MAT1320 I

MAT1320 LINEAR ALGEBRA EXERCISES I

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- 1. Let A be a 3×3 matrix such that the sum of its columns is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$. Then, which of the followings is equal to the matrix AB?
 - a) $\begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ 3)
- d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ e) None of them

 1) Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then, see that

on + and + and = 0, and + and = 0, and and of the matrix B are [-1], [1], [2],

Are elements of AB are of the form

K. (a₁₁+a₁₂+a₁₃), ½. (a₂₁+a₂₂+a₂₃),

Ł. (a₃₁+a₃₁+a₃₃) where & {-1,1,2}.

Then, AB=O metrix.

- 2) $X_{2,3} = X_{3,2} = X_{3,2} = X_{3,3} = X$
- $A_{LX3} \cdot C_{3X3} \cdot A_{3X4}^{T} = B_{4x4}$ $= B_{4x4}^{T}$
- 2. For the matrices $X_{2\times3}, Y_{2\times3}$ and $A_{4\times3}$, which of the followings is equal to the dimension of the matrix $\left[A\left(X^TY\right)^{-1}A^T\right]^T$?

- 3. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$, then which of the followings is equal to x?
 - a) -2 b) 2 c) $-\frac{1}{7}$ d) $\frac{7}{8}$ e) $\frac{8}{3}$
- 3) [1 × 1] [1 2 3] [1 4+4× 6+×] 0 2 3] [1 4+4× 6+×]
- $\begin{bmatrix} 1 \times 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} \times \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 + 1 \\ 1 & 2 \end{bmatrix}$

$$= x - L_1 - 6x + 12 + 2x = 0$$

$$\Rightarrow 8 = 3x = 1 \quad \boxed{x = \frac{8}{3}}$$

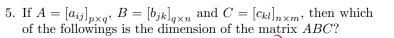
- 4) A.C.B.D=In proliting by A-1

 =) CBD=A-1 from left side

 =) CB=A-10-12 multiply by D'

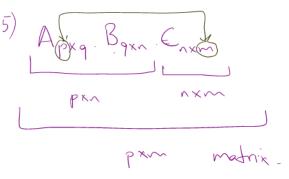
 from right side
 - of C = A-'D-'B-') multiply by B'
 from right side.
- = BOA
- 4. Let A, B, C and D be $n \times n$ matrices having the inverse. If $ACBD = I_n$, then which of the followings is equal to C^{-1} ?
 - a) BAD b) BDAe) C^{-1} may not be exist.
- c) DBA
- d) $A^{-1}D^{-1}B^{-1}$

a) (4×4) b) (4×3) c) (3×4) d) (2×2) e) (4×2)



- a) $q \times m$
- b) $q \times n$

- d) $m \times p$
- e) $p \times q$



6)
$$A^{2} = A \cdot A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$= 2A^{2} = 2 \cdot \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 0 & 2 \end{bmatrix}$$

$$= 2A^{2} + A - 5 \cdot 5 = \begin{bmatrix} 2 & -4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -5 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, then which of the followings is equal to

- c) $\begin{bmatrix} 2 & -5 \\ 0 & -2 \end{bmatrix}$
- d) $\begin{bmatrix} -2 & 5 \\ 0 & -2 \end{bmatrix}$ e) $\begin{bmatrix} 2 & 5 \\ 0 & -2 \end{bmatrix}$

- 7. Let the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ and the polynomial f(x) = 0 $2x^3 - x^2 + 3x + 7$ be given. Then, which of the followings is the matrix f(A)?

$$f(A) = 2A^{3} - A + 3A + 4 \cdot \frac{1}{2}$$

$$A^{2} = AA = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$$

$$A^{3} = A \cdot A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 12 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

8. Let $B = \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 2 \\ -1 & -1 \end{bmatrix}$. If $(AB)^{-1} =$ $(CB)^T$, then which of the following is equal to the matrix A^{-1} ?

- a) $\begin{bmatrix} -44 & 17 \\ 31 & -12 \end{bmatrix}$ b) $\begin{bmatrix} 44 & -17 \\ 31 & 12 \end{bmatrix}$ c) $\begin{bmatrix} 44 & -17 \\ 31 & -12 \end{bmatrix}$ d) $\begin{bmatrix} -44 & 17 \\ -31 & -12 \end{bmatrix}$ e) $\begin{bmatrix} -44 & -17 \\ 31 & 12 \end{bmatrix}$

$$(AB)^{-1} = (CB)^{T} \Rightarrow B^{-1}A^{-1} = (CB)^{T}$$

$$\Rightarrow A^{-1} = B(CB)^{T}$$

$$CB = \begin{bmatrix} 3 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 2 & -5 \end{bmatrix}$$

$$(CB)^{T} = \begin{bmatrix} -5 & 2 \\ 13 & -5 \end{bmatrix}$$

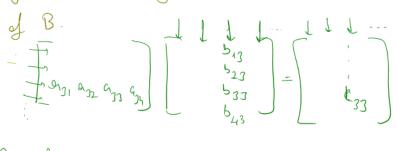
$$= A^{-1} = \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -5 & 2 \\ 13 & -5 \end{bmatrix} = \begin{bmatrix} 44 & -17 \\ 31 & -12 \end{bmatrix}$$

9. Let
$$A = \begin{bmatrix} -2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$$
. Which of the following is the

(3,3)-entry of the inverse matrix A^{-1} ? $\Rightarrow A A^{-1} = \mathcal{I}_{4}$ (a) 0) b) -2 c) 1 d) -1 e) 2

Multiplying the second row of A and the third column of A', we get 0.913+ 0.923+ 1.933+ 0.943= 0 =) \ 933=9

10) To find (313)-entry of the matrix AB, we need to calculate the product of third row of A and third column



931 = 2, 932 = 1, 933 = 6, 934 = 7 big = -1, b2g=1, b3g=1, b4g=4

7 Cgg = 2 - 1 + 1.1 + 6.4 + 7-4 = 51

10. Let $A = [a_{ij}]_{n \times m}$ and $B = [b_{ij}]_{m \times r}$ be two matrices where $a_{ij} = \begin{cases} i+j, & i \leq j \\ i-j & i>j \end{cases}$ and $b_{ij} = \begin{cases} 2i-j, & i < j \\ j+1, & i \geq j \end{cases}$. Then, which of the following is the (3,3)-entry of the matrix AB?

where AB?

Then, AB?

- 11. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}$, then which of the following is the entry \bar{b}_{24} ?

a) -2 b) 3 c) 8 d) -2 e) 11 $A_{1\times 2}$, $B_{m\times n} = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 1 & -1 & 0 & 3 \end{bmatrix}_{2\times n} = \begin{bmatrix} n & 2 & 1 \\ n & -1 & 0 & 3 \end{bmatrix}_{2\times n}$

= [3 4]. [b12 b13 b14] = [4 0 - 25] b21 b22 b23 (b24) = [4-4 0 3]

=> 3.b14 + 4.b24 = 5

-3/1.bn + 1.b2n = 3

b24=5-J.3 = -4

12. (D points) Let $B = \begin{bmatrix} -2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$. Which of the followings is the (4,4)-entry of the matrix B^{-1} ? $\Rightarrow BB \subseteq \mathcal{I}$ a) 0 b) -2 c) 1 d) -1 e) 2

[-2 10 -2] [b11 b12 b13 b14] 0 0 1 0 | b21 b22 b23 b24 | b31 b32 b33 b34 | b41 b42 b43 b44

Tile card row, fourth column: = b3h = 0. , third row, forth cham; but by = 0

3. bry + 2. bry = 1 = -bru=1 = 1/bru=-1