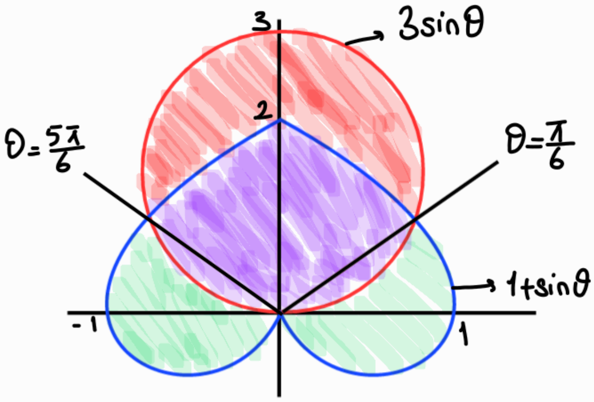
POLAR COORDINATES

1) Write the integral that gives the area

a) Inside $r=3\sin\theta$ Outside $r=1+\sin\theta$

b) Outside $r=3\sin\theta$ Inside $r=1+\sin\theta$

c) Common to $r=3\sin\theta$, $r=1+\sin\theta$



Intersection points

$3\sin\theta = 1+\sin\theta$

$2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

a) Red b) Green c) Purple

$$\text{a) } A = \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left[(3\sin\theta)^2-(1+\sin\theta)^2\right]d\theta \underset{\text{symmetry}}{=} \int_{\pi/6}^{\pi/2}\left[(3\sin\theta)^2-(1+\sin\theta)^2\right]d\theta$$

$$\text{b) } A \underset{\text{symmetry}}{=} \int_{-\pi/2}^{\pi/6}(1+\sin\theta)^2 d\theta - \int_{0}^{\pi/6}(3\sin\theta)^2 d\theta$$ (Without symmetry, don't forget $\frac{1}{2}$)

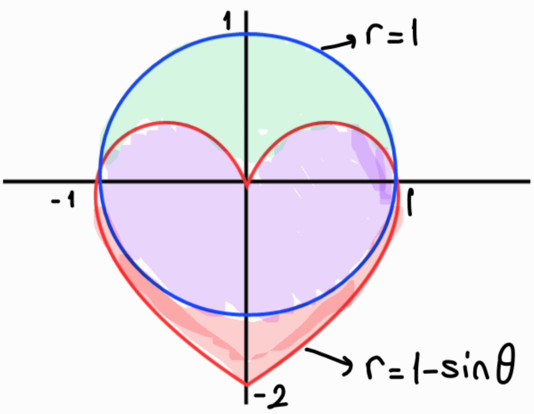
$$\text{c) } A \underset{\text{symmetry}}{=} \int_{0}^{\pi/6}(3\sin\theta)^2 d\theta + \int_{\pi/6}^{\pi/2}(1+\sin\theta)^2 d\theta$$

2) Write the integral that gives the area

a) Inside $r=1-\sin\theta$ Outside $r=1$

b) Outside $r=1-\sin\theta$ Inside $r=1$

c) Common to $r=1-\sin\theta$, $r=1$



a) Red $$\text{a) } A \underset{\text{symmetry}}{=} \int_{-\pi/2}^{0}\left[(1-\sin\theta)^2-1^2\right]d\theta$$

b) Green $$\text{b) } A \underset{\text{symmetry}}{=} \int_{0}^{\pi/2}\left[1^2-(1-\sin\theta)^2\right]d\theta$$

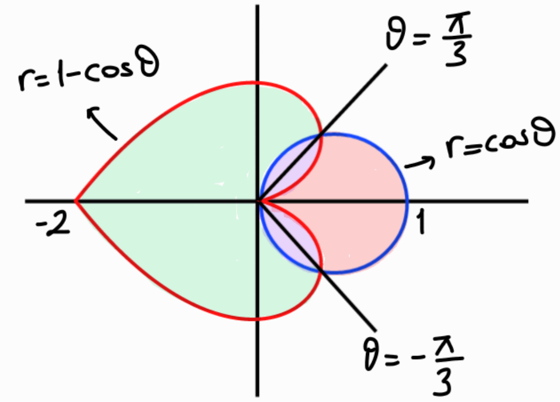
c) Purple $$\text{c) } A \underset{\text{symmetry}}{=} \int_{-\pi/2}^{0}1^2 d\theta + \int_{0}^{\pi/2}(1-\sin\theta)^2 d\theta$$

③ Write the integral that gives the area

a) Inside $r=\cos\theta$ Outside $r=1-\cos\theta$

b) Outside $r=\cos\theta$ Inside $r=1-\cos\theta$

c) Common to $r=\cos\theta$, $r=1-\cos\theta$



Intersection points

$\cos\theta = 1-\cos\theta$

$2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}$

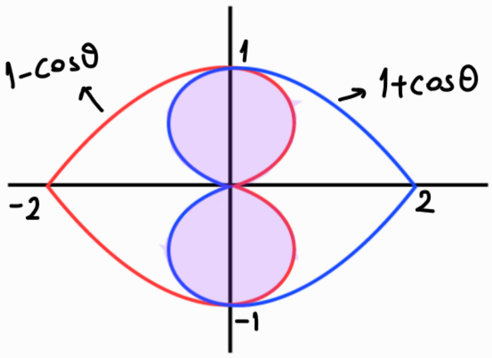
a) Red b) Green c) Purple

a) $A = \int_0^{\pi/2} \left[\cos^2\theta - (1-\cos\theta)^2\right] d\theta$ Symmetry

b) $A = \int_{\pi/3}^{\pi} (1-\cos\theta)^2 - \int_{\pi/3}^{\pi/2} \cos^2\theta \, d\theta$ Symmetry

c) $A = \int_0^{\pi/3} (1-\cos\theta)^2 + \int_{\pi/3}^{\pi/2} \cos^2\theta \, d\theta$ Symmetry

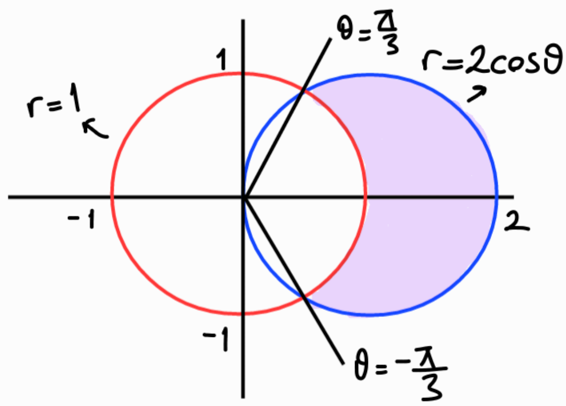
④ Write the integral that gives the area common to $r=1+\cos\theta$ and $r=1-\cos\theta$.



With symmetry

$$A = \int_0^{\pi/2} (1-\cos\theta)^2 + \int_{\pi/2}^{\pi} (1+\cos\theta)^2 d\theta$$

⑤ Write the integral that gives the area inside $r=2\cos\theta$ and outside $r=1$.



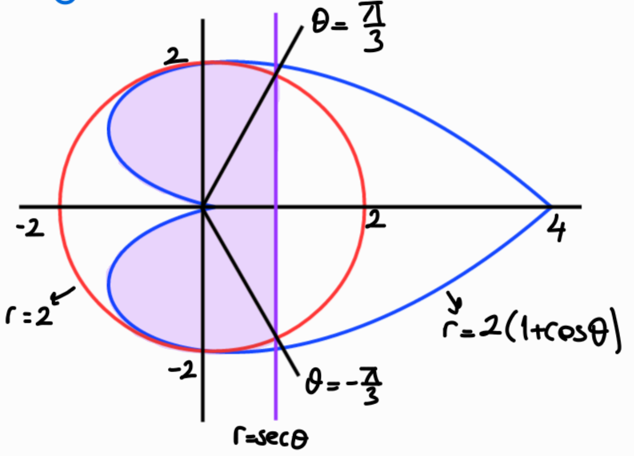
Intersection points

$2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}$

With symmetry

$$A = \int_0^{\pi/3} \left[(2\cos\theta)^2 - 1^2\right] d\theta$$

6 Write the integral that gives the area common to the circle $r=2$, the cardioid $r=2(1+\cos\theta)$ and bounded by the line $r=\sec\theta$



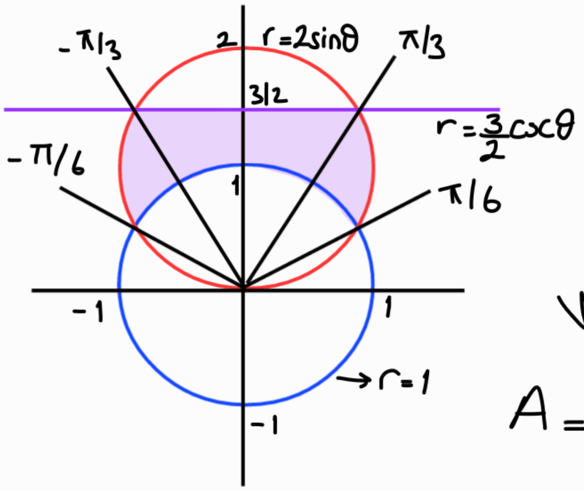
__Intersection points__

$\sec\theta=2 \Rightarrow \cos\theta=\frac{1}{2} \Rightarrow \theta=-\frac{\pi}{3}, \frac{\pi}{3}$

With symmetry:

$$A=\int_0^{\pi/3}(\sec\theta)^2\,d\theta+\int_{\pi/3}^{\pi/2}2^2\,d\theta+\int_{\pi/2}^{\pi}\left[2(1+\cos\theta)\right]^2\,d\theta$$

7 Write the integral that gives the area bounded by the circles $r=1$, $r=2\sin\theta$ and the line $r\sin\theta=\frac{3}{2}$.



__Intersection points__

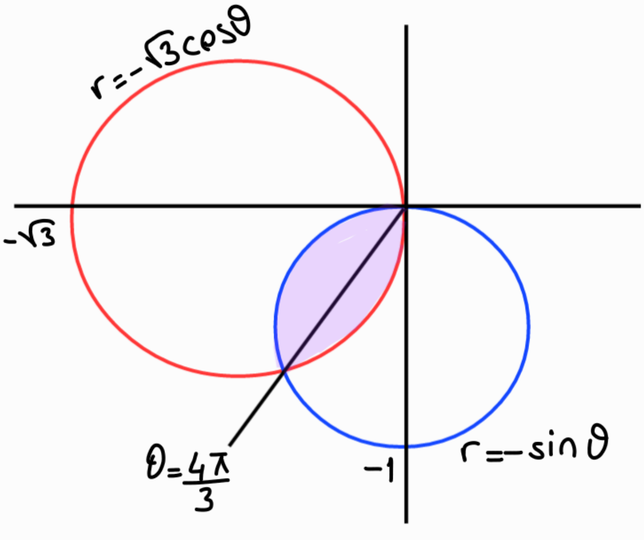
$2\sin\theta=1 \Rightarrow \sin\theta=\frac{1}{2} \Rightarrow \theta=-\frac{\pi}{6}, \frac{\pi}{6}$

$2\sin\theta=\frac{3}{2}\csc\theta \Rightarrow \sin^2\theta=\frac{3}{4} \Rightarrow \sin\theta=\frac{\sqrt{3}}{2} \Rightarrow \theta=-\frac{\pi}{3}, \frac{\pi}{3}$

With symmetry:

$$A=\int_{\pi/6}^{\pi/3}\left[(2\sin\theta)^2-1^2\right]d\theta+\int_{\pi/3}^{\pi/2}\left[\left(\frac{3}{2}\csc\theta\right)^2-1^2\right]d\theta$$

8 Write the integral that gives the area common to the circles $r=-\sqrt{3}\cos\theta$, $r=-\sin\theta$.



__Intersection point__

$-\sin\theta=-\sqrt{3}\cos\theta \Rightarrow \tan\theta=\sqrt{3} \Rightarrow \theta=\frac{4\pi}{3}$

$$A=\frac{1}{2}\left[\int_{\pi}^{4\pi/3}(-\sin\theta)^2\,d\theta+\int_{4\pi/3}^{3\pi/2}(-\sqrt{3}\cos\theta)^2\,d\theta\right]$$

Note that; symmetry does not exist.