MAT1320-Linear Algebra
Lecture Notes

System of Linear Equations, Gauss-Jordan Elimination Method

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System of Linear Equations

System of Linear Equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases} \tag{1}$$

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is a system of linear equations of m equations in n unknowns. The matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

is called the coefficient matrix of the system.

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Then for $\mathbf{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$ the system given in (1) can be written as

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Definition
The augmented matrix of the system is denoted by $[A|\mathbf{b}]$ and obtained by appending the vector of constants $\mathbf{b}$ as the $(n+1)$-th column to the coefficient matrix A.

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Example
The matrix A and its RREF C is given below.

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 & 2 \\ 1 & -1 & 0 & 1 & 0 \\ 2 & -2 & 0 & 2 & 0 \end{pmatrix} \sim C = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Then $\text{rank}(A) = 2$.

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   2.2 If $\text{rank}(A) = r < n$, i.e., the number of unknowns is greater than the number of equations, then the system has infinitely many solutions depending on $n - r$ parameters.

Example (Gauss-Jordan Elimination)
Let the system of linear equations $x - y + 3z = 1$, $y = -2x + 5$, $9z - x - 5y + 7 = 0$ be given.

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2. Write the augmented matrix of the given system and put it into RREF.
3. Find $rank(\mathbf{A})$ and $rank\left([\mathbf{A}|\mathbf{b}]\right)$.
4. If any, find all solutions of the system.

Çözüm:1

1. The given system can be arranged as follows.

$$\begin{array}{rrrcr} x & -y & +3z & = & 1 \\ 2x & +y & & = & 5 \\ -x & -5y & +9z & = & -7 \end{array}$$

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Then

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ -1 & -5 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -7 \end{bmatrix}$$

Çözüm:2

2. The augmented matrix of the system is $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 2 & 1 & 0 & 5 \\ -1 & -5 & 9 & -7 \end{array}\right]$.

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2. The augmented matrix of the system is $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 2 & 1 & 0 & 5 \\ -1 & -5 & 9 & -7 \end{array}\right]$.

Apply elementary row operations to the augmented matrix to get REF and RREF.

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for arbitrary parameter $z = t \in \mathbb{R}$ we have $\begin{array}{l} x = 2 - t \\ y = 1 + 2t \\ z = t \end{array}$. Then

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$ is the solution of the system.

Homogenous System of Linear Equations

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1. We rearrange the system as follows:

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2. The augmented matrix of the system is $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & -5 & 9 & 0 \end{array}\right]$.

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$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

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