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Article · January 2009

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Abstract: - The aim of the present paper is that of developing a computational technique of the Campbell diagram of rotors. This paper deals with bending in the field of mono-rotors, such as in compressors and turbines. This presentation concerns the use of the Campbell diagram and highlights some specific cases of its use.

Key-Words: - Rotor-bearing systems, dynamics, Campbell diagram, eigenvalues problem

1 Introduction

The Campbell diagram is one of the most important tools for understanding the dynamic behaviour of the rotating machines. It basically consists of a plot of the natural frequencies of the system as functions of the spin speed. Although being based on complete linearity, the Campbell diagram of the linearized model can yield many important information concerning a nonlinear rotating system. A critical speed of order k of a single – shaft rotor system is defined [2] as spin speed for which a multiple of that speed coincides with one of the system's natural frequencies of precession. The paper deals with bending only and consists of two main parts. The first part is a brief presentation of rotor equations and the second part gives models and examples.

2 Rotor Equations

The equations of motions of anisotropic rotor-bearing systems which consist of a flexible nonuniform axisymmetric shaft, rigid disk and anisotropic bearings are obtained, in second order form, by the finite element method. The model consists of a rotor treated as a continuous elastic shaft with several rigid disks, supported on an anisotropic elastic bearings. Consider that the dynamic equilibrium configuration of the rotor-bearing system the undeformed shaft is along the *x*-direction of an inertial *x, y, z* coordinate system. In the study of the lateral motion of the rotor, the displacement of any point is defined by two translations (v,w) and two rotations (φ_y,φ_z) [3], [2].

The model could use one of the following three beam finite element types:

beam C^1 finite element type based on Euler-Bernoulli beam model;

beam C^1 finite element type based on Timoshenko beam model;

beam C^0 isoperimetric finite element type based on Timoshenko beam model.

The beam finite element has two nodes. For the static analysis, a 2D problem, there are two degrees of freedom (DOF) per node, one displacement perpendicularly on the beam axis and the slope of the deformed beam. In the case of the dynamic analysis four degrees of freedom (DOF) per node are considered: two displacements and two slopes measured in two perpendicular planes containing the beam. The equations may be written as [1], [2], [3]

$$\mathbf{M}\,\ddot{\mathbf{q}} + (\mathbf{C} + \boldsymbol{\Omega}\,\mathbf{G})\,\dot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{F} \tag{1}$$

where **q** is the global displacement vector, whose upper half contains the nodal displacements in the y-x plane, while the lower half contains those in z-y plane, and where the positive definite matrix **M** is mass (inertia) matrix, the skew symmetric matrix **G** is gyroscopic matrix, and the nonsymmetric matrices **C** and **K** are called the damping and the stiffness matrices, respectively.

The matrices of **M, C, G, K, q,** and **F** consist of element matrices given as

$$\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}_{N\times N}, \mathbf{C} = \begin{bmatrix} c_{yy} & c_{yz} \\ c_{zy} & c_{zz} \end{bmatrix}_{N\times N}, \tag{2}$$

$$\mathbf{G} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}_{N\times N}, \mathbf{K} = \begin{bmatrix} k_{yy} & k_{yz} \\ k_{zy} & k_{zz} \end{bmatrix}_{N\times N}, \tag{3}$$

$$\mathbf{F} = \begin{Bmatrix} f_y(t) \\ f_z(t) \end{Bmatrix}_{N\times 1}, \mathbf{q}(t) = \begin{Bmatrix} q_y(t) \\ q_z(t) \end{Bmatrix}_{N\times 1} \tag{4}$$

where $N = 4n$, n is the number of nodes.

The equation of motion (1) can be rewritten in state space form as:

$$\mathbf{A}\,\dot{\mathbf{X}} + \mathbf{B}\,\mathbf{X} = \mathbf{Q} \tag{5}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{C} + \boldsymbol{\Omega}\mathbf{G} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{bmatrix} \tag{6}$$

$$\mathbf{Q} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{0} \end{Bmatrix}, \mathbf{X} = \begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix} \quad (7)$$

The $2N \times 2N$ matrices **A** and **B** are real but in general indefinite, nonsymmetric. The resulting system of equation (5) gives nonself-adjoint eigenvalue problem. The eigenvalue problem associated with Eq. (5) and its adjoint are given by

$$\lambda_{\mathrm{i}} \mathbf{A} \mathbf{u}_{\mathrm{i}} = \mathrm{B} \mathrm{u}_{\mathrm{i}} \quad (8)$$

$$\lambda_{\mathrm{k}} \mathbf{A}^{\mathrm{T}} \mathbf{v}_{\mathrm{k}} = \mathbf{B}^{\mathrm{T}} \mathrm{v}_{\mathrm{k}} \quad (\mathrm{i}, \mathrm{k} = 1,2,..,2\mathrm{N}) \quad (9)$$

where $\mathbf{u}_{\mathrm{i}}$ and $\mathbf{v}_{\mathrm{i}}$ are right and left eigenvectors, respectively. Equations (8), (9) has $2N$ eigensolutions, where N is the order of the system global matrices. They are purely real for over damped modes and appear in complex conjugate pairs for under damped or undamped modes of precession. In general, the eigenvalues are a function of the rotating assembly spin speed Ω and are of the form

$$\lambda_i = \alpha_i \pm j\omega_i \,, (i = 1,2,.., N) \quad (10)$$

The real part α_{i} of the eigenvalue is the damping constant, and the imaginary part ω_{i} is the damped natural frequency of whirl speed or precession speed.

It is well known that most vibrations in rotating machinery are induced by rotational-related source. For example, rotating unbalance is the major source of vibration synchronous to the rotational speed Ω; misalignment and cracks in shafts cause the vibration $i\Omega$ (i is a integer); ball bearing defects cause the vibration $n\Omega$ (n is a real number). A critical speed of order s of a single shaft rotor is defined as spin speed for which a multiple of that speed coincides with one of the systems natural frequencies of precession. The intersection of excitation frequencies line $\omega = \mathrm{s}\Omega$ with the natural frequency curve ω_{i} defines the critical speed n_i^{cr}. Thus, when $\Omega = \mathrm{n}_{\mathrm{i}}^{\mathrm{cr}}$, the excitation frequency $\mathrm{in}_{\mathrm{i}}^{\mathrm{cr}}$ creates a resonance condition [1], [2].

However, such a definition regarding the critical speed of rotor bearing systems, in particular, with rotational speed dependent parameters, is conceptually incorrect since the damped natural frequencies change with the rotational speed.

One approach for determining critical speeds is to generate the whirl speed map, include all excitation frequency lines of interest, and graphically note the intersections to obtain the critical speeds associated with each excitation. It is common practice to plots both whirl speed maps, i.e., the whirl frequencies and damping ratio ς_{i} (or in terms of the logarithmic decrement δ_{i}) versus the rotor spin speed Ω. In order to obtain a correct plotting of the Campbell diagrams, in case these diagrams are drown for a finite number of rotations, we should achieve a correspondence between the modal forms and the eigenvalues.

$$\varsigma_{\mathrm{i}} = -\frac{\alpha_{\mathrm{i}}}{\sqrt{\alpha_{\mathrm{i}}^2 + \omega_{\mathrm{i}}^2}} \cong -\frac{\alpha_{\mathrm{i}}}{\omega_{\mathrm{i}}}, \; \delta_{\mathrm{i}} = 2\pi\,\varsigma_{\mathrm{i}} \quad (11)$$

This is necessary to establish following the crossing of two curves of the Campbell diagram, to which modal form belongs the point. The ordering of the eigenvalues can be done using the orthogonality relations between the modal vectors [2].

In the present paper the authors have put in order the eigenvalues according to the fact that when changing the order of the eigenfrequencies the logarithmic decrement has a jump, Fig. 2.

In this paper, in order to obtain the critical speed and to plotting of the Campbell diagrams, the authors have used the computer program DIROPTIM [3].

The solution of the generalized eigenvalues problem has been found using the generalized canonical Hessenberg and Schur matriceal forms, FSRG [3].

3 Illustrative Examples

In this section we shall consider two numerical examples, for two rotor-bearings systems. The Campbell diagrams and logarithmic decrement obtained by the computer code written by the authors is further presented.

Example 1

In this example we consider the system from Fig. 1 with rotor data from table 1. In Fig. 2 and Fig. 2 the Campbell diagram and logarithmic decrement are given.

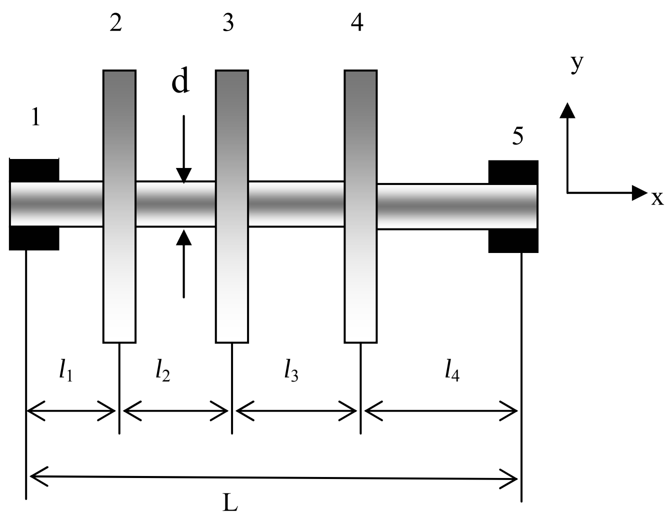

Fig.1 Rotor configuration

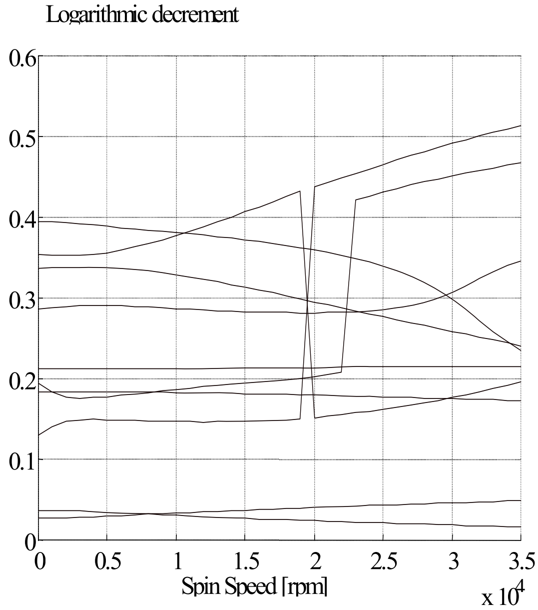

Fig.2 Logarithmic decrement

Shaft	Disk	Bearings
l_1 = 0.2 m, l_2 = 0.3 m	m_2 = 14.58 Kg, m_3 = 45.94 Kg, m_4 = 55.13 Kg	Station 1 k_{yy} = 7e7 N/m k_{zz} = 5e7 N/m
l_3 = 0.5 m, l_4 = 0.3 m	J_{T2} = 0.064 Kg m^2 J_{T3} = 0.498 Kg m^2 J_{T4} = 0.602 Kg m^2	c_{yy} = 7000 Ns/m c_{zz} = 4000 Ns/m Station 5
d = 0.1 m ρ=7800 Kg/m^3 E=2e11 N/m^2	J_{P2} = 0.123 Kg m^2 J_{P3} = 0.976 Kg m^2 J_{P4} = 1.171 Kg m^2	k_{yy} = 6e7 N/m k_{zz} = 4e7 N/m c_{yy} = 6000 Ns/m c_{zz} = 5000 Ns/m

Table 1 Rotor data

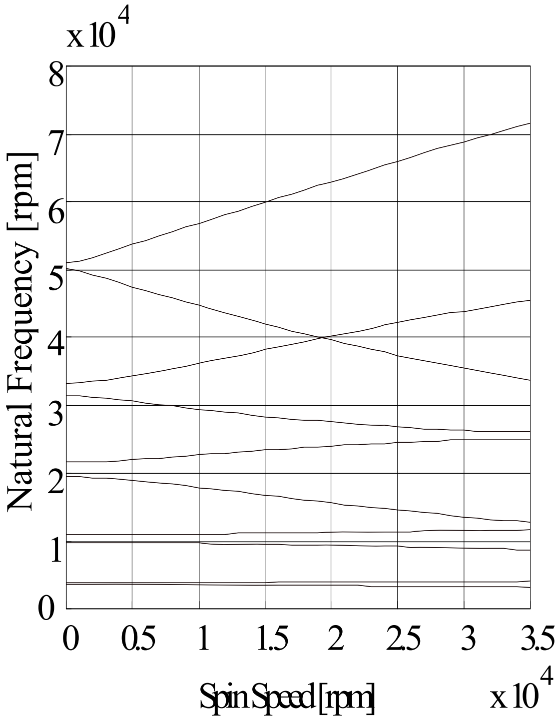

Fig.3 Campbell diagram

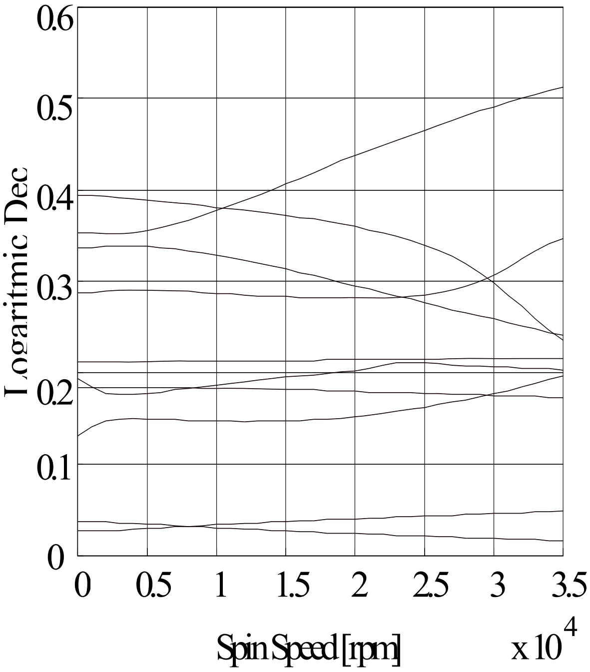

Fig.4 Logarithmic decrement

Example 2

We conside the simple rotor supported by ball rearings (s=0.3 m, a=0.4 m), Fig. 5 and rotor data given by Table 2.

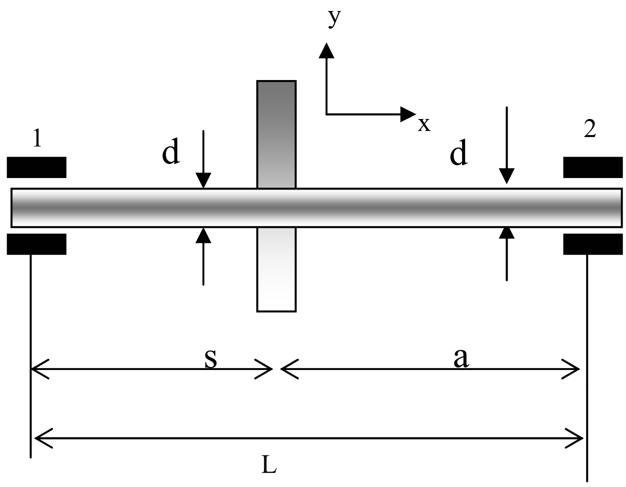

Fig.5 Rotor configuration

Shaft	Disk	Bearings
E = 2e15 N/m^2 ρ = 1000 Kg/m^3 d = 10 mm L = s+a = (0.3+0.4) m	m = 30 Kg J_T = 1.2 Kg m^2 J_P = 1.8 Kg m^2	Bearing (1) k_{yy} = 0.50e7 N/m k_{zz} = 0.70e7 N/m Bearing (2) k_{yy} = 0.50e7 N/m k_{zz} = 0.70e7 N/m

Table 2 Rotor data

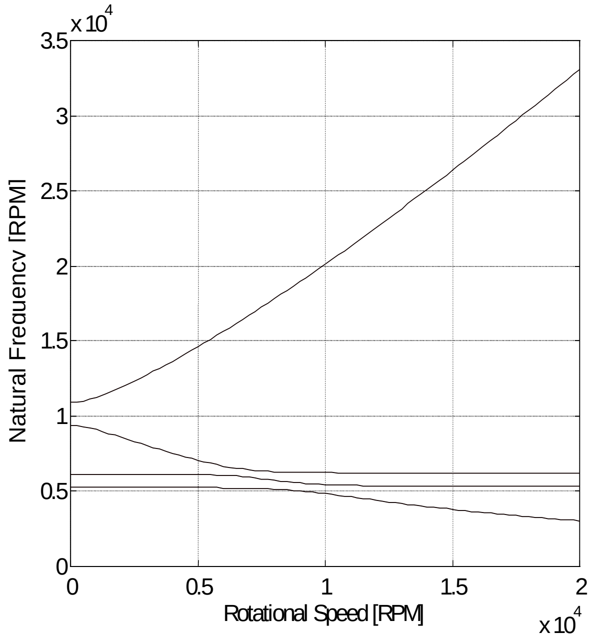

Fig.6 Natural frequency (Campbell diagram)

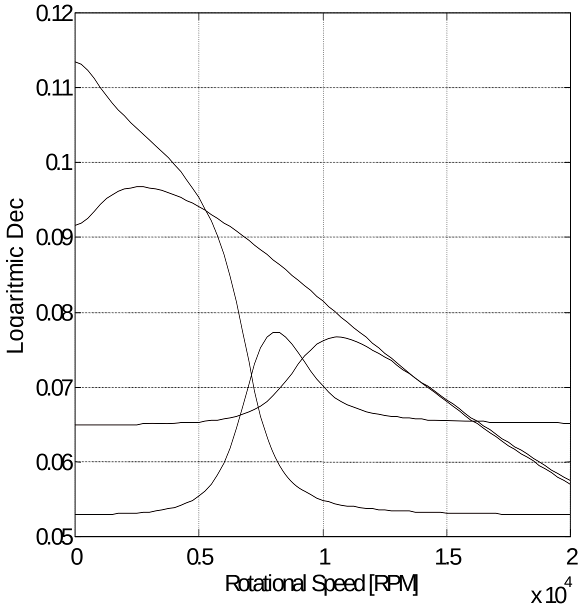

Fig.7 Logarithmic decrement

4 Conclusion

When the Campbell diagram and the mass unbalance responses are concerned, it should be pointed out that: - If the rotors are symmetric with low damping, mostly rotors supported by roller bearings, the interpretation concerning the intersection points is not straightforward. - If the rotor is not symmetric, as with many rotors because of fluid film bearings, and highly damped $0 < Q < 2.5$, the intersection points do not give any forbidden speed range. - If the rotor is not symmetric with $2.5 < Q$, the intersection points give forbidden ranges and it is then necessary to determine the mass unbalance responses and check the amplitudes.

References:

[1] Childs, D., *Turbomachinery Rotordynamics*,J. Wiley, New-York, 1993.

[2] Radeş, M., *Dynamics of Machinery*, Universitatea Politehnica Bucuresti, 1995.

[3] Nicoară, D.,Contributionto the external optimization of the continous mechanical systems and concentrated parameters systems, PhD. Thesis, “Transilvania” University of Brasov, 1998.