MAT1320 LINEAR ALGEBRA EXERCISES IV-V

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1.

The number of unknowns $n = 3$.

$$\begin{aligned} x + y - z &= 2 \\ x + 2y + z &= 3 \\ x + y + \left(a^2 - 5\right) z &= a \end{aligned}$$

If the above linear system of equations has unique solution, then which of the followings is the set of all possible values for a?

a) $a = 2$ b) $a \neq \pm 2$ c) $a = -2$ d) $a = \pm 2$

e) There is no such a number a.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right] \begin{array}{l} r_2 \to r_2 - r_1 \\ \longrightarrow \\ r_3 \to r_3 - r_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

If there is unique solution, then $\operatorname{rank}(A) = \operatorname{rank}(A:b)$ and $\operatorname{rank} A = 3$.

$\Rightarrow$ $0\ 0\ a^2-4$ can not be zero row.

$\Rightarrow$ $a^2 - 4 \neq 0 \Rightarrow a \neq \mp 2$.

2)

$$\left[\begin{array}{ccc} 4 & -2 & 7 \\ 8 & -3 & 10 \end{array}\right] \xrightarrow{r_2 \to r_2 - r_1} \left[\begin{array}{ccc} 4 & -2 & 7 \\ 0 & 1 & -4 \end{array}\right]$$

$$\xrightarrow{r_1 \to r_1 + 2r_2} \left[\begin{array}{ccc} 4 & 0 & -1 \\ 0 & 1 & -4 \end{array}\right] \xrightarrow{r_1 \cdot \frac{1}{4}} \left[\begin{array}{ccc} 1 & 0 & -1/4 \\ 0 & 1 & -4 \end{array}\right] \begin{array}{l} \longrightarrow x_1 - \frac{1}{4} x_3 = 0 \\ \longrightarrow x_2 - 4x_3 = 0 \end{array}$$

2.

$$\begin{aligned} 4x_1 - 2x_2 + 7x_3 &= 0 \\ 8x_1 - 3x_2 + 10x_3 &= 0 \end{aligned}$$

$(n = 3)$ - $(\operatorname{rank} A = 2) \neq 1$,

$\Rightarrow$ Infinite solutions depending on one parameter, say k.

Which of the followings is the solution of above homogeneous system of linear equations?

a) There exists only trivial (zero) solution.

b) $x_1 = \frac{1}{4}k,\ x_2 = 4k,\ x_3 = k,\ k \in \mathbb{R}$.

c) $x_1 = 4k,\ x_2 = \frac{1}{4}k,\ x_3 = k,\ k \in \mathbb{R}$.

d) $x_1 = k,\ x_2 = 4k,\ x_3 = k,\ k \in \mathbb{R}$.

e) The system is inconsistent.

Say $x_3 = k$. Then

$x_1 = \frac{1}{4}k$, $x_2 = 4k$, $x_3 = k$,

$k \in \mathbb{R}$

3.

$$\begin{aligned} 2x_1 + 3x_2 + 7x_3 &= 3 \\ -2x_1 - 4x_3 &= 1 \\ x_1 + 2x_2 + 4x_3 &= 4 \end{aligned}$$

Which of the followings is true for above linear system of equations?

I. The system is inconsistent.

II. The system has infinite solutions.

III. The system has unique solution.

a) Only I b) Only II c) Only III d) I and III

e) None of them

$$\underbrace{\left[\begin{array}{ccc} 2 & 3 & 7 \\ -2 & 0 & -4 \\ 1 & 2 & 4 \end{array}\right]}_{A} \underbrace{\left[\begin{array}{c} 3 \\ 1 \\ 4 \end{array}\right]}_{b} \xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 4 \\ -2 & 0 & -4 & 1 \\ 2 & 3 & 7 & 3 \end{array}\right]$$

$$\begin{array}{l} r_2 \to r_2 + 2r_1 \\ \longrightarrow \\ r_3 \to r_3 - 2r_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 4 \\ 0 & 4 & 4 & 9 \\ 0 & -1 & -1 & -5 \end{array}\right]$$

$$\xrightarrow{r_3 \to r_3 + \frac{1}{4} r_2} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 4 \\ 0 & 4 & 4 & 9 \\ 0 & 0 & 0 & -11/4 \end{array}\right]$$

$\Rightarrow \operatorname{rank} A = 3 \neq \operatorname{rank}(A:b) = 4$

$\Rightarrow$ No solution (inconsistent)