node-voltage method of circuit analysis to find the branch currents i_a, i_b, and i_c in the circuit shown in Fig. 4.8.

b) Find the power associated with each source, and state whether the source is delivering or absorbing power.

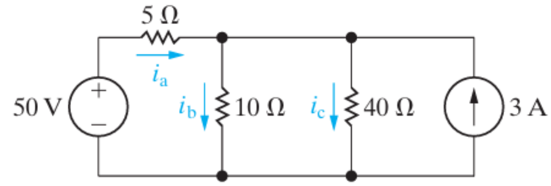


Figure 4.8 ▲ The circuit for Example 4.3.

Solution

a) We begin by noting that the circuit has two essential nodes; thus, we need to write a single KCL equation. **Step 1**: Identify the two essential nodes. **Step 2**: Select the lower node as the reference node and define the unknown node voltage as $_1$. Figure 4.9 illustrates these decisions. **Step 3**: Write a KCL equation at the nonreference essential node by summing the currents leaving node 1:

$$\frac{v_1 - 50}{5} + \frac{v_1}{10} + \frac{v_1}{40} - 3 = 0.$$

Step 4: Solve the equation for $_1$, giving

$$v_1 = 40 \text{ V}.$$

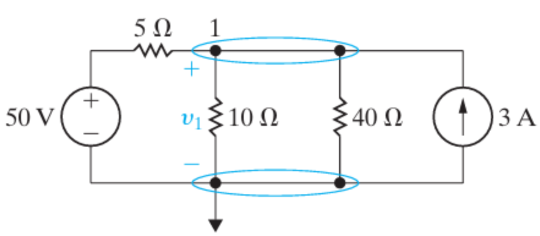


Figure 4.9 ▲ The circuit shown in Fig. 4.8 with a reference node and the unknown node voltage v_1.

Step 5: Use the node voltage $_1$ and Ohm's law to find the requested branch currents:

$$i_a = \frac{50 - v_1}{5} = \frac{50 - 40}{5} = 2 \text{ A},$$

$$i_b = \frac{v_1}{10} = \frac{40}{10} = 4 \text{ A},$$

$$i_c = \frac{v_1}{40} = \frac{40}{40} = 1 \text{ A}.$$

b) The power associated with the 50 V source is

$$p_{50V} = -50 i_a = -100 \text{ W (delivering)}.$$

The power associated with the 3 A source is

$$p_{3A} = -3v_1 = -3(40) = -120 \text{ W (delivering)}.$$

We check these calculations by noting that the total delivered power is 220 W. The total power absorbed by the three resistors is $4(5) + 16(10) + 1(40)$ or 220 W, which equals the total delivered power.

EXAMPLE 4.4 Using the Node-Voltage Method with Dependent Sources

Use the node-voltage method to find the power dissipated in the 5 Ω resistor in the circuit shown in Fig. 4.10.

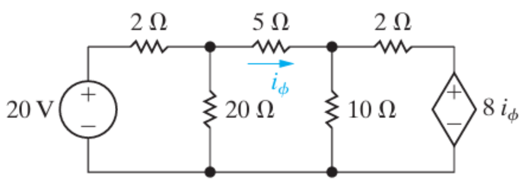


Figure 4.10 ▲ The circuit for Example 4.4.

Solution

Step 1: Identify the circuit's three essential nodes. We will need two KCL equations to describe the circuit.

Step 2: Since four branches terminate on the lower node, we select it as the reference node and label the node voltages at the remaining essential nodes. The results of the first two steps are shown in Fig. 4.11.

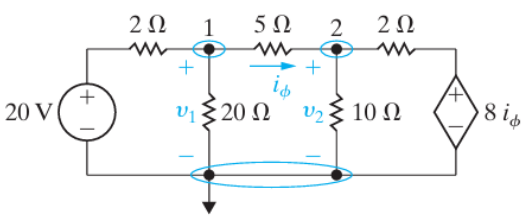


Figure 4.11 ▲ The circuit shown in Fig. 4.10, with a reference node and the node voltages.

Step 3: Generate the simultaneous equations by applying KCL at the nonreference essential nodes. Summing the currents leaving node 1 gives the equation

$$\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0.$$

Summing the currents leaving node 2 yields

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} + \frac{v_2 - 8i_\phi}{2} = 0.$$

As written, these two node-voltage equations contain three unknowns, namely, $_1$, $_2$, and i_ϕ. We need a third equation, which comes from the constraint imposed by the dependent source. This equation expresses the controlling current of the dependent source, i_ϕ, in terms of the node voltages, or

$$i_\phi = \frac{_1 - _2}{5}.$$

As you can see, we need to modify Step 3 in the node-voltage procedure to remind us to write a constraint equation whenever a dependent source is present.

Step 3: Write a KCL equation at each nonreference essential node. If the circuit contains dependent sources, write a dependent source constraint equation that defines the controlling voltage or current of the dependent source in terms of the node voltages.

The condensed form for Step 3 is shown in Analysis Method 4.2.

Step 4: Solve for $_1$, $_2$, and i_ϕ, giving

$$v_1 = 16\text{ V}, v_2 = 10\text{ V, and } i_\phi = 1.2\text{ A}.$$

Step 5: Use the node voltage values to find the current in the 5 Ω resistor and the power dissipated in that resistor:

$$i_\phi = \frac{v_1 - v_2}{5} = \frac{16 - 10}{5} = 1.2\text{ A},$$

$$p_{5\Omega} = 5i_\phi^2 = 5(1.2)^2 = 7.2\text{ W}.$$

A good exercise to build your problem-solving intuition is to reconsider this example, using node 2 as the reference node. Does it make the analysis easier or harder?

NODE-VOLTAGE METHOD

3. Write a KCL equation for every nonreference essential node.

- If there are dependent sources, write a constraint equation for each one.

Analysis Method 4.2 Modified Step 3 for the node-voltage method.

EXAMPLE 4.5 Node-Voltage Analysis of the Amplifier Circuit

Use the node-voltage method to find i_B in the amplifier circuit shown in Fig. 4.17.

Solution

Step 1: We identify the four essential nodes, which are labeled a, b, c, and d. **Step 2**: Choose node d as the reference node. Then label the voltages at the remaining three essential nodes. **Step 3**: Before writing equations, we notice two special cases. The voltage source V_{CC} in the branch connecting node a and the reference node constrains the voltage between those nodes, so $v_a = V_{CC}$, and the voltage source V_0 in the branch between nodes b and c constrains the voltage between those nodes and creates a supernode. The results of Steps 1 and 2 and the modifications prompted by the special cases are depicted in Fig. 4.18.

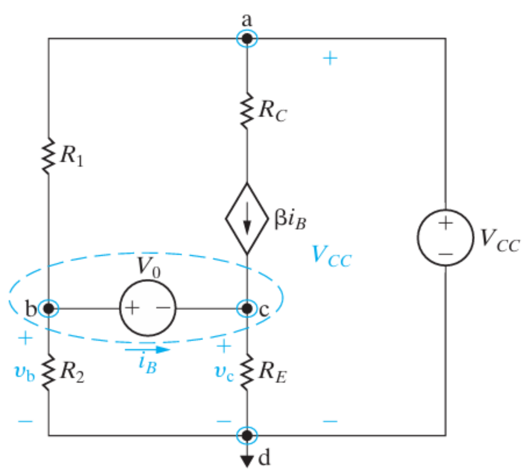


Figure 4.18 ▲ The circuit shown in Fig. 4.17, with voltages and the supernode identified.

Continuing Step 3, write the supernode KCL equation to give

$$\frac{v_b}{R_2} + \frac{v_b - V_{CC}}{R_1} + \frac{v_c}{R_E} - \beta i_B = 0. \qquad (4.10)$$

Now write the dependent source constraint equation, which defines the controlling current i_B in terms of the node voltages. Since i_B is the current in a voltage source, we cannot use Ohm's law, so instead, write a KCL equation at node c:

$$i_B = \frac{\text{c}}{R_E} - \beta i_B. \qquad (4.11)$$

The last part of Step 3 is the supernode constraint equation

$$v_b - v_c = V_0. \qquad (4.12)$$

Step 3 gave us three equations with three unknowns. To solve these equations, we use back-substitution to eliminate the variables v_b and v_c. Begin by rearranging Eq. 4.11 to give

$$i_B = \frac{v_c}{R_E(1 + \beta)}. \qquad (4.13)$$

Next, solve Eq. 4.12 for v_c to give

$$v_c = v_b - V_0. \qquad (4.14)$$

Substituting Eqs. 4.13 and 4.14 into Eq. 4.10 and rearranging yields

$$v_b\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{(1 + \beta)R_E}\right] = \frac{V_{CC}}{R_1} + \frac{V_0}{(1 + \beta)R_E}. \qquad (4.15)$$

Solving Eq. 4.15 for ${}_{\text{b}}$ yields

$$v_b = \frac{V_{CC}R_2(1 + \beta)R_E + V_0R_1R_2}{R_1R_2 + (1 + \beta)R_E(R_1 + R_2)}. \qquad (4.16)$$

You should verify that, when Eq. 4.16 is combined with Eqs. 4.13 and 4.14, the solution for i_B is

$$i_B = \frac{(V_{CC}R_2)/(R_1 + R_2) - V_0}{(R_1R_2)/(R_1 + R_2) + (1 + \beta)R_E}, \qquad (4.17)$$

which is identical to Eq. 2.21. (See Problem 4.31.) Using the node-voltage method to analyze this circuit reduces the problem from manipulating six simultaneous equations (see Problem 2.38) to manipulating three simultaneous equations.

EXAMPLE 4.6 Using the Mesh-Current Method

a) Use the mesh-current method to determine the power associated with each voltage source in the circuit shown in Fig. 4.21.

b) Calculate the voltage $_o$ across the 8 Ω resistor.

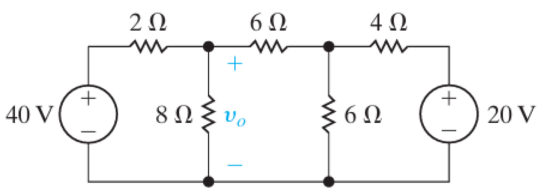


Figure 4.21 ▲ The circuit for Example 4.6.

Solution

a) **Step 1**: We identify the three meshes in the circuit and draw the mesh currents as directed curved arrows following the perimeter of each mesh. It is best to define all mesh currents in the same direction.

Step 2: Label the mesh currents; the results of the first two steps are depicted in Fig. 4.22.

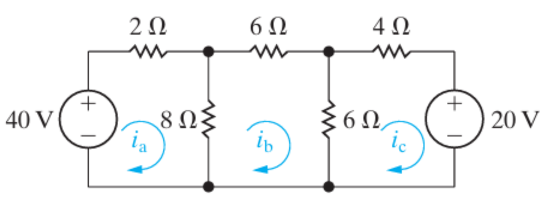


Figure 4.22 ▲ The three mesh currents used to analyze the circuit shown in Fig. 4.21.

Step 3: We use KVL to generate an equation for each mesh by summing the voltages in the direction of the mesh current. In the i_a mesh, start just below the 40 V source and sum the voltages in the clockwise direction to give

$$-40 + 2i_a + 8(i_a - i_b) = 0.$$

In the i_b mesh, start below the 8 Ω resistor and sum the voltages in the clockwise direction to give

$$8(i_b - i_a) + 6i_b + 6(i_b - i_c) = 0.$$

In the i_c mesh, start below the 6 Ω resistor and sum the voltages in the clockwise direction to give

$$6(i_c - i_b) + 4i_c + 20 = 0.$$

Step 4: Solve the three simultaneous mesh current equations from Step 3 to give

$$i_a = 5.6 \text{ A},$$

$$i_b = 2.0 \text{ A},$$

$$i_c = -0.80 \text{ A}.$$

Step 5: Use the mesh currents to find the power for each source. The mesh current i_a equals the branch current in the 40 V source, so the power associated with this source is

$$p_{40V} = -40i_a = -224 \text{ W}.$$

The minus sign means that this source is delivering power to the network. The current in the 20 V source equals the mesh current i_c; therefore

$$p_{20V} = 20i_c = -16 \text{ W}.$$

The 20 V source also is delivering power to the network.

b) The branch current in the 8 Ω resistor in the direction of the voltage drop $_o$ is $i_a - i_b$. Therefore

$$v_o = 8(i_a - i_b) = 8(3.6) = 28.8 \text{ V}.$$

EXAMPLE 4.7 Using the Mesh-Current Method with Dependent Sources

Use the mesh-current method to find the power dissipated in the 4 Ω resistor in the circuit shown in Fig. 4.23.

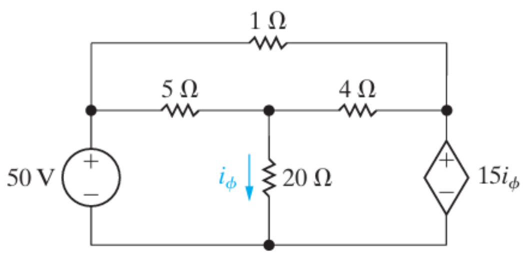


Figure 4.23 ▲ The circuit for Example 4.7.

Solution

Step 1: Begin by drawing the mesh currents in each of the three meshes.

Step 2: Label each mesh current. The resulting circuit is shown in Fig. 4.24.

Step 3: Write a KVL equation for each mesh by picking a starting point anywhere in the mesh and summing the voltages around the mesh in the direction of the mesh current. When you return to the starting point, set the sum equal to zero. The three mesh-current equations are

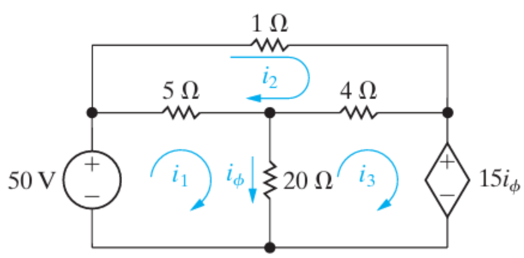


Figure 4.24 ▲ The circuit shown in Fig. 4.23 with the three mesh currents.

$$5(i_1 - i_2) + 20(i_1 - i_3) - 50 = 0,$$
$$5(i_2 - i_1) + 1i_2 + 4(i_2 - i_3) = 0,$$
$$20(i_3 - i_1) + 4(i_3 - i_2) + 15i_\phi = 0.$$

To complete Step 3, express the branch current controlling the dependent voltage source in terms of the mesh currents as

$$i_\phi = i_1 - i_3.$$

Step 4: Solve the four equations generated in Step 3 to find the four unknown currents:

$$i_1 = 29.6 \text{ A}, \quad i_2 = 26 \text{ A}, \quad i_3 = 28 \text{ A}, \quad i_\phi = 1.6 \text{ A}.$$

Step 5: Use the mesh currents to find the power for the 4 Ω resistor. The current in the 4 Ω resistor oriented from left to right is $i_3 - i_2$, or 2 A. Therefore, the power dissipated is

$$p_{4\Omega} = (i_3 - i_2)^2(4) = (2)^2(4) = 16 \text{ W}.$$

What if you had not been told to use the mesh-current method? Would you have chosen the node-voltage method? It reduces the problem to finding one unknown node voltage because of the presence of two voltage sources between essential nodes. We say more about making such choices in Section 4.8.

EXAMPLE 4.8 A Special Case in the Mesh-Current Method

Use the mesh-current method to find branch currents i_a, i_b, and i_c in the circuit for Example 4.3, repeated here as Fig. 4.25.

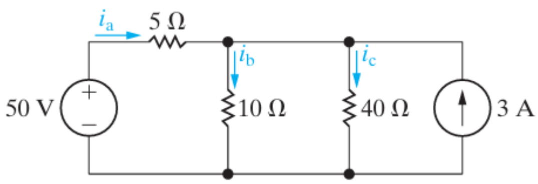


Figure 4.25 ▲ The circuit for Example 4.8.

Solution

Step 1: Use directed arrows that traverse the mesh perimeters to identify the three mesh currents.

Step 2: Label the mesh currents as i_1, i_2, and i_3. The modification in Step 2 reminds us to look for current sources, and the i_3 mesh has a current source that is not shared by any other mesh. Therefore, the i_3 mesh current equals the current supplied by the source. Note that i_3 and the current source are

in opposite directions, so the current in this mesh should be labeled −3 A. The results of Steps 1 and 2 are shown in Fig. 4.26.

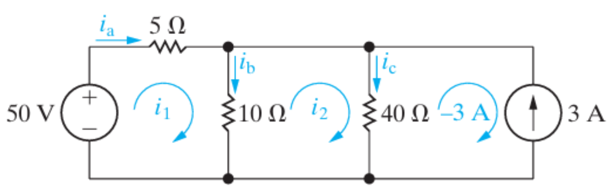


Figure 4.26 ▲ The circuit shown in Fig. 4.25 with the mesh currents identified and labeled.

Step 3: Write the KVL equations for the meshes whose mesh currents are unknown, which in this example are the i_1 and i_2 meshes. Remember to pick a starting point anywhere along the mesh, sum the voltages in the direction of the mesh current, and set the sum equal to zero when you return to the starting point. The resulting simultaneous mesh current equations are

$$-50 + 5i_1 + 10(i_1 - i_2) = 0 \quad \text{and}$$

$$10(i_2 - i_1) + 40(i_2 - (-3)) = 0.$$

Step 4: Solving the simultaneous mesh current equations gives

$$i_1 = 2\text{ A} \qquad \text{and} \qquad i_2 = -2\text{ A}.$$

Step 5: Finally, we use the mesh currents to calculate the branch currents in the circuit, i_a, i_b, and i_c.

$$i_a = i_1 = 2\text{ A},$$

$$i_b = i_1 - i_2 = 4\text{ A},$$

$$i_c = i_2 + 3 = 1\text{ A}.$$

These are the same branch current values as those calculated in Example 4.3. Which of the two circuit analysis methods is better when calculating the branch currents? Which method is better when calculating the power associated with the sources?

EXAMPLE 4.9 Mesh-Current Analysis of the Amplifier Circuit

Use the mesh-current method to find i_B for the amplifier circuit in Fig. 4.29.

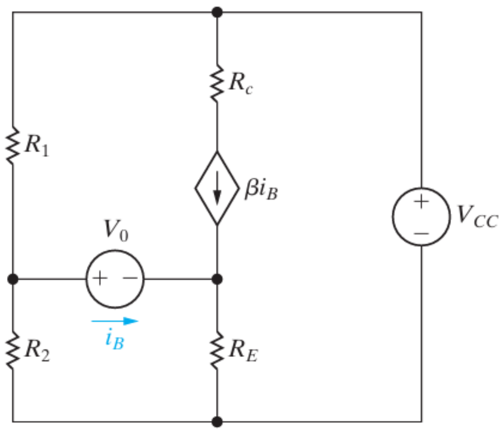


Figure 4.29 ▲ The circuit shown in Fig 2.24.

Solution

Step 1: Use directed arrows that traverse the mesh perimeters to identify the three mesh currents.

Step 2: Label the mesh currents as i_a, i_b, and i_c. Then recognize the current source that is shared between the i_a and i_c meshes. Combine these meshes, bypassing the branch with the shared current source, to create a supermesh. The result of the first two steps is the circuit shown in Fig. 4.30.

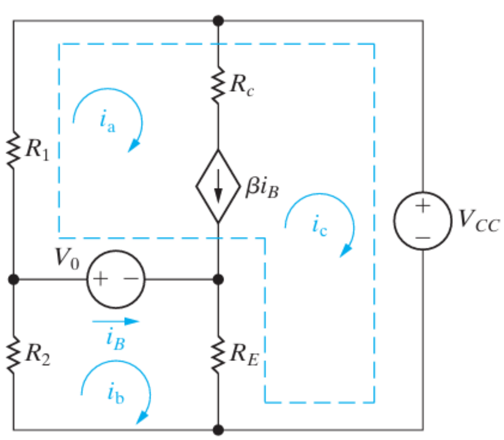


Figure 4.30 ▲ The circuit shown in Fig. 4.29, depicting the supermesh created by the presence of the dependent current source.

Step 3: Using KVL, sum the voltages around the supermesh in terms of the mesh currents i_a, i_b, and i_c to obtain

$$R_1 i_a + v_{CC} + R_E(i_c - i_b) - V_0 = 0. \quad (4.24)$$

The KVL equation for mesh b is

$$R_2 i_b + V_0 + R_E(i_b - i_c) = 0. \quad (4.25)$$

The constraint imposed by the dependent current source is

$$i_B = i_b - i_a. \quad (4.26)$$

The supermesh constraint equation is

$$\beta i_B = i_a - i_c. \quad (4.27)$$

Step 4: Use back-substitution to solve Eqs. 4.24–4.27. Start by combining Eqs. 4.26 and 4.27 to eliminate i_B and solve for i_c to give

$$i_c = (1 + \beta)i_a - \beta i_b. \quad (4.28)$$

We now use Eq. 4.28 to eliminate i_c from Eqs. 4.24 and 4.25:

$$[R_1 + (1 + \beta)R_E]i_a - (1 + \beta)R_E i_b = V_0 - V_{CC}, \quad (4.29)$$

$$-(1 + \beta)R_E i_a + [R_2 + (1 + \beta)R_E]i_b = -V_0. \quad (4.30)$$

You should verify that the solution of Eqs. 4.29 and 4.30 for i_a and i_b gives

$$i_a = \frac{V_0 R_2 - V_{CC}R_2 - V_{CC}(1 + \beta)R_E}{R_1 R_2 + (1 + \beta)R_E(R_1 + R_2)}, \quad (4.31)$$

$$i_b = \frac{-V_0 R_1 - (1 + \beta)R_E V_{CC}}{R_1 R_2 + (1 + \beta)R_E(R_1 + R_2)}. \quad (4.32)$$

Step 5: Use the two mesh currents from Eqs. 4.31 and 4.32, together with the definition for i_B in Eq. 4.26, to find i_B. You should verify that the result is the same as that given by Eq. 2.21.

EXAMPLE 4.10 Understanding the Node-Voltage Method Versus Mesh-Current Method

Find the power dissipated in the 300 Ω resistor in the circuit shown in Fig. 4.31.

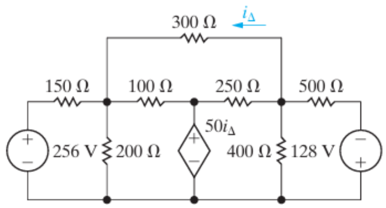


Figure 4.31 ▲ The circuit for Example 4.10.

Solution

To find the power dissipated in the 300 Ω resistor, we need to find either the current in the resistor or the voltage across it. The mesh-current method yields the current in the resistor; this approach requires solving five mesh equations, as depicted in Fig. 4.32, and a dependent source constraint equation, for a total of six simultaneous equations.

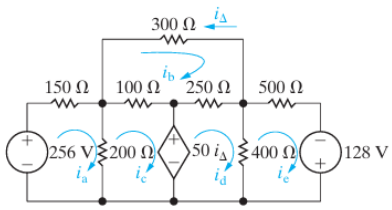


Figure 4.32 ▲ The circuit shown in Fig. 4.31, with the five mesh currents.

Let's now consider using the node-voltage method. The circuit has four essential nodes, and therefore only three node-voltage equations are required to describe the circuit. The dependent voltage source between two essential nodes forms a supernode, requiring a KCL equation and a supernode constraint equation. We have to sum the currents at the remaining essential node, and we need to write the dependent source constraint equation, for a total of four simultaneous equations. Thus, the node-voltage method is the more attractive approach.

Step 1: We begin by identifying the four essential nodes in the circuit of Fig. 4.31. The three black dots at the bottom of the circuit identify a single essential node, and the three black dots in the middle of the circuit are the three remaining essential nodes.

Step 2: Select a reference node. Two essential nodes in the circuit in Fig. 4.31 merit consideration. The first is the reference node in Fig. 4.33, where we also defined the three node voltages v_1, v_2, and v_3, and indicated that nodes 1 and 3 form a supernode because they are connected by a dependent voltage source. If the reference node in Fig. 4.33 is selected, one of the unknown node voltages is the voltage across the 300 Ω resistor, namely, $_2$ in Fig. 4.33. Once we know this voltage, we calculate the power in the 300 Ω resistor by using the expression

$$p_{300\Omega} = v_2^2/300.$$

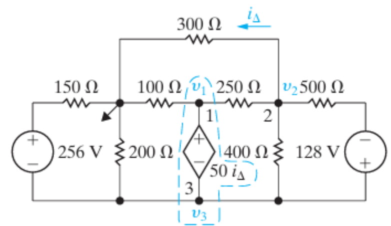


Figure 4.33 ▲ The circuit shown in Fig. 4.31, with a reference node.

The second node worth considering as the reference node is the bottom node in the circuit, as shown in Fig. 4.34. If this reference node is chosen, one of the unknown node voltages is eliminated because $v_b = 50i_\Delta$. We would need to write two KCL equations and a dependent source constraint equation, and solve these three simultaneous equations. However, to find either the current in the 300 Ω resistor or the voltage across it requires an additional calculation once we know the node voltages $_a$ and $_c$.

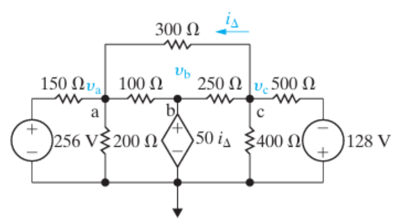


Figure 4.34 ▲ The circuit shown in Fig. 4.31 with an alternative reference node.

Step 3: We compare these two possible reference nodes by generating two sets of KCL equations and constraint equations. The first set pertains to the circuit shown in Fig. 4.33, and the second set is based on the circuit shown in Fig. 4.34.

- Set 1 (Fig. 4.33)
  At the supernode,

$$\frac{v_1}{100} + \frac{v_1 - v_2}{250} + \frac{v_3}{200} + \frac{v_3 - v_2}{400} + \frac{v_3 - (v_2 + 128)}{500} + \frac{v_3 + 256}{150} = 0.$$

At $_2$,

$$\frac{v_2}{300} + \frac{v_2 - v_1}{250} + \frac{v_2 - v_3}{400} + \frac{v_2 + 128 - v_3}{500} = 0.$$

The dependent source constraint equation is

The supernode constraint equation is

$$v_1 - v_3 = 50i_\Delta.$$

- Set 2 (Fig. 4.34); remember that $v_b = 50i_\Delta$.
  At $_a$,

$$\frac{v_a}{200} + \frac{v_a - 256}{150} + \frac{v_a - 50i_\Delta}{100} + \frac{v_a - v_c}{300} = 0.$$

At v_c,

$$\frac{v_c}{400} + \frac{v_c + 128}{500} + \frac{v_c - 50i_\Delta}{250} + \frac{v_c - v_a}{300} = 0.$$

The dependent source constraint equation is

$$i_\Delta = \frac{_c - _a}{300}.$$

Step 4: Solve each set of equations.

EXAMPLE 4.11 Comparing the Node-Voltage and Mesh-Current Methods

Find the voltage $_o$ in the circuit shown in Fig. 4.35.

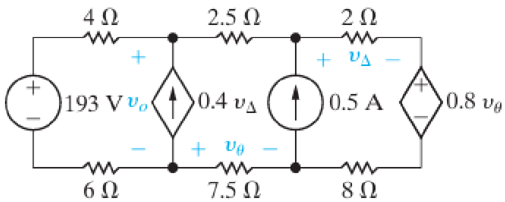


Figure 4.35 ▲ The circuit for Example 4.11.

Solution

We first consider using the mesh-current method.

Step 1: Identify the three mesh currents in the circuit using directed arrows that follow the mesh perimeters.

Step 2: Label the three mesh currents. Because there are two currents sources, each shared by two meshes, we can combine all three meshes into a single supermesh that traverses the perimeter of the entire circuit and avoids the two branches with current sources. The result of these two steps is shown in Fig. 4.36.

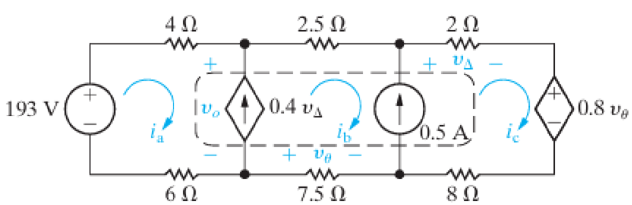


Figure 4.36 ▲ The circuit shown in Fig. 4.35 with the three mesh currents.

Step 3: Write the KCL equation for the supermesh:

$$-193 + 4i_a + 2.5i_b + 2i_c + 0.8v_\theta + 8i_c + 7.5i_b + 6i_a = 0.$$

The supermesh constraint equations are

$$i_b - i_a = 0.4v_\Delta \qquad \text{and} \qquad i_c - i_b = 0.5,$$

and the two dependent source constraint equations are

$$v_\Delta = 2i_c \qquad \text{and} \qquad v_\theta = -7.5i_b.$$

Step 4: We must solve the five simultaneous equations generated in Step 3.

Step 5: We need one additional equation to find v_o from the mesh current i_a:

$$v_o = 193 - 10i_a.$$

Now let's consider using the node-voltage method.

Step 1: There are four essential nodes in the circuit of Fig. 4.35, identified by the four black dots in the figure.

Step 2: We can make the unknown voltage v_o one of the three node voltages by choosing the bottom left node as the reference node. After labeling the remaining two node voltages, we have the circuit in Fig. 4.37.

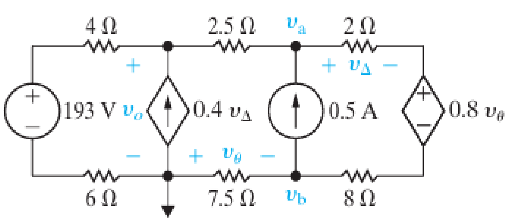


Figure 4.37 ▲ The circuit shown in Fig. 4.35 with node voltages.

Step 3: The KCL equations are

$$\frac{v_o - 193}{10} - 0.4v_\Delta + \frac{v_o - v_a}{2.5} = 0,$$

$$\frac{v_a - v_o}{2.5} - 0.5 + \frac{v_a - (v_b + 0.8v_\theta)}{10} = 0,$$

$$\frac{v_b}{7.5} + 0.5 + \frac{v_b + 0.8v_\theta - v_a}{10} = 0.$$

The dependent source constraint equations are

$$v_\theta = -v_b \quad \text{and} \quad v_\Delta = 2\left[\frac{v_a - (v_b + 0.8v_\theta)}{10}\right].$$

Step 4: Once we solve these five simultaneous equations, we have the value of v_o without writing an additional equation, so Step 5 is not needed.

Based on the comparison of the two methods, the node-voltage method involves a bit less work. You should verify that both approaches give $v_o = 173$ V.

EXAMPLE 4.12 Using Source Transformations to Solve a Circuit

Find the power associated with the 6 V source for the circuit shown in Fig. 4.39 and state whether the 6 V source is absorbing or delivering the power.

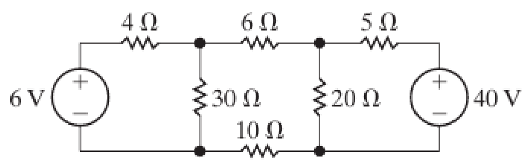


Figure 4.39 ▲ The circuit for Example 4.12.

Solution

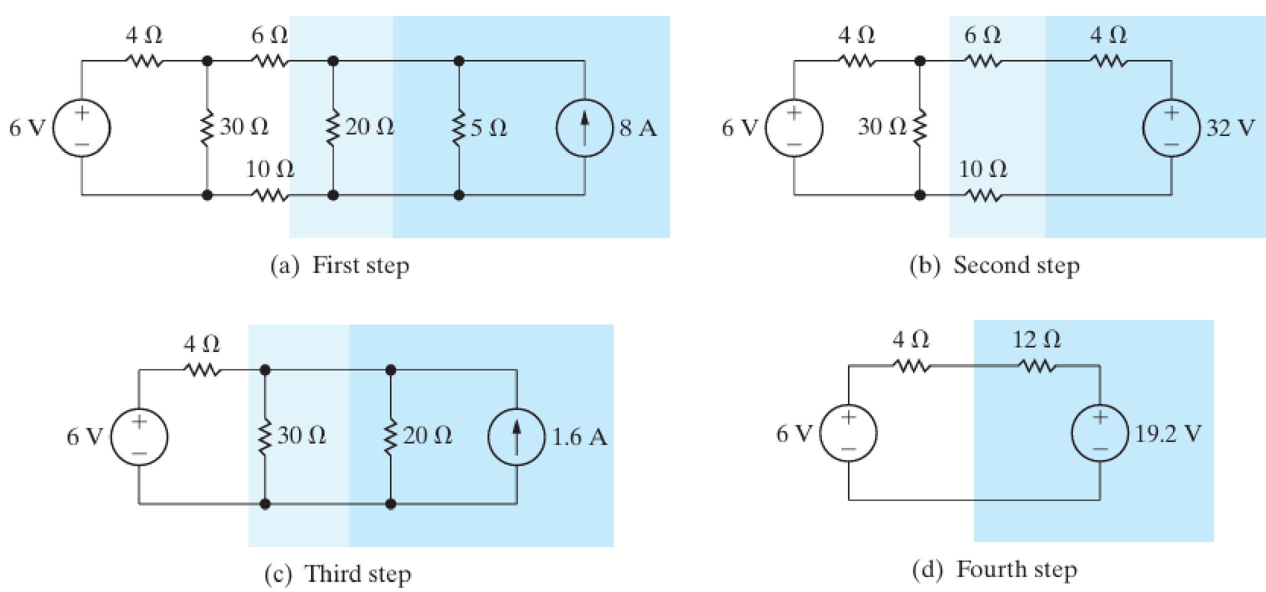
If we study the circuit shown in Fig. 4.39, we see ways to simplify the circuit by using source transformations. But we must simplify the circuit in a way that preserves the branch containing the 6 V source. Therefore, begin on the right side of the circuit with the branch containing the 40 V source. We can transform the 40 V source in series with the 5 Ω resistor into an 8 A current source in parallel with a 5 Ω resistor, as shown in Fig. 4.40(a).

Next, replace the parallel combination of the 20 Ω and 5 Ω resistors with a 4 Ω resistor. This 4 Ω resistor is in parallel with the 8 A source and therefore can be replaced with a 32 V source in series with a 4 Ω resistor, as shown in Fig. 4.40(b). The 32 V source is in series with 20 Ω of resistance and, hence, can be replaced by a current source of 1.6 A in parallel with 20 Ω, as shown in Fig. 4.40(c). The 20 Ω and 30 Ω parallel resistors can be reduced to a single 12 Ω resistor. The parallel combination of the 1.6 A current source and the 12 Ω resistor transforms into a voltage source of 19.2 V in series with 12 Ω. Figure 4.40(d) shows the result of this last transformation. The current in the direction of the voltage drop across the 6 V source is $(19.2 - 6)/16$, or 0.825 A. Therefore, the power associated with the 6 V source is

$$p_{6\text{V}} = (6)(0.825) = 4.95 \text{ W}$$

and the voltage source is absorbing power.

Practice your circuit analysis skills by using either the node-voltage method or the mesh-current method to solve this circuit and verify that you get the same answer.



a circuit with a resistor in series with a current source.

EXAMPLE 4.13 Using Special Source Transformation Techniques

a) Use source transformations to find the voltage $_o$ in the circuit shown in Fig. 4.42.

b) Find the power developed by the 250 V voltage source.

c) Find the power developed by the 8 A current source.

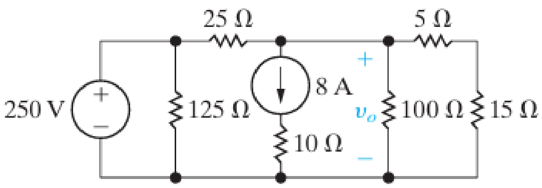


Figure 4.42 ▲ The circuit for Example 4.13.

Solution

a) We begin by removing the 125 Ω and 10 Ω resistors because the 125 Ω resistor is connected in parallel with the 250 V voltage source and the 10 Ω resistor is connected in series with the 8 A current source. We also combine the series-connected resistors in the rightmost branch into a single resistance of 20 Ω. Figure 4.43 shows the simplified circuit.

Now use a source transformation to replace the series-connected 250 V source and 25 Ω resistor with a 10 A source in parallel with the 25 Ω resistor, as shown in Fig. 4.44. We can then use Kirchhoff's current law to combine the parallel

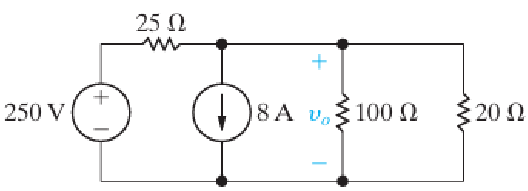


Figure 4.43 ▲ A simplified version of the circuit shown in Fig. 4.42.

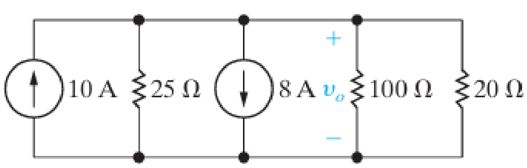


Figure 4.44 ▲ The circuit shown in Fig. 4.43 after a source transformation.

current sources into a single source. The parallel resistors combine into a single resistor. Figure 4.45 shows the result. Hence $v_o = 20$ V.

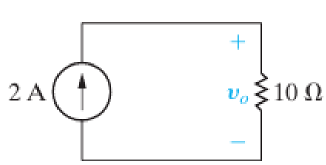


Figure 4.45 ▲ The circuit shown in Fig. 4.44 after combining sources and resistors.

b) We need to return to the original circuit in Fig. 4.42 to calculate the power associated with the sources. While a resistor connected in parallel with a voltage source or a resistor connected in series with a current source can be removed without affecting the terminal behavior of the circuit, these resistors play an important role in how the power is dissipated throughout the circuit. The current supplied by the 250 V source, represented as i_s, equals the current in the 125 Ω resistor plus the current in the 25 Ω resistor. Thus,

$$i_s = \frac{250}{125} + \frac{250 - 20}{25} = 11.2 \text{ A}.$$

Therefore, the power developed by the voltage source is

$$p_{250\text{V}}(\text{developed}) = (250)(11.2) = 2800 \text{ W}.$$

c) To find the power developed by the 8 A current source, we first find the voltage across the source. If we let $_s$ represent the voltage across the source, positive at the upper terminal of the source, we obtain

$$v_s + 8(10) = v_o = 20, \quad \text{or} \quad v_s = -60 \text{ V},$$

and the power developed by the 8 A source is 480 W. Note that the 125 Ω and 10 Ω resistors do not affect the value of $_o$ but do affect the power calculations. Check your power calculations by determining the power absorbed by all of the resistors in the circuit.

EXAMPLE 4.14 Finding a Thévenin Equivalent

Find the Thévenin equivalent of the circuit in Fig. 4.47.

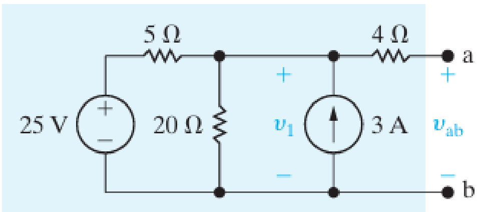


Figure 4.47 ▲ A circuit used to illustrate a Thévenin equivalent.

Solution

To find the Thévenin equivalent of the circuit shown in Fig. 4.47, we first calculate the open-circuit voltage $_{ab}$. Note that when the terminals a, b are open, there is no current in the 4 Ω resistor. Therefore the open-circuit voltage v_{ab} is identical to the voltage across the 3 A current source, labeled $_1$. We find the voltage by solving a single KCL equation. Choosing the lower node as the reference node, we get

$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0.$$

Solving for $_1$ yields

$$v_1 = 32 \text{ V} = V_{\text{Th}}.$$

Hence, the Thévenin voltage for the circuit is 32 V.

The next step is to place a short circuit across the terminals a and b and calculate the resulting short-circuit current. Figure 4.48 shows the circuit with the short in place. Note that the short-circuit current is

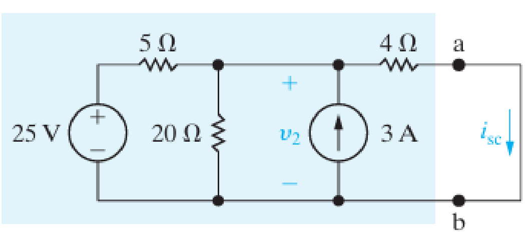


Figure 4.48 ▲ The circuit shown in Fig. 4.47 with terminals a and b short-circuited.

in the direction of the open-circuit voltage drop across the terminals a and b. If the short-circuit current is in the direction of the open-circuit voltage rise across the terminals, a minus sign must be inserted in. Eq. 4.37.

The short-circuit current (i_{sc}) is found easily once $_2$ is known. Therefore, the problem reduces to finding $_2$ with the short in place. Again, if we use the lower node as the reference node, the KCL equation at the node labeled $_2$ is

$$\frac{v_2 - 25}{5} + \frac{v_2}{20} - 3 + \frac{v_2}{4} = 0.$$

Solving for $_2$ gives

$$v_2 = 16 \text{ V}.$$

Hence, the short-circuit current is

$$i_{sc} = \frac{16}{4} = 4 \text{ A}.$$

We now find the Thévenin resistance by substituting the numerical values for the Thévenin voltage, V_{Th}, and the short-circuit current, i_{sc}, into Eq. 4.37:

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{i_{sc}} = \frac{32}{4} = 8 \ \Omega.$$

Figure 4.49 shows the Thévenin equivalent for the circuit shown in Fig. 4.45.

You should verify that, if a 24 Ω resistor is connected across the terminals a and b in Fig. 4.47, the voltage across the resistor will be 24 V and the current in the resistor will be 1 A, as would be the case with the Thévenin circuit in Fig. 4.49. This same equivalence between the circuits in Figs. 4.47 and 4.49 holds for any resistor value connected between nodes a and b.

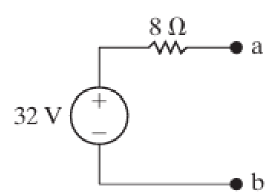


Figure 4.49 ▲ The Thévenin equivalent of the circuit shown in Fig. 4.47.

EXAMPLE 4.15 Finding a Norton Equivalent

Find the Norton equivalent of the circuit in Fig. 4.47 by making a series of source transformations.

Solution

We start on the left side of the circuit and transform the series-connected 25 V source and 5 Ω resistor to a parallel-connected 5 A source and 5 Ω resistor, as shown in Step 1 of Fig. 4.51. Use KCL to combine the parallel-connected 5 A and 3 A sources into a single 8 A source, and combine the parallel-connected 5 Ω and 25 Ω resistors into a single 4 Ω resistor, as shown in Step 2 of Fig. 4.51. Now transform the parallel-connected 8 A source and 4 Ω resistor to a series-connected 32 V source and 4 Ω resistor, and combine the two series-connected 4 Ω resistors into a single 8 Ω resistor, as shown in Step 3 of Fig. 4.51. Note that the result of Step 3 is the Thévenin equivalent circuit we derived in Example 4.12. Finally, transform the series-connected 32 V source and 8 Ω resistor into a parallel-connected 4 A source and 8 Ω resistor, which is the Norton equivalent circuit, as shown in Step 4 of Fig. 4.51.

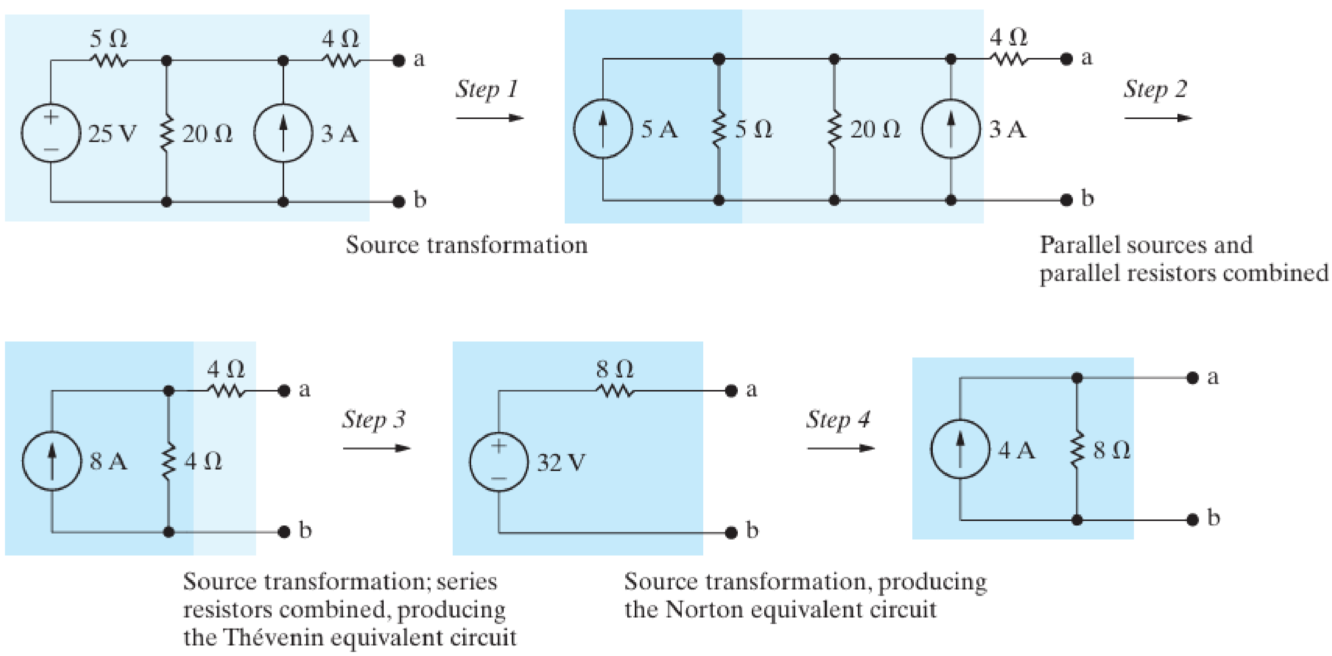


Figure 4.51 ▲ Step-by-step derivation of the Thévenin and Norton equivalents of the circuit shown in Fig. 4.47.

EXAMPLE 4.16 Finding the Thévenin Equivalent of a Circuit with a Dependent Source

Find the Thévenin equivalent for the circuit containing dependent sources shown in Fig. 4.52.

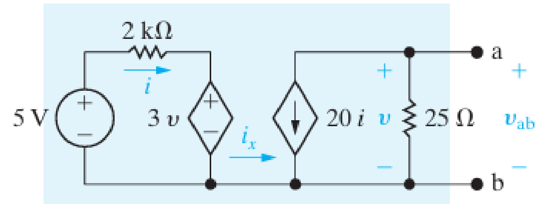


Figure 4.52 ▲ A circuit used to illustrate a Thévenin equivalent when the circuit contains dependent sources.

Solution

The first step in analyzing the circuit in Fig. 4.52 is to recognize that the current labeled i_x must be zero. (Note the absence of a return path for i_x to enter the left-hand portion of the circuit.) The open-circuit, or Thévenin, voltage is the voltage across the 25 Ω resistor. Since $i_x = 0$,

$$V_{Th} = v_{ab} = (-20i)(25) = -500i.$$

The current i is

$$i = \frac{5 - 3v}{2000} = \frac{5 - 3V_{Th}}{2000}.$$

In writing the equation for i, we recognize that the Thévenin voltage is identical to v. When we combine these two equations, we obtain

$$V_{Th} = -5 \text{ V}.$$

To calculate the short-circuit current, we place a short circuit across a and b. When the terminals a and b are shorted together, the control voltage is reduced to zero. Therefore, with the short in place, the circuit shown in Fig. 4.52 becomes the one shown in Fig. 4.53. With the short circuit shunting the 25 Ω resistor, all of the current from the dependent current source appears in the short, so

$$i_{sc} = -20i.$$

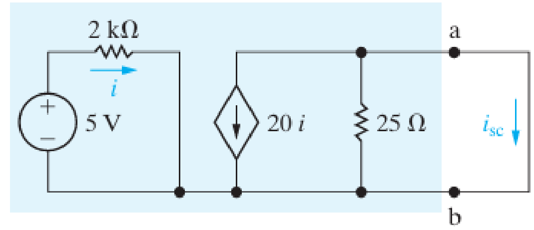


Figure 4.53 ▲ The circuit shown in Fig. 4.52 with terminals a and b short-circuited.

As the voltage controlling the dependent voltage source has been reduced to zero, the current controlling the dependent current source is

$$i = \frac{5}{2000} = 2.5 \text{ mA}.$$

Combining these two equations yields a short-circuit current of

$$i_{sc} = -20(2.5) = -50 \text{ mA}.$$

From i_{sc} and V_{Th} we get

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-5}{-0.05} = 100 \ \Omega.$$

Figure 4.54 illustrates the Thévenin equivalent for the circuit shown in Fig. 4.52. Note that the reference polarity marks on the Thévenin voltage source in Fig. 4.54 agree with the preceding equation for V_{Th}.

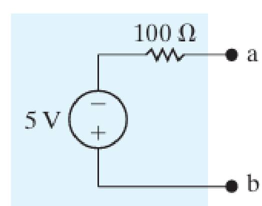


Figure 4.54 ▲ The Thévenin equivalent for the circuit shown in Fig. 4.52.

EXAMPLE 4.17 Finding the Thévenin Equivalent Resistance Directly from the Circuit

Find R_{Th} the circuit shown in Fig. 4.55.

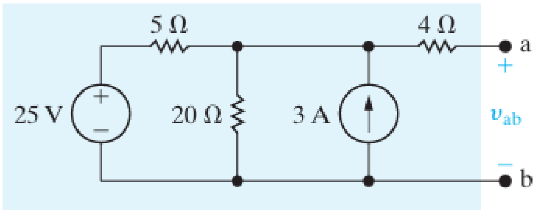


Figure 4.55 ▲ A circuit used to illustrate a Thévenin equivalent.

Solution

Deactivating the independent sources simplifies the circuit to the one shown in Fig. 4.56. The resistance seen looking into the terminals a and b is denoted R_{ab}, which consists of the 4 Ω resistor in series with the parallel combination of the 5 and 20 Ω resistors. Thus,

$$R_{ab} = R_{Th} = 4 + \frac{(5)(20)}{5 + 20} = 8\ \Omega.$$

Note that deriving R_{Th} directly from the circuit is much simpler than finding R_{Th} from Eq. 4.37, as we did in Example 4.14.

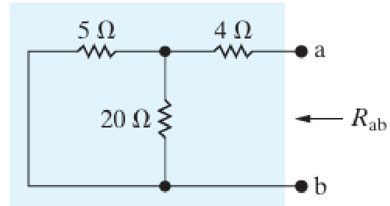


Figure 4.56 ▲ The circuit shown in Fig. 4.55 after deactivation of the independent sources.

EXAMPLE 4.18 Finding the Thévenin Equivalent Resistance Using a Test Source

Find the Thévenin resistance R_{Th} for the circuit in Fig. 4.52, using the test source method.

Solution

Begin by deactivating the independent voltage source and exciting the circuit from the terminals a and b with either a test voltage source or a test current source. If we apply a test voltage source, we will know the voltage of the dependent voltage source and hence the controlling current i. Therefore, we opt for the test voltage source. Figure 4.57 shows the circuit for computing the Thévenin resistance.

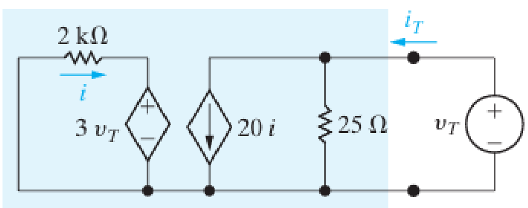


Figure 4.57 ▲ An alternative method for computing the Thévenin resistance.

The test voltage source is denoted $_T$, and the current that it delivers to the circuit is labeled i_T. To find the Thévenin resistance, we solve the circuit for the ratio of the voltage to the current at the test source; that is, $R_{\mathrm{Th}} = v_T/i_T$. From Fig. 4.57,

$$i_T = \frac{{}_T}{25} + 20i,$$

$$i = \frac{-3v_T}{2000}.$$

We then substitute the expression for i into the equation for i_T and solve the resulting equation for the ratio i_T/v_T:

$$i_T = \frac{v_T}{25} - \frac{60v_T}{2000},$$

$$\frac{i_T}{v_T} = \frac{1}{25} - \frac{6}{200} = \frac{50}{5000} = \frac{1}{100}.$$

The Thévenin resistance is the inverse of the ratio i_T/v_T, so

$$R_{\mathrm{Th}} = \frac{{}_T}{i_T} = 100\ \Omega.$$

EXAMPLE 4.19 Finding the Thévenin Equivalent of a Circuit with Dependent Sources and Resistors

Find the Norton equivalent for the circuit in Fig. 4.58.

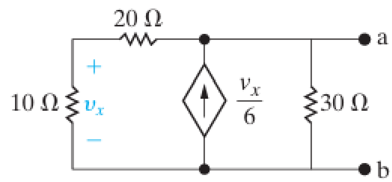


Figure 4.58 ▲ A circuit used to determine a Thévenin equivalent when the circuit contains only dependent sources and resistors.

Solution

The circuit in Fig. 4.58 has no independent sources. Therefore, the Norton equivalent current is zero, and the Norton equivalent circuit consists only of the Norton resistance, R_N. Applying a test source to the terminals a and b is the only way to determine R_N. We have applied a test current source, whose value is i_T, as shown in Fig. 4.59. Analyze this circuit to calculate the voltage across the test source, v_T, and then calculate the Norton equivalent resistance using the ratio of v_T to i_T.

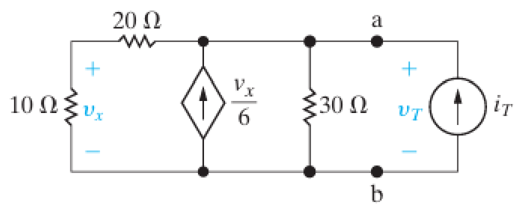


Figure 4.59 ▲ The circuit in Fig. 4.58 with a test current source.

Write a KCL equation at the top essential node to give

$$i_T = \frac{T}{20 + 10} - \frac{x}{6} + \frac{T}{30}.$$

Use voltage division to find the voltage across the 10 Ω resistor:

$$v_x = \frac{10}{20 + 10} v_T = \frac{v_T}{3}.$$

Then,

$$i_T = \frac{T}{30} - \frac{T}{18} + \frac{T}{30} = \frac{T}{90}.$$

Therefore, the Norton equivalent of the circuit in Fig. 4.58 is a single resistor whose resistance $R_N = v_T/i_T = 90\ \Omega$.

EXAMPLE 4.20 Using a Thévenin Equivalent to Analyze the Amplifier Circuit

Use a Thévenin equivalent of the left side of the amplifier circuit, shown in Fig. 4.60, to find the current i_B.

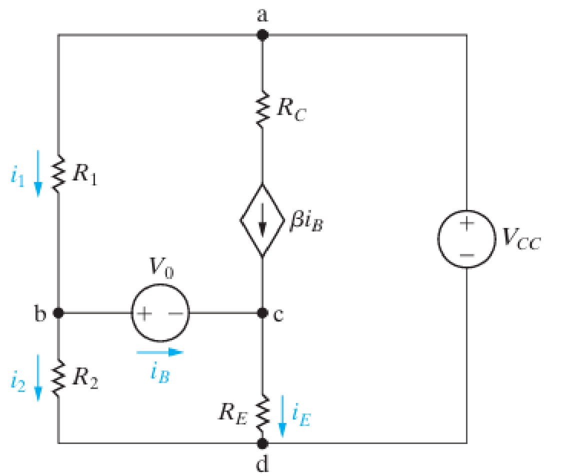


Figure 4.60 ▲ The application of a Thévenin equivalent in circuit analysis.

Solution

We redraw the circuit as shown in Fig. 4.61 to prepare to replace the subcircuit to the left of V_0 with its Thévenin equivalent. You should be able to determine that this modification has no effect on the branch currents i_1, i_2, i_B, and i_E. Then replace the circuit made up of V_{CC}, R_1, and R_2 with a Thévenin equivalent, with respect to the terminals b and d. The Thévenin voltage and resistance are

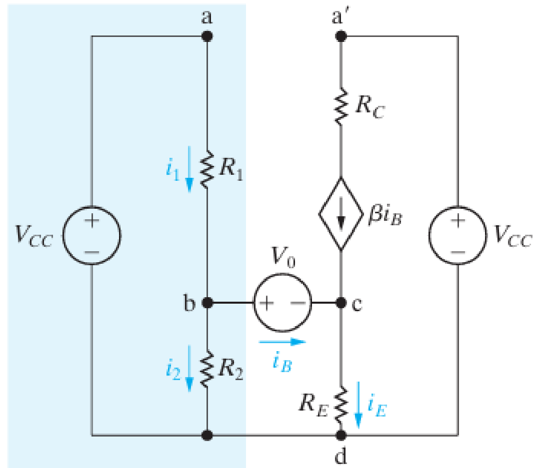


Figure 4.61 ▲ A modified version of the circuit shown in Fig. 4.60.

$$V_{\text{Th}} = \frac{V_{CC}R_2}{R_1 + R_2}, \tag{4.39}$$

$$R_{\text{Th}} = \frac{R_1 R_2}{R_1 + R_2}. \tag{4.40}$$

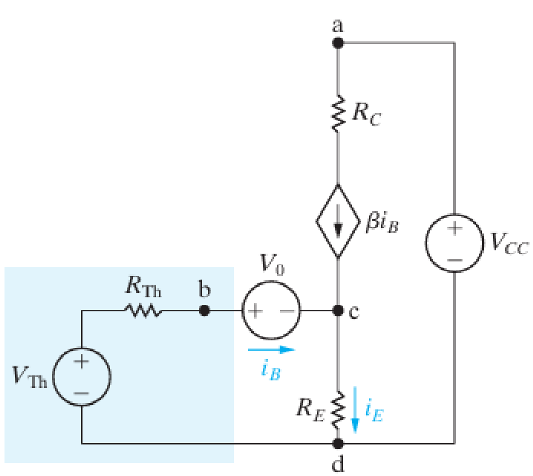


Figure 4.62 ▲ The circuit shown in Fig. 4.61 modified by a Thévenin equivalent.

With the Thévenin equivalent, the circuit in Fig. 4.61 becomes the one shown in Fig. 4.62.

We now derive an equation for i_B by summing the voltages around the left mesh. In writing this mesh equation, we recognize that $i_E = (1 + \beta)i_B$. Thus,

$$V_{\text{Th}} = R_{\text{Th}} i_B + V_0 + R_E(1 + \beta)i_B,$$

from which

$$i_B = \frac{V_{\text{Th}} - V_0}{R_{\text{Th}} + (1 + \beta)R_E}. \tag{4.41}$$

When we substitute Eqs. 4.39 and 4.40 into Eq. 4.41, we get the same expression obtained in Eq. 2.25. Note that once we have incorporated the Thévenin equivalent into the original circuit, we can obtain the solution for i_B by writing a single equation.

EXAMPLE 4.21 Calculating the Condition for Maximum Power Transfer

a) For the circuit shown in Fig. 4.65, find the value of R_L that results in maximum power being transferred to R_L.

b) Calculate the maximum power that can be delivered to R_L.

c) When R_L is adjusted for maximum power transfer, what percentage of the power delivered by the 360 V source reaches R_L?

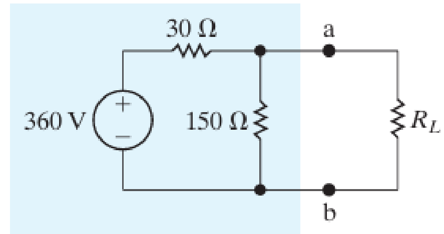


Figure 4.65 ▲ The circuit for Example 4.21.

Solution

a) The Thévenin voltage for the circuit to the left of the terminals a and b is

$$V_{\text{Th}} = \frac{150}{180}(360) = 300\text{ V}.$$

The Thévenin resistance is

$$R_{\text{Th}} = \frac{(150)(30)}{180} = 25\ \Omega$$

Replacing the circuit to the left of the terminals a and b with its Thévenin equivalent gives us the circuit shown in Fig. 4.66, so R_L must equal 25 Ω for maximum power transfer.

b) The maximum power that can be delivered to R_L is

$$p_{\max} = \left(\frac{300}{50}\right)^2(25) = 900\text{ W}.$$

c) When R_L equals 25 Ω, the voltage $_{\text{ab}}$ is

$$v_{\text{ab}} = \left(\frac{300}{50}\right)(25) = 150\text{ V}.$$

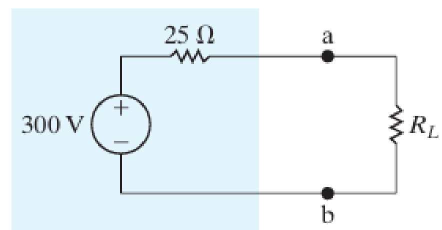


Figure 4.66 ▲ Reduction of the circuit shown in Fig. 4.66 by means of a Thévenin equivalent.

From Fig. 4.65, when $_{\text{ab}}$ equals 150 V, the current in the voltage source in the direction of the voltage rise across the source is

$$i_s = \frac{360 - 150}{30} = \frac{210}{30} = 7\text{ A}.$$

Therefore, the source is delivering 2520 W to the circuit, or

$$p_s = -i_s(360) = -2520\text{ W}.$$

The percentage of the source power delivered to the load is

$$\frac{900}{2520} \times 100 = 35.71\%.$$

EXAMPLE 4.22 Using Superposition to Solve a Circuit

Use the superposition principle to find the branch currents in the circuit shown in Fig. 4.67.

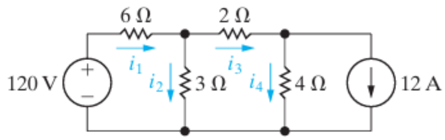


Figure 4.67 ▲ A circuit used to illustrate superposition.

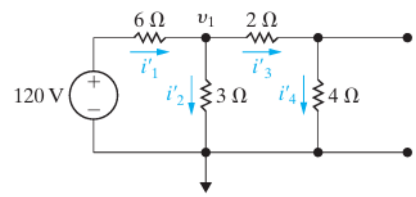


Figure 4.68 ▲ The circuit shown in Fig. 4.67 with the current source deactivated.

Solution

We begin by finding the branch currents resulting from the 120 V voltage source. We denote those currents with a prime. Replacing the ideal current source with an open circuit deactivates it; Fig. 4.68 shows this. The branch currents in this circuit are the result of only the voltage source.

We can easily find the branch currents in the circuit in Fig. 4.68 once we know the node voltage across the 3 Ω resistor. Denoting this voltage $_1$, we write

$$\frac{v_1 - 120}{6} + \frac{v_1}{3} + \frac{v_1}{2 + 4} = 0,$$

from which

$$v_1 = 30 \text{ V}.$$

Now we can write the expressions for the branch currents $i'_1 - i'_4$ directly:

$$i'_1 = \frac{120 - 30}{6} = 15 \text{ A},$$

$$i'_2 = \frac{30}{3} = 10 \text{ A},$$

$$i'_3 = i'_4 = \frac{30}{6} = 5 \text{ A}.$$

To find the component of the branch currents resulting from the current source, we deactivate the ideal voltage source and solve the circuit shown in Fig. 4.69. The double-prime notation for the currents indicates they are the components of the total current resulting from the ideal current source.

We determine the branch currents in the circuit shown in Fig. 4.69 by first solving for the node voltages across the 3 and 4 Ω resistors, respectively. Figure 4.70 shows the two node voltages.

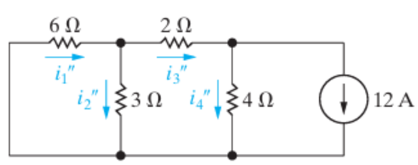


Figure 4.69 ▲ The circuit shown in Fig. 4.67 with the voltage source deactivated.

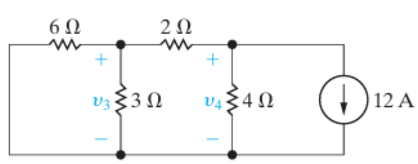


Figure 4.70 ▲ The circuit shown in Fig. 4.69 showing the node voltages v_3 and v_4.

The two KCL equations that describe the circuit are

$$\frac{_3}{3} + \frac{_3}{6} + \frac{_3 - \ _4}{2} = 0,$$

$$\frac{_4 - \ _3}{2} + \frac{v_4}{4} + 12 = 0.$$

Solving the simultaneous KCL equations for v_3 and $_4$, we get

$$v_3 = -12 \text{ V},$$
$$v_4 = -24 \text{ V}.$$

Now we can write the branch currents i''_1 through i''_4 directly in terms of the node voltages $_3$ and $_4$:

$$i''_1 = \frac{-v_3}{6} = \frac{12}{6} = 2 \text{ A},$$

$$i''_2 = \frac{v_3}{3} = \frac{-12}{3} = -4 \text{ A},$$

$$i''_3 = \frac{v_3 - v_4}{2} = \frac{-12 + 24}{2} = 6 \text{ A},$$

$$i''_4 = \frac{v_4}{4} = \frac{-24}{4} = -6 \text{ A}.$$

To find the branch currents in the original circuit, that is, the currents i_1, i_2, i_3, and i_4 in Fig. 4.67, we simply add the single-primed currents to the double-primed currents:

$$i_1 = i'_1 + i''_1 = 15 + 2 = 17 \text{ A},$$
$$i_2 = i'_2 + i''_2 = 10 - 4 = 6 \text{ A},$$
$$i_3 = i'_3 + i''_3 = 5 + 6 = 11 \text{ A},$$
$$i_4 = i'_4 + i''_4 = 5 - 6 = -1 \text{ A}.$$

You should verify that the currents i_1, i_2, i_3, and i_4 have the correct values for the branch currents in the circuit shown in Fig. 4.67.

EXAMPLE 4.23 Using Superposition to Solve a Circuit with Dependent Sources

Use the principle of superposition to find $_o$ in the circuit shown in Fig. 4.71.

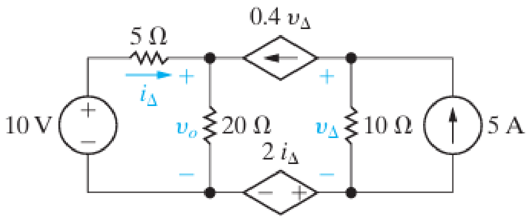


Figure 4.71 ▲ The circuit for Example 4.23.

Solution

We begin by finding the component of $_o$ resulting from the 10 V source. Figure 4.72 shows the circuit. With the 5 A source deactivated, v'_Δ must equal $(-0.4v'_\Delta)$ (10). Hence, v'_Δ must be zero, the branch containing the two dependent sources is open, and

$$v'_o = \frac{20}{25}(10) = 8\text{ V}.$$

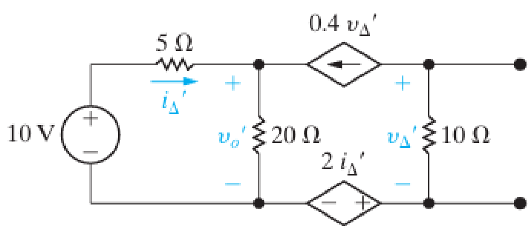


Figure 4.72 ▲ The circuit shown in Fig. 4.71 with the 5 A source deactivated.

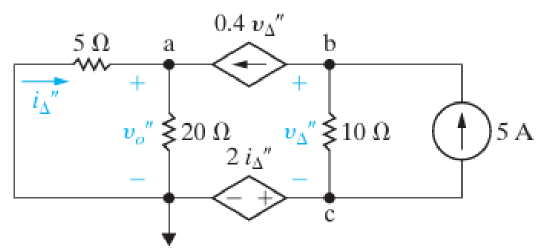


Figure 4.73 ▲ The circuit shown in Fig. 4.71 with the 10 V source deactivated.

When the 10 V source is deactivated, the circuit reduces to the one shown in Fig. 4.73. We have added a reference node and the node designations a, b, and c to aid the discussion. Summing the currents away from node a yields

$$\frac{v''_o}{20} + \frac{v''_o}{5} - 0.4v''_\Delta = 0, \quad \text{or} \quad 5v''_o - 8v''_\Delta = 0.$$

Summing the currents away from node b gives

$$0.4v''_\Delta + \frac{v_b - 2i''_\Delta}{10} - 5 = 0, \text{ or}$$

$$4v''_\Delta + v_b - 2i''_\Delta = 50.$$

We now use

$$v_b = 2i''_\Delta + v''_\Delta$$

to find the value for v''_Δ. Thus,

$$5v''_\Delta = 50, \quad \text{or} \quad v''_\Delta = 10\text{ V}.$$

From the node a equation,

$$5v''_0 = 80, \quad \text{or} \quad v''_0 = 16\text{ V}.$$

The value of $_o$ is the sum of v'_o and v''_o, or 24 V.

SELF-CHECK: Assess your understanding of this material by trying Chapter Problems 4.92 and 4.97.